Homework 6

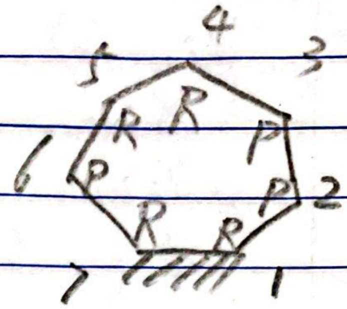
Problem 7.3

given $\alpha_{12} \rightarrow \alpha_{71}$

$a_{12} \rightarrow a_{71}$

$S_1, S_4, S_5, S_7, \theta_2, \theta_3, \theta_6$,

θ_7 (input angle)



(a) Solve for θ_1

① Find the fundamental / subsidiary formula that contains $\theta_2, \theta_3, \theta_6, \theta_7, \theta_1$. The fundamental equation I found in appendix is:

$$Z_{67123} = c_{45}$$

Z_{67123} can be expanded into:

$$Z_{67123} = S_{34}(X_{6712} S_3 + Y_{6712} C_3) + C_{34} Z_{6712}$$

$X_{6712}, Y_{6712}, Z_{6712}$ can be expanded into:

$$X_{6712} = X_{671} C_2 - Y_{671} S_2$$
$$Y_{6712} = C_{23}(X_{671} S_2 + Y_{671} C_2) - S_{23} Z_{671}$$
$$Z_{6712} = S_{23}(X_{671} S_2 + Y_{671} C_2) + C_{23} Z_{671}$$

$X_{671}, Y_{671}, Z_{671}$ can be expanded into:

$$X_{671} = X_{67} C_1 - Y_{67} S_1$$
$$Y_{671} = C_{12}(X_{67} S_1 + Y_{67} C_1) - S_{12} Z_{67}$$
$$Z_{671} = S_{12}(X_{67} S_1 + Y_{67} C_1) + C_{12} Z_{67}$$

X_{67}, Y_{67}, Z_{67} can be expanded into:

$$X_{67} = X_6 C_7 - Y_6 S_7$$
$$Y_{67} = C_{71}(X_6 S_7 + Y_6 C_7) - S_{71} Z_6$$
$$Z_{67} = S_{71}(X_6 S_7 + Y_6 C_7) + C_{71} Z_6$$

X_6, Y_6, Z_6 can be expanded into:

$$X_6 = S_{56} S_6$$
$$Y_6 = -(S_{67} C_{56} + C_{67} S_{56} C_6)$$
$$Z_6 = C_{67} C_{56} - S_{67} S_{56} C_6$$

where all quatities are known except for S_1, C_1.

② Rearrange the equation to form

$$A C_1 + B S_1 + D = 0$$

where A, B, D are constants.
The trigonometric solutions will be θ_{1A} and θ_{1B}, 2 solutions will be produced.

(b) Solve for θ_4
The values of θ_4 can be obtained from the following two spherical equations:

$$X_{67123} = S_{45} S_4$$
$$Y_{67123} = S_{45} C_4$$

X_{67123} and Y_{67123} can be expanded into two equations using patterns shown in part (a). $\theta_3, \theta_2, \theta_7, \theta_6$ are known. θ_{1A} and θ_{1B} were solved and they are plugged into the equations one at a time for two sets of unique solutions θ_{4A}, θ_{4B}. The two equations produce one unique solution for each θ_1 case. It can be solved in MATLAB by "[S_4, C_4]=A\b" and "θ_4 = atan2d(S_4, C_4)" and doing it twice using two θ_1 values.

(c) Solve for θ_5

The values of θ_5 can be obtained from the following two equations:

$$X_{56712} = S_{34} S_3$$
$$Y_{56712} = S_{34} C_3$$

X_{56712} and Y_{56712} can be expanded into two equations using patterns shown in part (a). $\theta_6, \theta_7, \theta_2, \theta_3$ are known. θ_{1A} and θ_{1B} are plugged into the equations for two sets of unique solutions θ_{5A}, θ_{5B} of the system of equations. (similar to part (b)).

(d) Solve for S_2, S_3, S_6.

The equation of vector loop is used:

* underbar represents vector

$S_1\underline{S_1} + a_{12}\underline{a_{12}} + S_2\underline{S_2} + a_{23}\underline{a_{23}} + S_3\underline{S_3} + a_{34}\underline{a_{34}} + S_4\underline{S_4} + a_{45}\underline{a_{45}} + S_5\underline{S_5} + a_{56}\underline{a_{56}} + S_6\underline{S_6} + a_{67}\underline{a_{67}} + S_7\underline{S_7} + a_{71}\underline{a_{71}} = 0$

where $S_1, S_4, S_5, S_7, a_{12} \rightarrow a_{71}$ are known. all vectors can be represented as values shown in appendix in direction cosines for spatial Heptagon. All values are known except S_2, S_3, S_6. Three equations with three unknowns can be solved with MATLAB using "$[S_2, S_3, S_6] = A \backslash b$". where A represents the 3 equations, b as a 3×1 vector that only contains 0 ($[0,0,0]^T$).

The equation can be expanded into: (with set 1)

$$S_1\begin{bmatrix}0\\0\\1\end{bmatrix} + a_{12}\begin{bmatrix}1\\0\\0\end{bmatrix} + S_2\begin{bmatrix}0\\-S_{12}\\C_{12}\end{bmatrix} + a_{23}\begin{bmatrix}C_2\\S_2C_{12}\\U_{21}\end{bmatrix} + S_3\begin{bmatrix}\bar{X}_2\\\bar{Y}_2\\\bar{Z}_2\end{bmatrix} +$$

$$a_{34}\begin{bmatrix}W_{32}\\-U^*_{321}\\U_{321}\end{bmatrix} + S_4\begin{bmatrix}X_{32}\\Y_{32}\\Z_{32}\end{bmatrix} + a_{45}\begin{bmatrix}W_{432}\\-U^*_{4321}\\U_{4321}\end{bmatrix} + S_5\begin{bmatrix}X_{432}\\Y_{432}\\Z_{432}\end{bmatrix} +$$

$$a_{56}\begin{bmatrix}W_{5432}\\-U^*_{54321}\\U_{54321}\end{bmatrix} + S_6\begin{bmatrix}X_{5432}\\Y_{5432}\\Z_{5432}\end{bmatrix} + a_{67}\begin{bmatrix}W_{65432}\\-U^*_{654321}\\U_{654321}\end{bmatrix} +$$

$$S_7\begin{bmatrix}X_{65432}\\Y_{65432}\\Z_{65432}\end{bmatrix} + a_{71}\begin{bmatrix}C_1\\-S_1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$