C1 Assignment 3 Report

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Abstract

1 Theory

The Lorenz system is the system of equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x)$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z$$

where $\sigma, \rho, \beta \in \mathbb{R}$ are given parameters. In this assignment, we use the values originally specified by Lorenz,

$$\sigma = 10,\ \beta = \frac{8}{3},\ \rho = 28.$$

For these values of σ, ρ, β, the system exhibits chaotic behaviour.

1.1 Schemes

We solve this ODE numerically using the Forward Euler method, the Runge-Kutta 4 method and the Modified Midpoint. These have global errors of order $O(h^1)$, $O(h^4)$ and $O(h^2)$ respectively.

2 Implementation

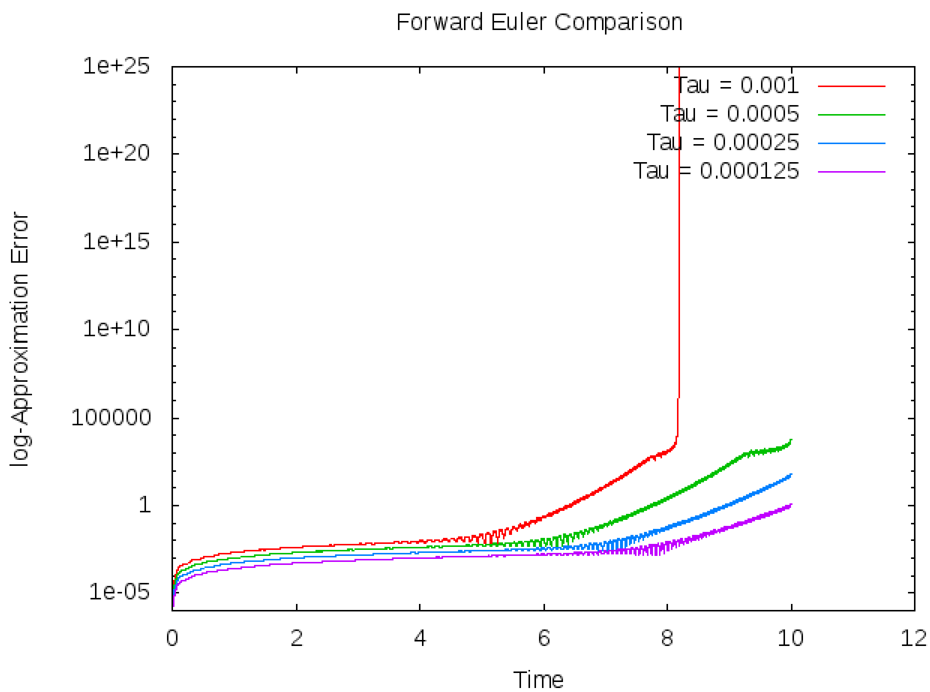
Rather than using the code from the last assignment, we have extensively used the ODEint library using three explicit schemes detailed above.

3 Results

Using the method of manufactured solutions, we calculated the L^∞-error for the various solutions.

3.1 Forward-Euler

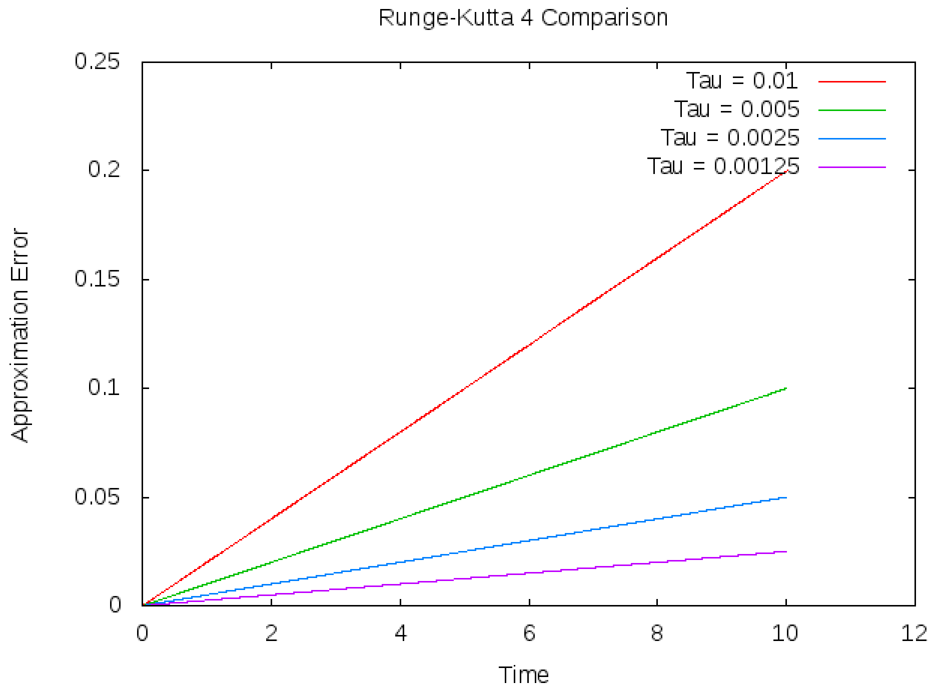
For the Forward Euler method, we tested the implementation for $\tau = 0.001, 0.0005, 0.00025$ and 0.000125. These were smaller time steps than for the Runge-Kutta 4 method as the order of the error is greater than for the other schemes tested. As there is no analytic solution to the ODE, we use the method of manufactured solutions to calculate the error.



As we expected reducing the time step leads to decreased error.

3.2 Runge-Kutta 4

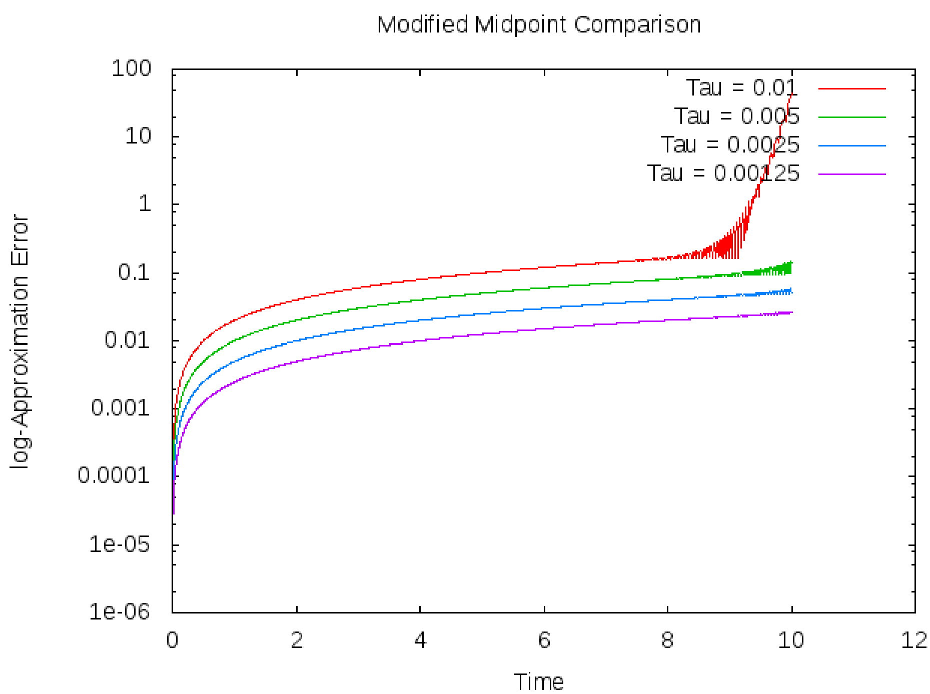
For the Runge-Kutta 4 method, we tested the implementation for $\tau = 0.01, 0.005, 0.0025$ and 0.00125.



As we expected reducing the time step leads to decreased error.

3.3 Midpoint

For the Modified Midpoint method, we tested the implementation for $\tau = 0.01, 0.005, 0.0025$ and 0.00125.



As we expected reducing the time step leads to decreased error.