C1 MA913 Scientific Computing 2015/2016

Assignment 3

Vector-valued IVP. Based on the program you handed in for Assignment 2, write a program to solve a system of ordinary differential equations

$$U'(t) = F(U(t), t) \quad U(0) = u_0$$

where $U : [0, T] \to \mathbb{R}^m$, $m \geq 1$.

There are different ways of extending the existing code to handle systems. Use some form of vector class to represent the approximations for the values of $U(t)$ and some matrix class for the Jacobian of F needed in the Newton scheme. The aim should be not to modify the interfaces used for the scalar case too much - if possible the implementation of the Runge-Kutta scheme itself should only require small changes. To this end provide the required operators on the vector and matrix class. Alternatively, the library ODEINT can be used (in this case more results for all the test cases are expected together with a deeper analysis and theoretical understanding).

Test cases:

First you should convince yourself that the tests from Assignment 2 (in dimension 1) still run correctly and the same results are obtained - you do not need to include this in the report...

Then solve the following problems (at least 1a or 1b and 2a or 2b):

Test 1a: use $T = 2$ and solve

$$\begin{aligned} u'(t) &= 998u(t) + 1998v(t) & u(0) &= 1 \\ v'(t) &= -999u(t) - 1999v(t) & v(0) &= 0\ . \end{aligned}$$

Exact solution is

$$\begin{aligned} u(t) &= 2\exp(-t) - \exp(-1000t) \\ v(t) &= -\exp(-t) + \exp(-1000t) \end{aligned}$$

Test 1b: use $T = 50$, $\lambda = 1000$ and $\mu = 10000$ and solve

$$\begin{aligned} u' &= q(t)/v(t) - (u(t) + h(t))u(t)v(t) & u(0) &= 0 \\ v' &= h(t)v(t)^2 & v(0) &= 10 \end{aligned}$$

with $q(t) = 1 - \cos(t)(\cos(t) - 1)$ and $h(t) = -\frac{3t^2-2t}{\mu}$

Exact solution is

$$\begin{aligned} u(t) &= \sin(t)\frac{(\lambda + t^3 - t^2)}{\mu} \\ v(t) &= \frac{\mu}{\lambda + t^3 - t^2} \end{aligned}$$

Test 2a: use $T = 300$ and solve

$$\begin{aligned} u'(t) &= v(t) & u(0) &= 2 \\ v'(t) &= 100(1 - u(t)^2)v(t) - u(t) & v(0) &= 0\ . \end{aligned}$$

No Exact solution available. Here you can try to use the method of manufactured solutions to check your results.

Test 2b (Lorenz attractor): use $T = 10, \beta = \frac{8}{3}, \sigma = 10, \rho = 28$ and solve

$$\begin{aligned} u'(t) &= -\beta u(t) + v(t)w(t) & u(0) &= 20 \\ v'(t) &= -\sigma v(t) + \sigma w(t) & v(0) &= 30 \\ w'(t) &= v(t)(\rho - u(t)) - w(t) & w(0) &= 10\ . \end{aligned}$$

No Exact solution available. Here you can try to use the method of manufactured solutions to check your results.

Testing your scheme: You can use the first two tests to compute the experimental order of convergence - just a warning: the test 1b is surprisingly tough on the schemes.

Next we should test the efficiency of the schemes. For that you can take either the first or the second test case (or both of course) and plot the number of F, DF evaluation needed versus the error produced by the different schemes. The evaluations should be measured including the Newton solve. To make the comparison simpler assume that an evaluation of DF is comparable to F.
The last two test cases are to show how even simple initial value problem can exhibit very complex structures (especially true for the Lorenz attractor). In the report it is fine to include some plots of the solution - but add some comments on how trustworthy you think the results are. A possible approach is to compare results obtained with two different step sizes...

Submission. For the submission follow the same instructions provided with assignment 1

Deadline: Wed 9 December 2015 6:00 pm