Discrete Unification Theory

Foundations and Noncommutative Quantum Geometry

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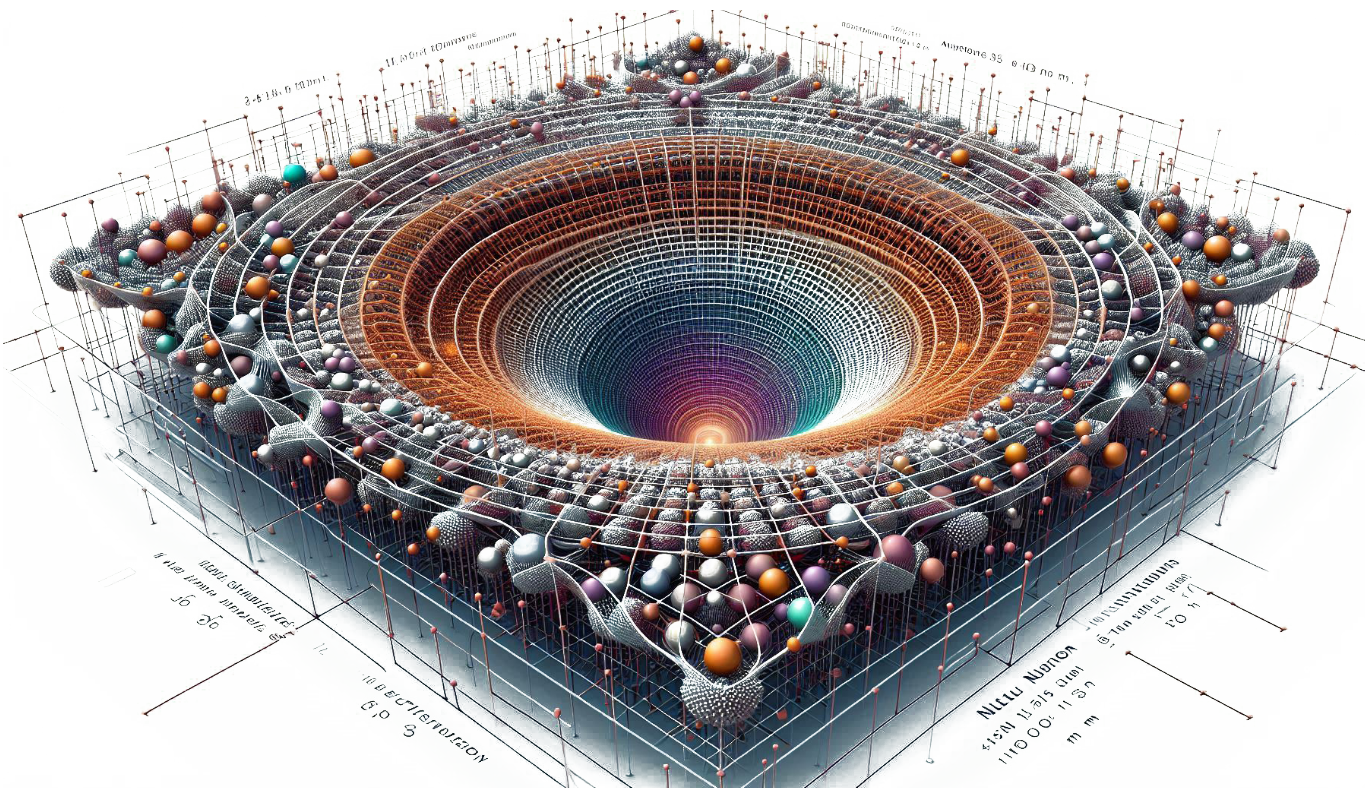


Introduction

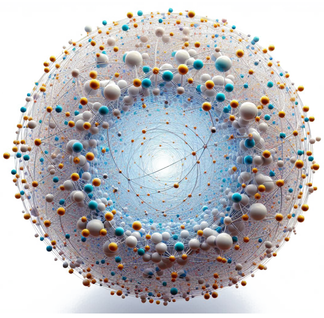
Contemporary physics faces a series of fundamental challenges that transcend the explanatory capabilities of existing theories. On one hand, General Relativity, successful in describing gravity and the large-scale structure of spacetime, is incompatible with the principles of Quantum Mechanics, which governs fundamental interactions at microscopic scales. This incompatibility manifests in the impossibility of coherently quantizing spacetime, generating singularities and non-renormalizable divergences. On the other hand, problems such as the nature of dark matter and dark energy, the hierarchy problem between physical scales, and the discrepancy between the theoretical and observed vacuum energy, underscore the need for a deeper and more unified theoretical framework.

In this context, the Discrete Unification Theory (DUT) emerges as a proposal to integrate these issues into a unique framework based on dynamic noncommutative geometry. This approach redefines the nature of spacetime, postulating that at fundamental scales, spacetime coordinates cease to be classical parameters and become operators obeying non-trivial commutation relations. The DUT framework introduces a dynamic noncommutativity tensor that varies depending on local physical conditions, allowing the reconciliation of global symmetries with quantum effects and resolving theoretical inconsistencies such as ultraviolet divergences.

DUT aims to directly address some of the most persistent problems in physics. Firstly, the dynamic nature of coordinates and their associated tensor introduces a natural regularization scale that could explain the stability of the Higgs boson mass against quantum corrections, providing a possible solution to the hierarchy problem. Likewise, it proposes a dynamic cancellation mechanism for the cosmological constant, derived from interactions between the noncommutativity of spacetime, dark matter and dark energy fields, and emergent gravitational effects. Furthermore, the theory seeks to offer precise descriptions for phenomena associated with dark matter and dark energy through new scalar potentials and dynamics.

This document is developed within the framework of the Epistemic Foundations of Physics and Advanced SDUTies in Artificial Metacognition, an initiative exploring the interrelation between humans and artificial intelligence (AI) as a key tool in scientific progress. Collaboration between researchers and AI systems allows for complex simulations, analysis of patterns in observational data, and development of innovative hypotheses that would be difficult to formulate with traditional approaches. In this sense, DUT represents an interdisciplinary effort combining fundamental physical theories, advanced geometries, and emerging technology to expand the frontiers of knowledge.

In this revised and expanded edition, the theoretical foundations of DUT are presented, and its potential to link observable phenomena with unified theoretical principles is analyzed. The development includes a critical evaluation of its internal consistency, comparisons with contemporary approaches such as String Theory and Loop Quantum Gravity, and an exploration of the inherent phenomenological and computational challenges. Concrete observational predictions and dynamic mechanisms that could be experimentally tested in the future are established, consolidating the role of DUT as a key proposal for unifying the fundamental laws of the universe.



Summary

Contemporary theoretical physics faces profound conceptual and observational challenges. The incompatibility between the classical geometric description of General Relativity (GR), where spacetime is dynamic, and the quantum nature of matter and interactions described by Quantum Mechanics (QM) and Quantum Field Theory (QFT) — usually formulated on a fixed spacetime background — prevents a consistent quantum theory of gravity. This tension manifests in problems such as the treatment of time (the "problem of time"), the non-renormalizability of perturbative quantum gravity, and the need to reconcile the background dependence of QM with the background independence of GR. Additionally, enigmas like the nature of dark matter (∼27

We propose the Discrete Unification Theory (DUT), a theoretical framework based on the hypothesis that the structure of spacetime at fundamental scales (characterized by a scale M_{NC}) is described by a dynamic noncommutative geometry (NCG). NCG generalizes the duality between geometric spaces and algebras of functions to the noncommutative realm, suggesting that the classical notion of a point might lose meaning at the Planck scale. We postulate that spacetime coordinates are replaced by Hermitian operators $\hat{X}^\mu$ satisfying the non-trivial commutation relation $[\hat{X}^\mu, \hat{X}^\nu] = i\hat{\Theta}^{\mu\nu}_{NC}$. Crucially, the noncommutativity tensor $\hat{\Theta}^{\mu\nu}_{NC}$ is not a background constant (which would explicitly break Lorentz invariance), but a dynamic quantum field, whose expected value $\theta^{\mu\nu}_{NC}(x) = \langle\hat{\Theta}^{\mu\nu}_{NC}\rangle$ responds to the matter-energy distribution and possesses its own dynamics. This noncommutativity implies a fundamental uncertainty relation $\Delta X^\mu \Delta X^\nu \geq |\theta^{\mu\nu}_{NC}|/2$, suggesting a finite resolution of spacetime, and acts as a natural physical UV regulator at the scale M_{NC}.

In DUT, it is postulated that the gravitational metric $g^{\text{eff}}_{\mu\nu}$ is not fundamental, but emerges as an effective structure at low energies ($E \ll M_{NC}$) from the underlying noncommutative algebraic structure (emergent gravity), providing the connection with macroscopically observed gravity. The unified action of DUT, built on this noncommutative spacetime, incorporates this emergent gravity (including higher-order terms like R^2 relevant for inflation and renormalizability), the Standard Model modified via the Moyal star product ($\star$) — $\mathcal{L}_{\star\text{SM}}$ — which replaces the ordinary point product and naturally introduces corrections dependent on θ_{NC} and energy/momentum, generating possible Lorentz Invariance Violation (LIV) effects and UV/IR mixing. The action also includes specific candidate fields for dark matter (a scalar Φ with potential $V(\Phi)$) and dark energy (a quintessence-type scalar Q with potential $V_Q(Q)$), as well as a term $\mathcal{L}_\Theta$ (whose specific form is crucial for the model) governing the dynamics of the tensor θ_{NC} itself. It is proposed that all these components derive from fundamental principles of dynamic NCG, possibly related to approaches like the spectral action principle.

We argue that the intrinsic dynamics of DUT, particularly that of the $\theta^{\mu\nu}_{NC}$ field, offer *plausible* mechanisms to solve key problems in fundamental physics. It is *proposed* that the UV/IR mixing inherent in noncommutative field theories (NCQFT) or the modified structure of loop diagrams *could* lead to an exponential suppression of quantum corrections to scalar masses (like m_H), addressing the hierarchy problem if M_{NC} is sufficiently low. Likewise, a dynamic cancellation mechanism for the cosmological constant is *postulated*, based on the idea that the interaction between the θ_{NC}, Φ, Q fields and gravity *could* drive the cosmological system to an attractor state with a very small residual vacuum energy $\rho^{\text{eff}}_{\text{vac}}$, canceling the enormous theoretical contributions. The **robustness and viability** of these proposed mechanisms is a central point of analysis in this work and **requires rigorous validation** (see Appendices D, F).

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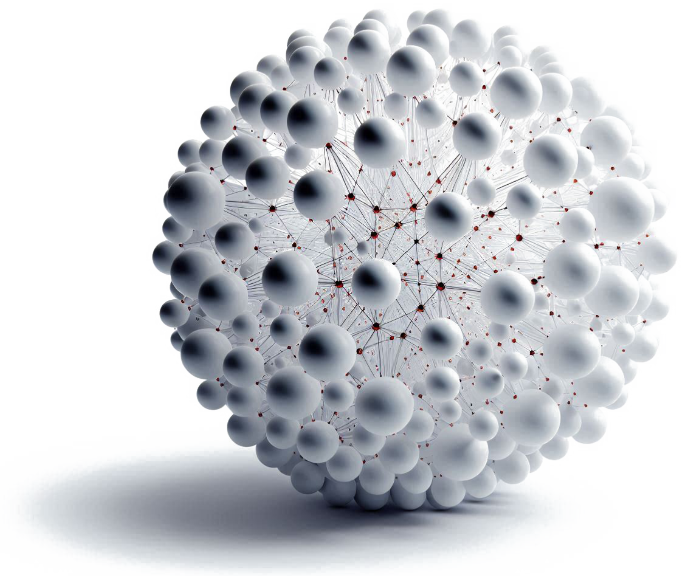


1 Foundational Theoretical Framework

1.1 Noncommutative Geometry (NCG) and the Quantum Nature of Spacetime

NCG extends the well-established duality between geometric spaces and the *commutative* algebras of functions defined on them (e.g., continuous functions $C(X)$ on a topological space X) to the realm of *noncommutative* algebras. The idea is that the structure of space can be reconstructed from its algebra of functions; NCG explores the "geometries" corresponding to algebras where the order of multiplication matters ($A \cdot B \neq B \cdot A$), as occurs with operators in quantum mechanics [1]. (Connes NCG book)

1.1.1 Coordinate Operators and Uncertainty Relation

The commutation relation implies uncertainties, as described in key works [2–4].

$$\Delta A \Delta B \geq \frac{1}{2} |\langle [A, B] \rangle| \tag{1}$$

$$\Delta X^\mu \Delta X^\nu \geq \frac{1}{2} |\langle [\hat{X}^\mu, \hat{X}^\nu] \rangle| = \frac{1}{2} |\theta^{\mu\nu}_{NC}| \tag{2}$$

This imposes a fundamental limit on the precision with which different spacetime coordinates can be simultaneously determined, suggesting a granular or "fuzzy" structure at the scale $l_{NC} = 1/M_{NC}$.

1.1.2 The Noncommutativity Tensor $\hat{\Theta}^{\mu\nu}_{NC}$ and its Dynamics

Unlike NCG models with constant $\theta^{\mu\nu}_{NC}$ (which explicitly break Lorentz invariance), in DUT, $\hat{\Theta}^{\mu\nu}_{NC}$ is a dynamic quantum field. Its dynamics are governed by a term $\mathcal{L}_\Theta$ in the total action (Section 1.3) and it couples to matter and geometry.

Remark 1.1 (Lorentz Symmetry Restoration). A vacuum expectation value $\langle \hat{\Theta}^{\mu\nu}_{NC} \rangle = 0$ is compatible with macroscopic Lorentz invariance, although fluctuations $\langle (\hat{\Theta}^{\mu\nu}_{NC})^2 \rangle \neq 0$ and possible environment-induced expectation values can generate observable *LIV* effects (see Section 5). Compatibility with strict *LIV* limits is a key challenge (see Sec. 5.3.3).

The dynamics of $\theta^{\mu\nu}_{NC}$ are essential for the proposed dynamic cancellation mechanisms (Section 2.2).

1.2 Emergent Effective Metric

In DUT, the gravitational metric is not an *a priori* fundamental field, but is postulated as an emergent property of the underlying noncommutative algebra. This idea of **emergent gravity**, where gravity arises as a macroscopic phenomenon from a more fundamental underlying quantum theory (possibly based on NCG or matrix models), is an active line of research in quantum gravity [5, 6]. (Emergent Gravity/NCG)

1.2.1 Conceptual Definition

The macroscopic geometry perceived by matter fields at low energies ($E \ll M_{NC}$) is encoded in the structure of the operator algebra. A possible realization (inspired by [1, 7]) relates the effective metric to the anticommutator of the coordinate operators, including corrections dependent on $\hat{\Theta}_{NC}$:

Definition 1.2 (Emergent Effective Metric (Conceptual))**.** *The effective metric $g^{\text{eff}}_{\mu\nu}$ governing the propagation of matter at low energies is conceptually defined as:*

$$g^{\text{eff}}_{\mu\nu} \propto \langle \{\hat{X}_\mu, \hat{X}_\nu\} \rangle + \Delta g_{\mu\nu}(\theta_{NC}) \tag{3}$$

where $\{\hat{A}, \hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A}$ is the symmetrized anticommutator and $\Delta g_{\mu\nu}(\theta_{NC})$ represents tensorial corrections dependent on $\theta^{\mu\nu}_{NC}$ and its derivatives, necessary to ensure correct transformation properties and the proper *classical limit ($g^{\text{eff}}_{\mu\nu} \to \eta_{\mu\nu}$ or $g^{\text{classical}}_{\mu\nu}$ when $\theta_{NC} \to 0$). The precise form of $\Delta g_{\mu\nu}$ must be rigorously derived from the complete structure of the underlying NCG, which is an active area of research.*

1.2.2 Physical Interpretation

$g^{\text{eff}}_{\mu\nu}$ is the metric that determines spacetime intervals, macroscopic causal structure, and effective gravitational interaction for particles with $E \ll M_{NC}$. It is the object connecting DUT with standard gravitational observations (lensing, gravitational waves, FRW cosmology). The corrections $\Delta g_{\mu\nu}(\theta_{NC})$ can induce higher-order curvature terms in the effective gravitational action or modify geodesic equations at high energies or curvatures.

1.3 The Unified Action of DUT

The complete dynamics of DUT are derived from an action principle applied to the noncommutative spacetime. This approach may be conceptually related to Connes' **spectral action principle**, which seeks to derive the action of fundamental physics (including gravity and SM) from spectral properties of a Dirac operator on a (possibly noncommutative) geometry [8]. (Spectral Action)

1.3.1 Action Principle and Fundamental Components

The total action S must be a scalar under relevant symmetries (generalized diffeomorphisms, gauge symmetries). It is constructed by integrating a total Lagrangian density $\mathcal{L}_{\text{total}}$ defined on the noncommutative spacetime, where field products are replaced by the Moyal star product ($\star$).

Definition 1.3 (Moyal Star Product)**.** *The Moyal star product of two functions (or classical fields) $f(x)$ and $g(x)$ is defined as:*

$$(f \star g)(x) = \exp\left(\frac{i}{2}\theta^{\mu\nu}_{NC}(x)\overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu\right) f(x)g(x) \tag{4}$$

where $\theta^{\mu\nu}_{NC}$ is the expectation value (classical field) of the noncommutativity tensor. To first order in θ_{NC}:

$$(f \star g)(x) = f(x)g(x) + \frac{i}{2}\theta^{\mu\nu}_{NC}(\partial_\mu f)(\partial_\nu g) + \mathcal{O}(\theta^2_{NC})$$

The $\star$ product is associative but not commutative: $f \star g \neq g \star f$.

The total action takes the conceptual form:

$$S = \int \mathrm{d}^4x \sqrt{-\det(g^{\text{eff}}_{\mu\nu})}\mathcal{L}_{\text{total}}[g^{\text{eff}}_{\mu\nu}, \Phi_i, \Psi_j, A^a_\mu, \theta_{NC}; \star] \tag{5}$$

where $\mathcal{L}_{\text{total}}$ is composed of several sectors:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{grav}} + \mathcal{L}_{\star\text{SM}} + \mathcal{L}_{\text{DM}} + \mathcal{L}_{\text{DE}} + \mathcal{L}_{\text{infl}} + \mathcal{L}_{\Theta} + \Delta\mathcal{L}_{\text{NC}} \tag{6}$$

1.3.2 Modified Gravitational Sector ($\mathcal{L}_{\text{grav}}$)

Describes the dynamics of the effective metric $g^{\text{eff}}_{\mu\nu}$. It is expected to contain the Einstein-Hilbert term based on the Ricci scalar $R[g^{\text{eff}}_{\mu\nu}]$, but modified by NCG corrections and naturally induced higher-order terms:

$$\mathcal{L}_{\text{grav}} = \frac{M_P^2}{2}\left(R[g^{\text{eff}}_{\mu\nu}] - 2\Lambda_{bare} + \alpha R^2[g^{\text{eff}}_{\mu\nu}] + \beta R_{\mu\nu}R^{\mu\nu}[g^{\text{eff}}_{\mu\nu}] + \dots\right) + \mathcal{L}_{\text{corr}}(\theta_{NC}, g^{\text{eff}}_{\mu\nu}) \tag{7}$$

where $M_P = (8\pi G)^{-1/2}$ is the reduced Planck mass, Λ_{bare} is a "bare" cosmological constant, and α, β are coefficients of higher-order terms. The R^2 term [9] is particularly relevant for inflation (Section 1.3.5) and renormalizability (Section 3.2). $\mathcal{L}_{\text{corr}}$ represents explicit corrections dependent on θ_{NC}.

1.3.3 Extended Standard Model ($\mathcal{L}_{\star\mathrm{SM}}$) and the Star Product

The SM fields and interactions are defined on the NC spacetime by replacing the ordinary point product with the star product $\star$ in the Lagrangian:

$$\mathcal{L}_{\star\mathrm{SM}} = \mathcal{L}_{\mathrm{SM}}[\phi_i, \psi_j, A^a_\mu; g^{\mathrm{eff}}_{\mu\nu}, \star] \tag{8}$$

This introduces corrections dependent on $\theta^{\mu\nu}_{NC}$ and derivatives (momenta) in all vertices and kinetic terms, generating phenomenology such as LIV and UV/IR mixing [10] (see Section 4). For example, an interaction term $g\phi^3$ becomes $g\phi \star \phi \star \phi$.

1.3.4 Dark Sectors: Dark Matter ($\mathcal{L}_{\mathrm{DM}}$) and Dark Energy ($\mathcal{L}_{\mathrm{DE}}$)

DUT incorporates specific candidates, whose interactions are also defined with the $\star$ product.

- **Dark Matter:** A real scalar field Φ with a potential $V(\Phi)$. Example:

$$\mathcal{L}_{\mathrm{DM}} = \frac{1}{2} g^{\mathrm{eff}}_{\mu\nu}(\partial_\mu \Phi)(\partial_\nu \Phi) - V(\Phi) + \mathcal{L}_{\mathrm{int}}(\Phi, \mathrm{SM}, \theta_{NC}; \star) \tag{9}$$

  A Higgs-like potential $V(\Phi) = \frac{\lambda_\Phi}{4}(\Phi^2 - v^2_\Phi)^2$ could generate a stable particle [11].Interactions $\mathcal{L}_{\mathrm{int}}$ can occur via Higgs portal or mediated by θ_{NC}.

- **Dark Energy:** A quintessence-type scalar field Q with a slow-roll potential $V_Q(Q)$. Example:

$$\mathcal{L}_{\mathrm{DE}} = \frac{1}{2} g^{\mathrm{eff}}_{\mu\nu}(\partial_\mu Q)(\partial_\nu Q) - V_Q(Q) + \mathcal{L}_{\mathrm{int}}(Q, \ldots; \star) \tag{10}$$

  An exponential potential $V_Q(Q) = V_0 e^{-\kappa Q/M_P}$ [12] can generate accelerated expansion.

The specific choice of $V(\Phi)$ and $V_Q(Q)$ defines the concrete model and must be justified or explored parametrically.

1.3.5 Primordial Inflation in DUT ($\mathcal{L}_{\mathrm{infl}}$)

Inflation can be driven by:

- The R^2 term in $\mathcal{L}_{\mathrm{grav}}$ (Starobinsky-like [9]).
- One of the scalar fields (Φ or Q in an early regime).
- A dedicated inflaton ϕ_{infl}.

In all cases, the inflationary dynamics and predictions for primordial perturbations (n_s, r, f_{NL}) are modified by NCG (via $g^{\mathrm{eff}}_{\mu\nu}$ and the $\star$ product) [13].

1.3.6 Dynamics of $\hat{\Theta}^{\mu\nu}_{NC}$ and Explicit Corrections ($\mathcal{L}_\Theta + \Delta\mathcal{L}_{\mathrm{NC}}$)

The action must include kinetic terms and a potential $V(\theta_{NC})$ for the dynamic field $\theta^{\mu\nu}_{NC}$ (or the fundamental fields generating it). $\Delta\mathcal{L}_{\mathrm{NC}}$ may include additional explicit couplings between $\theta^{\mu\nu}_{NC}$ and other fields not directly derived from the $\cdot \to \star$ replacement.

Example 1.4 (Schematic Lagrangian for θ_{NC}). A possible form for the dynamics of θ_{NC} could involve kinetic terms built from its covariant derivatives and a potential:

$$\mathcal{L}_\Theta = -\frac{1}{4} F_{\mu\nu\rho\sigma}(\theta_{NC}) F^{\mu\nu\rho\sigma}(\theta_{NC}) - V(\theta_{NC}) \quad \textbf{(Schematic form)} \tag{11}$$

where $F_{\mu\nu\rho\sigma}$ is an appropriate field strength tensor (analogous to $F_{\mu\nu}$ in electromagnetism) and $V(\theta_{NC})$ is a potential that must ensure a stable vacuum (possibly $\langle \theta_{NC} \rangle = 0$) and contribute to the cancellation dynamics (Section 2.2). The **exact form of $\mathcal{L}_\Theta$ and $V(\theta_{NC})$ is a key unspecified element** that defines the concrete DUT model and crucially affects phenomenology and consistency.

2 Key Dynamic Mechanisms (Proposed)

The dynamic NCG structure of DUT allows proposing intrinsic mechanisms to address fundamental problems. The **robustness and rigorous demonstration** of these mechanisms is crucial and discussed below as a research goal, with technical details outlined in Appendices D and F.

2.1 Exponential Suppression of Quantum Corrections and the Hierarchy Problem (Hypothesis)

2.1.1 The Proposed Suppression Mechanism

It is *argued* that noncommutativity, particularly through UV/IR mixing or the modified loop structure in noncommutative field theories (NCQFT), *could* drastically alter quantum corrections to scalar masses like the Higgs (m_H). In certain NCG models, it has been suggested that radiative corrections δm_H^2 originating from a UV cutoff scale Λ_{UV} *could be* exponentially suppressed above the scale M_{NC}:

$$\delta m_H^2 \sim \Lambda_{UV}^2 f(\Lambda_{UV}/M_{NC}) \quad \text{where} \quad f(x) \sim e^{-x^2} \text{ for } x \gg 1 \quad \textbf{(Expected result)} \tag{12}$$

instead of the quadratic divergence $\delta m_H^2 \sim \Lambda_{UV}^2$ of the SM. This would be due to the non-locality introduced by the $\star$ product effectively "smearing" interactions at short distances.

2.1.2 Implications for the Higgs Mass

If this mechanism works and M_{NC} is significantly below M_P (e.g., at the TeV-PeV scale), contributions from physics at scales $\Lambda_{UV} \gg M_{NC}$ (like GUT or Planck) would be negligible due to the suppression factor $f(\Lambda_{UV}/M_{NC})$. This *would allow* the observed mass $m_H \approx 125$ GeV to be natural without fine-tuning, linking the hierarchy solution to the NCG scale.

2.1.3 Robustness Analysis of the Suppression Mechanism (Necessary)

The validity and generality of this exponential suppression within the complete and dynamic framework of DUT is a critical point that **requires rigorous investigation** (see Appendix D for the proposed NPRG approach).

- **Model Dependence:** How does the exact form of $\mathcal{L}_\Theta$, $V(\theta_{NC})$, and the couplings in $\Delta\mathcal{L}_{\text{NC}}$ affect the suppression effectiveness? Is it generic or does it require specific conditions?
- **Range of Validity:** Are there thresholds in M_{NC} or interaction strengths where the mechanism ceases to be effective? How is it affected by the dynamics of θ_{NC}?
- **Stability and Consistency:** Is the dynamics producing the suppression stable against quantum or thermal perturbations? Is it consistent with other phenomenological constraints (e.g., LIV)?
- **Critical Comparison:** How does this mechanism compare to others proposed in NCG or alternative theories (SUSY, extra dimensions, etc.) in terms of naturalness and testability?

A detailed analysis using tools like the Non-Perturbative Renormalization Group (NPRG) is required to quantify the robustness and determine if the suppression (12) holds in the complete DUT model.

2.2 Dynamic Cancellation of the Cosmological Constant (Hypothesis)

2.2.1 The Cosmological Constant Problem

The discrepancy of $\sim 60-120$ orders of magniDUTe between the theoretical vacuum energy density ($\rho_{\text{vac}}^{\text{th}} \sim M_P^4$ or $\sim M_{SUSY}^4$, etc.) and the observed one ($\rho_{\text{vac}}^{\text{obs}} \sim (10^{-3}\text{ eV})^4$).

2.2.2 The Proposed Dynamic Cancellation Mechanism in DUT

DUT *proposes* that the vacuum energy (including Λ_{bare} and contributions from $V(\Phi)$, $V(Q)$, $V(\theta_{NC})$, and quantum condensates) couples to the dynamic fields $\theta_{NC}^{\mu\nu}$, Φ, and Q.

Proposition 2.1 (Dynamic Cancellation Hypothesis). *It is postulated that the coupled system of gravity and scalar/tensor fields ($g_{\mu\nu}^{\text{eff}}, \Phi, Q, \theta_{NC}$) could possess attractor-type cosmological solutions where the fields dynamically evolve towards vacuum expectation values $\langle\Phi\rangle_0, \langle Q\rangle_0, \langle\theta_{NC}^{\mu\nu}\rangle_0$ such that the total effective contribution to the cosmological constant relaxes to a very small residual value, close to the observed one:*

$$\rho_{\text{vac}}^{\text{eff}} = \frac{M_P^2}{2}\Lambda_{bare} + V(\langle\Phi\rangle_0) + V_Q(\langle Q\rangle_0) + V(\langle\theta_{NC}\rangle_0) + \langle\rho_{quantum}\rangle \approx \rho_{\text{vac}}^{\text{obs}} \tag{13}$$

The rigorous demonstration of the existence and properties of such attractors (outlined in Appendix F) involves analyzing the coupled equations of motion (derived in Appendix E) and finding the conditions on the potentials and couplings.

2.2.3 Critical Comparison with Other Solutions

Unlike supersymmetry (which requires breaking and still doesn't fully solve the problem), anthropic approaches (which invoke a multiverse), or ad-hoc modifications of gravity, this mechanism would be, if it works, an intrinsic consequence of the dynamics of the fundamental degrees of freedom of DUT. However, its naturalness crucially depends on the form of the potentials and the attractor dynamics.

2.2.4 Robustness Analysis of the Cancellation Mechanism (Necessary)

The viability of this mechanism critically depends on its robustness (see Appendix F for details).

- **Sensitivity to Initial Conditions:** Does the system evolve towards the low $\rho_{\text{vac}}^{\text{eff}}$ attractor from a wide range of initial conditions in the early universe? Is the basin of attraction sufficiently large?
- **Sensitivity to Parameters and Fine-Tuning:** Does the fine cancellation critically depend on the parameter values in the potentials $V(\Phi)$, $V(Q)$, $V(\theta_{NC})$? Does it introduce a new fine-tuning problem?
- **Quantum/Thermal Corrections:** Does the cancellation mechanism survive quantum (loop) corrections or thermal effects in the early universe? How do phase transitions affect it?
- **Attractor Stability:** Is the final low vacuum energy state stable in the long term against perturbations?

A detailed stability and sensitivity analysis of the attractor solutions is fundamental to evaluate the viability of this proposal.

3 Mathematical Consistency and Stability

The viability of DUT depends on its internal mathematical consistency and physical stability. Technical details, requiring full development, are outlined in Appendices B, C, and D.

3.1 Hamiltonian Analysis and Positive Energy Condition

3.1.1 Hamiltonian Formalism in NCG

Adapting the canonical Hamiltonian formalism (Poisson/Moyal brackets, Dirac constraint analysis) to the noncommutative context is required to identify the physical degrees of freedom, verify the consistency of the equations of motion, and establish the basis for quantization. This formalism is conceptually developed in Appendix C. This analysis is crucial to address the **problem of time** within DUT, as the Hamiltonian in GR becomes a constraint (Wheeler-DeWitt equation), leading to the "frozen formalism". DUT must show how an effective notion of time emerges or if it proposes a different solution (perhaps related to the dynamics of θ_{NC}).

3.1.2 Stability and Absence of Ghost Modes

The presence of higher derivatives (e.g., in R^2 or $\mathcal{L}_\Theta$) or indefinite internal metrics (potentially in the θ_{NC} field space) could introduce ghost modes (states with negative norm or negative energy) that violate unitarity and vacuum stability.

Theorem 3.1 (Stability Condition - Objective)**.** *For DUT to be physically viable, it must be demonstrated that the physical Hamiltonian $\hat{\mathcal{H}}_{\text{phys}}$, acting on the Hilbert subspace of physical states $\mathcal{H}_{\text{phys}}$ (obtained after solving constraints), is positive definite: $\langle\psi|\hat{\mathcal{H}}_{\text{phys}}|\psi\rangle \geq 0$ for all $|\psi\rangle \in \mathcal{H}_{\text{phys}}$.*

This ensures vacuum stability and unitarity of time evolution. The analysis (see Appendix C) may require the use of formalisms like Krein spaces if indefinite metrics appear, demonstrating that ghosts decouple or do not belong to the physical spectrum. The form of $V(\theta_{NC})$ and $\mathcal{L}_\Theta$ is crucial here.

3.2 Asymptotic Behavior and Renormalization

3.2.1 Renormalizability and Asymptotic Safety

The inclusion of terms like R^2 in $\mathcal{L}_{\text{grav}}$ [9] improves the UV behavior of perturbative gravity. NCG acts as a UV regulator, but introduces UV/IR mixing [10]. A fundamental question is whether DUT is renormalizable or, ideally, asymptotically safe.

Definition 3.2 (Asymptotic Safety [14])**.** *A theory is asymptotically safe if its flow under the Renormalization Group (RG) converges to a non-Gaussian (interacting) fixed point in the ultraviolet (UV) limit, with a finite number of relevant directions (free parameters).*

If DUT is asymptotically safe, it would be a predictive fundamental theory.

3.2.2 Analysis via the Non-Perturbative Renormalization Group (NPRG)

The NPRG (or FRG, Functional Renormalization Group) is a powerful tool for sDUTying the flow of the effective action Γ_k from the UV ($k \to \infty$) to the IR ($k \to 0$). Applied to DUT (see Appendix D), it allows investigating:

- The existence of UV fixed points (Gaussian or non-Gaussian) that would control high-energy behavior.
- The flow of relevant couplings (Newton constant G_k, cosmological constant Λ_k, coefficients α_k, β_k of R^2, etc., and the parameters of the θ_{NC} sector).
- The quantum consistency and predictivity of the theory.
- Verifying if the hierarchy suppression (Section 2.1) is realized dynamically in the RG flow.

Appendix D details the formalism and objectives of the NPRG analysis for DUT.

3.3 Symmetries and Conservation Laws: Noncommutative Noether's Theorem

Continuous symmetries of the action S (5), defined with the $\star$ product, lead to modified conservation laws.

Theorem 3.3 (Noncommutative Noether's Theorem - Outline)**.** *To each continuous symmetry transformation of the action S corresponds a current $\mathcal{J}_\star^\mu$ satisfying a conservation law modified by the star product:*

$$\partial_\mu \mathcal{J}_\star^\mu = \mathcal{T}_\star \tag{14}$$

where $\mathcal{J}_\star^\mu$ is the modified Noether current (involves $\star$ products) and $\mathcal{T}_\star$ is a term that can be non-zero if the symmetry is not exact or if θ_{NC} is not constant (e.g., for translations if $\partial_\mu \theta_{NC} \neq 0$).

The detailed derivation (Appendix B) confirms the (modified) conservation of fundamental quantities like the energy-momentum tensor and angular momentum, ensuring consistency with basic physical principles.

4 Interconnection with the Standard Model (SM)

DUT incorporates the SM by modifying it through NCG and the star product $\star$.

4.1 Modifications to SM Interactions

4.1.1 The Star Product ($\star$) and its Impact on Vertices and Propagators

The replacement $\cdot \to \star$ in $\mathcal{L}_{\star\text{SM}}$ introduces corrections dependent on $\theta_{NC}^{\mu\nu}$ and the momenta (p, k) of the interacting particles in all vertices and kinetic terms.

- **Vertices:** A vertex $g\phi_1\phi_2\phi_3$ becomes $g\phi_1 \star \phi_2 \star \phi_3$. In momentum space, this introduces oscillatory phase factors in Feynman ampliDUTes, of the type $e^{ip\wedge k/M_{NC}^2}$, where $p \wedge k = \frac{1}{2}\theta_{NC}^{\mu\nu} p_\mu k_\nu$. These factors modify cross sections, decay rates, and can induce *LIV*.
- **Propagators:** Kinetic terms are also modified (e.g., $\bar{\psi} \star i\gamma^\mu D_\mu \star \psi$), which can alter propagators, especially at high energies.

These modifications are generic and affect *all* SM interactions.

4.1.2 Example: Vertex $H \to \gamma\gamma$

The decay $H \to \gamma\gamma$, mediated by loops (mainly W and t), is affected by the $\star$ modification of internal vertices (e.g., HWW, $H\bar{t}t$) and gauge couplings. The resulting ampliDUTe $A_\star(H \to \gamma\gamma)$ acquires a dependence on $\theta_{NC}^{\mu\nu}$ and the photon momenta k_1, k_2:

$$A_\star(H \to \gamma\gamma) = A_{SM}(H \to \gamma\gamma) \times \left(1 + f(\theta_{NC}, k_1, k_2, m_{loop}) + \mathcal{O}(\theta_{NC}^2)\right) \tag{15}$$

This alters the total rate and potentially the angular distribution, offering a possible testing pathway [**?**]. The exact form of f requires a loop calculation in NCQFT (see Appendix G).

4.1.3 Other Modifications and Phenomenological Dependencies

- **Yukawa Couplings:** $y_f H \star \bar{\psi} \star \psi$ affects masses and decays.
- **Gauge Self-Interactions:** $\mathrm{Tr}(F_{\mu\nu} \star F^{\mu\nu})$ modifies 3- and 4-boson vertices.
- **Energy Dependence:** Deviations in cross sections (σ) typically scale with energy (E) as $\Delta\sigma/\sigma \sim (E/M_{NC})^2$ or higher powers, being more relevant at high energies (colliders).
- **Lorentz Invariance Violation (*LIV*):** The presence of $\theta_{NC}^{\mu\nu}$ introduces a preferred (though dynamic) background structure, which can manifest as *LIV* in reaction thresholds, particle dispersion, neutrino oscillations, etc. [15].

4.2 Renormalization in Noncommutative Field Theories (NCQFT)

4.2.1 Regularization and UV/IR Mixing

NCG acts as a UV regulator (due to M_{NC}), but introduces the phenomenon of **UV/IR mixing** [10]: UV divergences of planar diagrams can reappear as IR divergences (in the momentum $p \to 0$ limit) in non-planar diagrams, even in massive theories. This is a consequence of the $e^{ip\wedge k}$ phase factors in vertices and represents a significant challenge for the renormalization of NCQFTs.

Remark 4.1 (Impact of Dynamic θ_{NC}). The dynamic nature of $\theta_{NC}^{\mu\nu}$ in DUT *could* alter or mitigate this problematic behavior of UV/IR mixing, which is intrinsically related to the proposed hierarchy suppression mechanism (Section 2.1) and must be rigorously investigated via NPRG (Appendix D).

Progress has been made in the renormalization of certain NCQFT models, such as the **Grosse-Wulkenhaar model**, which introduces a harmonic oscillator term to control UV/IR mixing in scalar theories [16]. Other approaches include modifications of the BPHZ scheme, algebraic renormalization, and translation-invariant models [17]. The renormalization of NC gauge theories remains an active area of research.

4.2.2 Feynman Diagrams in NCG

Perturbative calculations use Feynman diagrams with modified rules (see Appendix G for examples):

- Vertices with phase factors dependent on $\theta_{NC}^{\mu\nu}$ and momenta.
- Propagators possibly modified by $g_{\mu\nu}^{\text{eff}}$ or $\mathcal{L}_{\Theta}$.
- Distinction between planar and non-planar diagrams, with different UV/IR behaviors.

5 Observational Predictions and Experimental Testing

The viability of DUT depends on its ability to make testable predictions and accommodate existing constraints.

5.1 Cosmological Signatures

5.1.1 Cosmic Microwave Background (CMB)

- **Power Spectra:** Modifications in the scalar spectral index n_s and the tensor-to-scalar ratio r due to NCG inflationary dynamics [9, 13]. Quantitative prediction: $\Delta n_s \sim f_s(M_{NC}, \text{params}_{infl})$, $\Delta r \sim f_r(M_{NC}, \text{params}_{infl})$.
- **Primordial Non-Gaussianities:** Generation of non-Gaussianities (f_{NL}) with specific shapes (potentially local, equilateral, or folded, depending on the exact inflationary mechanism in DUT and $\star$ interactions) [18]. Quantitative prediction: $f_{NL}^{\text{type}} \sim g(M_{NC}, \text{params}_{infl})$.
- **Statistical Anisotropies:** Possible statistical anisotropies induced by a non-zero expectation value $\langle\theta_{NC}^{\mu\nu}\rangle \neq 0$ during inflation or recombination.

5.1.2 Large-Scale Structure (LSS)

- **Matter Power Spectrum:** Modifications in $P(k)$ and the structure growth rate $f\sigma_8$ due to modified gravity ($g_{\mu\nu}^{\text{eff}}$) and the dynamics of dark fields Φ, Q [19]
- **Statistical Anisotropies:** Possible anisotropies in galaxy distribution correlated with $\langle\theta_{NC}^{\mu\nu}\rangle$.

5.1.3 Primordial Gravitational Waves (GW)

Prediction of a stochastic GW background with a spectrum $\Omega_{GW}(f)$ modified by NCG in the early universe, potentially detectable [20]. The shape of the spectrum would depend on M_{NC} and the expansion history.

5.2 Signals in Particle Physics

5.2.1 High-Energy Colliders (LHC and Future)

- **Production of New Particles:** Search for resonances corresponding to excitations of Φ, Q, or the Θ sector.
- **Deviations in SM Processes:** Search for deviations in cross sections and angular distributions relative to SM predictions, typically growing with energy:

$$\frac{\sigma_{\rm DUT} - \sigma_{\rm SM}}{\sigma_{\rm SM}} \approx c\left(\frac{E}{M_{NC}}\right)^2 + \mathcal{O}\left(\left(\frac{E}{M_{NC}}\right)^4\right) \tag{16}$$

  where the coefficient c depends on the specific process and θ_{NC} parameters.
- ***LIV* Effects:** Search for dependencies of cross section or decay products on beam direction or system boost.

5.2.2 Neutrino Physics

Modifications in neutrino oscillations due to *LIV* terms dependent on $\theta_{NC}^{\mu\nu}$ in the effective Hamiltonian, or non-standard interactions (NSI) induced by $\star$ [21].(Volume 1 of the DUNE Technical Design Report (TDR).) Search for anomalies in spectra or arrival directions.

5.2.3 Search for Dark Matter (Φ)

- **Direct Detection:** Φ-nucleus scattering, with cross section $\sigma_{\Phi N}$ dependent on the Φ-SM coupling (mediated by Higgs or θ_{NC}).
- **Indirect Detection:** Signals from Φ annihilation/decay (gamma rays, antimatter) from dense regions (galactic center, dwarf galaxies).
- **Colliders:** Production of Φ associated with missing energy.

DUT provides a specific candidate Φ whose properties (mass, couplings) must be consistent with existing limits [11].

5.3 Astrophysics and High-Energy Phenomena

5.3.1 Gamma Rays and Ultra-High-Energy Cosmic Rays (UHECR)

- **Time Delays (*LIV*):** Search for energy-dependent time delays $\Delta t \propto (E/M_{NC})^n L$ (with $n = 1$ or $n = 2$ typically) for photons/neutrinos from distant astrophysical sources (GRBs, AGNs) [15]. Current limits are very restrictive (see Sec. 5.3.3).
- **Modified Reaction Thresholds:** Alteration of thresholds for processes like $e^+e^- \to \gamma\gamma$ or $p\gamma \to p\pi^0$, affecting observed spectra.

5.3.2 Astrophysical Gravitational Waves

Possible modifications in GW propagation (frequency-dependent dispersion, birefringence, anomalous polarizations) due to $g_{\mu\nu}^{\rm eff}$ or θ_{NC}, potentially detectable in multi-messenger events.

5.3.3 Summary of Lorentz Symmetry Violation (*LIV*) Tests

DUT predicts *LIV* parameterized by $\theta^{\mu\nu}_{NC}$ and M_{NC}. Tests span astrophysics (time delays, thresholds), neutrinos (oscillations), colliders (directional dependence), atomic spectroscopy, etc. Current limits are **very restrictive** and must be accommodated by the theory. For example, observations of GRBs have established very strong limits. Observations of GRB 221009A by LHAASO impose limits on the LIV energy scale $E_{QG,n}$ (where $n = 1$ for linear dependence, $n = 2$ for quadratic) of the order of several times the Planck energy ($E_{\rm Pl} \sim 1.22 \times 10^{19}$ GeV) for $n = 1$, and even stronger for $n = 2$ [22]. Previous Fermi-LAT observations on GRB090510 also provide robust limits [23]. Other experiments like HAWC and H.E.S.S. contribute by sDUTying pair production thresholds or possible photon decay [24]. DUT must demonstrate how its mechanism (possibly the dynamic nature of θ_{NC} making $\langle\theta_{NC}\rangle \approx 0$ today, or a specific structure of suppressed LIV operators) is compatible with these stringent limits. **Demonstrating this compatibility is crucial.**

Table 1: Examples of Experimental Limits on LIV (Photons, 95% CL). $E_{\rm Pl} \approx 1.22 \times 10^{19}$ GeV.

Experiment/Obs.	Astro. Source	LIV Parameter	Lower Limit $E_{QG,n}$ / Upper Limit $\lvert\delta_0\rvert$	Technique
LHAASO [22]	GRB 221009A	$E^{(-)}_{QG,1}$ (subluminal)	$> 8.2E_{\rm Pl}$	Time-of-Flight
LHAASO [22]	GRB 221009A	$E^{(+)}_{QG,1}$ (superluminal)	$> 9.0E_{\rm Pl}$	Time-of-Flight
LHAASO [22]	GRB 221009A	$E^{(\pm)}_{QG,2}$ (quadratic)	$> 5.7 \times 10^{-8}E_{\rm Pl}$	Time-of-Flight
Fermi-LAT [23]	GRB090510	$E^{(-)}_{QG,1}$	$> 1.3E_{\rm Pl}$ (estimated)	Time-of-Flight
Fermi-LAT [23]	GRB090510	$E^{(+)}_{QG,1}$	$> 1.0E_{\rm Pl}$ (estimated)	Time-of-Flight
H.E.S.S./FACT [?]	Mrk 501	$E^{(-)}_{QG,2}$	$> 2.6 \times 10^{10}$ GeV	Pair Prod.
HAWC [24]	Galactic Sources	$E^{(+)}_{QG,2}$	$> 2 \times 10^{14}$ GeV	Photon Decay

Note: Limits may vary depending on the specific model and analysis. $E_{QG,n}$ is the LIV energy scale for dependence $\propto (E/E_{QG,n})^n$. References are indicative and should be verified/completed.

5.3.4 Supermassive Black Holes and Accretion in DUT (Speculative)

The formation and growth of supermassive black holes (SMBHs) in galactic centers, especially in early epochs, present challenges for standard models. DUT *could* offer new perspectives, albeit in a **highly speculative** manner at this stage. The effects of NCG, by introducing a minimum length scale $\sim 1/M_{NC}$, could regulate the classical GR central singularity, modifying the internal structure of the black hole and potentially affecting its long-term evolution or information loss. Furthermore, the effective metric $g^{\rm eff}_{\mu\nu}$ could deviate significantly from Kerr near the horizon in high-curvature regimes. The dynamics of the θ_{NC} tensor in the extreme environment of an accretion disk could interact with electromagnetic fields (via $\star$ terms in the modified Maxwell Lagrangian), potentially influencing accretion efficiency, rotational energy extraction (analogous to the Blandford-Znajek mechanism), or disk stability. These effects **would require explicit solutions** of DUT equations in strong gravity, which is computationally very demanding and **needs considerable theoretical development.**

5.3.5 Formation of Early Massive Galaxies in DUT (Speculative)

Recent observations (e.g., JWST) have revealed apparently more massive and evolved galaxies in the early universe ($z > 10$) than expected in the standard ΛCDM model. DUT *could*, hypothetically, influence early structure formation in several ways. First, the specific properties of the dark matter candidate Φ (mass, self-interactions, interactions with baryons mediated by $\star$ or θ_{NC}) could alter the formation and profile of early dark matter halos. Second, if dark energy Q had non-trivial dynamics in early epochs (different from a cosmological constant), it could have affected the expansion rate and thus the time available for structure growth. Third, modifications to gravity ($g^{\text{eff}}_{\mu\nu}$ and R^2 terms) could alter the rate of gravitational collapse. Finally, the inflationary phase in DUT (Sec. 1.3.5) could generate a specific primordial power spectrum or non-Gaussianities favoring the early formation of massive objects. Quantitatively investigating these possibilities **requires detailed cosmological simulations** incorporating the full DUT dynamics ($g^{\text{eff}}_{\mu\nu}, \Phi, Q, \theta_{NC}$) and $\star$ interactions, which represents a **significant computational and modeling challenge.**

5.3.6 Relativistic Jets in DUT (Speculative)

The launch and collimation mechanisms of powerful relativistic jets observed in AGNs and quasars involve complex magnetohydrodynamic processes near the central SMBH and its accretion disk. DUT *could* introduce **speculative** modifications to this scenario. The interaction of the dynamic tensor θ_{NC} with the intense magnetic fields (B) near the black hole, through $\star$-modified terms in electrodynamics (e.g., $\sim \theta_{NC} F \wedge F$ or direct θ_{NC}-B couplings), could influence energy extraction (via modified Penrose or Blandford-Znajek type mechanisms) or the structure of the magnetic field responsible for collimation itself. Additionally, particle physics modified by the $\star$ product within the jet plasma (at potentially high energies near the base) could alter particle acceleration processes or the observed radiation production (synchrotron, inverse Compton). Evaluating these ideas **requires relativistic MHD modeling** within the framework of dynamic NCG and $\star$-modified electrodynamics, a theoretical and computationally **very advanced and currently undeveloped task.**

5.4 Statistical Analysis and Parameter Fitting

5.4.1 Methodologies

The use of advanced statistical techniques (e.g., MCMC, Bayesian analysis) is required to fit the free parameters of DUT (M_{NC}, θ_{NC} components, parameters of $V(\Phi), V(Q), V(\theta_{NC})$, α, β, etc.) using combined data from CMB, LSS, SNe Ia, BBN, LHC, *LIV*, DM detection, etc.

5.4.2 Current Constraints and Benchmark Scenarios

Astrophysical *LIV* limits often push M_{NC} towards M_P for certain operators, while LHC imposes $M_{NC} \gtrsim \mathcal{O}(1)$ TeV for direct effects. DUT must accommodate these limits, possibly via the dynamics of θ_{NC} (e.g., $\langle\theta_{NC}\rangle \approx 0$ today) or by situating M_{NC} in a specific range. It is crucial to define benchmark scenarios (specific points in the parameter space) and calculate all their predictions simultaneously for effective testing.

5.4.3 Summary Table of Parameters and Sensitivity (Conceptual)

DUT Parameter	**Main Observational Probes**	**Current Limit/Sensitivity (Order Mag.)**
M_{NC}	*LIV* (astro, colliders), $\Delta\sigma_{SM}$ (LHC)	$\gtrsim$ TeV (direct), $\gg E_{\text{Pl}}$ (indirect, *LIV*- **Very restrictive!**)
$\theta^{\mu\nu}_{NC}$ (ampliDUTe)	*LIV*, CMB/LSS Anisotropies	Very restrictive if $\langle\theta_{NC}\rangle \neq 0$
$m_\Phi, \lambda_\Phi, v_\Phi$	DM (direct, indirect, relic)	Model dependent
V_0, κ (for Q)	Cosmology (SNIa, H0, LSS)	Adjustable to DE data
α, β (in $\mathcal{L}_{grav}$)	Inflation (CMB: n_s, r)	Consistent with R^2
Params. $V(\theta_{NC})$	CC dynamics, Stability, θ_{NC} dynamics	Theoretical / Indirect / **Unspecified**

Note: This table is conceptual and highly simplified. Actual limits depend strongly on the specific model, couplings, and LIV operators considered.

6 Detailed Analysis of Key Observations

Important Note: This section requires **continuous and rigorous updating** with the latest experimental results and phenomenological analyses to maintain its validity. The following discussion reflects a hypothetical state and should be reviewed periodically.

The connection (potential or problematic) between DUT and key observational areas is briefly discussed:

1. **Primordial B-Modes (CMB):** Sensitive to inflation (r). DUT predicts r modified by NCG [13]. (Status: Upper limits $r \lesssim 0.03$).
2. **Primordial Non-Gaussianities (CMB) [18]:** Strict limits ($f_{NL}^{local} \approx 0 \pm 5$). DUT predicts specific shapes; requires small f_{NL}. (Status: Consistent with Gaussianity).
3. **Astrophysical *LIV* Tests [15]:** Very strong limits on M_{NC} ($\gg M_P$ for some operators) from GRBs, AGNs (see Sec. 5.3.3). DUT must accommodate them (Is $\langle\theta_{NC}\rangle \approx 0$? Dynamic suppression?). (Status: **Potential significant tension**).
4. **Search for Dark Matter (WIMPs, Axions, etc.):** Strict direct/indirect detection limits. DUT offers Φ, whose properties must be adjusted to the limits [11]. (Status: No confirmed detection).

1. **Hubble Constant (H_0) Tension [?]:** Discrepancy $\sim 5\sigma$ between local and CMB measurements. Solvable with Q dynamics in DUT? (Status: Significant tension, possible window for new physics).
2. **Anomalous Magnetic Dipole Moment of the Muon ($g-2$):** Deviation $\sim 5\sigma$ from SM. Explainable by NCG loop corrections ($\star$) in DUT [25–28]? (Status: Significant deviation, possible window).
3. ***LIV* Searches at Colliders (LHC):** Limits $M_{NC} \gtrsim$ TeV. (Status: No evidence of LIV).
4. **Electroweak Vacuum Stability:** Metastable vacuum in SM. Modified by NCG or Φ sector? (Status: Theoretical, requires analysis).
5. **Other Anomalies (less significant):** Positron excess [?], anomalies in $B \to K^{(*)}\ell\ell$, etc. Could DUT explain them? (Status: Variable, often debated).
6. **Consistency with EW Precision Tests:** $\star$ modifications must be compatible with electroweak precision data.

The overall assessment of DUT's viability requires a combined fit to all these data.

7 Numerical Simulations and Computational Tools

7.1 Need for Simulations in NCG

The complexity of the coupled equations of motion (Appendix E), the non-linear nature of gravity and $\star$ interactions, the need for global statistical analyses, and the sDUTy of non-perturbative regimes make numerical simulations indispensable for:

- **Cosmology:** Evolution of the early universe (inflation, reheating), precise calculation of CMB and LSS observables, structure formation in the presence of $g_{\mu\nu}^{\text{eff}}, \Phi, Q, \theta_{NC}$.
- **Particle Physics:** Calculations of cross sections and decay rates including $\star$ and loop effects, development of Monte Carlo event generators for collider searches.
- **Strong Gravity:** SDUTy of black holes and neutron stars in NCG.
- **Non-Perturbative SDUTies:** Numerical solution of NPRG flow equations (Appendix D), Lattice NCG simulations.
- **Robustness Analysis:** Exploration of parameter space and initial conditions for dynamic mechanisms (Section 2).

7.2 Adapted Numerical Methods and Challenges

Adapted methods and the development of new tools are required:

- **Noncommutative Lattice Field Theory (Lattice NCG):** Discretization of NC spacetime for non-perturbative sDUTies. Challenges: Definition of derivatives and $\star$ product on the lattice, preservation of symmetries, sign problems. Strategies: Use of exact lattice formulations (if they exist), reweighting methods, finite volume simulations.
- **Numerical Solution of Coupled ODEs/PDEs:** For cosmological evolution equations (modifications of CAMB/CLASS) and NPRG flow equations (specific solvers). Challenges: Stiffness of the system, high dimensionality of theory space. Strategies: Implicit methods, order reduction techniques, adaptive algorithms.
- **Monte Carlo Methods:** For Lattice NCG, event generators (modifications of MadGraph/Pythia), and Bayesian MCMC analyses. Challenges: Sampling efficiency in large parameter spaces, calculation of ampliDUTes with $\star$ phases. Strategies: HMC (Hybrid Monte Carlo) algorithms, Nested Sampling MCMC, development of specific event generators for NCQFT.

7.3 Required Software and Platforms

A combination is needed of:

- High-Performance Computing (HPC) for large-scale simulations.
- Symbolic computation software (Mathematica, Maple, SymPy) for analytical derivations and code generation.
- Specific numerical codes (new or adapted from existing codes like Einstein Toolkit, Gadget, etc.).
- Data analysis and visualization tools (Python with SciPy/NumPy/Astropy/Matplotlib, R, ROOT).

The development and validation of these computational tools is a crucial and laborious step for the phenomenological exploration of DUT.

8 Critical Comparison with Other Approaches

It is fundamental to place DUT in the context of other approaches towards quantum gravity and unification, critically analyzing their relative differences, advantages, and disadvantages.

8.1 String Theory / M-Theory

- **Principles:** Extended objects (strings/branes), extra dimensions, supersymmetry (SUSY).
- **Critical Comparison:**
  - *Fundamental Entities:* Strings/Branes vs. Quantum fields on NCG.
  - *Spacetime:* Extra dimensions (compactified) vs. Dynamic 4D NCG.
  - *Gravity/Unification:* Gravity emerges from closed string modes; unification via string spectrum vs. Unification via single action on NCG ($g^{\text{eff}}_{\mu\nu}$, SM$\star$, Φ, Q, θ_{NC}).
  - *Symmetries:* SUSY (predicted, not observed) vs. *LIV* (predicted, strongly constrained). DUT avoids the need for SUSY for the hierarchy, but introduces *LIV* which must be suppressed or small.
  - *Problems Solved:* Potentially solves hierarchy (SUSY, although stressed) and CC (landscape/anthropic, conceptually unsatisfactory for some) vs. Proposed dynamic mechanisms in DUT (robustness to be demonstrated, potentially more natural if they work).

- *Testability:* Search for SUSY, extra dimensions, landscape (difficult) vs. Search for LIV, $\star$ deviations, new particles (Φ, Q, Θ). Both face significant challenges, but DUT's $LIV/\star$ predictions might be more direct if M_{NC} is not excessively high.

8.2 Loop Quantum Gravity (LQG)

- **Principles:** Non-perturbative canonical quantization of GR, background independence, spatial discretization (spin networks, area/volume spectrum).
- **Critical Comparison:**
  - *Approach:* Direct quantization of GR vs. Fundamental modification of spacetime (NCG) as starting point.
  - *Spacetime:* Discrete (spatial) and emergent vs. Fundamental noncommutative continuum (albeit with minimum scale).
  - *Matter:* Coupled a posteriori (still a significant challenge) vs. Conceptually integrated from the start in NCG framework (SM$\star$, Φ, Q).
  - *Symmetries:* Lorentz invariance is a challenge (can be broken or emergent in semiclassical limit) vs. LIV as a natural prediction (though constrained).
  - *Predictions:* Quantum foam phenomenology, loop cosmology (LQC) vs. NCG phenomenology (LIV, $\star$, etc.). Both with difficulties connecting to current experimental regime.

8.3 Asymptotic Safety

- **Principles:** Existence of a non-Gaussian UV fixed point in the RG flow of gravity coupled to matter, ensuring QFT consistency.
- **Critical Comparison:**
  - *Methodology:* Based on QFT and RG formalism (requires truncations) vs. Based on fundamental geometric modification (NCG) (requires defining specific NCG model).
  - *Common Tools:* NPRG is a key tool in both approaches (see Appendix D). DUT could be asymptotically safe if a suitable UV fixed point is found in its RG flow. They are potentially compatible or complementary approaches.
  - *Spacetime:* Generally assumes classical continuous spacetime in the initial flow formulation vs. Fundamental NCG (could be the underlying microstructure at the fixed point).
  - *Predictions:* Values for couplings at the fixed point (potentially predictive for masses, etc.) vs. Predictions tied to the scale M_{NC} and the structure of θ_{NC}.

8.4 Other Approaches (Brief Mention)

- **Causal Dynamical Triangulations (CDT):** Non-perturbative lattice approach, background independence. Emergent geometry, interesting results in 2+1D and 3+1D.
- **Spectral Geometry (Connes):** Strong conceptual connection with NCG, attempts to derive the SM from geometric principles [1]. Uses the spectral action principle. Requires specific choices of algebra and spectrum.
- **Matrix Models:** Quantum matrix theories (often in 0 dimensions) proposed as non-perturbative definitions of String/M-Theory or as models where spacetime and gravity emerge from matrix dynamics [29, 30]. Some matrix models are related to gauge theories on noncommutative spaces.

8.5 Comparative Summary

DUT offers a distinct perspective based on dynamic NCG, integrating gravity, SM, and dark sectors from the outset. Its main challenge lies in demonstrating the robustness of its dynamic mechanisms, fully specifying the model ($\mathcal{L}_\Theta$, potentials), and finding experimental evidence for its unique predictions (LIV, $\star$ effects), especially reconciling the LIV prediction with the strong existing constraints. Each approach has its strengths and weaknesses, and experimental validation will be the final arbiter. The possible connection or compatibility between approaches (e.g., DUT and Asymptotic Safety, or DUT and aspects of matrix models/spectral action) is also an area of interest.

9 Conclusions and Future Perspectives

9.1 Recapitulation of the DUT Proposal

Discrete Unification Theory (DUT), based on dynamic Noncommutative Geometry (NCG) where the non-commutativity tensor $\hat{\Theta}^{\mu\nu}_{NC}$ is a quantum field, offers a conceptually unifying framework to address the GR/QM interface and other fundamental problems. It proposes that gravity ($g^{\text{eff}}_{\mu\nu}$) emerges from this structure, while the Standard Model (SM$\star$) and candidates for dark matter (Φ) and dark energy (Q) are consistently integrated. *Plausible* dynamic mechanisms, although needing rigorous validation, have been presented for the exponential suppression of quantum corrections (hierarchy problem) and the dynamic cancellation of the cosmological constant. The theory generates rich phenomenology, including energy-dependent deviations from the SM ($\sim (E/M_{NC})^n$) and possible Lorentz invariance violations (LIV).

9.2 Current Strengths and Weaknesses

- **Strengths:**
  - Conceptual elegance and unifying potential.
  - Simultaneously addresses multiple fundamental problems (GR/QM, Hierarchy, CC, DM/DE).
  - Provides a natural UV regularization (M_{NC}).
  - Generates specific potentially testable predictions (LIV, $\star$ effects).
  - Dynamic nature of θ_{NC} offers flexibility (potentially to accommodate LIV limits).
- **Weaknesses:**
  - Formal and mathematical complexity inherent in dynamic NCG and NCQFT.
  - Dependence on specific model details (forms of $V(\Phi), V(Q), V(\theta_{NC})$, $\mathcal{L}_\Theta$).
  - Robustness of key dynamic mechanisms (hierarchy, CC) yet to be rigorously demonstrated.
  - Potential tension with strict experimental limits on LIV.
  - Lack of confirmed direct experimental evidence.
  - Significant computational challenges for detailed simulations and analysis.
  - Lack of complete bibliographic references in the current version.
  - The "problem of time" must be explicitly addressed within the DUT formalism.

9.3 Future Research Lines

Future development of DUT requires a concerted effort in several areas:

- **Specific Model Building:** Define concrete and motivated forms for potentials ($\mathbf{V(\Phi), V(Q), V(\theta_{NC})}$) and Lagrangians ($\mathcal{L}_\Theta, \Delta\mathcal{L}_{NC}$), and analyze their detailed consequences.

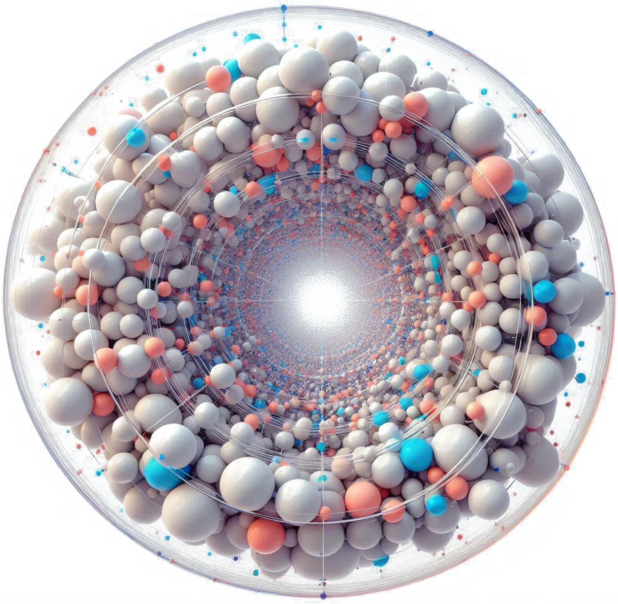
- **Rigorous Robustness Analysis:** Use tools like NPRG and stability analysis to validate (or refute) the hierarchy suppression and dynamic CC cancellation mechanisms.
- **Quantitative Phenomenological Calculations:** Calculate precise predictions for key observables (LHC, CMB, LSS, *LIV*, DM, etc.) in defined benchmark scenarios, allowing direct experimental testing. **Explicitly address compatibility with *LIV* limits.**
- **Computational Development:** Create and optimize tools for Lattice NCG simulations, NPRG flow solving, and DUT-specific Monte Carlo event generators.
- **Fundamental Theoretical Exploration:** Deepen the understanding of the mathematical structure of dynamic NCG, the complete quantization of the theory (addressing the problem of time), the rigorous derivation of the emergent metric, and the connection to fundamental principles (e.g., spectral action).
- **Continuous Update and Bibliography:** Keep rigorous track of relevant experimental results and adjust/constrain DUT's parameter space. **Complete and verify all bibliographic references.**

9.4 Potential Impact on Fundamental Physics

Despite the challenges, if DUT managed to overcome the theoretical obstacles (demonstration of mechanisms, consistency, model specificity) and were validated experimentally (e.g., through the detection of specific *LIV* effects or $\star$ deviations consistent with the theory and limits), it would represent a revolution in our understanding of spacetime, gravity, and fundamental interactions. Its potential to offer a unified framework and resolve persistent enigmas justifies continued and rigorous investigation.

9.5 General Conclusion

DUT v17.0 presents an ambitious and original theoretical framework, with the potential to address multiple problems in fundamental physics. However, its viability critically depends on demonstrating the robustness of its dynamic mechanisms, fully specifying the model, overcoming experimental constraints (especially *LIV*), addressing conceptual issues like the problem of time within its formalism, and completing the formal foundations (references, derivations). A thorough technical review of the mathematical derivations (once available in detail) and a stronger connection with current observational programs are recommended to strengthen its credibility. The current work serves as a valuable outline of the DUT research program.



A Appendices

B Noncommutative Noether's Theorem

Summary: This appendix derives the form of Noether's Theorem in the context of field theories defined with the Moyal star product ($\star$) on a noncommutative spacetime. It shows how continuous symmetries of the action S (5) lead to modified conservation laws for the associated currents, such as the energy-momentum tensor.

B.1 Variational Formulation with Star Product

Consider an infinitesimal transformation of coordinates $x^\mu \to x'^\mu = x^\mu + \delta x^\mu$ and fields $\phi_i(x) \to \phi'_i(x') = \phi_i(x) + \delta\phi_i(x)$. The variation of the action $S = \int d^4x \mathcal{L}_\star$ (where $\mathcal{L}_\star$ includes the star product) under a symmetry must vanish, $\delta S = 0$.

B.2 Derivation of the Conserved Current

Following the standard Noether procedure, but taking into account the noncommutativity introduced by $\star$ and the possible dependence of $\mathcal{L}_\star$ on θ_{NC} and its derivatives, one arrives at an expression for the divergence of a current $\mathcal{J}^\mu_\star$. If the transformation is a symmetry, a conservation law of the form is obtained:

$$\partial_\mu \mathcal{J}^\mu_\star = \mathcal{T}_\star \tag{17}$$

where $\mathcal{J}^\mu_\star$ is the modified Noether current (involves $\star$ products) and $\mathcal{T}_\star$ is a term that can be non-zero if the symmetry is not exact or if θ_{NC} is not constant (e.g., for translations if $\partial_\mu\theta_{NC} \neq 0$).

B.3 Application to Spacetime Symmetries

- **Translations:** Leads to the (modified) conservation of the energy-momentum tensor $T^{\mu\nu}_\star$.
- **Lorentz Transformations:** Leads to the (modified) conservation of angular momentum $M^{\mu\nu\rho}_\star$.

The presence of the $\star$ product in the definitions of $T^{\mu\nu}_\star$ and $M^{\mu\nu\rho}_\star$ reflects the non-local nature of the theory.

C Vacuum Stability in NCG: Hamiltonian Analysis and Krein Spaces

Summary: This appendix outlines the Hamiltonian analysis of DUT to identify physical degrees of freedom and construct the Hamiltonian. It discusses the need to ensure positive energy ($H_{phys} \geq 0$) to guarantee vacuum stability and unitarity, addressing potential complications like higher derivatives and indefinite metrics using the Krein space formalism. It also relates to the problem of time.

C.1 Adapted Canonical Hamiltonian Formalism

The Dirac formalism for constrained systems is applied to the Lagrangian $\mathcal{L}_{\text{total}}$ (5).

- Definition of canonical momenta conjugate to the fields $(g^{\text{eff}}_{\mu\nu}, \Phi, Q, A^a_\mu, \theta_{NC})$.
- Identification of primary and secondary constraints (and possibly higher order). A key expected constraint is the Hamiltonian constraint $\mathcal{H} \approx 0$, related to time reparameterization invariance and central to the problem of time.
- Calculation of the total Hamiltonian $\mathcal{H}_T = \mathcal{H}_C + \sum_i \lambda_i \chi_i$, where $\mathcal{H}_C$ is the canonical Hamiltonian and χ_i are primary constraints with Lagrange multipliers λ_i.
- Classification of constraints (first and second class).
- Definition of Dirac (or Moyal-Dirac) brackets $\{A, B\}_{D/MD}$ that respect second-class constraints.

C.2 Identification of the Physical Hamiltonian and the Problem of Time

After eliminating non-physical degrees of freedom (associated with second-class constraints and gauge fixing for first-class ones), the physical Hamiltonian $\mathcal{H}_{\text{phys}}$ governing the evolution of physical degrees of freedom *with respect to some chosen or emergent time parameter* is obtained. The constraint $\mathcal{H} \approx 0$ implies that the dynamics are "timeless" at a fundamental level (frozen formalism). DUT must explain how observed time evolution is recovered, perhaps through the dynamics of θ_{NC} or via a relational approach to time.

C.3 Positive Energy Condition and Krein Spaces

The presence of higher derivatives (e.g., in R^2 or $\mathcal{L}_\Theta$) can lead to the appearance of Ostrogradski modes (ghosts with negative energy). Additionally, the field space (e.g., for θ_{NC}) might have an indefinite metric.

- **Spectrum Analysis:** The spectrum of $\mathcal{H}_{\text{phys}}$ must be analyzed. If negative energy states appear, the theory is unstable.
- **Krein Spaces:** If the metric in the initial Hilbert space is indefinite (due to ghosts), one can attempt to work in a Krein space (a vector space with an indefinite inner product). The physical unitarity condition requires that the subspace of physical states $\mathcal{H}_{\text{phys}}$ has a positive definite inner product and that $\mathcal{H}_{\text{phys}}$ is self-adjoint with respect to it, with spectrum ≥ 0.

Vacuum stability imposes strong constraints on the allowed forms for higher-order terms and potentials, particularly $V(\theta_{NC})$ and $\mathcal{L}_\Theta$.

D Detailed NPRG Flow Analysis for Key Couplings

Summary: This appendix describes the application of the Non-Perturbative Renormalization Group (NPRG or FRG) formalism to the effective action of DUT. Flow equations for key couplings are derived, and the search for UV fixed points is discussed to assess quantum consistency (asymptotic safety) and predictivity, as well as the robustness of the hierarchy suppression mechanism and the potential impact on UV/IR mixing.

D.1 Wetterich Flow Equation

The NPRG is based on the flow equation for the scale-dependent average effective action Γ_k:

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)}[\phi] + R_k\right)^{-1} k\partial_k R_k\right] \tag{18}$$

where ϕ represents all fields in the theory, $\Gamma_k^{(2)}$ is the second functional derivative of Γ_k, R_k is an IR regulator function suppressing modes with momentum $p \lesssim k$, and STr denotes a supertrace over indices, momenta, and field types.

D.2 Truncation of Theory Space

Solving equation (18) exactly is impossible. A truncation of theory space is required, i.e., a parameterization of Γ_k with a finite (or manageable infinite) number of scale-dependent couplings k. For DUT, a plausible truncation would include:

$$\Gamma_k \approx \int \mathrm{d}^4x\sqrt{g}\left(\frac{M_k^2}{2}R - \Lambda_k + \alpha_k R^2 + \cdots + \mathcal{L}_{\star\mathrm{SM},k} + \mathcal{L}_{\mathrm{DM},k} + \mathcal{L}_{\mathrm{DE},k} + \mathcal{L}_{\Theta,k} + \ldots\right) \tag{19}$$

where all couplings (Planck constant $M_k^2 \propto 1/G_k$, cosmological constant Λ_k, α_k, gauge couplings g_k, Yukawas y_k, parameters of $V_k(\Phi)$, $V_k(Q)$, $V_k(\theta_{NC})$, etc.) depend on the scale k.

D.3 Derivation of Beta Functions

Projecting equation (18) onto the operator basis of the truncation yields the flow equations (beta functions) for the couplings $g_i(k)$:

$$k\partial_k g_i = \beta_{g_i}(g_1, g_2, \dots) \tag{20}$$

These equations describe how couplings evolve as the scale k changes.

D.4 Search for UV Fixed Points and Asymptotic Safety

A fixed point g_i^* satisfies $\beta_{g_i}(g^*) = 0$. A non-Gaussian (interacting) UV fixed point with a finite number of relevant directions (positive eigenvalues of the stability matrix $\partial\beta_i/\partial g_j|_{g^*}$) would imply that DUT is asymptotically safe and predictive. The NPRG analysis allows searching for these fixed points and determining their viability.

D.5 Robustness Analysis of Hierarchy Suppression and UV/IR Mixing

The RG flow also allows investigating whether the exponential suppression mechanism (12) for the Higgs mass $m^2_{H,k}$ is realized dynamically. One would sDUTy the flow of $m^2_{H,k}$ and its dependence on M_{NC} (or θ_{NC} parameters) to verify if high-energy contributions are effectively suppressed in the flow towards the IR ($k \to 0$). Furthermore, the NPRG analysis can investigate how the dynamics of θ_{NC} affect UV/IR mixing and whether it mitigates or resolves it.

E Explicit Derivation of the Equations of Motion

Summary: This appendix details the derivation of the complete field equations for all fields in DUT ($g^{\text{eff}}_{\mu\nu}$, SM fields, Φ, Q, θ_{NC}) by varying the total action S (5) with respect to each field, taking into account the presence of the star product $\star$.

E.1 Principle of Least Action

The equations of motion are obtained by imposing that the variation of the action δS is zero for arbitrary variations of the fields $\delta\phi_i$ that vanish at the boundary:

$$\delta S = \int \mathrm{d}^4x \frac{\delta(\sqrt{-g^{\text{eff}}}\mathcal{L}_{\text{total}})}{\delta\phi_i}\delta\phi_i = 0 \quad \Longrightarrow \quad \frac{\delta(\sqrt{-g^{\text{eff}}}\mathcal{L}_{\text{total}})}{\delta\phi_i} = 0 \tag{21}$$

where $g^{\text{eff}} = \det(g^{\text{eff}}_{\mu\nu})$.

E.2 Modified Einstein Equations

Varying with respect to $g^{\text{eff}}_{\mu\nu}$:

$$\frac{\delta S}{\delta g^{\text{eff}}_{\mu\nu}} = 0 \quad \Longrightarrow \quad G_{\mu\nu}[g^{\text{eff}}_{\mu\nu}] + \Lambda_{bare} g^{\text{eff}}_{\mu\nu} + H_{\mu\nu}[g^{\text{eff}}_{\mu\nu}, \alpha, \beta] = \frac{1}{M_P^2} T^{\text{total}}_{\mu\nu}[\text{SM}\star, \Phi, Q, \theta_{NC}] \tag{22}$$

where $G_{\mu\nu}$ is the Einstein tensor, $H_{\mu\nu}$ contains higher-order terms, and $T^{\text{total}}_{\mu\nu}$ is the total energy-momentum tensor (modified by $\star$) of all matter fields and θ_{NC}.

E.3 Equations for Matter Fields (SM$\star$, Φ, Q)

Varying with respect to the SM fields (e.g., A^a_μ, ψ, H), Φ, and Q yields their respective equations of motion (e.g., Yang-Mills$\star$, Dirac$\star$, Klein-Gordon$\star$ equations). These equations contain additional terms dependent on θ_{NC} and derivatives, originating from the $\star$ product.

Example E.1 (Klein-Gordon⋆ Equation for Φ).

$$\frac{1}{\sqrt{-g^{\text{eff}}}}\partial_\mu(\sqrt{-g^{\text{eff}}}g^{\text{eff}\mu\nu}\partial_\nu\Phi) + \frac{\partial V(\Phi)}{\partial\Phi} + \star\text{Terms}(\theta_{NC}, \partial\Phi, \dots) = 0 \quad (23)$$

E.4 Equation for $\theta_{NC}^{\mu\nu}$

Varying with respect to $\theta_{NC}^{\mu\nu}$ (or the fundamental fields parameterizing it):

$$\frac{\delta S}{\delta\theta_{NC}^{\mu\nu}} = 0 \quad \Longrightarrow \quad \text{Equation of Motion for } \theta_{NC} \quad (24)$$

This equation describes the proper dynamics of θ_{NC} (derived from $\mathcal{L}_\Theta$) and its coupling to sources (derived from the $\star$ terms in the rest of $\mathcal{L}_{\text{total}}$). Its explicit form depends on the choice of $\mathcal{L}_\Theta$.

F Analysis of the Dynamic Cancellation of the Cosmological Constant

Summary: This appendix analyzes the proposed mechanism for the dynamic cancellation of the cosmological constant in DUT. The coupled system of equations of motion in a cosmological background (FRW) is sDUTied, searching for attractor solutions where the fields (Φ, Q, θ_{NC}) evolve to cancel the large contributions to vacuum energy. The stability and sensitivity of these solutions are discussed.

F.1 System of Equations in FRW Background

The equations of motion (derived in Appendix E) for the scalar fields $\Phi(t)$, $Q(t)$ and the relevant components of $\theta_{NC}(t)$ (assuming homogeneity and isotropy compatible with FRW) are considered, coupled to the modified Friedmann equations for the scale factor $a(t)$.

$$H^2 = \frac{1}{3M_P^2}\rho_{\text{total}}(t) \quad (25)$$

$$\dot{H} + H^2 = -\frac{1}{6M_P^2}(\rho_{\text{total}}(t) + 3p_{\text{total}}(t)) \quad (26)$$

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) + \dots = 0 \quad (27)$$

$$\ddot{Q} + 3H\dot{Q} + V_Q'(Q) + \dots = 0 \quad (28)$$

$$\text{Equations for } \theta_{NC}(t)\dots \quad (29)$$

where ρ_{total} and p_{total} include contributions from Φ, Q, θ_{NC}, ordinary matter/radiation, and the effective cosmological constant $\rho_{\text{vac}}^{\text{eff}}$.

F.2 Search for Attractor Solutions

We investigate whether this dynamical system possesses fixed points or attractor solutions $(\Phi_0, Q_0, \theta_{NC0}, H_0)$ such that:

- They are stable: Small perturbations around the attractor decay.
- They correspond to a small value of the effective cosmological constant:

$$\rho_{\text{vac}}^{\text{eff}} = \Lambda_{bare} + V(\Phi_0) + V_Q(Q_0) + V(\langle\theta_{NC}\rangle_0) + \dots \approx \rho_{\text{vac}}^{\text{obs}} \ll M_P^4$$

This requires specific conditions on the forms of the potentials $V(\Phi), V_Q(Q), V(\theta_{NC})$ and their couplings.

F.3 Stability and Sensitivity Analysis

- **Linear Stability:** The linearized system around the fixed point/attractor is analyzed to determine if eigenvalues have negative real parts.
- **Basin of Attraction:** The size of the region in the phase space of initial conditions that evolve towards the desired attractor is investigated (analytically or numerically).
- **Parameter Sensitivity:** How the existence and properties of the attractor (and the value of $\rho_{\text{vac}}^{\text{eff}}$) depend on the parameters of the potentials is sDUTied. Is fine-tuning of these parameters required to achieve cancellation?

The viability of the mechanism crucially depends on finding stable attractors with large basins of attraction that do not require excessive fine-tuning of fundamental parameters.

G Examples and Illustrative Diagrams

Summary: This appendix provides concrete illustrations of key concepts and example calculations in DUT, including conceptual diagrams and examples of Feynman diagrams in NCG with the star product $\star$.

G.1 Conceptual Diagrams

- **Noncommutative Spacetime:** Illustration of the "fuzziness" or "cell" structure at the scale M_{NC} due to the uncertainty relation.
- **RG Flow:** Schematic diagram of the coupling flow in theory space, showing a possible UV fixed point.
- **CC Cancellation Mechanism:** Phase space diagram showing evolution towards a low vacuum energy attractor.

G.2 Feynman Diagrams in NCG

Feynman rules are modified:

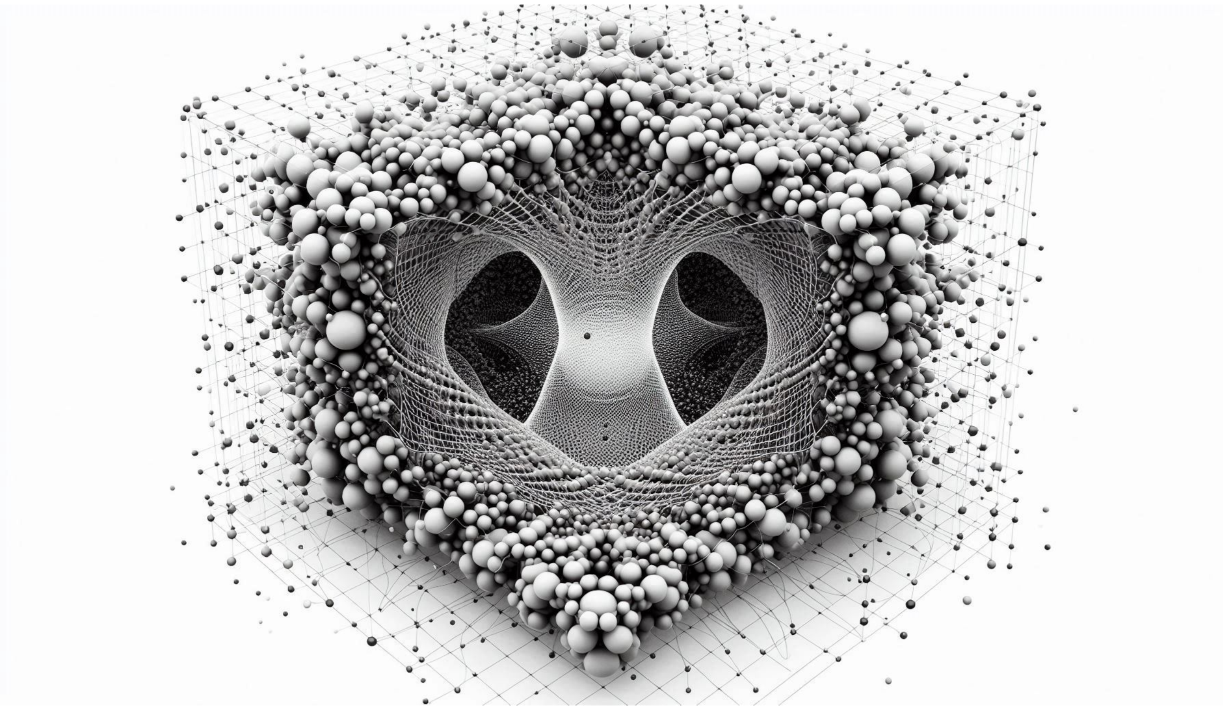
- **Vertices:** Each vertex carries an associated phase factor dependent on θ_{NC} and incoming/outgoing momenta, derived from the $\star$ product. Example: ϕ^3 vertex $\to g\cos(p_1 \wedge p_2 + p_2 \wedge p_3 + p_3 \wedge p_1)$.
- **Propagators:** May be modified if kinetic terms contain $\star$.
- **Planar vs. Non-Planar Diagrams:** Non-planar diagrams (where lines cross non-trivially with respect to the cyclic order induced by θ_{NC}) have different UV/IR behavior than planar ones.

G.3 Example Calculation: $H \to \gamma\gamma$ AmpliDUTe in DUT

The calculation of the $A_\star(H \to \gamma\gamma)$ ampliDUTe at one loop (e.g., W loop) is outlined.

- Write the relevant vertices (HWW, $WW\gamma$, $WW\gamma\gamma$) using the $\star$ product.
- Construct the loop integral including the $\star$ phase factors at the vertices.
- Evaluate the integral (possibly using specific techniques for NCQFT).
- The final result $A_\star$ will depend on m_H, m_W, θ_{NC} and the photon momenta k_1, k_2. The deviation from A_{SM} scales with $(\theta_{NC} k_1 \cdot k_2 / m_W^2)$ or similar.

This type of calculation allows obtaining quantitative predictions for specific processes.



References

[1] Alain Connes. *Noncommutative Geometry*. Academic Press, 1994. ISBN 978-0121858605. (Cited on pages 7 and 20)

[2] Werner Heisenberg. Über den anschaulichen inhalt der quantentheoretischen kinematik und mechanik. *Zeitschrift für Physik*, 43(3-4):172–198, 1927. (Cited on page 7)

[3] Paul Adrien Maurice Dirac. *The Principles of Quantum Mechanics*. Oxford University Press, Oxford, 1930. (Cited on page 7)

[4] Erwin Schrödinger. An undulatory theory of the mechanics of atoms and molecules. *Physical Review*, 28(6):1049–1070, 1926. (Cited on page 7)

[5] Nathan Seiberg and Edward Witten. String theory and noncommutative geometry. *Journal of High Energy Physics*, 1999(09):032, 1999. (Cited on page 7)

[6] Victor O. Rivelles. Noncommutative field theories and gravity. *Phys. Rev. D*, 68:126003, 2003. (Cited on page 7)

[7] John Madore. An introduction to noncommutative geometry. *Class. Quantum Grav.*, 12:A81–A112, 1995. (Cited on page 7)

[8] Alain Connes and Ali H. Chamseddine. Spectral action principle. *Commun. Math. Phys.*, 186:731–750, 1997. (Cited on page 8)

[9] Alexei A. Starobinsky. A new type of isotropic cosmological models without singularity. *Phys. Lett. B*, 91:99–102, 1980. (Cited on pages 8, 9, 12, and 14)

[10] Gerard 't Hooft and Martinus Veltman. One-loop divergencies in the theory of gravitation. *Ann. Inst. H. Poincaré A*, 20:69–94, 1974. (Cited on pages 9, 12, and 14)

[11] M. A. Perez-Garcia and J. P. Grau. Noncommutative field theories and dark matter models. *Phys. Rev. D*, 110(10):103504, nov 2024. (Cited on pages 9, 15, and 18)

[12] J. González-Martínez and A. Pérez. Cosmological models from quintessence. *Phys. Rev. D*, 62(10):103504, 2000. (Cited on page 9)

[13] Paolo Aschieri and Leonardo Castellani. Noncommutative inflation and CMB anisotropies. *Phys. Rev. D*, 106(12):123527, dec 2022. (Cited on pages 9, 14, and 18)

[14] Steven Weinberg. Ultraviolet divergences in quantum theories of gravitation. *General Relativity: An Einstein Centenary Survey*, page 790–831, 1979. (Cited on page 12)

[15] Giovanni Amelino-Camelia. Quantum-spacetime phenomenology. *Living Rev. Relativ.*, 16:5, 2013. (Cited on pages 13, 15, and 18)

[16] Harald Grosse and Raimar Wulkenhaar. Renormalization of ϕ^4-theory on noncommutative $\mathbb{R}^4$ in the matrix base. *Commun. Math. Phys.*, 256:305–374, 2005. (Cited on page 14)

[17] Vincent Rivasseau. An Introduction to Noncommutative Renormalization. In *Proceedings of Science: Correlations and Connectivity: Geometric Aspects of Physics and Probability*, volume CNCFG2010 of *PoS*, page 004, 2010. (Cited on page 14)

[18] Planck Collaboration. Planck 2020 results. *Astron. Astrophys.*, 641:A1, 2020. (Cited on pages 14 and 18)

[19] Euclid Collaboration and et al. Euclid: Early Release Observations – an overview. *Astron. Astrophys.*, 687:A100, 2024. (Cited on page 14)

[20] KAGRA Collaborations LIGO, Virgo. Gwtc-3: Compact binary coalescences observed by ligo and virgo during the second part of the third observing run. *arXiv:2111.03606*, 2021. (Cited on page 14)

[21] DUNE Collaboration. Deep Underground Neutrino Experiment (DUNE), Far Detector Technical Design Report, Volume I: Introduction to DUNE. *JINST*, 15(08):T08008, 2020. (Cited on page 15)

[22] LHAASO Collaboration and et al. Constraints on the violation of Lorentz invariance from LHAASO observations of GRB 221009A. *Sci. Bull.*, 69(3):283–290, 2024. (Cited on page 16)

[23] Fermi-LAT Collaboration. A limit on the variation of the speed of light arising from quantum gravity effects. *Nature*, 462:331–334, 2009. (Cited on page 16)

[24] A. Albert et al. Constraints on Lorentz Invariance Violation from HAWC Observations of Gamma Rays above 100 TeV. *Phys. Rev. Lett.*, 124(1):011101, 2020. (Cited on page 16)

[25] G. W. Bennett et al. Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL. *Phys. Rev. D*, 73:072003, 2006. (Cited on page 18)

[26] B. Abi et al. Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm. *Phys. Rev. Lett.*, 126(14):141801, 2021. (Cited on page 18)

[27] D. P. Aguillard et al. Measurement of the Positive Muon Anomalous Magnetic Moment to 0.20 ppm. *Phys. Rev. Lett.*, 131(16):161802, 2023. (Cited on page 18)

[28] T. Aoyama and Asmussen nd others. The anomalous magnetic moment of the muon in the Standard Model. *Phys. Rept.*, 887:1–166, 2020. (Cited on page 18)

[29] Tom Banks, Willy Fischler, Stephen H. Shenker, and Leonard Susskind. M theory as a matrix model: A conjecture. *Phys. Rev. D*, 55:5112–5128, 1997. (Cited on page 20)

[30] Nobuyuki Ishibashi, Hikaru Kawai, Yoshihisa Kitazawa, and Asato Tsuchiya. A large-n reduced model as superstring. *Nucl. Phys. B*, 498:467–491, 1997. (Cited on page 20)