Survey of Flocking Algorithms in Multi-agent Systems

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Abstract

Flocking behaviour in Multi-agent Systems (MAS) has attracted tremendous attention amongst researchers in the recent past due to its potential applications in various fields where distributed work environment is desired. The flocking algorithms have the potential to introduce self-organizing, self-healing and self-configuring capabilities in the functioning of a distributed system. The flocking algorithms exploit various artificial intelligence techniques, mathematical potential functions and geometric approaches to realize the global objectives by controlling local parameters. The main parameters of characterization of any flocking algorithm consist of mathematical models of agents, their hierarchical or flat control structures and the control approach by which these agents are controlled to exhibit flocking behaviour along with any type of formational constraints. A rigorous survey study of flocking algorithms for agents in MAS in the perspective of various instances of agents shows that there lies a huge scope for the researchers to apply, experiment and analyse various techniques locally to achieve global objectives. This paper surveys the flocking algorithms in perspective of these parameters.

Keywords: *Flocking Algorithms, Multi-agent systems, Formation control, Leader-follower flocking, Wireless Sensor Networks, Multi-robot systems*

1. Introduction

An agent is any living or non-living, virtual or physical computational quantity which demonstrates autonomous behaviour and which is reactive, proactive and has social ability to communicate. Hence the examples of agents include living organisms, robots, sensors, autonomous vehicles and even software programs which have above mentioned properties. Such multiple homogeneous/ heterogeneous agents together form a multi-agent system which can build distributed complex system to solve large scale problems that are distributed in nature.

Formally a Multi-agent System can be defined as a collection of a number of agents that

1) interact through communication,
2) act/react in an environment
3) have different "spheres of influence" which may coincide, and
4) are linked by organizational relationships. [1]

Multi-agent system approach of problem solving offers several advantages to a system that is distributed in nature such as flexibility, robustness, reliability, efficiency and speed, maintainability, reusability and reduced cost.
Flocking is one of the behavioural properties of multi-agent systems. Flocking is a form of collective behaviour of large number of interacting agents with a common group objective [2]. Reynolds introduced three heuristic rules that led to the creation of the first computer animation of flocking, which are termed as *flocking rules of Reynolds* [3].

The important aspects of these flocking rules represent the *cohesion*, *separation* and *alignment* features of a flock, described as *Flock Centering, Obstacle Avoidance and Velocity Matching* respectively. Amongst the multiple agents in a flock, a *Flock Centering* rule attempts to let an agent stay close to the nearby flock mates. An *Obstacle Avoidance* rule will guide an agent to avoid collisions with nearby flock mates. The *Velocity Matching* rule will let the agents in the flock to match velocity with nearby flock mates. Figure 1 presents the *flocking rules* which collectively lead to flocking behaviour.

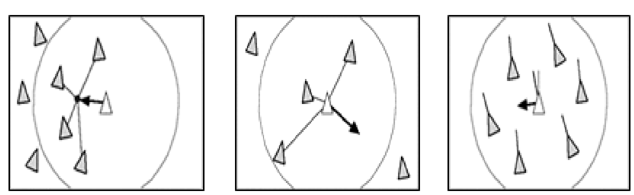

Figure 1: Flocking Rules

The flocking algorithms have the potential to introduce self-organizing, self-healing and self-configuring capabilities in the functioning of a distributed system. The flocking algorithms exploit various artificial intelligence techniques, potential functions and geometric approaches to realize the global objectives by controlling local parameters of a flock-member. These capabilities are governed by *cohesion, separation* and *alignment* features of a flock. Hence, in this paper we survey and study various techniques and approaches proposed and experimented to achieve the desired global optimum goal by handling important parameters locally in the flocking algorithms. In this paper, we present *mathematical models, control structures* and *control approaches* for agents that belong to MAS to design and implement flocking algorithms.

The paper is organized as follows. Section 2 gives an overview of flocking algorithms for multi-agent systems. Various mathematical models that are used for describing agent behaviour are explained in section 3. Section 4 explains various control structures for multi-agent systems exhibiting flocking behaviour. Section 5 gives overview the control approaches that control the flock so as to satisfy all the three flocking rules, and Section 6 concludes the paper.

2. Flocking Phenomenon in Multi-Agent Systems

In nature, flocking phenomenon can be found in many living organisms like ants, birds, fishes and even in crowds. Multi-robot systems, wireless mobile sensor networks, UVs are man-made artificial hardware multi-agent systems and software modules, data packets are software multi-agent systems.
In these systems each individual has specified limited capability within its local environment, however all of them collectively are governed by simple flocking rules. Hence they can be treated as a single entity to carry out distributed tasks.
The flocking rules imposed on a multi-agent system lead to *emergent collective behavior* which demonstrates perfectly *adaptive nature* of the system to achieve a common objective.
Inspired by these two potential strengths, flocking phenomenon has been researched widely to solve scalable and robust problems in distributed environments. These distributed fields include engineering, finance, psychology etc. In the viewpoint of multi-agent systems, applications of flocking algorithms for various instances of agents can be effectively demonstrated using robots, mobile sensor networks, unmanned vehicles and software agents.

2.1 Multi-Robot Systems

A swarm of simple robots offer many advantages in terms of efficiency and fault-tolerance. Multiple robots can be used for tasks where life of human may be in danger. The flocking of robots include *cooperative movement* so as to stay close to each other and maintain some desired formation while moving. Thus a robot flock can behave as a single entity to achieve common global objective by controlling each robot using set of local rules.
Flock of robots can perform effective works like area exploration, autonomous navigation for deployment, surveillance, search and rescue operation etc. Some desirable characteristics for robots in flocks of multi-agent system which can be deduced from these application areas are fault tolerance, adaptability, obstacle avoidance, low cost, small size etc.
In multi-robot systems, faults observed in the robots can be temporary or permanent. There is no recovery from the failure in the case of the permanent failure. In [4] and [5], the authors have studied and proposed fault-tolerant flocking algorithms for multi-robot system in asynchronous and semi-synchronous environments respectively with robots having permanent failures. A leader-follower control strategy is assumed in which follower robots follow a single leader robot which has some different set of rules than the follower robots. It is observed that while permanent failures are dealt in literature substantially, fault tolerance capability in crash-recovery mode has much scope of research work.
Adaptability is another important desirable characteristic for collective robotic systems working in continuously changing constrained environment. In [6], such adaptive flocking algorithm is proposed which uses regular triangle structure formation in which robots are constrained with no assignment of individual identification numbers and a pre-determined leader, no memory of previous actions, and no means of direct communications. It also demonstrates maneuverability of the flock.
Obstacle avoidance capability is studied in [7] for the task of area exploration using robot flock for finding a target in presence of obstacles using the concepts of utility-cost value of a region and bid value for a robot.

2.2 Wireless Mobile Sensors Networks

Wireless sensor Networks are ad hoc networks of small, cooperative sensor nodes (agents) which have computational, communication and mobility capabilities and highly constrained resources like memory and energy. These nodes may interact with each other in order to exchange information or to forward data extracted from the environment. Sensor networks provide low cost, low power, multifunctional capability to carry out tasks like sensing, data processing and communication.

Mobile wireless sensor networks potentially try to improve *Quality of service*. Flocking phenomenon can be used as an approach to overcome issues which arise while dealing higher QoS oriented wireless mobile sensor networks by treating the sensor nodes as agents. Some of the desired qualities to maintain high QoS are bandwidth optimization oriented efficient routing and congestion control protocols, self-organization and self-healing capabilities, connectivity maintenance and maximum area coverage.
Congestion in sensor network is major bottleneck while achieving necessary QoS. A flock-based congestion control (Flock-CC) approach is proposed in [8]. The main idea of the proposed Flock-CC model is to guide packets to form groups or flocks, and flow towards a global attractor, while avoiding obstacles such as congestion and failing nodes regions.
A flocking algorithm for self-organization and control of the backbone nodes in Hierarchical heterogeneous wireless sensor network (WSN) by gathering local information from end nodes is proposed in [9].
A mobile ad hoc network (MANET) is a special type of sensor network which consists of an autonomous collection of mobile user nodes that communicate over wireless links. Main challenge in controlling MANET is mobility of nodes. Recently a flocking approach to maintain network connectivity between user nodes in MANET by introducing special agent nodes is introduced in [10].These special agent nodes proactively relocate themselves so as to adapt with continuously changing network topology.
The bandwidth of the wireless medium in MANET is very limited and is shared by all nodes. Hence efficient routing is required. Flocking behaviour based routing algorithms routing in ad hoc network are studied in [11].In one of the algorithm studied in [11],like birds share information about velocity and direction, history of last encounters between nodes is maintained and is shared among network to have information about the current network topology which helps in efficient routing. Flocking algorithm based models for area coverage and self-deployment for multi-target tracking are proposed for sensor networks [12, 13].

2.3 Software Agents

Intelligent software agents are characterized as a software which can perform specific tasks on the behalf of user, can interact and adapt with the changing environment in beneficial manner, has mobility capability and have a degree of intelligence so as to perform some of its tasks autonomously.
Software agents can be software modules, data packets, program capsules etc. Flocking phenomenon facilitates grouping of these agents based on desired application specific similarity property of their associated objects.
Anomalies in data streams can be detected using scalable flocking algorithms based on concept of similarity in linked data objects [14]. A swarm intelligence based recommender system (FlockRecom) based on the collaborative behaviour of bird flocks for generating Top-N recommendations is studied in [15]. Each agent of the flock represents a user and based on similarity between agents, the updating of speed of individual agents is done to eventually form clusters.
Document clustering algorithm based on flocking approach is proposed in [16] which has each document object associated to each agent. Each agent in [16] is governed by the alignment, separation, cohesion and feature similarity and dissimilarity rules, to move in the virtual space to form a cluster. Time-varying data visualization using flocking strategy is presented in [17].

2.4 Unmanned Vehicles (UVs)

Unmanned ground, air, underwater vehicles can be regarded as class of multi-agent system with multiple objectives. The important desirable characteristics of the flock of UVs (agents) can be deduced from its potential applications like military surveillance, target tracking, reconnaissance, communication, remote sensing, and transport. These characteristics include maximum area coverage, safety, efficient navigation, communication capabilities etc.
Efficient navigation and communication is major necessity of an UV multi-agent system. A flocking algorithm approach is proposed in [18] in which the co-operation amongst UAV-UGV helps to detect a moving target. Use of flocking behavior induced in UAVs for maintaining of connectivity between mobile ground nodes for the uneven terrain where direct connectivity is not guaranteed, is demonstrated in [19].
Maximum area coverage, one of the necessary property is studied with dynamic and reliable coverage control algorithm with heterogeneous vehicles with vehicle-to-vehicle collision avoidance [20]. In [20], Control laws are developed for coordination and coverage vehicles for maintaining strong communication links by including a flocking component in them.
Safety issue related to obstacle avoidance capabilities of UVs is studied in [20, 21]. An intelligent multi-objective transportation system like cooperative autonomous driving algorithm taking into consideration safety measures is proposed in [21].

2.5 Others

Flocking algorithms are used for shape-constrained flock animation system in virtual environments [22]. Crowd behaviour can also be modelled using flocks [23]. Simulation flocking has been introduced in [24] as a method for generating simulation input i.e. input modelling.

3. Mathematical Models for Agents in Flocks of Multi-Agent Systems

Agents in flocks of multi-agent systems are governed by various mathematical models of agent/vehicle dynamics which contribute effectively in formation control, co-ordination aspects of multi-agent systems. These mathematical models explain a collective behavior of agents which emerges from simple flocking rules. In this section we will survey some of the mathematical models considered in the literature for implementation of flocking algorithms of multi-agent systems.

3.1 Self-propelled Particle Model

Self-propelled particles/ Vicsek model is a concept used to model swarm behaviour. In this model each particle functions individually as an autonomous agent. A swarm is modelled a collection of these particles that move with a constant speed adapt with the average direction of motion of the other particles in their local neighbourhood at each time increment [25].
Discrete-time model of 'n' autonomous agents all moving in the plane with the same speed but with different headings is proposed in [25]. At time $t=0$, particles are randomly distributed having same absolute velocity v and random direction θ.
x_i(t) is the position of the particle at time instant t. According to model, the velocities v_i of the particle i is determined at each time step and the position of the i particle is updated according to the rule

$$x_i(t+1) = x_i(t) + v_i(t) * \Delta t \qquad (1)$$

Here the velocity of a particle $v_i(\text{t+1})$ was constructed to have an absolute value v and direction given by the angle $\theta(\text{t+1})$.This angle was obtained from the expression (2).

$$\theta(t+1) = \{\theta(t)\}_r + \Delta\theta \qquad (2)$$

Where $\{\theta(t)\}_r$ denotes the average direction of the velocities of particles including particle i within a circle of radius r surrounding the given particle. The average direction was given by $\tan^{-1}$ [sin $\{\theta(t)\}_r$ / cos $\{\theta(t)\}_r$].
A theoretical explanation of the behaviour observed of model in [25] is provided in [26] along with a modified version consisting of one leader and n followers.

3.2 Single Integrator Model

Single integrator model for an agent in multi-agent systems is one of the simple models as it ignores the lower-level dynamics of the individual agents. Hence it is also called as Higher-Level model or kinematic model.
In single integrator model, each agent i has the following dynamics [27]:

$$\dot{q} = u_i \ , \ i \in N \qquad (3)$$

Where N is the set of all agents in the group, u_i is the control input vector of the agent i, and $\dot{q}_i$ is the change in position of agent. The dot represents the derivative with respect to time.
Single integrator model is used when model of the agent is not the main concern for the work. It is used in [27] as the main concern was to analyse the system for some other constraints (elliptical shape of agents) which are not related to dynamics of the agents.

3.3 Double Integrator Model

Double integrator model is used commonly for modelling of individual agent in multi-agent system for control and co-ordination. It is also called as Point Mass model. Single Integrator model is a special case of double integrator model. Equation (4) describes the dynamics of the model [2].

$$\begin{aligned} \dot{q} &= p_i \\ \dot{p} &= u_i \end{aligned} \qquad (4)$$

Where q_i is position and p_i is the velocity of agent i and u_i is acceleration input to the agent i.
This model is used more frequently in the literature as it is more relevant to engineering systems than any other model [28-30].

3.4 Non-holonomic Unicycle model

Non-holonomic unicycle model is another mathematical model used in literature for 2-D space. It is considered in [31, 32] which is given by (5).

$$\begin{aligned} x_i &= v\cos\theta_i \\ y_i &= v\sin\theta_i \\ \dot{\theta}_i &= u_i \\ \dot{v}_i &= a_i \end{aligned} \qquad (5)$$

Where v is velocity, a_i is acceleration, u_i is angular velocity of agent i and θ_i is the heading of agent. x_i and y_i represent the position of the agent.
Many mobile robots used for experimentation in the laboratories obey this model [33].

3.5 Dubins' Vehicle Model

Dubins' vehicle model is a special case of non-holonomic model where only first three equations from (5) are considered. The vehicle model used in [34] is given by

$$x_i = v\cos\theta_i$$
$$x_i = v\cos\theta_i$$
$$\dot{\theta}_i = u_i \qquad (6)$$

Where v is constant velocity, u_i is angular velocity of agent i and θ_i is the heading of agent i. Position of agent is represented by x_i and y_i.

It is useful in flocking algorithms in which each vehicle is required to move with a constant translational speed due to imposed special constraints [35, 36].

4. Control Structures for Agents in Flocks of Multi-Agent Systems

In flocking algorithms for multi agent systems, global and local knowledge distribution among agents play a vital role for controlling the flocking behaviour. Due to non-uniform knowledge distribution environment and the need of controlling the flocking phenomenon externally, various structures are proposed and studied in the literature.

Figure 2 provides various classes for control structures used for agents performing flocking phenomenon.

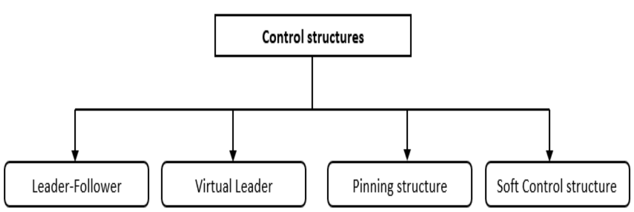

Figure 2: Control Structures

4.1 Leader-Follower Structure

Leader-follower control structure consists two kind of agents, leader and followers. Leader agent has some greater capability than followers and acts independently of them. Leader can be controlled externally. The follower can identify the leader from other followers. The followers are required to follow the leader wherever it goes, while maintaining the formation.

In a leader–follower flocking system, one/few agents is/are flock leaders who have global knowledge of a desired path, while others are followers who can communicate with neighbours but do not have the global knowledge.[37].

Multi-robot algorithms generally follow this type of structure with leader robot having knowledge of path or have better resources than follower robots [4, 5]. A flocking algorithm with multi-leader tracking for multi-agent systems is presented in [38].

4.2 Virtual Leader Structure

Virtual leader structure is first used in [2] for accommodating the effect of obstacles and collective objective. Virtual leader is not a physical agent present in the system but it is a moving reference point (virtual agent) that influences agents in its neighbourhood is used to guide the flock.

In [29] it is shown that even for minority of informed agents and varying velocity of virtual leader, the flocking is stable. An implementation of flocking control laws on wheeled mobile robots has been considered in [39]. For a multi-objective flocking scenario, multiple virtual leader environment is possible. Multiple virtual leader following in such an environment is studied in [40]. It should be mentioned that in pure virtual leader structure, all follower agents have the knowledge about the leader [2].

4.3 Pinning structure

In dynamic network, direct control of each agent is not necessary to have stable flock behaviour. Hence there is no need to have knowledge about the leader to all the agents. Only fraction of the agents may be pinned agents which have information.

These pinned agents are called informed agents in the literature [41]. Only these agents have navigational feedback term. It has been shown in [42] that even if some agents are informed about the leader, the flocking phenomenon can take place satisfactorily.

4.4 Soft Control structure

Soft control structure contains a special agent, which can be controlled externally. This special agent is treated as an ordinary agent by other agents in the flock [43].

Local rules of flock remain unaltered in this control structure. Soft control can be used as efficient intervene method for complex system. Use of a special agent called 'shill' to control self-propelled particle model [25] externally is presented in [43].

5. Formation Control of Agents in Flocks of Multi-Agent Systems

Formation of agents in flocks specifies a well-structured collection of number of agents with more deterministic dynamics. Hence formation control is one of the key considerations while designing a flocking algorithm.
Formation flocking is required in various practical applications of flocking algorithms like area coverage using multi-agents, formation flying, cooperative transportation, sensor networks deployment, collective robot movement as well as combat intelligence, surveillance, and reconnaissance. All these application include movement of flock with desired formation. Hence design of formation flocking algorithm requires to consider navigation component along with cohesion, separation and alignment components.
Formation control of agents in flocks of multi-agent systems includes tasks like formation acquisition, formation maintenance, formation stabilization, formation reconfiguration if needed. The formations can either be regular or irregular geometric shapes like line, circle, polygons and voronoi patterns or of lattice type. Characteristic requirements of individual flocking algorithm can impose some special constraints on the formation of flock. These constraints include control structure, shape, size, synchronization, fault tolerance and communication capabilities and sensor range of the agent and nature of the path being tracked.
Synchronization and fault tolerance issues in flocks are considered in multi-robot environment in [4, 5]. A regular polygon structure formation flocking in presence of faulty robots while considering asynchronous model for robots is considered in [4].Work in [4] is extended in [5] so as to have rotation in formation. A behavioural approach is introduced in [44] which considers formations like line, column, and diamond wedge.
In flocking behaviour, geometric formation is achieved by group of agents either with or without flock reference. Formation control without group reference is termed as formation producing while formation tracking in literature while formation control with a group reference is termed as formation tracking [46]. Group reference represents collective objective of the flock. Algorithms following virtual leader structure such as [29] use artificial potential function approach to track the desired trajectory while avoiding inter agent collision. Virtual leader in [29] acts as a group reference for formation tracking. Flocking of a group of agents is investigated in under fixed and switching interaction graphs, where a group of agents moves cohesively without group reference is studied in [28,47].
Geometric approach can be used to control of a flock in which local interactions are done by observing the position of the neighbouring agents so as to maintain a uniform distance, shape and adapt the heading. In [45], Voronoi partitions are used to assign navigation function to achieve desirable geographical distribution of the vehicles.

6. Conclusion

Flocking algorithms in MASs have the potential to introduce self-organizing, self-healing and self-configuring capabilities in the functioning of decentralized and distributed systems. They incorporate various artificial intelligence techniques, mathematical potential functions and geometric approaches to realize the global objectives by controlling local parameters.
Flocking algorithms comprise of crucial parameters that represent a *model of an agent*, *structure of a flock* that define *the knowledge distribution of the flock* and *the control methods* by which a flock is controlled. In the view of various instances of agents in MAS, they are employed to achieve various objectives like improvement in QoS, coverage maximization, reliability, reachability, fault-tolerance, energy conservation, etc. With the tight constraints on the communication bandwidth and energy resources, the processes of autonomous connectivity, network/agent addressing and routing structures based on the formation structures are the critical parameters for realization of MAS.

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