Numerical Methods for Hyperbolic PDEs
Homework 7

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Exercise 1

See extra pdf.

Exercise 2

Since this is simply the linear transport equation, the exact solution is

$$u(t) = u_0(x-t) = \begin{cases} -1, & x < t \\ 1, & x > 0 \end{cases}$$

(a)

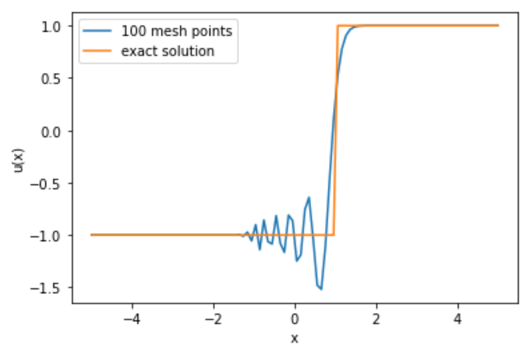

Figure 1: Numerical solutions with the the Lax-Wendroff scheme at time $t = 1$

*Immatriculation Nr. 23-942-030
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We observe that Lax-Wendroff has oscillations before the shock. Beam-Warming has slightly less problems with oscillations and they occur after the shock.

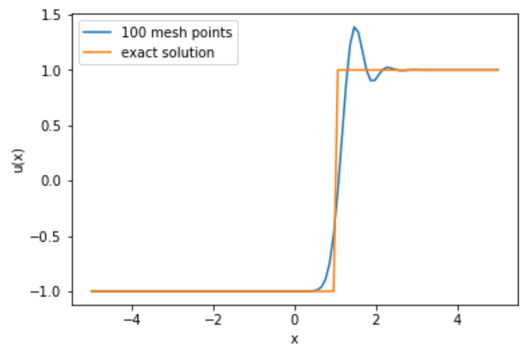

Figure 2: Numerical solutions with the the Beam-Warming scheme at time $t = 1$

(b)

Note that we use the explicit Euler rule to approximate the ODE, resulting in the standard first-order monotone finite volume scheme form

$$\bar{U}_j^{n+1} = \bar{U}_j^n - \frac{\Delta t}{\Delta x}\Big(F_{j+1/2}^n - F_{j-1/2}^n\Big).$$

For $\bar{U}_0$ we explicitly calculate

$$\bar{U}(x,0) = \frac{1}{\Delta x}\int_{x_{j-1/2}}^{x_{j+1/2}} u_0(x) = \begin{cases} -1, & x < -\frac{\Delta x}{2} \\ \frac{2x}{\Delta x}, & -\frac{\Delta x}{2} < x < \frac{\Delta x}{2} \\ 1, & x > \frac{\Delta x}{2} \end{cases}$$

We note that the numerical solution with the Lax-Friedrichs scheme is very diffusive and not at all very accurate. At 100 mesh points the graph seems

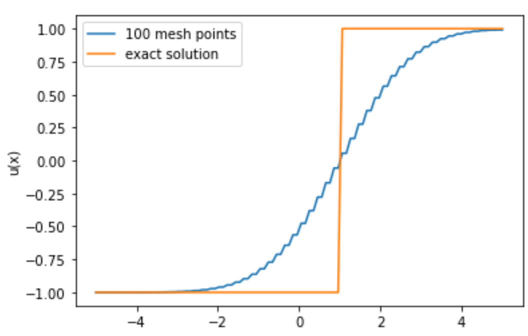

Figure 3: Numerical solutions with the the Lax-Friedrichs scheme at time $t = 1$