dzisiaj 2

zad. 3.

$$\begin{cases} \dot{x}_1 = 3x_2 + x_1x_2 + x_2^2 - u^2 \\ \dot{x}_2 = -2x_1 + e^{-x_2}\sin x_1 + 2u \\ y = x_1 + u \end{cases}$$

$$\begin{cases} \dot{x} = F(x,u) \\ y = G(x,u) \end{cases} \longrightarrow \begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

$$F(x,u) = \begin{bmatrix} f_1(x(t), u(t)) \\ f_2(x(t), u(t)) \end{bmatrix} = \begin{bmatrix} 3x_2 + x_1x_2 + x_2^2 - u^2 \\ -2x_1 + e^{-x_2}(\sin x_1) + 2u \end{bmatrix}$$

$$G(x,u) = \left[g(x(t), u(t)) \right] = \left[x_1 + u \right]$$

$$u \longrightarrow \boxed{\dot{x}_1, \dot{x}_2} \longrightarrow y$$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix} \qquad C = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} \end{bmatrix} \qquad D = \begin{bmatrix} \frac{\partial g_1}{\partial u} \end{bmatrix}$$

$$\frac{\partial f_1}{\partial x_1} = x_2$$

$$\frac{\partial f_1}{\partial x_2} = 3 + x_1 + 2x_2$$

$$\frac{\partial f_2}{\partial x_1} = -2 + e^{-x_2}\cos x_1$$

$$\frac{\partial f_2}{\partial x_2} = -e^{-x_2}\sin x_1$$

$$\frac{\partial f_1}{\partial u} = -2u$$

$$\frac{\partial f_2}{\partial u} = 2$$

$$\frac{\partial g_1}{\partial x_1} = 1$$

$$\frac{\partial g_1}{\partial x_2} = 0$$

$$\frac{\partial g_1}{\partial u} = 1$$

$$A = \begin{bmatrix} x_2 & 3 + x_1 + 2x_2 \\ -2 + e^{-x_2}\cos x_1 & -e^{-x_2}\sin x_1 \end{bmatrix}$$

$$B = \begin{vmatrix} -2u \\ u \end{vmatrix}$$

$$\bar{x} = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$$

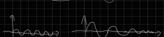
$$\bar{u} = \begin{vmatrix} 2 \end{vmatrix}$$

$$C = \begin{vmatrix} 1 & 0 \end{vmatrix} \left\{ \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} \right\} + 1u$$

Lista 3

TRANSMITANCJA WIDMOWA

$$u \longrightarrow \boxed{K(j\omega)} \longrightarrow y$$



$$K(j\omega) = K(s)\Big|_{s=j\omega} = \frac{L(j\omega)}{M(j\omega)}$$

$$K(s) = \frac{L(s)}{M(s)}$$

Np. $K(s) = \frac{1}{s+1} \rightarrow K(j\omega) = \frac{1}{j\omega + 1}$

ω - pulsacja $\qquad \omega = 2\pi f \qquad f = \frac{\omega}{2\pi}$

$$K(j\omega) = P(\omega) + jQ(\omega)$$

cz. rzeczywista, cz. urojona

1) Charakterystyka amplitudo - fazowa

2) Char. amplitudowa $A(\omega) = |K(j\omega)| = \sqrt{P(\omega)^2 + Q^2(\omega)}$

3) Ch. fazowa $\varphi(\omega) = \arg|K(j\omega)| = \mathrm{arctg}\left(\frac{Q(\omega)}{P(\omega)}\right)$

Lista 3

zad. 1

a) $$K(s) = \frac{k}{s(sT + 1)}$$

k - współczynnik wzmocnienia

T - stała czasowa

$$K(j\omega) = \frac{k}{j\omega(j\omega T + 1)}$$

$$K(j\omega) = \underline{P(\omega)} + j\underline{Q(\omega)}$$

$$K(j\omega) = \frac{k}{-\omega^2 T + j\omega} = \frac{k}{j\omega - \omega^2 T} \cdot \frac{j\omega + \omega^2 T}{j\omega + \omega^2 T} =$$

l. sprzężona

$j^2 = -1$

$$= \frac{kj\omega + k\omega^2 T}{-\omega^2 - \omega^4 T^2} =$$

$$= \frac{k\omega^2 T + j\omega k}{-\omega^2(1 + \omega^2 T^2)} =$$

$$= \underbrace{\frac{k\cancel{\omega^2} T}{-\cancel{\omega^2}(1 + \omega^2 T^2)}}_{P(\omega)} + \underbrace{\frac{j\cancel{\omega} k}{-\cancel{\omega^2}(1 + \omega^2 T^2)}}_{Q(\omega)}$$

$$P(\omega) = -\frac{kT}{(1 + \omega^2 T^2)}$$

$$Q(\omega) = -\frac{k}{\omega(1 + \omega^2 T^2)}$$

$$\omega \to \infty$$

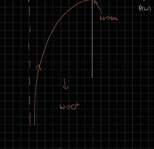
$$\omega \to 0$$

$$\lim_{\omega \to \infty} Q(\omega) = \frac{k}{\omega(1 + \omega^2 T^2)} = 0$$

$$\lim_{\omega \to 0^+} Q(\omega) = -\frac{k}{\omega(1 + \omega^2 T^2)} = -\infty$$

$$\lim_{\omega \to \infty} P(\omega) = \lim_{\omega \to \infty} -\frac{kT}{(1 + \omega^2 T^2)} = 0$$

$$\lim_{\omega \to 0^+} P(\omega) = \lim_{\omega \to 0} -\frac{kT}{(1 + \omega^2 T^2)} = -kT$$



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3 listy