Probability Game Report

1. How to Play

Materials:

A standard 52 deck of cards

4 dice, each labeled as a suit from a standard deck of cards

Instructions:

1. Remove all the face cards and jokers from the deck.
2. Player 1 randomly chooses one of the dice, for example the dice of hearts, then rolls it remembering the number.
3. Next player 2 fans out the remaining 40 cards from the deck, and player 1 randomly chooses one card, for example the 6 of spades.
4. If the suit of the die that was picked and the suit of the card that was picked are the same, then Player 1's score is equal to their dice roll multiplied by the number of the card they picked (Ace is 1). If the two suits are different, then Player 1's score is equal to the sum of the dice roll and the number of the card picked. For example, if a 6 of hearts was rolled and a 4 of hearts was picked, Player 1's score would be 24 (6x4 = 24). However, if a 6 of hearts was rolled and a 4 of diamonds was picked, then Player 1's score would be 10 (6 + 4 = 10).
5. The dice that was chosen by Player 1 is removed from the game, and the card that was chosen by Player 1 is removed from the game. Now there are 3 dice to choose from and 39 cards to choose from.

6. Player 2 repeats the same process as player 1, but randomly chooses one of the 3 dice instead of the 4 dice, and one of the 39 cards instead of the 40 cards.
7. Player 2's score is calculated the same way as Player 1's.
8. The player with a higher score wins (if the score is a tie, player 1 wins).

2. Calculating the Theoretical Probability of Player 1 Winning

Part 1

Calculating the Probability of Each Score: Player 1

Step 1: List out all outcomes of die and cards and their corresponding scores

	Di Numbers					
Card #	1	2	3	4	5	6 S
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12
7	8	9	10	11	12	13
8	9	10	11	12	13	14
9	10	11	12	13	14	15
10	11	12	13	14	15	16

Table 1

Table 1 lists out all the possible scores when adding the values of the dice roll and the card chosen. These outcomes would occur when the suit of the dice roll and the suit of the card are different.

	Di Numbers					
Card #	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36
7	7	14	21	28	35	42
8	8	16	24	32	40	48
9	9	18	27	36	45	54
10	10	20	30	40	50	60

Table 2

Table 2 lists out all the possible scores when multiplying the values of the dice roll and the card chosen. These outcomes would occur when thes suit of the dice roll and the suit of the card are the same.

Step 2: Calculate the probabilities of each score being achieved by the first player.

There are 60 different possible outcomes that could occur between 1 die and 10 cards (6 sides * 10 cards = 60). Table 1 shows us all the 60 different outcomes when in the case of the game, the die and the card chosen are different suits, hence we add these two numbers to get Player 1's score. Table 2 does the same thing except, when in the case of the game, the die and the card chosen are the same suits, hence we multiply their two values to get the final score. However, we don't have 1 possible die, we have four possible dice. To get the total number of possible outcomes for 4 dice, each with 6 different sides, and 40 possible cards, we multiply 4 by 6 by 40, which gives us a total of 960 different outcomes. However, when we count the number of possibilities in both of the tables, we only get 120.

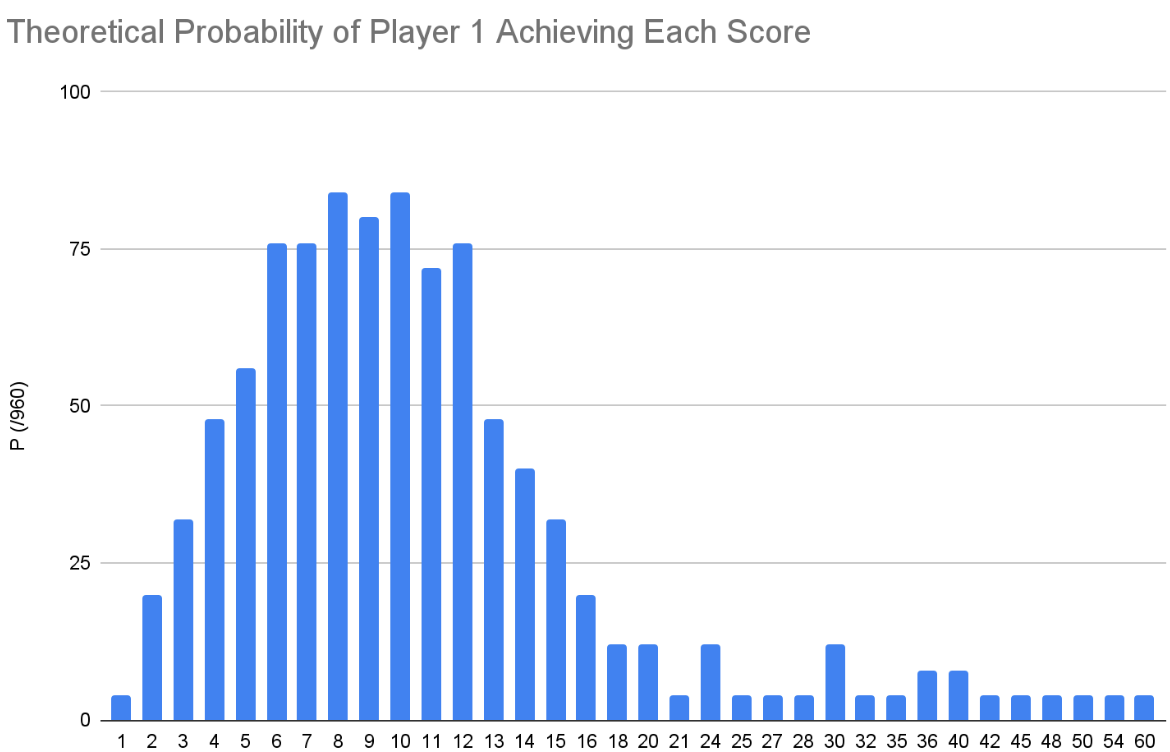
In table 1, we looked at the possible outcomes between one die and 10 cards with the die and card picked being different suits, for example the dice of spades and all the hearts 1-10, in the deck. However, there are two other suits that could be paired with the dice of spades, diamonds and clubs. Therefore, for each die, there are three possible ways it could get the values in table 1. Because there are 4 different dice, to get the total number of outcomes in table 1, we multiply 60 (the number of outcomes in Table 1), by 4 (the number of die), by 3 (the number of suits each die can be paired with), for a total of 720 different outcomes.

In table 2, we looked at the possible outcomes when the die and the card are the same suits, and got a total of 60 outcomes, for example the dice of spades and all the spades 1-10 in the deck. In this case, each die only has one corresponding suit that can achieve the values in table 2, however there are still 4 die. Therefore the total number of outcomes in table 2 is equal to 60 (the number of outcomes), times 4 (the number of dice), for a total of 240 different outcomes.

Adding the total outcomes in table 1 (720), and table 2 (240), we get the total outcomes overall, 960, like we calculated before. Next we need to calculate how likely each score is to occur for player 1. We do this by counting the number of times each score appears in table 1, then multiplying it by 12 because, like we just calculated, there are 12 different ways to get the scores in table 1 (# number of die, 4 * 3, #of suits = 12). For example, the score 2 appears once in table 1, therefore we multiply it by 12, 1*12 = 12. We do the same thing for player 2, except multiplying by 4 because there are 4 different dice. For example, the score 2 appears twice, 2 * 4 = 8. Then we add the number of outcomes in which a score appears in table 2 to the number of outcomes a score appears in table 1 divide by 960 (# of outcomes) to get the overall probability of each score occurring. In the case of getting a score of 2, we add 12 and 8 = 20, so the probability of getting a score of 2 is 20/960.

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/960)	4	20	32	48	56	76	76	84	80	84	72	76	48	40	32	20	12	
																		Player 1 Chance of Scores
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/960)	12	4	12	4	4	4	12	4	4	8	8	4	4	4	4	4	4	

Table 3



Part 2

Calculating the Probability of Each Score: Player 2

After the first players turn, the card and the die they used are removed. This causes the number of outcomes, and therefore the probability of each score being achieved to change depending on which card was removed and which die was removed. For player 2, the total number of outcomes is equal to 6x3x39 = 702 [(#'s on the die) (# of die) (#of cards)].

To calculate the probability of player 2 getting any score we need to find the number of ways player 2 can achieve each score, similarly to what we did for player one. However, this time there are two cases. One in which player 1 chose a die and a card that were the same suit, and one in which player 1 chose a die and a card that were of different suits. We'll calculate each of these individually, and then add them together like we did with player 1.

Step 1: Possible scores when player 1's card and die were the same suit

To do this, we'd start by remaking Table 3, however this time we'd take into account that there are only 3 dice possible, so each of the scores would need to be divided by 4/3. Originally all the values in Table 3 were multiplied by 4 due to there being 4 die, but due to there only being 3 die now we divide by 4/3 to compensate for this. This gives us a table demonstrating all the scores probabilities if there were only 3 dice.

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18
P (/720)	3	15	24	36	42	57	57	63	60	63	54	57	36	30	24	15	9
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60
P (/720)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3

Table 4

Next, we make 10 Tables each corresponding to the probability of Player 2 getting a score when 1 of the cards from 1 to 10 is removed from the deck. Assuming the card and the die chosen by player 1 were the same suit, when player 2 gets a die and a card of the same suit, the probabilities don't change from Table 4, because the dice that was used to multiply with the card that was removed was also removed, so it doesn't change the probability.(If the dice of hearts was removed, none of the cards of hearts can multiply, so it doesn't matter if a card of hearts was also removed). However, when player 2 gets a card and a dice that were of different suits, the probability does change from Table 4. This is due to each score that was the result of an addition between a dice roll and a card will have 1 less way to happen. For example, say the die of clubs and the ace of clubs were removed from the deck. Then say that player 2 rolled a die of hearts and got a 1.If no cards were removed from the deck, there are 3 ways for them to get a score of 2 after this, if their card isn't a heart. They could draw a ace of clubs, spades, or diamonds. Each of these outcomes would result in adding 1, the card value, to 1, the dice roll, giving a score of 2. However, since the ace of clubs was removed, there are now only 2 outcomes where player 2 gets a score of 2 when rolling a 1 of hearts, drawing an ace of spades or ace of diamonds. The outcomes where they get a 2 decreased by 1 in this situation. However, an identical situation could happen with the two other dice that are still in the deck, the die of diamonds and the die of spades. Because there are 2 corresponding outcomes to the one mentioned, the total possible outcomes where they get a 2 will decrease by 3. If we extend this logic, we see that each score that involved an ace to occur would have its total possible outcomes reduced by 3. We can make a table with the probabilities of each score happening when an ace is removed. Compared to Table 4, this table would have the scores of 2,3,4,5,6, and 7 all reduced by 3 because an ace could be added to the scores 1 through 6 on a die to achieve these scores.

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	12	21	33	39	54	54	63	60	63	54	57	36	30	24	15	9	
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3	Ace Removed (Same Suit)

This process can be repeated for the rest of the card numbers 1 through 10 that could be removed from the deck by player 1, with the pattern repeating. (if a 2 was removed, the # of outcomes of the scores 3,4,5,6,7,8 would go down by 3, if a 3 was removed, the # of outcomes of the scores 4,5,6,7,8,9 would go down by 3, etc.)

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	15	21	33	39	54	54	60	60	63	54	57	36	30	24	15	9	
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3	Two Removed (Same Suit)

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	15	24	33	39	54	54	60	57	63	54	57	36	30	24	15	9	
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3	Three Removed (Same Suit)

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	Four Removed (Same Suit)
P (/702)	3	15	24	36	39	54	54	60	57	60	54	57	36	30	24	15	9	
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3	

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	Five Removed (Same Suit)
P (/702)	3	15	24	36	42	54	54	60	57	60	51	57	36	30	24	15	9	
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3	

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	Six Removed (Same Suit)
P (/702)	3	15	24	36	42	57	54	60	57	60	51	54	36	30	24	15	9	
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3	

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	Seven Removed (Same Suit)
P (/702)	3	15	24	36	42	57	57	60	57	60	51	54	33	30	24	15	9	
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3	

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	Eight Removed (Same Suit)
P (/702)	3	15	24	36	42	57	57	63	57	60	51	54	33	27	24	15	9	
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3	

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	15	24	36	42	57	57	63	60	60	51	54	33	27	21	15	9	
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	Nine Removed (Same Suit)
P (/702)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3	

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	15	24	36	42	57	57	63	60	63	51	54	33	27	21	12	9	
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	10 Removed (Same Suit)
P (/702)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3	

Step 2: Possible scores when player 1's die and card were different suits

We again start with the values in Table 4 due to there still being one less die.

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18
P (/720)	3	15	24	36	42	57	57	63	60	63	54	57	36	30	24	15	9
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60
P (/720)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3

Table 4

In this scenario, the number of possible ways to multiply to a given score will decrease by 1 if one of its factors was removed by player 1, because if it has one less factor it will have one less possibility of happening. For example, say a die of hearts and a Ace of spades were removed from the deck. If player 2 rolled a 1 of spades, they would normally have one possible way to get a score of 1, drawing an ace of spades. However, because the ace of spades was removed, they now can no longer get a score of 1 when they get a dice of spades. They can still get a score of 1 if they get a die of clubs or diamonds, but not spades, decreasing the possibilities to get 1 by 1. This is true for all scores that ace is a factor of, in this case the scores 1-6, so each of the scores 1 to 6 would have their values in Table 4 reduced by 1.

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18
P (/714)	2	14	23	35	41	56	57	63	60	63	54	57	36	30	24	15	9
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60
P (/714)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3

Table 5 (Ace multiplication values removed)

However, as seen in the above Table 5, the number of possible outcomes is at 714, when it should be 702, because 3x6x39 = 702. This is because we also need to take into account how the number of possible ways to add to a given score changes. When the dice and the card chosen by player 1 are different suits, the number of ways to add to a given score for player 2 decreases by 2 if the card number removed is used to achieve the given score. This is due to the fact that the two suits that didn't lose a card lose one of there addends for each score, whereas the suit that did lose a card did not lose any addends. For example, say a die of hearts and a Ace of spades were removed from the deck. If player 2 rolled a 1 of spades, there would be 3 ways for them to achieve a score of 2: drawing an ace of hearts, diamonds, or clubs. No possibilities lost so far compared to table 4. However, if player 2 rolled a 1 of diamonds, there would only be 2 ways for them to achieve a score of 2: drawing an ace of hearts or an ace of clubs (they can't draw the ace of spades because it was removed). This repeats itself in the case of player 2 rolling a 1 of clubs instead, with there still only being 2 ways for them to reach a score of 2: ace of diamonds, ace of hearts (no ace of spades). If we extend this logic, we see that each score that involves adding 1 (ace) to a number on the die (1-6) would have its total possible outcomes reduced by 2. In this case, the scores 2-7 fit this criteria, so the values for the scores 2-7 in table 5 would each be subtracted by 2. Now we can make a table with the probabilities of each score happening when an ace is removed, in the case of the dice removed and the card removed being different suits.

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	2	12	21	33	39	54	55	63	60	63	54	57	36	30	24	15	9	
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	Ace Removed (Different Suit)
P (/702)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3	

This process can be repeated for the rest of the card numbers 1 through 10 that could be removed from the deck by player 1, with the pattern repeating. If the score is a multiple from 1-6 of the card that was removed, the value in Table 4 decreases by 1, if the score is a result of adding the card that was removed to a number from 1-6, the value in Table 4 decreases by 2. If the score is a result of both, the value in table 4 decreases by 3.

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	14	22	33	40	54	55	60	60	62	54	56	36	30	24	15	9	
																		Two Removed (Different Suit)
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3	

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	15	23	34	40	54	55	61	57	63	54	56	36	30	23	15	8	
																		Three Removed (Different Suit)
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3	

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	15	24	35	40	55	55	60	58	61	54	56	36	30	24	14	9	
																		Four Removed (Different Suit)
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	8	3	8	3	3	3	9	3	3	6	6	3	3	3	3	3	3	

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	15	24	36	41	55	55	61	58	60	52	57	36	30	23	15	9	
																		Five Removed (Different Suit)
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	8	3	9	2	3	3	8	3	3	6	6	3	3	3	3	3	3	

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	15	24	36	42	56	55	61	58	61	52	54	36	30	24	15	8	
																		Six Removed (Different Suit)
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	8	3	3	3	8	3	3	5	6	3	3	3	3	3	3	

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	15	24	36	42	57	56	61	58	61	52	55	34	29	24	15	9	
																		Seven Removed (Different Suit)
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	2	9	3	3	2	9	3	2	6	6	2	3	3	3	3	3	

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	15	24	36	42	57	57	62	58	61	52	55	34	28	24	14	9	
																		Eight Removed (Different Suit)
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	8	3	3	3	9	2	3	6	5	3	3	2	3	3	3	

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	15	24	36	42	57	57	63	59	61	52	55	34	28	22	15	8	
																		Nine Removed (Different Suit)
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	9	3	2	3	9	3	3	5	6	3	2	3	3	2	3	

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	15	24	36	42	57	57	63	60	62	52	55	34	28	22	13	9	
																		Ten Removed (Different Suit)
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	8	3	9	3	3	3	8	3	3	6	5	3	3	3	2	3	2	

Part 3:

Calculate the probability of Player 1 winning given each combination of initial card and die.

Step 1: Probability of winning when player 1 chooses a die and a card of the same suit

Now that we can calculate the probability of each score occurring in each possible situation we need to calculate the odds of player 1 winning based on the cards chosen and their score. The probability of player 1 winning will be equal to the sum of the probabilities of player 2 achieving a score below or equal to player 1's score. There are two cases we have to consider; player 1 getting a die and a card of the same suit, so their score is a result of a multiplication, and player 1 getting a die and a card of different suit, so their score is a result of an addition. For example, if player 1 rolled a 6 of spades and drew a 9 of spades, their score would be equal to 54. Because player 1 got a die and a card of the same suit, and that card was a 9, we'd add up all the scores below 54 in the "9 removed (same suit)" table. (All the scores in yellow)

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	
P (/702)	3	15	24	36	42	57	57	63	60	60	51	54	33	27	21	15	9	
																		Nine Removed (Same Suit)
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	9	3	3	3	9	3	3	6	6	3	3	3	3	3	3	

These scores add up to 699, and this indicates that if Player 1 rolled a 6 of spades and drew a 9 of spades they would have a 699/702 chance of winning. We repeat this process for each combination of die and card that were the same suit.This gives us the below table, which indicates the probability of player 1 winning in each case where player 1's die and card were the same suit.

Same Suit Probability of Player 1 Winning

	Di Numbers					
Card #	1	2	3	4	5	6
1	3	15	36	69	108	162
2	18	72	165	279	402	513
3	42	168	339	513	603	627
4	78	285	513	618	647	648
5	120	405	603	636	651	666
6	177	513	627	648	666	678
7	234	579	639	639	672	687
8	297	618	648	669	684	693
9	357	627	654	678	690	699
10	420	636	666	684	696	702

Then, we repeat the same process, except in the cases where player 1 chose a card and a die that were different suits. The only difference this time being that instead of using the “Same Suit” tables, we’d use the “Different Suit” tables. For example, if player 1 rolled a 5 of hearts and chose a 6 of diamonds we’d use the “6 removed (different suit) table. In this case, because the suits are different we’d add the value of 5 and 6 to find that player 1 has a score of 11. Then, in the below table we’d add up the probabilities of player 2 getting a score below or equal to 11.

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	Six Removed (Different Suit)
P (/702)	3	15	24	36	42	56	55	61	58	61	52	54	36	30	24	15	8	
Score	20	21	24	25	27	28	30	32	35	36	40	42	45	48	50	54	60	
P (/702)	9	3	8	3	3	3	8	3	3	5	6	3	3	3	3	3	3	

These scores add up to 463, and this indicates that if player 1 rolled a 5 of hearts and chose a 6 of diamonds they would have 463/702 chance of winning. We repeat this process for each combination of die and card that were different suits. This gives us the below table, which indicates the probability of player 1 winning in each case where player 1’s die and card were different suits.

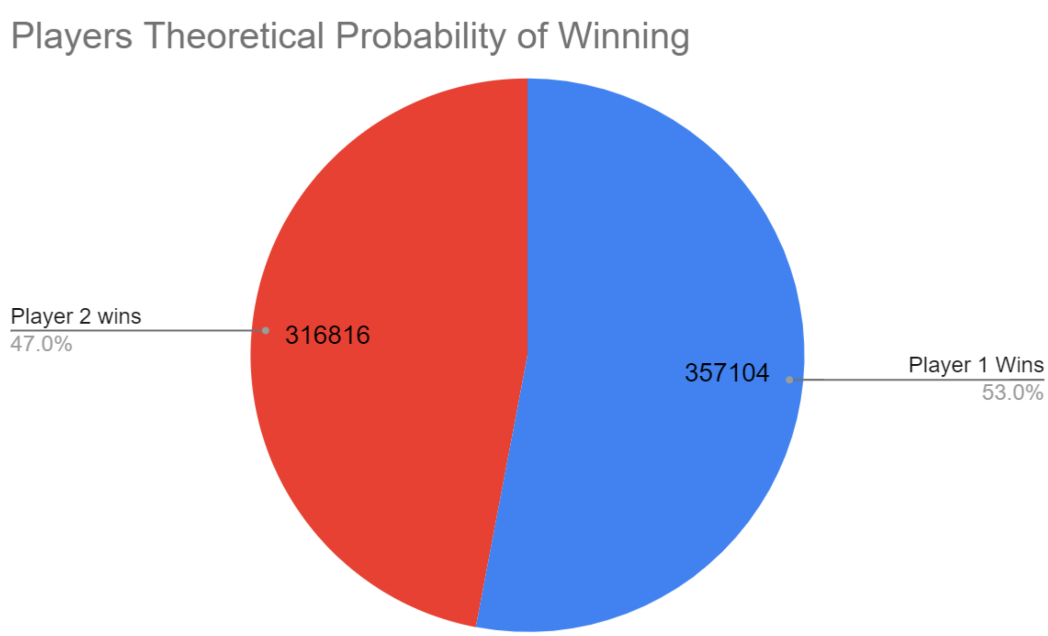
Different Suit Probability of Player 1 Winning

	Di Numbers					
Card #	1	2	3	4	5	6
1	14	35	68	107	161	216
2	39	72	112	166	221	281
3	75	115	169	224	285	342
4	117	172	227	287	345	406
5	174	229	290	348	408	460
6	231	292	350	411	463	517
7	294	352	413	465	520	554
8	354	415	467	522	556	584
9	417	469	524	558	586	608
10	471	526	560	588	610	623

Finally, we have the probabilities of player 1 winning given any initial die and any initial card. Now we can find the total number of times player 1 wins out of all the different outcomes. First, we need to calculate how many different outcomes there are. Since there are 960 different costumes for player 1, (as we calculated in part 1,40x4x6), and

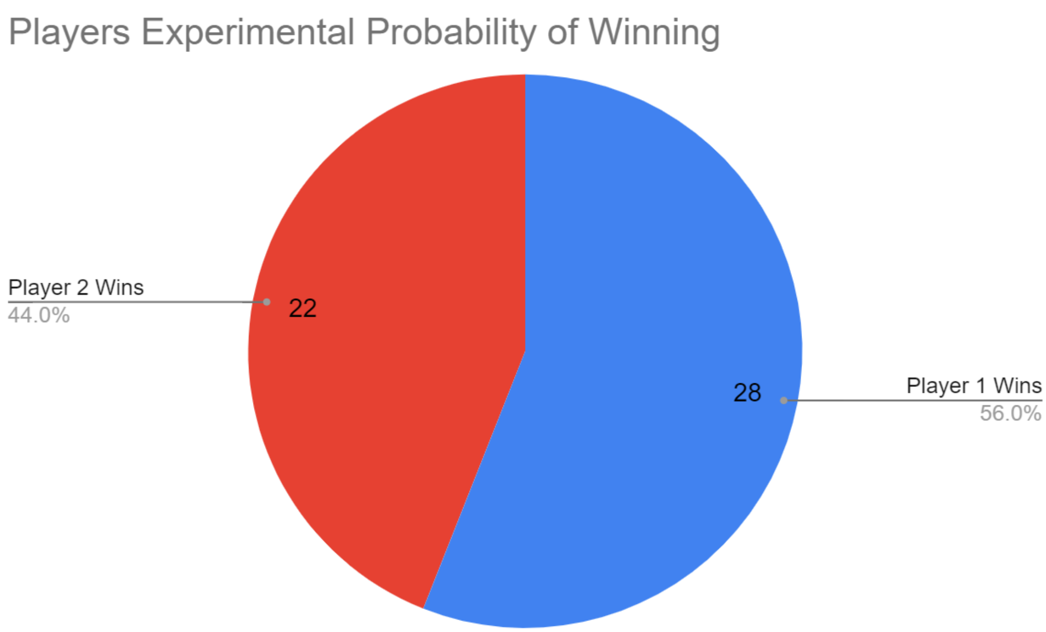
702 outcomes for player 2, (as we calculated in part 2, 39x6x3), the total number of overall outcomes is 673920 (960x702). Next, we add up all the numbers in the two tables displaying how many times each player wins in each scenario. For the Different suit table (player 1 chose a die and a card that were different suits), player 1 wins 20465 times. However, due to there being 4 possible dice with different suits, with each die being able to pair with 3 different cards (like we calculated in part 1), we need to multiply this value by 12. Therefore, if player 1 chose a die and card that were different suits, they win in 20465x3x4 cases, = 245580 cases.
For the Same Suit Table (player one chose a die and a card that were the same suit), player 1 wins 27881 times. However, again there are 4 possible ways this could happen, due to their being 4 different suits. Therefore we must multiply this by 4 to get the number of cases where player 1 wins, 27881 x 4 = 111524 cases. Now to get the overall chance of player 1 winning, we add together all the cases where player 1 wins, then divide by the total number of cases.
(111524 + 245580) / 673920 ≈ 0.5299 = 52.99%

Therefore, player 1 wins in 52.99% of games, making the game slightly favored in player 1's side, meaning the game is **not fair.**



3.Calculating Experimental Probability:

Our experimental probability was calculated by performing 50 trials of the game and then recording which player won each time. Out of these 50 trials, 28 of them were won by Player 1, and 22 of them were won by player 2. To calculate the experimental probability of player 1 winning we do 28/50 x 100% = 56% Due to there being an overwhelming number of possible outcomes for this game, our results would've been much more accurate with a larger number of trials, however this wasn't practically possible. Instead, a program was created in Python to perform 1000000 trials of the game, and record which player won each time. Out of those 1000000 trials, player 1 won 530189 of them, whereas player 2 won 469811 of them. Calculating the experimental probability of player 1 winning for these trials we do 530189/1000000 x 100% ≈ 53.02% chance of player 1 winning, which is much closer to the theoretical probability due to the larger number of trials. The code for the program that was used can be found here.



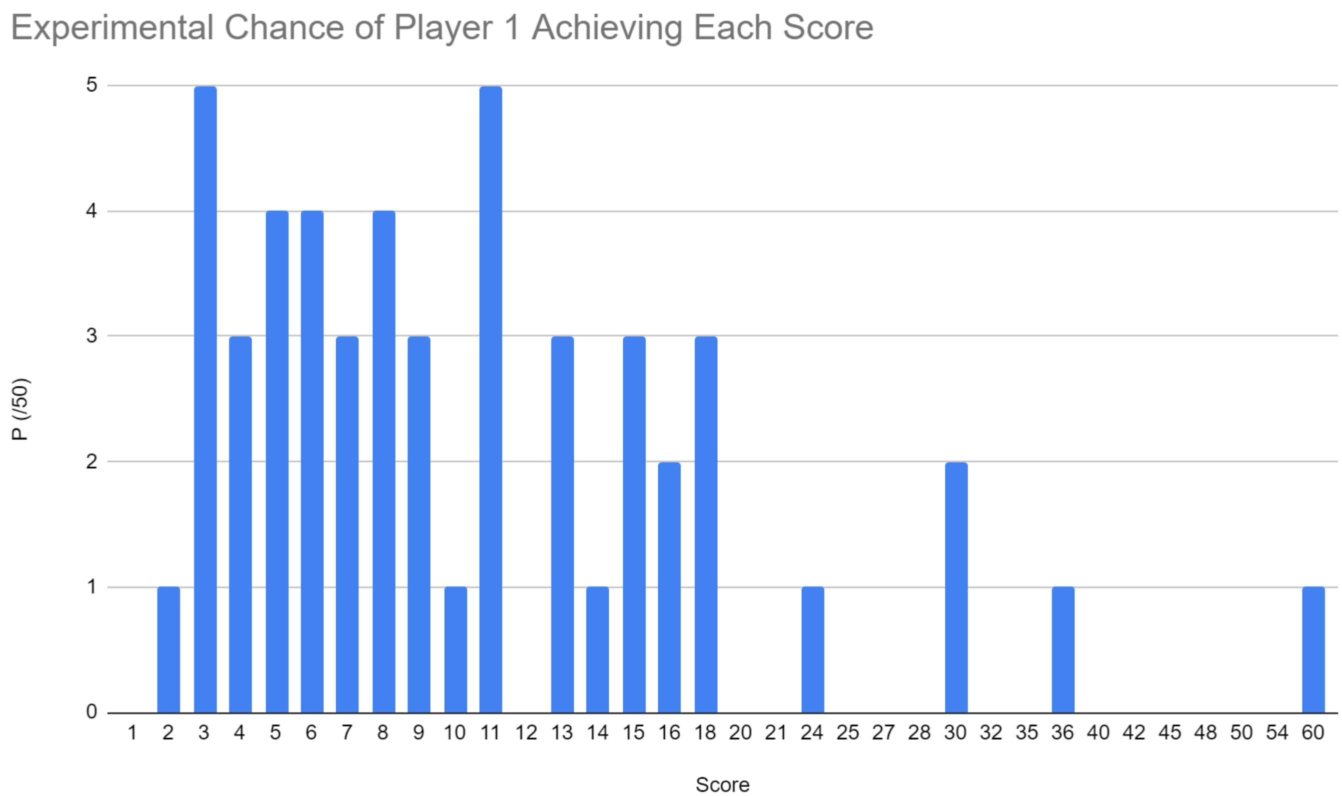
Scores (Manual Trials)

Player 1 Score	Player 2 Score
11	9
9	7
3	16

Player 1 Score	Player 2 Score
8	4
2	20
5	12
30	5
13	54
18	4
7	10
18	6
5	27
13	9
11	12
16	40
24	1
3	3
4	7
9	14
5	8
6	2
3	4
36	12
4	14
6	6
15	2
3	18

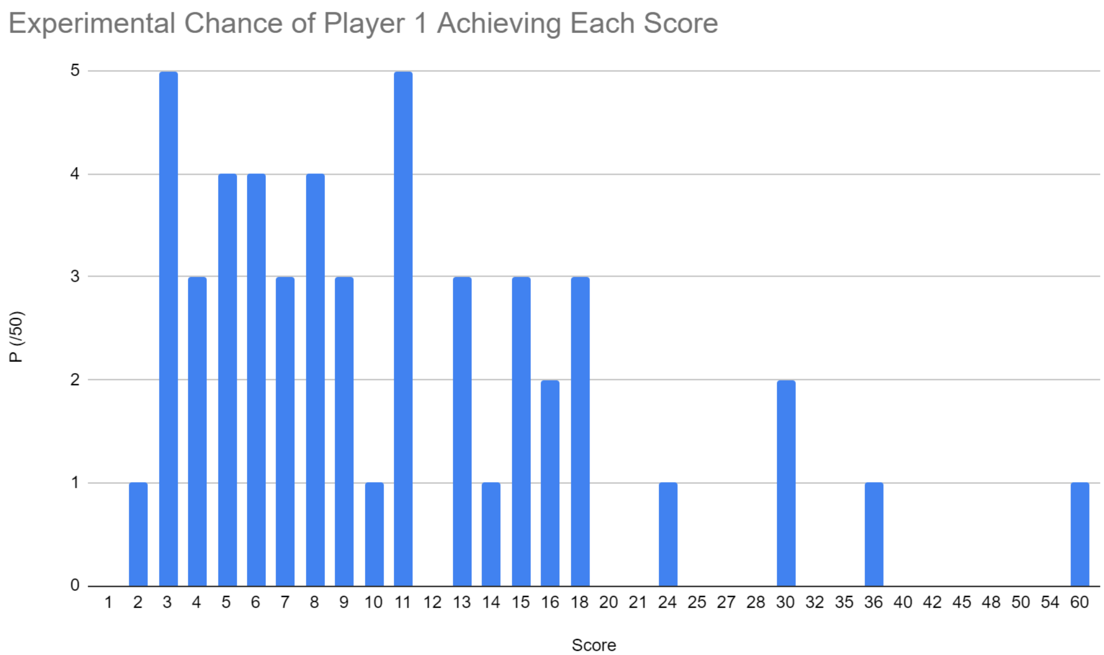
Player 1 Score	Player 2 Score
13	7
5	20
16	3
11	6
15	35
9	4
6	3
4	14
60	10
8	6
18	1
7	10
15	3
14	9
6	2
30	9
11	4
8	5
7	9
8	10
10	11
11	13
3	3

Based on the values above, we can additionally graph the probabilities of player 1 achieving any score, similarly to how we did it theoretically. Taking the number of times player 1 achieved each score and graphing it.



Conclusion:

Comparing the graphs of the theoretical probability of player 1 achieving each score, and the experimental probability, we see that they follow the same general trend, with there being one big cluster of higher likelihood outcomes and then a wide range of less likely outcomes. The experimental probability doesn't exactly match the theoretical probability due to the low number of trials compared to the total number of outcomes. 50 trials to represent 960 outcomes isn't a very good sample size, and the experimental probability would've been more accurate if more trials were conducted.



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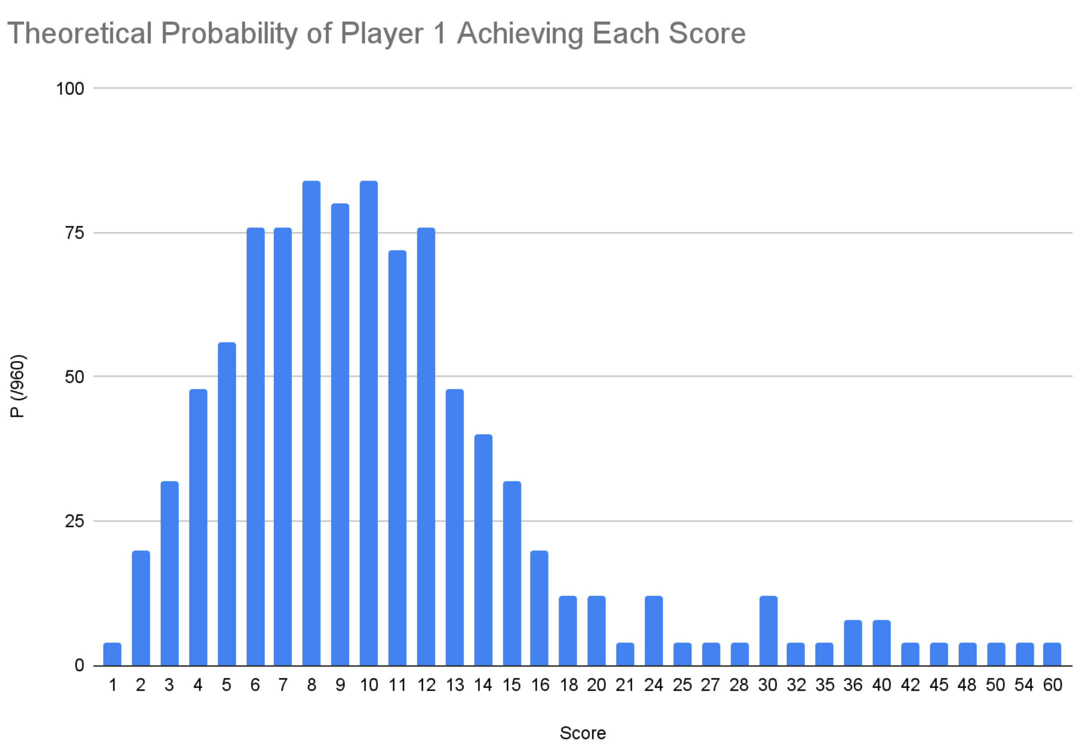
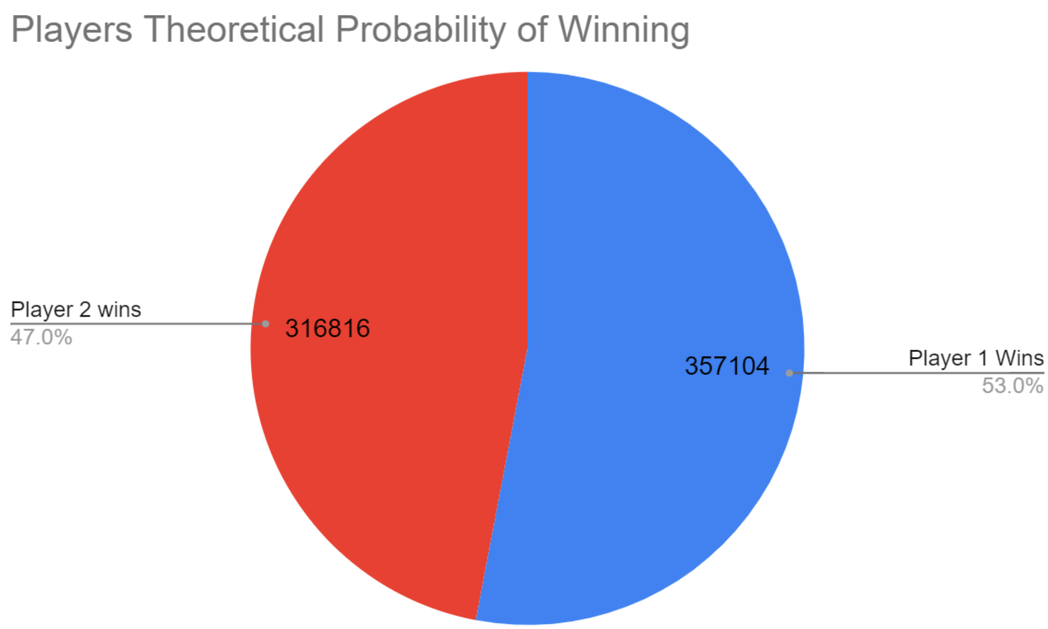
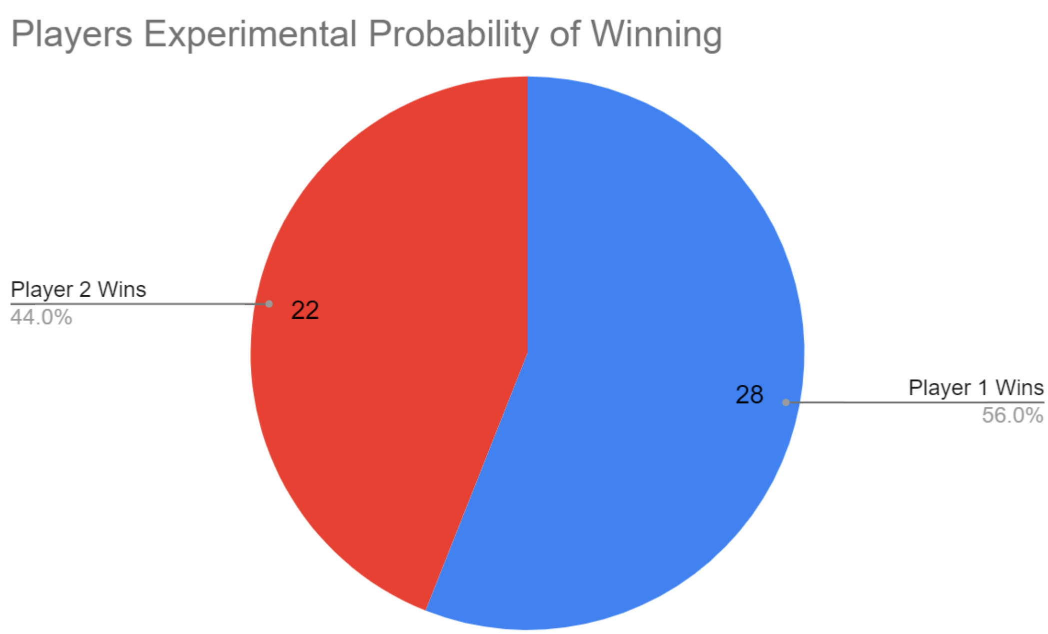


When comparing the experimental and theoretical probabilities of player 1 winning, we can see that there is a fairly similar probability, with the experimental probability having player 1 have a 3% higher probability of winning compared to the theoretical probability.





Again, the experimental probability would certainly have been more accurate given more trials took place, as is evident from the 1000000 trials taken place by the python code that gave nearly the same experimental probability as the theoretical probability. Overall, we can conclusively conclude that the game is **not fair,** due to both the experimental as well as theoretical probability showing that Player 1 has a higher chance of winning than Player 2.