

метод каскающего спуска

$$M_0(0.5; 0.5)$$

шаг 1:

$$\frac{df(M_0)}{d\lambda_1} = 3\lambda_2 - 2\lambda_1\lambda_2 - \lambda_1^2 = 0.75$$

$$\frac{df(M_0)}{d\lambda_2} = 3\lambda_1 - \lambda_2^2 - 2\lambda_2\lambda_1 = 0.75$$

$$\lambda_{11} = \lambda_{10} - h_1 \frac{df(M_0)}{d\lambda_1} = 0.5 - h_1 \cdot 0.75$$

$$\lambda_{21} = 0.5 - h_1 \frac{df(M_0)}{d\lambda_2} = 0.5 - h_1 \cdot 0.75$$

$$f(\lambda_{21}, \lambda_{22}) = 3(0.5 - h_1 \cdot 0.75)^2 - (0.5 - h_1 \cdot 0.75)^3 - (0.5 - h_1 \cdot 0.75)^3 = 3(0.5 - h_1 \cdot 0.75)^2 - 2(0.5 - h_1 \cdot 0.75)^3$$

$$\frac{df}{dh_1} = 0 \quad \text{или} \quad h_1 = -\frac{2}{3} \approx -0.667$$

шаг 2:

$$\lambda_{11} = 0.5 + 0.667 \cdot 0.75 = 1.00025$$

$$\lambda_{21} = 0.5 + 0.667 \cdot 0.75 = 1.00025$$

$$M_1 \left(\begin{pmatrix} 1.00025 \\ 1.00025 \end{pmatrix} \right)$$

$$\frac{df}{d\lambda_1} = -0.00075 \quad \frac{df}{d\lambda_2} = -0.00075$$

$$\lambda_{31} = \lambda_{21} - h_2 \cdot (-0.00075) = 1.00025 + h_2 \cdot 0.00075$$

$$f(x_{32}, x_{33}) = 3(1.00025 + h_2 \cdot 0.00075)^2 - 2(1.00025 + h_2 \cdot 0.00075)^3$$

$$\frac{df}{dh_2} = 0 \Rightarrow h_2 = -0.33$$

$$x_{31} = 1.00025 - 0.33 \cdot 0.00075 = 1.0000025$$

~~x_{32}~~

~~x_{33}~~

Метод Градiente

$$f(x_1, x_2) = 3x_1x_2 - x_1^2x_2 - x_1x_2^2$$

$$\frac{df}{dx_1} = 3x_2 - 2x_1x_2 - x_2^2$$

$$\frac{df}{dx_2} = 3x_1 - x_1^2 - 2x_1x_2$$

$$X^{(0)} = (0.5; 0.5)$$

$$f = 0.5$$

$$\nabla f = \begin{pmatrix} 0.75 \\ 0.75 \end{pmatrix} \quad \lambda = 0.25$$

Шаг 1:

$$x^{(1)} = x^{(0)} + \lambda \nabla f(X^{(0)}) = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} + \lambda \begin{pmatrix} 0.75 \\ 0.75 \end{pmatrix} = \begin{pmatrix} 0.6875 \\ 0.6875 \end{pmatrix}$$

$$f(X^{(1)}) = 0.768$$

$$|f(X^{(1)}) - f(X^{(0)})| = |0.768 - 0.5| = 0.268$$

Шаг 2:

$$x^{(2)} = x^{(1)} + \lambda \nabla f(X^{(1)}) = \begin{pmatrix} 0.6875 \\ 0.6875 \end{pmatrix} + 0.25 \begin{pmatrix} 0.645 \\ 0.645 \end{pmatrix} = \begin{pmatrix} 0.849 \\ 0.849 \end{pmatrix}$$

$$f(X^{(2)}) = 0.938$$

Шаг 3:

$$X^{(3)} = X^{(2)} + \lambda \nabla f(X^{(2)}) = \begin{pmatrix} 0.849 \\ 0.849 \end{pmatrix} + 0.25 \begin{pmatrix} 0.385 \\ 0.385 \end{pmatrix} =$$

$$= \begin{pmatrix} 0.945 \\ 0.945 \end{pmatrix}$$

$$f(X^{(3)}) = 0.991$$