Problem Chosen	2024	Team Control Number
C	**MCM/ICM Summary Sheet**	**2405978**

How On Earth Does The Momentum Control The Match?

summary

In various sports competitions, momentum can have a significant psychological impact on players. Our research aims to quantitatively analyze the relationship between momentum and the tennis matches.In Task 1: The aim is to establish a model to determine which player performs better and to what extent during a specific period of the match. In this task, we primarily consider the impact of serving on player performance. We use **EWM** to assign weights to holding serve and breaking serve, then establish an average point differential model to evaluate player performance. Subsequently, we using **Excel** to visualize the average point differential model This allows us to determine which player performs better based on the positive or negative values on the y-axis and the magnitude of these values.

In Task 2: The aim is to investigate whether momentum fluctuations are related to player success and establish a model to determine this relationship. In this task, we quantified player success and momentum fluctuations separately. We quantify success based on players' cumulative points. When establishing the momentum quantification model, we first use the **AHP** and **conduct consistency checks** to determine weights.Then, we use **Python** to fit the quantified success model and quantified momentum model ,achieving a high fit of **83%**. This indicates a strong correlation between player success and momentum.

In Task 3: This aims to determine the most significant factors influencing momentum fluctuations and establish a model to predict momentum fluctuations in matches. In this task, we use the **EWM** to calculate the weights. We identify that the most relevant factors for momentum fluctuations are the **aces, untouchable shots, and break points**. To predict momentum fluctuations, we apply the **K-means clustering method** to group sample data into five stages of momentum predetermined. We visualize the results of the K-means clustering model using **Python**. To predict momentum fluctuations during matches, we calculate the **Mahalanobis distance** between the coordinates of data points and the centroids in the K-means plot in real-time during match progression.

In Task 4: The aim is to test the predictive ability of the established model and evaluate its generality. In this task, we input the test dataset into the K-means clustering model and determine the momentum stages before and after each score point. We use **0-1 normalization** to standardize the data.We then evaluate the model using **binary classification model evaluation methods**. Through **Excel**, we calculate the precision and recall of the model, and find that the momentum prediction model achieves **an accuracy of 65%** for tennis matches, indicating good predictive ability. For table tennis matches, the momentum prediction **accuracy reaches 62.8%**. Overall, this model has some universality for predicting momentum in individual sports matches.

Keywords: Entropy Weighting Method(EWM), Analytic Hierarchy Process(AHP), Consistency Check, K-means Clustering, Binary Classification Model Evaluation.

contents

1 Introduction

1.1 Problem Background

In sports competitions, the outcome of the competition is not only determined by the strength level of the athletes, the momentum[1] of the athletes in all phases of the competition is also one of the most important reasons for determining the outcome of the competition. Momentum is defined as a two-way link concept, which can be categorized into positive and negative momentum, and positive and negative momentum are the two obvious mechanisms for generating momentum fluctuations, which will greatly affect the athletes' psychology and the result of the competition.

Either type of momentum affects the athlete's psychology to a great extent. In order to help athletes win sports competitions, it is important to accurately quantify momentum so that they can utilize momentum fluctuations to flexibly change their game strategies.

1.2 Restatement of the Problem

In these problems, we need to analyze the athletes' game data to determine whether momentum affects the course of the game, and build mathematical models to quantify the momentum and predict the trend of the momentum. Considering the background information and restricted conditions identified in the problem statement, we need to solve the following problems. identified in the problem statement, we need to solve the following problems.

- **Problem 1:** Determine which player is performing better at a given time, and how well.
- **Problem 2:** Build a model to evaluate whether a player's game fluctuations and probability of success are determined by momentum.
- **Problem 3:** Use the provided game data to build a model that predicts shifts in momentum during a game and identifies the most important factors that lead to shifts in momentum. Also provide advice to players based on differences in momentum swings.
- **Problem 4:** Test the model in other tennis matches and evaluate the model accordingly. Test the model's generalizability to other types of sports.
- **Problem 5:** Summarize the results and advise the coaches accordingly so that the players are better prepared for the matches.

1.3 Our Work

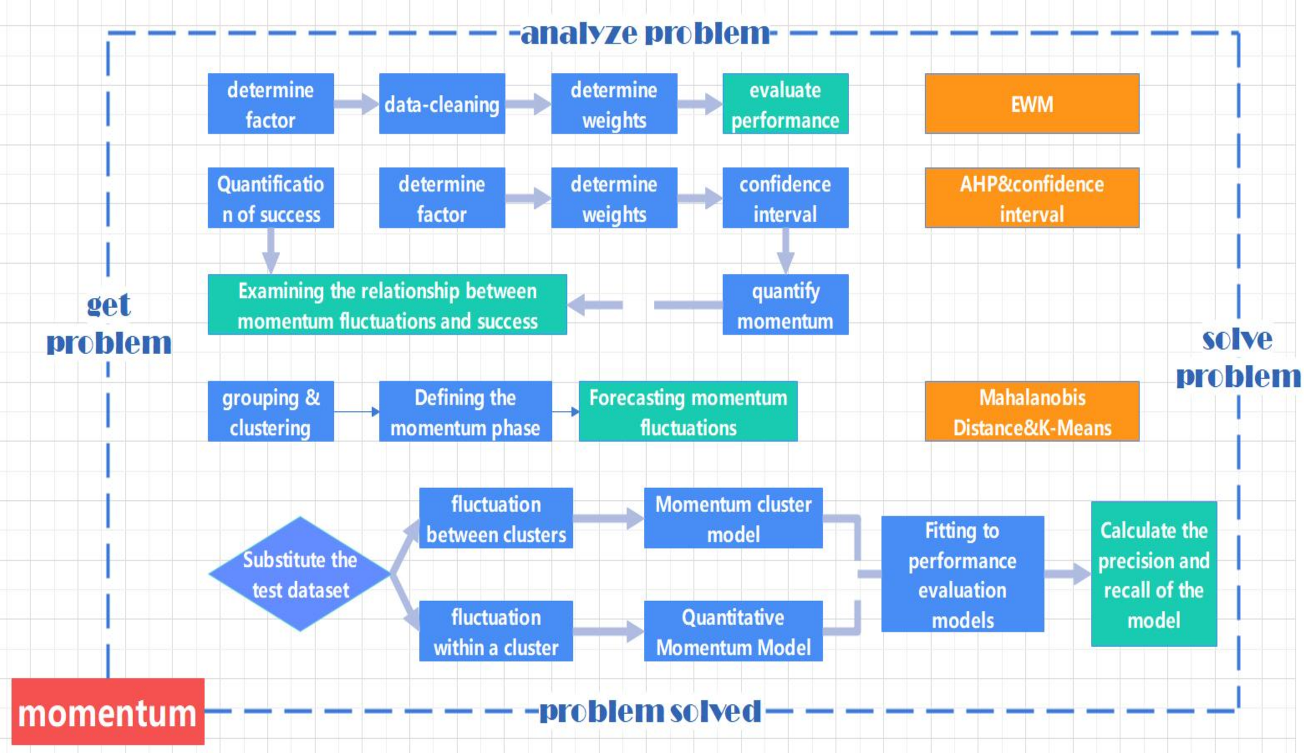

Figure 1: Our Work

2 Assumptions and Justifications

Considering those practical problems always contain many complex factors, first of all, we need to make reasonable assumptions to simplify the model, and each hypothesis is closely followed by its corresponding explanation:

●**Assumption 1:**Time between points scored and lost in a game t_0 has split the total time of the game, so $t_0 = \frac{T}{334}$.

- **Justification:** Since the number of rounds in each GAME varies and the round times vary greatly, it can lead to subsequent modeling that does not allow for a quantitative assessment of how well a player performs in a given time period.

●**Assumption 2:** Using binary method to measure the changes in momentum clusters, changes are represented as 1, no changes as 0.

●**Assumption 3:**Using a binary distribution to assess whether intra-cluster changes affect player performance.

-**Justification:** Most cluster changes occur between adjacent clusters, and whether a change occurs can also be represented in binary, which simplifies calculations and improves efficiency.

●**Assumption 4:**If the player's performance in the next round is less than or equal to the player's performance in the previous round, it is consistently recorded as 0, indicating a potential decline in future match situation. If it is greater, it is recorded as 1, indicating a potential improvement in situation.

-Justification: To more conveniently determine the correlation between intra-cluster momentum changes and match trends, a counter calculates that when player performance remains unchanged, the number of times the situation declines is significantly greater than the number of times it improves.

3 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbol	Description	Unit
P_{a1}	Player1's actual score	
P_{a2}	Player1's actual score	
N_{sei}	number of set already won by the player numbered i	
N_{gi}	number of game already won by the player numbered i	
N_{sci}	Points already scored by player numbered i	
N_{tb-7i}	Points scored by the player numbered i in the Tie-Breaker-7	
N_{tb-10i}	Points scored by the player numbered i in the Tie-Breaker-10	
m_j	Scores of hold-shots and break-shots after calculating weights	

4 Problem 1

4.1 Data preprocessing

Before data analysis,the availability of data must be guaranteed.

- `Data-cleaning` Since the scoring system in tennis is very special (scores are divided into Love/15/30/40/AD), in order to make it easy to calculate and establish the model, we standardize the data and each round is recorded as plus or minus one point（Love=0，15=1，30=2，40=3，AD is regarded as the player having scored 4 points).

4.2 Modeling of player performance

4.2.1 Tennis rules and cumulative scoring

It is known that in a singles tennis match, a game is won when a side wins by four points, and if both of the players have won three points, a side has to win by two points in order to end the game. A set is won by winning six games, and in the case of a 5:5 tiebreak, two games must be won by one side to end the set. At Wimbledon, the first player to score 7 points in a tie-break wins the game (must score 2 consecutive points), and in a tie-break other than the first 4 sets, the first player to score 10 points wins the match (must score 2 consecutive points).

- It therefore follows that when the tiebreaker is in the first four sets, player i's current cumulative score is:

$$P_{ai} = 24N_{sei} + 4N_{gi} + N_{sci} + \frac{1}{7}N_{tb-7i} \tag{1}$$

- When the tiebreaker is in the fifth set, player i's current cumulative score is.

$$P_{ai} = 24N_{sei} + 4N_{gi} + N_{sci} + \frac{1}{10}N_{tb-10i} \tag{2}$$

4.2.2 Entropy Weighting Method (EWM)

In tennis, the player who serves has a higher probability of winning the set, that is, hitting a hold (the player scores in the round in which he serves) has a higher probability to win the round than hitting a break (the player scores in the round in which the opponent serves), so the entropy weighting method is applied to find the weights of the effect of holds and breaks on a player's performance by analyzing the cumulative holds versus cumulative breaks of a given set of a match by the two players in a match up to a certain point of scoring.

- Step1 Data 0-1 standardization

The cumulative number of holds versus the cumulative number of breaks prior to each scoring point in the ten matches were selected and the data were standardized so that the data were distributed within 0 to 1.

- Step2 Calculating indicator variability

Assuming that there are n objects to be evaluated and m evaluation indicators, a non-negative matrix $X = (x_{ij})_{n\times m}$ is obtained after data normalization. Calculate the probability matrix P ,and each element in it is $p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{n} x_{ij}} \left(i = 1,2,3,...n; j = 1,2,3,...m\right)$

- Step3 Calculating information entropy

The information entropy is

$$e_j = -\frac{\sum_{i=1}^{n} p_{ij} \ln p_{ij}}{\ln n}$$

Find the corresponding information entropy for the cumulative number of holds and cumulative number of breaks for each of the two players.

- Step3 Calculate weights

Applying the weighting formula in the entropy weighting method:

$$w_j = \frac{1-e_j}{m - \sum_{j=1}^{m} e_j}$$

Determine how much weight the cumulative number of holds and the cumulative number of breaks of the two players have in judging the player's performance:

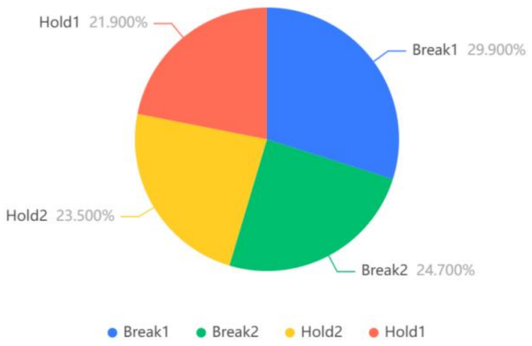

Figure 2: Weight of scoring type

By summing the weight of the hold and break of two players separately, we can determine that the weight of hold is $\frac{9}{20}$,and the weight of break is $\frac{11}{20}$

.So we set the weight matrix $A=\left(\frac{9}{20}\,\frac{11}{20}\right)^T$

4.2.3 Quantifying player performance

As of the scoring point j （j=1,2,...,334）

- The matrix of scoring situations for player1 is：

$$B_{1j}=\left(m_{1j}\ m_{2j}\right)^T$$

- The matrix of scoring situations for player2 is：

$$B_{2j}=\left(m_{3j}\ m_{4j}\right)^T$$

Thus, the difference between the cumulative scores of player1 and player2 is：

$$\Delta P_{wj}=P_{w1j}-P_{w2j}=A^T(B_{1j}-B_{2j}) \tag{3}$$

From the given data, it is known that there were 334 scoring points in the final match, and in order to accurately estimate how well player i performed in a given time period, take the average change in score differential over the seven scoring points near that time period, and the average score difference formula is：

$$AP_{wij}=\begin{cases}(-1)^{i+1}AP_{i(j+3)},j<4\\(-1)^{i+1}\dfrac{\Delta P_{wi(j+3)}-\Delta P_{wi(j-3)}}{6t_0},3<j<332\\(-1)^{i+1}AP_{i(j-3)},j>331\end{cases} \tag{4}$$

Since the average score difference is a vector, the positive and negative directions can indicate which player is performing better at a given time period, and the absolute value of the average score difference can indicate how well that player is performing.

4.3 The Solution of Model 1

The dataset of 2023-wimbledon-1701 is selected as the test set for Model 1. A line graph of the average score difference between Carlos Alcaraz(player1) and Novak Djokovic(player2) over the time of the game allows us to observe which player is performing better at a given time：

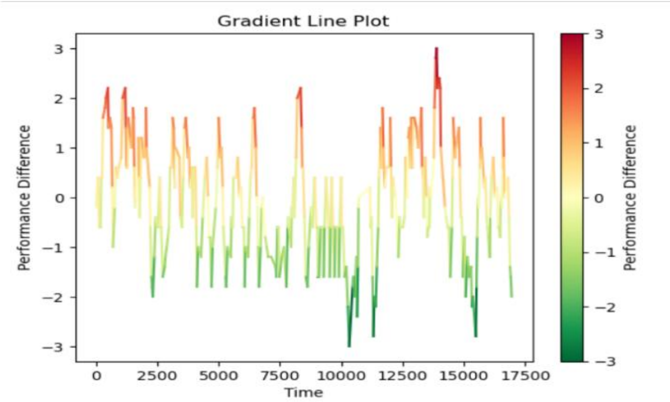

Figure 3: Difference in score between player 1 and player 2[2]

In this case, when the fold line is above the timeline, it is the time when Carlos Alcaraz performed better, and when the fold line is below the timeline, it is the time when Novak Djokovic performed better.The absolute value of the average score difference can determine how well a player has performed

5 Problem 2

5.1 Quantitative Approaches to Players' Succes and Momentum Fluctuation

We believe that there is a correlation between the success and the momentum of a player. To test our conjecture, we will quantify players' momentum and success with two separate functions and analyze the correlation between the two functions.

5.1.1 Quantifying success

Since the absolute criterion for judging which player wins is the player's score, and the criterion interval for judging a player's success is the whole match, instead of a game or a set, the cumulative score of a player before score point is used as an indicator to quantify the player's success.

5.1.2 Quantifying Momentum -Analytic Hierarchy Process (AHP)

The idea of Analytic Hierarchy Process is to first determine the weight of each criterion on the objective and the weight of each program on each criterion, applying a consistency test to determine that the pairwise comparison array is within the tolerance of inconsistency, and then synthesize the two to obtain the weight of the program on the objective[3].

Momentum has a huge impact on a player's level of play in tennis, both physical and psychological. Momentum is not a variable that can be accurately calculated, but in order to better quantify the momentum, we decided, based on our knowledge of the tennis game and review of the data, to weight the following eight factors: hold, break, ACE, untouchable shot, double fault, unforced errors, the opponent's scoring of two consecutive points, and the distance of the running game.

At the same time, we give the 2nd to 4th factors (break, ACE, untouchable shot) the same weight for momentum impact and the 5th to 7th factors(double fault, unforced errors, the opponent's scoring of two consecutive points) the same weight for momentum impact. Therefore, we can categorize the above eight influencing factors into four types of evaluation criteria X_1, X_2, X_3, X_4 :

- X_1 :hold
- X_2 :break、ACE、untouchable shot
- X_3 :double fault, unforced errors, the opponent's scoring of two consecutive points
- X_4 : the distance of the running game.

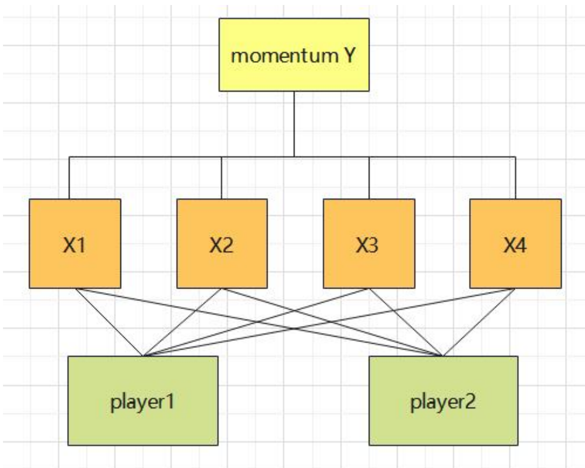

Figure 4: Hierarchical diagram of player momentum

In order to determine the weights of the lower elements (X_1, X_2, X_3, X_4) to the upper elements(momentum Y), the matrix of contrasts and eigenvectors need to be utilized and then perform a consistency test to determine the weights.

- **Step1 matrix of contrasts and eigenvectors**

In determining the weights of the four criteria on the upper element (momentum), in order to reduce the difficulty of comparing the four criteria with different properties, we decided to use two-by-two comparison with each other, And relative scales are used in comparisons, Denote the ratio of the importance of X_i and X_j to Y by a_{ij}

The result of the two-by-two pairwise importance ratio of the four elements is represented by the pairwise comparison matrix:

$$A=(a_{ij})_{4\times4}\ ,\quad a_{ij}>0\ \ \mathrm{CR}=\frac{\mathrm{CI}}{\mathrm{RI}}=\frac{0.09667}{0.9}=0.0925<0.1 \tag{5}$$

Based on our knowledge of the game of tennis and subjective evaluations, In terms of the significance of the four dimensions for the change in potential energy, we compare the significance of X_1,X_2,X_3,X_4 two by two, resulting into a comparison matrix as:

$$A=\begin{pmatrix} 1 & \frac{1}{2} & -1 & -\frac{1}{2} \\ 2 & 1 & -1 & -\frac{2}{5} \\ -1 & -1 & 1 & \frac{2}{5} \\ -2 & -\frac{5}{2} & \frac{5}{2} & 1 \end{pmatrix}$$

From the above ratios in the pairwise comparison array, it can be noticed that there are inconsistencies in pairwise comparison array. Take the eigenvector $w=(w_1,w_2,w_3,w_4)^T$,and the eigenvector satisfy $\sum_{j=1}^{4}w_j=1$,If the pairwise comparison array $A=(a_{ij})_{4\times4}$ is within the tolerance of inconsistency, then using the eigenvector corresponding to the largest eigenroot λ of the pairwise comparison array A as the weight vector w ,and w satisfy:

$$Aw=\lambda w \tag{6}$$

Using MATLAB to compute the eigenvectors:

$$w=(0.35599545,0.57712764,-0.65534288,-0.3327\,4111)^T$$

- **Step2 consistency test**

In the above process, we have assumed that pairwise comparison arrays are within the tolerance of inconsistency, so now we need to define the scope of inconsistency of pairwise comparison arrays, Introducing consistency indicators CI :

$$CI=\frac{\lambda-n}{n-1} \tag{1}$$

n	3	4	5	6	7	8	9	10
RI	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

Figure 5: Stochastic Consistency Indicators

Introducing stochastic consistency metrics RI ,when the consistency ratio CR satisfies:

$$CR = \frac{CI}{RI} < 0.1$$

Then the pairwise comparison array is said to be within the tolerance of inconsistency.

In this model, there are four dimensions that have an impact on the momentum (n = 4.) According to the numerical plot of the stochastic consistency indicator, it can be seen that $RI = 0.9$,Maximum characteristic root $\lambda \approx 4.25$,sot that the consistency ratio satisfies

Therefore, the pairwise comparison array is within the tolerance of inconsistency.

- **Step3 Define the weights**

The consistency test of the pairwise comparison array passes, so the normalized eigenvectors can be determined as:

$$w = (0.35599545, 0.57712764, -0.65534288, -0.33274111)^T$$

And this eigenvector can be used as a weight vector in four dimensions X_1, X_2, X_3, X_4 .

Thus we can obtain the formula that measures the player's momentum at the jth scoring point:

$$M = 0.356\delta_{hj} + 0.577(\delta_{bj} + \delta_{ACEj} + \delta_{utbj}) - 0.655(\delta_{dfj} + \delta_{unfj} + \delta_{oj}) - 0.333\Sigma_{dj} \tag{7}$$

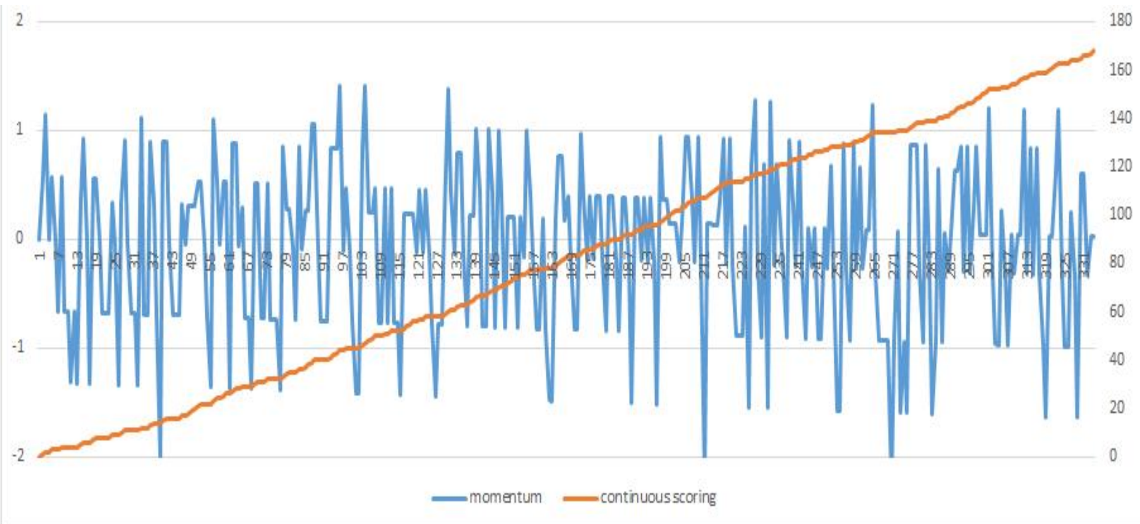

Figure 6: Momentum's effect on continuous scoring

5.2 Methods of Comparing a Player's Success and Momentum fluctuation

We assume that an increase in a player's momentum at scoring point j can greatly increase the probability of scoring in scoring point $j+1$, so we first define the matrix of the relationship between a player's increase in momentum and score $C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

- $c_{11} = 1$ represents a player with increased momentum and scores
- $c_{12} = 0$ indicates a player with decreased momentum but also scores,
- c_{21} represents a player with increased momentum but loses points
- $c_{22} = 1$ represents a player with decreased momentum and loses points.

To verify that players' momentum fluctuations and successes were not random, we counted cases where the actual situation matched our predictions at each scoring point.

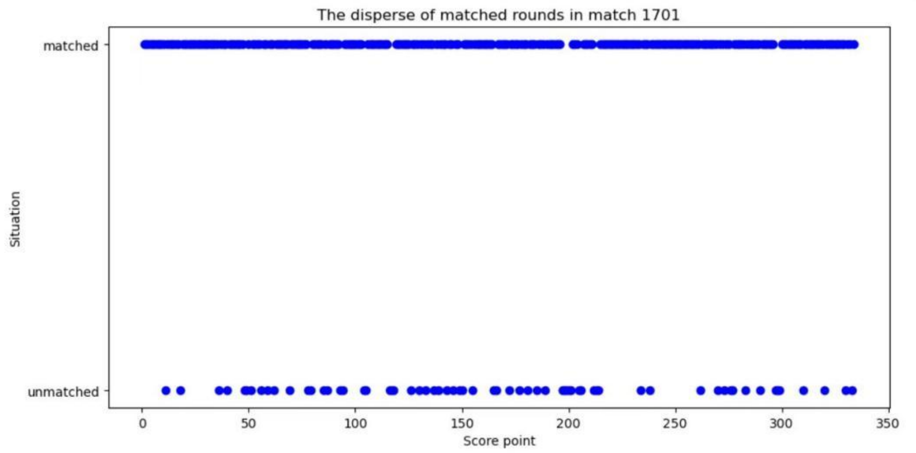

Figure 7: The disperse of matched rounds in match 1701

The statistics show that the number of scoring points where the actual situation matches the predicted situation is 83% of the number of all scoring points.

Thus we can infer that players' match fluctuations are strongly correlated with success.

6 Problem3

6.1Flow chart

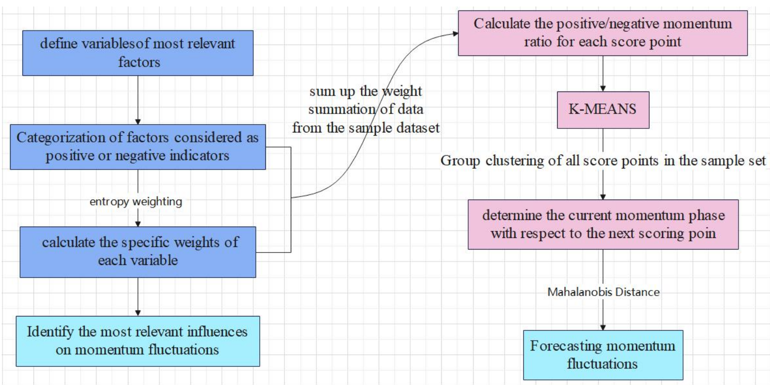

Figure 8: Flow chat of problem 3

6.2 Predictive momentum fluctuation model

6.2.1 Identify factors that affect momentum

Considering the strong relationship factors and removing the weak relationship factors, we decided to include ACE, double faults, unforced errors, untouchable shots, holds of serve, breaks of serve, opponent's points, and physical fitness[4] in the consideration of momentum fluctuation.

6.2.2 EWM

In order to determine the influence of each factor on the player's momentum fluctuation, and to identify the most relevant influencing factors on the momentum fluctuation, we decided to still use the Entropy Weighting Method(EWM)to calculate the weights of each factor, in which the positive indicators that will play a role in increasing the momentum are: ace, hold serve, break serve, untouchable shot, and the negative indicators that will play a role in weakening the momentum are: double fault, unforced error physical fitness, and opponent's points. The weights are calculated as follows:

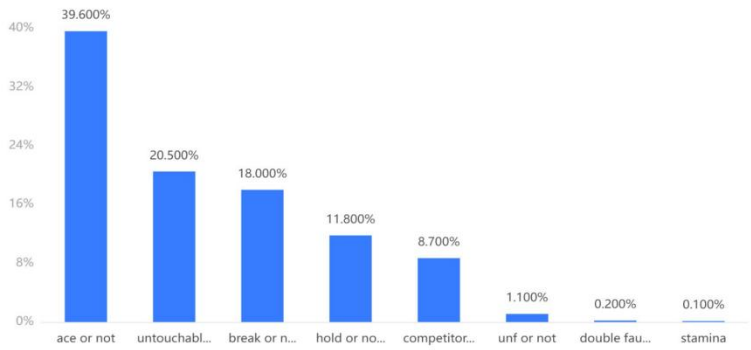

Figure 9: Weights of factors

From the graph, it can be determined that the most relevant influences on momentum fluctuations are ACE shots, untouchable shots, hold serves and break serves.

6.2.3 K-means Clustering Model

K-means Clustering Model is a common unsupervised learning algorithm mainly used for clustering analysis.[5] The goal of this algorithm is to group many data by dividing the dataset into K different clusters, so that the data points within the clusters are more similar to each other and less similar between different clusters.

We define **pos** as the weighted sum of positive indicators and **neg** as the weighted sum of negative indicators, establishing two-dimensional coordinates associated with positive and negative indicators, and apply the K-means Clustering Model to group the momentum at different stages.

Step1: To determine the value of K

To provide as complete a picture as possible of how player's momentum fluctuates throughout the match, we divide the momentum experienced by a player into five phases, as follows:[6]

Phase 1：when momentum is totally against a player
Phase 2：when momentum is turning against a player
Phase 3：when momentum is neutral
Phase 4：when momentum is in a player's favor
Phase 5：when momentum is totally with a player

Therefore we determine the value of K is 5, and divide all data into 5 clusters: C_1, C_2, C_3, C_4, C_5

Step2: To optimize goals

● By iterating over the centers of mass of each cluster $\mu_i = \frac{1}{|C_i|}\sum_{x \in C_i} x$, to make sure that the clusters satisfy the optimization objective: $\min \sum_{i=1}^{k} \sum_{x \in c_i} dist(c_i, x)^2$

The following figure shows the K-means plot of the grouping after it has been processed for center of mass optimization:

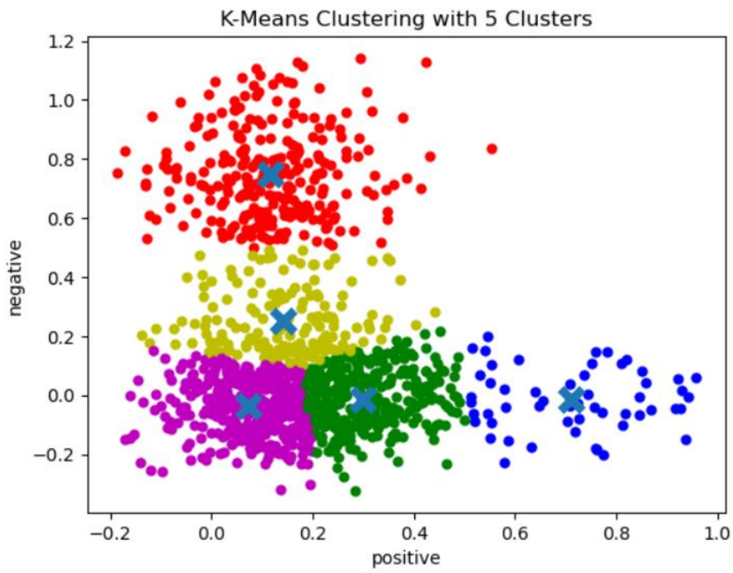

Figure 10: K-means

Combined with the five phases of momentum divided in Step 1, we can confirm that:

- The red cluster represents the phase one of the momentum, and its center of mass coordinates is (0.12372487049254663, 0.29309587923358327)
- The yellow cluster represents the phase two of the momentum, and its center of mass coordinates is (0.12985140819535976, 0.7380935485474919)
- The purple cluster represents the phase three of the momentum, and its center of mass coordinates is (0.1299576451180319, 0.0001787451116204153)
- The green cluster represents the phase four of the momentum, and its center of mass

coordinates is (0.3625475771432825, -0.0001275869305675803)

- The blue cluster represents the phase five of the momentum, and its center of mass coordinates is (0.799503913659468, 0.0001438909681804046)

6.2.4 Define the momentum phase

Mahalanobis Distance is a measure of variability between samples that takes into account the covariance structure of the data set, so the distance between samples can be better measured when the data are correlated or on different scales.

Given a specific dataset for the test set, we assume that each sample is an n-dimensional vector. Mahalanobis Distance can be defined as the distance between two vectors, so we should consider the covariance matrix of them.

We define the correlated matrix of the ith center of mass μ_i and data set x is S. So the Mahalanobis Distance between x and μ_i can be calculated by the following formula:

$$D(x,\mu_i)=\sqrt{(x-\mu_i)^T S^{-1}(x-\mu_i)}$$

The Mahalanobis Distance takes into account the structure of the covariance between the data, so it can be an effective method to measure the distance in cases where the data sets have different scales or there is a correlation. Typically, if the Mahalanobis Distance between two samples is small, they are less differences between them in feature space. Calculate the Mahalanobis Distances between the current score point and the five centers of mass in the K-means plot, and group the current score point into the same cluster as the center of mass with the smallest Mahalanobis Distance.

6.2.5 Forecast momentum fluctuations

During the course of the match, update the grouping of the current scoring points in the K-means plot promptly. We view momentum fluctuates when the clusters between the next scoring point and the previous scoring point are different, i.e., when the phase where the momentum changes.

As the match progresses, the Mahalanobis Distance of the scoring points from each center of mass are calculated promptly, and all scoring points are grouped promptly, so that it is possible to calculate whether the momentum fluctuates and how fluctuates when the next scoring point occurs.

6.3 Advise players based on momentum fluctuations

In tennis, momentum as one of the important factors affecting players' psychology and the game situation, its fluctuation has a significant impact on the outcome of the match. In order to win the game, the players should make full use of the momentum at different phases, and adopt different offensive and defensive strategies corresponding to the momentum at different phases. The strategies that should be adopted by the players to deal with the different types of momentum at different phases of the match are elaborated as:

- When the game just started

At this time, the momentum of both players is at phase 3, i.e. both players have equal momentum. In the case of equal momentum, you should keep calm, make full use of every scoring opportunity, do not leave cracks to the opponent, but also look for the opponent's loopholes in a short period of time, flexibly adjust the strategy through the initial judgment of the opponent's defects, maintain a steady baseline play, and wait for the opponent's mistakes or create scoring opportunities. In the weight assignment, consecutive goals have a certain percentage of weight, which means that scoring consecutively in the early game can increase one's momentum, and players usually show higher confidence after gaining momentum, which is beneficial to the increase of momentum.

- When the early scoring dominates

The score of early match has a very significant impact on the fluctuation of the player's potential energy, if the score of early game has an advantage, the player’s power increases significantly, the momentum begins to tilt in their favor. At this time he should be moderately strengthened offensive and seize the small advantage of the early points to expand the advantage of a more aggressive strategy. After the advantage is expanded, it is easier to score consecutive points, which can be utilized to keep the pressure on the opponent and force the opponent to make mistakes in playing. At the same time, it should be noted that even with the increased momentum in the early stages, it is important to remain focused and patient, avoiding mistakes in capturing large positional advantages due to overconfidence after capturing small positional advantages.

- When the early scoring fails to dominate

If the early scoring fails to dominate and the momentum in the match starts to go against the player, the player needs to adjust his strategy in time to avoid further deterioration of the momentum and try to turn the situation around. Firstly, player needs to control his emotion and remains calm to avoid further errors caused by nervousness or frustration. Secondly, since unforced errors and double faults hold a certain weight in the negative indicators affecting the fluctuation of momentum, player should be alert to prevent the increase of unforced errors and double faults due to their own negative psychological factors and the aggressive attacking strategies of their opponents, which will result in the player never being able to increase his momentum. At the same time, the serve is the only aspect of tennis that can be fully controlled, and changing the serve strategy can help to change the momentum of the match by trying to change the speed of the serve, the angle of the serve, and the depth of the serve in order to find the weaknesses of the opponent.

- When the momentum is relatively better in the middle of the match

If the player's momentum gradually increases after the adjustment of the mindset as well as the offensive and defensive strategies in the early stage, and the momentum is gradually in the upper hand in comparison with the opponent, it means that the existing offensive and defensive strategies are correct, and it is necessary to consolidate the existing advantages, and to maintain the pressure on the opponent through the stable serve and the high-quality return. At the same time, do not use a fixed strategy, you can change the style of playing the, the use of slicing, high ball and other techniques to disrupt the opponent's rhythm, so you can take the initiative to control the pace of the game, the use of momentum to suppress the opponent, to avoid the opponent's counterattack.

- When the momentum dominates overwhelmingly late in the match

When the player occupies an absolute advantage in the momentum in the late stage of the match, the player needs to maintain mental concentration, even with an absolute advantage, he couldn't let down his guard, and should continue to maintain the concentration of the game. At the same time, in the late stage of the game, both sides of the physical energy are exhausted seriously, players need to reasonably allocate physical energy to avoid unnecessary consumption. Finally, use the absolute advantage of momentum to end the game as soon as possible to avoid the reversal of momentum that may result in the reversal of the match.

7 Problem 4

7.1 Verification and Testing of the Model

We have established two models in the first two questions, here referred to as Model 2 and Model 3. Model 2 is mainly used for quantitative researches on the size of momentum, which can be obtained by providing data such as hold and break to get a real-time momentum for a player. Model 3 uses the KMEANS model to obtain the positive and negative values of momentum, thereby finding which phase the current momentum is in. The main idea of this question is to combine Model 2 and Model 3 for predicting the trend in following games and conducting verification. Here, we use "momentum" and "momentum change" as our prediction tools, and compare the actual match results with our prediction results by taking whether the next round scores as the actual match situation.

7.2 Is There a Relationship Between Momentum and Prediction Results?

It indicates that the momentum of athletes often changes before their performance changes, which proves that there is a correlation between momentum and the predicted trend results. However, regarding the "momentum change" among the three prediction factors, the issue to be addressed is: should we use the intra-cluster variation or the inter-cluster variation of momentum as the standard for prediction, or should we combine both?

7.2.1 Decision on Prediction and Its Criteria

In conjunction with the problem statement, we plan to use the 2023 Wimbledon Final match between Carlos Alcaraz and Novak Djokovic for prediction.

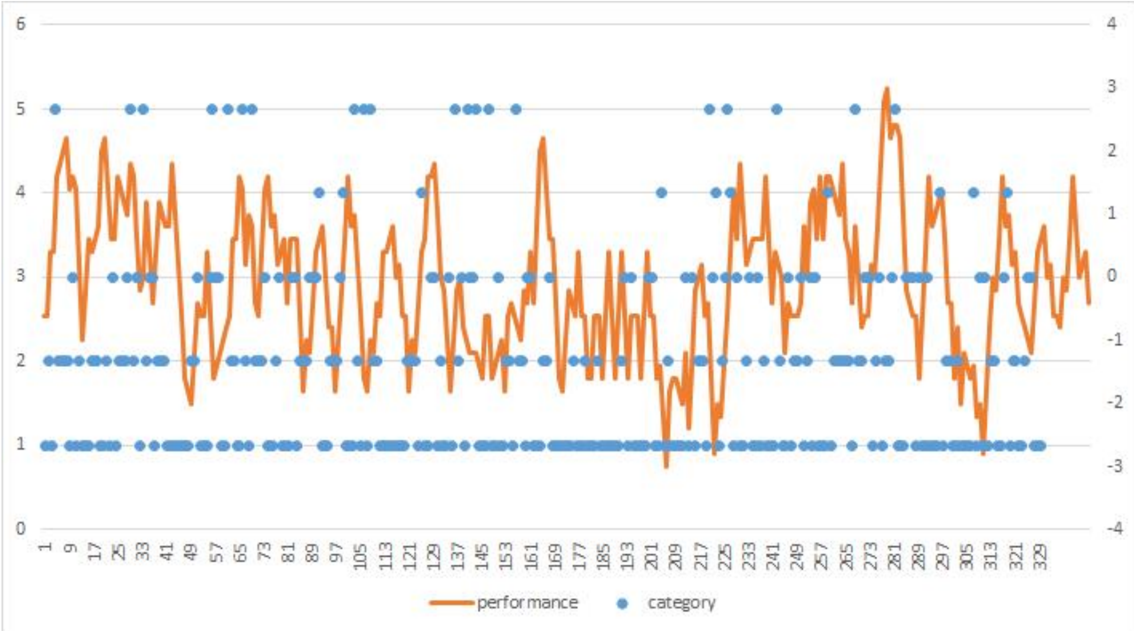

Figure 11: Effect of momentum category on performance

Based on the charts above and the images in the appendix, it is evident that when an athlete's consecutive momentum shifts within the cluster (category stays or not=1), according to Excel data analysis, there is only a 17% probability (percentage of number of "1"s in "if stability affects performance" in all total number of data) that the athlete's performance will fluctuate within an acceptable range (within ±1%) (we use the percentage of difference of current performance and next performance in the current performance to define it). Therefore, it is believed here that there is no strong correlation between the athlete's momentum shifts within the cluster and the final prediction trend of the athlete's performance. Hence, we only observe and verify the match trend when there is a change between clusters.

To make the verification results more convincing and accurate, from the perspective of prediction, we limit the range of prediction results by considering the direction of change in both the athlete's momentum size and the cluster class number of momentum.

For example, if the momentum increases while the cluster class number of momentum also increases, then it is predicted as a better prospect. Additionally, since the two changes we defined are both binary distributions (0 or 1), we introduce F1 score, precision, and recall to evaluate this binary classification model, where:

prediction/reality	suppose a raise in circumstance	predict a decline in circumstance
raise in fact	TRUE POSITIVE(TP)	FALSE NEGATIVE(FN)
decline in fact	FALSE POSITIVE(FP)	TRUE NEGATIVE(TN)

Figure 12: The definitions

- Definition:

(1) If the athlete's momentum increases and the cluster class number of momentum

increases, and the athlete scores in the next round, it is a true positive.

(2) If the athlete's momentum increases and the cluster class number of momentum increases, but the athlete does not score in the next round, it is a false positive.

(3) If the athlete's momentum decreases and the cluster class number of momentum decreases, but the athlete scores in the next round, it is a false negative.

(4) If the athlete's momentum decreases and the cluster class number of momentum decreases, and the athlete does not score in the next round, it is a true negative.

- Precision ratio: $\frac{TP}{TP+FP}$
- Recall ratio: $\frac{TP}{TP+FN}$
- $F_1 = \frac{2\times(PRE\times RE)}{PRE+RE}$

Final F1 score for the last match is 65%,indicating that the combination of the models from the second and third questions (after removing invalid data) provides a relatively accurate prediction of future match trends, thus validating the models.

7.2.2 Reasons for the possibly insufficient fit: :

(1) Integration of serve depth, stroke depth, and ball type with player's dominant hand

(2) Consideration of the energy consumption during strokes inferred from ball speed $E=\frac{1}{2}mv^2$ in the K-means model.

(3) Insufficient features used in the model may fail to accurately describe the complexity of the game. More relevant features such as player skill level, physical condition, pre-match preparation, and court conditions should be considered.

(4) Data may contain missing, outlier, or erroneous values, affecting the training and prediction effectiveness of the model. Data cleaning and preprocessing are necessary to ensure data quality.

(5) Insufficient sample size may hinder the training of an accurate model, especially for complex problems. Gathering more data can improve the model's generalization capability.

(6) Dynamic factors during the game, such as player status, game strategy, injury situation, may affect the game outcome but have not been included in the model.

(7) Interactions between different players may influence the game outcome, such as player rivalry history, tactical styles, etc.

(8) Time series factors may be crucial in predicting sports game outcomes, such as player historical performance, season trends.

To further validate the accuracy of the models in predicting other sports games in the second and third questions, we obtained data from the men's table tennis singles 1/8 finals at the 2016 Rio Olympics through web scraping.

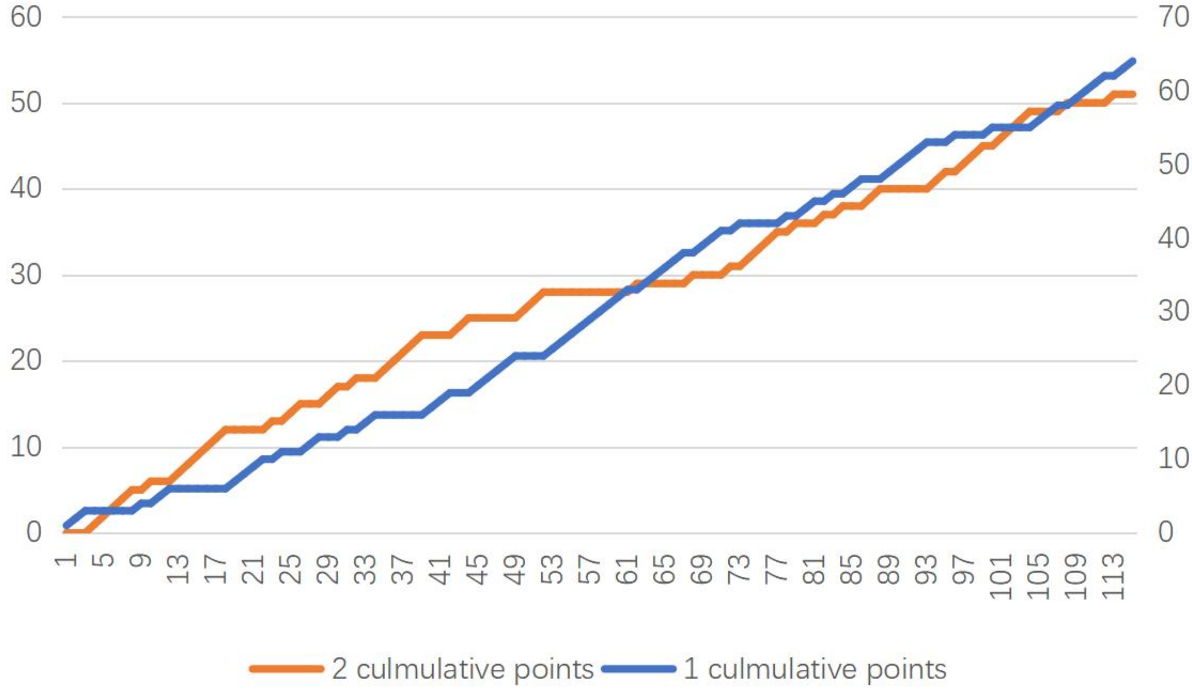

Figure 13: Contrasting culmulative points

Similar operations as in the first question were conducted, and the combination of the two models achieved an accuracy rate of 62.8% for this table tennis match. The reasons for the lower accuracy in predicting table tennis matches compared to tennis matches by 2.2% may include:

1.Physical fitness has a greater impact on tennis players' performance in matches, while it has a smaller impact on table tennis players.

2.In this table tennis match, there is minimal occurrence of unforced errors and double faults, so errors may arise in the calculation and classification of momentum.

3.Tennis and table tennis differ in rules, court size, and match pace, making features used for predicting tennis matches potentially unsuitable for table tennis matches.

4.Data distributions in tennis and table tennis matches may vary, including player skill levels, match intensity, and competition among players.

5.Tennis and table tennis matches may exhibit different time series features during matches; for instance, table tennis matches may have faster ball speeds and shorter rounds, necessitating consideration of distinct time series features.

6.In table tennis matches, local features such as ball speed, spin, and ball placement may have a greater impact on match outcomes, whereas global features such as player skills and fitness may be more crucial in tennis matches.

8 Future work

8.1 Strengths

Our model offers the following strengths:

- .We applied the K-means clustering model. The K-means model is simpler and more efficient than other categorical clustering models, and we exhaustively categorized momentum into five phases, which covers almost all possible momentum scenarios throughout the game, making the detection of momentum fluctuations more significant.

- In the process of quantifying the momentum, we used the consistency test to check the inconsistency tolerance of the pairwise comparison array, which makes the distribution of the weights of the factors affecting the momentum more reasonable and reliable, and also makes the final fitting results satisfactory.
- When building the model, we are relatively more concerned about the uniform momentum influencing factors in all ball games, which makes our model more generalized, not only applicable to the momentum prediction of tennis games, but also more applicable to the momentum prediction of other categories of sports.

8.2 Weaknesses

- When considering the influencing factors affecting momentum, a significant portion of the force majeure factors cannot be accurately predicted, which results in a model fit that is not particularly excellent, and in the future it is possible to go through a large number of databases to consider as many of these factors as possible
- When using K-means to predict momentum fluctuations, only large fluctuations between groups can be predicted, while for small fluctuations within groups, the prediction ability is not good, and in the future, the grouping can be more refined to make the prediction more accurate.

9 References

[1] Arjun Goyal and Jeffrey S. Simonoff.Hot Racquet or Not?An Exploration of Momentum in Grand Slam Tennis Matches.

[2] Lionel Page University of Westminster.The momentum effect in competitions:field evidence from tennis matches Working paper.

[3] Trevor Hastie,Robert Tibshirani,Jerome Friedman.The Elements of Statistical Learning

[4] Bai Zhentao.An Analysis of the Physical Consumption of Players in Tennis Men's Singles Matches and the Indicators related to Match Outcomes

[5] Ian H.Witten,Eibe Frank,Mark A.Hall.Data Mining: Practical Machine Learning Tools and Techniques》

[6] Allistair Higham.Momentum:the hidden force

MEMORANDUM

To: coaches

From: Team#2405978

Subject: qualitative and quantitative momentum effects on decision making of matches

Date: February 6,2024

DEAR COACH,

We are honored to inform you that we have built a quantitative momentum calculating model based on eight factors and tried to give you the earnest decision about how to prepare players to respond to events that impact the flow of play during a tennis match. Our model, strategy to analyze the situations and results are described below.

In our model, we have developed K-means Qualitative Analyzing method that helps you classify which group should you categorize your momentum in.

- **Construction idea**

1. Take all present 8 relative data as input(such as break or not)
2. Build a prediction model to get the value of 'positive' and 'negative' column
3. Put your 'positive' and 'negative' value in the program and get your specific category of the momentum which is directly influenced by the data you have put in.
4. Put 8 relative data into the AHP model and then get the exact value of the current momentum.
5. Collect your value and category's fluctuation columns and utilize them with the column of 'score or not in the next round'.

- **K-means model**

The core idea of the whole model is to make decisions based on the previous two rounds' momentum to predict what would happen next, so correct and accurate model to get the rough range of momentum is very critical. This model takes players' previous matches' into consideration and will give you the best output.

- **AHP model**

After getting the rough range of momentum, you could get the quantitative values of momentums. Combining two models to get the final answer that whether to call timeout or not based on previous data of matches would be most paralleled to the reality, which is up to 65% goodness of fit .Furthermore, the value of fit between momentum calculated by AHP model and reality is actually high up to 83%!

11 References

[7] Arjun Goyal and Jeffrey S. Simonoff.Hot Racquet or Not?An Exploration of Momentum in Grand Slam Tennis Matches.

[8] Lionel Page University of Westminster.The momentum effect in competitions:field evidence from tennis matches Working paper.

[9] Trevor Hastie,Robert Tibshirani,Jerome Friedman.The Elements of Statistical Learning

[10] Bai Zhentao.An Analysis of the Physical Consumption of Players in Tennis Men's Singles Matches and the Indicators related to Match Outcomes

[11] Ian H.Witten,Eibe Frank,Mark A.Hall.Data Mining: Practical Machine Learning Tools and Techniques》

[12] Allistair Higham.Momentum:the hidden force