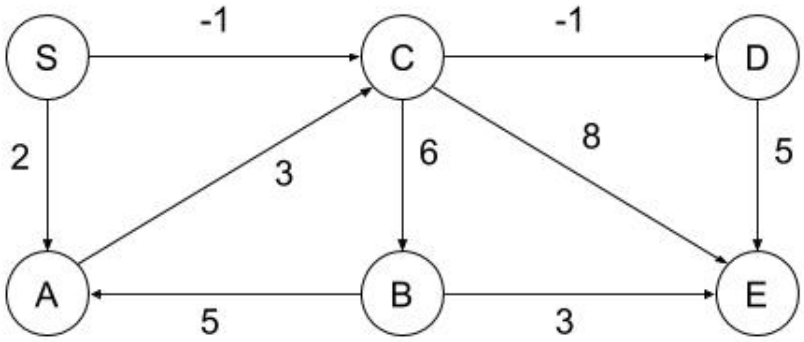
MIDTERM PRACTICE PROBLEMS

1. True/False: Dijkstra's algorithm will return the correct shortest paths from S to all other vertices when run on the graph given below.



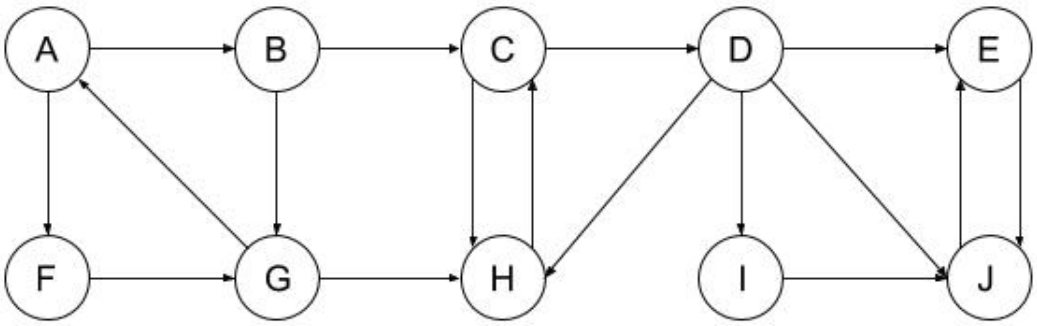
2. True/False: If $f(n) = \mathcal{O}(g(n))$, then $f(5n)$ is $\mathcal{O}(g(5n))$.

3. Suppose we are given an array A containing n elements such that the first p elements are in increasing order and the rest are in decreasing order. Give an algorithm to find the k^{th} smallest element in the array in $\mathcal{O}(log\ n)$ time.

4. You are given a weighted directed graph $G = (V, E, w)$ and the shortest path distances $\delta(s, u)$ from a source vertex s to every other vertex in G. With this information, give an algorithm to find a shortest path from s to a given vertex t in $\mathcal{O}(V + E)$ time. Note the "a given vertex t" in the problem statement.

5. Given a directed acyclic graph $G = (V, E)$ and a vertex u, design an algorithm that outputs all vertices $S \subseteq V$ such that for all $v \in S$, there is an even-length simple path from u to v in G. (A simple path is a path with all distinct vertices.)

6. Run the strongly connected components algorithm on the following directed graph. Whenever there is a choice of vertices to explore, always pick the one that is alphabetically first.



   (a) In what order are the strongly connected components (SCCs) found?

   (b) Which are source SCCs and which are sink SCCs?

   (c) Draw the metagraph (each meta-node is an SCC of G).