Q1 explanation about modeling

1. A student in a Modeling and Data Analysis course downloaded monthly San Diego temperature data covering the previous 2 years (n=24 data points). For each of the following methods, comment on whether it could potentially be an appropriate way of modeling the data. Explain your answer

a. Polynomial regression with polynomial order k=2?

```
No, the curve when k =2 does not match the sin/cos like funciton of the 2 ye
ars temperature data.
```

b. Polynomial regression with order k=24?

```
No, highly possible to be overfitting and uninterpretable
```

c. Polynomial regression with order k=48?

```
No, highly possible to be overfit and uninterpretable
```

d. Spline regression with 35 knots?

```
No, can not match the sin/cos like funciton of the 2 years temperature data.
```

Q2 Principal component (PC) regression.

Principal component (PC) regression. Using the anesthesia dataset from last week, we will perform PCA regression.

In [8]:

```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.stats import pearsonr
import statsmodels.formula.api as smf
from sklearn.model_selection import KFold
```

executed in 14.9s, finished 17:31:55 2018-11-20

In [334]:

```
df = pd.read_csv('anesthesia.csv')
df.head()
```

executed in 118ms, finished 21:25:09 2018-11-20

Out[334]:

	Time	F0Hz_1	F1Hz_2	F3Hz_3	F4Hz_4	F5Hz_5	F6Hz_6	F8Hz_7	F9Hz_8	F1
0	5.004	3.115293	1.676500	1.097419	0.900837	0.537178	0.454494	0.512818	-0.131658	-0.1
1	15.004	2.864158	1.499845	0.879378	1.020294	0.281333	0.722017	0.086080	0.080071	0.1
2	25.004	2.039253	1.057344	0.163134	0.351954	0.149567	0.325558	0.231917	0.284673	-0.0
3	35.004	2.417074	0.348083	0.582521	0.468952	0.176949	0.116783	0.200230	0.166558	0.1
4	45.004	2.507836	1.036731	0.622822	0.436470	0.465713	0.703881	0.048926	-0.327707	0.0

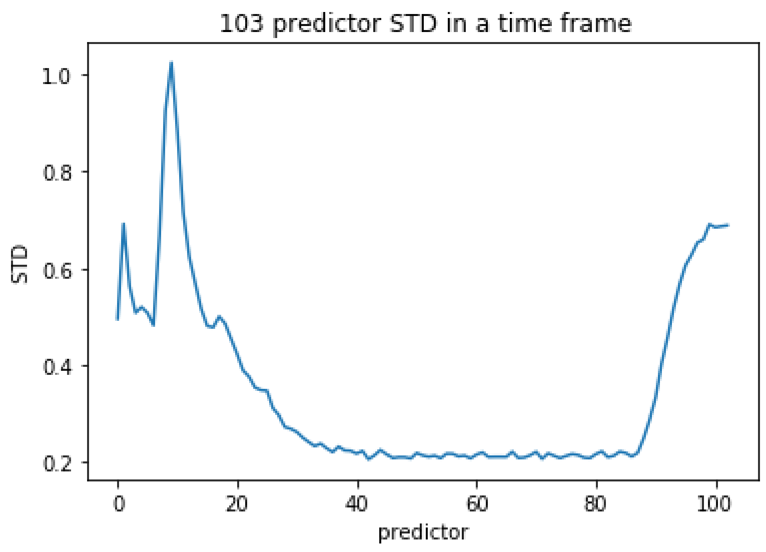
5 rows × 105 columns

a) Compute the standard deviation (s.d.) of each of the predictors and make a plot showing them. Which frequency band has the largest s.d.?

In [113]:

```
# get all EEG power columns
x_cols = [col for col in df.columns if col not in ['Time', 'BehaviorResponse']]

std_list = []

for col in x_cols:
     std_list.append(np.std(df[col] ))

x = range(103)
plt.plot( x, std_list )
plt.title(" 103 predictor STD in a time frame")
plt.xlabel(" predictor ")
plt.ylabel(" STD ")
plt.show()
```

executed in 341ms, finished 19:50:45 2018-11-20



In [119]:

```python
print "the largest STD: ", max(std_list)
print "And the frequency brand is ", np.argsort(-np.array(std_list))[0]
```

executed in 10ms, finished 19:53:24 2018-11-20

```
the largest STD:  1.02438689155
And the frequency brand is  9
```

b

In [148]:

```python
S_matrix = df.copy()

for col_idx in range(1,104):
    col_label = df.columns[col_idx]
    S_matrix[col_label] = df.iloc[:,col_idx] / std_list[col_idx-1]

print "after nomalization"
S_matrix.head()
```

executed in 134ms, finished 20:07:59 2018-11-20

```
after nomalization
```

Out[148]:

	Time	F0Hz_1	F1Hz_2	F3Hz_3	F4Hz_4	F5Hz_5	F6Hz_6	F8Hz_7	F9Hz_8	F1
0	5.004	6.291898	2.424252	1.951658	1.775085	1.034106	0.896090	1.064482	-0.197641	-0.1
1	15.004	5.784684	2.168805	1.563892	2.010472	0.541587	1.423545	0.178680	0.120201	0.1
2	25.004	4.118641	1.528940	0.290118	0.693520	0.287928	0.641878	0.481401	0.427343	-0.0
3	35.004	4.881719	0.503335	1.035959	0.924062	0.340640	0.230252	0.415628	0.250033	0.2
4	45.004	5.065028	1.499133	1.107631	0.860057	0.896531	1.387788	0.101558	-0.491945	0.0

5 rows × 105 columns

In [163]:

```python
for col in range(1,104):
    if round(np.std(S_matrix[S_matrix.columns[col]])) == 1:
        pass
    else:
        print "error"
print "verified"
```

executed in 28ms, finished 20:12:36 2018-11-20

```
verified
```

c. (3 points) Compute the top 20 principal components (PCs) of the predictors. Make a plot showing the value of PC1 as a function of time throughout the experiment. Make similar plot showing the values of PC2 and PC3 vs. time

In [203]:

```
from sklearn.decomposition import PCA

data = S_matrix.iloc[:,1:104]
print data.shape
```

executed in 11ms, finished 20:24:42 2018-11-20

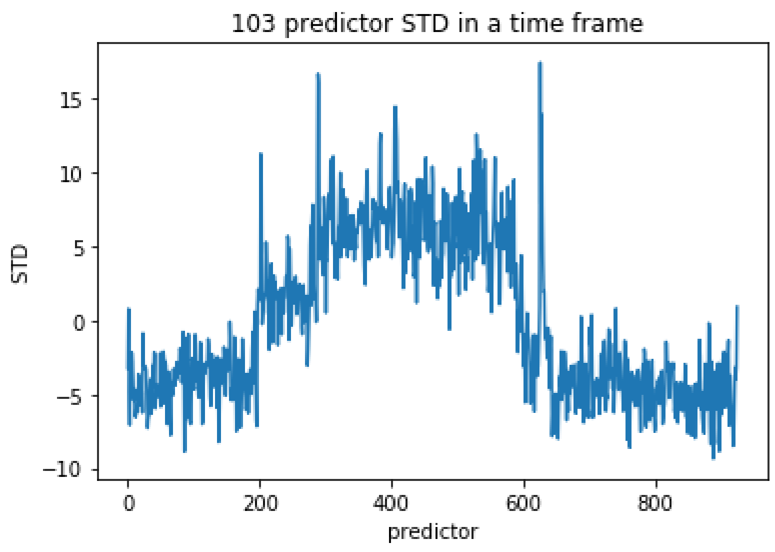
```
(926, 103)
```

In [207]:

```
pca = PCA(n_components=20)
new_data = pca.fit_transform(data)
print new_data.shape

x = range(926)
plt.plot( x, new_data[:,0] )
plt.title(" PC1 VS Time")
plt.xlabel(" time ")
plt.ylabel(" PC1 ")
plt.show()
```

executed in 284ms, finished 20:26:31 2018-11-20

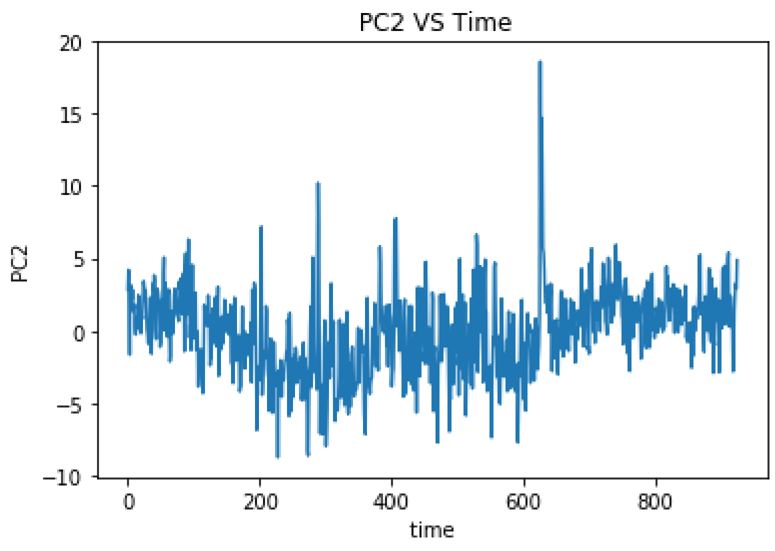
```
(926, 20)
```



In [208]:

```
x = range(926)
plt.plot( x, new_data[:,1] )
plt.title(" PC2 VS Time")
plt.xlabel(" time ")
plt.ylabel(" PC2 ")
plt.show()
```

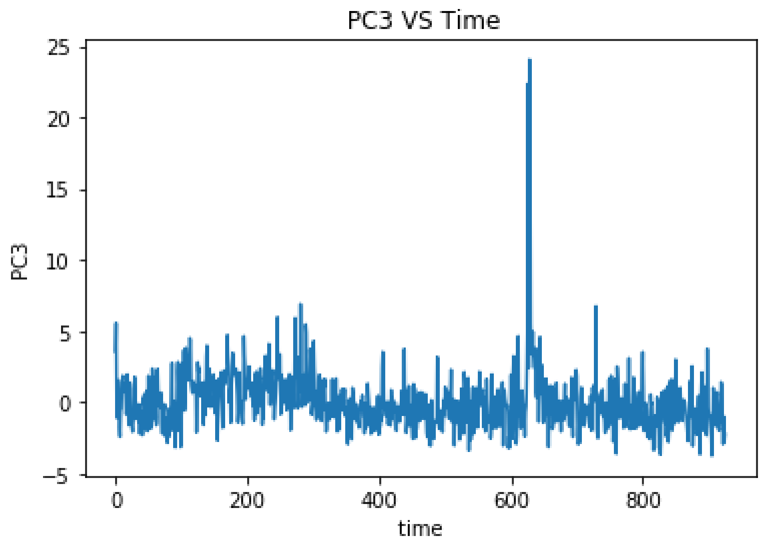
executed in 299ms, finished 20:27:15 2018-11-20



In [209]:

```
x = range(926)
plt.plot( x, new_data[:,2] )
plt.title(" PC3 VS Time")
plt.xlabel(" time ")
plt.ylabel(" PC3 ")
plt.show()
```

executed in 269ms, finished 20:27:25 2018-11-20



d. (4 points) Run PC regression by fitting a series of linear models using the top k=1, 2, 3, …, 20 principal components. Use 10fold crossvalidation to estimate the test set error for each linear model. Make a plot showing the MSE for training and test data as a function of k.

In [511]:

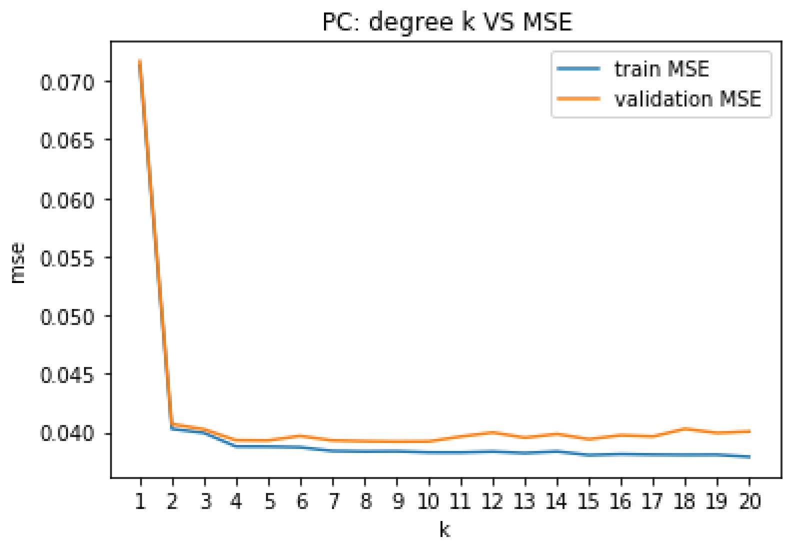
```python
from sklearn import linear_model
# from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_squared_error
from sklearn.model_selection import KFold

train_mse_list = []
test_mse_list = []

for k in range(1,21):

    pca = PCA(n_components=k)
    new_data = pca.fit_transform(data)

    train_mse_temp = []
    test_mse_temp = []

    #define K fold
    kf = KFold (n_splits= 10 , shuffle = True)

    for train_idx, test_indx in kf.split(new_data):

        X_train, X_test = new_data[train_idx], new_data[test_indx]

        y_train, y_test = df['BehaviorResponse'][train_idx], df['BehaviorRespo

        # Create linear regression object
        regr = linear_model.LinearRegression()

        # Train the model using the training sets
        regr.fit(X_train , y_train)

        # Make prediction using both train and test data
        pred_train =  regr.predict(X_train)
        pred_test = regr.predict(X_test)

        train_mse = mean_squared_error(pred_train, y_train)
        test_mse = mean_squared_error(pred_test, y_test)

        train_mse_temp.append(train_mse)
        test_mse_temp.append(test_mse)

    train_mse_list.append(np.mean(train_mse_temp))
    test_mse_list.append(np.mean(test_mse_temp))

```

executed in 1.86s, finished 23:10:00 2018-11-20

In [514]:

```python
x = range (1,21)
plt.xticks(range(1,21))
plt.plot(x, train_mse_list , label ='train MSE')
plt.plot(x, test_mse_list , label ='validation MSE')
plt.title(" PC: degree k VS MSE")
plt.xlabel(" k ")
plt.ylabel(" mse ")
plt.legend()
plt.show()
```

executed in 434ms, finished 23:10:39 2018-11-20



e. (1 point) Based on these results, what value of k would you select to provide the best predictive accuracy?

based on the result, I think both k=4 is good because it might overfit if we add more predictors and this is a reasonable transition spot where the MSE is low as well

f. (1 point) Conceptual question (no coding required): Consider a linear model that includes just the top PC (k=1). Which frequencies contribute to this model's prediction? Explain

the 10th(9th starting from index 0) frequency contribute most to this model's prediction because for this one its variance is the MAX . PC 1 caputrues the max var of any linear comb of the predictors.

Q 3. Polynomial and spline regression

try to fit smooth curves to the timecourse of the behavioral response variable in the Anesthesia dataset.

a. (1 point) Plot BehaviorResponse vs. time.

In [368]:

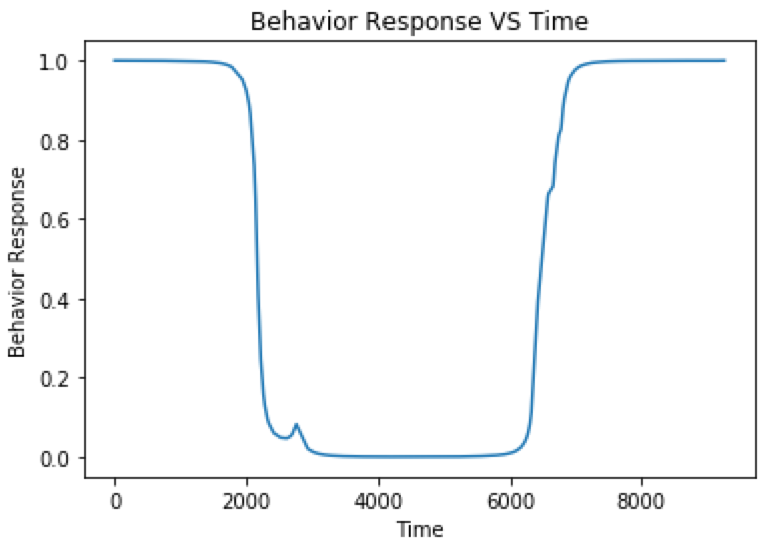
```
# Part A
# Make a plot of Time Vs. BehaviorResponse for the anesthesia data.
time = df['Time']
BehaviorResponse = df['BehaviorResponse']
```

executed in 8ms, finished 21:42:14 2018-11-20

In [369]:

```
plt.plot( time, BehaviorResponse)
plt.title("Behavior Response VS Time")
plt.xlabel("Time")
plt.ylabel("Behavior Response")
plt.show()

```

executed in 232ms, finished 21:42:15 2018-11-20



b) Fit a linear model of the form: , where is BehaviorResponse y = β0 + β1x y and x is time. Plot the resulting prediction, yˆ , as a function of time and overlay it on top of the true data. Is this model too simple or too complex for this dataset?

In [535]:

```python
from sklearn import linear_model

cosnt_df = pd.Series(np.ones(926))
new_time = pd.concat([time, cosnt_df], axis=1)

# Create linear regression object
regr = linear_model.LinearRegression()

# Train the model using the training sets
regr.fit(new_time , BehaviorResponse)

# Make predictions using the testing set
pred_y = regr.predict(new_time)

print regr.coef_

plt.plot( time, BehaviorResponse)
plt.plot( time , pred_y , 'red')
plt.title(" time VS pred_y ")
plt.xlabel(" time ")
plt.ylabel(" pred_y ")
plt.show()
```

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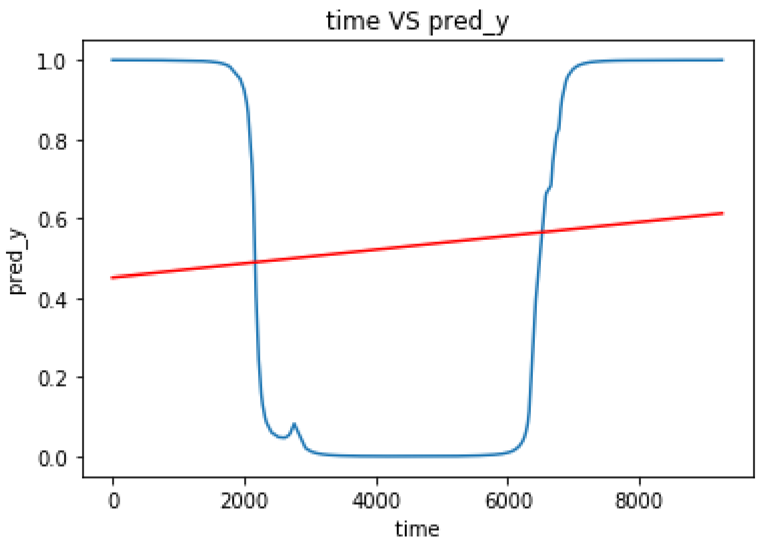
```
[1.74252205e-05 0.00000000e+00]
```



Is this model too simple or too complex for this dataset?

Obviously this model is too simple because it does not fit the dataset well

c. (2 points) Fit polynomials of order 2 (quadratic), 3 (cubic), 5, 10 and 20. For each polynomial, plot yˆ vs. time.

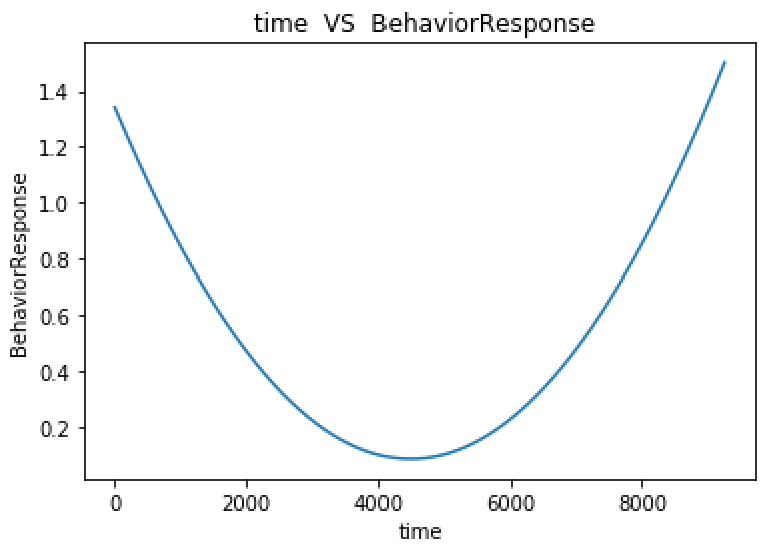
In [460]:

```
print "polynomials of order 2"

weights = np.polyfit(time, BehaviorResponse, 2)
model = np.poly1d(weights)
y_plot = model(time)
plt.plot(time, y_plot)
plt.title(" time  VS  BehaviorResponse")
plt.xlabel("time")
plt.ylabel('BehaviorResponse')
plt.show()
```

executed in 365ms, finished 22:31:41 2018-11-20

```
polynomials of order 2
```



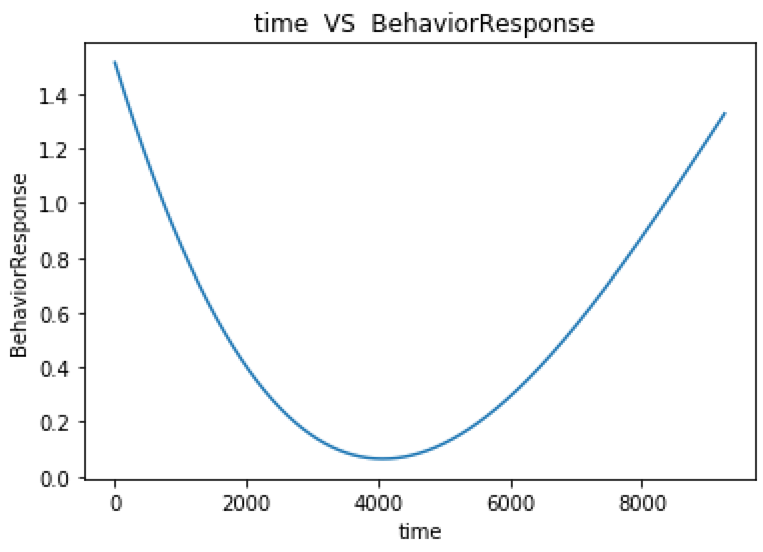
In [461]:

```
print "polynomials of order 2"

weights = np.polyfit(time, BehaviorResponse, 3)
model = np.poly1d(weights)
y_plot = model(time)
plt.plot(time, y_plot)
plt.title(" time  VS  BehaviorResponse")
plt.xlabel("time")
plt.ylabel('BehaviorResponse')
plt.show()
```

executed in 355ms, finished 22:31:44 2018-11-20

```
polynomials of order 2
```



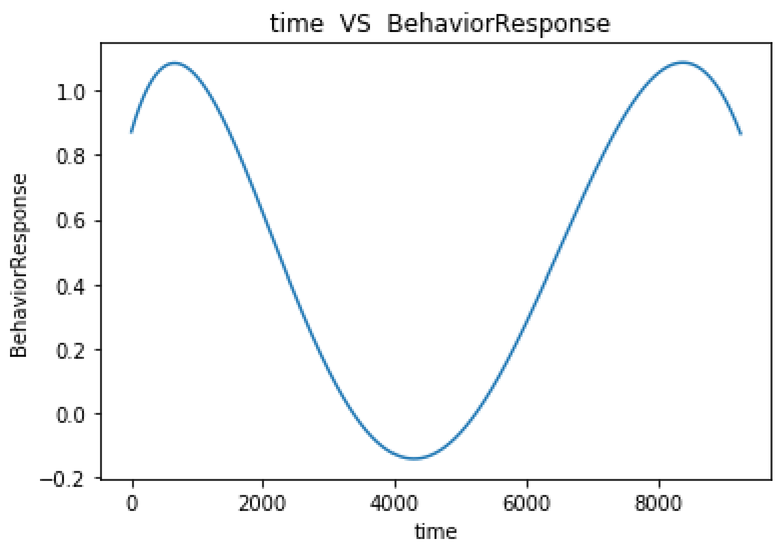
In [462]:

```
print "polynomials of order 5"

weights = np.polyfit(time, BehaviorResponse, 5)
model = np.poly1d(weights)
y_plot = model(time)
plt.plot(time, y_plot)
plt.title(" time  VS  BehaviorResponse")
plt.xlabel("time")
plt.ylabel('BehaviorResponse')
plt.show()
```

executed in 464ms, finished 22:31:47 2018-11-20

```
polynomials of order 5
```



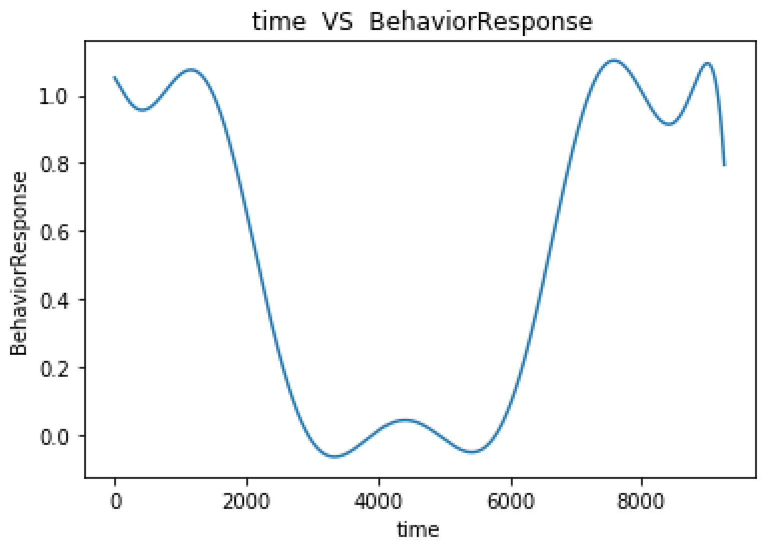
In [463]:

```
print "polynomials of order 10"

weights = np.polyfit(time, BehaviorResponse, 10)
model = np.poly1d(weights)
y_plot = model(time)
plt.plot(time, y_plot)
plt.title(" time  VS  BehaviorResponse")
plt.xlabel("time")
plt.ylabel('BehaviorResponse')
plt.show()
```

executed in 328ms, finished 22:31:50 2018-11-20

```
polynomials of order 10
```



In [524]:

```
print "polynomials of order 20"

weights = np.polyfit(time, BehaviorResponse, 20)
model = np.poly1d(weights)
y_plot = model(time)

plt.plot(time, y_plot)
plt.title(" time  VS  BehaviorResponse")
plt.xlabel("time")
plt.ylabel('BehaviorResponse')
plt.show()
```

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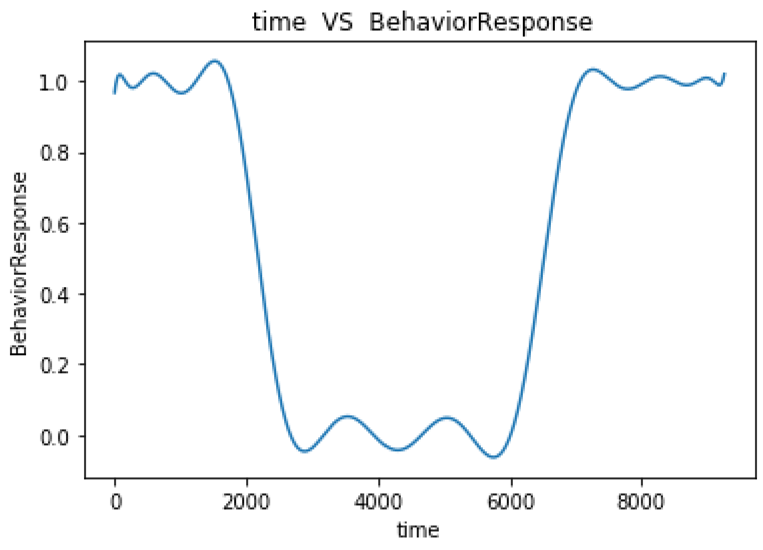
```
polynomials of order 20

/Users/xuzhaokai/anaconda2/lib/python2.7/site-packages/ipykernel_launc
her.py:3: RankWarning: Polyfit may be poorly conditioned
  This is separate from the ipykernel package so we can avoid doing im
ports until
```



d. (3 points) Using 10 fold crossvalidation, determine the testing MSE for all polynomials of order 1 up to 20. Plot the training and test MSE vs. model order.

In [548]:

```python
train_mse_list = []
test_mse_list = []

for k in range(1,21):

    train_mse_temp = []
    test_mse_temp = []

    #define K fold
    kf = KFold (n_splits= 10 , shuffle = True)

    for train_idx, test_indx in kf.split(df['Time']):

        X_train, X_test = df['Time'][train_idx], df['Time'][test_indx]

        y_train, y_test = df['BehaviorResponse'][train_idx], df['BehaviorRespo

        # training
        weights = np.polyfit(X_train.reshape(-1), y_train, k)
        model = np.poly1d(weights)

        # Make prediction using both train and test data
        pred_train =  model(X_train)
        pred_test = model(X_test)

        train_mse = mean_squared_error(pred_train, y_train)
        test_mse = mean_squared_error(pred_test, y_test)

        train_mse_temp.append(train_mse)
        test_mse_temp.append(test_mse)

    train_mse_list.append(np.mean(train_mse_temp))
    test_mse_list.append(np.mean(test_mse_temp))

```

executed in 3.47s, finished 23:37:38 2018-11-20

```
/Users/xuzhaokai/anaconda2/lib/python2.7/site-packages/ipykernel_launc
her.py:19: FutureWarning: reshape is deprecated and will raise in a su
bsequent release. Please use .values.reshape(...) instead
/Users/xuzhaokai/anaconda2/lib/python2.7/site-packages/ipykernel_launc
her.py:19: RankWarning: Polyfit may be poorly conditioned
/Users/xuzhaokai/anaconda2/lib/python2.7/site-packages/ipykernel_launc
her.py:19: RankWarning: Polyfit may be poorly conditioned
/Users/xuzhaokai/anaconda2/lib/python2.7/site-packages/ipykernel_launc
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her.py:19: RankWarning: Polyfit may be poorly conditioned
```

```
/Users/xuzhaokai/anaconda2/lib/python2.7/site-packages/ipykernel_launc
her.py:19: RankWarning: Polyfit may be poorly conditioned
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her.py:19: RankWarning: Polyfit may be poorly conditioned
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her.py:19: RankWarning: Polyfit may be poorly conditioned
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her.py:19: RankWarning: Polyfit may be poorly conditioned
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her.py:19: RankWarning: Polyfit may be poorly conditioned
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her.py:19: RankWarning: Polyfit may be poorly conditioned
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her.py:19: RankWarning: Polyfit may be poorly conditioned
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her.py:19: RankWarning: Polyfit may be poorly conditioned
/Users/xuzhaokai/anaconda2/lib/python2.7/site-packages/ipykernel_launc
her.py:19: RankWarning: Polyfit may be poorly conditioned
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her.py:19: RankWarning: Polyfit may be poorly conditioned
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her.py:19: RankWarning: Polyfit may be poorly conditioned
/Users/xuzhaokai/anaconda2/lib/python2.7/site-packages/ipykernel_launc
her.py:19: RankWarning: Polyfit may be poorly conditioned
/Users/xuzhaokai/anaconda2/lib/python2.7/site-packages/ipykernel_launc
her.py:19: RankWarning: Polyfit may be poorly conditioned
/Users/xuzhaokai/anaconda2/lib/python2.7/site-packages/ipykernel_launc
her.py:19: RankWarning: Polyfit may be poorly conditioned
/Users/xuzhaokai/anaconda2/lib/python2.7/site-packages/ipykernel_launc
her.py:19: RankWarning: Polyfit may be poorly conditioned
```
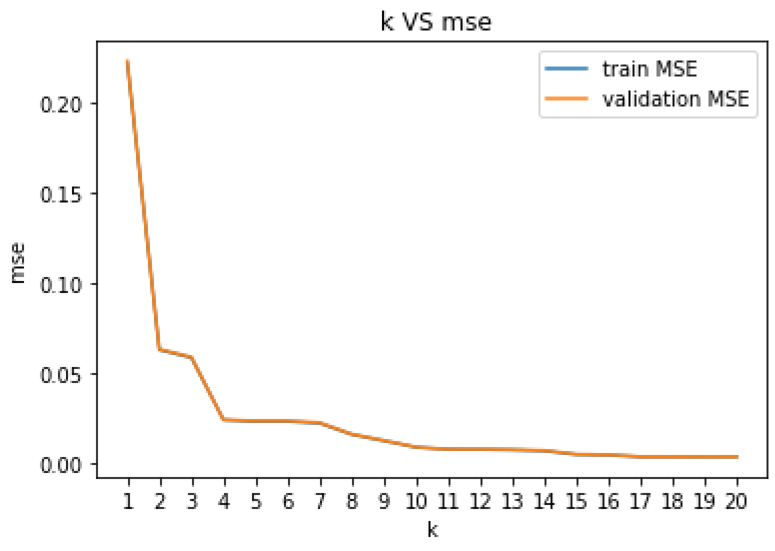

In [549]:

```
x = range (1,21)
plt.xticks(range(1,21))
plt.plot(x, train_mse_list , label ='train MSE')
plt.plot(x, test_mse_list , label ='validation MSE')
plt.title(" k VS mse")
plt.xlabel(" k ")
plt.ylabel(" mse ")
plt.legend()
plt.show()
```

executed in 754ms, finished 23:37:41 2018-11-20



In [551]:

```
print "min train MSE: ", np.argsort(train_mse_list)[0]
print "min val MSE: ", np.argsort(test_mse_list)[0]
```

executed in 9ms, finished 23:39:10 2018-11-20

```
min train MSE:  16
min val MSE:  18
```

e . (1 point) Based on these results, what model order would you recommend for fitting these data?

Based on the above plot, I think using a degree of 16--18 poly to fit the data should get a good result because this have a good balance btw train and test acc

In [ ]:

```

```