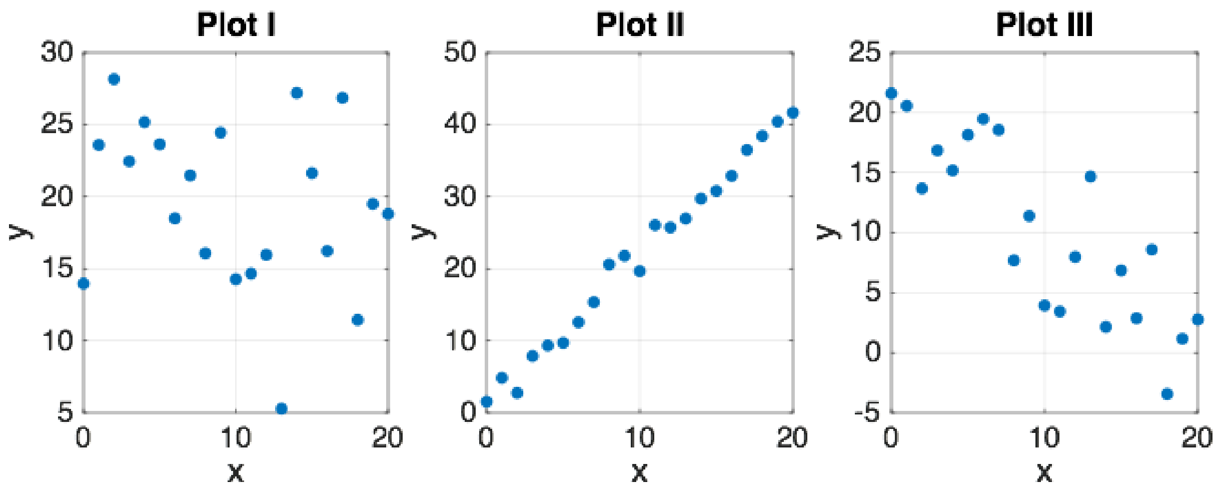
Cogs 109: Modeling and Data Analysis

Homework 2

Due Friday 10/13 in class



1. For each of the three data sets plotted above (I, II and III), answer the following:
   a. (3 points) Does the data show a positive or negative correlation between x and y?

      No correlation, positive, and negative, respectively.

   b. (3 points) Which function (equation) best describes each data set?
      i. $f(x) = 1 + 2x + \varepsilon$

         Plot II

      ii. $f(x) = 20 + \varepsilon$

         Plot I

      iii. $f(x) = 20 - x + \varepsilon$

         Plot III

   c. (3 points) Which regression table corresponds to each plot?

i.

	Estimate	SE	tStat	pValue
(Intercept)	1.2478	0.61327	2.0347	0.056077
x1	2.0417	0.052459	38.92	1.3891e-19

Plot II

ii.

	Estimate	SE	tStat	pValue
(Intercept)	20.273	1.7491	11.591	4.6458e-10
x1	-1.0082	0.14962	-6.7383	1.9406e-06

Plot III

iii.

	Estimate	SE	tStat	pValue
(Intercept)	21.808	2.4438	8.9236	3.1883e-08
x1	-0.2323	0.20905	-1.1112	0.28033

Plot I

2. (A: 3 points, B: 1 point, C: 2 points) ISLR chapter 3, problem 3 (page 120)

   A) For a fixed value of IQ and GPA, males earn more on average than females provided that the GPA is high enough.

   B) 137.1

   C) False. We use the p-value to determine whether a regression coefficient is significant or not.

3. (2 points for each part) ISLR chapter 3, problem 4 (pages 120-121)

   A) The cubic model would have a lower RSS because more flexible models always have a lower RSS in the training data.

   B) If the true model is linear, then the cubic model would have a higher RSS in our testing data because of overfitting.

   C) Again, the cubic model will have a lower RSS because more flexible models always have a lower RSS in the training data.

   D) Because we do not know the true model for the data, we cannot say.

4. **UPDATED:** In this problem, we will simulate a dataset and use multiple linear regression to investigate it. Imagine we conduct a survey of N=100 students and ask them how much time per week they spend on work (x_1) and how much time on play (x_2). We also ask them about their overall level of satisfaction (y), which we take to be the outcome. Download the dataset HW2.csv from the course website, which contains these data.

   a. (3 points) Make a scatter plot showing y vs. x_1. Comment on the relationship between these variables: do they appear correlated (positively or negatively)? Is their relationship linear or non-linear?

   b. (4 points) Fit a simple linear regression of y vs. x_1. In MATLAB, you could use the function `regress` or `fitlm`. Report the estimated intercept and slope, and make a plot showing the data points together with the regression line. Is there a statistically significant effect of x_1 on y? NOTE: The Matlab function `regress` sdf

   c. (1 point) What is the 95% confidence interval for the slope of x_1?

   d. (2 points) Now fit a multiple linear regression with x_1 and x_2 as independent variables. Report a table with the regression results (similar to Table 3.9 on page 88 in ISLR). Which parameters have a statistically significant effect?

   e. (2 points) Make a scatter plot showing y vs. $\hat{y}$, the predicted value of y.

f. (3 points) Create a categorical variable with 3 levels called WorkType, where WorkType="Idle" for $x_1 < 10,$ WorkType="Diligent" for $10 \le x_1 < 30$, and WorkType="Workaholic" for $x_1 \geq 30$. Fit a linear regression of y against WorkType and x_2 , and report the regression table.

g. (2 point) In part (f) you should have obtained two different coefficients for WorkType corresponding to different "levels" of this categorical variable. What is your interpretation of the term corresponding to WorkType=Workaholic?

HW2Solutions

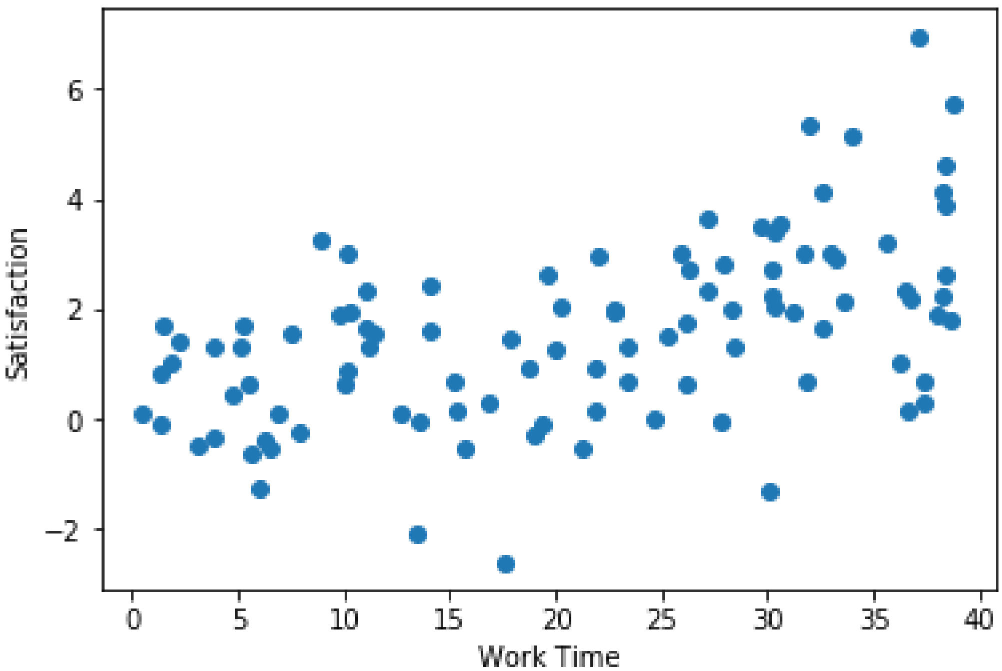
October 19, 2018

```
In [1]: import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        import statsmodels.api as sm
        from patsy import dmatrices
```

```
In [2]: df = pd.read_csv('HW2.csv', header=None, names=["x1", "x2", "y"])

        print(df.head())
```

```
        x1       x2       y
0  32.5890   6.4873  4.1549
1  36.2320  31.7710  1.0401
2   5.0795  12.4490  1.3170
3  36.5350  21.1410  2.3423
4  25.2940   6.6259  1.5134
```

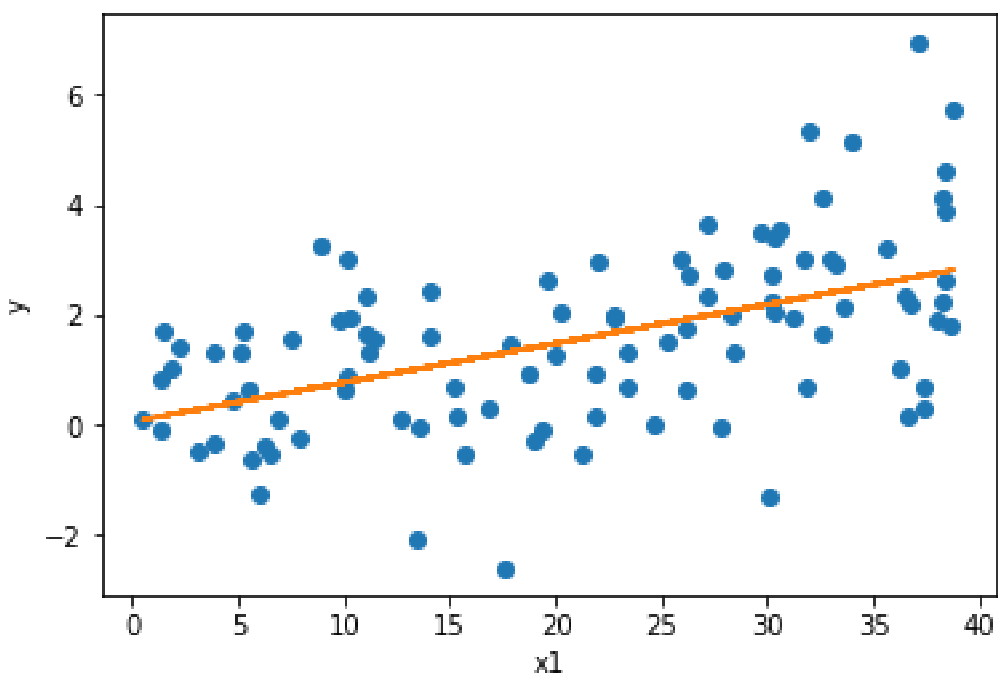
```
In [3]: # 4 (a)
        fig, ax = plt.subplots()
        ax.scatter(df.x1, df.y)
        ax.set_xlabel('Work Time')
        ax.set_ylabel('Satisfaction')
        plt.show()
```




```
In [4]: # 4 (b) (c)
        ### do linear regression
        # setup input data
        y, X = dmatrices('y ~ x1', data=df, return_type='dataframe')
        # print(y.head())
        # print(X.head())
        # describe model
        mod = sm.OLS(y, X)
        # fit model
        res = mod.fit()
        # look at results
        print(res.summary())
        yhat = np.dot(X.values, res.params.values)
        fig, ax = plt.subplots()
        ax.scatter(df.x1, df.y)
        ax.plot(df.x1, yhat, color='C1')
        ax.set_xlabel('x1')
        ax.set_ylabel('y')
        plt.show()
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.263
Model:                            OLS   Adj. R-squared:                  0.255
Method:                 Least Squares   F-statistic:                     34.90
```

```
Date:                Fri, 19 Oct 2018   Prob (F-statistic):          5.04e-08
Time:                        22:35:36   Log-Likelihood:               -176.08
No. Observations:                 100   AIC:                            356.2
Df Residuals:                      98   BIC:                            361.4
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      0.0604      0.291      0.207      0.836      -0.517       0.638
x1             0.0711      0.012      5.907      0.000       0.047       0.095
==============================================================================
Omnibus:                        1.420   Durbin-Watson:                  2.256
Prob(Omnibus):                  0.492   Jarque-Bera (JB):               0.913
Skew:                          -0.025   Prob(JB):                       0.634
Kurtosis:                       3.465   Cond. No.                        49.6
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

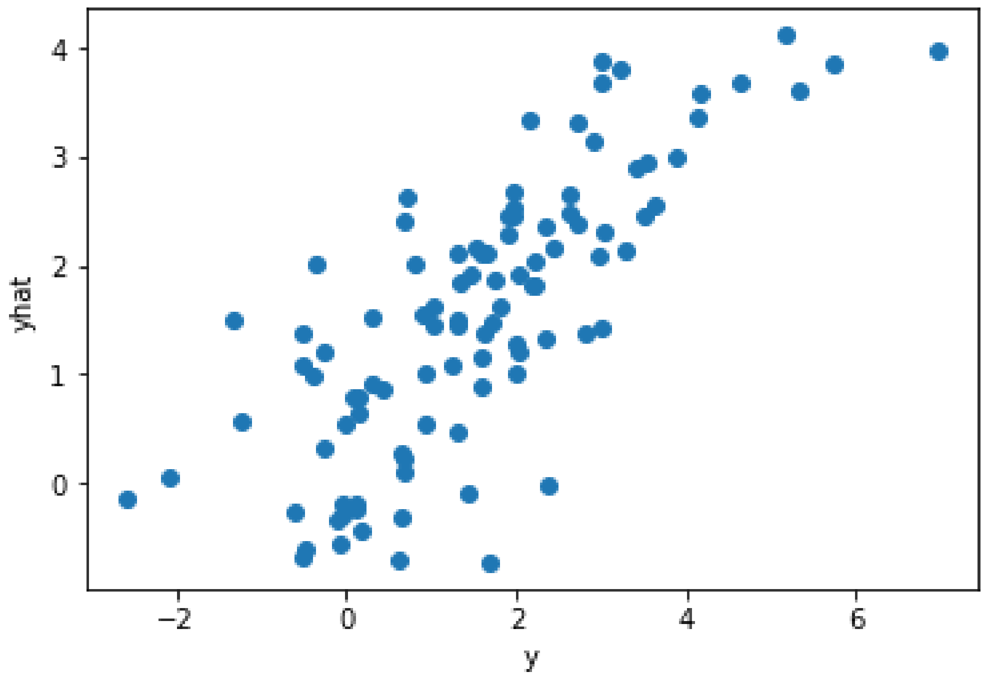


In [5]:
```
# 4 (d)
### do multi-linear regression
3
```

```python
# setup input data
y, X = dmatrices('y ~ x1 + x2', data=df, return_type='dataframe')
# print(y.head())
# print(X.head())
# describe model
mod = sm.OLS(y, X)
# fit model
res = mod.fit()
# look at results
print(res.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.575
Model:                            OLS   Adj. R-squared:                  0.566
Method:                 Least Squares   F-statistic:                     65.64
Date:                Fri, 19 Oct 2018   Prob (F-statistic):           9.39e-19
Time:                        22:35:36   Log-Likelihood:                -148.52
No. Observations:                 100   AIC:                             303.0
Df Residuals:                      97   BIC:                             310.8
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      1.8659      0.308      6.053      0.000       1.254       2.478
x1             0.0571      0.009      6.119      0.000       0.039       0.076
x2            -0.0808      0.010     -8.446      0.000      -0.100      -0.062
==============================================================================
Omnibus:                        1.629   Durbin-Watson:                   2.210
Prob(Omnibus):                  0.443   Jarque-Bera (JB):                1.117
Skew:                           0.076   Prob(JB):                        0.572
Kurtosis:                       3.495   Cond. No.                         85.7
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

In [9]:
```python
# 4 (e)
yhat = res.predict(X.values)
4

fig, ax = plt.subplots()
ax.plot(y, yhat, 'o')
ax.set_xlabel('y')
ax.set_ylabel('yhat')
plt.show()
```




```
In [10]: # 4(f)
         WorkType = []
         for item in df.x1:
             if item < 10:
                 WorkType.append('Idle')
             elif 10 <= item < 30:
                 WorkType.append('Diligent')
             elif item >=30:
                 WorkType.append('Workaholic')

         print(WorkType)
         df['WorkType'] = WorkType
         print(df.head())
```

```
['Workaholic', 'Workaholic', 'Idle', 'Workaholic', 'Diligent', 'Idle', 'Diligent', 'Diligent',
        x1       x2       y    WorkType
0  32.5890   6.4873  4.1549  Workaholic
1  36.2320  31.7710  1.0401  Workaholic
2   5.0795  12.4490  1.3170        Idle
3  36.5350  21.1410  2.3423  Workaholic
4  25.2940   6.6259  1.5134    Diligent
```

```
In [11]: # 4(f)
```

```python
### do linear regression with categorical variables
# setup input data
y, X = dmatrices('y ~ WorkType + x2', data=df, return_type='dataframe')
print(y.head())
print(X.head())
# describe model
mod = sm.OLS(y, X)
# fit model
res = mod.fit()
# look at results
print(res.summary())
```

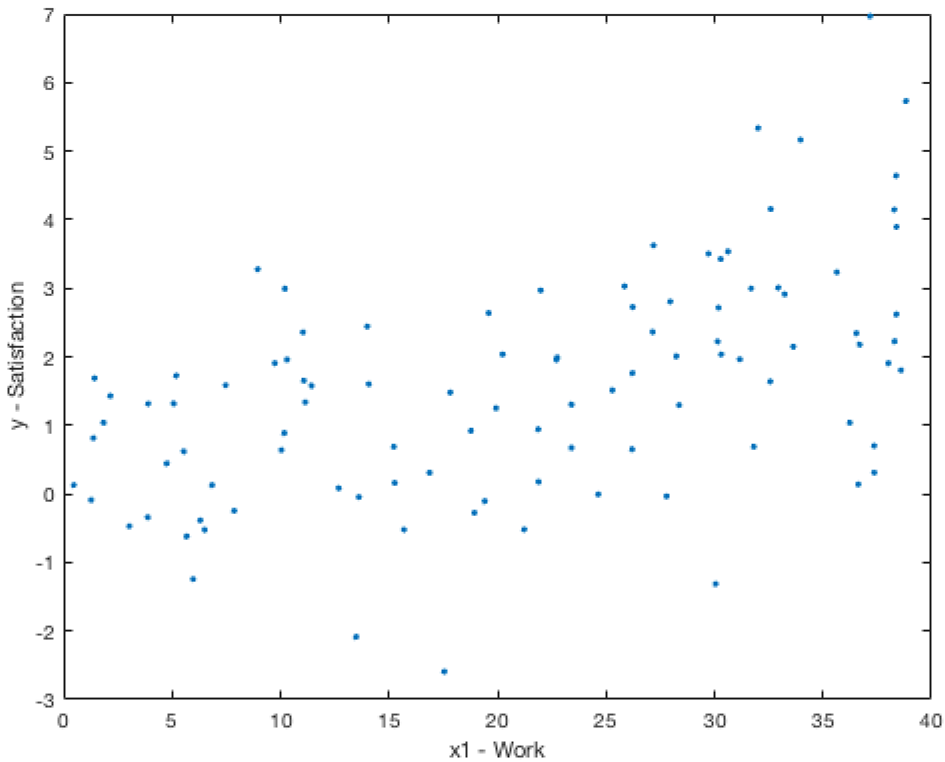
```
        y
0  4.1549
1  1.0401
2  1.3170
3  2.3423
4  1.5134
   Intercept  WorkType[T.Idle]  WorkType[T.Workaholic]       x2
0        1.0               0.0                     1.0   6.4873
1        1.0               0.0                     1.0  31.7710
2        1.0               1.0                     0.0  12.4490
3        1.0               0.0                     1.0  21.1410
4        1.0               0.0                     0.0   6.6259
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.588
Model:                            OLS   Adj. R-squared:                  0.575
Method:                 Least Squares   F-statistic:                     45.71
Date:                Fri, 19 Oct 2018   Prob (F-statistic):           1.94e-18
Time:                        22:35:44   Log-Likelihood:                -146.95
No. Observations:                 100   AIC:                             301.9
Df Residuals:                      96   BIC:                             312.3
Df Model:                           3
Covariance Type:            nonrobust
==========================================================================================
                             coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------------------
Intercept                  2.7356      0.233     11.760      0.000       2.274       3.197
WorkType[T.Idle]          -0.2262      0.282     -0.803      0.424      -0.785       0.333
WorkType[T.Workaholic]     1.4040      0.247      5.678      0.000       0.913       1.895
x2                        -0.0842      0.009     -8.893      0.000      -0.103      -0.065
==============================================================================
Omnibus:                        1.628   Durbin-Watson:                   2.027
Prob(Omnibus):                  0.443   Jarque-Bera (JB):                1.088
Skew:                          -0.123   Prob(JB):                        0.580
Kurtosis:                       3.447   Cond. No.                         68.6
==============================================================================
```

```
Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Table of Contents

Problem 4a

```
clear
clf
data = readtable('HW2.csv');
data.Properties.VariableNames = {'x1','x2','y'};

figure(1)
plot(data.x1,data.y,'.')
xlabel('x1 - Work')
ylabel('y - Satisfaction')
```



4b

```
p = fitlm(data,'y~x1')
yhat_x1 = predict(p,data);
```

```
p = 


Linear regression model:
    y ~ 1 + x1

Estimated Coefficients:
                   Estimate       SE        tStat       pValue  
                   ________    ________    _______    __________

    (Intercept)    0.060392     0.29112    0.20745       0.83609
    x1             0.071053    0.012028     5.9072    5.0376e-08


Number of observations: 100, Error degrees of freedom: 98
Root Mean Squared Error: 1.42
R-squared: 0.263,  Adjusted R-Squared 0.255
F-statistic vs. constant model: 34.9, p-value = 5.04e-08
```

4c

```
confidence = coefCI(p)
```

```
confidence =

   -0.5173    0.6381
    0.0472    0.0949
```

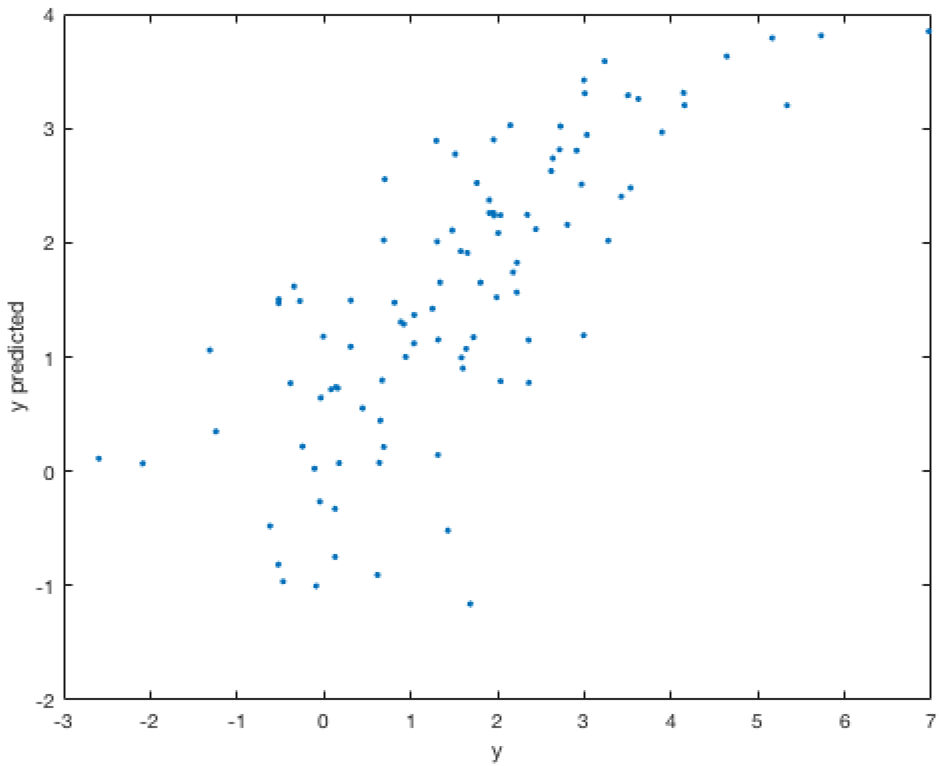
4d

```
p = fitlm(data,'y~x1+x2')
yhat_x1_x2 = predict(p,data);
```

```
p = 


Linear regression model:
    y ~ 1 + x1 + x2

Estimated Coefficients:
                   Estimate        SE         tStat       pValue  
                   _________    _________    _______    __________

    (Intercept)       1.8659      0.30828     6.0528     2.683e-08
    x1              0.057066    0.0093256     6.1193     1.987e-08
    x2             -0.080764    0.0095621    -8.4462    2.9961e-13


Number of observations: 100, Error degrees of freedom: 97
Root Mean Squared Error: 1.08
R-squared: 0.575,  Adjusted R-Squared 0.566
F-statistic vs. constant model: 65.6, p-value = 9.39e-19
```

4e

```
figure(2); clf
plot(data.y, yhat_x1_x2, '.')
hold on
%plot([0,6],[0,6],'k-')
xlabel('y')
ylabel('y predicted')
```



4f

```
N = size(data,1);
data.WorkType = repmat({'Idle'},N,1);
data.WorkType(data.x1>=10 & data.x1<30) = {'Diligent'};
data.WorkType(data.x1>=30) = {'Workaholic'};
p = fitlm(data,'y~1+WorkType+x2')
```

```
p =


Linear regression model:
    y ~ 1 + x2 + WorkType

Estimated Coefficients:
                         Estimate        SE         tStat        pValue

                         _________    _________    _______
 __________

    (Intercept)             4.1396      0.24919     16.612
 5.4727e-30
    x2                   -0.084184    0.0094664    -8.8929
 3.5502e-14
```

```
    WorkType_Idle          -1.6302      0.30179    -5.4017
 4.7861e-07
    WorkType_Diligent       -1.404      0.24728    -5.6777
 1.4511e-07


Number of observations: 100, Error degrees of freedom: 96
Root Mean Squared Error: 1.07
R-squared: 0.588,  Adjusted R-Squared 0.575
F-statistic vs. constant model: 45.7, p-value = 1.94e-18
```

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