Katie ROC Regression

jdt

2023-09-24

load packages

```
library("tidyverse"); theme_set(theme_classic())
```

```
## -- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
## v dplyr     1.1.2     v readr     2.1.4
## v forcats   1.0.0     v stringr   1.5.0
## v ggplot2   3.4.3     v tibble    3.2.1
## v lubridate 1.9.2     v tidyr     1.3.0
## v purrr     1.0.2
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```
library("boot")
library("pROC")
```

```
## Type 'citation("pROC")' for a citation.
##
## Attaching package: 'pROC'
##
## The following objects are masked from 'package:stats':
##
##     cov, smooth, var
```

```
library("mvtnorm")
library("quantreg")
```

```
## Loading required package: SparseM
##
## Attaching package: 'SparseM'
##
## The following object is masked from 'package:base':
##
##     backsolve
```

```
library("betareg")
library("gamlss")
```

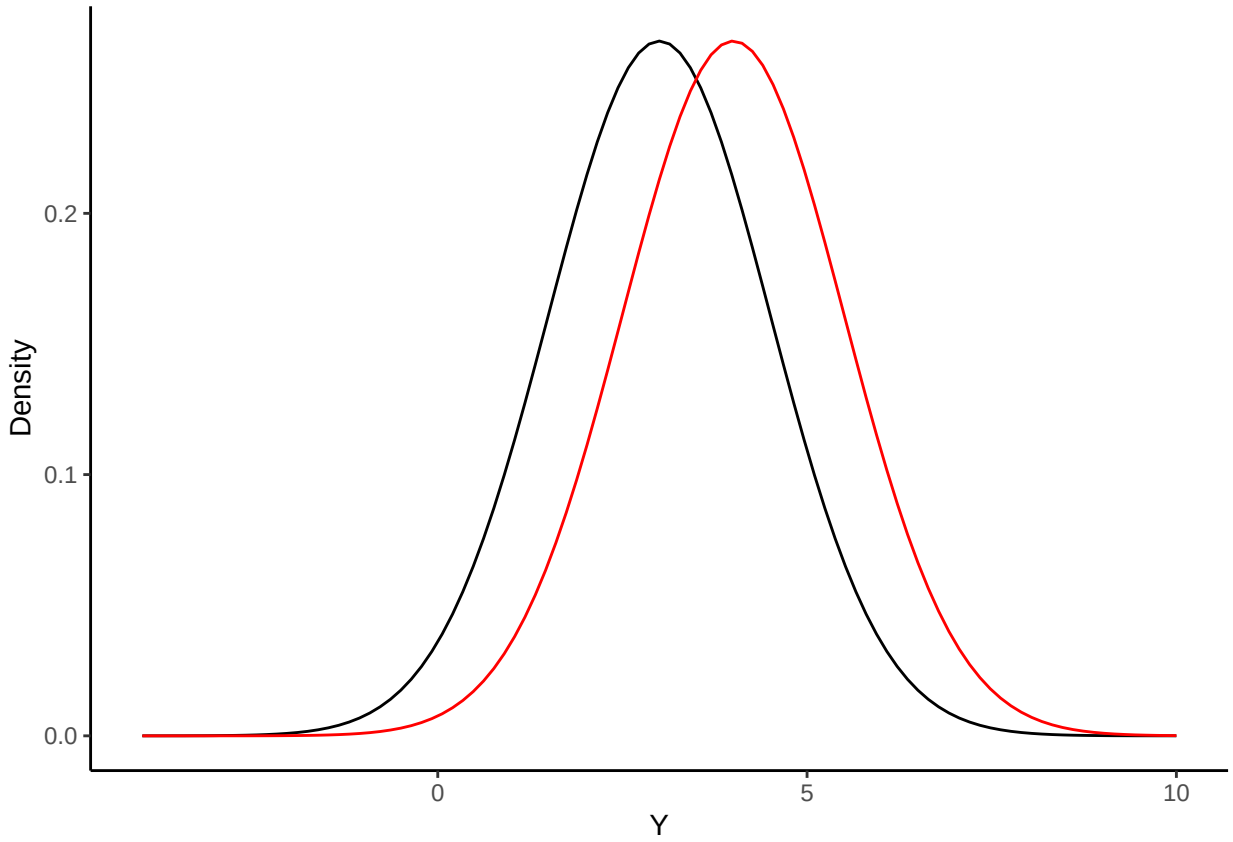
```
## Loading required package: splines
## Loading required package: gamlss.data
##
## Attaching package: 'gamlss.data'
```

```
##
## The following object is masked from 'package:boot':
##
##     aids
##
## The following object is masked from 'package:datasets':
##
##     sleep
##
## Loading required package: gamlss.dist
## Loading required package: nlme
##
## Attaching package: 'nlme'
##
## The following object is masked from 'package:dplyr':
##
##     collapse
##
## Loading required package: parallel
##  **********   GAMLSS Version 5.4-18  **********
## For more on GAMLSS look at https://www.gamlss.com/
## Type gamlssNews() to see new features/changes/bug fixes.
```

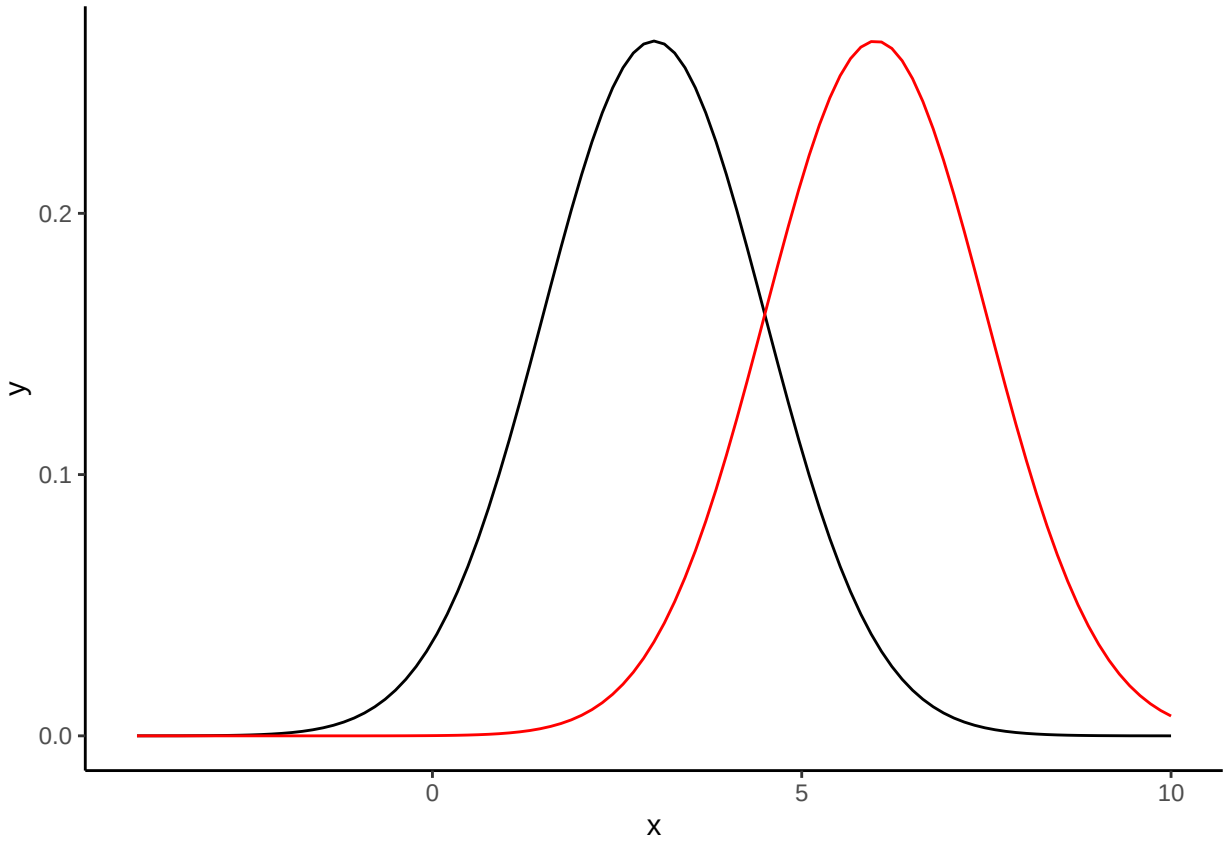
```
library("ggplot2")
```

Simulate binormal data

```
# density functions for Y1 and Y0
Norm_dens <- function(Y, c, d, sd_e){
  dnorm(Y, c + d * .5, sd_e) # mean of U(0, 1) r.v. is 0.5
}

# SCENARIO 1: normal distributions with minimal separation
SC1_H <- list(c = 1.5, d = 3, sd = 1.5)
SC1_D <- list(c = 2, d = 4, sd = 1.5)

ggplot(data.frame(x = c(-4, 10)), aes(x)) +
  stat_function(fun = Norm_dens, args = SC1_H) +
  stat_function(fun = Norm_dens, args = SC1_D, color = "red") +
  xlab("Y") +
  ylab("Density")
```




```
# SCENARIO 2: normal distributions with considerable separation
SC2_H <- SC1_H
SC2_D <- list(c = 3, d = 6, sd = 1.5)

ggplot(data.frame(x = c(-4, 10)), aes(x)) +
  stat_function(fun = Norm_dens, args = SC2_H) +
  stat_function(fun = Norm_dens, args = SC2_D, color = "red")
```

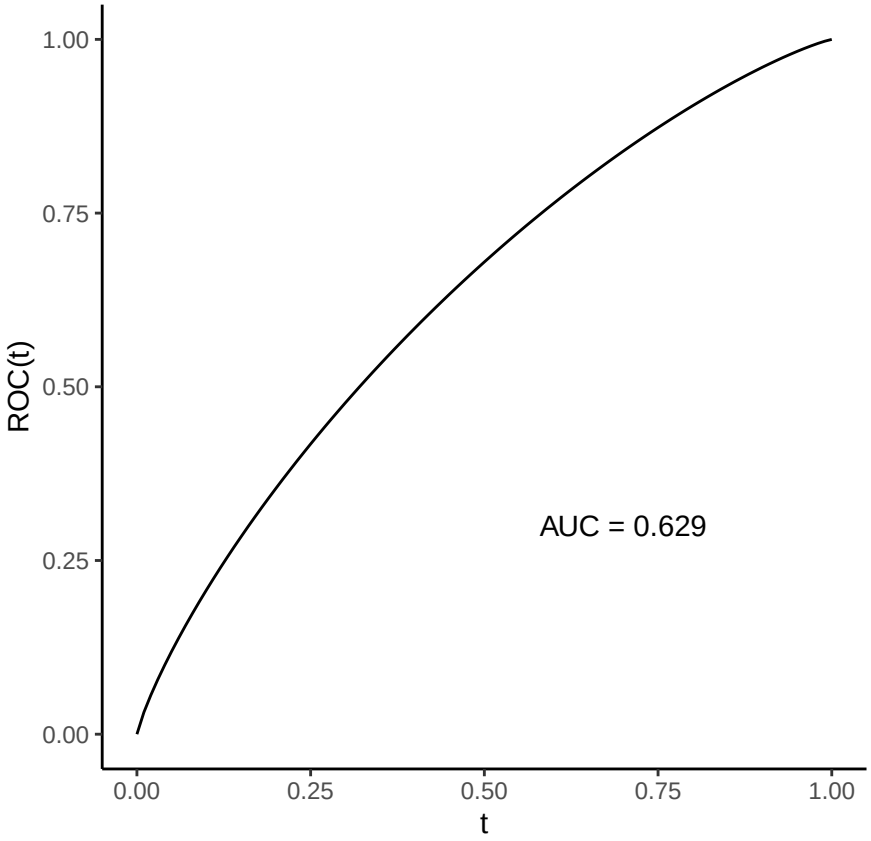



```r
# binormal ROC curves for each scenario

# True covariate-adjusted binormal ROC curve
true_binormal_ROC <- function(t, c1, d1, sd1, c0, d0, sd0, X0){
  a <- (c1 + d1 * X0 - (c0 + d0 * X0)) / sd1
  b <- sd0 / sd1
  pnorm(a + b * qnorm(t))
}

# True covariate-adjusted binormal AUC
true_binormal_AUC <- function(c1, d1, sd1, c0, d0, sd0, X0){
  a <- (c1 + d1 * X0 - (c0 + d0 * X0)) / sd1
  b <- sd0 / sd1
  pnorm(a / sqrt(1 + b^2))
}

SC1_H_v <- unlist(SC1_H)
SC1_D_v <- unlist(SC1_D)
```

SCENARIO 1: TRUE ROC curve and AUC

```r
ggplot(data.frame(t = c(0, 1)), aes(t)) +
  stat_function(fun = true_binormal_ROC, args = list(SC1_D_v[1], SC1_D_v[2],
  SC1_D_v[3], SC1_H_v[1], SC1_H_v[2], SC1_H_v[3], X0 = 0.2)) +
  annotate("text", x = 0.7, y = 0.3,
           label = paste0("AUC = ", true_binormal_AUC(SC1_D_v[1],
```

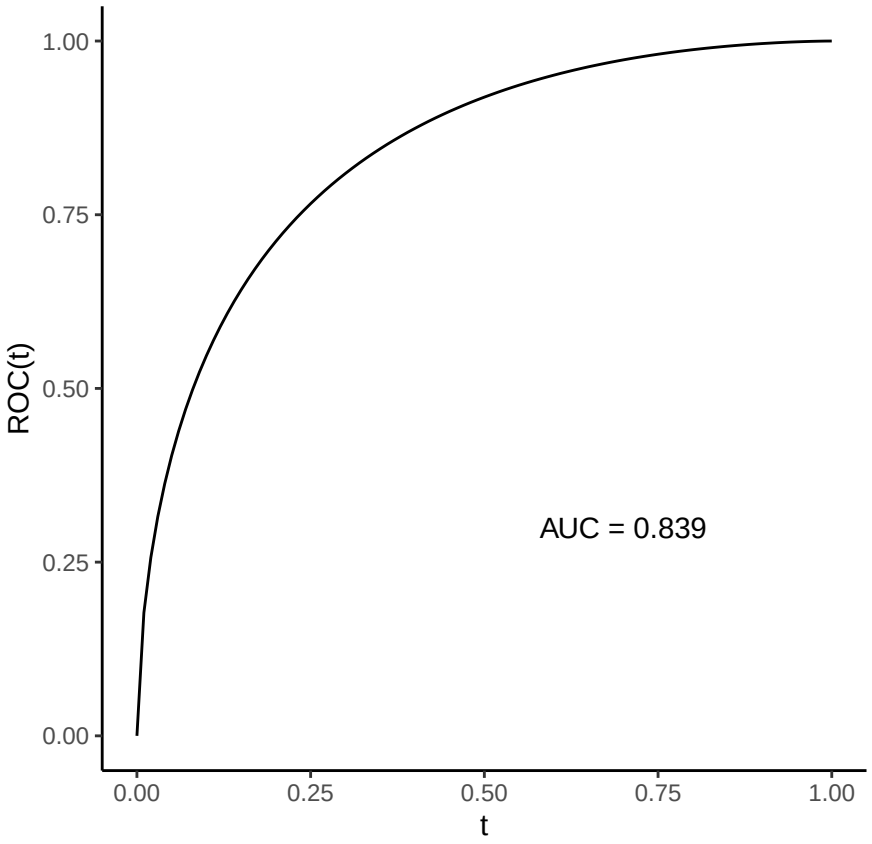
```
          SC1_D_v[2], SC1_D_v[3],
          SC1_H_v[1], SC1_H_v[2], SC1_H_v[3],
          X0 = 0.2) %>% round(3))) +
  ylab("ROC(t)") +
  coord_fixed(ratio = 1)
```




```
SC2_H_v <- unlist(SC2_H)
SC2_D_v <- unlist(SC2_D)
```

SCENARIO 2: TRUE ROC curve and AUC

```
ggplot(data.frame(t = c(0, 1)), aes(t)) +
  stat_function(fun = true_binormal_ROC,
  args = list(SC2_D_v[1], SC2_D_v[2], SC2_D_v[3], SC2_H_v[1],
  SC2_H_v[2], SC2_H_v[3], X0 = 0.2)) +
  annotate("text", x = 0.7, y = 0.3,
           label = paste0("AUC = ",
           true_binormal_AUC(SC2_D_v[1], SC2_D_v[2], SC2_D_v[3],
           SC2_H_v[1], SC2_H_v[2], SC2_H_v[3],
           X0 = 0.2) %>% round(3))) +
  ylab("ROC(t)") +
  coord_fixed(ratio = 1)
```




```
# function to generate binormal data
gen_Norm_data <- function(c0, d0, sd_e0, n0, c1, d1, sd_e1, n1){
  x <- runif(n1)
  eps0 <- rnorm(n0, 0, sd_e0)
  eps1 <- rnorm(n1, 0, sd_e1)
  y0 <- c0 + d0 * x + eps0
  y1 <- c1 + d1 * x + eps1

  data.frame(y = c(y0, y1), dis = as.factor(c(rep(0, n0), rep(1, n1))),
             x = rep(x, 2))
}

n_T <- 50
t <- seq(1/n_T, (n_T - 1)/n_T, 1/n_T) # set of FPRs considered
```

```
set.seed(2433)
# generate 1,000 datasets  # Changed to 100
dats <- replicate(100, gen_Norm_data(1.5, 3, 1.5, 200, 2, 4, 1.5, 200),
simplify = FALSE)
```

2. Functions to calculate ROC for each method

A. parametric distribution-free method (PDF)

```
PDF_FUN <- function(data, t){
  df0 <- filter(data, dis == 0)
```

```
  df1 <- filter(data, dis == 1)

  n1 <- nrow(df1)
  # quantile regression to estimate reference survival
  qr0 <- rq(y ~ x, data = df0, tau = t)

  # covariate adjusted survival for diseased observations
  pred1 <- predict.rq(qr0, newdata = df1)

  Inv_t <- qnorm(rev(t)) # normal quantiles of FPRs
  nq <- length(t)        # number of quantiles

  # reshaping the data
  t_pred1 <- t(pred1)
  col_pred1 <- c(t_pred1)
  col_t <- rep(Inv_t, n1)
  col_y1 <- rep(df1$y, each = nq)
  col_x <- rep(df1$x, each = nq)
  col_uit <- as.numeric(col_y1 >= col_pred1)

  probitData <- data.frame("fdbar" = col_pred1, "phiInv" = col_t,
  "disRes" = col_y1, "covX" = col_x, "uit" = col_uit)

  probMod <- glm(uit ~ phiInv + covX, family = binomial(link = "probit"),
                 data = probitData)
  probMod$coefficients
}
```

B. Beta method.

function that transforms PVs, so they're in the open unit interval

```
dep_trans_betareg <- function(y){
  n_obs <- sum(!(is.na(y)))
  (y * (n_obs - 1) + 0.5) / n_obs
}

Beta_FUN <- function(data, t){
  df0 <- filter(data, dis == 0)
  df1 <- filter(data, dis == 1)

  Y_D <- pull(df1, y)
  n1 <- nrow(df1)

  # quantile regression to estimate reference survival
  qr0 <- rq(y ~ x, data = df0, tau = t)

  # covariate adjusted survival for diseased observations
  pred1 <- predict.rq(qr0, newdata = df1)
  t_pred1 <- t(pred1)
  pred3 <- lapply(seq_len(ncol(t_pred1)), function(i) t_pred1[,i])
  PV_dis <- mapply(function(x, y) mean(x >= y), x = pred3, y = Y_D)
```

```
  datdis <- data.frame(Y_D, x = df1$x, PV_dis)

  if(any(datdis$PV_dis == 0 | datdis$PV_dis == 1)){ # transform PVs
    SUM0 <- sum(datdis$PV_dis == 0) # keep PVs = 0
    SUM1 <- sum(datdis$PV_dis == 1) # keep track of PVs = 1
    BetaMod <- betareg(dep_trans_betareg(PV_dis) ~ x, data = datdis,
                       link.phi = "identity", link = "logit")

    list(BetaMod$coefficients, SUM0 = SUM0, SUM1 = SUM1)
  }else{
    BetaMod <- betareg(PV_dis ~ x, data = datdis,
                       link.phi = "identity", link = "logit")

    list(BetaMod$coefficients)
  }
}
```

C. Inflated Beta method.

```
InfBeta_FUN <- function(data, t){
  df0 <- filter(data, dis == 0)
  df1 <- filter(data, dis == 1)

  Y_D <- pull(df1, y)
  n1 <- nrow(df1)

  # quantile regression to estimate reference survival
  qr0 <- rq(y ~ x, data = df0, tau = t)

  # covariate adjusted survival for diseased observations
  pred1 <- predict.rq(qr0, newdata = df1)
  t_pred1 <- t(pred1)
  pred3 <- lapply(seq_len(ncol(t_pred1)), function(i) t_pred1[,i])
  PV_dis <- mapply(function(x, y) mean(x >= y), x = pred3, y = Y_D)
  disdat <<- data.frame(Y_D, x = df1$x, PV_dis)

  if(any(disdat$PV_dis == 0) & any(disdat$PV_dis == 1)){ ## BEINF Beta Inflated
    SUM0 <- sum(disdat$PV_dis == 0)
    SUM1 <- sum(disdat$PV_dis == 1)
    Beta_InfMod <<- gamlss(PV_dis ~ x, data = disdat, family = BEINF)
    list(Beta_InfMod$mu.coefficients,
         Beta_InfMod$sigma.coefficients,
         Beta_InfMod$nu.coefficients,
         Beta_InfMod$tau.coefficients, SUM0 = SUM0, SUM1 = SUM1)
  }else if(any(disdat$PV_dis == 0)){      ## BEINF0 Beta Inflated zero
    SUM0 <- sum(disdat$PV_dis == 0)
    Beta_0Inf <<- gamlss(PV_dis ~ x, data = disdat, family = BEINF0)
    list(Beta_0Inf$mu.coefficients,
         Beta_0Inf$sigma.coefficients,
         Beta_0Inf$nu.coefficients, SUM0 = SUM0, SUM1 = 0)
  }else if(any(disdat$PV_dis == 1)){     ## BEINF1 Beta Inflated one
    SUM1 <- sum(disdat$PV_dis == 1)
```

```
    Beta_1Inf <- gamlss(PV_dis ~ x, data = disdat, family = BEINF1)
    list(Beta_1Inf$mu.coefficients,
         Beta_1Inf$sigma.coefficients,
         Beta_1Inf$nu.coefficients, SUM0 = 0, SUM1 = SUM1)
  }else{                                ## Regular beta regression
    BetaMod <- betareg(PV_dis ~ x, data = disdat,
                       link.phi = "identity", link = "logit")
    list(BetaMod$coefficients, SUM0 = 0, SUM1 = 0)
  }
}
```

3. Calculate ROC and compute MSE

for each of the 1,000(100) data sets generated under Scenario 1, I obtain the regression coefficients using the three different methods.

```
PDF_coefs <- sapply(dats, PDF_FUN, t = t, simplify = FALSE)
Beta_coefs <- sapply(dats, Beta_FUN, t = t, simplify = FALSE)
```

```
## Warning in betareg.fit(X, Y, Z, weights, offset, link, link.phi, type,
## control): no valid starting value for precision parameter found, using 1
## instead

## Warning in betareg.fit(X, Y, Z, weights, offset, link, link.phi, type,
## control): no valid starting value for precision parameter found, using 1
## instead

## Warning in betareg.fit(X, Y, Z, weights, offset, link, link.phi, type,
## control): no valid starting value for precision parameter found, using 1
## instead
```

```
InfBeta_coefs <- sapply(dats, InfBeta_FUN, t = t, simplify = FALSE)
```

```
## GAMLSS-RS iteration 1: Global Deviance = 134.9024
## GAMLSS-RS iteration 2: Global Deviance = 133.1062
## GAMLSS-RS iteration 3: Global Deviance = 133.1058
## GAMLSS-RS iteration 1: Global Deviance = 19.3515
## GAMLSS-RS iteration 2: Global Deviance = 18.2684
## GAMLSS-RS iteration 3: Global Deviance = 18.2647
## GAMLSS-RS iteration 4: Global Deviance = 18.2647
## GAMLSS-RS iteration 1: Global Deviance = 87.9845
## GAMLSS-RS iteration 2: Global Deviance = 86.7859
## GAMLSS-RS iteration 3: Global Deviance = 86.7858
## GAMLSS-RS iteration 1: Global Deviance = 41.0125
## GAMLSS-RS iteration 2: Global Deviance = 41.0021
## GAMLSS-RS iteration 3: Global Deviance = 41.002
## GAMLSS-RS iteration 1: Global Deviance = 18.4149
## GAMLSS-RS iteration 2: Global Deviance = 18.4099
## GAMLSS-RS iteration 3: Global Deviance = 18.4099
## GAMLSS-RS iteration 1: Global Deviance = 28.3447
## GAMLSS-RS iteration 2: Global Deviance = 26.7162
## GAMLSS-RS iteration 3: Global Deviance = 26.708
```

```
## GAMLSS-RS iteration 4: Global Deviance = 26.7079
## GAMLSS-RS iteration 1: Global Deviance = 26.2817
## GAMLSS-RS iteration 2: Global Deviance = 26.2705
## GAMLSS-RS iteration 3: Global Deviance = 26.2704
## GAMLSS-RS iteration 1: Global Deviance = 41.0097
## GAMLSS-RS iteration 2: Global Deviance = 40.7052
## GAMLSS-RS iteration 3: Global Deviance = 40.7037
## GAMLSS-RS iteration 4: Global Deviance = 40.7037
## GAMLSS-RS iteration 1: Global Deviance = 84.8962
## GAMLSS-RS iteration 2: Global Deviance = 83.7876
## GAMLSS-RS iteration 3: Global Deviance = 83.7876
## GAMLSS-RS iteration 1: Global Deviance = 31.48
## GAMLSS-RS iteration 2: Global Deviance = 30.5611
## GAMLSS-RS iteration 3: Global Deviance = 30.5597
## GAMLSS-RS iteration 4: Global Deviance = 30.5597
## GAMLSS-RS iteration 1: Global Deviance = 92.8681
## GAMLSS-RS iteration 2: Global Deviance = 92.3975
## GAMLSS-RS iteration 3: Global Deviance = 92.3959
## GAMLSS-RS iteration 4: Global Deviance = 92.3959
## GAMLSS-RS iteration 1: Global Deviance = 118.65
## GAMLSS-RS iteration 2: Global Deviance = 118.1033
## GAMLSS-RS iteration 3: Global Deviance = 118.1006
## GAMLSS-RS iteration 4: Global Deviance = 118.1006
## GAMLSS-RS iteration 1: Global Deviance = 106.2089
## GAMLSS-RS iteration 2: Global Deviance = 106.0952
## GAMLSS-RS iteration 3: Global Deviance = 106.0951
## GAMLSS-RS iteration 1: Global Deviance = 114.3532
## GAMLSS-RS iteration 2: Global Deviance = 112.9238
## GAMLSS-RS iteration 3: Global Deviance = 112.9234
## GAMLSS-RS iteration 1: Global Deviance = 97.8149
## GAMLSS-RS iteration 2: Global Deviance = 96.0419
## GAMLSS-RS iteration 3: Global Deviance = 96.0401
## GAMLSS-RS iteration 4: Global Deviance = 96.0401
## GAMLSS-RS iteration 1: Global Deviance = 101.4521
## GAMLSS-RS iteration 2: Global Deviance = 99.7581
## GAMLSS-RS iteration 3: Global Deviance = 99.7565
## GAMLSS-RS iteration 4: Global Deviance = 99.7565
## GAMLSS-RS iteration 1: Global Deviance = -43.067
## GAMLSS-RS iteration 2: Global Deviance = -43.3937
## GAMLSS-RS iteration 3: Global Deviance = -43.4014
## GAMLSS-RS iteration 4: Global Deviance = -43.4016
## GAMLSS-RS iteration 1: Global Deviance = 72.357
## GAMLSS-RS iteration 2: Global Deviance = 72.3324
## GAMLSS-RS iteration 3: Global Deviance = 72.3322
## GAMLSS-RS iteration 1: Global Deviance = 78.5028
## GAMLSS-RS iteration 2: Global Deviance = 77.4404
## GAMLSS-RS iteration 3: Global Deviance = 77.4403
## GAMLSS-RS iteration 1: Global Deviance = 93.6514
## GAMLSS-RS iteration 2: Global Deviance = 92.4069
## GAMLSS-RS iteration 3: Global Deviance = 92.4067
## GAMLSS-RS iteration 1: Global Deviance = 60.1781
## GAMLSS-RS iteration 2: Global Deviance = 59.8404
## GAMLSS-RS iteration 3: Global Deviance = 59.8388
## GAMLSS-RS iteration 4: Global Deviance = 59.8388
```

```
## GAMLSS-RS iteration 1: Global Deviance = 123.7646
## GAMLSS-RS iteration 2: Global Deviance = 122.1798
## GAMLSS-RS iteration 3: Global Deviance = 122.1798
## GAMLSS-RS iteration 1: Global Deviance = 112.0994
## GAMLSS-RS iteration 2: Global Deviance = 110.7838
## GAMLSS-RS iteration 3: Global Deviance = 110.7838
## GAMLSS-RS iteration 1: Global Deviance = 140.1886
## GAMLSS-RS iteration 2: Global Deviance = 138.7291
## GAMLSS-RS iteration 3: Global Deviance = 138.729
## GAMLSS-RS iteration 1: Global Deviance = 54.0998
## GAMLSS-RS iteration 2: Global Deviance = 53.0242
## GAMLSS-RS iteration 3: Global Deviance = 53.0233
## GAMLSS-RS iteration 1: Global Deviance = 60.9578
## GAMLSS-RS iteration 2: Global Deviance = 59.9036
## GAMLSS-RS iteration 3: Global Deviance = 59.903
## GAMLSS-RS iteration 1: Global Deviance = 75.7332
## GAMLSS-RS iteration 2: Global Deviance = 75.7316
## GAMLSS-RS iteration 3: Global Deviance = 75.7316
## GAMLSS-RS iteration 1: Global Deviance = 16.3696
## GAMLSS-RS iteration 2: Global Deviance = 14.1355
## GAMLSS-RS iteration 3: Global Deviance = 14.1199
## GAMLSS-RS iteration 4: Global Deviance = 14.1197
## GAMLSS-RS iteration 1: Global Deviance = 109.4689
## GAMLSS-RS iteration 2: Global Deviance = 108.0911
## GAMLSS-RS iteration 3: Global Deviance = 108.0911
## GAMLSS-RS iteration 1: Global Deviance = 78.1218
## GAMLSS-RS iteration 2: Global Deviance = 78.1055
## GAMLSS-RS iteration 3: Global Deviance = 78.1054
## GAMLSS-RS iteration 1: Global Deviance = 100.1407
## GAMLSS-RS iteration 2: Global Deviance = 99.8871
## GAMLSS-RS iteration 3: Global Deviance = 99.8859
## GAMLSS-RS iteration 4: Global Deviance = 99.8859
## GAMLSS-RS iteration 1: Global Deviance = 62.0324
## GAMLSS-RS iteration 2: Global Deviance = 61.6513
## GAMLSS-RS iteration 3: Global Deviance = 61.6479
## GAMLSS-RS iteration 4: Global Deviance = 61.6479
## GAMLSS-RS iteration 1: Global Deviance = 15.3465
## GAMLSS-RS iteration 2: Global Deviance = 14.582
## GAMLSS-RS iteration 3: Global Deviance = 14.5806
## GAMLSS-RS iteration 4: Global Deviance = 14.5806
## GAMLSS-RS iteration 1: Global Deviance = 120.9527
## GAMLSS-RS iteration 2: Global Deviance = 119.3626
## GAMLSS-RS iteration 3: Global Deviance = 119.3619
## GAMLSS-RS iteration 1: Global Deviance = 94.4705
## GAMLSS-RS iteration 2: Global Deviance = 92.245
## GAMLSS-RS iteration 3: Global Deviance = 92.2387
## GAMLSS-RS iteration 4: Global Deviance = 92.2387
## GAMLSS-RS iteration 1: Global Deviance = 103.2243
## GAMLSS-RS iteration 2: Global Deviance = 103.2113
## GAMLSS-RS iteration 3: Global Deviance = 103.2113
## GAMLSS-RS iteration 1: Global Deviance = 73.8872
## GAMLSS-RS iteration 2: Global Deviance = 72.9774
## GAMLSS-RS iteration 3: Global Deviance = 72.9726
## GAMLSS-RS iteration 4: Global Deviance = 72.9726
```

```
## GAMLSS-RS iteration 1: Global Deviance = 85.348
## GAMLSS-RS iteration 2: Global Deviance = 84.0727
## GAMLSS-RS iteration 3: Global Deviance = 84.0727
## GAMLSS-RS iteration 1: Global Deviance = 42.6132
## GAMLSS-RS iteration 2: Global Deviance = 41.4786
## GAMLSS-RS iteration 3: Global Deviance = 41.4779
## GAMLSS-RS iteration 1: Global Deviance = 120.4827
## GAMLSS-RS iteration 2: Global Deviance = 118.7354
## GAMLSS-RS iteration 3: Global Deviance = 118.7349
## GAMLSS-RS iteration 1: Global Deviance = -18.6828
## GAMLSS-RS iteration 2: Global Deviance = -18.7225
## GAMLSS-RS iteration 3: Global Deviance = -18.723
## GAMLSS-RS iteration 1: Global Deviance = 118.8613
## GAMLSS-RS iteration 2: Global Deviance = 117.0578
## GAMLSS-RS iteration 3: Global Deviance = 117.0561
## GAMLSS-RS iteration 4: Global Deviance = 117.0561
## GAMLSS-RS iteration 1: Global Deviance = 100.1866
## GAMLSS-RS iteration 2: Global Deviance = 99.8929
## GAMLSS-RS iteration 3: Global Deviance = 99.8921
## GAMLSS-RS iteration 1: Global Deviance = 36.2818
## GAMLSS-RS iteration 2: Global Deviance = 34.9474
## GAMLSS-RS iteration 3: Global Deviance = 34.9394
## GAMLSS-RS iteration 4: Global Deviance = 34.9393
## GAMLSS-RS iteration 1: Global Deviance = 88.1138
## GAMLSS-RS iteration 2: Global Deviance = 86.9365
## GAMLSS-RS iteration 3: Global Deviance = 86.9365
## GAMLSS-RS iteration 1: Global Deviance = -5.0278
## GAMLSS-RS iteration 2: Global Deviance = -5.5414
## GAMLSS-RS iteration 3: Global Deviance = -5.5424
## GAMLSS-RS iteration 4: Global Deviance = -5.5425
## GAMLSS-RS iteration 1: Global Deviance = 91.1876
## GAMLSS-RS iteration 2: Global Deviance = 91.0774
## GAMLSS-RS iteration 3: Global Deviance = 91.0771
## GAMLSS-RS iteration 1: Global Deviance = 16.2104
## GAMLSS-RS iteration 2: Global Deviance = 16.0968
## GAMLSS-RS iteration 3: Global Deviance = 16.0961
## GAMLSS-RS iteration 1: Global Deviance = 145.3847
## GAMLSS-RS iteration 2: Global Deviance = 144.2104
## GAMLSS-RS iteration 3: Global Deviance = 144.2103
## GAMLSS-RS iteration 1: Global Deviance = 67.8374
## GAMLSS-RS iteration 2: Global Deviance = 66.7894
## GAMLSS-RS iteration 3: Global Deviance = 66.7884
## GAMLSS-RS iteration 1: Global Deviance = -49.3336
## GAMLSS-RS iteration 2: Global Deviance = -49.9528
## GAMLSS-RS iteration 3: Global Deviance = -49.9576
## GAMLSS-RS iteration 4: Global Deviance = -49.9577
## GAMLSS-RS iteration 1: Global Deviance = -28.7628
## GAMLSS-RS iteration 2: Global Deviance = -28.8725
## GAMLSS-RS iteration 3: Global Deviance = -28.8738
## GAMLSS-RS iteration 4: Global Deviance = -28.8738
## GAMLSS-RS iteration 1: Global Deviance = 78.3612
## GAMLSS-RS iteration 2: Global Deviance = 78.36
## GAMLSS-RS iteration 3: Global Deviance = 78.36
## GAMLSS-RS iteration 1: Global Deviance = 126.3878
```

```
## GAMLSS-RS iteration 2: Global Deviance = 124.9339
## GAMLSS-RS iteration 3: Global Deviance = 124.9338
## GAMLSS-RS iteration 1: Global Deviance = 110.8487
## GAMLSS-RS iteration 2: Global Deviance = 109.3417
## GAMLSS-RS iteration 3: Global Deviance = 109.3416
## GAMLSS-RS iteration 1: Global Deviance = 68.3993
## GAMLSS-RS iteration 2: Global Deviance = 67.5629
## GAMLSS-RS iteration 3: Global Deviance = 67.5627
## GAMLSS-RS iteration 1: Global Deviance = 119.3782
## GAMLSS-RS iteration 2: Global Deviance = 117.8298
## GAMLSS-RS iteration 3: Global Deviance = 117.8296
## GAMLSS-RS iteration 1: Global Deviance = 73.3253
## GAMLSS-RS iteration 2: Global Deviance = 73.1616
## GAMLSS-RS iteration 3: Global Deviance = 73.1609
## GAMLSS-RS iteration 1: Global Deviance = 96.2819
## GAMLSS-RS iteration 2: Global Deviance = 95.1364
## GAMLSS-RS iteration 3: Global Deviance = 95.1363
## GAMLSS-RS iteration 1: Global Deviance = 118.5078
## GAMLSS-RS iteration 2: Global Deviance = 117.2035
## GAMLSS-RS iteration 3: Global Deviance = 117.2026
## GAMLSS-RS iteration 1: Global Deviance = 139.4901
## GAMLSS-RS iteration 2: Global Deviance = 137.5205
## GAMLSS-RS iteration 3: Global Deviance = 137.52
## GAMLSS-RS iteration 1: Global Deviance = 22.9294
## GAMLSS-RS iteration 2: Global Deviance = 22.8751
## GAMLSS-RS iteration 3: Global Deviance = 22.8748
## GAMLSS-RS iteration 1: Global Deviance = 82.156
## GAMLSS-RS iteration 2: Global Deviance = 79.1051
## GAMLSS-RS iteration 3: Global Deviance = 79.0851
## GAMLSS-RS iteration 4: Global Deviance = 79.0848
## GAMLSS-RS iteration 1: Global Deviance = 56.6965
## GAMLSS-RS iteration 2: Global Deviance = 56.1628
## GAMLSS-RS iteration 3: Global Deviance = 56.1599
## GAMLSS-RS iteration 4: Global Deviance = 56.1598
## GAMLSS-RS iteration 1: Global Deviance = 68.2834
## GAMLSS-RS iteration 2: Global Deviance = 68.1219
## GAMLSS-RS iteration 3: Global Deviance = 68.1202
## GAMLSS-RS iteration 4: Global Deviance = 68.1201
## GAMLSS-RS iteration 1: Global Deviance = 87.5921
## GAMLSS-RS iteration 2: Global Deviance = 87.47
## GAMLSS-RS iteration 3: Global Deviance = 87.4687
## GAMLSS-RS iteration 4: Global Deviance = 87.4687
## GAMLSS-RS iteration 1: Global Deviance = 80.7886
## GAMLSS-RS iteration 2: Global Deviance = 79.7167
## GAMLSS-RS iteration 3: Global Deviance = 79.7159
## GAMLSS-RS iteration 1: Global Deviance = 139.5889
## GAMLSS-RS iteration 2: Global Deviance = 139.568
## GAMLSS-RS iteration 3: Global Deviance = 139.5679
## GAMLSS-RS iteration 1: Global Deviance = 73.5202
## GAMLSS-RS iteration 2: Global Deviance = 72.6564
## GAMLSS-RS iteration 3: Global Deviance = 72.6516
## GAMLSS-RS iteration 4: Global Deviance = 72.6515
## GAMLSS-RS iteration 1: Global Deviance = 33.3694
## GAMLSS-RS iteration 2: Global Deviance = 33.3529
```

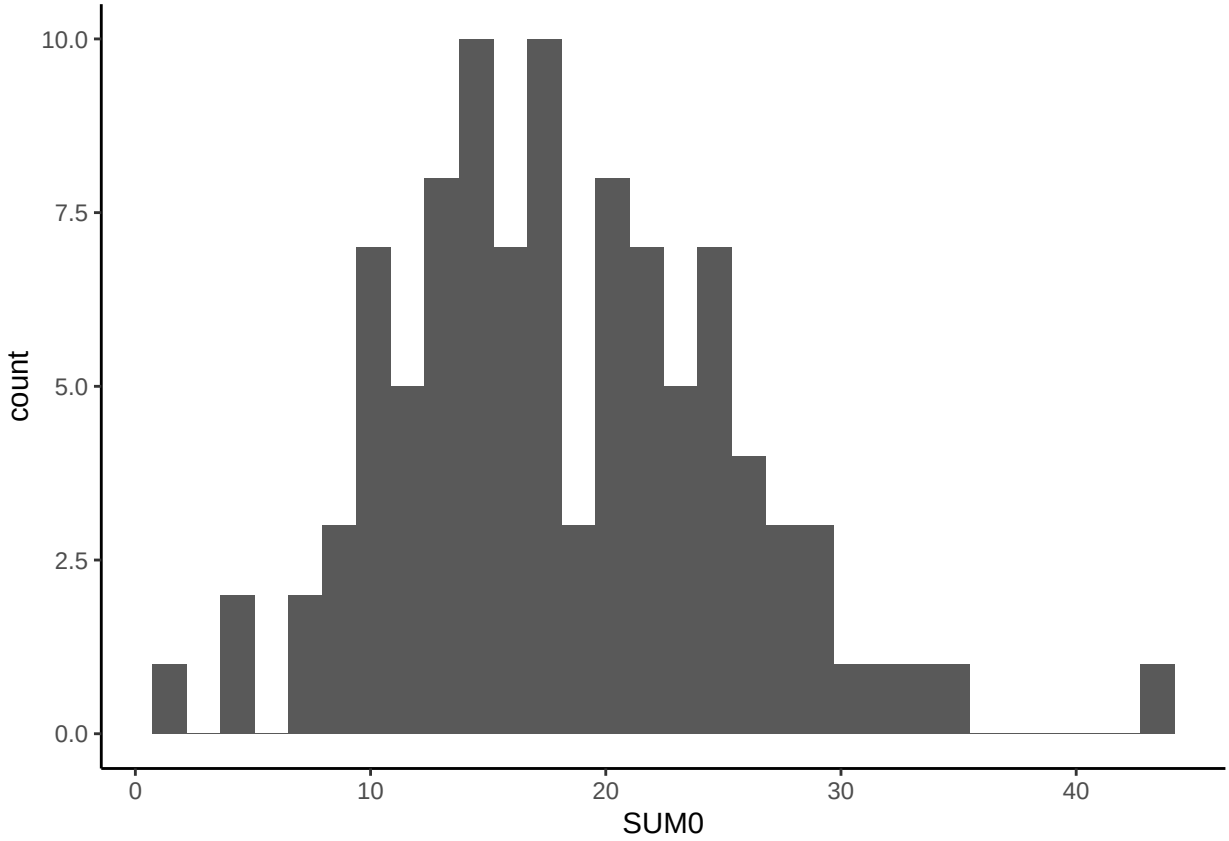
```
## GAMLSS-RS iteration 3: Global Deviance = 33.3528
## GAMLSS-RS iteration 1: Global Deviance = 77.4086
## GAMLSS-RS iteration 2: Global Deviance = 76.4523
## GAMLSS-RS iteration 3: Global Deviance = 76.4523
## GAMLSS-RS iteration 1: Global Deviance = 45.8072
## GAMLSS-RS iteration 2: Global Deviance = 44.648
## GAMLSS-RS iteration 3: Global Deviance = 44.648
## GAMLSS-RS iteration 1: Global Deviance = 61.814
## GAMLSS-RS iteration 2: Global Deviance = 59.8761
## GAMLSS-RS iteration 3: Global Deviance = 59.8706
## GAMLSS-RS iteration 4: Global Deviance = 59.8706
## GAMLSS-RS iteration 1: Global Deviance = 46.751
## GAMLSS-RS iteration 2: Global Deviance = 46.5151
## GAMLSS-RS iteration 3: Global Deviance = 46.5142
## GAMLSS-RS iteration 1: Global Deviance = 155.0189
## GAMLSS-RS iteration 2: Global Deviance = 153.5295
## GAMLSS-RS iteration 3: Global Deviance = 153.5291
## GAMLSS-RS iteration 1: Global Deviance = 21.1593
## GAMLSS-RS iteration 2: Global Deviance = 20.2653
## GAMLSS-RS iteration 3: Global Deviance = 20.2624
## GAMLSS-RS iteration 4: Global Deviance = 20.2624
## GAMLSS-RS iteration 1: Global Deviance = 92.1856
## GAMLSS-RS iteration 2: Global Deviance = 91.8716
## GAMLSS-RS iteration 3: Global Deviance = 91.8706
## GAMLSS-RS iteration 4: Global Deviance = 91.8706
## GAMLSS-RS iteration 1: Global Deviance = 44.0148
## GAMLSS-RS iteration 2: Global Deviance = 42.812
## GAMLSS-RS iteration 3: Global Deviance = 42.8114
## GAMLSS-RS iteration 1: Global Deviance = 26.508
## GAMLSS-RS iteration 2: Global Deviance = 25.2579
## GAMLSS-RS iteration 3: Global Deviance = 25.2534
## GAMLSS-RS iteration 4: Global Deviance = 25.2534
## GAMLSS-RS iteration 1: Global Deviance = 32.7467
## GAMLSS-RS iteration 2: Global Deviance = 32.5894
## GAMLSS-RS iteration 3: Global Deviance = 32.5888
## GAMLSS-RS iteration 1: Global Deviance = 46.8133
## GAMLSS-RS iteration 2: Global Deviance = 45.4837
## GAMLSS-RS iteration 3: Global Deviance = 45.4804
## GAMLSS-RS iteration 4: Global Deviance = 45.4804
## GAMLSS-RS iteration 1: Global Deviance = 15.4005
## GAMLSS-RS iteration 2: Global Deviance = 15.3998
## GAMLSS-RS iteration 1: Global Deviance = 106.7655
## GAMLSS-RS iteration 2: Global Deviance = 105.4694
## GAMLSS-RS iteration 3: Global Deviance = 105.4692
## GAMLSS-RS iteration 1: Global Deviance = -60.0421
## GAMLSS-RS iteration 2: Global Deviance = -60.088
## GAMLSS-RS iteration 3: Global Deviance = -60.0887
## GAMLSS-RS iteration 1: Global Deviance = 233.0264
## GAMLSS-RS iteration 2: Global Deviance = 230.8214
## GAMLSS-RS iteration 3: Global Deviance = 230.8209
## GAMLSS-RS iteration 1: Global Deviance = 84.9578
## GAMLSS-RS iteration 2: Global Deviance = 83.8522
## GAMLSS-RS iteration 3: Global Deviance = 83.8512
## GAMLSS-RS iteration 1: Global Deviance = 29.8095
```

```
## GAMLSS-RS iteration 2: Global Deviance = 28.7547
## GAMLSS-RS iteration 3: Global Deviance = 28.7524
## GAMLSS-RS iteration 4: Global Deviance = 28.7524
## GAMLSS-RS iteration 1: Global Deviance = 113.7874
## GAMLSS-RS iteration 2: Global Deviance = 113.611
## GAMLSS-RS iteration 3: Global Deviance = 113.6101
## GAMLSS-RS iteration 1: Global Deviance = 76.4495
## GAMLSS-RS iteration 2: Global Deviance = 76.4243
## GAMLSS-RS iteration 3: Global Deviance = 76.424
## GAMLSS-RS iteration 1: Global Deviance = 108.8919
## GAMLSS-RS iteration 2: Global Deviance = 107.0765
## GAMLSS-RS iteration 3: Global Deviance = 107.072
## GAMLSS-RS iteration 4: Global Deviance = 107.072
## GAMLSS-RS iteration 1: Global Deviance = 44.8253
## GAMLSS-RS iteration 2: Global Deviance = 44.2831
## GAMLSS-RS iteration 3: Global Deviance = 44.28
## GAMLSS-RS iteration 4: Global Deviance = 44.2799
## GAMLSS-RS iteration 1: Global Deviance = 87.6962
## GAMLSS-RS iteration 2: Global Deviance = 86.1239
## GAMLSS-RS iteration 3: Global Deviance = 86.1226
## GAMLSS-RS iteration 4: Global Deviance = 86.1226
## GAMLSS-RS iteration 1: Global Deviance = 23.0936
## GAMLSS-RS iteration 2: Global Deviance = 22.2465
## GAMLSS-RS iteration 3: Global Deviance = 22.2456
## GAMLSS-RS iteration 1: Global Deviance = 86.4255
## GAMLSS-RS iteration 2: Global Deviance = 84.9967
## GAMLSS-RS iteration 3: Global Deviance = 84.9961
## GAMLSS-RS iteration 1: Global Deviance = 80.3232
## GAMLSS-RS iteration 2: Global Deviance = 80.1449
## GAMLSS-RS iteration 3: Global Deviance = 80.1445
## GAMLSS-RS iteration 1: Global Deviance = 166.0474
## GAMLSS-RS iteration 2: Global Deviance = 164.2062
## GAMLSS-RS iteration 3: Global Deviance = 164.2049
## GAMLSS-RS iteration 4: Global Deviance = 164.2049
## GAMLSS-RS iteration 1: Global Deviance = 119.8173
## GAMLSS-RS iteration 2: Global Deviance = 119.5052
## GAMLSS-RS iteration 3: Global Deviance = 119.5028
## GAMLSS-RS iteration 4: Global Deviance = 119.5028
## GAMLSS-RS iteration 1: Global Deviance = 106.3074
## GAMLSS-RS iteration 2: Global Deviance = 104.4095
## GAMLSS-RS iteration 3: Global Deviance = 104.404
## GAMLSS-RS iteration 4: Global Deviance = 104.404
## GAMLSS-RS iteration 1: Global Deviance = 42.6226
## GAMLSS-RS iteration 2: Global Deviance = 41.6267
## GAMLSS-RS iteration 3: Global Deviance = 41.6254
## GAMLSS-RS iteration 4: Global Deviance = 41.6254
## GAMLSS-RS iteration 1: Global Deviance = -2.2289
## GAMLSS-RS iteration 2: Global Deviance = -2.4173
## GAMLSS-RS iteration 3: Global Deviance = -2.4184
## GAMLSS-RS iteration 4: Global Deviance = -2.4184
```

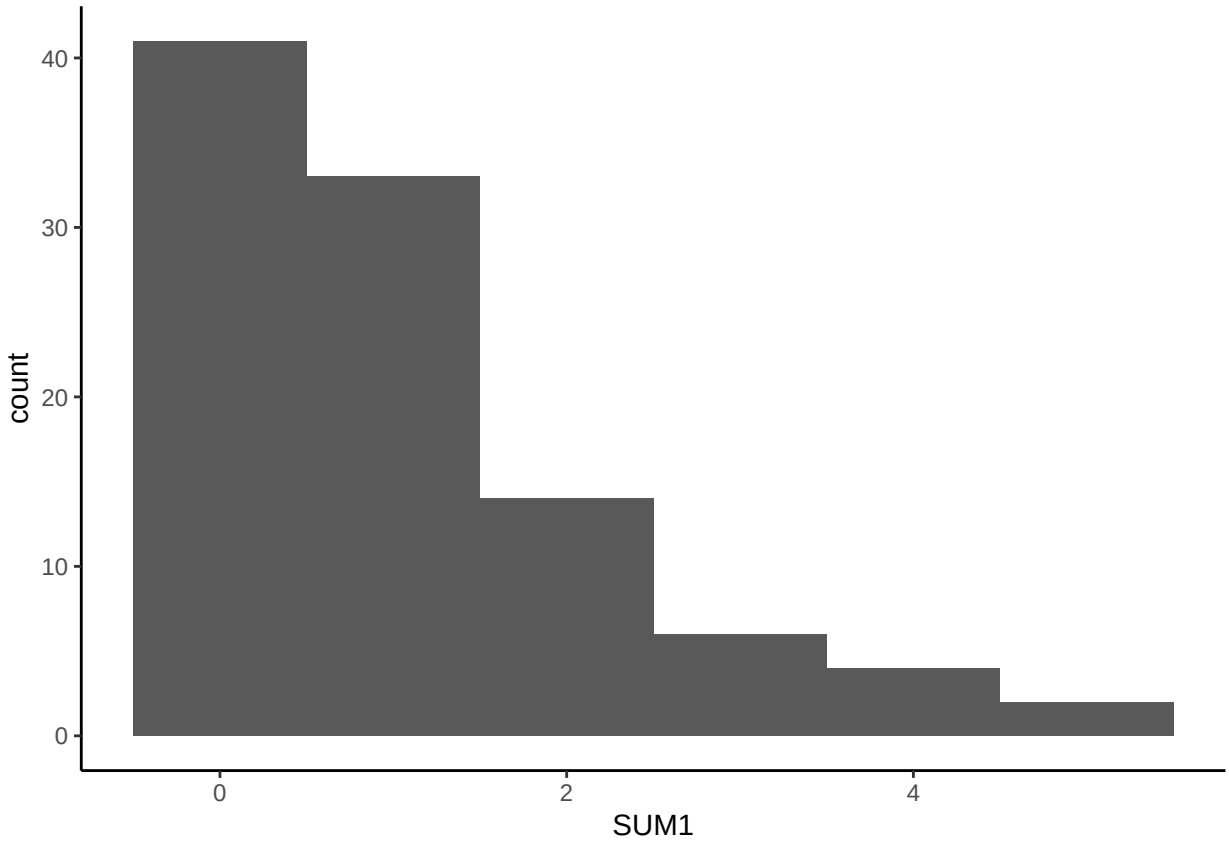
```
# histograms of the counts of PVs that are zero from the 1,000 data sets.

# get number of PVS that are 0 and 1 for each data set
```

```r
getNums <- function(coefs, SUM0, SUM1){
  coefs <- unlist(coefs)
  coefs[c(SUM0, SUM1)]
}

PVs_0and1 <- sapply(Beta_coefs, getNums, "SUM0", "SUM1")
df_PVs <- data.frame(t(PVs_0and1))

# histogram of PVs = 0
ggplot(df_PVs, aes(SUM0)) +
  geom_histogram()
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




```r
# histogram of PVs = 1
ggplot(df_PVs, aes(SUM1))  +
  geom_histogram(bins = 6)
```




```r
# Determine the difference between the estimator of the covarariate-specific ROC curve and the true ROC

# TRUE ROC(t) values for t = 0, 1/50, ..., 1.
T_vals <- mapply(true_binormal_ROC, t = c(0, t, 1),
X0 = rep(c(0, t, 1), each = length(t) + 2),
MoreArgs = list(c1 = 2, d1 = 4, sd1 = 1.5, c0 = 1.5, d0 = 3, sd0 = 1.5))

# MSE wrapper function
wrapper <- function(coef_vals, ROC_fun){
  mapply(ROC_fun, t = c(0, t, 1), X_val = rep(c(0, t, 1), each = length(t) + 2),
         MoreArgs = list(coef_vals))
}

# function to get PDF ROC for particular t and X_val
binorm_ROC <- function(coefs, t, X_val){
  coefs <- unlist(coefs)
  ga_1 <- coefs[[1]]
  ga_2 <- coefs[[2]]
  Be <- coefs[[3]]
  pnorm(ga_1 + ga_2 * qnorm(t) + Be * X_val)
}

PDF_vals <- sapply(PDF_coefs, wrapper, ROC_fun = binorm_ROC)

MSE_PDF <- apply(PDF_vals, 2, function(x) sum((x - T_vals)^2)/n_T^2)
summary(MSE_PDF)
```

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## 0.0001381 0.0005944 0.0012612 0.0019256 0.0027949 0.0081150
```

```r
# function to get Beta ROC for particular t and X_val
Beta_ROC <- function(coefs, t, X_val){
  coefs <- unlist(coefs)
  Be_hat0 <- coefs[[1]]
  Be_hat1 <- coefs[[2]]
  phi_hat <- coefs[[3]]

  a_hat <- phi_hat/(1 + exp(-Be_hat0 -X_val * Be_hat1))
  b_hat <- phi_hat * (1 - 1/(1 + exp(-Be_hat0 -X_val * Be_hat1)))
  pbeta(t, a_hat, b_hat)
}

Be_vals <- sapply(Beta_coefs, wrapper, ROC_fun = Beta_ROC)

MSE_Be <- apply(Be_vals, 2, function(x) sum((x - T_vals)^2)/n_T^2)
summary(MSE_Be)
```

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## 6.355e-05 7.394e-04 1.400e-03 1.897e-03 2.550e-03 1.140e-02
```

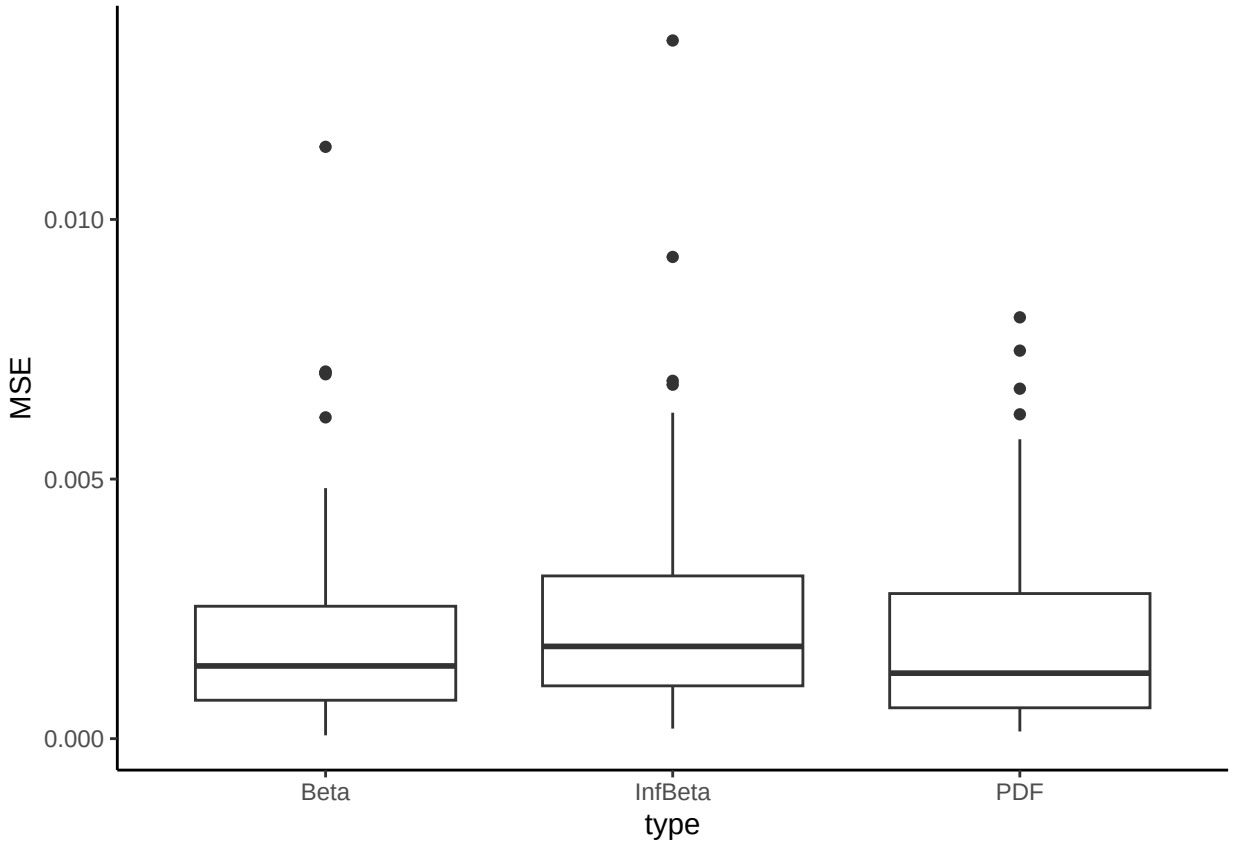
```r
# function to get Inf Beta ROC for particular t and X_val
InfBeta_ROC <- function(coefs, t, X_val){
  coefs <- unlist(coefs)

  if(coefs["SUM0"] > 0 | coefs["SUM1"] > 0){ ## BEINF/BEINF0/BEINF1
    mu.c1 <- coefs[[1]]
    mu.c2 <- coefs[[2]]
    mu <- 1 / (1 + exp(-mu.c1 - X_val * mu.c2))
    sig.c <- coefs[[3]]
    sig <- 1 / (1 + exp(-sig.c))
    nu.c <- coefs[[4]]
    nu <- exp(nu.c)
    if(coefs["SUM0"] > 0 & coefs["SUM1"] > 0){ ## BEINF cdf
      tau.c <- coefs[[5]]
      tau <- exp(tau.c)
      pBEINF(t, mu, sig, nu, tau)
    }else if(coefs["SUM0"] > 0){               ## BEINF0 cdf
      pBEINF0(t, mu, sig, nu)
    }else if(coefs["SUM1"] > 0){               ## BEINF1 cdf
      pBEINF1(t, mu, sig, nu)
    }
  }else{                                       ## Beta cdf
    Be_hat0 <- coefs[[1]]
    Be_hat1 <- coefs[[2]]
    phi_hat <- coefs[[3]]

    a_hat <- phi_hat/(1 + exp(-Be_hat0 -X_val * Be_hat1))
    b_hat <- phi_hat * (1 - 1/(1 + exp(-Be_hat0 -X_val * Be_hat1)))
    pbeta(t, a_hat, b_hat)
  }
}
```

```r
outputInf <- sapply(InfBeta_coefs, wrapper, ROC_fun = InfBeta_ROC)
MSE_Inf <- apply(outputInf, 2, function(x) sum((x - T_vals)^2)/n_T^2)
summary(MSE_Inf)
```

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## 0.0001935 0.0010171 0.0017764 0.0023723 0.0031343 0.0134468
```

Boxplots of the MSE values for each method

```r
datMSE <- data.frame(MSE = c(MSE_PDF, MSE_Be, MSE_Inf),
type = c(rep("PDF", length(MSE_PDF)), rep("Beta", length(MSE_Be)),
rep("InfBeta", length(MSE_Inf))))

ggplot(datMSE, aes(type, MSE)) +
  geom_boxplot()
```




```r
## Repeat process of finding the ROCs and MSEs for 1,000 data sets except this time under Scenario 2, wi

# generate 1,000 data sets  # chnaged to 100
dats2 <- replicate(100, gen_Norm_data(1.5, 3, 1.5, 200, 3, 6, 1.5, 200),
simplify = FALSE)

# ROC regression coefficients for multiple datasets
PDF_coefs2 <- sapply(dats2, PDF_FUN, t = t, simplify = FALSE)
Beta_coefs2 <- sapply(dats2, Beta_FUN, t = t, simplify = FALSE)
InfBeta_coefs2 <- sapply(dats2, InfBeta_FUN, t = t)
```

```
## GAMLSS-RS iteration 1: Global Deviance = 140.6906
## GAMLSS-RS iteration 2: Global Deviance = 136.2218
## GAMLSS-RS iteration 3: Global Deviance = 135.8069
## GAMLSS-RS iteration 4: Global Deviance = 135.7787
## GAMLSS-RS iteration 5: Global Deviance = 135.7769
## GAMLSS-RS iteration 6: Global Deviance = 135.7768
## GAMLSS-RS iteration 1: Global Deviance = 154.3608
## GAMLSS-RS iteration 2: Global Deviance = 150.8503
## GAMLSS-RS iteration 3: Global Deviance = 150.5523
## GAMLSS-RS iteration 4: Global Deviance = 150.5343
## GAMLSS-RS iteration 5: Global Deviance = 150.5333
## GAMLSS-RS iteration 1: Global Deviance = 20.6857
## GAMLSS-RS iteration 2: Global Deviance = 0.6446
## GAMLSS-RS iteration 3: Global Deviance = -2.8434
## GAMLSS-RS iteration 4: Global Deviance = -3.28
## GAMLSS-RS iteration 5: Global Deviance = -3.322
## GAMLSS-RS iteration 6: Global Deviance = -3.3256
## GAMLSS-RS iteration 7: Global Deviance = -3.3259
## GAMLSS-RS iteration 1: Global Deviance = 172.3839
## GAMLSS-RS iteration 2: Global Deviance = 172.3646
## GAMLSS-RS iteration 3: Global Deviance = 172.3634
## GAMLSS-RS iteration 4: Global Deviance = 172.3634
## GAMLSS-RS iteration 1: Global Deviance = 68.4234
## GAMLSS-RS iteration 2: Global Deviance = 63.0502
## GAMLSS-RS iteration 3: Global Deviance = 62.5861
## GAMLSS-RS iteration 4: Global Deviance = 62.5567
## GAMLSS-RS iteration 5: Global Deviance = 62.555
## GAMLSS-RS iteration 6: Global Deviance = 62.5549
## GAMLSS-RS iteration 1: Global Deviance = 154.1945
## GAMLSS-RS iteration 2: Global Deviance = 146.1439
## GAMLSS-RS iteration 3: Global Deviance = 145.5789
## GAMLSS-RS iteration 4: Global Deviance = 145.5578
## GAMLSS-RS iteration 5: Global Deviance = 145.5572
## GAMLSS-RS iteration 1: Global Deviance = 188.9861
## GAMLSS-RS iteration 2: Global Deviance = 188.637
## GAMLSS-RS iteration 3: Global Deviance = 188.6213
## GAMLSS-RS iteration 4: Global Deviance = 188.6206
## GAMLSS-RS iteration 1: Global Deviance = 159.7813
## GAMLSS-RS iteration 2: Global Deviance = 159.728
## GAMLSS-RS iteration 3: Global Deviance = 159.7243
## GAMLSS-RS iteration 4: Global Deviance = 159.7241
## GAMLSS-RS iteration 1: Global Deviance = 112.438
## GAMLSS-RS iteration 2: Global Deviance = 103.0357
## GAMLSS-RS iteration 3: Global Deviance = 102.1519
## GAMLSS-RS iteration 4: Global Deviance = 102.0625
## GAMLSS-RS iteration 5: Global Deviance = 102.0547
## GAMLSS-RS iteration 6: Global Deviance = 102.054
## GAMLSS-RS iteration 1: Global Deviance = 69.1369
## GAMLSS-RS iteration 2: Global Deviance = 63.0054
## GAMLSS-RS iteration 3: Global Deviance = 62.3496
## GAMLSS-RS iteration 4: Global Deviance = 62.2981
## GAMLSS-RS iteration 5: Global Deviance = 62.2945
## GAMLSS-RS iteration 6: Global Deviance = 62.2942
## GAMLSS-RS iteration 1: Global Deviance = 128.8754
```

```
## GAMLSS-RS iteration 2: Global Deviance = 127.8745
## GAMLSS-RS iteration 3: Global Deviance = 127.7597
## GAMLSS-RS iteration 4: Global Deviance = 127.7481
## GAMLSS-RS iteration 5: Global Deviance = 127.747
## GAMLSS-RS iteration 6: Global Deviance = 127.7469
## GAMLSS-RS iteration 1: Global Deviance = 76.2606
## GAMLSS-RS iteration 2: Global Deviance = 67.6015
## GAMLSS-RS iteration 3: Global Deviance = 66.4131
## GAMLSS-RS iteration 4: Global Deviance = 66.3043
## GAMLSS-RS iteration 5: Global Deviance = 66.2958
## GAMLSS-RS iteration 6: Global Deviance = 66.2952
## GAMLSS-RS iteration 1: Global Deviance = 158.7487
## GAMLSS-RS iteration 2: Global Deviance = 158.1968
## GAMLSS-RS iteration 3: Global Deviance = 158.1579
## GAMLSS-RS iteration 4: Global Deviance = 158.1554
## GAMLSS-RS iteration 5: Global Deviance = 158.1552
## GAMLSS-RS iteration 1: Global Deviance = 149.1124
## GAMLSS-RS iteration 2: Global Deviance = 146.1326
## GAMLSS-RS iteration 3: Global Deviance = 145.9554
## GAMLSS-RS iteration 4: Global Deviance = 145.9475
## GAMLSS-RS iteration 5: Global Deviance = 145.9472
## GAMLSS-RS iteration 1: Global Deviance = 124.4556
## GAMLSS-RS iteration 2: Global Deviance = 120.2124
## GAMLSS-RS iteration 3: Global Deviance = 119.8066
## GAMLSS-RS iteration 4: Global Deviance = 119.7782
## GAMLSS-RS iteration 5: Global Deviance = 119.7764
## GAMLSS-RS iteration 6: Global Deviance = 119.7763
## GAMLSS-RS iteration 1: Global Deviance = 53.8116
## GAMLSS-RS iteration 2: Global Deviance = 42.2522
## GAMLSS-RS iteration 3: Global Deviance = 40.7677
## GAMLSS-RS iteration 4: Global Deviance = 40.6489
## GAMLSS-RS iteration 5: Global Deviance = 40.6411
## GAMLSS-RS iteration 6: Global Deviance = 40.6406
## GAMLSS-RS iteration 1: Global Deviance = 126.4086
## GAMLSS-RS iteration 2: Global Deviance = 123.7756
## GAMLSS-RS iteration 3: Global Deviance = 123.5021
## GAMLSS-RS iteration 4: Global Deviance = 123.4795
## GAMLSS-RS iteration 5: Global Deviance = 123.4778
## GAMLSS-RS iteration 6: Global Deviance = 123.4777
## GAMLSS-RS iteration 1: Global Deviance = 200.6959
## GAMLSS-RS iteration 2: Global Deviance = 200.2874
## GAMLSS-RS iteration 3: Global Deviance = 200.2764
## GAMLSS-RS iteration 4: Global Deviance = 200.2762
## GAMLSS-RS iteration 1: Global Deviance = 166.303
## GAMLSS-RS iteration 2: Global Deviance = 163.0137
## GAMLSS-RS iteration 3: Global Deviance = 162.8315
## GAMLSS-RS iteration 4: Global Deviance = 162.8245
## GAMLSS-RS iteration 5: Global Deviance = 162.8242
## GAMLSS-RS iteration 1: Global Deviance = 110.1264
## GAMLSS-RS iteration 2: Global Deviance = 107.6288
## GAMLSS-RS iteration 3: Global Deviance = 107.3978
## GAMLSS-RS iteration 4: Global Deviance = 107.3805
## GAMLSS-RS iteration 5: Global Deviance = 107.3793
## GAMLSS-RS iteration 6: Global Deviance = 107.3792
```

```
## GAMLSS-RS iteration 1: Global Deviance = 163.0156
## GAMLSS-RS iteration 2: Global Deviance = 156.1196
## GAMLSS-RS iteration 3: Global Deviance = 155.7213
## GAMLSS-RS iteration 4: Global Deviance = 155.6932
## GAMLSS-RS iteration 5: Global Deviance = 155.6914
## GAMLSS-RS iteration 6: Global Deviance = 155.6913
## GAMLSS-RS iteration 1: Global Deviance = 41.9671
## GAMLSS-RS iteration 2: Global Deviance = 26.7449
## GAMLSS-RS iteration 3: Global Deviance = 24.1551
## GAMLSS-RS iteration 4: Global Deviance = 23.8921
## GAMLSS-RS iteration 5: Global Deviance = 23.8712
## GAMLSS-RS iteration 6: Global Deviance = 23.8697
## GAMLSS-RS iteration 7: Global Deviance = 23.8696
## GAMLSS-RS iteration 1: Global Deviance = 173.7922
## GAMLSS-RS iteration 2: Global Deviance = 171.9863
## GAMLSS-RS iteration 3: Global Deviance = 171.9042
## GAMLSS-RS iteration 4: Global Deviance = 171.9013
## GAMLSS-RS iteration 5: Global Deviance = 171.9012
## GAMLSS-RS iteration 1: Global Deviance = 103.1586
## GAMLSS-RS iteration 2: Global Deviance = 97.7891
## GAMLSS-RS iteration 3: Global Deviance = 97.1502
## GAMLSS-RS iteration 4: Global Deviance = 97.0954
## GAMLSS-RS iteration 5: Global Deviance = 97.0913
## GAMLSS-RS iteration 6: Global Deviance = 97.091
## GAMLSS-RS iteration 1: Global Deviance = 128.0818
## GAMLSS-RS iteration 2: Global Deviance = 127.7302
## GAMLSS-RS iteration 3: Global Deviance = 127.7038
## GAMLSS-RS iteration 4: Global Deviance = 127.702
## GAMLSS-RS iteration 5: Global Deviance = 127.7019
## GAMLSS-RS iteration 1: Global Deviance = 150.9206
## GAMLSS-RS iteration 2: Global Deviance = 150.4108
## GAMLSS-RS iteration 3: Global Deviance = 150.3653
## GAMLSS-RS iteration 4: Global Deviance = 150.3617
## GAMLSS-RS iteration 5: Global Deviance = 150.3614
## GAMLSS-RS iteration 1: Global Deviance = 143.1733
## GAMLSS-RS iteration 2: Global Deviance = 137.7014
## GAMLSS-RS iteration 3: Global Deviance = 137.2834
## GAMLSS-RS iteration 4: Global Deviance = 137.2625
## GAMLSS-RS iteration 5: Global Deviance = 137.2616
## GAMLSS-RS iteration 1: Global Deviance = 168.9441
## GAMLSS-RS iteration 2: Global Deviance = 168.7737
## GAMLSS-RS iteration 3: Global Deviance = 168.7621
## GAMLSS-RS iteration 4: Global Deviance = 168.7614
## GAMLSS-RS iteration 1: Global Deviance = 180.2369
## GAMLSS-RS iteration 2: Global Deviance = 178.6595
## GAMLSS-RS iteration 3: Global Deviance = 178.5763
## GAMLSS-RS iteration 4: Global Deviance = 178.5729
## GAMLSS-RS iteration 5: Global Deviance = 178.5728
## GAMLSS-RS iteration 1: Global Deviance = 188.0731
## GAMLSS-RS iteration 2: Global Deviance = 188.0474
## GAMLSS-RS iteration 3: Global Deviance = 188.046
## GAMLSS-RS iteration 4: Global Deviance = 188.0459
## GAMLSS-RS iteration 1: Global Deviance = 84.2224
## GAMLSS-RS iteration 2: Global Deviance = 75.133
```

```
## GAMLSS-RS iteration 3: Global Deviance = 73.842
## GAMLSS-RS iteration 4: Global Deviance = 73.7217
## GAMLSS-RS iteration 5: Global Deviance = 73.7123
## GAMLSS-RS iteration 6: Global Deviance = 73.7116
## GAMLSS-RS iteration 1: Global Deviance = 148.139
## GAMLSS-RS iteration 2: Global Deviance = 146.2103
## GAMLSS-RS iteration 3: Global Deviance = 146.0688
## GAMLSS-RS iteration 4: Global Deviance = 146.0605
## GAMLSS-RS iteration 5: Global Deviance = 146.06
## GAMLSS-RS iteration 1: Global Deviance = 184.2271
## GAMLSS-RS iteration 2: Global Deviance = 180.4022
## GAMLSS-RS iteration 3: Global Deviance = 180.3624
## GAMLSS-RS iteration 4: Global Deviance = 180.3604
## GAMLSS-RS iteration 5: Global Deviance = 180.3603
## GAMLSS-RS iteration 1: Global Deviance = 178.4944
## GAMLSS-RS iteration 2: Global Deviance = 176.7145
## GAMLSS-RS iteration 3: Global Deviance = 176.6326
## GAMLSS-RS iteration 4: Global Deviance = 176.6297
## GAMLSS-RS iteration 5: Global Deviance = 176.6296
## GAMLSS-RS iteration 1: Global Deviance = 184.2767
## GAMLSS-RS iteration 2: Global Deviance = 184.2761
## GAMLSS-RS iteration 1: Global Deviance = 126.8069
## GAMLSS-RS iteration 2: Global Deviance = 123.9303
## GAMLSS-RS iteration 3: Global Deviance = 123.686
## GAMLSS-RS iteration 4: Global Deviance = 123.6698
## GAMLSS-RS iteration 5: Global Deviance = 123.6688
## GAMLSS-RS iteration 1: Global Deviance = 192.5131
## GAMLSS-RS iteration 2: Global Deviance = 192.2039
## GAMLSS-RS iteration 3: Global Deviance = 192.1925
## GAMLSS-RS iteration 4: Global Deviance = 192.1921
## GAMLSS-RS iteration 1: Global Deviance = 79.2328
## GAMLSS-RS iteration 2: Global Deviance = 75.4026
## GAMLSS-RS iteration 3: Global Deviance = 74.9275
## GAMLSS-RS iteration 4: Global Deviance = 74.8813
## GAMLSS-RS iteration 5: Global Deviance = 74.8772
## GAMLSS-RS iteration 6: Global Deviance = 74.8769
## GAMLSS-RS iteration 1: Global Deviance = 150.32
## GAMLSS-RS iteration 2: Global Deviance = 147.9141
## GAMLSS-RS iteration 3: Global Deviance = 147.7326
## GAMLSS-RS iteration 4: Global Deviance = 147.7216
## GAMLSS-RS iteration 5: Global Deviance = 147.721
## GAMLSS-RS iteration 1: Global Deviance = 169.5538
## GAMLSS-RS iteration 2: Global Deviance = 167.4819
## GAMLSS-RS iteration 3: Global Deviance = 167.3364
## GAMLSS-RS iteration 4: Global Deviance = 167.3288
## GAMLSS-RS iteration 5: Global Deviance = 167.3284
## GAMLSS-RS iteration 1: Global Deviance = 140.7331
## GAMLSS-RS iteration 2: Global Deviance = 139.0937
## GAMLSS-RS iteration 3: Global Deviance = 138.9639
## GAMLSS-RS iteration 4: Global Deviance = 138.9553
## GAMLSS-RS iteration 5: Global Deviance = 138.9548
## GAMLSS-RS iteration 1: Global Deviance = 57.7493
## GAMLSS-RS iteration 2: Global Deviance = 51.6951
## GAMLSS-RS iteration 3: Global Deviance = 50.6525
```

```
## GAMLSS-RS iteration 4: Global Deviance = 50.5152
## GAMLSS-RS iteration 5: Global Deviance = 50.499
## GAMLSS-RS iteration 6: Global Deviance = 50.4971
## GAMLSS-RS iteration 7: Global Deviance = 50.4969
## GAMLSS-RS iteration 1: Global Deviance = 145.3013
## GAMLSS-RS iteration 2: Global Deviance = 142.8021
## GAMLSS-RS iteration 3: Global Deviance = 142.619
## GAMLSS-RS iteration 4: Global Deviance = 142.6086
## GAMLSS-RS iteration 5: Global Deviance = 142.608
## GAMLSS-RS iteration 1: Global Deviance = 171.7468
## GAMLSS-RS iteration 2: Global Deviance = 171.7457
## GAMLSS-RS iteration 3: Global Deviance = 171.7456
## GAMLSS-RS iteration 1: Global Deviance = 44.2497
## GAMLSS-RS iteration 2: Global Deviance = 31.5827
## GAMLSS-RS iteration 3: Global Deviance = 29.3333
## GAMLSS-RS iteration 4: Global Deviance = 29.0731
## GAMLSS-RS iteration 5: Global Deviance = 29.0484
## GAMLSS-RS iteration 6: Global Deviance = 29.0462
## GAMLSS-RS iteration 7: Global Deviance = 29.046
## GAMLSS-RS iteration 1: Global Deviance = -26.7993
## GAMLSS-RS iteration 2: Global Deviance = -31.3696
## GAMLSS-RS iteration 3: Global Deviance = -31.9581
## GAMLSS-RS iteration 4: Global Deviance = -32.0185
## GAMLSS-RS iteration 5: Global Deviance = -32.0242
## GAMLSS-RS iteration 6: Global Deviance = -32.0248
## GAMLSS-RS iteration 1: Global Deviance = 136.828
## GAMLSS-RS iteration 2: Global Deviance = 136.8251
## GAMLSS-RS iteration 3: Global Deviance = 136.825
## GAMLSS-RS iteration 1: Global Deviance = 124.8058
## GAMLSS-RS iteration 2: Global Deviance = 123.4019
## GAMLSS-RS iteration 3: Global Deviance = 123.2577
## GAMLSS-RS iteration 4: Global Deviance = 123.245
## GAMLSS-RS iteration 5: Global Deviance = 123.244
## GAMLSS-RS iteration 6: Global Deviance = 123.2439
## GAMLSS-RS iteration 1: Global Deviance = 85.9657
## GAMLSS-RS iteration 2: Global Deviance = 81.6218
## GAMLSS-RS iteration 3: Global Deviance = 81.135
## GAMLSS-RS iteration 4: Global Deviance = 81.0937
## GAMLSS-RS iteration 5: Global Deviance = 81.0905
## GAMLSS-RS iteration 6: Global Deviance = 81.0903
## GAMLSS-RS iteration 1: Global Deviance = 150.9344
## GAMLSS-RS iteration 2: Global Deviance = 145.7215
## GAMLSS-RS iteration 3: Global Deviance = 145.526
## GAMLSS-RS iteration 4: Global Deviance = 145.511
## GAMLSS-RS iteration 5: Global Deviance = 145.5099
## GAMLSS-RS iteration 6: Global Deviance = 145.5098
## GAMLSS-RS iteration 1: Global Deviance = 148.5385
## GAMLSS-RS iteration 2: Global Deviance = 146.4678
## GAMLSS-RS iteration 3: Global Deviance = 146.2935
## GAMLSS-RS iteration 4: Global Deviance = 146.2818
## GAMLSS-RS iteration 5: Global Deviance = 146.2811
## GAMLSS-RS iteration 1: Global Deviance = 109.9594
## GAMLSS-RS iteration 2: Global Deviance = 103.3689
## GAMLSS-RS iteration 3: Global Deviance = 102.6478
```

```
## GAMLSS-RS iteration 4: Global Deviance = 102.594
## GAMLSS-RS iteration 5: Global Deviance = 102.5905
## GAMLSS-RS iteration 6: Global Deviance = 102.5903
## GAMLSS-RS iteration 1: Global Deviance = 136.5956
## GAMLSS-RS iteration 2: Global Deviance = 136.5803
## GAMLSS-RS iteration 3: Global Deviance = 136.5791
## GAMLSS-RS iteration 4: Global Deviance = 136.579
## GAMLSS-RS iteration 1: Global Deviance = 81.393
## GAMLSS-RS iteration 2: Global Deviance = 77.6824
## GAMLSS-RS iteration 3: Global Deviance = 77.254
## GAMLSS-RS iteration 4: Global Deviance = 77.2153
## GAMLSS-RS iteration 5: Global Deviance = 77.2121
## GAMLSS-RS iteration 6: Global Deviance = 77.2118
## GAMLSS-RS iteration 1: Global Deviance = 73.7488
## GAMLSS-RS iteration 2: Global Deviance = 72.1195
## GAMLSS-RS iteration 3: Global Deviance = 71.8842
## GAMLSS-RS iteration 4: Global Deviance = 71.8546
## GAMLSS-RS iteration 5: Global Deviance = 71.8511
## GAMLSS-RS iteration 6: Global Deviance = 71.8507
## GAMLSS-RS iteration 1: Global Deviance = 166.9719
## GAMLSS-RS iteration 2: Global Deviance = 166.078
## GAMLSS-RS iteration 3: Global Deviance = 166.0129
## GAMLSS-RS iteration 4: Global Deviance = 166.0089
## GAMLSS-RS iteration 5: Global Deviance = 166.0086
## GAMLSS-RS iteration 1: Global Deviance = 105.7648
## GAMLSS-RS iteration 2: Global Deviance = 100.8399
## GAMLSS-RS iteration 3: Global Deviance = 100.1831
## GAMLSS-RS iteration 4: Global Deviance = 100.1185
## GAMLSS-RS iteration 5: Global Deviance = 100.1129
## GAMLSS-RS iteration 6: Global Deviance = 100.1124
## GAMLSS-RS iteration 1: Global Deviance = 139.7166
## GAMLSS-RS iteration 2: Global Deviance = 138.1315
## GAMLSS-RS iteration 3: Global Deviance = 137.9819
## GAMLSS-RS iteration 4: Global Deviance = 137.9701
## GAMLSS-RS iteration 5: Global Deviance = 137.9692
## GAMLSS-RS iteration 1: Global Deviance = 75.1063
## GAMLSS-RS iteration 2: Global Deviance = 65.8401
## GAMLSS-RS iteration 3: Global Deviance = 64.1204
## GAMLSS-RS iteration 4: Global Deviance = 63.9052
## GAMLSS-RS iteration 5: Global Deviance = 63.883
## GAMLSS-RS iteration 6: Global Deviance = 63.8809
## GAMLSS-RS iteration 7: Global Deviance = 63.8807
## GAMLSS-RS iteration 1: Global Deviance = 68.7275
## GAMLSS-RS iteration 2: Global Deviance = 64.6998
## GAMLSS-RS iteration 3: Global Deviance = 64.0535
## GAMLSS-RS iteration 4: Global Deviance = 63.9713
## GAMLSS-RS iteration 5: Global Deviance = 63.9619
## GAMLSS-RS iteration 6: Global Deviance = 63.9608
## GAMLSS-RS iteration 7: Global Deviance = 63.9607
## GAMLSS-RS iteration 1: Global Deviance = 151.088
## GAMLSS-RS iteration 2: Global Deviance = 148.8564
## GAMLSS-RS iteration 3: Global Deviance = 148.647
## GAMLSS-RS iteration 4: Global Deviance = 148.6314
## GAMLSS-RS iteration 5: Global Deviance = 148.6303
```

```
## GAMLSS-RS iteration 6: Global Deviance = 148.6302
## GAMLSS-RS iteration 1: Global Deviance = 97.5498
## GAMLSS-RS iteration 2: Global Deviance = 96.5877
## GAMLSS-RS iteration 3: Global Deviance = 96.4896
## GAMLSS-RS iteration 4: Global Deviance = 96.4806
## GAMLSS-RS iteration 5: Global Deviance = 96.4798
## GAMLSS-RS iteration 1: Global Deviance = 107.7938
## GAMLSS-RS iteration 2: Global Deviance = 104.6168
## GAMLSS-RS iteration 3: Global Deviance = 104.35
## GAMLSS-RS iteration 4: Global Deviance = 104.3324
## GAMLSS-RS iteration 5: Global Deviance = 104.3313
## GAMLSS-RS iteration 6: Global Deviance = 104.3313
## GAMLSS-RS iteration 1: Global Deviance = 162.0905
## GAMLSS-RS iteration 2: Global Deviance = 159.2476
## GAMLSS-RS iteration 3: Global Deviance = 159.0405
## GAMLSS-RS iteration 4: Global Deviance = 159.0294
## GAMLSS-RS iteration 5: Global Deviance = 159.0289
## GAMLSS-RS iteration 1: Global Deviance = 87.0947
## GAMLSS-RS iteration 2: Global Deviance = 81.9065
## GAMLSS-RS iteration 3: Global Deviance = 81.0149
## GAMLSS-RS iteration 4: Global Deviance = 80.8982
## GAMLSS-RS iteration 5: Global Deviance = 80.8847
## GAMLSS-RS iteration 6: Global Deviance = 80.8832
## GAMLSS-RS iteration 7: Global Deviance = 80.883
## GAMLSS-RS iteration 1: Global Deviance = 117.8511
## GAMLSS-RS iteration 2: Global Deviance = 113.7987
## GAMLSS-RS iteration 3: Global Deviance = 113.4928
## GAMLSS-RS iteration 4: Global Deviance = 113.4758
## GAMLSS-RS iteration 5: Global Deviance = 113.475
## GAMLSS-RS iteration 1: Global Deviance = 93.218
## GAMLSS-RS iteration 2: Global Deviance = 86.8627
## GAMLSS-RS iteration 3: Global Deviance = 85.9648
## GAMLSS-RS iteration 4: Global Deviance = 85.8747
## GAMLSS-RS iteration 5: Global Deviance = 85.8669
## GAMLSS-RS iteration 6: Global Deviance = 85.8662
## GAMLSS-RS iteration 1: Global Deviance = 73.911
## GAMLSS-RS iteration 2: Global Deviance = 65.5134
## GAMLSS-RS iteration 3: Global Deviance = 64.3155
## GAMLSS-RS iteration 4: Global Deviance = 64.1996
## GAMLSS-RS iteration 5: Global Deviance = 64.1901
## GAMLSS-RS iteration 6: Global Deviance = 64.1893
## GAMLSS-RS iteration 1: Global Deviance = 52.9897
## GAMLSS-RS iteration 2: Global Deviance = 48.2413
## GAMLSS-RS iteration 3: Global Deviance = 47.4435
## GAMLSS-RS iteration 4: Global Deviance = 47.3382
## GAMLSS-RS iteration 5: Global Deviance = 47.3257
## GAMLSS-RS iteration 6: Global Deviance = 47.3243
## GAMLSS-RS iteration 7: Global Deviance = 47.3241
## GAMLSS-RS iteration 1: Global Deviance = 157.5059
## GAMLSS-RS iteration 2: Global Deviance = 156.3569
## GAMLSS-RS iteration 3: Global Deviance = 156.2789
## GAMLSS-RS iteration 4: Global Deviance = 156.2744
## GAMLSS-RS iteration 5: Global Deviance = 156.2741
## GAMLSS-RS iteration 1: Global Deviance = 175.9177
```

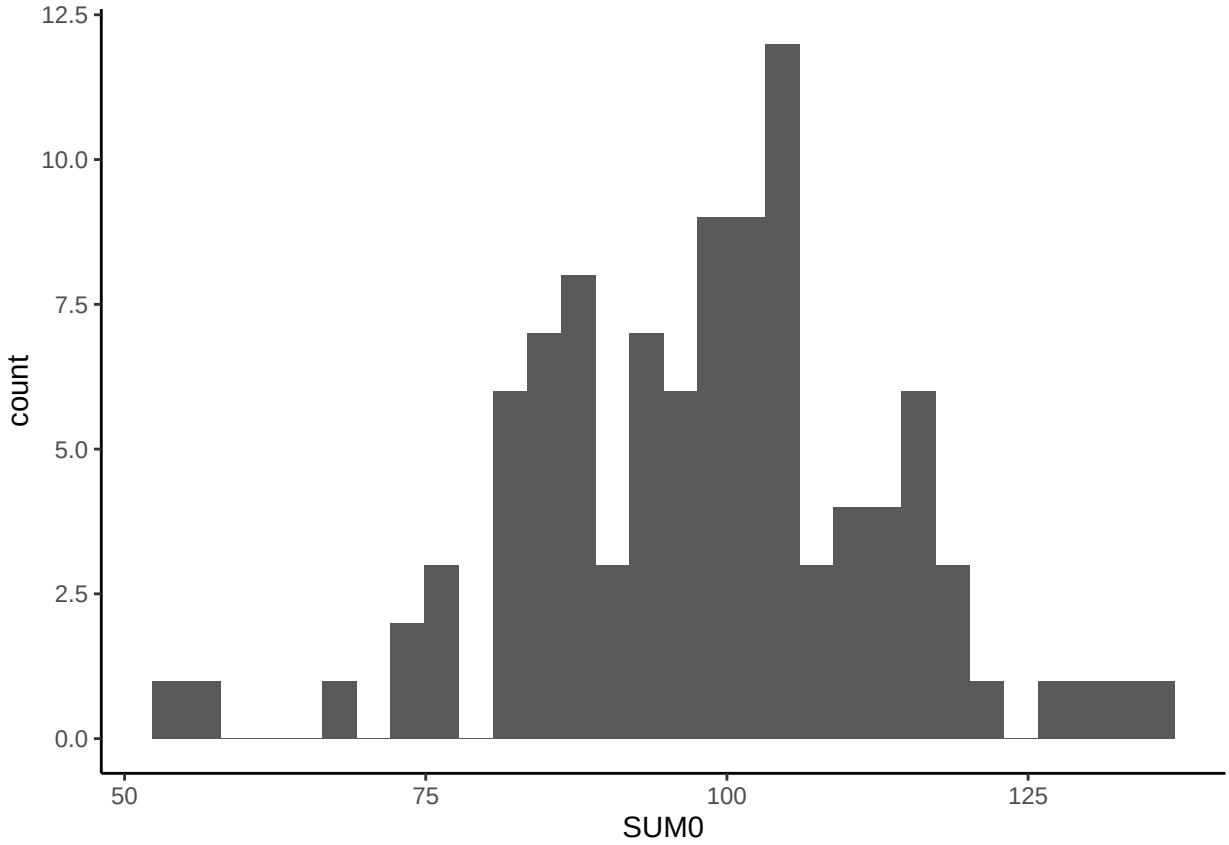
```
## GAMLSS-RS iteration 2: Global Deviance = 175.6107
## GAMLSS-RS iteration 3: Global Deviance = 175.5931
## GAMLSS-RS iteration 4: Global Deviance = 175.5921
## GAMLSS-RS iteration 1: Global Deviance = 103.6497
## GAMLSS-RS iteration 2: Global Deviance = 96.0455
## GAMLSS-RS iteration 3: Global Deviance = 95.2302
## GAMLSS-RS iteration 4: Global Deviance = 95.1729
## GAMLSS-RS iteration 5: Global Deviance = 95.1695
## GAMLSS-RS iteration 6: Global Deviance = 95.1693
## GAMLSS-RS iteration 1: Global Deviance = 113.9233
## GAMLSS-RS iteration 2: Global Deviance = 107.783
## GAMLSS-RS iteration 3: Global Deviance = 107.2305
## GAMLSS-RS iteration 4: Global Deviance = 107.197
## GAMLSS-RS iteration 5: Global Deviance = 107.1952
## GAMLSS-RS iteration 6: Global Deviance = 107.1951
## GAMLSS-RS iteration 1: Global Deviance = 140.1256
## GAMLSS-RS iteration 2: Global Deviance = 134.1626
## GAMLSS-RS iteration 3: Global Deviance = 133.5741
## GAMLSS-RS iteration 4: Global Deviance = 133.5364
## GAMLSS-RS iteration 5: Global Deviance = 133.5343
## GAMLSS-RS iteration 6: Global Deviance = 133.5342
## GAMLSS-RS iteration 1: Global Deviance = 180.9677
## GAMLSS-RS iteration 2: Global Deviance = 180.5126
## GAMLSS-RS iteration 3: Global Deviance = 180.4903
## GAMLSS-RS iteration 4: Global Deviance = 180.4893
## GAMLSS-RS iteration 1: Global Deviance = 122.3958
## GAMLSS-RS iteration 2: Global Deviance = 116.1451
## GAMLSS-RS iteration 3: Global Deviance = 115.6216
## GAMLSS-RS iteration 4: Global Deviance = 115.5926
## GAMLSS-RS iteration 5: Global Deviance = 115.5912
## GAMLSS-RS iteration 6: Global Deviance = 115.5911
## GAMLSS-RS iteration 1: Global Deviance = 125.4913
## GAMLSS-RS iteration 2: Global Deviance = 124.409
## GAMLSS-RS iteration 3: Global Deviance = 124.2555
## GAMLSS-RS iteration 4: Global Deviance = 124.2365
## GAMLSS-RS iteration 5: Global Deviance = 124.2343
## GAMLSS-RS iteration 6: Global Deviance = 124.234
## GAMLSS-RS iteration 1: Global Deviance = 180.6364
## GAMLSS-RS iteration 2: Global Deviance = 180.2263
## GAMLSS-RS iteration 3: Global Deviance = 180.2025
## GAMLSS-RS iteration 4: Global Deviance = 180.2013
## GAMLSS-RS iteration 5: Global Deviance = 180.2012
## GAMLSS-RS iteration 1: Global Deviance = 124.9016
## GAMLSS-RS iteration 2: Global Deviance = 123.3582
## GAMLSS-RS iteration 3: Global Deviance = 123.2222
## GAMLSS-RS iteration 4: Global Deviance = 123.2121
## GAMLSS-RS iteration 5: Global Deviance = 123.2114
## GAMLSS-RS iteration 1: Global Deviance = 132.3482
## GAMLSS-RS iteration 2: Global Deviance = 129.1007
## GAMLSS-RS iteration 3: Global Deviance = 128.7566
## GAMLSS-RS iteration 4: Global Deviance = 128.7288
## GAMLSS-RS iteration 5: Global Deviance = 128.7268
## GAMLSS-RS iteration 6: Global Deviance = 128.7266
## GAMLSS-RS iteration 1: Global Deviance = 154.76
```

```
## GAMLSS-RS iteration 2: Global Deviance = 148.5595
## GAMLSS-RS iteration 3: Global Deviance = 148.3369
## GAMLSS-RS iteration 4: Global Deviance = 148.3245
## GAMLSS-RS iteration 5: Global Deviance = 148.3239
## GAMLSS-RS iteration 1: Global Deviance = 106.9696
## GAMLSS-RS iteration 2: Global Deviance = 97.0441
## GAMLSS-RS iteration 3: Global Deviance = 95.844
## GAMLSS-RS iteration 4: Global Deviance = 95.7567
## GAMLSS-RS iteration 5: Global Deviance = 95.7515
## GAMLSS-RS iteration 6: Global Deviance = 95.7512
## GAMLSS-RS iteration 1: Global Deviance = 182.0358
## GAMLSS-RS iteration 2: Global Deviance = 180.5825
## GAMLSS-RS iteration 3: Global Deviance = 180.5142
## GAMLSS-RS iteration 4: Global Deviance = 180.5116
## GAMLSS-RS iteration 5: Global Deviance = 180.5115
## GAMLSS-RS iteration 1: Global Deviance = 118.5416
## GAMLSS-RS iteration 2: Global Deviance = 115.7322
## GAMLSS-RS iteration 3: Global Deviance = 115.4766
## GAMLSS-RS iteration 4: Global Deviance = 115.4583
## GAMLSS-RS iteration 5: Global Deviance = 115.4571
## GAMLSS-RS iteration 6: Global Deviance = 115.457
## GAMLSS-RS iteration 1: Global Deviance = 172.1897
## GAMLSS-RS iteration 2: Global Deviance = 165.7465
## GAMLSS-RS iteration 3: Global Deviance = 165.4964
## GAMLSS-RS iteration 4: Global Deviance = 165.4832
## GAMLSS-RS iteration 5: Global Deviance = 165.4826
## GAMLSS-RS iteration 1: Global Deviance = 134.2616
## GAMLSS-RS iteration 2: Global Deviance = 130.0933
## GAMLSS-RS iteration 3: Global Deviance = 129.6923
## GAMLSS-RS iteration 4: Global Deviance = 129.6645
## GAMLSS-RS iteration 5: Global Deviance = 129.6628
## GAMLSS-RS iteration 6: Global Deviance = 129.6627
## GAMLSS-RS iteration 1: Global Deviance = 107.5408
## GAMLSS-RS iteration 2: Global Deviance = 107.4628
## GAMLSS-RS iteration 3: Global Deviance = 107.4559
## GAMLSS-RS iteration 4: Global Deviance = 107.4553
## GAMLSS-RS iteration 1: Global Deviance = 152.2983
## GAMLSS-RS iteration 2: Global Deviance = 152.2914
## GAMLSS-RS iteration 3: Global Deviance = 152.2909
## GAMLSS-RS iteration 1: Global Deviance = 71.6267
## GAMLSS-RS iteration 2: Global Deviance = 63.0415
## GAMLSS-RS iteration 3: Global Deviance = 61.9207
## GAMLSS-RS iteration 4: Global Deviance = 61.8207
## GAMLSS-RS iteration 5: Global Deviance = 61.813
## GAMLSS-RS iteration 6: Global Deviance = 61.8124
## GAMLSS-RS iteration 1: Global Deviance = -13.4836
## GAMLSS-RS iteration 2: Global Deviance = -41.6068
## GAMLSS-RS iteration 3: Global Deviance = -48.3958
## GAMLSS-RS iteration 4: Global Deviance = -49.2653
## GAMLSS-RS iteration 5: Global Deviance = -49.3421
## GAMLSS-RS iteration 6: Global Deviance = -49.348
## GAMLSS-RS iteration 7: Global Deviance = -49.3484
## GAMLSS-RS iteration 1: Global Deviance = 190.1469
## GAMLSS-RS iteration 2: Global Deviance = 190.1456
```

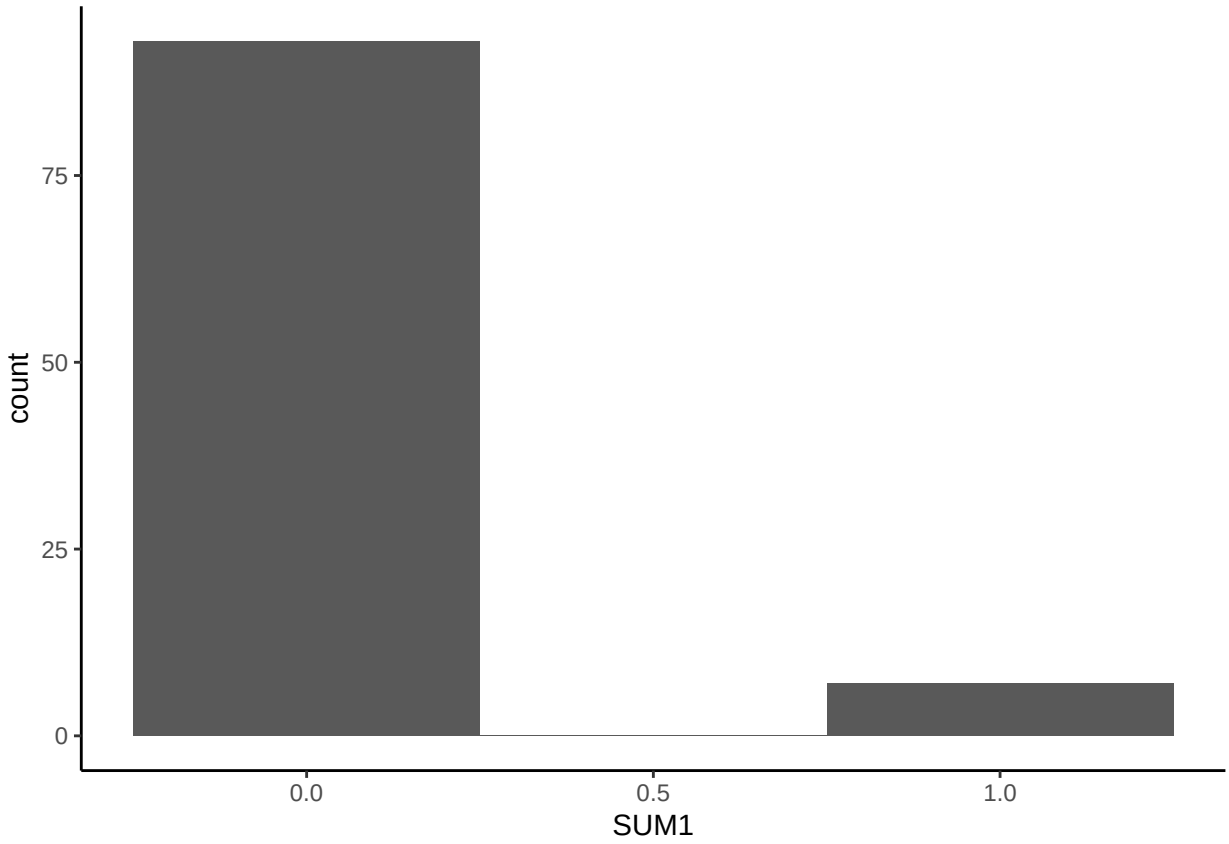
```
## GAMLSS-RS iteration 3: Global Deviance = 190.1456
## GAMLSS-RS iteration 1: Global Deviance = 196.9091
## GAMLSS-RS iteration 2: Global Deviance = 195.9961
## GAMLSS-RS iteration 3: Global Deviance = 195.9682
## GAMLSS-RS iteration 4: Global Deviance = 195.9675
## GAMLSS-RS iteration 1: Global Deviance = 150.1368
## GAMLSS-RS iteration 2: Global Deviance = 149.3097
## GAMLSS-RS iteration 3: Global Deviance = 149.243
## GAMLSS-RS iteration 4: Global Deviance = 149.2382
## GAMLSS-RS iteration 5: Global Deviance = 149.2379
## GAMLSS-RS iteration 1: Global Deviance = 110.4196
## GAMLSS-RS iteration 2: Global Deviance = 110.0116
## GAMLSS-RS iteration 3: Global Deviance = 109.9768
## GAMLSS-RS iteration 4: Global Deviance = 109.974
## GAMLSS-RS iteration 5: Global Deviance = 109.9738
## GAMLSS-RS iteration 1: Global Deviance = 125.5791
## GAMLSS-RS iteration 2: Global Deviance = 125.531
## GAMLSS-RS iteration 3: Global Deviance = 125.5271
## GAMLSS-RS iteration 4: Global Deviance = 125.5267
## GAMLSS-RS iteration 1: Global Deviance = 164.1563
## GAMLSS-RS iteration 2: Global Deviance = 163.4089
## GAMLSS-RS iteration 3: Global Deviance = 163.3528
## GAMLSS-RS iteration 4: Global Deviance = 163.3491
## GAMLSS-RS iteration 5: Global Deviance = 163.3489
## GAMLSS-RS iteration 1: Global Deviance = 122.6169
## GAMLSS-RS iteration 2: Global Deviance = 119.9881
## GAMLSS-RS iteration 3: Global Deviance = 119.6103
## GAMLSS-RS iteration 4: Global Deviance = 119.5667
## GAMLSS-RS iteration 5: Global Deviance = 119.5621
## GAMLSS-RS iteration 6: Global Deviance = 119.5616
## GAMLSS-RS iteration 1: Global Deviance = 2.5314
## GAMLSS-RS iteration 2: Global Deviance = 0.788
## GAMLSS-RS iteration 3: Global Deviance = 0.5841
## GAMLSS-RS iteration 4: Global Deviance = 0.5632
## GAMLSS-RS iteration 5: Global Deviance = 0.5612
## GAMLSS-RS iteration 6: Global Deviance = 0.561
## GAMLSS-RS iteration 1: Global Deviance = 137.191
## GAMLSS-RS iteration 2: Global Deviance = 132.9136
## GAMLSS-RS iteration 3: Global Deviance = 132.5114
## GAMLSS-RS iteration 4: Global Deviance = 132.4845
## GAMLSS-RS iteration 5: Global Deviance = 132.4829
## GAMLSS-RS iteration 6: Global Deviance = 132.4828
## GAMLSS-RS iteration 1: Global Deviance = 128.9848
## GAMLSS-RS iteration 2: Global Deviance = 126.8916
## GAMLSS-RS iteration 3: Global Deviance = 126.7344
## GAMLSS-RS iteration 4: Global Deviance = 126.7248
## GAMLSS-RS iteration 5: Global Deviance = 126.7242
```

```r
PVs_0and1_2 <- sapply(Beta_coefs2, getNums, "SUM0", "SUM1")
df_PVs2 <- data.frame(t(PVs_0and1_2))

# histogram of PVs = 0
ggplot(df_PVs2, aes(SUM0)) +
  geom_histogram()
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




```
# histogram of PVs = 1
ggplot(df_PVs2, aes(SUM1))  +
  geom_histogram(bins = 3)
```




```
# TRUE ROC(t) values for t = 0, 1/50, ..., 1.
T_vals2 <- mapply(true_binormal_ROC, t = c(0, t, 1), X0 = rep(c(0, t, 1),
each = length(t) + 2),
MoreArgs = list(c1 = 3, d1 = 6, sd1 = 1.5, c0 = 1.5, d0 = 3, sd0 = 1.5))

PDF_vals2 <- sapply(PDF_coefs2, wrapper, ROC_fun = binorm_ROC)

MSEs2 <- apply(PDF_vals2, 2, function(x) sum((x - T_vals2)^2)/n_T^2)
summary(MSEs2)
```

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## 1.824e-05 3.801e-04 7.465e-04 9.840e-04 1.281e-03 4.318e-03
```

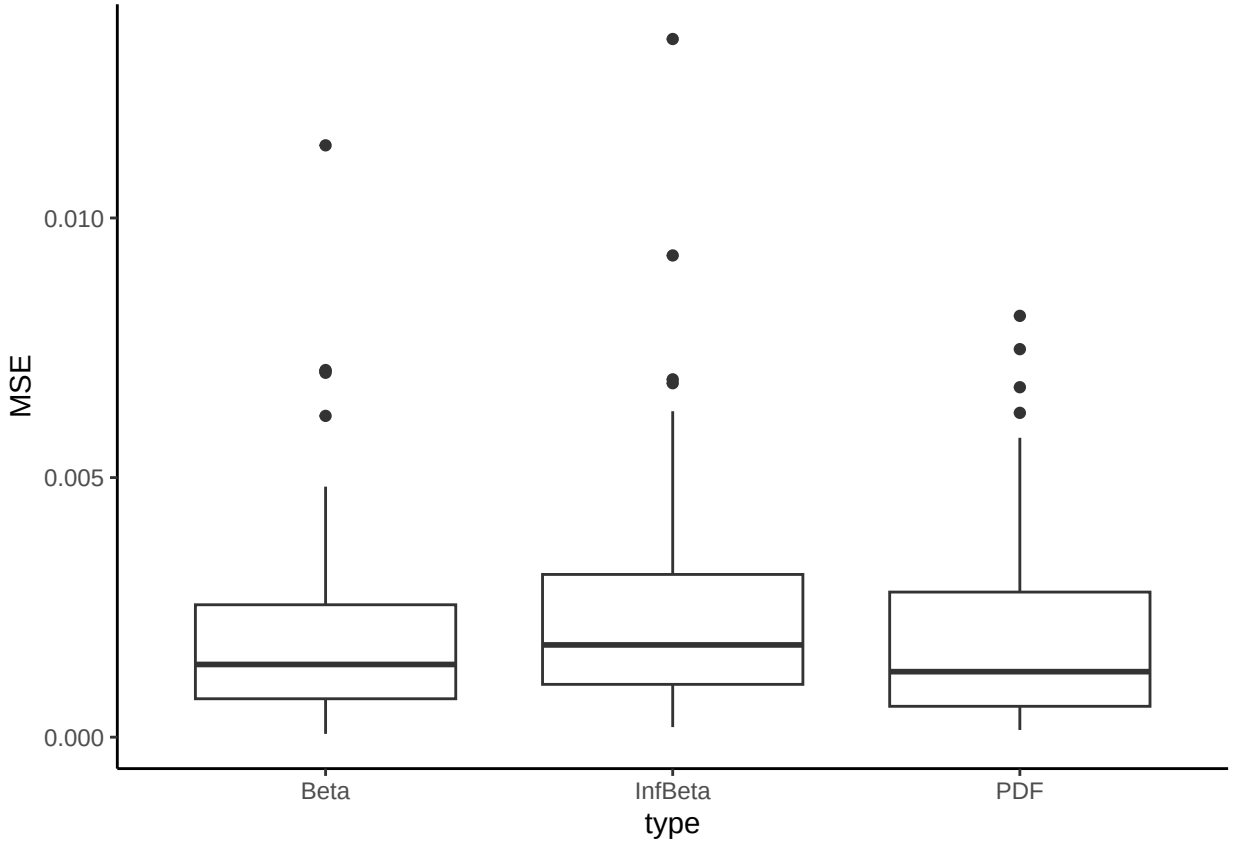
```
outputBe2 <- sapply(Beta_coefs2, wrapper, ROC_fun = Beta_ROC)

MSE_Bes2 <- apply(outputBe2, 2, function(x) sum((x - T_vals2)^2)/n_T^2)
summary(MSE_Bes2)
```

```
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## 0.001261 0.002512 0.003059 0.003221 0.003964 0.007552
```

```
outputInf2 <- sapply(InfBeta_coefs2, wrapper, ROC_fun = InfBeta_ROC)
MSE_Inf2 <- apply(outputInf2, 2, function(x) sum((x - T_vals2)^2)/n_T^2)
summary(MSE_Inf2)
```

```
##     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## 0.004332 0.008086 0.009726 0.010243 0.012035 0.020518
```

```r
# Boxplots of the MSE values for each method

datMSE <- data.frame(MSE = c(MSE_PDF, MSE_Be, MSE_Inf),
type = c(rep("PDF", length(MSE_PDF)), rep("Beta", length(MSE_Be)),
rep("InfBeta", length(MSE_Inf))))

ggplot(datMSE, aes(type, MSE)) +
  geom_boxplot()
```



#Application using the South African Heaert Disease Data

```r
# load packages
library("tidyverse"); theme_set(theme_classic())
library("pROC")
library("quantreg")
library("betareg")

# load data
dat <- read.csv("SAHeart_Disease.csv")
dat$row.names <- NULL
dat$famhist <- ifelse(dat$famhist == "Present", 1, 0) %>% factor()

# Empirical AUC of ldl
roc(dat$chd, dat$ldl)
```

```
## Setting levels: control = 0, case = 1
```

```
## Setting direction: controls < cases
```

```
##
## Call:
## roc.default(response = dat$chd, predictor = dat$ldl)
##
## Data: dat$ldl in 302 controls (dat$chd 0) < 160 cases (dat$chd 1).
## Area under the curve: 0.6643
```

```
# selecting variables for ROC regression
dat_mod <- dplyr::select(dat, chd, age, ldl, famhist, tobacco)

n_T <- 50
t <- seq(1/n_T, (n_T - 1)/n_T, 1/n_T) # set of FPRs

# 1. PDF ROC on SA chd data set
PDF_fun <- function(data, t){
  df0 <- filter(data, chd == 0)
  df1 <- filter(data, chd == 1)

  n1 <- nrow(df1)
  # quantile regression to estimate reference survival
  qr0 <- rq(ldl ~ famhist + age + tobacco, data = df0, tau = t)

  # covariate adjusted survival for diseased observations
  pred1 <- predict.rq(qr0, newdata = df1)

  Inv_t <- qnorm(rev(t))  # normal quantiles of FPRs
  nq <- length(t)         # number of quantiles

  # reshaping the data
  t_pred1 <- t(pred1)
  col_pred1 <- c(t_pred1)
  col_t <- rep(Inv_t, n1)
  col_ldl <- rep(df1$ldl, each = nq)
  col_age <- rep(df1$age, each = nq)
  col_tobacco <- rep(df1$tobacco, each = nq)
  col_famhist <- rep(df1$famhist, each = nq)
  col_uit <- as.numeric(col_ldl >= col_pred1)

  probitData <- data.frame(fdbar = col_pred1, phiInv = col_t,
                           ldl = col_ldl, age = col_age,
                           tobacco = col_tobacco, famhist = col_famhist,
                           uit = col_uit)
  probMod <- glm(uit ~ phiInv + famhist + age + tobacco,
                 family = binomial(link = "probit"), data = probitData)
  summary(probMod)
}

coefs_PDF <- PDF_fun(dat_mod, t = t)$coefficients[,1]   # coefficients
coefs_PDF
```

```
##  (Intercept)       phiInv     famhist1          age      tobacco
##  0.431500639  0.974357399  0.499319779 -0.003623718 -0.047432066
```

```
# bootstrapping SEs
B <- 100    #changed to 100
```

```r
results <- matrix(NA ,nrow = B, ncol = 5)
for(b in 1:B){
  boot_dat <- dat_mod[sample(nrow(dat_mod), nrow(dat_mod),
                             replace = TRUE),]
  results[b,] <- PDF_fun(boot_dat, t = t)$coefficients[,2]
}

colMeans(results) # Bootstrap SEs for coefficients
```

```
## [1] 0.082764871 0.022398332 0.035062941 0.001709852 0.003346367
```

```r
# get z statistics and p-values
get_sigs <- function(estimate, std_error){
  z <- estimate / std_error
  c(z, ifelse(z > 0, pnorm(z, lower.tail = FALSE) * 2,
         pnorm(z) * 2))
}
map2(.x = coefs_PDF,
     .y = colMeans(results), get_sigs)
```

```
## $`(Intercept)`
## [1] 5.213572e+00 1.852383e-07
##
## $phiInv
## [1] 43.50134  0.00000
##
## $famhist1
## [1] 1.424067e+01 5.123347e-46
##
## $age
## [1] -2.11931654  0.03406372
##
## $tobacco
## [1] -1.417420e+01  1.323465e-45
```

```r
# Estimate of AUC: age = 45, famhist = 1, tobacco = 5
pnorm((coefs_PDF[1] + coefs_PDF[3] + coefs_PDF[4] * 45 +
         coefs_PDF[5] * 5)/sqrt(1 + coefs_PDF[2]^2))
```

```
## (Intercept)
##   0.6480371
```

```r
# 2. Beta ROC on SA chd data set

# function that transforms PVs, so they're in the open unit interval
dep_trans_betareg <- function(y){
  n_obs <- sum(!(is.na(y)))
  (y * (n_obs - 1) + 0.5) / n_obs
}

Beta_FUN <- function(data, t){
  df0 <- filter(data, chd == 0)
  df1 <- filter(data, chd == 1)

  Y_ldl <- pull(df1, ldl)
  n1 <- nrow(df1)
```

```r
  # quantile regression to estimate reference survival
  qr0 <<- rq(ldl ~ famhist + age + tobacco, data = df0, tau = t)

  # covariate adjusted survival for diseased observations
  pred1 <<- predict.rq(qr0, newdata = df1)
  t_pred1 <- t(pred1)
  pred3 <- lapply(seq_len(ncol(t_pred1)), function(i) t_pred1[,i])
  PV_dis <<- mapply(function(x, y) mean(x >= y), x = pred3, y = Y_ldl)
  datdis <<- data.frame(Y_ldl, age = df1$age,
                        famhist = df1$famhist, tobacco = df1$tobacco,
                        PV_dis)

  if(any(datdis$PV_dis == 0 | datdis$PV_dis == 1)){ # transform PVs if any are 0 or 1
    SUM0 <- sum(datdis$PV_dis == 0)
    SUM1 <- sum(datdis$PV_dis == 1)
    BetaMod <<- betareg(dep_trans_betareg(PV_dis) ~ famhist + age + tobacco,
                        data = datdis,
                        link.phi = "identity", link = "logit")

    BetaMod$coefficients
  }else{
    BetaMod <- betareg(PV_dis ~ famhist + age + tobacco, data = datdis,
                        link.phi = "identity", link = "logit")

    BetaMod$coefficients
  }
}

coefs_Be <- Beta_FUN(dat_mod, t = t)    # coefficients
coefs_Be
```

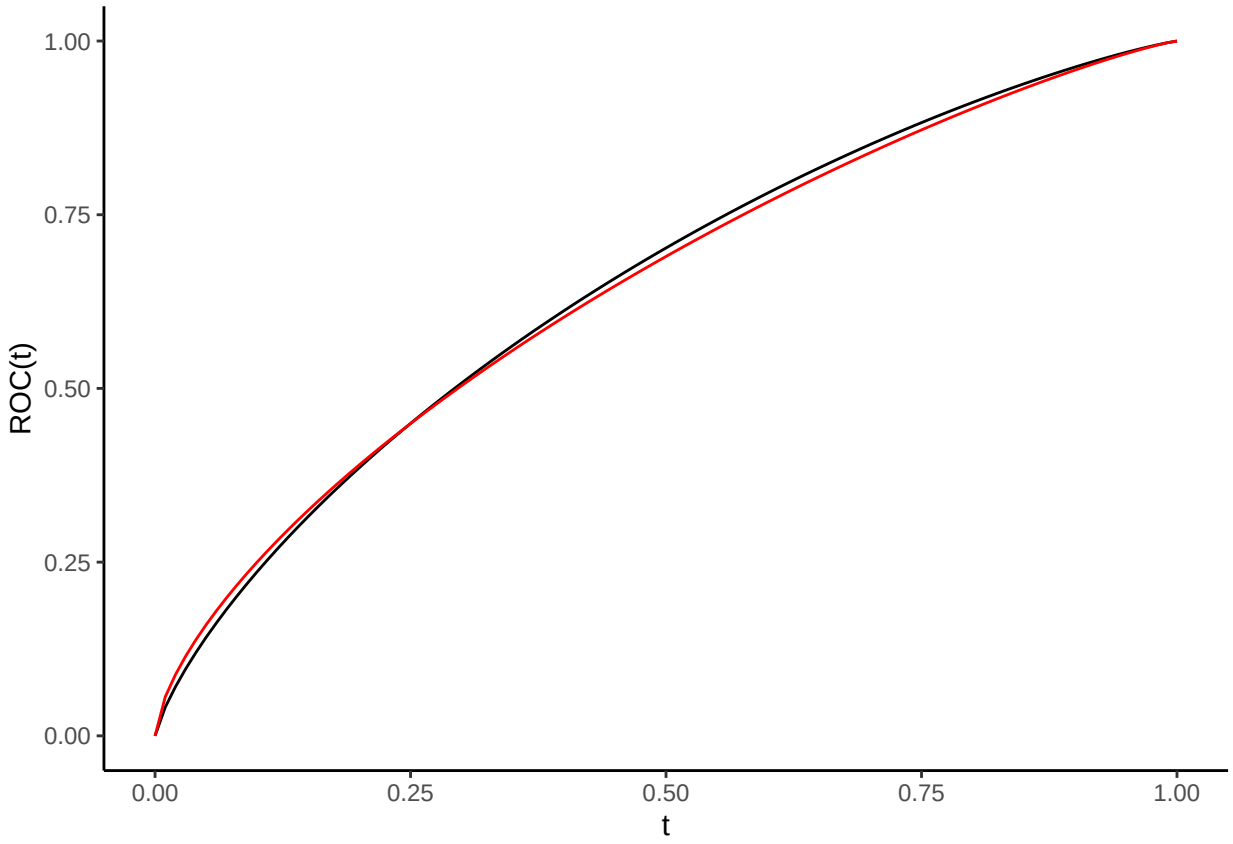
```
## $mean
##   (Intercept)      famhist1           age       tobacco
## -0.3767570521 -0.4953297874  0.0001815581  0.0539334928
##
## $precision
##    (phi)
## 1.833178
```

```r
# function to get Beta AUC
Beta_AUC <- function(coefs, X_val = matrix(c(1, 1, 45, 5), nrow = 4)){
  coefs <- unlist(coefs)
  Be_hat0 <- coefs[[1]]
  Be_hat_fh <- coefs[[2]]
  Be_hat_age <- coefs[[3]]
  Be_hat_tob <- coefs[[4]]
  phi_hat <- coefs[[5]]

  Be <- matrix(c(Be_hat0, Be_hat_fh, Be_hat_age,
                 Be_hat_tob), ncol = 4)

  a_hat <- phi_hat/(1 + exp(-Be %*% X_val))
  b_hat <- phi_hat * (1 - 1/(1 + exp(-Be %*% X_val)))
  # pbeta(t, a_hat, b_hat)    ## ROC
```

```r
  1 - a_hat / (a_hat + b_hat) ## AUC
}

Beta_AUC(coefs_Be)
```

```
##           [,1]
## [1,] 0.6443395
```

```r
# Plotting ROC curves for PDF method and Beta method

Beta_ROC <- function(coefs, t, X_val = matrix(c(1, 1, 45, 5), nrow = 4)){
  coefs <- unlist(coefs)
  Be_hat0 <- coefs[[1]]
  Be_hat_fh <- coefs[[2]]
  Be_hat_age <- coefs[[3]]
  Be_hat_tob <- coefs[[4]]
  phi_hat <- coefs[[5]]

  Be <- matrix(c(Be_hat0, Be_hat_fh, Be_hat_age,
                 Be_hat_tob), ncol = 4)

  a_hat <- phi_hat/(1 + exp(-Be %*% X_val))
  b_hat <- phi_hat * (1 - 1/(1 + exp(-Be %*% X_val)))

  pbeta(t, a_hat, b_hat)    ## ROC
}

binorm_ROC <- function(coefs, t, X_val = matrix(c(1, 45, 5), nrow = 3)){
  ga_1 <- coefs[1]
  ga_2 <- coefs[2]
  Be <- coefs[c(3, 4, 5)] %>% matrix %>% t

  pnorm(ga_1 + ga_2 * qnorm(t) + c(Be %*% X_val))
}

# plot of ROC curves
ggplot(data.frame(t = c(0, 1)), aes(t)) +
  stat_function(fun = binorm_ROC, args = list(coefs = coefs_PDF)) +
  stat_function(fun = Beta_ROC, args = list(coefs = coefs_Be),
                color = "red") +
  ylab("ROC(t)")
```


1.00
0.75
0.50
0.25
0.00
ROC(t)
0.00
0.25
0.50
0.75
1.00
t