Assignment #1: Time Plots, Filters, Covariance, and Stationarity

STA 5362: Time Series Analysis

Fall Semester

1. Plot each of the following time series. Decide on the best visual presentation (with dots for each observation or without, proper dates on the x-axis, etc), and then comment on the features that you notice in the plots.

   (a) Yealy average sunspot numbers from 1700 to 1988. Load with the following code: `data(sunspot.year)`. For more information, see `help(sunspot.year)`.

   (b) The Texas Water Development Board (TWDB) maintains a database on water reservoirs across Texas. Plot the `percent_full` variable, which is the ratio of conservation storage to conservation capacity expressed as a percentage. Load the data with the following code:

   ```
   www <- "https://www.waterdatafortexas.org/reservoirs/statewide.csv"
   water <- read.csv(file=www, header=T, skip=29)
   ```

   (c) Daily stock price returns of Amazon, calculated as the difference between today's ending price and the prior day's ending price. To get this data, you will need the `quantmod` package and the following code.

   ```
   price <- getSymbols("AMZN", src="yahoo")
   amznReturn <- diff(AMZN[,4])
   ```

2. Let $W_t \sim \mathrm{N}(0,\sigma_W^2)$ be white noise, and consider the filtered version of white noise X_t defined by

$$X_t = a_{-1}W_{t+1} + a_0W_t + a_1W_{t-1},$$

   where a_{-1}, a_0, and a_1 are real numbers.

   (a) Find an expression for the autocovariance $\gamma_X(s,t)$ and autocorrelation $\rho_X(s,t)$ functions. Simplify $\gamma_X(s,t)$ and $\rho_X(s,t)$ for $a_{-1} = 0.25, a_0 = 0.5$, and $a_1 = 0.25$.

(b) Generate a realization of length $n = 500$ for the white noise series W_t. Use the `filter` function to compute X_t for $a_{-1} = 0.25, a_0 = 0.5$ and $a_1 = 0.25$. Asuume $\sigma_W^2 = 1$. Supply the R code.

(c) Plot the simulated realizations of W_t and X_t on the same axes. Use different colors and a legend to distinguish the two series. Compare and contrast w_t with x_t.

(d) Use the `acf` function to compute and plot the sample autocorrelation function for X_t. (Don't forget to make the vertical axes go from -1 to 1.) Superimpose the exact expression for the (exact) autocorrelation function from part (a). What do you notice about the sample autocorrelation funtion as an estimator of the true autocorrelation function?

3. Schumay & Stoffer #1.2

4. Schumay & Stoffer #1.3

5. Schumay & Stoffer #1.5

6. Schumay & Stoffer #1.6.

   Note: For the "simplified version" of the autocovariance, focus only on

$$Y_t = \frac{1}{2q+1} \sum_{j=-\infty}^{\infty} a_j W_{t-j},$$

   where $a_j = 1$ for $j = 0, \pm 1, \ldots, \pm q$, and is zero otherwise. You want to show that the autocovariance function of Y is

$$\gamma_Y(h) = \frac{\sigma^2 (2q + 1 - |h|)}{(2q+1)^2},$$

   for $h = 0, \pm 1, \ldots, \pm 2q$ and zero for $|h| > 2q$.