Structured Plasma Waves

Internship Report

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February 2025

Abstract

This report summarizes the work conducted during an internship at the Group of Lasers and Plasmas. This work focused on studying and simulating 1D electrostatic plasma waves. Initially, a theoretical Python code was developed to explore wavepacket formation and propagation with arbitrary plane wave distributions and parameters. Then, this tool was transposed to a more realistic simulation environment, using the particle-in-cell (PIC) code ZPIC. Special attention was given to numerical stability and parameter tuning, aiming to accurately simulate the propagation and evolution of structured plasma wavepackets.

1 Introduction

During this internship, the primary objective was to investigate the behavior of 1D electrostatic plasma waves. Plasma oscillations were analyzed in a simplified framework, where all relevant physical quantities (density n, velocity v, electric field E) were split into equilibrium (n_0, v_0, E_0) and small perturbation (n_1, v_1, E_1) components. The latter were assumed to evolve sinusoidally according to the real part of $\psi_1(x,t) = \psi_1 e^{i(kx-wt)}$. The linearization of the continuity, momentum, and Poisson's equations provided the following relations:

$$im\omega v_1 = eE_1 \ , \tag{1}$$

$$i\omega n_1 = n_0 ikv_1 \ , \tag{2}$$

$$ik\varepsilon_0 E_1 = -en_1 \ . \tag{3}$$

Here, similarly to the procedure described in section 4.3 of [1], we assumed $\nabla n_0 = v_0 = E_0 = 0$ and also $\frac{\partial n_0}{\partial t} = \frac{\partial v_0}{\partial t} = \frac{\partial E_0}{\partial t} = 0$. For electron plasma waves, the standard dispersion relation corresponds to

$$\omega^2 = \omega_p^2 + \frac{3}{2}k^2 v_{\text{th}}^2 \ , \tag{4}$$

with $\omega_p = \sqrt{\frac{n_o e^2}{\epsilon_0 m}}$ being the plasma frequency.

2 Methods and Simulations

2.1 Theoretical 1D Code

An initial theoretical Python code was developed, following the normalizations in [2], to study the evolution and field structure of arbitrary plasma wavepackets. The wavepackets are constructed by superposing sinusoidal modes over a range of wavenumbers k, each weighted by a Gaussian envelope centered at k_0 with width σ_k. By setting

$$n_1(x,t) = \sum_{k=k_i}^{k_f} \frac{\Delta k}{\sqrt{2\pi\sigma_k^2}} \exp\left[-\frac{(k-k_0)^2}{2\sigma_k^2}\right] \cos(kx - \omega t),$$

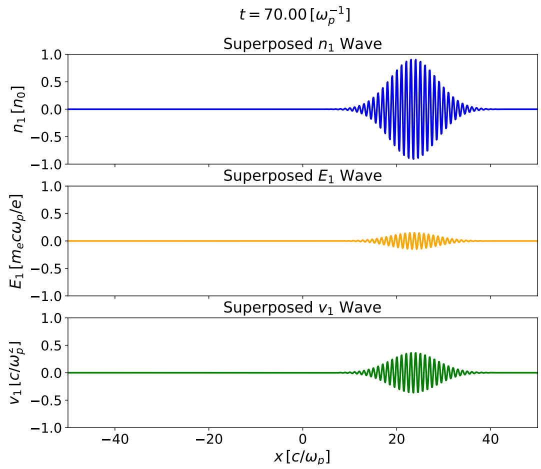

Figure 1: Snapshot of the wavepacket at $t = 70\,[\omega_p^{-1}]$ for $u_{th} = 0.3c$ and box size = 100. The wavepacket, which was initially centered at $x = 0$, is shown in terms of the density perturbation n_1 (blue), electric field E_1 (red), and velocity v_1 (green).

(with $\Delta k = \frac{2\pi}{\text{box size}}$) we recovered the expressions for $v_1(x,t)$ and $E_1(x,t)$ with eqs. (2) and (3). In the cold plasma limit ($u_{\text{th}} = 0$), the relation is dispersionless, and also the group velocity is zero. In contrast, for finite u_{th}, $\omega(k)$ depends on k, causing each Fourier component to oscillate at a slightly different frequency. This causes the wavepacket to spread (widen) over time and its peak - corresponding to the central wavenumber k_0 - approximately propagates at the group velocity $v_g = \frac{d\omega}{dk}|_{k=k_0}$, as illustrated by Fig.1 and by the waterfall plot in Fig.2.

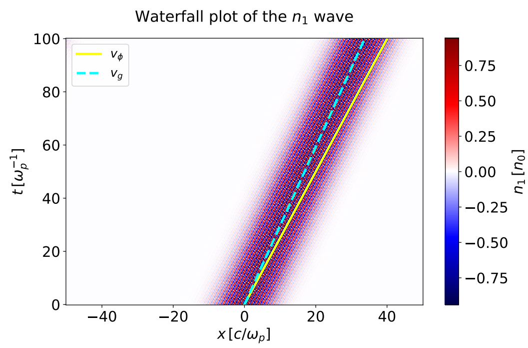

Figure 2: Waterfall plot of the density perturbation n_1 as a function of position x and $\omega_p t$ for the case $u_{th} = 0.3c$. The yellow line represents the theoretical phase velocity v_ϕ, while the cyan line marks the group velocity v_g evaluated at the central wavenumber k_0.

2.2 Particle-in-Cell (PIC) Approach

The next step consisted of transposing the theoretical code to a more realistic simulation environment using the particle-in-cell (PIC) code ZPIC [3]. In PIC simulations, a discrete spatial grid is used to compute self-consistent electric fields, which are then applied to advance the charged particles in time. This approach captures phenomena such as wavebreaking and nonlinear effects arising from particle motion more accurately. Before studying the wavepacket evolution, we focused on understanding how the most relevant simulation parameters influenced the lifetime of the waves (i.e., the duration for which their overall shape was preserved). For this, we examined a single wave and systematically adjusted parameters like the number of cells nx, box size box, time step dt, particles per cell ppc, and wavenumber k. Once we understood the impact of each parameter and how to maximize the wave's lifetime, the goal was to guide and propagate wavepackets via controlled plasma interfaces.

3 Results

3.1 Single Wave Analysis

For a single sinusoidal wave in ZPIC, higher values of nx, dt, and ppc noticeably improved wave stability, allowing the perturbations to survive longer before transitioning to noise. Moreover, we checked that lower values of n_1 (amplitude of the perturbation) enhance stability, as expected. Finally, we investigated the impact of box size through two approaches: one where the number of wavelengths a inside the box was kept constant, and another where the wavenumber $k = a\frac{2\pi}{\text{box}}$ was fixed (which varies the number of wavelengths). The former was inconclusive, but the latter showed that larger box sizes slightly improve stability.

3.2 Wavepacket analysis

Wavepacket simulations with non-zero thermal velocities are currently being conducted. Incorporating thermal velocity in the simulations increases numerical heating and noise, causing the wavepackets to break down more rapidly, as shown in Fig. 3. While increasing the resolution and the number of particles per cell (ppc) can help mitigate (but not fully solve) these issues, it also leads to significantly longer computational times, making it hard to perform systematic studies. Thus, our current goal is to ensure that the wavepacket maintains its overall shape long enough to clearly observe its propagation.

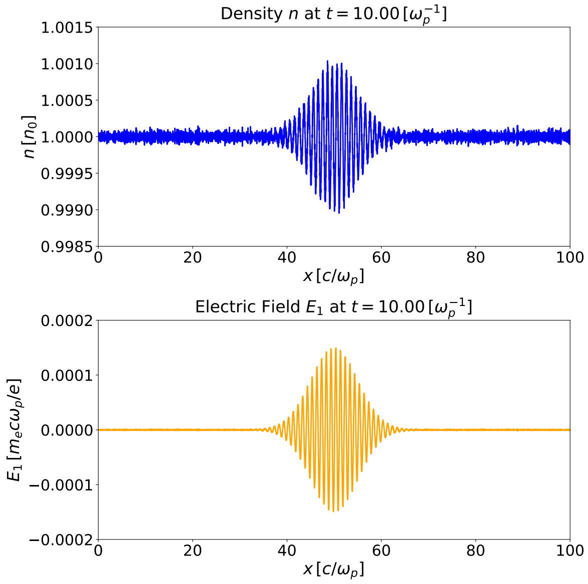

Figure 3: Snapshot of the density n (top) and electric field E_1 (bottom) from a PIC simulation using ZPIC with $u_{\text{th}} = 10^{-4}c$. Although the simulation is still in its early stages, numerical noise effects are already evident.

4 Conclusions and Future Goals

This internship involved studying 1D plasma waves, beginning with a simple theoretical model and advancing to a PIC simulation. The results show that the choice of the simulation parameters plays a key role in maintaining the overall shape of the waves. Additionally, the difficulty of simulating plasmas with non-zero temperatures was highlighted. Future work will focus on exploring new methods to reduce heating and noise effects, as well as transposing the existing code to OSIRIS [4], to take advantage of its advanced PIC capabilities.

References

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[2] Zpic units - pic codes for plasma physics education. https://zpic-plasma.github.io/about/units/. Accessed: 2025-02-22.

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[4] R. A. Fonseca et al. Osiris: A three-dimensional, fully relativistic particle in cell code for modeling plasma based accelerators. In *Lecture Notes in Computer Science, vol. 2331*, pages 342–351. Springer, 2002.