Assignment-2

DS 211, Fall 2022: Numerical Optimization

Submission instructions:

- The assignment is to be submitted in ONE single notebook.

- Submit the .ipynb file through this Teams Assignment.

- The submitted notebook should be a fresh file.

- If your IISc email ID is $< username > @iisc.ac.in$, then name the file $< username > \_Assgn\_2$ E.g. akankshamr_Assgn_2 for my email ID akankshamr@iisc.ac.in

- Make sure you follow the question-wise instructions for each problem.

- Before submission, execute the 'Restart and Run all' command from the Runtime/Kernel tab. Verify that there are no errors and that you are getting the output you expect.

Problem 1

Conjugate Gradient

(a) `1.5 marks` Code the Conjugate Gradient Method and apply it to solve a simple Hx=b system, with H's dimension as 10×10.

(b) `1.5 marks` Construct matrices with various eigenvalue distributions (clustered and non-clustered) and apply the above CG method to them. Comment on the behaviour of the CG method in terms of convergence.

Problem 2

Linear Programming

(a) `1.0 marks` Your start-up will face the cash requirements shown in Table 1 in the next eight quarters (positive entries represent cash needs while negative entries represent cash surpluses).
The company has three borrowing possibilities.

Table 1: This is the caption for the second table.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
100	500	100	-600	-500	200	600	-900

- a 2-year loan available at the beginning of Q1, with a 1% interest per quarter.

- The other two borrowing opportunities are available at the beginning of every quarter: a 6-month loan with a 1.8% interest per quarter, and a quarterly loan with a 2.5% interest for the quarter.

(b) `1.0 marks` Write a program that implements the revised simplex algorithm.

(c) `1.0 marks` Solve the above LP using your program and report the final results.

(d) `0.5 marks` Use PuLP and scipy.optimize.linprog to verify your answer. Change the optimization algorithm used by the solver - try interior point, revised-simplex and dual-simplex. Comment on findings.

Problem 3

Quadratic Programming

(a) `0.5 marks` Consider the problem

$$min \ x_1^2 + 2x_2^2 - 2x_1 - 6x_2 - 2x_1x_2,$$

$$s.t. \ x_1 + x_2 \leq 2, \ -x_1 + 2x_2 \leq 2, \ x_1, \ x_2 \geq 0.$$

Solve geometrically (Plot the geometry and solution points to visualize)

(b) `1.0 marks` Implement the active set method for quadratic programs (Algo 16.3 in Nocedal and Wright (2006)).

(c) `1.0 marks` Choose three initial starting points: one in the interior of the feasible region, one at the vertex, and one at a non-vertex point on the boundary of the feasible region. Comment on the convergence properties.

(d) `1.0 marks` Use CVXOPT and solve the above QP. What algorithm does CVXOPT use to solve QP? Understand and comment on your findings.