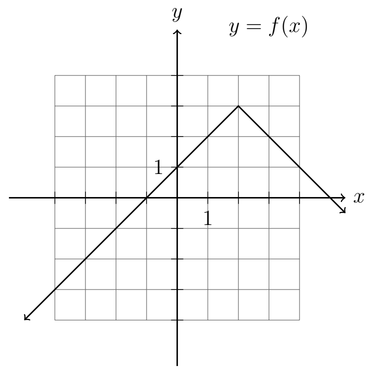
[4] **1.** Consider the function given by the graph on the right below:

(a) State the range of $f(x)$ in interval notation.

(b) Evaluate: $(f \circ f)(1)$

(c) Does f have an inverse? Why or why not?

(d) Which of the following could be a formula for $f(x)$?: (circle one)



(a) $f(x) = -|x+2|+1$ (b) $f(x) = -|x-2|+1$ (c) $f(x) = 3-|x+2|$ (d) $f(x) = 3-|x-2|$

[4] **2.** Use long division to calculate $(15x^4 - 13x^3 + 21x + 7) \div (5x + 4)$.
Write your conclusion in the form $\dfrac{15x^4 - 13x^3 + 21x + 7}{5x + 4} = Q(x) + \dfrac{R(x)}{D(x)}$
where $Q(x)$ is the quotient, $R(x)$ is the remainder, and $D(x)$ is the divisor.

3. Factor the following completely:

[2] (a) $6x^2 - 11x - 10$

[3] (b) $x^5 + 3x^3 - 8x^2 - 24$

[3] (c) $2x^4 - 16x^2 + 32$

[4] **4.** Given the function $f(x) = 2x^2 - 2x - 4$,

(a) Find the coordinates of all axis intercepts,

(b) Find the coordinates of the vertex,

(c) Sketch a graph on the given axes and label the points found in the previous parts.

[5] **5.** Given $f(x) = \dfrac{3}{(x-5)(x+4)}$ and $g(x) = \dfrac{1}{(x+1)(x+4)}$;

(a) Simplify the difference: $(f - g)(x)$.

(b) State the domain of $(f - g)(x)$.

6. Given $f(x) = \dfrac{5-x}{3x-2}$ and $g(x) = \dfrac{2}{x+1}$

[3] (a) Find $f^{-1}(x)$.

[3] (b) Simplify $f(g(x))$.

7. Solve the following equations for x:

[5] (a) $\dfrac{2x}{x+4} - \dfrac{1}{x-3} = \dfrac{26 - 11x}{x^2 + x - 12}$

[4] (b) $x - \sqrt{3x-8} = 4$

[4] **8.** Solve the following inequality. Express your answer in interval notation. $\dfrac{x}{9-x^2} \geqslant 0$

[4] **9.** State the domain of the function: $f(x) = \dfrac{(x-1)\sqrt{x-2}}{(x-3)\sqrt{4-x}}$

[3] **10.** Simplify: $\dfrac{\sqrt{x^9}}{\sqrt[3]{x^2}\sqrt[6]{x^5}}$

[3] **11.** If \$3000 is invested at 5% interest compounded monthly, what is the value after 7 years? (Round your answer to the nearest cent; i.e. two decimal places.)

[1] **12.** Use the change of base formula to calculate the following to three decimal places: $\log_3(500)$

[5] **13.** Given $f(x) = 4 - 2^{x+1}$

(a) State the equation of any asymptote of $f(x)$.

(b) Find the coordinates of all axis intercepts of f.

(c) Sketch $f(x)$ on the axes provided. Label all the intercepts and asymptote(s) properly.

[3] **14.** Given $f(x) = 4 - 2^{x+1}$, find $f^{-1}(x)$.

[3] **15.** Rewrite the following expression in terms of the simplest possible logarithms: $\ln\left(\dfrac{e^3}{(x+5)\sqrt{x}}\right)$

16. Solve for x:

[4] (a) $\log_2(x^2-9) - \log_2(-x) = 3$

[4] (b) $e^{x^2} \cdot e^{4x} = 1$

[3] **17.** An *acute* angle θ satisfies: $\tan\theta = \frac{2}{5}$. Find the exact value of $\sec\theta$.

[3] **18.** Find all angles in $[0°, 360°)$ such that $\cos\theta = -0.85$. (Give two decimal places.)

[3] **19.** Find the exact value (no decimal approximations) of all angles in $[0, 2\pi)$ such that $\csc\theta = \sqrt{2}$.

[2] **20.** Find the exact value (no decimals) of $\cot(5\pi/6)$.

[4] **21.** Prove the identity: $\csc x(\csc x - \cot x) = \dfrac{1}{1+\cos x}$

[3] **22.** State the amplitude, period, and sketch at least two cycles of the function $f(x) = 2\cos\left(\frac{x}{3}\right)$.

[4] **23.** A triangle has sides of length a, b and c across from angles of measure A, B, and C respectively. Suppose $A = 15°$, $b = 9$ and $c = 5$ Find a, B and C. Give your answers rounded to two decimal places.

[4] **24.** You are standing 10m from the base of a vertical climbing wall watching your friend climb up. How far up has your friend climbed in the time between when the angle of elevation is 20° and when it is 50°? (Answer in metres to 2 decimal places.)

END OF EXAM (Answers on next page.)

Answers:

1.(a) $R = (-\infty, 3]$

1(b) $f(f(1)) = 3$

1(c) No. It fails the horizontal line test.

1(d) $f(x) = 3 - |x - 2|$

2. $3x^3 - 5x^2 + 4x + 1 + \frac{3}{5x+4}$

3(a) $(3x + 2)(2x - 5)$

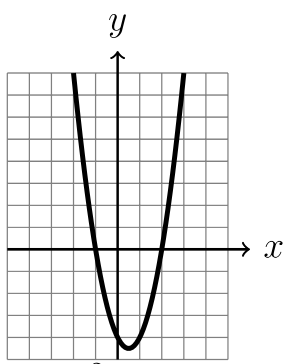
3(b) $(x - 2)(x^2 + 2x + 4)(x^2 + 3)$

3(c) $2(x - 2)^2(x + 2)^2$

4(a) y-int: $(0, -4)$, x-int's: $(-1, 0), (2, 0)$

4(b) vertex: $(\frac{1}{2}, -\frac{9}{2})$

4(c)



5(a) $\dfrac{2}{(x-5)(x+1)}$

5(b) $\mathbb{R}\backslash\{-4, -1, 5\}$

6(a) $f^{-1}(x) = \frac{2x+5}{3x+1}$

6(b) $f(g(x)) = \frac{5x+3}{4-2x}$

7(a) $x = -5$

7(b) $x = 8$

8. $(-\infty, -3) \cup [0, 3)$

9. $[2, 3) \cup (3, 4)$

10. x^3

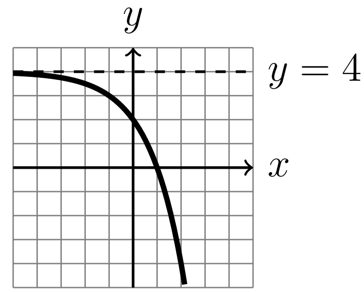
11. \$4254.11

12. 5.657

13(a) H.A. at $y = 4$

13(b) y-int: $(0, 2)$, x-int: $(1, 0)$

13(c)



14. $f^{-1}(x) = \log_2(4 - x) - 1$

15. $3 - \ln(x + 5) - \frac{1}{2}\ln x$

16(a) $x = -9$

16(b) $x = -4, x = 0$

17. $\sec\theta = \frac{\sqrt{29}}{5}$

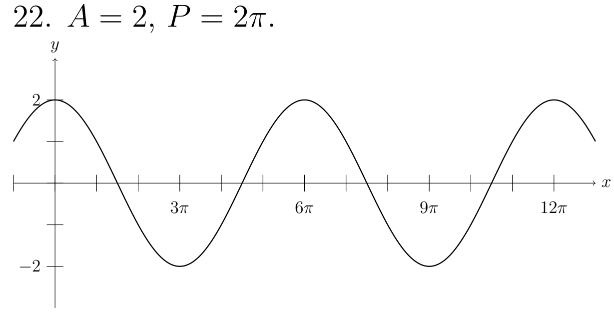
18. $\theta \approx 148.21°$, $\theta \approx 211.79°$

19. $\theta = \frac{\pi}{4}, \theta = \frac{3\pi}{4}$

20. $\cot(5\pi/6) = -\sqrt{3}$

21. Left$= \dfrac{1}{\sin x}\left(\dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x}\right)$
$= \dfrac{1 - \cos x}{\sin^2 x} = \dfrac{1 - \cos x}{1 - \cos^2 x}$
$= \dfrac{1 - \cos x}{(1 - \cos x)(1 + \cos x)} = \dfrac{1}{1 + \cos x}$ =Right.

22. $A = 2$, $P = 2\pi$.



23. $a \approx 4.37, C \approx 17.24°, B \approx 147.76°$

24. 8.28m.