

1. Evaluate the following integrals.

(a) $\int \frac{2x^{-2} + 35\sqrt[4]{x^7} + 1}{5x} dx$

(b) $\int_{-1}^0 (2 - x)e^{-x} dx$

(c) $\int \frac{\ln(x+2)}{x+2} dx$

(d) $\int_0^{\frac{\pi}{6}} \frac{9 \cos(3x)}{\sqrt{4 - 3 \sin(3x)}} dx$

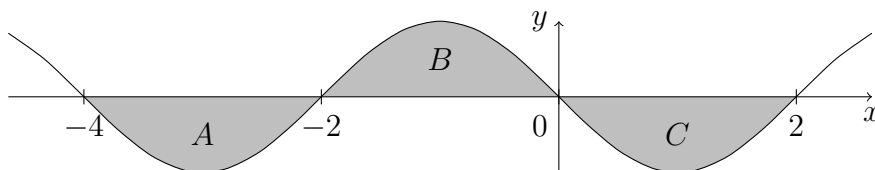
(e) $\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}} dx$

2. Given $f'(x) = \sqrt{x} \left(3\sqrt{x^3} - \frac{10}{x} \right)$ and $f(4) = 10$, find $f(x)$.

3. Evaluate $\int_0^2 (3x^2 + 5x + 1) dx$ using Riemann sums. You can use the formulas below:

$$\sum_{k=1}^n c = c \cdot n, \quad \sum_{k=1}^n k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}.$$

4. Each of the regions A , B , and C bounded by the graph of f and the x -axis has an area of 3.



Use the graph of f and the given area of each region to find the value of

(a) $\int_{-4}^2 f(x) dx$

(b) $\int_{-4}^2 |f(x)| dx$

(c) $\int_{-4}^2 (f(x) + 2x + 5) dx$

5. Find the area enclosed by the curves of: $f(x) = 3x^2 + 10$ and $g(x) = 5x^2 - 8x - 14$.

6. Given the demand function $p = \sqrt{125 - 5x}$ and the supply function $p = x + 5$, find the equilibrium point, and find the consumer surplus.

7. Evaluate each limit. Use L'Hopital's rule when it applies and is appropriate.

(a) $\lim_{x \rightarrow 0} \frac{4x - \sin(4x)}{x - \sin(x)}$

(b) $\lim_{x \rightarrow 1} \frac{5x - 5e^{x-1}}{(x+1)^2}$

8. Find the general term a_n for the given sequence: $\left\{ -\frac{3}{5}, -\frac{9}{7}, -\frac{27}{9}, -\frac{81}{11}, \dots \right\}$

9. Determine whether each sequence diverges or converges.

If the sequence converges, find its limit.

(a) $\left\{ \frac{\sqrt{25n^2 + 7}}{-3 + 4n} \right\}$

(b) $\left\{ \frac{n + (-1)^n}{n} \right\}$

10. Determine whether each integral converges or diverges. If the integral converges, find its value.

(a) $\int_1^{\infty} \frac{e^{-1/x}}{x^2} dx$

(b) $\int_3^5 \frac{x}{\sqrt{x^2 - 9}} dx$

11. Last year, a certain country had an income distribution described by $f(x) = \frac{5}{8}x^4 + \frac{3}{8}x$.

(a) Compute $f(0.4)$ and interpret your result.

(b) Find the Gini index for last year.

(c) After a change in government, the new Gini index for the country is 0.428.

Is the new income distribution more or less equitable?

12. Find the function y that satisfies the differential equation $\frac{dy}{dx} = 4e^{2x}y^3$ with initial conditions $y(0) = 1$, $y > 0$.

13. The Harkonnens captured 4 sandworms from the planet Arakis to reproduce them in the hopes of creating a new spice source. The population of sandworms is growing at a rate proportional to the square root of its current population. The population began with 4 sandworms and after 10 years there were 16 sandworms.

(a) Write the differential equation and initial conditions to represent this situation.

(b) How large is the population after 20 years?

14. Suppose the average time (in minutes) spent on hold during a call with SlowPoke Cable company is exponentially distributed with probability density function

$$f(t) = \frac{1}{10}e^{-t/10}, \quad (0 < t < \infty)$$

What is the probability that a caller will wait on hold between 15 minutes and an hour?

Give your answer to four decimal places.

15. Find the mean, variance, and standard deviation of the random variable X associated with the probability density function $f(x) = \frac{3}{8}x^2$ over the interval $[0, 2]$.

16. The weights of coffee packages sold at Starbreakers coffee shop are normally distributed with a mean of 500 grams and a standard deviation of 2.83 grams.

If a package of the coffee is selected at random from the shelf, what is the probability that it weighs between 497 and 505 grams?

Give your answer to four decimal places.

Answers:

1. (a) $-\frac{1}{5x^2} - 4\sqrt[4]{x^7} + \frac{1}{5} \ln|x| + C$ (d) 2
(b) $2e - 1$
(c) $\frac{1}{2} \ln^2(x+2) + C$ (e) $2 \tan(\sqrt{x}) + C$
2. $f(x) = x^3 - 20\sqrt{x} - 14$
3. 20
4. (a) -3 (b) 9 (c) 15
5. $\frac{512}{3}$
6. (5, 10), \$3.01
7. (a) 64 (b) 0
8. $a_n = \frac{3^n}{-3 - 2n}$
9. (a) converges to $\frac{5}{4}$ (b) converges to 1
10. (a) converges to $1 - \frac{1}{e}$ (b) converges to 4
11. (a) 0.166. The lowest 40% of the people receive 16.6% of the total income.
(b) 0.375
(c) $0.428 > 0.375$. The new income distribution is less equitable.
12. $y = \sqrt{\frac{1}{5 - 4e^{2x}}}$
13. (a) $\frac{dP}{dt} = k\sqrt{P}$ $P(0) = 4, P(10) = 16$ (b) 36 sandworms.
14. 0.2207
15. $\mu = \frac{3}{2} = 1.5, \text{Var}(X) = \frac{3}{20} = 0.15, \sigma = 0.3872$
16. 0.8170