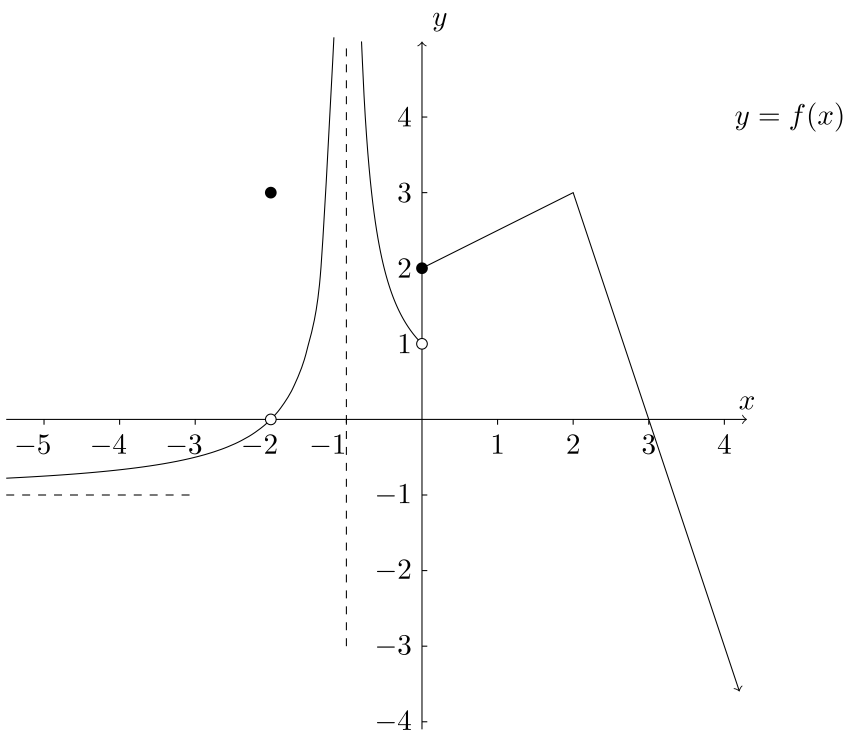
(Marks)

(6) 1. Given the graph of f below, evaluate each of the following. Use ∞, $-\infty$ or "does not exist" where appropriate.



a) $\lim\limits_{x \to -1} f(x) =$ b) $\lim\limits_{x \to 0^+} f(x) =$ c) $f(0) =$

d) $\lim\limits_{x \to 0^-} f(x) =$ e) $\lim\limits_{x \to -2} f(x) =$ f) $f(-2) =$

g) $\lim\limits_{h \to 0^-} \dfrac{f(2+h) - f(2)}{h} =$ h) $\lim\limits_{x \to 3^+} \dfrac{5}{f(x)} =$

i) List all the x-values (if any) of x where f is not continuous.

j) List all the x -values (if any) of x where f is continuous but not differentiable.

(18) 2. Evaluate the following limits. Use ∞, $-\infty$ or "does not exist" where appropriate.

a) $\lim\limits_{x \to 2} \dfrac{2x^2 - x - 6}{x^3 - 8}$

b) $\lim\limits_{x \to -5} \dfrac{x+3}{x^2 - 25}$

c) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{4x^2 - x}}{3x + 2}$

(Marks)

d) $\lim_{x\to 4} \dfrac{\sqrt{2x+1}-3}{x-4}$

e) $\lim_{x\to 1} \dfrac{\dfrac{1}{x+3}-\dfrac{1}{4x}}{x-1}$

f) $\lim_{x\to 3^-} \dfrac{|3-x|}{9-x^2}$

(3) 3. Find the value(s) for k (if any) such that the following function is continuous everywhere.

$$f(x) = \begin{cases} k^2 - 3x - 1 & \text{if} \quad x \le 3 \\ kx + 8 & \text{if} \quad x > 3 \end{cases}$$

(5) 4. Find the point(s) of discontinuity for the following function. Justify using the definition of continuity.

$$f(x) = \begin{cases} \dfrac{e^x}{x^2-1} & \text{if } x \le 0 \\ \sqrt{5-x} & \text{if } 0 < x < 5 \\ 0 & \text{if } x \ge 5 \end{cases}$$

(1) 5. (a) State the limit definition of the derivative.

(3) (b) Use the **limit definition of the derivative** to differentiate $f(x) = x^3$.

(20) 6. Find the derivative for each of the following functions. Do not simplify.

a) $y = \dfrac{4}{x^3} - e^{\ln x} + \sqrt{5x} - 3\pi^4$

b) $y = \csc(e^{x^3-2x})$

c) $y = \dfrac{3\tan^4 x}{\sqrt{5-2^x}}$

d) $y = \sqrt[3]{x}\cot(x)\log_5 x$

e) $y = \ln\left(\dfrac{\sqrt{x}\, e^{\sec x}}{7^x(5x-2)^3}\right)$ **Hint:** Use properties of ln to simplify the function first.

f) $y = 2x^{\sin x}$

(4) 7. Find the critical numbers (x-coordinates only) on the curve $y = x(x-8)^{1/3}$.

(Marks)

(5) 8. Let $e^{xy} - x^3 + \sin(y) = y$.

a) Find $\dfrac{dy}{dx}$ b) Find the equation of the tangent line to the curve at $(1, 0)$

(3) 9. Find the 3^{rd} derivative of the function $y = (x^2 + x)(2x - 3) + \sin(3x)$.

(3) 10. Find the absolute (global) extrema of $f(x) = 3x^4 + 4x^3 - 36x^2 - 5$ on the interval $[-1, 4]$.

(10) 11. Given $f(x) = \dfrac{x}{(1-x)^2}$ with $f'(x) = \dfrac{x+1}{(1-x)^3}$ and $f''(x) = \dfrac{4+2x}{(1-x)^4}$,

(a) Find the domain of f.

(b) Find the x- and y- intercepts (if any).

(c) Find the vertical and horizontal asymptotes (if any).

(d) Give the intervals where $f(x)$ is increasing and decreasing, and the relative extrema (if any).

(e) Give the intervals where $f(x)$ is concave up and concave down, and the points of inflection (if any).

(f) Sketch a labelled graph of $f(x)$.

(5) 12. A rectangular play yard is to be constructed along the side of a house by erecting a fence on three sides, using the house wall as the fourth side of the play yard. Find the dimensions that produce the play yard of maximum area if 320 meters of fence is available for the project.

(5) 13. A teapot has a cost function of $C(x) = 10x + 2500$ in dollars, where x is the number of units produced. The demand equation for these teapots is $p = -0.001x + 18$.

a) Determine the production level that will maximize profit.

b) At this production level, which of the following statements must be true? Circle one answer.
I: Marginal Revenue > Marginal Cost
II: Marginal Revenue = Marginal Cost
III: Marginal Revenue < Marginal Cost
IV: Demand is unit (or unitary) elastic

(4) 14. Sally's Canoes rents out canoes hourly during the summer. Sally prices an hourly rental at \$30 and typically rents out 120 canoes in a day. Through market research, she noticed that for every \$0.50 she lowers the price, she can rent 3 additional canoes.
a) What price should she set to maximize her daily revenue?
b) What is that maximum revenue?

(5) 15. The demand for Jack's backpacks is given by $x = 8100 - 3p^2$ where x is the quantity demanded.

a) Determine the price elasticity of demand function.
b) What is the price elasticity of demand when the price is \$20? Is demand elastic, inelastic or unit elastic?
c) At the price of \$20, what percent price change would increase the demand by 8%?
d) Determine the price that would maximize the revenue.