(Marks)

(4×3) 1. For each of the following, calculate the derivative $\dfrac{dy}{dx}$.
You do not have to simplify fully. However, your answers should not be unnecessarily complicated

(a) $y = \dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$

(b) $y = \tan^5(e^{-3x^2})$

(c) $y = \ln\left(\dfrac{x^2\sqrt{e^{2x}-4}}{(\sin 2x)^3}\right)$

(d) $\ln(x^2y) = xy^2 - \cos y$

(2×4) 2. For each of the following, find the second derivative $\dfrac{d^2y}{dx^2}$. *Simplify your answers as much as possible.*

(a) $y = \tan^{-1} e^x$

(b) $y = x\left(e^{x^2}\right)$

(4) 3. Find the slope of the line which is normal to $x^2 + 4y^2 - 7x + 6 = 0$ at the point $(2, -1)$.

(4) 4. Find an equation for the line which is tangent to $y = (2x+1)^2$ and whose slope is 8.

(5) 5. Use Newton's Method to solve $x^2 - \cos x = 3$ accurate to four decimal places. Start with a guess of $x = 2$.

(4) 6. The electrical potential on the xy plane is given by $V = x + y$. A parabola lying in the plane has the equation $y = x^2 - 3x + 4$. Find the point on the parabola where the potential, V is a minimum.

(4) 7. A missile is fired from an aircraft so that its altitude is given by $h = 15\,000 - t^3 + 30t^2 + 900t$ where h is in metres and $t\,(\geq 0)$ is time in seconds. Find the maximum altitude of the missile.

(10×3) 8. Evaluate the following integrals.

(a) $\displaystyle\int \left(4x^7 - \frac{7}{x^4} + \frac{1}{\sqrt[4]{x^7}} + \frac{4}{7x} - e^7\right) dx$

(b) $\displaystyle\int (\sin 2x)\, e^{\cos 2x}\, dx$

(c) $\displaystyle\int \frac{\sin(1+\sqrt{x})}{\sqrt{x}}\, dx$

(d) $\displaystyle\int_1^5 (\sqrt{2x-1})^3\, dx$

(e) $\displaystyle\int \frac{\sec^2(\ln x)}{x}\, dx$

(f) $\displaystyle\int x^2 e^{-x}\, dx$

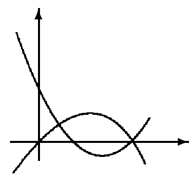
(g) $\displaystyle\int x^3 \ln 3x\, dx$

(h) $\displaystyle\int \tan^{-1} x\, dx$

(i) $\displaystyle\int \frac{dx}{\sqrt{25-16x^2}}$

(j) $\displaystyle\int \frac{x\, dx}{4+x^4}$

9. Find the area contained between the graphs $y = 4x - x^2$ and $y = x^2 - 6x + 8$. (4)



10. Taking $n = 4$, approximate to four decimal places $\displaystyle\int_{0.5}^{1.7} \sqrt{7-2x^2}\, dx$ using (5)

(a) the trapezoidal rule (b) Simpson's rule

11. A motor vehicle travelling at 28 m/s brakes with deceleration given by $a(t) = 0.5t - 8$ m/s^2 where t is the time (in s). (4)

(a) How long will it take to come to a full halt?

(b) What distance will have been travelled during that time?

12. Evaluate the coefficients a_0 and b_2 of the Fourier Series for $f(x) = \begin{cases} -x & -\pi \leq x < 0 \\ 0 & 0 \leq x < \pi \end{cases}$ (4)

13. Determine which if any is a solution to the differential equation $y'' + 9y = 6y' + 9$. *Justify your answers.* (2)

(a) $y = Ae^{3x} + 1$ (b) $y = 3xe^{3x} + 1$

14. Find the particular solution to the differential equation $y' + \dfrac{2y}{x} = x^2$ given $y(-1) = 2$. (4)

15. Find a general solution to the following differential equations. (2×3

(a) $yy'\sqrt{1-x^2} = \sqrt{1-y^2}$

(b) $y' + 2y = e^x$

(Marks)

ANSWERS

1.
(a) $\dfrac{\left(\frac{1}{\sqrt{1-x^2}}\right)\sqrt{1-x^2}-\sin^{-1}x\left(\frac{-2x}{2\sqrt{1-x^2}}\right)}{1-x^2}$

(b) $5\tan^4(e^{-3x^2})\sec^2(e^{-3x^2})(e^{-3x^2})(-6x)$

(c) $\dfrac{2}{x}+\dfrac{1}{2}\dfrac{2e^{2x}}{e^{2x}-4}-\dfrac{6\cos 2x}{\sin 2x}$

(d) $\dfrac{y^2-2/x}{1/y-\sin y-2xy}$

2.
(a) $\dfrac{e^x(1-e^{2x})}{(1+e^{2x})^2}$

(b) $e^{x^2}(6x+4x^3)$

3. $-3/8$

4. $y=8x$

5. $x=1.6957$

6. $(1,2)$

7. 42,000 m

8.
(a) $\dfrac{1}{2}x^8+\dfrac{7}{3}x^{-3}-\dfrac{4}{3}x^{-3/4}+\dfrac{4}{7}\ln|x|-xe^7+C$

(b) $-\dfrac{1}{2}e^{\cos 2x}+C$

(c) $-2\cos(1+\sqrt{x})+C$

(d) $\dfrac{242}{5}$

(e) $\tan(\ln x)+C$

(f) $-e^{-x}(x^2+2x+2)+C$

(g) $\dfrac{x^4}{4}\ln 3x-\dfrac{x^4}{16}+C$

(h) $x\tan^{-1}x-\dfrac{1}{2}\ln|1+x^2|+C$

(i) $\dfrac{1}{4}\sin^{-1}\dfrac{4x}{5}+C$

(j) $\dfrac{1}{4}\tan^{-1}\dfrac{x^2}{2}+C$

9. 9

10. (a) 2.4341, (b) 2.4521

11. (a) 4 s, (b) 160/3 m

12. $a_0=\pi/4 \quad b_2=1/2$

13. Both (a) and (b) are solutions.

14. $y=\dfrac{x^3}{5}+\dfrac{11}{5x^2}$

15.
(a) $-\sqrt{1-y^2}=\sin^{-1}x+C$

(b) $y=\dfrac{e^x}{3}+\dfrac{C}{e^{2x}}$