

**Problems:**

- (7) 1. Given the graph of the function  $f$  below, evaluate each of the following expressions. Use  $\infty$ ,  $-\infty$ , or  $DNE$  as appropriate.

(a)  $\lim_{x \rightarrow -\infty} f(x)$

(d)  $\lim_{x \rightarrow 3} f(x)$

(g)  $\lim_{x \rightarrow 10^+} f(x)$

(b)  $f(3)$

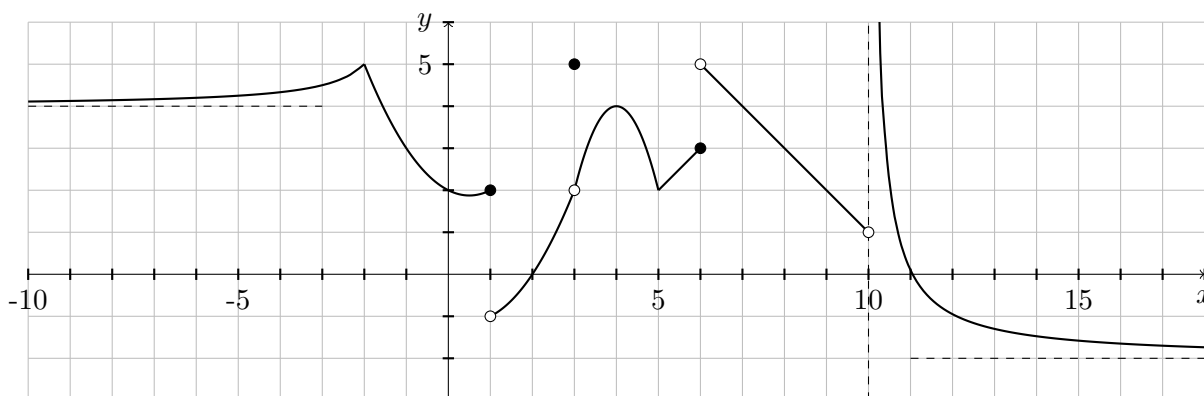
(e)  $\lim_{x \rightarrow 6^+} f(x)$

(h)  $f'(4)$

(c)  $\lim_{x \rightarrow 1} f(x)$

(f)  $\lim_{x \rightarrow 6^-} (x^2 - 3f(x))$

(i)  $\lim_{h \rightarrow 0} \frac{f(8+h) - f(8)}{h}$

(j) Find all values of  $x$  where  $f$  is not continuous.(k) Find all values of  $x$  where  $f$  is continuous but not differentiable.

2. Evaluate the following limits.

(3) (a)  $\lim_{x \rightarrow -3} \frac{2x^2 + x - 15}{x^3 - 9x}$

(4) (c)  $\lim_{x \rightarrow -3^-} \frac{|x+3| + 2x + 6}{4x + 12}$

(3) (e)  $\lim_{x \rightarrow 5^+} \frac{x-6}{2x-10}$

(4) (b)  $\lim_{t \rightarrow 2} \frac{\sqrt{4t+1} - 3}{t^2 - 5t + 6}$

(3) (d)  $\lim_{x \rightarrow -\infty} \frac{(5-2x)^3(x+3)^2}{7x^5 + 2x^3 + 17}$

- (5) 3. Find the value(s) of  $x$  at which the following function is not continuous. Justify your answers by referencing the definition of continuity.

$$f(x) = \begin{cases} \frac{24x}{(x-3)(x+5)} & \text{if } x < 1 \\ x^2 + 2 & \text{if } 1 \leq x \leq 3 \\ \frac{11x}{3(x-2)} & \text{if } x > 3 \end{cases}$$

- (4) 4. Find the value(s) of  $k$  for which the following function is continuous everywhere.

$$g(x) = \begin{cases} x^2 - x + k & \text{if } x < 3 \\ \frac{k^2 - 3}{x-1} & \text{if } x \geq 3 \end{cases}$$

- (3) 5. (a) Use the limit definition of the derivative to find the derivative of  $f(x) = \frac{3}{2x+5}$ .

- (1) (b) Find an equation of the tangent line to the graph of  $f(x)$  at  $x = -2$ .

6. Find  $dy/dx$ . Do not simplify your answer.

(2) (a)  $y = 12^x - \frac{5}{\sqrt[3]{x^4}} + \log_{15} x - \sin 2$

(3) (e)  $x + e^{xy} = 3y + 2$

(3) (b)  $y = (e^{3x} + 2)^5 + \log_4(3x^7 + 1)$

(4) (f)  $y = \frac{\cos^3(x)\sqrt{1+4x}}{x(x^2+5)^6}$

(3) (c)  $y = \frac{\cot(5x)}{\sqrt{x^3+1} + x^e}$

(3) (g)  $y = 2 \cdot x^{1+\tan(2x)}$

(3) (d)  $y = \ln(\sec(e^{5x}))$

(5) 7. Calculate the 95<sup>th</sup> derivative of  $f(x) = (x+1)^7 + \cos(2x+3) + e^{-x}$ .

8. Let  $f(x) = \sqrt[3]{(x^2 - 64)^2}$ .

(4) (a) Find all critical numbers of  $f$ .

(2) (b) Find the absolute extrema of  $f$  on the interval  $[-10, 0]$ .

9. A printer of academic books has a demand function of  $p(x) = \frac{-0.01}{3}x^2 - 0.5x + 800 + \frac{30}{x}$  and an average cost function of  $\overline{C}(x) = 776 + \frac{3000}{x}$ , where  $x$  is the number of books printed per day ( $0 < x \leq 300$ ).

(1) (a) Find a formula for the revenue function  $R(x)$ .

(2) (b) Determine the marginal revenue when  $x = 225$  books and explain what it represents.

(1) (c) Find a formula for the profit function  $P(x)$ .

(3) (d) Determine the number of books per day that should be printed in order to maximize profit and find the maximum profit.

(10) 10. Given the function

$$f(x) = \frac{(x-1)^2}{x^2+1}$$

$$f'(x) = \frac{2(x^2-1)}{(x^2+1)^2}$$

$$f''(x) = \frac{-4x(x^2-3)}{(x^2+1)^3}$$

Find the domain, any  $x$ - and  $y$ -intercepts, any vertical or horizontal asymptotes, the intervals of increase and decrease, any local extrema, the intervals of concavity and any points of inflection, and use all of this information to sketch a graph of the function, labelling all relevant points.

(5) 11. A box with a square base and no top must have a volume of  $4\text{m}^3$ . What should be the dimensions of the box in order to minimize its surface area?

(4) 12. Karli sells her hand-knitted mittens to raise money for the student assistance fund. From experience, she knows that she will sell 18 pairs if the price is \$20 per pair. She expects that each time she increases the price by \$2 per pair, she will sell one fewer pair of mittens. What price should she charge to maximize the revenue? What is the maximum revenue?

13. Due to climate disruptions in major avocado-producing regions, the prices have surged. For a large grocery chain, the following function shows the relationship between the demand in the number of avocados ( $x$ ) and the price per avocado ( $p$ ):

$$x = (25 - 4p)^3$$

(2) (a) Find the price elasticity of demand function.

(1) (b) When the price is  $p = \$2$  per avocado, is the demand elastic, inelastic, or unit elastic?

(1) (c) If we increase the price by 3% from \$2, what would be the effect on the demand for avocados?

(1) (d) What price  $p$  should be set by this grocery chain in order to maximize their revenue?

**Answers:**

1. (a) 4 (d) 2 (g)  $\infty$  (j)  $x = 1, 3, 6, 10$   
 (b) 5 (e) 5 (h) 0  
 (c) DNE (f) 27 (i) -1 (k)  $x = -2, 5$

2. (a)  $-11/18$  (b)  $-2/3$  (c)  $1/4$  (d)  $-8/7$  (e)  $-\infty$

3.  $x = -5, 1$

4.  $k = -3, 5$

5. (a)  $f'(x) = \frac{-6}{(2x+5)^2}$  (b)  $y = -6x - 9$

6. (a)  $12^x \ln(12) + \frac{20}{3}x^{-7/3} + \frac{1}{x \ln(15)}$

(b)  $15e^{3x}(e^{3x} + 2) + \frac{21x^6}{(3x^7 + 1) \ln(4)}$

(c)  $\frac{-5 \csc^2(5x)(\sqrt{x^3 + 1} + x^e) - \cot(5x)(\frac{3}{2}x^2(x^3 + 1)^{-1/2} + ex^{e-1})}{(\sqrt{x^3 + 1} + x^e)^2}$

(d)  $5e^{5x} \tan(e^{5x})$

(e)  $\frac{1 + ye^{xy}}{3 - xe^{xy}}$

(f)  $\frac{\cos^3 x \sqrt{1 + 4x}}{x(x^2 + 5)^6} \left[ -3 \tan x + \frac{2}{1 + 4x} - \frac{1}{x} - \frac{12x}{x^2 + 5} \right]$

(g)  $2 \cdot x^{1+\tan(2x)} \left[ 2 \sec^2 x \ln x + \frac{1 + \tan(2x)}{x} \right]$

7.  $2^{95} \sin(2x + 3) - e^{-x}$

8. (a)  $x = -8, 0, 8$  (b) Min: 0 Max: 16

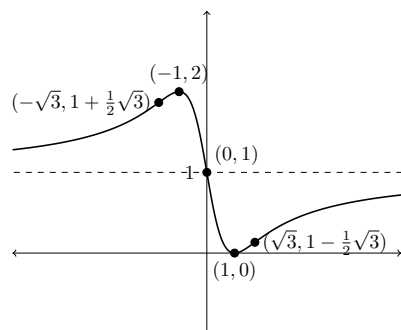
9. (a)  $R(x) = \frac{-0.01}{3}x^3 - 0.5x^2 + 800x + 30$

(b)  $R'(225) = 68.75$ . Increasing production from 225 to 226 will increase revenue by approximately 68.75.

(c)  $P(x) = \frac{-0.01}{3}x^3 - 0.5x^2 + 24x - 2970$

(d) 20 books/week,  $P(20) = -2746.67$

10.



11.  $2 \times 2 \times 1$

12.  $p = \$28$ ;  $R(28) = \$392$

13. (a)  $\pm \frac{12p}{4p - 25}$

(b)  $\pm 1.41$ . Demand is elastic.

(c) Decrease by  $\sim 4.24\%$

(d)  $\sim \$1.56$