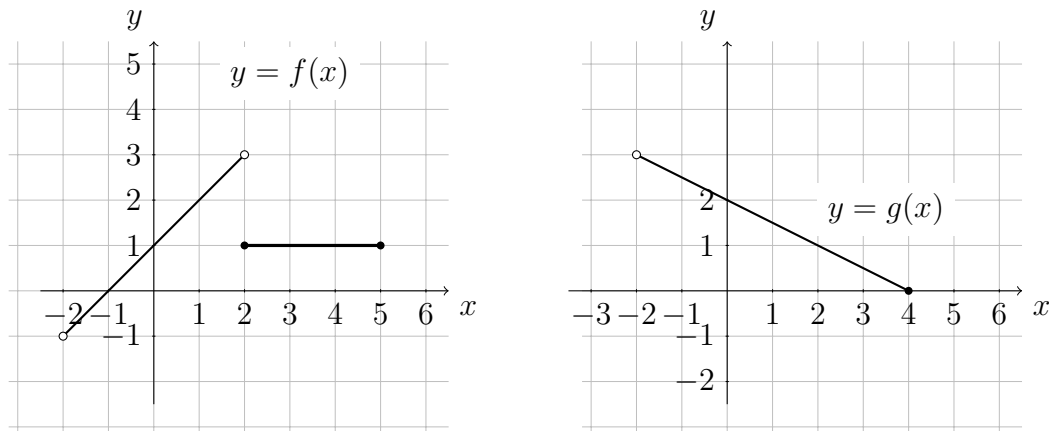**1.** (7 points) The graphs of the functions f and g are given below. Use these to answer the following questions.

(a) State the domain and range of f

(b) If possible, evaluate the following or state that it is undefined:

    i. $f(2)$

    ii. $(f + 2g)(0)$

    iii. $(f \circ g)(4)$

    iv. $g^{-1}(2)$

    v. $(f/g)(4)$

(c) Is f invertible? Justify your answer.

2. (4 points) Suppose f is defined by $f(x) = \begin{cases} 1 - 2x & \text{if } x < 1 \\ x - 1 & \text{if } x \geq 1 \end{cases}$.

(a) Evaluate $f(1)$.

(b) Sketch the graph of f using the grid provided below.

(c) Label any intercept(s) that occur on the graph of f.

(d) State the range of f.

3. (4 points) Factor each of the following expressions completely.

(a) $2x^7(x - 3) - 16x^4(x - 3)$

(b) $3x^4 - 26x^2 - 9$

4. (2 points) Perform the following polynomial division, writing your final answer in the form $p(x) + \dfrac{r(x)}{q(x)}$ where $p(x), r(x)$, and $q(x)$ are polynomials.

$$\frac{6x^3 - 8x + 5}{2x - 4}$$

5. (5 points) Consider the function $f(x) = -\dfrac{1}{2}x^2 + 4x - 6$.

(a) Put $f(x)$ into the form $f(x) = a(x - h)^2 + k$ by completing the square.

(b) Identify any intercept(s) that occur.

(c) Identify the vertex.

(d) Use the grid below to sketch $y = f(x)$.

6. (6 points) Solve each equation for x.

(a) $\dfrac{1}{x+4} - \dfrac{x+3}{x^2 + 2x - 8} = 1$

(b) $x - \sqrt{6 - 3x} = 2$

7. (3 points) Find all values of x that satisfy the following inequality.

$$\frac{2x^2 - 18}{x} \geq 0$$

8. (5 points) Let $f(x) = \dfrac{3x - 1}{8 - x}$ and $g(x) = 3 - 5x$. Compute and simplify expressions for the following functions.

(a) $(f \circ g)(x)$

(b) $f^{-1}(x)$

9. (3 points) Simplify the following expression.$\qquad$$\dfrac{\dfrac{3}{x - 2} + \dfrac{x}{x^2 - 4}}{1 - \dfrac{1}{3x + 6}}$

10. (3 points) Simplify the following expression.$\qquad$$\dfrac{4x^2 - 4x + 1}{x^2 + 2x - 3} \div \dfrac{6x^2 + x - 2}{x^3 + 27}$

11. (2 points) Simplify and reduce the radical expression below.

$\dfrac{\sqrt[3]{-81x^7}\sqrt{y^6}}{\sqrt[3]{8x^2 y^3}}$

12. (2 points) Rationalize the **numerator** of the expression below and simplify the result.

$\dfrac{\sqrt{3} - 1}{2 + 2\sqrt{3}}$

13. (4 points) Let $f(x) = -\sqrt{4 - x}$.

(a) State the domain and range of f.

(b) Identify any intercept(s) that occur.

(c) Use the grid provided below to sketch $y = f(x)$.

14. (3 points) State the domain of the function $f(x) = \dfrac{x - 1}{1 - \sqrt{x + 1}}$.

15. (2 points) If \$4000 is invested at 2.5% interest compounded quarterly, find the value of the investment after 15 years. Give your answer to the nearest cent.

16. (3 points) Write the following expression in terms of the simplest possible logarithms.

$$\log\left(\frac{100\sqrt{x^5 y}}{4 - z^2}\right)$$

17. (3 points) Combine the terms of following expression into a single logarithm and simplify the result.

$$\frac{2}{3}\ln[(x+2)y] + \frac{1}{3}\ln(x^9 y) - 6\ln(xy)$$

18. (4 points) Given $f(x) = 1 - 3^{x+2}$, answer the following.

(a) Identify any intercept(s) that occur.

(b) Write the equation of any asymptotes.

(c) Use the grid below to sketch $y = f(x)$.

19. (4 points) Given $f(x) = \log_2(-x) - 1$, answer the following.

(a) Identify any intercept(s) that occur.

(b) Write the equation of any asymptotes.

(c) Use the grid below to sketch $y = f(x)$.

20. (1 point) Evaluate $\log_4(70)$. Round your answer to three decimal places.

21. (6 points) Solve each equation for x.

(a) $3 + \log_5 10 + \log_5 x = \log_5(3x - 1) + 4$

(b) $\dfrac{7^{2x}}{3(5^{2x+1})} = 1$ (Give your answer as an exact value).

22. (2 points) Given that $\cos\theta = \dfrac{7}{9}$ for an acute angle θ, find the other five trigonometric functions of θ. (Give exact values.)

23. (4 points) Find all angles in $[0°, 360°)$ that satisfy each given equation. Round your answers to three decimal places.

(a) $\sin\theta = -\dfrac{3}{11}$

(b) $\sec\theta = 7$

24. (2 points) Find the exact value of $\tan(2\pi/3)$.

25. (4 points) Find all angles in $[0, 2\pi)$ that satisfy each given statement. (Give exact values.)

(a) $\sec\theta$ is undefined.

(b) $2\cos^2\theta - \cos\theta - 1 = 0$

26. (3 points) Identify the amplitude, period, and sketch at least two cycles of the function $f(x) = -3\cos(\pi x)$.

27. (3 points) Suppose that an industrial ramp for a construction site forms a $14°$ angle with the ground. If the ramp needs to reach a 4 meter high platform, how far back from the platform must the ramp begin? Give your answers in meters to two decimal places.

28. (3 points) Simplify the expression below as much as possible.

$$\frac{\csc x - \cos x \cot x}{\csc x - \sin x}$$

29. (3 points) Consider a triangle with sides of length a, b, c across from angles A, B, C respectively. If $a = 5$, $B = 15°$, and $c = 2$, find A, b, and C. (Draw the triangle to solve the problem, and round your answers to 2 decimal places.)

**Answers:**

1. (a) Domain: $(-2, 5]$ and range: $(-1, 3)$
   (b)   i. 1
        ii. 5
        iii. 1
        iv. 0
        v. Undefined
   (c) No. The graph of f does not pass the horizontal line test; in other words, f is not a one-to-one function so it is not invertible.

2. (a) 0
   (b) See below.
   (c) x-intercepts are $(1, 0)$ and $(\frac{1}{2}, 0)$; y-intercept is $(0, 1)$
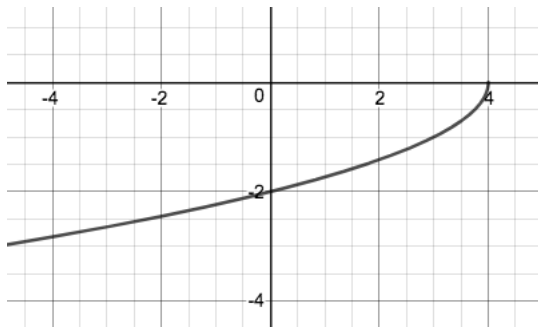   (d) Range: $(-1, \infty)$
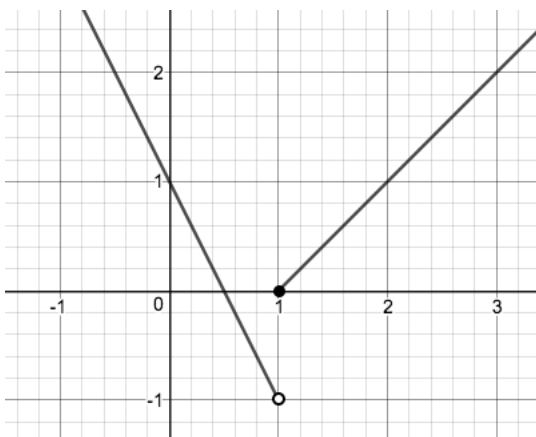
3. (a) $2x^4(x - 3)(x - 2)(x^2 + 2x + 4)$
   (b) $(3x^2 + 1)(x - 3)(x + 3)$

4. $3x^2 + 6x + 8 + \dfrac{37}{2x - 4}$

5. (a) $f(x) = -\dfrac{1}{2}(x-4)^2 + 2$

(b) x-intercepts at $(6,0)$ and $(2,0)$; y-intercept at $(0,-6)$

(c) Vertex $(4,2)$

(d) See below.
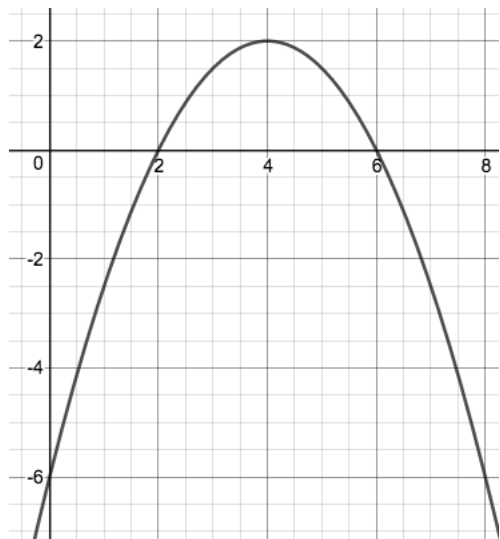
6. (a) $x = -3$ or 1

(b) $x = 2$

7. $[-3,0) \cup [3,\infty)$

8. (a) $(f \circ g)(x) = \dfrac{8 - 15x}{5 + 5x}$

(b) $f^{-1}(x) = \dfrac{8x + 1}{x + 3}$

9. $\dfrac{6(2x + 3)}{(3x + 5)(x - 2)}$

10. $\dfrac{(2x - 1)(x^2 - 3x + 9)}{(x - 1)(3x + 2)}$

11. $\dfrac{-3\sqrt[3]{3}x^2 y^2}{2\sqrt[3]{x}}$

12. $\dfrac{1}{(1 + \sqrt{3})^2}$

13. (a) Domain: $(-\infty, 4]$ and range: $(-\infty, 0]$

(b) x-intercept at $(4,0)$ and y-intercept at $(0,-2)$

(c) See below.
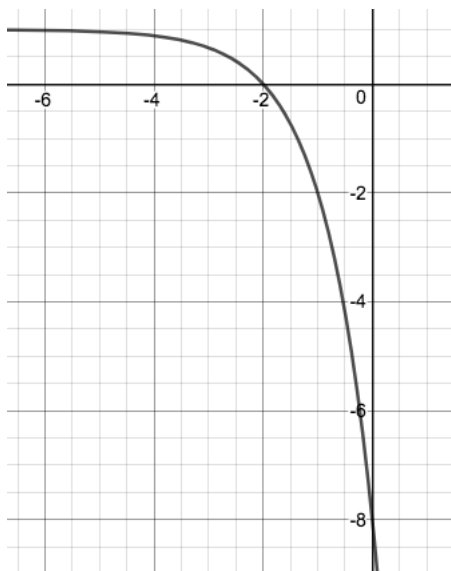
14. $[-1, 0) \cup (0, \infty)$

15. $\$5813.18$

16. $2 + \dfrac{5}{2}\log x + \dfrac{1}{2}\log y - \log(2 - x) - \log(2 + z)$

17. $\ln\left[\dfrac{(x+2)^{2/3}}{x^3 y^5}\right]$

18. (a) x-intercept at $(-2, 0)$ and y-intercept at $(0, -8)$
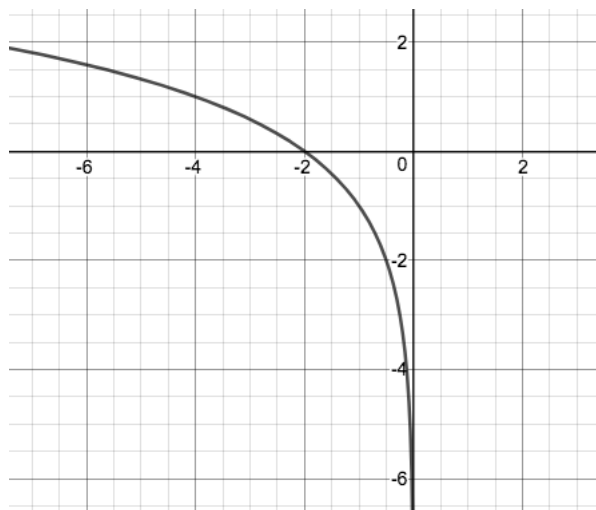
    (b) Horizontal asymptote: $y = 1$

    (c) See below.

19. (a) x-intercept at $(-2, 0)$; no y-intercept

    (b) Vertical asymptote: $x = 0$

    (c) See below.

20. 3.065

21. (a) $x = 1$

(b) $x = \dfrac{\ln 15}{\ln 49 - \ln 25}$

22. $\sin\theta = \dfrac{\sqrt{32}}{9}$, $\csc\theta = \dfrac{9}{\sqrt{32}}$, $\tan\theta = \dfrac{\sqrt{32}}{7}$, $\cot\theta = \dfrac{7}{\sqrt{32}}$, $\sec\theta = \dfrac{9}{7}$
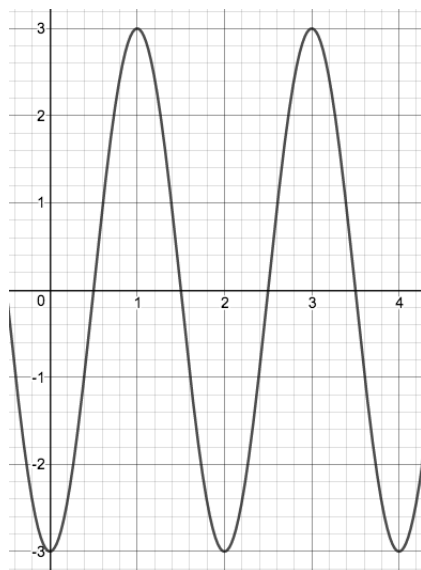
23. (a) $195.827°$ and $344.173°$

(b) $81.787°$ and $278.213°$

24. $-\sqrt{3}$

25. (a) $\dfrac{\pi}{2}$ and $\dfrac{3\pi}{2}$

(b) $\dfrac{2\pi}{3}$, $\dfrac{4\pi}{3}$ and 0

26. Amplitude is 3, period is 2. See graph below.

27. 16.04 meters

28. $\tan^2 x$

29. $A \approx 155.42°$, $C \approx 9.56°$, $b \approx 3.11$