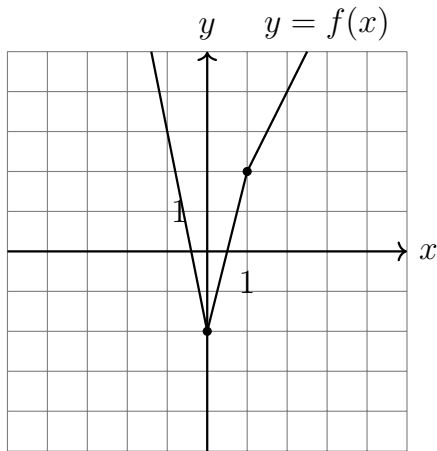


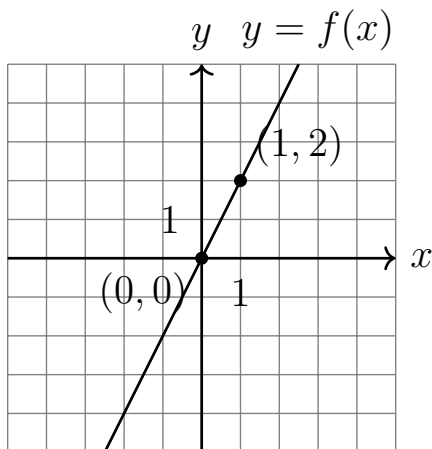
1. (8 points) The function  $f$  is defined by the graph below. The function  $g$  is defined by the formula

$$g(x) = 2 - x.$$



Find

- a)  $f(-1)$                       b)  $f^{-1}(-2)$   
 c)  $(f \circ g)(0)$                 d)  $\frac{f}{g}(1)$   
 e)  $(f^{-1} \circ g)(4)$             f)  $(f - 2g)(0)$   
 g) the domain of  $f$             h) the range of  $f$ .
2. (2 points) Sketch the graph of the inverse function  $y = f^{-1}(x)$  for the following graph of a function  $y = f(x)$ .



3. (1 point) Find the inverse function of  $f(x) = 3x + 2$ .

4. (4 points) Graph the piecewise defined function

$$f(x) = \begin{cases} (x+2)^2 & \text{if } x \leq -2 \\ -2x+3 & \text{if } -2 < x \leq 4 \end{cases}$$

5. (8 points) Solve the following quadratic equations by the method stated.
- a) Solve  $6x^2 - 13x + 6 = 0$  by factoring.  
 b) Solve  $4x^2 + 24x + 1 = 0$  by completing the square.  
 c) Solve  $3x^2 - 5x = 2$  by using the quadratic formula.  
 d) Solve  $7x^2 + 2x - 1 = x^2 + x + 1$  by the method of your choice.
6. (3 points) Simplify so that each exponent is positive and each variable appears at most once.

$$\left( \left( \frac{3x^2y^{-3}}{x^3y^{-2}} \right)^2 \cdot \frac{xyz}{9} \right)^{-1}$$

7. (5 points) Given the quadratic function

$$f(x) = x^2 - 8x + 12$$

- a) find the coordinates of all intercepts;  
 b) find the coordinates of the vertex;  
 c) sketch a graph of the function using the information from the previous parts.
8. (3 points) Perform the following polynomial division. Write your final answer in the form  $Q(x) + \frac{R(x)}{D(x)}$  where  $Q(x)$ ,  $R(x)$ , and  $D(x)$  are polynomials.

$$\frac{4x^4 - 6x^2 + 5}{2x + 8}$$

9. (3 points) Solve the following inequality. State your answer in interval notation.

$$\frac{2x^3 + x^2}{(x+6)(x-5)} \geq 0$$

10. (4 points) Simplify the following expression:

$$\frac{\frac{x+1}{x-1} - \frac{x-1}{x+2}}{\frac{15x+3}{x^2-1}}$$

11. (4 points) Given  $f(x) = \frac{(x-3)(x+2)}{x+5}$  and  $g(x) = \frac{(x+2)(x+6)}{x^2-25}$ ,

a) find the domain of the function  $\left(\frac{f}{g}\right)(x)$ ;

b) simplify the quotient  $\left(\frac{f}{g}\right)(x)$ .

12. (3 points) Simplify the following expression as much as possible.

$$\sqrt[3]{\frac{8x^{\frac{7}{2}}y^{10}}{27y\sqrt{x^{19}}}}$$

13. (3 points) Solve the following equation for  $x$ . Make sure to verify your answer.

$$\sqrt{7x+9} - \sqrt{3x+1} = 6$$

14. (5 points) Given the function  $f(x) = 1 - \sqrt{x+4}$ ,

a) find the  $x$ - and  $y$ -intercepts;

b) sketch the graph;

c) state the domain of  $f$ ;

d) state the range of  $f$ .

15. (3 points) State the domain of the following function in interval notation.

$$f(x) = \frac{x+1}{x} + \frac{\sqrt{x+2}}{1-x}$$

16. (2 points) Frank places \$3000 in a tax-free savings account at an annual interest rate of 5% compounded semi-annually. Find the value of the investment after 12 years. Round your answer to the nearest cent.

17. (1 point) Use your calculator to evaluate  $\log_7(50)$  accurate to three decimal places.

18. (5 points) For the function  $f(x) = 8 - 2^{x+1}$ ,

a) find the  $x$ - and  $y$ -intercepts and one other point;

b) find the equation of the asymptote;

c) sketch the graph.

19. (2 points) Simplify the following expression:

$$e^{3\ln(x+1)-5\ln(x)}$$

20. (2 points) Evaluate the following expression:

$$\log_b\left(\frac{1}{\sqrt[7]{b^{12}}}\right)$$

21. (3 points) Express as a single logarithm and simplify:

$$\frac{7}{3}\ln(xyz^2) - \frac{1}{3}\ln(yz) - \frac{4}{3}\ln(xz)$$

22. (3 points) Solve the following equation for  $x$ :

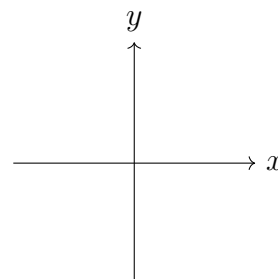
$$\log_2(x-4) - \log_2(12) = 3 - \log_2(x+6)$$

23. (3 points) Solve the following equation for  $x$ . Express your answer in the form  $x = \frac{\ln A}{\ln B}$ .

$$5^{(2x+1)} = 3 \cdot 4^{(x-1)}$$

24. (3 points) Find all angles  $\theta$  in the interval  $[0, 360^\circ)$  such that  $\cos \theta = \frac{2}{3}$ . Give your answers accurate to two decimal places.

25. (3 points) In the cartesian plane below, sketch the angle  $\theta$  in standard position whose terminal side contains the point  $(-1, -2)$  and evaluate the following trigonometric ratios.

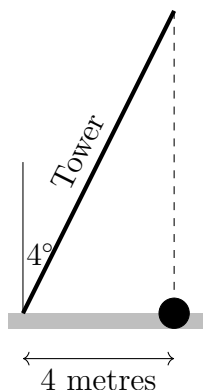


$$\sin \theta = \quad \sec \theta =$$

$$\cos \theta = \quad \csc \theta =$$

$$\tan \theta = \quad \cot \theta =$$

- 26.** (4 points) The Leaning Tower of Pisa in Italy is a famous historical landmark known for its distinctive tilt. As a surveyor tasked with measuring the tower's height, you drop a stone from the top and measure the distance from where the stone landed to the tower's base. The distance from the stone to the base of the tower is 4 metres, and we can measure the angle of inclination of the tower to be 4 degrees.

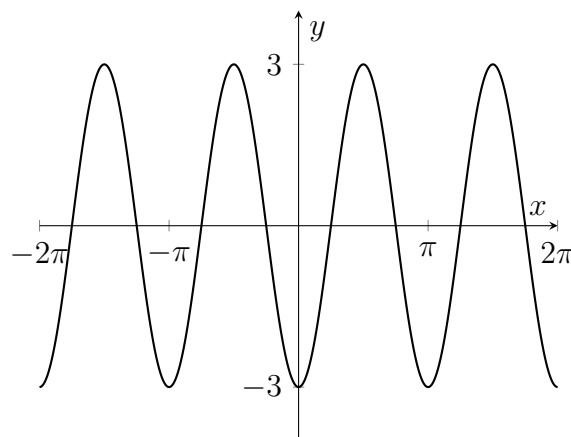


With this data, calculate the height of the tower to the nearest metre.

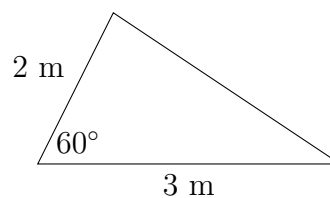
- 27.** (4 points) Complete the table below following the example:

Deg	Rad	Sine	Cosine	Tangent
$30^\circ$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$120^\circ$				
	$\frac{5\pi}{4}$			

- 28.** (3 points) Consider the graph of a sinusoidal function below and answer the questions.



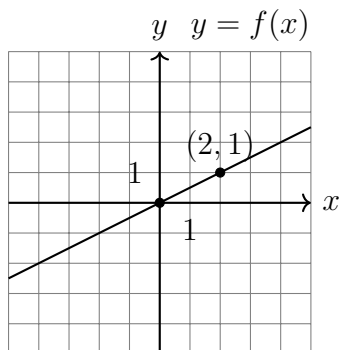
- a) What is the amplitude of the function?  
 b) What is the period of the function?  
 d) Which of these functions corresponds to the graph?
- a)  $y = -3 \sin\left(\frac{1}{2}x\right)$   
 b)  $y = 3 \sin(2x)$   
 c)  $y = 3 \cos\left(\frac{1}{2}x\right)$   
 d)  $y = -3 \cos(2x)$
- 29.** (3 points) Solve the triangle below, i.e., determine the lengths of all sides and the measures of all angles.



## Answers.

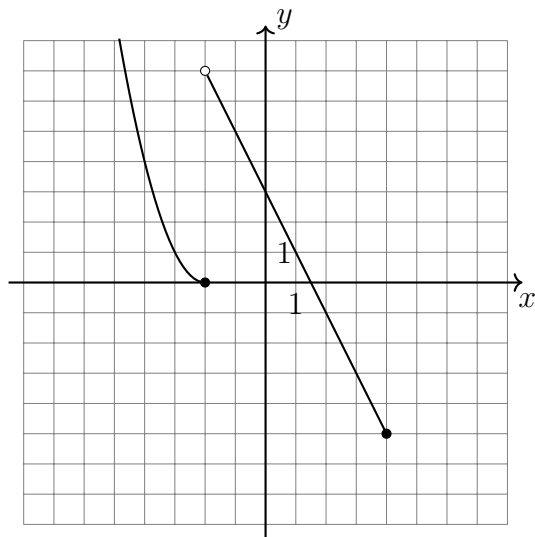
1. a) 3                      b) 0                      c) 4  
      d) 2                      e) 0                      f) -6  
      g)  $(-\infty, \infty)$       h)  $[-2, \infty)$

2.



3.  $f^{-1}(x) = \frac{x-2}{3}$

4.

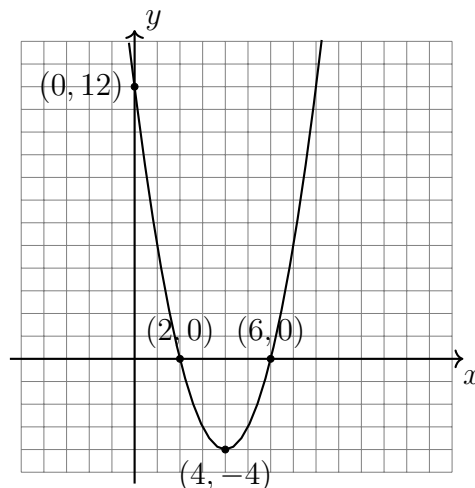


5. a)  $x = \frac{2}{3}, x = \frac{3}{2}$                       b)  $x = \pm \frac{\sqrt{35}}{2} - 3$   
      c)  $x = 2, x = -\frac{1}{3}$                       d)  $x = \frac{1}{2}, x = -\frac{2}{3}$

6.  $\frac{xy}{z}$

7. a)  $(0, 12), (2, 0), (6, 0)$   
      b)  $(4, -4)$

c)



8.  $2x^3 - 8x^2 + 29x - 116 + \frac{933}{2x+8}$

9.  $\left(-6, -\frac{1}{2}\right] \cup \{0\} \cup (5, \infty)$

10.  $\frac{x+1}{3(x+2)}$

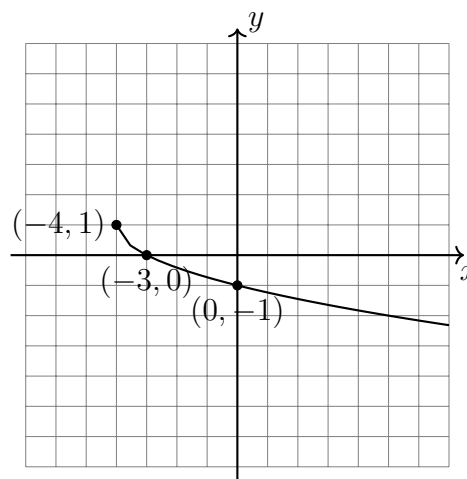
11. a)  $D = \mathbb{R} \setminus \{-6, -5, -2, 5\}$

b)  $\frac{(x-3)(x-5)}{x+6}$

12.  $\frac{2y^3}{3x^2}$

13.  $x = 40$  ( $x = 1$  is not a solution)

14. a)  $(0, -1), (-3, 0)$   
      b)



c)  $[-4, \infty)$

d)  $(-\infty, 1]$

15.  $[-2, 0) \cup (0, 1) \cup (1, \infty)$

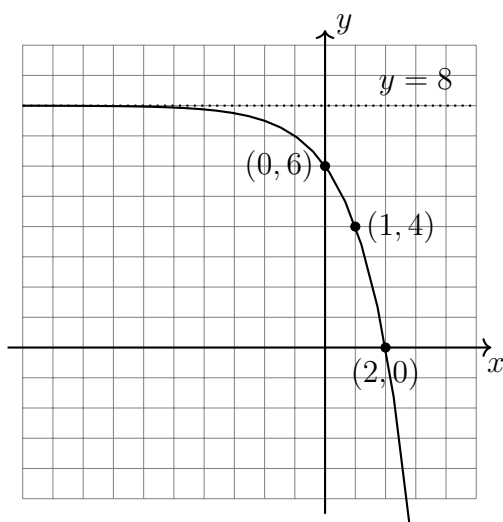
16. \$5426.18

17.  $\log_7(50) = 2.010$

18. a)  $(0, 6), (2, 0), (1, 4), (-1, 7), (3, -8)$

b)  $y = 8$

c)



19.  $\frac{(x+1)^3}{x^5}$

20.  $-\frac{12}{7}$

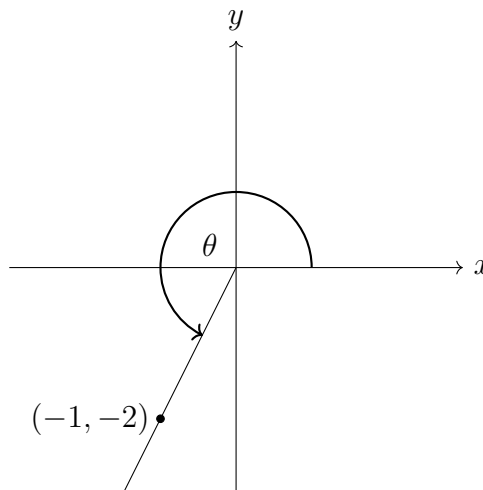
21.  $\ln(xy^2z^3)$

22.  $x = 10$  ( $x = -12$  is not a solution)

23.  $x = \frac{\ln\left(\frac{3}{20}\right)}{\ln\left(\frac{25}{4}\right)}$

24.  $\theta_1 = 48.19^\circ, \theta_2 = 311.81^\circ$

25.



$$\sin \theta = -\frac{2\sqrt{5}}{5} \quad \sec \theta = -\frac{\sqrt{5}}{2}$$

$$\cos \theta = -\frac{\sqrt{5}}{5} \quad \csc \theta = -\sqrt{5}$$

$$\tan \theta = 2 \quad \cot \theta = \frac{1}{2}$$

26. 57m

27.

Deg	Rad	Sine	Cosine	Tangent
$30^\circ$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$120^\circ$	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
$225^\circ$	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1

28. a) amplitude = 3

b) period =  $\pi$

c)  $y = -3 \cos(2x)$

29.  $c = 2.66, B = 40.89^\circ, C = 79.11^\circ$