1. Evaluate the following integrals.

(a) (5 points) $\int x\sqrt{x+5}\,dx$

(b) (5 points) $\int \frac{x-2}{x^2+6x+25}\,dx$

(c) (5 points) $\int x\cos(2x/5)\,dx$

(d) (5 points) $\int_0^{5/2} \frac{x^2}{\sqrt{25-x^2}}\,dx$

(e) (5 points) $\int \tan^2(x)\sec^4(x)\,dx$

(f) (5 points) $\int \frac{1}{e^x(e^{2x}+1)}\,dx$

2. Evaluate the following limits.

(a) (4 points) $\lim_{x\to 0} \frac{x-2\arctan(x)}{x+5\arcsin(x)}$

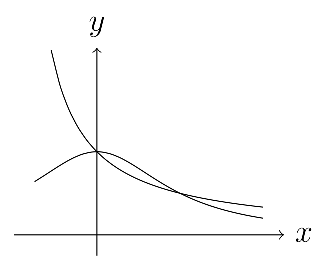
(b) (4 points) $\lim_{x\to\infty} \left(\sec\left(\frac{5}{x}\right)\right)^{x^2}$

3. Evaluate each improper integral or show it diverges.

(a) (5 points) $\int_1^e \frac{dx}{x\ln(x)}$

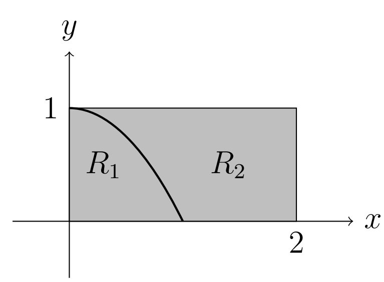
(b) (5 points) $\int_1^\infty \frac{dx}{x(2x+5)}$

4. (4 points) Find the area of the region bounded by the curves given by $y = \frac{1}{x+1}$ and $y = \frac{1}{x^2+1}$.



5. (5 points) Solve the differential equation $\frac{dy}{dx} = y^2\ln(x)$ with $y(1) = -1$.

6. In the figure a shaded rectangle is divided into two regions, R_1 and R_2, by the curve $y = 1 - x^2$. Write down, *but do not evaluate*, an integral for the volume of the solid of revolution obtained by



(a) (2 points) rotating R_1 about the x-axis

(b) (2 points) rotating R_1 about the line $y = 2$

(c) (2 points) rotating R_2 about the line $x = 2$.

7. (5 points) Find the length of the curve given by $y = \sqrt{4 - x^2}$ for $0 \le x \le 1$.

8. (3 points) Determine the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2 + 5^n}{5^{2n}}$.

9. Determine whether the series converges or diverges.

(a) (3 points) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan(n)}{n}$

(b) (3 points) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{5^n + n}$

(c) (3 points) $\displaystyle\sum_{n=1}^{\infty} n^2 \sin\left(\frac{1}{n^5}\right)$

10. Determine whether the series converges absolutely, converges conditionally, or diverges.

(a) (3 points) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n+2}{n+5}$

(b) (3 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$

(c) (3 points) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(2n+1)!}{5^n (n!)^2}$

11. (5 points) Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n n}(x+1)^n$.

12. (5 points) Find the Taylor series for $f(x) = \dfrac{1}{(x+1)^2}$ centred at 4. Give your answer using sigma notation.

13. (1 point) If $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi/2)}{n^2} x^n$, what is the coefficient of x^{25} in the Maclaurin series for $f'(x)$?

Answers:

1. (a) $\frac{2}{5}(x+5)^{5/2} - \frac{10}{3}(x+5)^{3/2} + C$

(b) $\frac{1}{2}\ln(x^2+6x+25) - \frac{5}{4}\arctan\left(\frac{x+3}{4}\right) + C$

(c) $\frac{5}{2}x\sin\left(\frac{2x}{5}\right) + \frac{25}{4}\cos\left(\frac{2x}{5}\right) + C$

(d) $\frac{25}{24}(2\pi - 3\sqrt{3})$

(e) $\frac{1}{3}\tan^3(x) + \frac{1}{5}\tan^5(x) + C$

(f) $-e^{-x} - \tan(e^x) + C$

2. (a) $-\frac{1}{6}$

(b) $e^{25/2}$

3. (a) diverges (to ∞)

(b) $\frac{1}{5}\ln\left(\frac{7}{2}\right)$

4. $\frac{\pi}{4} - \ln(2)$

5. $y = -\frac{1}{x\ln(x) - x + 2}$.

6. (a) $\int_0^1 \pi(1-x^2)^2 dx$ or $\int_0^1 2\pi y\sqrt{1-y}dy$

(b) $\int_0^1 \pi[4-(1+x^2)^2]dx$ or $\int_0^1 2\pi(2-y)\sqrt{1-y}dy$

(c) $\int_0^1 \pi(2-\sqrt{1-y})^2 dy$

7. $\frac{\pi}{3}$

8. $\frac{1}{3}$

9. (a) diverges

(b) converges

(c) converges

10. (a) diverges

(b) converges conditionally

(c) converges absolutely

11. $(-3, 1]$

12. $\sum_{n=0}^{\infty} \frac{(-1)^n(n+1)}{5^{n+2}}(x-4)^n$

13. $-\frac{1}{26}$