Department of Mathematics
Final Examination

23 May 2019
9h–12h

Algebra & Trigonometry
201-015-50

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Student number: ─────────────────────

Instructor: ─────────────────────

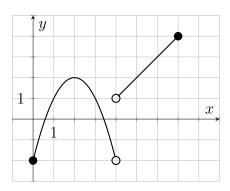## Instructions

1. Do not open this booklet before the examination begins.

2. Check that this booklet contains 8 pages, excluding this cover page.

3. Write all of your solutions in this booklet and show all supporting work.

4. If the space provided is not sufficient, continue the solution on the opposite page.

5. The only calculators allowed on this exam are models beginning with Sharp EL-531.

6. Please double check that you have no cell phone or other prohibited device on your person. (If you do have such a device present it to a teacher.)

1   Multiple choice [16%]

For each of the following questions, there is only one correct answer. Circle your choice. If two choices are selected for the same question, no marks will be awarded.
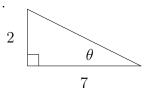
1. Consider the graph of the function f, shown below.

(a) (1 point) What is the domain of f?

        A. $[0, 7]$

        B. $[-2, 4]$

        C. $[0, 4) \cup (4, 7]$

        D. $(-2, 4]$

(b) (1 point) What is the range of f?

        A. $[0, 7]$

        B. $[-2, 4]$

        C. $[0, 4) \cup (4, 7]$

        D. $(-2, 4]$

(c) (1 point) Evaluate $f(3)$.

    A. 0   B. 1   C. 6   D. Undefined.

(d) (1 point) Evaluate $(f \circ f)(7)$.

    A. –2   B. 1   C. 16   D. Undefined.

(e) (1 point) How many solution(s) does the equation $f(x) = 0$ have?

    A. 0   B. 1   C. 2   D. 3

(f) (1 point) Is the function f invertible?
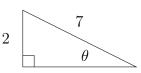
    A. Yes   B. No

2. (1 point) If $g(x) = \dfrac{1}{1 - 3x}$, then $g(x + h)$ is

A. $\dfrac{1}{1 - 3x} + h$

B. $\dfrac{1}{1 - 3x + h}$

C. $\dfrac{1}{1 - 3x + 3h}$

D. $\dfrac{1}{1 - 3(x + h)}$

3. (1 point) Let $f(x) = \dfrac{(x - 1)(x + 3)}{(x + 1)(2 - x)}$. The domain of f is:

A. $\{-1, 2\}$

B. $\mathbb{R}\backslash\{-1, 2\}$

C. $\{-3, -1, 1, 2\}$

D. $\mathbb{R}\backslash\{-3, -1, 1, 2\}$

4. (1 point) In order to rationalize the denominator of $\dfrac{3 - x}{\sqrt{x + 1} - 2}$ one needs to:

A. multiply the numerator and denominator by $\left(\sqrt{x + 1} + 2\right)$.

B. multiply the numerator and denominator by $\left(\sqrt{x + 1} - 2\right)$.

C. multiply the numerator and denominator by $\left(\sqrt{x + 1}\right)$.

D. square the numerator and denominator.

5. (1 point) The reduced form of the expression $\sqrt{6}\sqrt{12}$ is

A. $3\sqrt{8}$

B. $6\sqrt{2}$

C. $9\sqrt{8}$

D. $2\sqrt{6}\sqrt{3}$

6. (1 point) If $\log x = \log y + 3 \log z$, then

    A. $x = y + 3z$

    B. $x = y + z^3$

    C. $x = yz^3$

    D. $x = (yz)^3$

7. (1 point) Consider the function $f(x) = \left(\dfrac{1}{3}\right)^x$. As x becomes larger, the value of $f(x)$ will

    A. get larger as well.

    B. get closer to zero.

    C. become negative.

    D. None of the above.

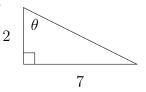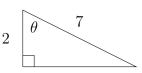**8.** (1 point) If $\sec \theta = \dfrac{7}{2}$. Choose the triangle that correctly represents this situation:
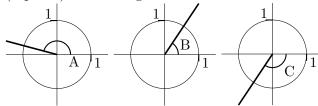

A. .

B. .

C. .

D. .

9. (1 point) Three angles are shown below. Order the values of $\sin A$, $\sin B$, and $\sin C$ in increasing order.

    A. $\sin A < \sin B < \sin C$

B. $\sin C < \sin B < \sin A$

C. $\sin B < \sin A < \sin C$

D. $\sin C < \sin A < \sin B$

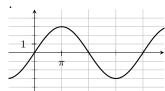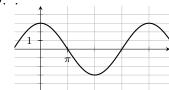**10.** (1 point) Which one of the following graphs corresponds to the function $f(x) = 3\sin(2x)$.
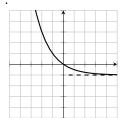
A. .

B. .
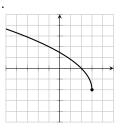

C. .

D. .

2 Short Answer [26%]
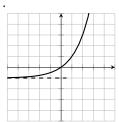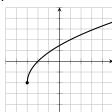
1. Consider the following four graphs.

A. .

B. .

C. .

D. .

(a) (2 points) The graph of the curve $y = 2\sqrt{3-x} - 2$ is _____.

(b) (2 points) The graph of the curve $y = 2^{-x} - 1$ is _____.

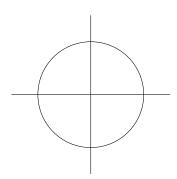**2.** (4 points) Determine which of the following statements are true for all values of x. **Check all that apply.**

○ $\sqrt{x^2 + 3} = x + \sqrt{3}$

○ $\dfrac{x}{x^2 + 3} = \dfrac{1}{x + 3}$

○ $\ln(x + 3) = \ln x + \ln 3$

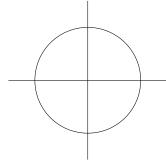○ $2^{x+3} = 2^x 2^3$

3. (2 points) The simplified form of $(\sqrt{x})(\sqrt[3]{x^2})$ is _____.

4. (2 points) If $\sqrt{3x - 2} = 5$, then $x =$ _____.

5. (2 points) The solution of the equation $\log_2 x = 5$ is _____.

6. (2 points) The value of $\log_5(25^{2019})$ is _____.

7. (2 points) The radian measure of the angle $-330°$ is _____.

8. (2 points) Sketch the angle $\frac{-3\pi}{4}$ in the following graph.

9. (2 points) The exact value of $\sin\left(\frac{-3\pi}{4}\right)$ is _____.

10. (2 points) Sketch the angle(s) $\theta \in [0, 360°)$ that satisfy the equation $\cos(\theta) = \frac{-1}{3}$.
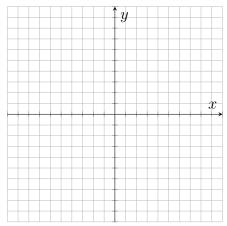
11. (3 points) Solve the equation $\cos(\theta) = \frac{-1}{3}$.

3 Long Answer [47%]

1. (a) (2 points) Find the x and y intercepts of the function $g(x) = \frac{1}{2}\left(x^2 - 4x - 5\right)$.
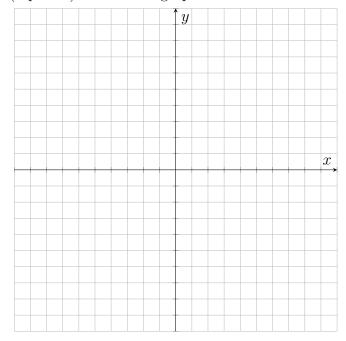
(b) (2 points) Find the vertex of the graph of g by completing the square.

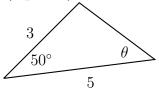(c) (2 points) Sketch the graph of the function g.

2. (2 points) Factor $2x^3 - 3x^2 - 8x + 12$ completely.

3. (2 points) Factor $27x^3 + 8$ completely.

4. (3 points) Add and simplify $\dfrac{x}{x^2 - 4} + \dfrac{3}{5x - 10}$.

5. (3 points) Find the inverse of the function $f(x) = \dfrac{x}{3 - 2x}$.

6. (3 points) Simplify the expression $\dfrac{\dfrac{x}{2} - \dfrac{2}{x}}{x - 2}$.

7. (6 points) Solve the following inequality: $\dfrac{x^2 + 2x - 3}{x - 4x^2} \geq 0$.

8. (a) (2 points) Solve the equation $4u^2 - 9u + 2 = 0$.

   (b) (2 points) Now, solve $4x^4 - 9x^2 + 2 = 0$.

   (c) (2 points) Solve the equation $4 \cdot e^{2x} - 9 \cdot e^x + 2 = 0$.

9. Let $h(x) = 3 \log_2(x + 4) - 9$.

   (a) (2 points) Find the x and y intercepts of the graph of h.

   (b) (2 points) Does the graph of h have any asymptotes? If so, find their equation.

   (c) (2 points) Sketch the graph of the function h.

10. (a) (2 points) Factor the polynomial $9x^2 - 3x - 2$ completely.

   (b) (5 points) Solve the following equation: $\log_2(3x + 4) + \log_2(3x - 1) = 1 + \log_2(6x - 1)$.

11. (3 points) In the triangle below, find the value of θ.

4 Applications [11%]

1. Luke invests $2500 at an annual nominal rate of 7%, compounded monthly.

(a) (2 points) What will be the value of his investment two years later?

(b) (3 points) If he wants to buy a lightsaber at $4000, how long will he have to wait before he can buy it?

2. Cath is standing on the side of a road that is 7m wide, and is looking directly across. Moreover, if she turns her head 21° degrees to the left, she sees a deer on the other side of the road; if she then moves her head 13° further, she sees a road sign.

(a) (3 points) Sketch the situation.

(b) (3 points) How far apart are the deer and the road sign?