[15] **1.** Evaluate the following limits, if they exist. If a limit doesn't exist, write $\infty, -\infty$ or DNE as appropriate. Justify with appropriate calculations.

(a) $\displaystyle\lim_{x\to-3} \frac{x+\sqrt{6-x}}{x+3}$

(b) $\displaystyle\lim_{x\to2^-} \frac{3}{x^2-x-2}$

(c) $\displaystyle\lim_{x\to0} \frac{\tan^2(x)}{x\sin(x)}$

(d) $\displaystyle\lim_{x\to-\infty} \frac{7x^3+\sqrt{x^6+1}}{(3x^2-x)(1+2x)}$

(e) $\displaystyle\lim_{x\to\infty} \frac{2\cos(3x)}{1+5x}$ (Hint: you may use the squeeze theorem.)

[5] **2.** Find and classify (as removable, jump or infinite) all discontinuities of

$$f(x) = \begin{cases} \dfrac{5x^2+4x-1}{x-3} & \text{if } x \leqslant 1 \text{ and} \\ \dfrac{x^2-9}{x^2-5x+6} & \text{if } x > 1. \end{cases}$$

[5] **3.** Use the limit definition of the derivative to find $f'(x)$ where $f(x) = \dfrac{5}{x^2-2}$.

[13] **4.** Find $\dfrac{dy}{dx}$ for each of the following. **Do not simplify your answers.**

(a) $y = \dfrac{5}{x} + \pi^e + \log_3(x) + \arcsin(x)$

(b) $y = \dfrac{7+e^x}{\tan^3(x)}$

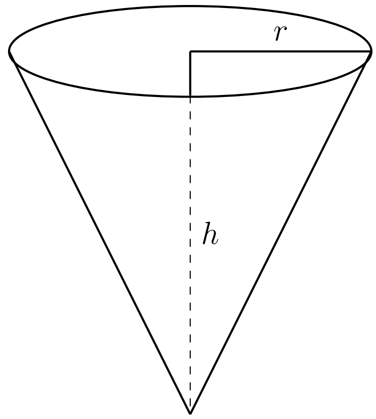
(c) $y = \cos^2\Big((3x^2+1)^4\Big)$

(d) $y = 5(2x+3)^{2-x}$

[4] **5.** Given the equation $e^{xy}+3 = \dfrac{x}{\sin(y)}$, find $\dfrac{dy}{dx}$.

[4] **6.** Consider the curve given by $y = x - \sqrt{4x-8}$. Give an equation for the tangent line at $x = 6$.

[6] **7.** An inverted cone that is 20 cm tall with a radius of 8 cm is initially full of water. The water drains at a constant rate of 15 cm^3 per second. We want to find the rate at which the water level is falling when the water is halfway down the cone. (The volume of a cone is given by $V = \frac{1}{3}\pi h r^2$.)



[12] **8.** Given

$$f(x) = \frac{-4(x+1)(x+4)}{(x-2)^2} \quad , \quad f'(x) = \frac{36(x+2)}{(x-2)^3} \quad , \quad f''(x) = \frac{-72(x+4)}{(x-2)^4},$$

sketch the graph of $f(x)$ and clearly list

(i) domain of $f(x)$

(ii) x-intercepts and y-intercept of f

(iii) vertical and horizontal asymptotes of f

(iv) relative (local) extrema of f

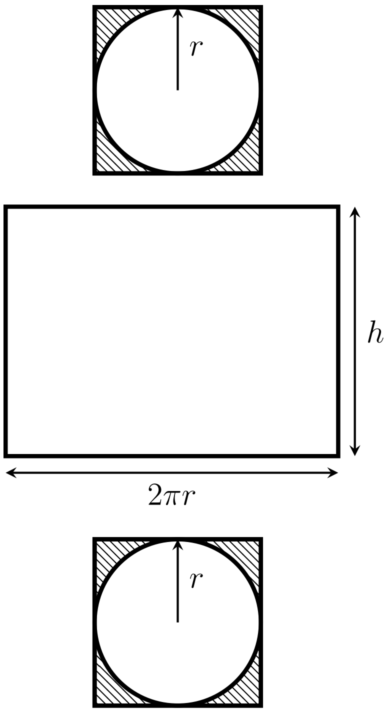
(v) point(s) of inflection of f

(vi) intervals where f is increasing, decreasing, concave upward, concave downward.

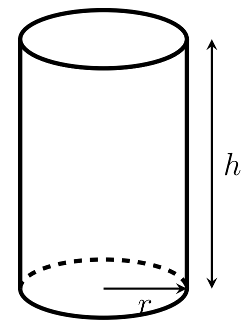
[6] **9.** A cylindrical can is constructed from a rectangular sheet of metal (for its side wall) and two circles that have been cut from square pieces (for its top and bottom.)

If the can's volume must be 1000 cm^3, find the dimensions of the can that will minimize the total amount of starting material (the rectangle and both squares).

The volume of a cylinder is given by $V = \pi r^2 h$.



Final Product



[5] **10.** Evaluate $\displaystyle\int_0^4 (x^2 - 4x)\, dx$, using the definition of the integral as a limit of Riemann sums.

The following summation formulas are provided for reference.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

[12] **11.** Evaluate the integrals below.

(a) $\displaystyle\int (4\sqrt{x^3} + 2\sin(x) - e^{2025})\, dx$

(b) $\displaystyle\int \frac{(3x-5)^2}{x}\, dx$

(c) $\displaystyle\int \big(\cot(x) + \csc^3(x)\big) \sin(x)\, dx$

(d) $\displaystyle\int_{1/2}^{\sqrt{3}/2} \frac{6\, dx}{\sqrt{1-x^2}}$

[5] **12.** Consider a particle that moves along the number line with velocity $v(t) = e^t + 6t^2$ and initial position $s(0) = 3$. Find the position function $s(t)$ of the particle.

[5] **13.** Find the area of the region between the curves $y = \sin x$ and $y = \cos x$ for $0 \leqslant x \leqslant \pi$. (Hint: the two curves intersect at $x = \pi/4$.)

[3] **14.** Let $f(x) = \displaystyle\int_0^x \sqrt[3]{t-8}\, dt$. Over what interval(s) is $f(x)$ increasing?

 Question 14 continues on the next page.

Answers:

1. (a) $5/6$ (b) $-\infty$ (c) 1 (d) 1 (e) 0

2. infinite discontinuity at $x = 2$ and removable discontinuity at $x = 3$

3. $\frac{-10x}{(x^2-2)^2}$

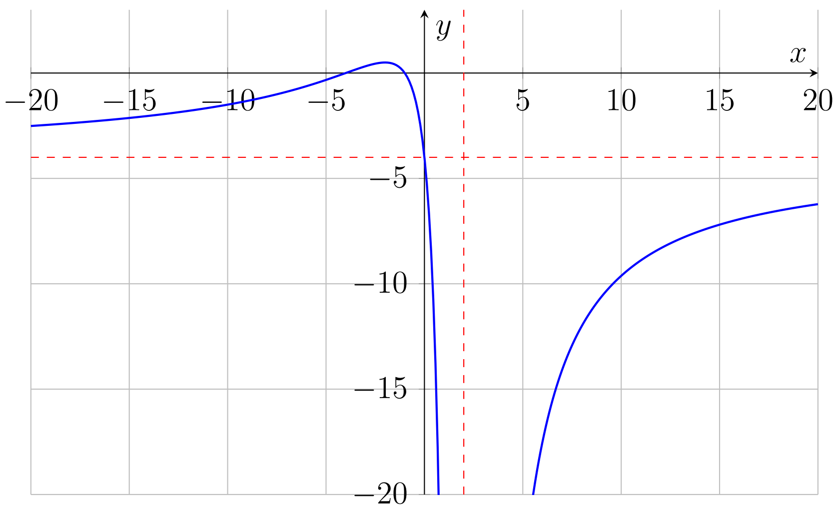
4. (a) $\frac{-5}{x^2} + \frac{1}{x \ln 3} + \frac{1}{\sqrt{1-x^2}}$ (b) $\frac{e^x \tan^3(x) - 3(7+e^x)\tan^2(x)\sec^2(x)}{\tan^6(x)}$ (c) $-48\cos((3x^2+1)^4)\sin((3x^2+1)^4)(3x^2+1)^3 x$ (d) $5(2x+3)^{2-x}\left(-\ln(2x+3) + \frac{4-2x}{2x+3}\right)$

5. $\frac{\sin(y) - y\sin^2(y)e^{xy}}{x\cos(y) + x\sin^2(y)e^{xy}}$

6. $y = \frac{x}{2} - 1$

7. $\frac{15}{16\pi}$ cm/s

8. (i) domain $= \mathbb{R}\backslash\{2\}$ (ii) x-intercepts: $(-1, 0)$ and $(-4, 0)$, y-intercept: $(0, -4)$ (iii) v.a.: $x = 2$, h.a.: $y = -4$ (iv) local max. at $x = -2$, no local min. (increasing on $(-\infty, -2)$ and $(2, \infty)$, decreasing on $(-2, 2)$) (v) point of inflection at $x = -4$ (concave up on $(-\infty, -4)$, concave down on $(-4, 2)$ and $(2, \infty)$)



9. $r = 5$, $h = 40/\pi$

10. $-32/3$

11. (a) $\frac{8}{5}x5/2 - 2\cos(x) - e^{2025}x + C$ (b) $\frac{9}{2}x^2 - 30x + 25\ln|x| + C$ (c) $\sin(x) - \cot(x) + C$ (d) π

12. $s(t) = e^t + 2t^3 + 2$

13. $2\sqrt{2}$

14. $(8, \infty)$