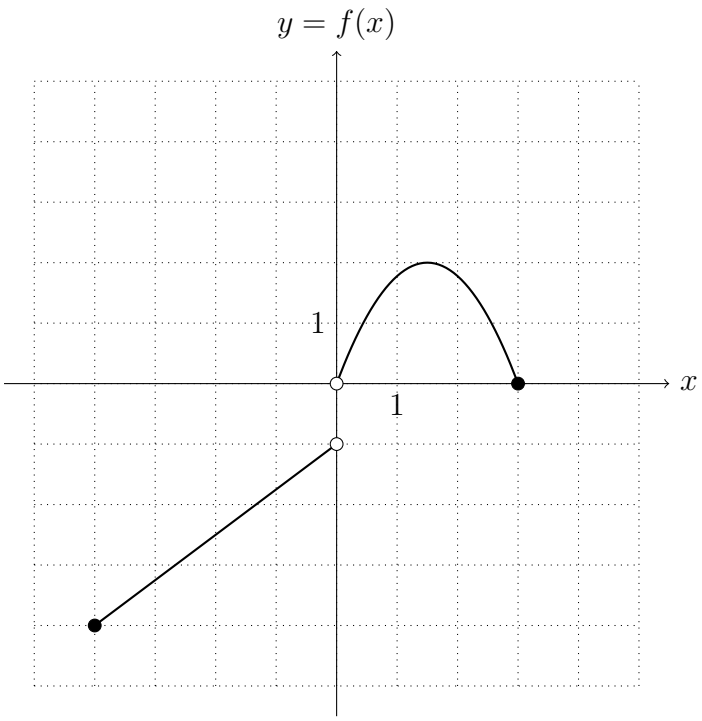
1. (4 points) Determine the domain and range of the function f graphed below.

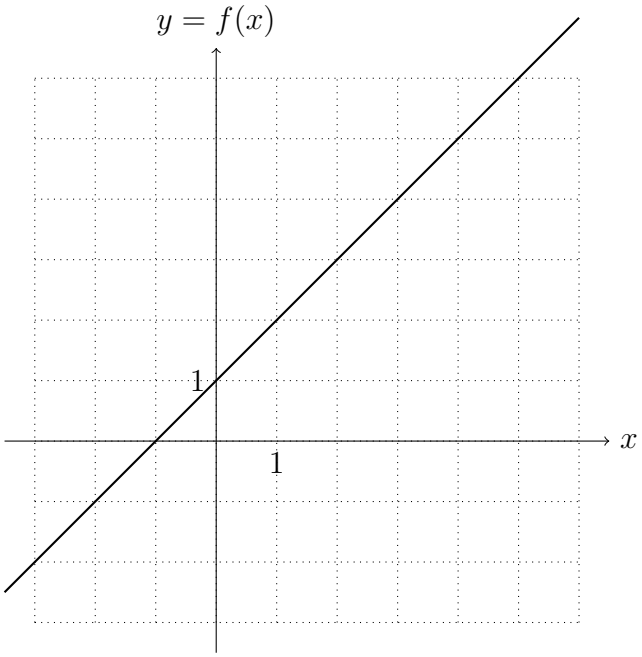


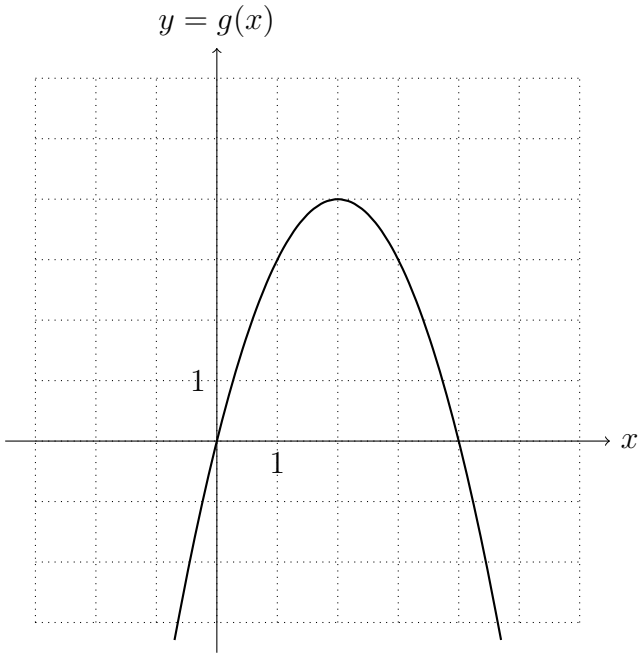
(a) Domain:

(b) Range:

(c) Does f have an inverse? Justify your answer.

2. (4 points) Consider the functions f and g given by the graphs below.





Evaluate:

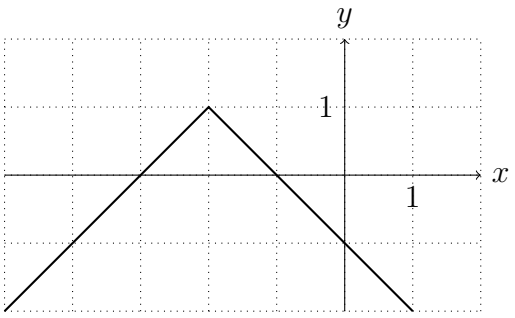
(a) $(f + g)(2) =$

(b) $(f - g)(3) =$

(c) $(f \circ g)(2) =$

(d) $(g \circ f)(3) =$

3. (2 points) Select the function that corresponds to the graph below



A. $y = -|x+2|+1$

B. $y = -|x+2|-1$

C. $y = -|x-2|+1$

D. $y = -|x-2|-1$

4. (4 points) Use long division to calculate $(x^3 + x^2 - x + 2) \div (x - 2)$

5. Determine the inverse function $f^{-1}(x)$ of the following functions:

(a) (3 points) $f(x) = \dfrac{2-x}{3x-5}$

(b) (2 points) $f(x) = 1 + e^{x-1}$

6. Determine the domain of the following functions:

(a) (2 points) $f(x) = \dfrac{x+1}{x^2+2x-3}$

(b) (3 points) $f(x) = \dfrac{\sqrt{x-2}+1}{\sqrt{3-x}}$

7. (5 points) Let $f(x) = -2x^2 + 4x + 6$

(a) Determine the coordinates of the axis intercepts.

(b) Determine the coordinates of the vertex of the parabola.

(c) Sketch a graph labelling the intercepts and the vertex.

8. Solve the following equations:

(a) (3 points) $x^4 + 2x^2 = 3x^3$

(b) (3 points) $x^3 - x^2 + x = 1$

(c) (3 points) $x^4 - 5x^2 + 4 = 0$

(d) (3 points) $\ln x + \ln(x-1) = \ln(4x+6)$

(e) (3 points) $3^{2x+5} = 27$

(f) (3 points) $2 \cdot 3^x = 5$ (Round the answer to three decimal places)

(g) (4 points) $\dfrac{x}{x+1} = \dfrac{12}{x^2-x-2} - \dfrac{4}{x-2}$

(h) (4 points) $\sqrt{x^2+32} - \sqrt{x^2-33} = 5$

9. Simplify the following expressions:

(a) (3 points) $\dfrac{\sqrt[3]{x^{17}y^{-9/4}}}{x^{2/3}\sqrt[4]{y}}$ (Answer must be expressed with positive exponents)

(b) (3 points) $\dfrac{\frac{x-3}{x+1}+x}{\frac{6}{x+1}-x}$

10. (3 points) Solve the inequality: $\dfrac{x^2-x-1}{x-1} \geqslant 1$. Write the answer in interval notation.

11. (3 points) Use properties of logarithms to rewrite the following expression as a combination of $\ln x$, $\ln 5$, and $\ln(x-5)$:

$$\ln\left(\frac{x^3-5x^2}{5}\right)$$

12. (5 points) Let $f(x) = 1 - \log_3(x+1)$.

(a) Determine the coordinates of the axis intercepts.

(b) State the equation of the asymptote of the graph.

(c) Sketch a graph labelling the intercepts and the asymptote.

13. (2 points) I need \$15 000 in 18 years. My bank offers an investment fund that earn 5% interest rate of compounded **monthly**. How much should I invest now to have that amount in the future? Approximate the answer to the nearest cent.

14. (3 points) How long does it take to **double** an investment made at 5% interest rate compounded **monthly**? Approximate the answer to the nearest year.

15. (1 point) Convert $\dfrac{11\pi}{4}$ radians to degrees.

16. (1 point) Convert $240°$ to radians.

17. (3 points) If $\sec\theta = \frac{3}{2}$ and $\tan\theta < 0$, then:

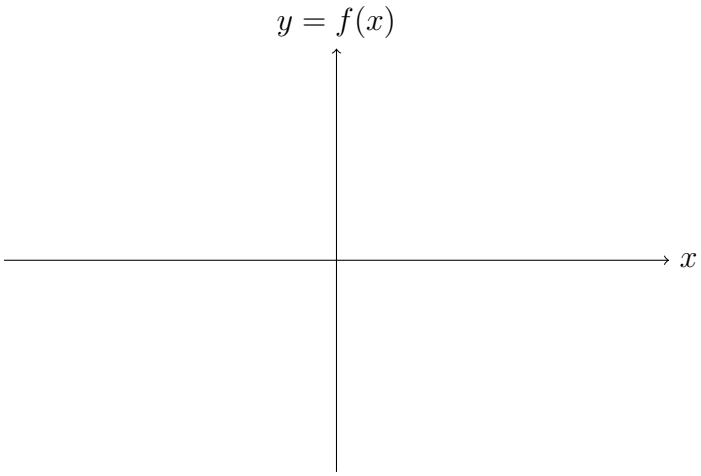
(a) At which quadrant is θ located?

(b) $\sin\theta =$

(c) $\cot\theta =$

18. (3 points) State the amplitude, the period, and sketch at least two cycles of the function

$$f(x) = -3\sin(2x)$$



(a) Amplitude:

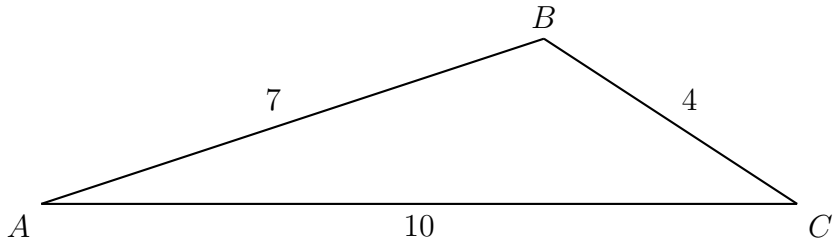
(b) Period:

19. (3 points) Determine all values of θ in the interval $[0, 2\pi)$ that satisfy $\cot\theta = \sqrt{3}$. Provide all solutions using exact values (involving π).

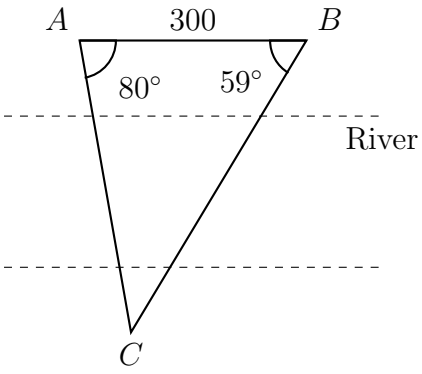
20. (3 points) Determine all values of θ in the interval $[0, 360^\circ)$ that satisfy $\sec\theta = -3$. Use the calculator to approximate your answers to the nearest degree.

21. (3 points) Prove that $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

22. (3 points) Determine the measure of all angles of the triangle below. Approximate your answer to the nearest degree.



23. (3 points) To find the distance across a river, a surveyor selects two reference points, A and B, which are 300 metres apart on the same side of the river. The surveyor then selects a third reference point, C, on the opposite side of the river and determines the angles as shown in the figure. Approximate the distance from A to C to the nearest metre.



Total: 100 points

This is the end of the examination.

Answers

1. (a) Domain: $[-4, 0) \cup (0, 3]$

(b) Range: $[-4, -1) \cup [0, 2]$

(c) f does not have an inverse because it is not one-to-one as it fails the horizontal line test.

2. (a) $(f + g)(2) = 7$

(b) $(f - g)(3) = 1$

(c) $(f \circ g)(2) = 5$

(d) $(g \circ f(3) = 0$

3. A. $y = -|x + 2| + 1$

4. Quotient: $x^2 + 3x + 5$

Remainder: 12

5. (a) $f^{-1}(x) = \dfrac{5x + 2}{3x + 1}$

(b) $f^{-1}(x) = \ln(x - 1) + 1$

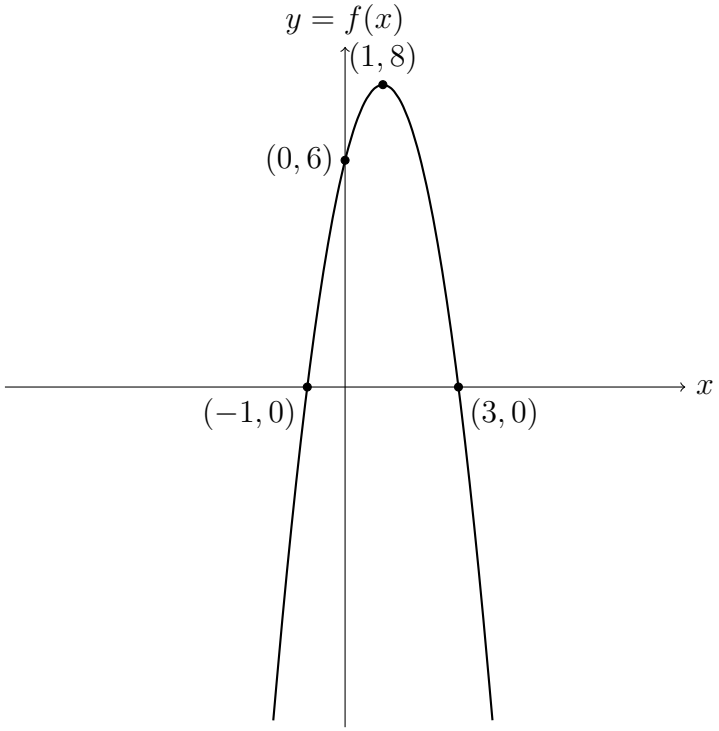
6. (a) Domain: $\mathbb{R} \setminus \{-3, 1\}$

(b) Domain: $[2, 3)$

7. (a) x-intercepts: $(-1, 0)$ and $(3, 0)$
y-intercept: $(0, 6)$

(b) Vertex: $(1, 8)$

(c)



8. (a) $x = 0$, $x = 1$, and $x = 2$

(b) $x = 1$

(c) $x = -2$, $x = -1$, $x = 1$ and $x = 2$

(d) $x = 6$

(e) $x = -1$

(f) $x = \dfrac{\ln\left(\frac{5}{2}\right)}{\ln 3} \approx 0.834$

(g) $x = -4$

(h) $x = -7$ and $x = 7$

9. (a) $\dfrac{x^5}{y}$

(b) $\dfrac{1-x}{x-2}$

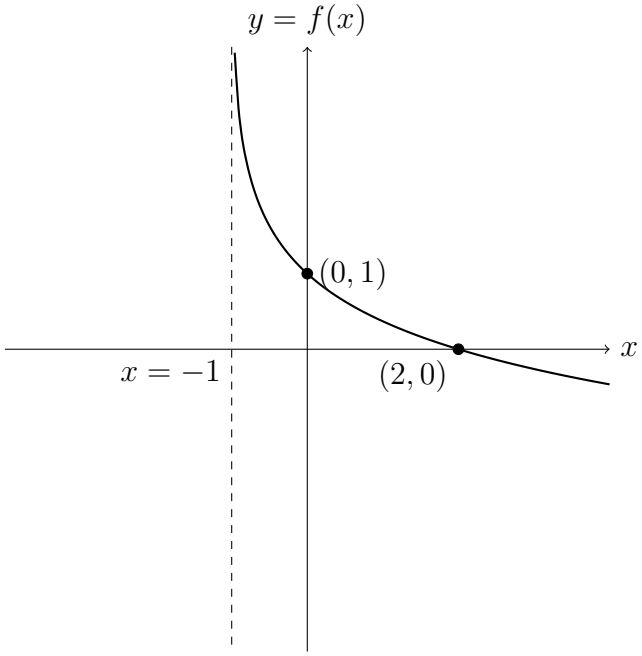
10. $[0, 1) \cup [2, \infty)$

11. $\ln(x-5) + 2\ln x - \ln 5$

12. (a) x-intercept: $(2, 0)$
y-intercept: $(0, 1)$

(b) Asymptote: $x = -1$

(c)



13. \$6110

14. 14 years

15. 495°

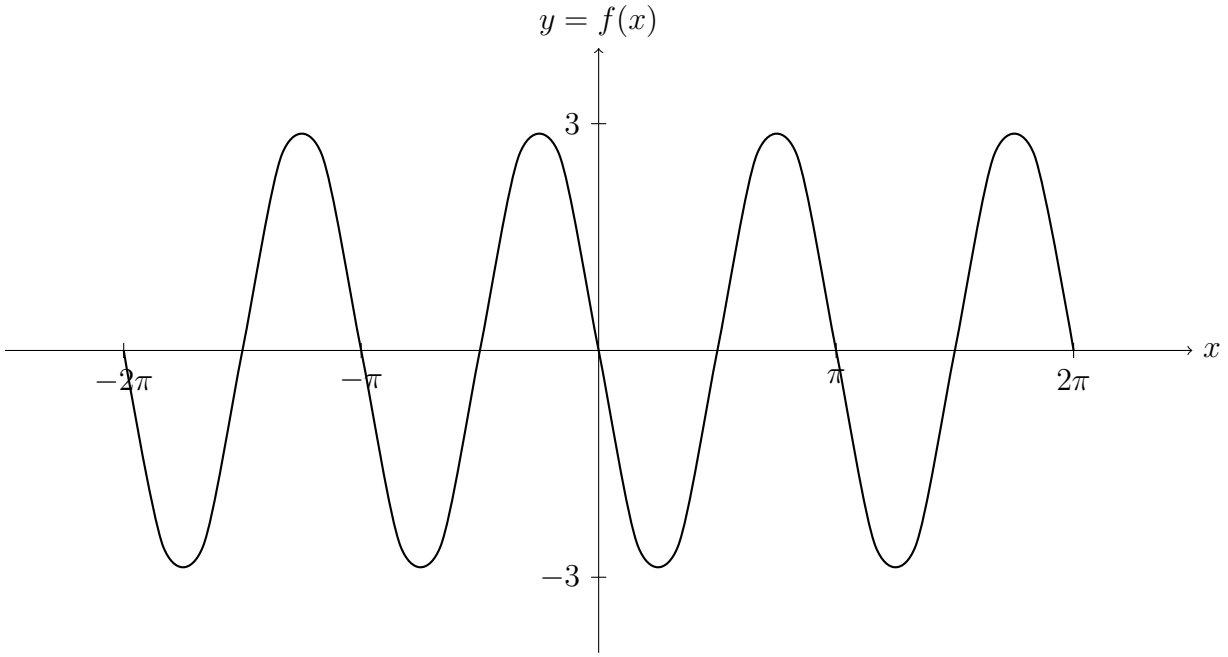
16. $\dfrac{4\pi}{3}$

17. (a) Fourth quadrant

(b) $\sin\theta = \dfrac{-\sqrt{5}}{3}$

(c) $\cot\theta = \dfrac{2}{-\sqrt{5}}$

18.



(a) Amplitude: 3

(b) Period: π

19. $\theta = \dfrac{\pi}{6}$ and $\theta = \dfrac{7\pi}{6}$

20. $\theta \approx 109°$ and $\theta \approx 251°$

21.

$$\begin{aligned}
\tan^2 x - \sin^2 x &= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x \\
&= \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x} \\
&= \frac{\sin^2 x(1 - \cos^2 x)}{\cos^2 x} \\
&= \frac{\sin^2 x(\sin^2 x)}{\cos^2 x} \\
&= \tan^2 x \sin^2 x
\end{aligned}$$

22. $A \approx 18°$, $B \approx 33°$ and $C \approx 129°$

23. 392 metres