1. (4 points) Evaluate each limit. Use L'Hopital's rule when it applies and is appropriate.

(a) $\displaystyle \lim_{x \to 0} \frac{e^x - e^{-x} - 2\sin(x)}{4x^2}$

(b) $\displaystyle \lim_{x \to \infty} \frac{\ln(x + e^x)}{x}$

2. (2 points) Find the general term a_n for the given sequence: $\left\{ \dfrac{1}{9}, \ -\dfrac{2}{7}, \ \dfrac{4}{5}, \ -\dfrac{8}{3}, \ \cdots \right\}$

3. (4 points) Determine whether each sequence diverges or converges.
If the sequence converges, find its limit.

(a) $\left\{ \dfrac{e^{1-n} + 2}{1 + e^{n+1}} \right\}$

(b) $\left\{ (-1)^n \sqrt{\dfrac{2n^3}{3 + 8n^3}} \right\}$

4. (24 points) Evaluate the following integrals.

(a) $\displaystyle \int \frac{\sqrt[3]{x^2} + 4x^3 \sin(x) - (5x)^2}{x^3} dx$

(b) $\displaystyle \int_1^e \frac{10}{3x(1 + \ln(x))^3} dx$

(c) $\displaystyle \int (x^2 + 1) \ln(4x) dx$

(d) $\displaystyle \int (x - 1)^2 e^{-3x} dx$

(e) $\displaystyle \int \sec^2(x) 5^{(3\tan(x)+2)} dx$

5. (4 points) Given the following graph of f, find:

(a) $\displaystyle \int_{-5}^3 f(x) \, dx$

(b) $\displaystyle \int_0^{-5} f(x) \, dx$

(c) $\displaystyle \int_{-3}^3 2f(x) \, dx$

(d) $\displaystyle \int_0^5 |f(x)| \, dx$
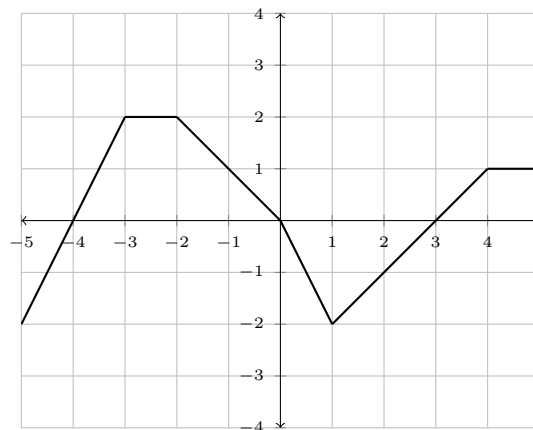
6. (3 points) Given the integral $\displaystyle \int_{-5}^1 |2x + 6| dx$,

(a) Sketch the graph of $f(x) = |2x + 6|$ and shade the region of the graph that corresponds to the definite integral.

(b) Evaluate the definite integral by interpreting it in terms of areas.

7. (5 points) Evaluate $\displaystyle\int_0^4 (4x^3 - 3x - 2)\, dx$ using Riemann sums. You can use the formulas below:

$$\sum_{k=1}^{n} c = c \cdot n, \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

8. (5 points) Find the area enclosed by the curves of: $f(x) = x^3 + 3x^2 - 4$ and $g(x) = 3x^2 - 5$ between $x = -2$ and $x = 2$.

9. (10 points) Determine whether each improper integral converges or diverges. If the integral converges, find its value.

(a) $\displaystyle\int_0^{\pi/2} \frac{\sin(x)}{\sqrt{4 - 4\cos(x)}}\, dx$

(b) $\displaystyle\int_{-\infty}^{0} \frac{e^{4x}}{(3e^{4x} + 3)^2}\, dx$

10. (5 points) Given the demand function $p = \dfrac{112}{x+2}$ and the supply function $p = 2x + 6$, find the equilibrium point, and find the consumer surplus.

11. (5 points) Librarians in one country have an income distribution described by $f(x) = \dfrac{2}{7}x^3 + \dfrac{5}{7}x$.

(a) Compute $f(0.8)$ and interpret your result.

(b) Find the Gini index for librarians.

(c) A government clerk mistakenly entered this income distribution function for the librarians:
$f(x) = \dfrac{2}{7}x^2 + \dfrac{5}{7}x$.
Calculate the resulting Gini coefficent. Would this clerical error make the income distribution of librarians appear more or less equitable than it really is?

12. (4 points) Given $f'(x) = \dfrac{1 + 3\sqrt{x}}{\sqrt{x}}$ and $f(4) = 12$, find $f(x)$.

13. (4 points) Find the function y that satisfies the differential equation $\dfrac{dy}{dx} = xy - 2y$ with initial condition $y(0) = 3$.

14. (6 points) The number of an invasive alien species on the planet Dagobah is growing at a rate proportional to the product of its population P and time t in years. Initially, there were 50 who arrived on Dagobah. After two years, there were 200.

(a) Write the differential equation and initial conditions to represent this situation.

(b) How many years will it take for the population to reach 12800?

15. (9 points) Given that $f(x) = \dfrac{3}{16}\sqrt{x}$ is a probability density function for the random variable X on the interval $[0, 4]$, find the following:

(a) $P(X \le 2)$

(b) $P(X = 3)$

(c) The mean of X.

(d) The starndard deviation of X.

16. (6 points) The grade point averages of the students attending a large college are normally distributed with mean 2.4 and standard deviation 0.8.

    (a) Students who make the Honour Roll must attain a G.P.A. of at least 3.0.
        What percentage of students made the Honour Roll?

    (b) Alex made the Dean's List this semester, having attained a G.P.A. in the top 3%.
        What was her G.P.A.?

Answers:

1. (a) 0   (b) 1

2. $a_n = \dfrac{(-2)^{n-1}}{11 - 2n}$

3. (a) converges to 0   (b) diverges

4. (a) $-\dfrac{3}{4\sqrt[3]{x^4}} - 4\cos(x) - 25\ln|x| + C$ (d) $-\dfrac{(x-1)^2}{3}e^{-3x} - \dfrac{2(x-1)}{9}e^{-3x} - \dfrac{2}{27}e^{-3x} + C$

    (b) $\dfrac{5}{4}$

    (c) $\ln(4x)\left(\dfrac{x^3}{3} + x\right) - \dfrac{x^3}{9} - x + C$ (e) $\dfrac{5^{(3\tan(x)+2)}}{3\ln(5)} + C$
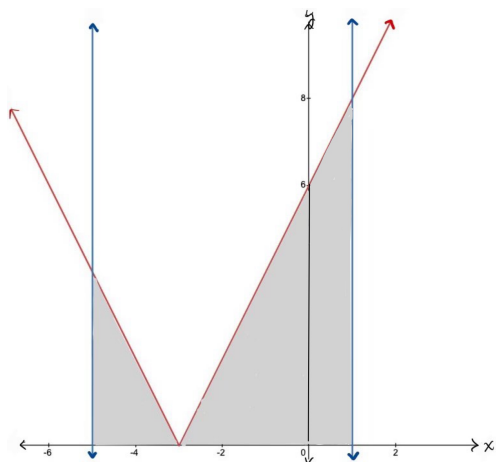
5. (a) 1   (b) -4   (c) 2   (d) $\dfrac{9}{2}$

6. (a) Graph:

    (b) 20

7. 224

8. $\dfrac{19}{2}$

9. (a) converges to 1

(b) converges to $\dfrac{1}{72}$

10. $(5, 16)$, \$60.31

11. (a) 0.7177. The poorest 80% of the librarians receive 71.77% of the total income.

(b) 0.1429

(c) 0.0952

   $0.1429 > 0.0952$: The incorrect income distribution would make the income appear more equitable.

12. $f(x) = 2\sqrt{x} + 3x - 4$

13. $y = 3e^{x^2/2 - 2x}$

14. (a) $\dfrac{dP}{dt} = kPt \quad P(0) = 50, P(2) = 200$

(b) 4 years.

15. (a) 0.3536

(c) $\mu = \dfrac{12}{5} = 2.4,$

(b) 0

(d) $\sigma = \sqrt{\frac{192}{175}} = 1.0474$

16. (a) 22.66%

(b) 3.904