1. Evaluate the following integrals.

(a) $\displaystyle\int (x^2 - 2x)e^{x/3}\,dx$

(b) $\displaystyle\int_{\pi/4}^{\pi/2} \frac{\csc^2(x)}{\sqrt{4 - \cot^2(x)}}\,dx$

(c) $\displaystyle\int \frac{\sin^3(\sqrt{x})\cos^2(\sqrt{x})}{\sqrt{x}}\,dx$

(d) $\displaystyle\int \frac{1}{(4x^2 - 9)^{3/2}}\,dx$

(e) $\displaystyle\int x\arctan(x)\,dx$

(f) $\displaystyle\int \frac{4x + 16}{(x - 2)(x + 1)(x - 5)}\,dx$

2. Evaluate the following limits.

(a) $\displaystyle\lim_{x\to 0} \frac{\cos(x) - 1}{x\arctan(x)}$

(b) $\displaystyle\lim_{x\to 0} \left(1 + \frac{1}{x^2}\right)^{x^3}$

3. Set up the integral, but you do not need to evaluate that equals the area bounded between $y = x^2 + 1$ and $y = x + 3$.

4. Find the volume created by rotating $f(x)$ around the x-axis from $x = 0$ to $x = 4$, when $f(x)$ is given as

(a) $f(x) = \sqrt{x}$

(b) $f(x) = \begin{cases} 0 & x \le 0 \\ x & 0 < x < 2 \\ 2x - 2 & 2 \le x \end{cases}$

5. Solve the differential equation $\dfrac{dy}{dx} = \dfrac{e^y}{\csc^2(x)}$ where $y(\pi) = 0$. Give an explicit solution for y.

6. A cup of hot coffee with a temperature of 80°C is placed in a room that has a constant temperature of 20°C. The coffee cools according to Newton's Law of Cooling, which states that the rate of change of temperature of some object (coffee) is proportional to the temperature difference between the object and its surroundings. Five minutes later, the coffee has cooled to 50°C.

(a) Write a differential equation that models the cooling of the coffee.

(b) Solve the differential equation to find an explicit expression for the temperature as a function of time.

(c) Determine the temperature of the coffee after 10 minutes. Simplify your answer.

7. Given $\displaystyle\sum_{n=2}^{\infty} \frac{(-3)^{n-1}}{3^{2n}}$

(a) Show that the series converges.

(b) Find the sum of the series.

8. Determine whether the following series converge of diverge. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{2n+4}{3n-1}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\ln(n^3+1)}{n+1}\right)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)+5}{\sqrt{16n-1}}$

9. Determine whether the following series are absolutely convergent, conditionally convergent or divergent. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(n+3)!}{(n^2+3)e^{2n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+10}$

10. Find the interval of convergence of the following power series.

$\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{4^{n+1}\sqrt{n}}$

11. Find a power series representation for the function and determine the interval of convergence for $f(x) = \dfrac{9}{3x+2}$

12. Find Maclaurin series for $g(x) = \ln(7-x)$ using known series and determine the radius of convergence.

13. Use the definition of a Taylor series to find the first three nonzero terms of the series for $g(x) = \sqrt{x}$ centred at $a = 4$.

14. Given f is continuous on $[0,3]$ and $\displaystyle\int_0^3 f(x)\,dx = 4$, evaluate the following definite integral. Simplify your answer.

$\displaystyle\int_1^{e^3} \frac{2+f(\ln(x))}{x}\,dx$

15. Fill in the following blanks with the word *must*, *might* or *cannot*, as appropriate.

(a) If $\lim\limits_{n\to\infty} a_n = 0$, then $\sum a_n$ _____ converge.

(b) If $a_n > 0$ and $\sum a_n$ converges, then $\sum (-1)^n a_n$ _____ converge.

(c) If the sequence $\{a_1, a_2, a_3, a_4, ...\}$ diverges, then the sequence $\{a_2, a_4, a_6, a_8, ...\}$ _____ diverge.

Answers

1. (a) $(3x^2 - 24x + 72)e^{x/3} + C$

(b) $\pi/6$

(c) $\dfrac{2\cos^5(\sqrt{x})}{5} - \dfrac{2\cos^3(\sqrt{x})}{3} + c$

(d) $\dfrac{-x}{9\sqrt{4x^2-9}} + c$

(e) $\frac{1}{2}[x^2 \arctan(x) - x + \arctan(x)] + c$

(f) $\frac{-8}{3}\ln|x-2| + 2\ln|x-5| + \frac{2}{3}\ln|x+1| + c$

2. (a) $-1/2$

(b) 1

3. $\displaystyle\int_1^2 x^2 - x - 2 \; dx$

4. (a) 8π

(b) $\dfrac{112\pi}{3}$

5. $y = -\ln\left(\dfrac{\sin(2x) - 2x + 4 + 2\pi}{4}\right)$

6. (a) $\dfrac{dT}{dt} = k(T - 20)$

(b) $T = 60\left(\dfrac{1}{2}\right)^{t/5} + 20$

(c) $T = 35°C$

7. (a) $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = 1/3 < 1$ Therefore, the series converges by the Ratio Test

(b) $-1/36$

8. (a) Diverges be the Test for Divergence

(b) Converges by the Root Test

(c) Diverges by the Comparison Test

9. (a) Diverges by the Ratio Test

(b) Conditionally Convergent by the Limit Comparison Test and the Alternating Series Test

10. $[-5, 3)$

11. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n 3^{2+n}}{2^{1+n}}$ when $|x| < \dfrac{2}{3}$

12. $g(x) = \ln(7) - \left(\sum_{n=1}^{\infty} \frac{x^n}{7^n n} \right)$ when $R = 7$

13. $2 + \frac{x-4}{4} - \frac{(x-4)^2}{64} + ...$

14. 10

15. (a) might

(b) must

(c) might