1. Evaluate the following integrals.

(3) (a) $\displaystyle\int \big(1+\tan(x)\big)^3 \sec^2(x)\,dx$

(5) (b) $\displaystyle\int x\sec^2(x)\,dx$

(5) (c) $\displaystyle\int \cos^8(2x)\,\sin^3(2x)\,dx$

(6) (d) $\displaystyle\int_0^{1/2} x\,\arcsin(x)\,dx$

(6) (e) $\displaystyle\int \frac{3x^2+1}{x^2+2x-3}\,dx$

(5) (f) $\displaystyle\int \frac{dx}{x^2\sqrt{9x^2-4}}$

(5) (g) $\displaystyle\int \frac{x}{x^2+4x+5}\,dx$

2. Evaluate the following limits.

(4) (a) $\displaystyle\lim_{x\to 0^+}\left(\frac{1}{e^x-1}-\frac{1}{x}\right)$

(4) (b) $\displaystyle\lim_{x\to 0}(\cos x)^{3/x^2}$

(5) **3.** Find the area of the region enclosed by $y=-x$ and $y=2-x^2$.

(5) **4.** Solve the differential equation

$$\ln(y)\,\frac{dy}{dx}=x^3\,y$$

given that $y=e$ when $x=0$. **Express y in terms of x.**

(6) **5.** Sugar dissolves in water at a rate proportional to the amount remaining. Suppose that there was 50 kg of sugar present initially, and at the end of 5 hours, only 20 kg was left.

(a) Write a differential equation (with initial condition) that models this situation.

(b) Solve the differential equation to find an explicit expression for the remaining amount of sugar as a function of time.

(c) How long will it take until 8 kg of sugar remains?

(4) **6.** Determine whether the sequence converges or diverges. If the sequence converges, find its limit; otherwise, explain why it diverges.

(a) $a_n = n\,\sin\left(\dfrac{\pi}{n}\right)$

(b) $b_n = (-1)^n\left(\dfrac{n^2+n}{3n^2+4}\right)$

(5) **7.** Find the sum of the following telescoping series.

$$\sum_{n=2}^{\infty}\frac{2}{n^2-1}$$

8. Determine whether the series converges or diverges. Justify your answer and state the test that you use.

(2) (a) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^3}\right)$

(3) (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2\,5^n}{(2n)!}$

(2) (c) $\displaystyle\sum_{n=2}^{\infty} \frac{\sec^2(n)}{n-1}$

9. Determine whether the series is absolutely convergent, conditionally convergent, or divergent. Justify your answer.

(3) (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{4n^3-7}{5n^3+3n^2}\right)^n$

(4) (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2+5}}$

(6) **10.** Find the radius and interval of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n(2x-1)^n}{7^n\,\sqrt{n+1}}$$

(6) **11.** (a) Find the Maclaurin series and state the radius of convergence for the following function.

$$f(x) = \frac{1}{1+4x}$$

(b) Write down the first four non-zero terms of the series in Part (a).

(c) Use your answer from Part (a) to find the Maclaurin series for the following function.

$$g(x) = \frac{1}{(1+4x)^2}$$

(3) **12.** Using known power series, find the Maclaurin series for the following function.

$$f(x) = x\sin(x^4)$$

(3) **13.** Determine whether the statement is **Always True**, **Always False**, or **Sometimes True**.

(a) If $\displaystyle\sum_{n=0}^{\infty} c_n 5^n$ converges, then $\displaystyle\sum_{n=0}^{\infty} c_n(-4)^n$ converges. ______________

(b) If $\displaystyle\sum_{n=0}^{\infty} c_n 5^n$ converges, then $\displaystyle\sum_{n=0}^{\infty} c_n(-5)^n$ converges. ______________

(c) If $\displaystyle\lim_{n\to\infty} a_n = 1$, then the series $\displaystyle\sum_{n=1}^{\infty} (1-a_n)^n$ converges. ______________

Answers

1. (a) $\dfrac{(1+\tan(x))^4}{4} + C$

(b) $x\tan(x) + \ln|\cos(x)| + C$

(c) $\dfrac{1}{2}\left(\dfrac{\cos^{11}(2x)}{11} - \dfrac{\cos^{9}(2x)}{9}\right) + C$

(d) $\dfrac{\sqrt{3}}{16} - \dfrac{\pi}{48}$

(e) $3x - 7\ln|x+3| + \ln|x-1| + C$

(f) $\dfrac{\sqrt{9x^2-4}}{4x} + C$

(g) $\dfrac{1}{2}\ln\left(x^2+4x+5\right) - 2\arctan(x+2) + C$

2. (a) $-\dfrac{1}{2}$

(b) $e^{-3/2}$

3. $\dfrac{9}{2}$

4. $e^{\sqrt{x^4/2+1}}$

5. (a) $\dfrac{dy}{dt} = ky \qquad y(0) = 50 \qquad y(5) = 20$

(b) $y = 50\left(\dfrac{2}{5}\right)^{t/5}$

(c) $t = 10$ hours

6. (a) π

(b) The sequence diverges as it oscillates between $-1/3$ and $1/3$.

7. $\dfrac{3}{2}$

8. (a) The series diverges by Test for Divergence.

(b) The series converges by Ratio Test.

(c) The series diverges by Direct Copmarison Test.

9. (a) Absolutely Convergent (by Root Test)

(b) Conditionally Convergent

10. $R = \dfrac{7}{2}$ and I.C.$=(-3, 4]$

11. (a) $f(x) = \displaystyle\sum_{n=0}^{\infty} (-1)^n 4^n x^n$ with $R = \dfrac{1}{4}$

(b) $f(x) = 1 - 4x + 16x^2 - 64x^3 + \dots$

(c) $g(x) = -\dfrac{f'(x)}{4} = \displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} n 4^{n-1} x^{n-1} = \sum_{n=0}^{\infty} (-1)^n (n+1) 4^n x^n$ with $R = \dfrac{1}{4}$

12. $x \sin(x^4) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{8n+5}}{(2n+1)!}$ with $R = \infty$

13. (a) Always True

(b) Sometimes True

(c) Always True