

1. Evaluate each limit. Use L'Hopital's rule when it applies and is appropriate.

(a) $\lim_{x \rightarrow \infty} \frac{\ln(x)}{1 + \sqrt{x}}$

(b) $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin(x)}$

2. Find the general term a_n for the given sequence: $\left\{-\frac{7}{5}, \frac{11}{15}, -\frac{1}{3}, \frac{19}{135}, -\frac{23}{405}, \dots\right\}$

3. Determine whether each sequence diverges or converges.

If the sequence converges, find its limit.

(a) $\left\{\frac{4^{n+1}}{1 + 9^n}\right\}$

(b) $\left\{(-1)^{n+1} \frac{n^2 + 4}{\sqrt{n + 4n^4}}\right\}$

4. Evaluate the following integrals.

(a) $\int \frac{5\sqrt[4]{x^3} + x^2 e^{5x} - 8x}{4x^2} dx$

(d) $\int \frac{6 \ln(4x)}{x^3} dx$

(b) $\int \frac{\cos(4 + 3 \ln(x))}{2x} dx$

(e) $\int_1^6 \frac{e^{4+1/x}}{x^2} dx$

(c) $\int_{-1}^4 |6 - 2x| dx$

(f) $\int (x + 1)^2 e^{3x} dx$

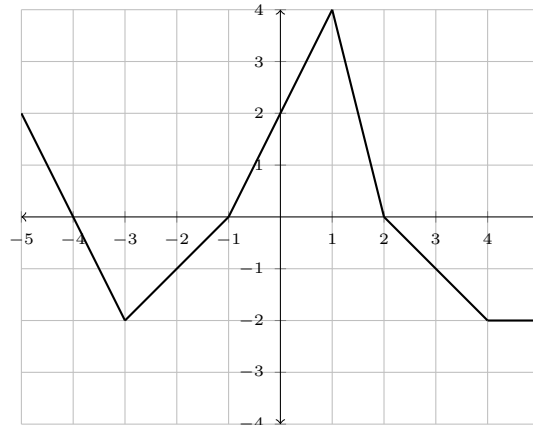
5. Given the following graph of f , find:

(a) $\int_{-5}^2 f(x) dx$

(b) $\int_{-1}^5 3f(x) dx$

(c) $\int_5^{-1} f(x) dx$

(d) $\int_{-5}^5 |f(x)| dx$



6. Find the area enclosed by the curves of:

$$f(x) = x^2 - 7x + 6, \quad g(x) = -x^2 + 5x - 10, \quad x = -1 \text{ and } x = 3.$$

7. Evaluate $\int_0^2 (2x^3 - 3x^2 + 5) dx$ using Riemann sums.

8. Determine whether each improper integral converges or diverges. If the integral converges, find its value.

(a) $\int_2^\infty \frac{3x + 2}{\sqrt[3]{(3x^2 + 4x + 1)^5}} dx$

(b) $\int_1^e \frac{3}{x \sqrt{\ln(x)}} dx$

9. For the past several years, college professors had an income distribution described by $f(x) = \frac{3}{8}x^2 + \frac{5}{8}x$.
- Compute $f(0.6)$ and interpret your result.
 - Calculate the coefficient of inequality.
 - A change to the pay scale seems to have affected the distribution of incomes for these professors. After the change, the Lorenz curve for college professors is now described by $f(x) = \frac{3}{7}x^2 + \frac{4}{7}x$. Calculate the new coefficient of inequality.
 - Does the new pay scale make the distribution of incomes for professors more or less equitable than it was before?
10. The demand function for a product is $p = -x^2 + 140$ and its supply function is $p = x^2 + 6x$. Find the equilibrium point, the consumer surplus, and the producer surplus.
11. Given $f''(x) = \frac{x^2 - 6x^3 + 48x^4}{x^2}$, $f(2) = 63$, and $f'(1) = 16$, find $f(x)$.
12. The number of followers N of one influencer is growing at a rate proportional to the square of time t (the number of years since 2020). In 2020, there were 500 followers. By 2023, there were 2500.
- Write the differential equation and initial conditions to represent this situation.
 - How many followers will there be in 2026?
 - In what year will the number of followers reach 75 000?
13. Find the function y that satisfies the differential equation $\frac{dy}{dx} = \frac{2x + 3\sec^2(x)}{2y}$ with initial condition $y(0) = -5$.
14. Given the probability density function $f(x) = \frac{2k}{\sqrt[3]{x}}$ defined on $(1 \leq x \leq 8)$, find the value of the constant k .
15. Given that $f(x) = \frac{3}{32}(4x - x^2)$ is a probability density function for the random variable X on the interval $[0, 4]$, find the following:
- $P(X < 1)$
 - $P(X > 1)$
 - The mean of X
 - The variance of X
16. Suppose a university professor with many years of experience teaching a specific course has compiled all the individual grades from his past students. It was determined that his students' grades are normally distributed with a mean of 70 and a standard deviation of 12.
- Suppose you are taking his course next semester. What is the probability that you will get a grade between 70 and 85?
 - Your friend will take the course with you and really wants to score in the top 2%. What grade would she need to achieve her goal?

Answers:

1. (a) 0 (b) 2
2. $a_n = \frac{4n+3}{(-5)(-3)^{n-1}}$
3. (a) converges to 0 (b) diverges
4. (a) $-5x^{-1/4} + \frac{1}{20}e^{5x} - 2\ln|x| + C$ (d) $-3x^{-2}\ln(4x) - \frac{3}{2}x^{-2} + C$
(b) $\frac{1}{6}\sin(4+3\ln(x)) + C$ (e) $-e^{\frac{25}{6}} + e^5$
(c) 17 (f) $\frac{1}{3}(x+1)^2e^{3x} - \frac{2}{9}(x+1)e^{3x} + \frac{2}{27}e^{3x} + C$
5. (a) 4 (b) 6 (c) -2 (d) 14
6. $\frac{112}{3}$
7. 10
8. (a) converges to $\frac{3}{4(21)^{2/3}}$ (b) converges to 6
9. (a) 0.510. The poorest 60% of the college professors received 51.0% of the total income.
(b) 0.125
(c) 0.143
(d) $0.143 > 0.125$, so the new income distribution is less equitable.
10. (7, 91) CS= \$228.67 PS=\$375.67
11. $f(x) = 4x^4 - x^3 + \frac{1}{2}x^2 + 2x + 1$
12. (a) $\frac{dN}{dt} = kt^2$ $N(0) = 500, N(3) = 2500$
(b) 16 500
(c) 2030
13. $y = \sqrt{x^2 + 3\tan(x) + 25}$
14. $k = \frac{1}{9}$
15. (a) 0.15625 (c) $\mu = 2.0$
(b) 0.84375 (d) $\sigma^2 = 0.8$
16. (a) 39.44% (b) 94.6