1. (30 points) Evaluate the following integrals.

(a) $\displaystyle\int \frac{5x^2+x+22}{(3x+6)(x^2+4)}\,dx$

(b) $\displaystyle\int_1^4 \frac{(\sqrt{x}+1)^3}{\sqrt{x}}\,dx$

(c) $\displaystyle\int e^{\sqrt{x}}\,dx$

(d) $\displaystyle\int_0^{\pi/4} \sec^4(\theta)\tan^4(\theta)\,d\theta$

(e) $\displaystyle\int x\,\mathrm{arcsec}(x)\,dx$

(f) $\displaystyle\int \sqrt{4-x^2}\,dx$

2. (6 points) Evaluate the following limits. If using l'Hospital's rule, justify why it may be used.

(a) $\displaystyle\lim_{x\to 0^+} xe^{\frac{1}{x}}$

(b) $\displaystyle\lim_{x\to 0} (x+1)^{\cot x}$

3. (10 points) For each of the following improper integrals, either evaluate it or show that it diverges.

(a) $\displaystyle\int_0^{\pi/2} (\sec(x)\tan(x)-\sec^2(x))\,dx$

(b) $\displaystyle\int_9^{\infty} \frac{1}{x^2-8x+41}\,dx$

4. (5 points) Find the area of the region bounded by the curves $y=x^3-8x$ and $y=x$.

5. (4 points) Let $\mathcal{R}$ be the region bounded by the curves $x=y^2$ and $x=y$. Set up, but **do not evaluate** an integral representing the volume of the solid obtained by rotating the region $\mathcal{R}$ about

(a) the y-axis.

(b) the line $y=2$.

6. (5 points) Find the length of the curve $y=\dfrac{1}{6}x^3+\dfrac{1}{2x}$ between $x=1$ and $x=3$.

7. (4 points) Solve the differential equation $\dfrac{dy}{dx}=\dfrac{x}{y(1+x)}$ with the initial condition $y(0)=-2$. Express y explicitly as a function of x and fully simplify your answer.

8. (5 points) A bacteria culture grows at a rate proportional to the number of bacteria present. Initially, the culture contains 1000 bacteria. After 3 hours, the population grows to 8000 bacteria.

(a) Set up a differential equation with the initial conditions describing the population growth.

(b) Find an expression for the number of bacteria as a function of time t.

(c) Find the time when the population reaches 100 000 bacteria.

9. (3 points) Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{3+4^n}{7^n}$

10. (3 points) Determine whether the sequence with a general term $a_n=\ln(n+1)-\ln(n)$ converges or diverges. Justify your answer.

11. (9 points) Determine whether each of the following series converges or diverges. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+5}{n^3+3n+7}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+7)^n}{7^{n^2}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2+\sin(n)}{n^4}$

12. (6 points) Determine whether each of the following series converges absolutely, converges conditionally, or diverges. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ln(n)}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n+e^{-n}}$

13. (5 points) Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(x-5)^n}{n\,2^n}$.

14. (5 points) Let $f(x)=\dfrac{1}{(3-x)^2}$.

(a) Write the first four nonzero terms of the Maclaurin series for $f(x)$.

(b) Find a formula for the $n-$th term of the series, and express the series in sigma notation.

ANSWERS

1. (a) $\frac{5}{3}\ln|3x+6| + \frac{1}{6}\arctan(\frac{x}{2}) + C$

(b) $\frac{65}{2}$

(c) $2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$

(d) $\frac{12}{35}$

(e) $\dfrac{x^2\text{arcsec}(x)}{2} - \frac{1}{2}\sqrt{x^2-1} + C$

(f) $2\arcsin(\frac{x}{2}) + \frac{x\sqrt{4-x^2}}{2} + C$

2. (a) ∞

(b) e

3. (a) -1

(b) $\frac{\pi}{20}$

4. $\frac{81}{2}$

5. (a) $V = \displaystyle\int_0^1 \pi[y^2 - y^4]\ dy$

(b) $V = \displaystyle\int_0^1 2\pi(2-y)(y-y^2)\ dy$

6. $\frac{14}{3}$

7. $y = -\sqrt{2x - 2\ln|x+1| + 4}$

8. (a) $\frac{dN}{dt} = kN,\ \ N(0) = 1000,\ \ N(3) = 8000$

(b) $N(t) = 1000(2)^t$

(c) $t = \frac{\ln 100}{\ln 2}$

9. $\frac{35}{6}$

10. $\lim\limits_{n\to\infty} a_n = 0$, conv.

11. (a) div.

(b) conv.

(c) conv.

12. (a) conditionally conv.

(b) absolutely conv.

13. $R = 2$, IoC $(3, 7]$

14. (a) $\dfrac{1}{(3-x)^2} = \frac{1}{9} + \frac{2}{27}x + \frac{3}{81}x^2 + \frac{4}{243}x^3 + \ldots$

(b) $\dfrac{1}{(3-x)^2} = \displaystyle\sum_{n=0}^{\infty} \frac{n+1}{3^{n+2}}x^n$