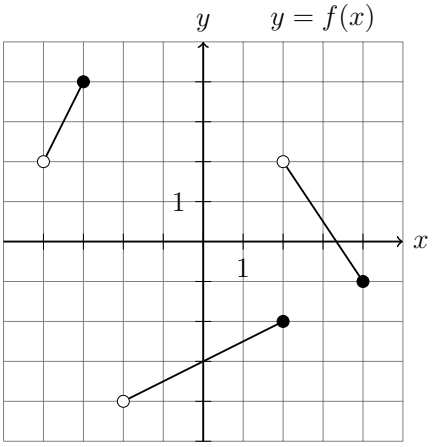
[7] **1.** Let $y = f(x)$ be given by the graph below on the right:

(a) State the domain of f.

(b) State the range of f.

(c) Evaluate: $(f \circ f)(0)$

(d) Evaluate: $f^{-1}(-1)$



(e) Using the graph, fill in the blanks to complete the piecewise definition for the function $f(x)$ below:

$$f(x) = \begin{cases} 2x + 10 & \text{if } -4 < x \leqslant -3 \\ \underline{\qquad\qquad} & \text{if } -2 < x \leqslant 2 \\ -\dfrac{3}{2}x + 5 & \text{if } \underline{\qquad\qquad} \end{cases}$$

[5] **2.** Consider the quadratic function $f(x) = -x^2 + 2x + 3$.

(a) Find the x- and y-intercepts of the function. State the coordinates of each intercept.

(b) Find the coordinates of the vertex of the parabola.

(c) Graph the function on the axes provided. Label all the intercepts and vertex properly.

[4] **3.** Use long division to calculate $(3x^3 - 4x^2 + 7x - 8) \div (x - 2)$.
Write your conclusion in the form

$$\frac{3x^3 - 4x^2 + 7x - 8}{x - 2} = Q(x) + \frac{R(x)}{D(x)}$$

where $Q(x)$ is the quotient, $R(x)$ is the remainder, and $D(x)$ is the divisor.

[5] **4.** Let $f(x) = \dfrac{2x^2 - x - 3}{x^3 + 2x^2}$ and $g(x) = \dfrac{x^4 + 8x}{6x - 4x^2}$. Find and simplify $(f \cdot g)(x)$.

5. Let $f(x) = \dfrac{7x}{x - 2}$.

[3] (a) Find $f^{-1}(x)$.

[3] (b) Find and simplify $f\left(\dfrac{1}{x^2}\right)$.

6. Solve the inequalities:

[3] (a) $x^3 - 3x^2 - 4x + 12 < 0$

[3] (b) $\dfrac{5x-2}{(x-2)(3-x)} \geqslant 0$

7. Solve for x:

[4] (a) $x^2(x^2-2) = 3$

[5] (b) $2 + \dfrac{39x+96}{(x-4)(x+3)} = \dfrac{-x}{x+3}$

[4] (c) $x + \sqrt{3-2x} = 0$

[3] **8.** Simplify: $\sqrt[5]{\sqrt[3]{x}\sqrt{x}}$
(Assume all variables are positive and leave only positive exponents in your answer.)

[3] **9.** State the domain in interval notation: $\dfrac{x-2}{\sqrt{x+5}} + \sqrt{3-x}$

[2] **10.** If \$200 is invested at 4% interest compounded monthly, what is the value after 2 years?
(Round your answer to the nearest cent.)

[2] **11.** If $\log_x(y) = 10$ and $\ln(y) = 3$, find the value of $\ln(x)$.

[6] **12.** Given $f(x) = \log_2(x+2) - 3$

(a) Evaluate $f(-1)$.

(b) State the equation of any asymptote of $f(x)$.

(c) Find the coordinates of all axis intercepts of f.

(d) Sketch $f(x)$. Label all the intercepts and asymptote(s) properly.

(e) Find $f^{-1}(x)$.

[4] **13.** Express in terms of the simplest possible logarithms.
$\ln\left(\dfrac{4x^3\sqrt{x-1}}{(x+1)^2e^x}\right)$

14. Solve for x. (Give exact values.)

[5] (a) $2(6)^{2x} = \left(\dfrac{1}{2}\right)^{x+1}$

[4] (b) $\log_3(x+3) - 1 = \log_3(x)$

[5] **15.** The terminal side of angle θ in standard position is in the *second quadrant* and $\sin\theta = \frac{1}{3}$.

(a) Give an exact value for $\tan\theta$.

(b) Give an exact value for $\sec\theta$.

(c) Use a calculator to find angle θ in the interval $[0°, 360°)$ accurate to two decimal places.

[2] **16.** Without a calculator, evaluate $\cot(120°)$. Show your work and give an exact value.

[3] **17.** Without a calculator, find all angles θ in the interval $[0, 2\pi)$ such that $\cos\theta = -\frac{\sqrt{3}}{2}$. Show your work.

[3] **18.** From the top of a 100m cliff a boat is spotted with an angle of depression of 10°. How far is the boat from the base of the cliff? (Round your answer to the nearest metre.)

[3] **19.** Simplify as much as possible: $(1 - \cos^2 x)(1 + \cot^2 x)$.

[3] **20.** Prove the identity: $\dfrac{\sec x - \cos x}{\sin x} = \tan x$

[3] **21.** Identify the amplitude, period, and sketch at least two cycles of the function $f(x) = -\sin(\pi x)$.

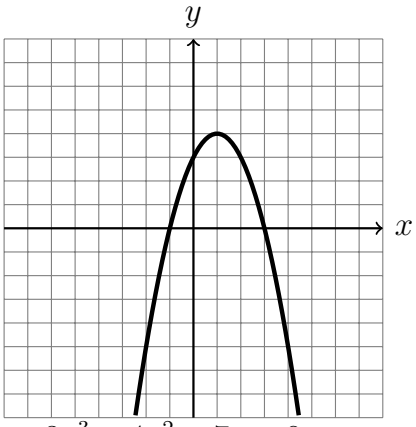
[3] **22.** A triangle has sides of length a, b and c across from angles of measure A, B, and C respectively. Suppose $a = 5, B = 70°$, and $C = 80°$. Find A, b and c. Give your answers rounded to two decimal places.

End of Exam (Answers on next page.)

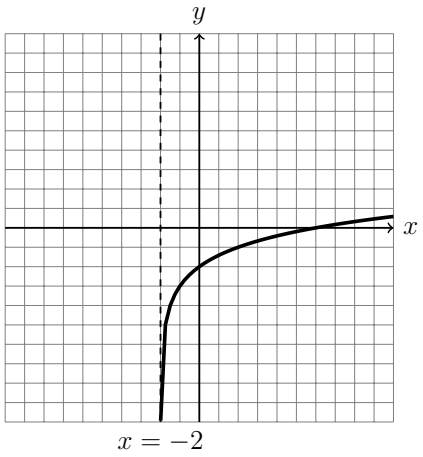
ANSWERS:

1.(a) $D = (-4, -3] \cup (-2, 4]$
1.(b) $R = (-4, -2] \cup [-1, 2) \cup (2, 4]$
1.(c) $(f \circ f)(0) = 4$
1.(d) $f^{-1}(-1) = 4$
1. (e) $f(x) = \begin{cases} 2x + 10 & \text{if } -4 < x \leqslant -3 \\ \frac{x}{2} - 3 & \text{if } -2 < x \leqslant 2 \\ -\frac{3}{2}x + 5 & \text{if } 2 < x \leqslant 4 \end{cases}$

2.(a) y-int: $(0, 3)$, x-int's: $(-1, 0)$ and $(3, 0)$.
2.(b) vertex: $(1, 4)$.
2.(c)

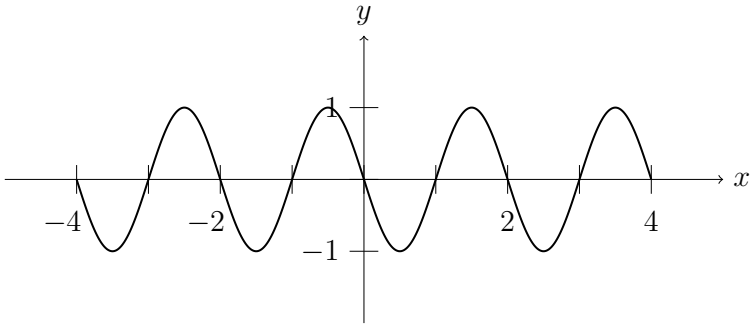


3. $\dfrac{3x^3 - 4x^2 + 7x - 8}{x - 2} = 3x^2 + 2x + 11 + \dfrac{14}{x - 2}$
4. $(f \cdot g)(x) = \dfrac{-(x + 1)(x^2 - 2x + 4)}{2x^2}$
5.(a) $f^{-1}(x) = \dfrac{2x}{x - 7}$
5.(b) $f\left(\frac{1}{x^2}\right) = \dfrac{7}{1 - 2x^2}$
6.(a) $(-\infty, -2) \cup (2, 3)$
6.(b) $(-\infty, \frac{2}{5}] \cup (2, 3)$
7.(a) $x = \pm\sqrt{3}$
7.(b) $x = -8$ (reject -3).
7.(c) $x = -3$ (reject 1).
8. $x^{1/6}$
9. $(-5, 3]$
10. \$216.63
11. $\ln x = \frac{3}{10}$
12.(a) $f(-1) = -3$
12.(b) V.A. at $x = -2$
12.(c) y-int: $(0, -2)$, x-int: $(6, 0)$.
12.(d)



12.(e) $f^{-1}(x) = 2^{(x+3)} - 2$
13. $\ln 4 + 3\ln x + \frac{1}{2}\ln(x - 1) - 2\ln(x + 1) - x$
14.(a) $x = \frac{\ln 4}{\ln(1/72)}$
14.(b) $x = \frac{3}{2}$
15.(a) $\tan\theta = -\dfrac{1}{2\sqrt{2}} = -\dfrac{\sqrt{2}}{4}$
15.(b) $\sec\theta = -\dfrac{3}{2\sqrt{2}} = -\dfrac{3\sqrt{2}}{4}$
15.(c) $\theta \approx 160.53°$
16. $\cot\theta = -\dfrac{1}{\sqrt{3}}$
17. $\theta = \dfrac{5\pi}{6}, \theta = \dfrac{7\pi}{6}$
18. 567m
19. $(1 - \cos^2 x)(1 + \cot^2 x) = 1$
20. $\dfrac{\sec x - \cos}{\sin x} = \dfrac{\frac{1}{\cos x} - \cos x}{\sin x}$
$= \dfrac{1 - \cos^2 x}{\sin x \cos x} = \dfrac{\sin^2 x}{\sin x \cos x}$
$= \dfrac{\sin x}{\cos x} = \tan x$

21. $A = 1, P = 2$.



22. $A = 30°$, $b \approx 9.40$, $c \approx 9.85$.