1. (30) Evaluate the following integrals.

(a) $\displaystyle\int (x+1)\sin(3x)\,dx$

(b) $\displaystyle\int (6x-1)\sqrt{3x+1}\,dx$

(c) $\displaystyle\int \cos^2(4x)\sin^3(4x)\,dx$

(d) $\displaystyle\int \frac{3x^2+10x+6}{x(x-3)(2x+1)}\,dx$

(e) $\displaystyle\int \frac{2}{x\sqrt{x^2+4}}\,dx$

(f) $\displaystyle\int_1^2 \frac{5}{\sqrt{3+2x-x^2}}\,dx$

2. (8) Evaluate the following limits.

(a) $\displaystyle\lim_{x\to 1^+}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$

(b) $\displaystyle\lim_{x\to\infty} x^{\frac{1}{e^x}}$

3. (5) Evaluate the following improper integral.
$$\int_2^\infty \frac{5}{(2x-3)^4}\,dx$$

4. (5) Find the volume of the solid whose base is the region enclosed by $y=1-x^2$ and x-axis, and the cross-sections perpendicular to the y-axis are squares.

5. (8) A radioactive isotope with an initial mass of 100 mg decays at a rate that is proportional to its mass. Five years later, its mass is 60 mg.

(a) Write a differential equation (with initial condition) that models this situation.

(b) Solve the differential equation to find an explicit expression for the remaining amount of isotope as a function of time.

(c) What is the amount of isotope remaining after 10 years?

6. (5) Solve the differential equation
$$\frac{dy}{dx}=y^2\,e^{-x}$$
given that $y=\frac{1}{2}$ when $x=0$. **Express y in terms of x.**

7. (4) Find the sum of the following series.
$$\sum_{n=0}^{\infty}\frac{2^n-3}{3^{2n+1}}$$

8. (6) Determine whether the sequence converges or diverges. If the sequence converges, find its limit; otherwise, explain why it diverges.

(a) $a_n = (-1)^n e^{\frac{n^2+1}{2n^2-1}}$

(b) $b_n = \dfrac{(\ln n)^2}{n}$

9. (12) Determine whether the series converges or diverges. Justify your answer and state the test that you use.

(a) $\displaystyle\sum_{n=2}^{\infty} \arctan\left(\frac{n}{n-1}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\pi^{2n}}{(n+1)!}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2+\sin n}{2n^3+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+5}}$

10. (7) Find the radius and interval of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n (x+1)^n}{2^n \,(n+1)}$$

11. (7)

(a) Find a power series representation for the function $f(x) = \ln(2-x)$ and determine the radius of convergence.

(b) Use your answer from part (a) to evaluate the indefinite integral as a power series.

$$\int x^2 \ln(2-x)\, dx$$

12. (3) Using known power series, find the Maclaurin series for the following function.

$$f(x) = xe^{x^2}$$

<u>ANSWERS</u>

1. (a) $-\dfrac{1}{3}(x+1)\cos(3x) + \dfrac{1}{9}\sin(3x) + C$

(b) $\dfrac{4}{15}(3x+1)^{\frac{5}{2}} - \dfrac{2}{3}(3x+1)^{\frac{3}{2}} + C$

(c) $\dfrac{1}{20}\cos^5(4x) - \dfrac{1}{12}\cos^3(4x) + C$

(d) $-2\ln|x| + 3\ln|x-3| + \dfrac{1}{2}\ln|2x+1| + C$

(e) $\ln \left| \dfrac{\sqrt{x^2 + 4} - 2}{x} \right| + C$

(f) $\dfrac{5\pi}{6}$

2. (a) $\dfrac{1}{2}$

(b) 1

3. converges to $\dfrac{5}{6}$

4. 2

5. (a) $\frac{dM}{dt} = -kM$ with $M(0) = 100$, $M(5) = 60$

(b) $M(t) = 100 \left(\dfrac{3}{5} \right)^{\frac{t}{5}}$

(c) 36mg

6. $y = \dfrac{1}{e^{-x} + 1}$

7. $-\dfrac{39}{56}$

8. (a) diverges, limit DNE

(b) converges to 0

9. (a) diverges (test for divergence)

(b) converges (ratio test)

(c) converges (direct comparison)

(d) converges (alternating series test)

10. R $= 2$, IoC $(-3, 1)$

11. (a) $\ln 2 - \displaystyle\sum_{n=1}^{\infty} \dfrac{x^n}{2^n \, n}$, $R = 2$

(b) $C + \dfrac{x^3 \ln 2}{3} - \displaystyle\sum_{n=1}^{\infty} \dfrac{x^{n+3}}{2^n \, n(n+3)}$

12. $\displaystyle\sum_{n=0}^{\infty} \dfrac{x^{2n+1}}{n!}$