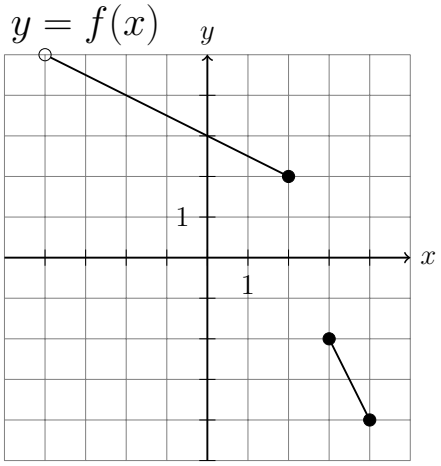
1. Let $y = f(x)$ be given by the graph:



[2] (a) State the domain of f.

[2] (b) State the range of f.

[2] (c) Evaluate: $(f \circ f)(3)$

[1] (d) Evaluate: $f^{-1}(3)$

[2] (e) From the graph, fill in the blanks to complete the piecewise definition for the function $f(x)$ below:

$$f(x) = \begin{cases} \_\_\_\_\_ & \text{if } \_\_\_\_\_ \\ -2x + 4 & \text{if } 3 \leqslant x \leqslant 4 \end{cases}$$

[3] **2.** Factor Completely: $x^5 + 2x^3 + 8x^2 + 16$

3. Solve for x:

[3] (a) $x^2 + 8x = 1$

[3] (b) $x^4 - x^2 - 20 = 0$

[4] **4.** Solve for x. (Express your answer in interval notation.)

$$\frac{(x-3)^2}{x(x+2)} > 0$$

[5] **5.** Given the quadratic function: $p(x) = 2x^2 - 8x + 6$

(a) find the coordinates of all intercepts;

(b) find the coordinates of the vertex;

(c) sketch a graph labeling the coordinates of the points in the previous two parts.

[3] **6.** Use long division to divide:
(Express your answer in the form $Q(x) + \frac{R(x)}{D(x)}$.)

$$\frac{x^3 + 4x^2 + 3}{x + 2}$$

[6] **7.** Given: $f(x) = \dfrac{21x}{x^2 - 2x - 15}$ and $g(x) = \dfrac{3x}{x - 5}$

(a) state the domain of $(f/g)(x)$;

(b) simplify $(f/g)(x)$;

(c) find a formula for $g^{-1}(x)$.

[4] **8.** Given: $f(x) = \dfrac{x}{x^2 - 5x + 4}$ and $g(x) = \dfrac{1}{x - 4}$

(a) state the domain of $(f - g)(x)$;

(b) simplify $(f - g)(x)$.

[4] **9.** Simplify: $\dfrac{\frac{2}{x-4} - \frac{2}{x-2}}{\frac{7}{x-4} - \frac{5}{x-2}}$

[4] **10.** Solve for x: $\sqrt{3x + 7} - \sqrt{x + 6} = 1$

[3] **11.** State the domain in interval notation: $f(x) = \sqrt{5 - x} + \dfrac{1}{\sqrt{3x + 2}}$

[2] **12.** Suppose \$8000 is invested at an annual interest rate of 4%. Find the value after 20 years if the interest is compounded monthly. (Round your answers to nearest cent.)

[2] **13.** Evaluate: $\log_b\left(\dfrac{1}{\sqrt[3]{b}}\right)$

[2] **14.** Use a calculator to evaluate: $\log_{11}(2)$
(Give three decimal places.)

[2] **15.** Simplify: $e^{(5\ln(x^2))}$

[4] **16.** Express as a single logarithm and simplify:
$\dfrac{1}{2}\ln(xy) - \dfrac{3}{2}\ln(yz) - \dfrac{5}{2}\ln(xz)$

[2] **17.** Given $f(x) = 1 - 2^{(1-x)}$, find $f^{-1}(x)$.

[5] **18.** Solve for x: $7 \cdot 2^{(x-1)} = 3^x$
(Express your answer in the form $x = \frac{\ln A}{\ln B}$)

[4] **19.** For the function $f(x) = -\log_2(1 - x) + 1$,

(a) find the the x and y-intercepts and one other point;

(b) find the equation of the asymptote;

(c) sketch the graph.

[2] **20.** Find all angles θ in the interval $[0°, 360°)$ for which $\sec(\theta)$ is undefined.

[3] **21.** (a) Sketch the angle $\theta = \dfrac{7\pi}{6}$ in standard position.

(b) Find the exact value of $\cos\left(\dfrac{7\pi}{6}\right)$.

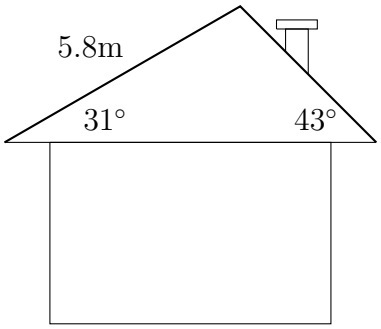
[3] **22.** Find all angles θ in the interval $[0°, 360°)$ that satisfy the equation: $\csc(\theta) = -4$.
(Round your answers to two decimal places.)

[3] **23.** Find all angles θ in the interval $[0, 2\pi)$ that satisfy the equation: $\cot(\theta) = -\sqrt{3}$.
(Give an exact value.)

[3] **24.** State the amplitude, period, and sketch at least two cycles of the function $f(x) = 3\cos(2x)$.

[4] **25.** Prove the identity $\dfrac{\sin(x)}{1+\cos(x)} = \csc(x) - \cot(x)$.

[3] **26.** The figure below shows a house with an asymmetric roof. The left side of the roof is 5.8m long and makes an angle 31° with the ceiling. The right side of the roof is at an angle 43° with the ceiling. How long is the right side of the roof? Answer in metres rounded to two decimal places.



[5] **27.** Consider a triangle with angles of measure $A = 50°$, B, and C, across from sides of length $a = 6$, $b = 7$, and c respectively.

(a) sketch all possible triangles with this data,

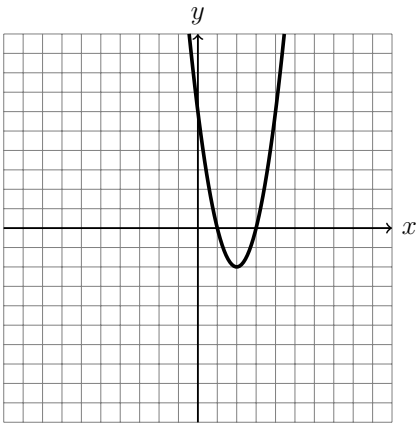
(b) find the angles B and C and the missing side length c for each of the sketched scenarios in part (a).

Round your answers to two decimal places.

End of Exam
Answers on next page.

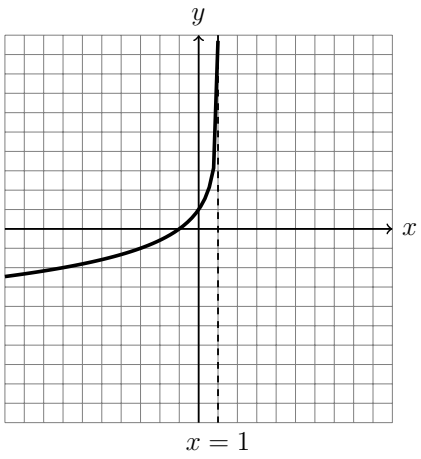
ANSWERS:

1.(a) $D = (-4, 2] \cup [3, 4]$
1.(b) $R = [-4, -2] \cup [2, 5)$
1.(c) 4
1.(d) 0
1.(e) $f(x) = \begin{cases} -\frac{1}{2}x + 3 \text{ if } -4 < x \leqslant 2 \\ -2x + 4 \text{ if } 3 \leqslant x \leqslant 4 \end{cases}$
2. $(x + 2)(x^2 - 2x + 4)(x^2 + 2)$
3.(a) $x = -4 \pm \sqrt{17}$
3.(b) $x = \pm\sqrt{5}$
4. $(-\infty, -2) \cup (0, 3) \cup (3, \infty)$
5.(a) y-int: $(0, 6)$, x-int's: $(1, 0)$ and $(3, 0)$.
5.(b) vertex: $(2, -2)$.
5.(c)

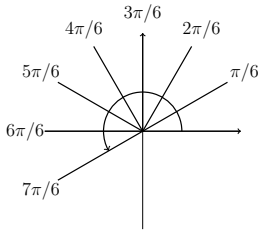


6. $x^2 + 2x - 4 + \frac{11}{x+2}$
7.(a) $D = \mathbb{R}\backslash\{-3, 0, 5\}$
7.(b) $\frac{7}{x+3}$
7.(c) $g^{-1}(x) = \frac{5x}{x-3}$
8.(a) $D = \mathbb{R}\backslash\{1, 4\}$
8.(b) $\frac{1}{(x-4)(x-1)}$
9. $\frac{2}{x+3}$
10. $x = 3$, (Reject -2)
11. $D = (-2/3, 5]$
12. \$17780.66
13. $-1/3$
14. 0.289
15. x^{10}
16. $\ln\left(\frac{1}{x^2yz^4}\right)$
17. $f^{-1}(x) = 1 - \log_2(1 - x)$
18. $x = \frac{\ln(7/2)}{\ln(3/2)}$

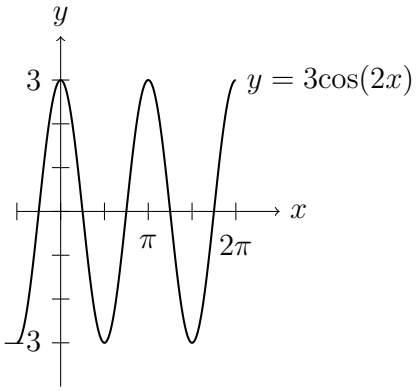
19.(a) y-int: $(0, 1)$, x-int's: $(-3, 1)$.
some other points $(-3, 1)$ or $(1/2, 2)$.
19.(b) Vertical asymptote at $x = 1$
19.(c)



20. $\theta = 90°, 270°$
21.(a)



21.(b) $\cos(7\pi/6) = -\sqrt{3}/2$
22. $\theta = 194.48°, 345.52°$
23. $\theta = 5\pi/6, 11\pi/6$.
24. $A = 3$, $P = \pi$



25. $\csc x - \cot x = \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \frac{1-\cos x}{\sin x} \cdot \frac{1+\cos x}{1+\cos x}$
$= \frac{1-\cos^2 x}{\sin x(1+\cos x)} = \frac{\sin^2 x}{\sin x(1+\cos x)} = \frac{\sin x}{1+\cos x}$ ✓

26. 4.38m
27. Solution 1: $B \approx 63.34°, C \approx 66.66°, c \approx 7.19$.
Solution 2: $B \approx 116.66°, C \approx 13.34°, c \approx 1.81$.