(8) **1.** Evaluate the following limits.

(a) $\displaystyle\lim_{x\to\pi^-} \frac{2\sin x}{1-\cos x}$

(b) $\displaystyle\lim_{x\to\infty} \frac{x^2+x}{e^x+1}$

(c) $\displaystyle\lim_{x\to 0^+} (\sin x)(\ln x)$

(6) **2.** Find y'. **Do not simplify your answer.**

(a) $y = \arctan(\ln(2x)) - 3\tan(e^{3x})$

(b) $y = \sin^{-1}(x^2)\cos^3(5x) - \sqrt{7}$

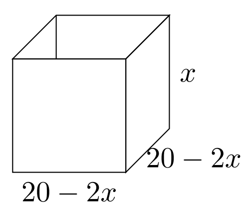
(c) $y = \sqrt{\sec^{-1}(\cot(2x))}$

(5) **3.** Use Newton's method to find the root of the function, $f(x) = x^4 + 2x^3 - 2x - 4$ that lies between $x \in [1, 2]$, accurate to **4 decimal places** starting with $x_1 = 1.3$.

(5) **4.** The volume and radius of a cylinder are increasing at a rate of 50π cm^3/s and 2 cm/s respectively. At what rate is the height of the cylinder changing $\dfrac{dh}{dt}$, when the volume is 36π cm^3 and the radius is 3 cm? (Recall: $V = \pi r^2 h$)

(5) **5.** An open box is to be made from a square piece of cardboard whose sides are 20.0 in long, by cutting equal squares from the corners and bending up the sides. Determine the size of the square that is to be cut out so that the volume of the box may be a maximum.

Figure 2: Box



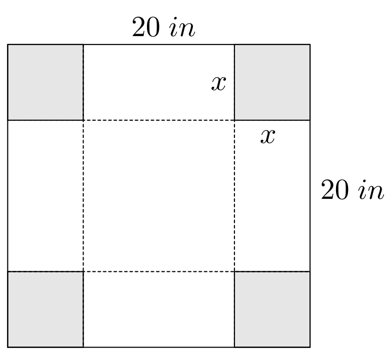


Figure 1: Square cardboard

6. Given $f(x) = \dfrac{4x-4}{x^2}$, $f'(x) = \dfrac{8-4x}{x^3}$, $f''(x) = \dfrac{8x-24}{x^4}$.
Find (if any):

(1) (a) The x and y intercept(s).

(1) (b) The vertical and horizontal asymptotes.

(1) (c) The critical numbers.

(1) (d) Local (relative) extrema.

(2) (e) Intervals on which f is increasing or decreasing.

(2) (f) Intervals of upward or downward concavity.

(1) (g) The inflection points.

(3) (h) Sketch the graph of f.

(5) **7.** Use the trapezoidal rule with $N = 5$ to estimate the value of $\displaystyle\int_0^{10} \sqrt{x^2+1}\,dx$ (give your answer to 4 decimals).

(4) **8.** During each cycle, the velocity v (in mm/s) of a piston is $v = 6t - 6t^2$, where t is the time (in secondes).Find the displacement S of the piston after $0.75\ s$ if the initial displacement is zero.

(4) **9.** Find the area of the region $\mathcal{R}$ between the two graphs, $y = 3x - x^2$ and $y = x$.

(5) **10.** Set up (your integral must include the limits of integration and the integrand) **but do not evaluate** the integral to find the volume generated by revolving the region bounded by $y = 3x - x^2, y = x$ about

(a) x-axis

(b) $y = -2$

(30) **11.** Integrate the following integrals.

(a) $\int (3x^5 + \frac{1}{x} - 6e^{3x} + 4)\, dx$

(b) $\int \frac{3}{x\sqrt{1 + \ln x}}\, dx$

(c) $\int \cos^2 x \sin^3 x\, dx$

(d) $\int \frac{\sqrt{16 - x^2}}{x^2}\, dx$

(e) $\int \frac{2x^2 - 4x + 14}{(x+1)(x^2+9)}\, dx$

(f) $\int_0^1 \arctan(x)\, dx$

(2) **12.** Determine whether $y = e^{-x^2}$ is a solution of the differential equation $\frac{dy}{dx} + 2xy = 0$.

(4) **13.** Solve the differential equation $\frac{dp}{dx} = \sqrt{\frac{p}{x}}$ for p.

(5) **14.** Find the particular solution of the differential equation $xdy = y \ln y dx$ if $y = e$ when $x = 2$.

(5) **15.** Solve the following first order linear differential equation for y, $x^2 dy - 3y dx = -3dx$ with initial condition $y(1) = 2$.

(5) **16.** Given the function

$$f(x) = \begin{cases} -1, & \text{if } -\pi \leq x < 0 \\ 1, & \text{if } 0 \leq x < \pi \end{cases}$$

(a) Show that a_0 and a_1 are equal to zero.

(b) Find the first two non-zero coefficients $(b_1, a_2, b_2 ...)$ of the Fourier series for the function above and write the functions expansion.

ANSWERS

1. (a) 0 (b) 0 (c) 0

2. (a) $y' = \dfrac{1}{1+\ln^2(2x)} \cdot \dfrac{2}{2x} - 3\sec^2(e^{3x}) \cdot e^{3x} \cdot 3$

(b) $y' = \dfrac{1}{\sqrt{1-x^4}} \cdot 2x\cos^3(5x) + \sin^{-1}(x^2)3\cos^2(5x) \cdot (-\sin(5x) \cdot 5$

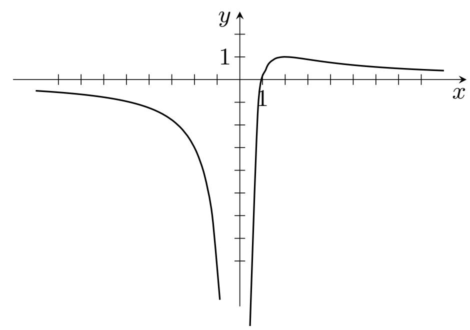
(c) $y' = \dfrac{1}{2\sqrt{\sec^{-1}(\cot(2x))}} \cdot \dfrac{1}{\cot(2x)\sqrt{\cot^2(2x)-1}} \cdot (-\csc^2(2x)) \cdot 2$

3. 1.2599

4. $\frac{2}{9}$ cm/s

5. $\frac{10}{3}$ in by $\frac{10}{3}$ in

6.



(a) $(1, 0)$, no y intercept.

(b) V.A $x = 0$, H.A $y = 0$

(c) $x = 2$

(d) Local max $y = 1$ at $x = 2$, no local min.

(e) incr. $(0, 2)$, decr. $(-\infty, 0)$ and $(2, \infty)$

(f) concavity up $(3, \infty)$, concavity down $(-\infty, 0)$ and $(0, 3)$

(g) I.P $(3, \frac{8}{9})$

(h)

7. 52.0583

8. 0.84375 mm

9. $\dfrac{4}{3}$

10. (a) $V = \displaystyle\int_0^2 [\pi(3x - x^2)^2 - \pi x^2]\, dx$

(b) $V = \displaystyle\int_0^2 [\pi(3x - x^2 + 2)^2 - \pi(x + 2)^2]\, dx$

11. (a) $\dfrac{x^6}{2} + \ln|x| - 2e^{3x} + 4x + C$

(b) $6\sqrt{1 + \ln x} + C$

(c) $\dfrac{1}{5}\cos^5 x - \dfrac{1}{3}\cos^3 x + C$

(d) $\dfrac{\sqrt{16 - x^2}}{x} - \sin^{-1}\left(\dfrac{x}{4}\right) + C$

(e) $2\ln|x + 1| - \dfrac{4}{3}\arctan\left(\dfrac{x}{3}\right) + C$

(f) $\dfrac{\pi}{4} - \dfrac{1}{2}\ln 2$

12. $y = e^{-x^2}$ is a solution of $\dfrac{dy}{dx} + 2xy = 0$.

13. $p = (\sqrt{x} + C)^2$

14. $y = e^{\frac{1}{2}x}$

15. $y = 1 + e^{(3-\frac{3}{x})}$

16. (a) $a_0 = \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{0} -1\ dx + \dfrac{1}{2\pi}\int_{0}^{\pi}\ dx = 0,\qquad a_1 = \dfrac{1}{\pi}\int_{-\pi}^{0} -\cos x\ dx + \dfrac{1}{\pi}\int_{0}^{\pi} \cos x\ dx = 0$

(b) $b_1 = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{0} -\sin x\ dx + \dfrac{1}{\pi}\int_{0}^{\pi} \sin x\ dx = \dfrac{4}{\pi}$

$a_2 = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{0} -\cos(2x)\ dx + \dfrac{1}{\pi}\int_{0}^{\pi} \cos(2x)\ dx = 0$

$b_2 = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{0} -\sin(2x)\ dx + \dfrac{1}{\pi}\int_{0}^{\pi} \sin(2x)\ dx = 0$

$b_3 = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{0} -\sin(3x)\ dx + \dfrac{1}{\pi}\int_{0}^{\pi} \sin(3x)\ dx = \dfrac{4}{3\pi}$

$f(x) \approx \frac{4}{\pi}\sin x + \frac{4}{3\pi}\sin(3x) + ...$

 This is the end of the examination.