1. (6 points) Evaluate the following limits using l'Hospital's rule where appropriate.

   (a) $\lim\limits_{x\to\infty} \dfrac{4e^{3x}+2x^2}{6x-e^{3x}}$

   (b) $\lim\limits_{x\to\frac{\pi}{3}^-} \dfrac{\cos\left(x-\frac{\pi}{3}\right)+2x-7}{\cos\left(2x-\frac{2\pi}{3}\right)}$

   (c) $\lim\limits_{x\to-\infty} \dfrac{2+e^{-2x-1}}{5+e^{-3x}}$

2. (2 points) Find the general term $a_n$ for the given sequence: $\left\{-\dfrac{5}{6},\ \dfrac{8}{18},\ -\dfrac{11}{54},\ \dfrac{14}{162},\ \ldots\right\}$

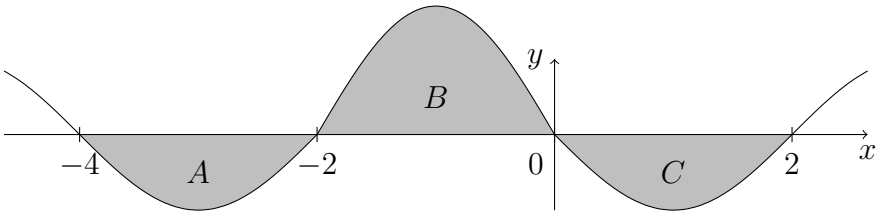
3. (4 points) Determine whether each sequence converges or diverges. If the sequence converges, find its limit.

   (a) $\left\{\dfrac{\sqrt{16n^4+5n+1}}{1-5n^2}\right\}$

   (b) $\left\{\dfrac{(-1)^n\cdot n}{2n+1}\right\}$

4. (3 points) Anastasia is working on an essay. On the first day, she types 300 words, and each day after that, she increases her daily word count by 45 words. After 21 days, what is the total number of words Anastasia has typed?

5. (3 points) Regions $A$, $B$, and $C$ bounded by the graph of $f$ and the $x$-axis have the following areas: region $A$ has an area of 2, region $B$ has an area of 3, and region $C$ has an area of 2.




   Use the graph of $f$ and the given area of each region to find the value of

   (a) $\displaystyle\int_{-4}^{0} f(x)dx$ (b) $\displaystyle\int_{-4}^{2} |f(x)|dx$ (c) $\displaystyle\int_{-4}^{2} (2f(x)-3x)\ dx$

6. (6 points) Evaluate the integral $\displaystyle\int_0^2 (2-3x+5x^2)\ dx$ using Riemann sums.
   You may make use of the formulas below.

$$\sum_{k=1}^{n} c = c\cdot n, \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

7. (5 points) Find the area enclosed by the curves of:

$$f(x) = -6x^2+5x \quad \text{and} \quad g(x) = x^3-4x^2-3x.$$

8. (25 points) Evaluate each integral.

   (a) $\displaystyle\int \left(\frac{7x^2 - 9x^3e^x + 5}{x^3} - 8^x\right) dx$

   (b) $\displaystyle\int \frac{\ln(x) + 1}{2x} dx$

   (c) $\displaystyle\int \frac{\ln x}{x^2} dx$

   (d) $\displaystyle\int (3x + 2)\cos(5x) dx$

   (e) $\displaystyle\int_0^4 \frac{3x}{\sqrt{x^2 + 9}} dx$

9. (10 points) Determine whether the following improper integrals converge or diverge. If an integral converges, find its value.

   (a) $\displaystyle\int_{-\infty}^{0} \frac{e^{3x}}{\sqrt{4 + e^{3x}}}\, dx$

   (b) $\displaystyle\int_0^{\pi/2} \frac{3\cos(x)}{[\sin(x) - 1]^2}\, dx$

10. (5 points) Given the demand function $p = -x^2 + 1000$ and the supply function $p = 5x + 250$, find the equilibrium point and the consumer surplus.

11. (5 points) In a study, it was found that the Lorenz curve for the distribution of income of financial planners was described by the function $f(x) = \frac{4}{7}x^2 + \frac{3}{7}x$ and that of accountants by the function $g(x) = \frac{3}{10}x^3 + \frac{7}{10}x^2$.

    (a) Compute the coefficient of inequality for each Lorenz curve.

    (b) Which profession has a more equitable income distribution? Briefly justify your answer.

12. (3 points) Given $f''(x) = \frac{3}{\sqrt{x}} + 2,\ f'(4) = 0,\ f(0) = 7$, find $f(x)$.

13. (5 points) Solve the initial value problem: $\displaystyle\frac{dy}{dx} = \frac{\sqrt{x}}{4y}, \quad y(9) = 4.$

14. (6 points) You should not have invested in your friend's company. The initial value of your investment was \$200. It decreases at a rate proportional to the square of its current value. After two years it's down to \$40. What's the value of your investment after a total of 12 years?

15. (4 points) After it leaves the warehouse, the time until a package is delivered is given by the probability density function $f(x) = \frac{1}{24}e^{\frac{-x}{24}} \quad 0 \le x < \infty$ where $x$ is the time in hours until the package is delivered. What is the probability that it takes more than 36 hours to deliver a randomly selected package?

16. (3 points) In India, the yearly total monsoon rainfall is approximately normally distributed with a mean of 852mm and a standard deviation of 82mm. Find the probability that a year's total monsoon rainfall is more than 977mm.

17. (5 points) Given that $f(x) = x^{-\frac{3}{2}}$ is a probability density function for the random variable X on the interval $[1, 4]$, find the following:

    (a) The mean of $X$.

    (b) The variance of $X$.

    (c) The standard deviation of $X$.

 Question 17 continues on the next page.

Answers:

1. (a) $-4$
   (b) $\frac{2\pi}{3} - 6$
   (c) 0
2. $\dfrac{3n+2}{-6(-3)^{n-1}}$
3. (a) $-\frac{4}{5}$
   (b) Diverges
4. 15750 words
5. (a) 1
   (b) 7
   (c) 16
6. $\frac{34}{3}$
7. $\frac{148}{3}$
8. (a) $7\ln|x| - 9e^x + \frac{5x^{-2}}{-2} - \frac{8^x}{\ln 8}$+C
   (b) $\frac{1}{4}\left[\ln x + 1\right]^2 + C$
   (c) $-\frac{1}{x}\ln x - \frac{1}{x} + C$
   (d) $\frac{1}{5}(3x+2)\sin(5x) + \frac{3}{25}\cos(5x) + C$
   (e) 6
9. (a) $\frac{2}{3}\sqrt{5} - \frac{4}{3}$ Converges
   (b) Diverges
10. Equilibrium point (25,375), CS= $10,416.67$
11. (a) $L_f \approx 0.19$, $L_g \approx 0.38$
    (b) The financial planners because their coefficient of inequality is closer to 0.
12. $f(x) = 4x^{\frac{3}{2}} + x^2 - 20x + 7$
13. $y = \sqrt{\frac{1}{3}x^{\frac{3}{2}} + 7}$
14. $V = \frac{200}{2t+1}$, $V(12) = \$8$
15. $P(x > 36) \approx 0.223$
16. $P(x > 977) = 0.0643$
17. (a) $\mu = 2$
    (b) Var(X)=$\frac{2}{3}$
    (c) $\sigma = \sqrt{\frac{2}{3}}$