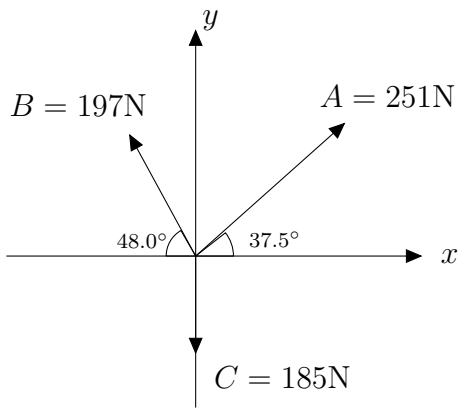
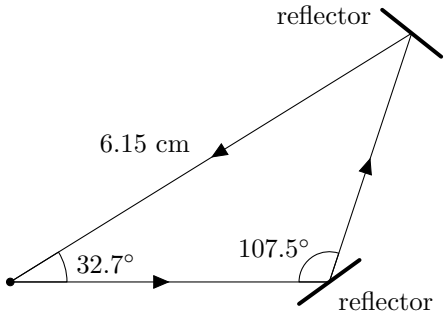
(4) 1. A medical supply company has 1150 workers-hours for production (P), maintenance (M), and inspection (I). Using this and other factors, the number of hours used for each operation, P, M and I, is found by solving the following system of equations. **Use Cramer's Rule to find P only**.

$$\begin{aligned} P + M + I &= 1150 \\ P - 4I &= -100 \\ P - 6M &= 50 \end{aligned}$$

(6) 2. Find the magnitude and direction of the resultant vector obtained by adding vectors A, B and C. (Round Appropriately)



(5) 3. Find the total length of the path of the laser beam shown in the picture. (Round appropriately)



(6) 4. The pressure P (in pascal Pa) in a pipe varies over time. Four times an hour, the pressure oscillates from a low of 60 to a high of 210 and then back to a low of 60. The pressure in the pipe at time $t = 0$ is 60 Pa. Let $P(t)$ denote the pressure in pipe at time t **minutes**.

(a) What is the frequency of the function $P(t)$?

(b) What is the period of the function $P(t)$?

(c) What is the amplitude of the function $P(t)$?

(d) What is the mid-line of the function $P(t)$?

(e) Find a possible equation for the function $P(t)$.

(6) 5. Solve the following equations for x such that $0 \leq x < 2\pi$:

(a) $\tan^2(x) + 3 = 2\sec^2(x)$

(b) $\cos(2x) - \cos(x) = 0$

(6) 6. Find all the possible solutions of the following equations:

(a) $3^{2x-7} = (27)^{3x}$

(b) $\log_3(x) + \log_3(x-8) = 2$

(3) 7. Evaluate the following and write your answer in **polar form** $(r\angle\theta)$.

(a) $\dfrac{(16\angle 45°)(5\angle 207°)}{10\angle 55°}$

(b) $(\sqrt{2}\angle 16°)^{10}$

(9) 8. Evaluate the following, and give your answer in **rectangular form** $(x + yj)$.

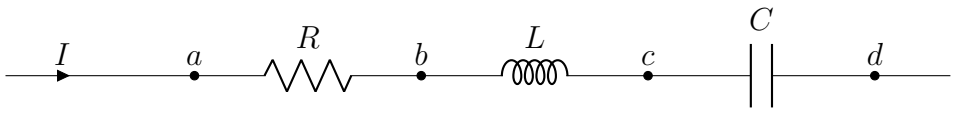
(a) $(5j^{17} + 3j^{52})(2j^{11} - 5j^{16})$

(b) $\dfrac{1+2j}{3-4j}$

(c) $(1 - \sqrt{3}j)^8$

(4) 9. Find all 3 cube roots of $-1 - j$. Give your answers in **polar form** $(r\angle\theta)$.

(7) 10. Consider the following electrical circuit:



- The current is $I = 7.00$ A (with a frequency of 75.0Hz).
- The reactance of the inductor is $X_L = 12.0\ \Omega$.
- The resistance is $R = 20.0\ \Omega$.
- The capacitance is $C = 50.0\ \mu$F.

Determine the following (Round your answers appropriately.):

(a) the reactance of the capacitor X_C,

(b) the impedance Z,

(c) the magnitude of the voltage across the RLC combination (between points a and d).

(d) Does the voltage lead or lag the current, and by what angle?

(5) 11. For the function f given in the diagram below, find each of the following, indicating *DNE* or ∞ or $-\infty$ or *undefined*, as appropriate.

$\lim_{x\to -1^+} f(x) =$

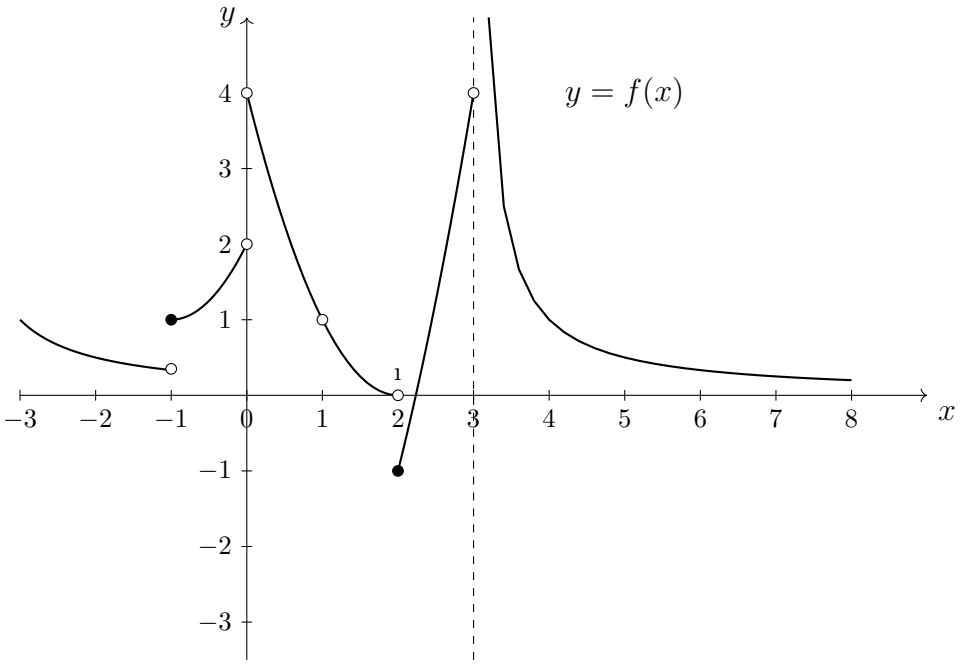
$\lim_{x\to 0} f(x) =$

$\lim_{x\to 2^-} f(x) =$

$\lim_{x\to 3^+} f(x) =$

$\lim_{x\to 1} f(x) =$

$f(1) =$



x-values of points of discontinuity are: _____

(11) 12. Evaluate the following limits:

(a) $\lim_{x\to 5} \dfrac{x^2-25}{2x^2-9x-5}$

(b) $\lim_{x\to 3} \dfrac{x^3-27}{x^2-9}$

(c) $\lim_{x\to -\infty} \dfrac{\sqrt{49x^6+4x^4+x}}{9x^3+4x+1}$

(d) $\lim_{x\to 1^-} \dfrac{x^2+27}{x-1}$

(4) 13. Find the derivative of $f(x) = \dfrac{x}{x-3}$ **using only the limit definition of the derivative**.

(15) 14. Find the derivative of the following functions. **Do not simplify your answers**.

(a) $y = x^6 + 6^x + \dfrac{3}{\sqrt[5]{x}} + \log_5 x + e^{\pi}$

(b) $y = e^{3x^5+2x^2}\tan(x^3+x+5)$

(c) $y = \dfrac{\sec(x)+\cot(x+1)}{1+\cos(3x^5+8)}$

(d) $y = (1+\sin^5(x^6+1))^9$

(e) $y = \ln\left(\dfrac{e^{3x}(x^5+2x^3+6)^9}{(x+5)^8\sqrt[7]{x-8}}\right)$ (**Simplify first using properties of the logarithm**.)

(5) 15. Consider the implicit equation $x^2y^3 - x^3y^2 = 12$.

(a) Find y' using implicit differentiation.

(b) Find an equation of the tangent line to the curve at $(-1, 2)$.

(4) 16. Let $f(x) = x^2\sqrt[3]{3x+1}$.

(a) Find $f'(x)$ and simplify your answer.

(b) Find any value(s) of x for which the tangent line is horizontal.

ANSWERS

1. $P = \dfrac{-27200}{-34} = 800$
2. $R = 133$ N and $\theta = 59.5°$
3. 13.76 cm
4. (a) $f = \dfrac{1}{900}$ Hz
   (b) $p = 15$ min
   (c) $|a| = 75$ Pa
   (d) $y = 135$
   (e) $P(t) = 75\sin\left(\dfrac{2\pi t}{15} + \dfrac{3\pi}{2}\right) + 135$
5. (a) $x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$
   (b) $x = 0, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$
6. (a) $x = -1$
   (b) $x = 9$
7. (a) $8\angle 197°$
   (b) $32\angle 160°$
8. (a) $-5 - 31j$
   (b) $-\dfrac{1}{5} + \dfrac{2}{5}j$
   (c) $128 - 128\sqrt{3}\,j$
9. $2^{1/6}\angle\dfrac{5\pi}{12}, \; 2^{1/6}\angle\dfrac{13\pi}{12}, \; 2^{1/6}\angle\dfrac{21\pi}{12}$
10. (a) $X_C = 42.4\ \Omega$
    (b) $z = 20.0 - 30.4j$
    (c) $|V_{RLC}| = 255$ V
    (d) $\theta = -56.7°$ (Voltage lags current by 56.7°.)

11. $\lim_{x\to -1^+} f(x) = 1$ $\lim_{x\to 0} f(x) = \text{DNE}$ $\lim_{x\to 2^-} f(x) = 0$
$\lim_{x\to 3^+} f(x) = \infty$ $\lim_{x\to 1} f(x) = 1$ $f(1) = \text{und}$
x-values of points of discontinuity are: $-1, 0, 1, 2, 3$

12. (a) $\dfrac{10}{11}$
(b) $\dfrac{9}{2}$
(c) $-\dfrac{7}{9}$
(d) $-\infty$

13. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\dfrac{x+h}{x+h-3} - \dfrac{x}{x-3}}{h} = \cdots = \dfrac{-3}{(x+h-3)(x-3)} \qquad (h \neq 0)$

Therefore the limit as h approaches zero is $f'(x) = \dfrac{-3}{(x-3)^2}$

14. (a) $y' = 6x^5 + 6^x \ln 6 - \dfrac{3}{5}x^{-6/5} + \dfrac{1}{x \ln 5}$
(b) $y' = e^{3x^5+2x^2}(15x^4+4x)\tan(x^3+x+5) + e^{3x^5+2x^2}\sec^2(x^3+x+5)(3x^2+1)$
(c) $y' = \dfrac{(\sec x \tan x - \csc^2(x+1))(1+\cos(3x^5+8)) - (\sec x + \cot(x+1))(-15x^4\sin(3x^5+8))}{(1+\cos(3x^5+8))^2}$
(d) $y' = 270x^5(1+\sin^5(x^6+1))^8 \sin^4(x^6+1)\cos(x^6+1)$
(e) $y' = 3 + \dfrac{9(5x^4+6x^2)}{x^5+2x^3+6} - \dfrac{8}{x+5} - \dfrac{1}{7(x-8)}$

15. (a) $y' = \dfrac{3x^2y^2 - 2xy^3}{3x^2y^2 - 2x^3y}$
(b) $y = \dfrac{7}{4}\,x + \dfrac{15}{4}$

16. (a) $y' = \dfrac{7x^2+2x}{(3x+1)^{2/3}}$
(b) The tangent line is horizontal at $x = -\dfrac{2}{7}$ and $x = 0$.