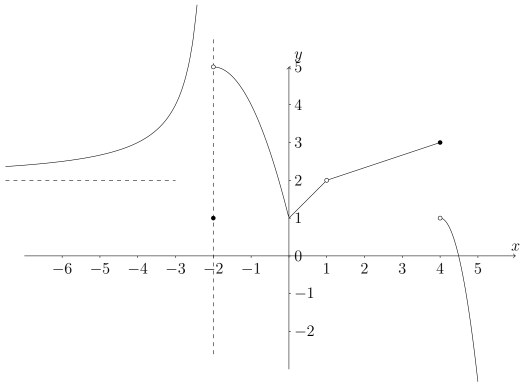
1. (7 points) Given the graph of the function $f(x)$ below, answer the following questions. Use $\infty, -\infty$, or dne as appropriate.



(a) $\lim_{x\to-\infty} f(x) =$

(b) $\lim_{x\to-2^-} f(x) =$

(c) $\lim_{x\to-2^+} f(x) =$

(d) $f(-2) =$

(e) $\lim_{x\to 1} f(x) =$

(f) $f(4) =$

(g) $\lim_{x\to 4^+} f(x) =$

(h) $\lim_{h\to 0} \dfrac{f(3+h)-f(3)}{h} =$

(i) List the $x-$value(s) of the discontinuities of $f(x)$ and justify.

(j) List the $x-$values(s) at which the function is continuous but not differentiable. Explain your answer.

2. (18 points) Evaluate the following limits:

(a) $\lim_{x\to 3} \dfrac{4x^2-15x+9}{x^2+2x-15}$

(b) $\lim_{x\to -3^-} \dfrac{4x+12}{|6+2x|}$

(c) $\lim_{x\to 2} \dfrac{x^2-4}{x-\sqrt{3x-2}}$

(d) $\lim_{x\to 4} \dfrac{\frac{x+1}{x-2}-\frac{10}{x}}{x^2-x-12}$

(e) $\lim_{x\to\infty} \dfrac{(2x+4)^2(4-x^2)}{(2-x)^3(1-3x)}$

(f) $\lim_{x\to 5^-} \dfrac{x^2+25}{x-5}$

3. (5 points) Given the function below, find the x-value(s) where the function is discontinuous. Justify your answers using the definition of continuity.

$$f(x) = \begin{cases} \frac{1}{x^2-4} & : x \le 1 \\ \frac{x-3}{x^2+5x-24} & : 1 < x < 5 \\ \frac{1}{2x+3} & : x \ge 5 \end{cases}$$

4. (3 points) Find the values of k for which $f(x)$ is continuous on $\mathbb{R}$.

$$f(x) = \begin{cases} 2x^2+6k+1 & : x \le 2 \\ k^2x+13 & : x > 2 \end{cases}$$

5. (2 points) **State True or False and briefly explain:**
If a function f is defined at $x = a$, then $\lim_{x \to a} f(x)$ exists.
Explain your answer, and feel free to use a graph if needed.

6. (4 points) Let $f(x) = 3x^2 - 2$

(a) Use the limit definition of the derivative to find the derivative of f.

(b) Find the equation of the tangent line to $f(x) = 3x^2 - 2$ at $x = 2$

7. (17 points) Find $\dfrac{dy}{dx}$ for each of the following. Do not simplify your answers.

(a) $y = e^x - 5^x + \sqrt[3]{x^2} + \sec(x) - \log_3 x + 5\pi^2$

(b) $y = \dfrac{x^2 + e^{\cos(x)}}{(3x^4 + 4)^2}$

(c) $y = \cot\left(5\sqrt[3]{x^2 + 4}\right)$

(d) $y = (\csc x)^{\ln x}$

(e) $(x^3 + y)^6 = x^2 y - 9$

8. (4 points) For $y = \ln\left[\dfrac{(2x+5)^3 e^{5x}}{\tan^2 x}\right]$:

(a) Use the laws of logarithmic functions to completely simplify y.

(b) Find $\dfrac{dy}{dx}$.

9. (4 points) Given the equation $2x^2 y + \dfrac{x}{y} = 3x + 4y$,
find an equation of the tangent line to the curve at the point $(2, 1)$.

10. (4 points) Find the x points at which the tangent line to the graph of $f(x) = (x^2 + 4)^2(\frac{1}{2}x - 3)^4$ is horizontal.

11. (3 points) Given $y = \sin(2x + 3) + e^{3x-1} + x^{15}$, find the 93$^{\text{rd}}$ derivative of y.

12. (4 points) Find the absolute extrema of the function $g(x) = x - 6\sqrt[3]{x^2}$ on the interval $[-1, 1]$.

13. (11 points) Given $f(x) = \dfrac{-9(x+2)}{(x+3)^2}$ $\quad f'(x) = \dfrac{9(x+1)}{(x+3)^3}$ $\quad f''(x) = \dfrac{-18x}{(x+3)^4}$

(a) Find the domain of f,

(b) Find the x- and y-intercepts of f,

(c) Find any vertical and horizontal asymptotes of f,

(d) Find the intervals of increase and decrease of f,

(e) Find any local extrema of f,

(f) Find the intervals of concavity of f,

(g) Find any points of inflection of f,

 Question 13 continues on the next page.

(h) Use your answers from the previous parts to sketch a graph of f on the grid below. Choose the scale of your axes carefully. Show all relevant information in the graph.

14. (5 points) A local gym sells discounted monthly gym memberships to John Abbott students. From experience, they know that 20 John Abbott students will purchase a monthly membership if the price is \$70 per month. Each time they lower the membership by \$5, 10 more John Abbott students buy a monthly membership.

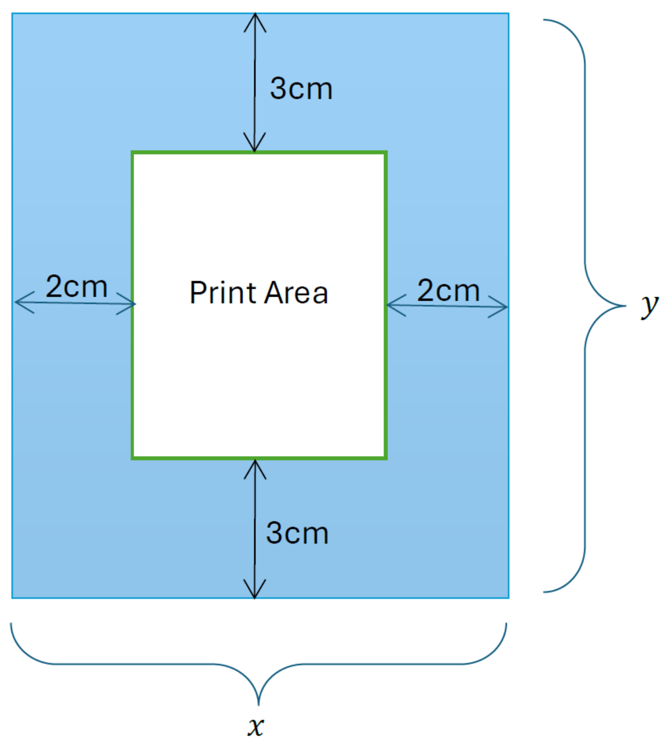
(a) What price should the gym charge for a monthly membership in order to maximize the revenue from these memberships?

(b) What is the maximum revenue obtained from these monthly memberships?

15. (4 points) You are starting a lawn maintenance company to help cover your university tuition costs. To advertise your business, you plan to purchase a rectangular advertisement space in a magazine.

The magazine requires that the top and bottom margins (each 3 cm) and the side margins (each 2 cm) be left blank. The total area of the advertisement space, including these margins, is 216 cm^2.

What dimensions of the advertisement space (including the margins) will maximize the printing area?



16. (5 points) The demand function for a new phone cover is given by $x = \frac{1}{4}(225 - p^2)$ where x is the quantity demanded (measured in hundreds) per week and p is the unit price.

(a) Calculate the price elasticity of demand when the price $p = \$10$.

(b) To increase revenue, should the price be increased or decreased from \$10 per unit?

(c) At what price is the demand unitary?

(d) What price would maximize revenue? $p =$ __________

(e) What is the maximum revenue?

 Question 16 continues on the next page.

Answers:

1. (a) 2 (b) ∞ (c) 5 (d) 1 (e) 2 (f) 3 (g) 1 (h) $\frac{1}{3}$ (i) $x = -2, 1, 4$ (j) $x = 1$

2. (a) 8 (b) -2 (c) 16 (d) $-\frac{1}{56}$

3. f is discontinuous at $x = -2, 1, 3$

4. $k = 1, 2$

5. False since the one-sided limits could not be equal, and hence the two-sided limit would not exist.

6. (a) $f'(x) = 6x$
   (b) $y = 12x - 14$
   (c) $y' = -\frac{10}{3}x \csc\left(5\sqrt[3]{x^2+4}\right)(x^2+4)^{-\frac{2}{3}}$
   (d) $y' = (\csc x)^{\ln x}(\frac{1}{x}\ln(\csc x) - \ln x \cot x)$

7. (a) $y' = e^x - 5^x \ln 5 + \frac{2}{3}x^{-\frac{1}{3}} + \sec x \tan x - \frac{1}{x \ln 3}$
   (b) $y' = \frac{(2x - e^{\cos x}\sin x)(3x^4+4)^2 - 24x^3(x^2+e^{\cos x})(3x^4+4)}{(3x^4)+4)^4}$
   (e) $y' = \frac{2xy - 18x^2(x^3+y)^5}{6(x^3+y)^5 - x^2}$

8. $y' = \frac{6}{2x+5} + 5 - 2\frac{\sec^2 x}{\tan x}$

9. $y = \frac{7}{3}x - \frac{11}{3}$

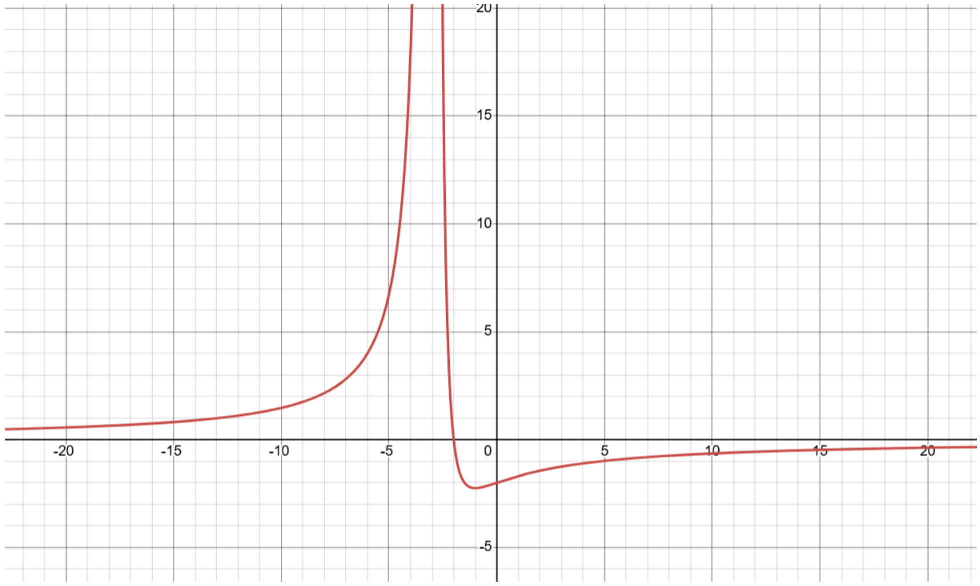
10. $x = 6, 2, 1$

11. $y^{(93)} = 2^{93}\cos(2x+3) + 3^{93}e^{3x-1}$

12. Abs. min $f(-1) = -7$ and Abs. max $f(0) = 0$.

13. (a) $\mathbb{R} - \{-3\}$
    (b) $(0, -2), (-2, 0)$
    (c) Vertical asymptote $x = -3$ and horizontal asymptote $y = 0$
    (d) Inc $(-\infty, -3) \cup (-1, 0) \cup (0, \infty)$. Dec. $(-3, -1)$
    (e) Local min $f(-1) = -\frac{9}{4}$. No local max
    (f) I.P $(0, -2)$

 Question 16 continues on the next page.

(g) 

14. (a) The price = \$40
    (b) \$ 3200

15. $x = 12, y = 18$

16. (a) $E(p) = \frac{2p^2}{225-p^2}$. $E(10) = 1.6 > 1$ Elastic.
    (b) The price should be decreased slightly
    (c) $p = \sqrt{75} \approx \$8.66$
    (d) \$8.66
    (e) \$324.76