1. (7 points) Given the graph of f below, evaluate each of the following.
Use ∞, $-\infty$ or "does not exist" where appropriate.

(a) $\lim\limits_{x\to-\infty} f(x) =$

(b) $\lim\limits_{x\to-3} f(x) =$

(c) $\lim\limits_{x\to0^+} f(x) =$

(d) $\lim\limits_{x\to0} f(x) =$

(e) $\lim\limits_{x\to4} f(x) =$

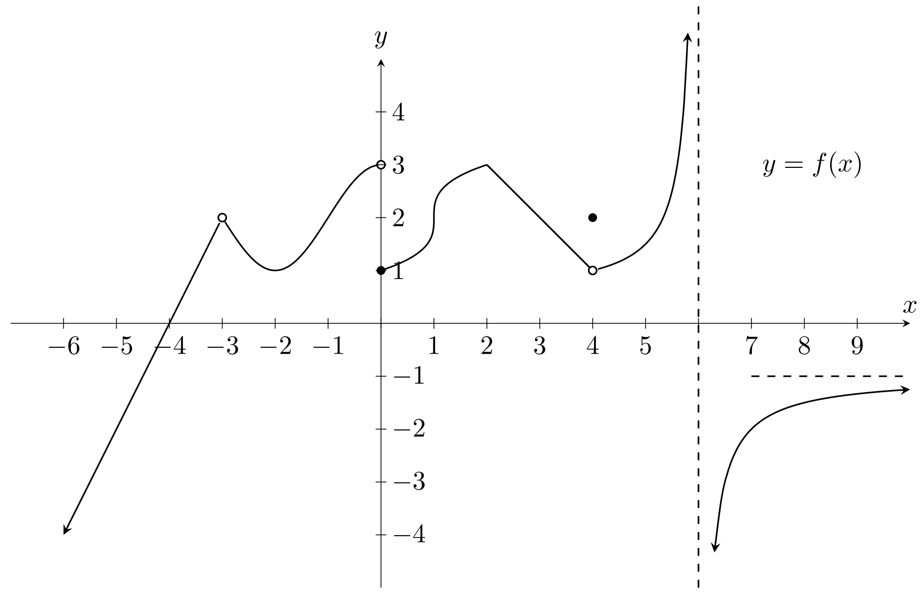
(f) $f(4) =$

(g) $\lim\limits_{x\to6^-} f(x) =$

(h) $f'(-2) =$

(i) $\lim\limits_{h\to0} \dfrac{f(3+h)-f(3)}{h} =$

(j) $\lim\limits_{x\to\infty} f(x) =$



(k) List all value(s) of x where f is discontinuous.

(l) List all value(s) of x where f is continuous but not differentiable.

2. (13 points) Evaluate the following limits.
Use ∞, $-\infty$ or "does not exist" where appropriate.

(a) $\lim\limits_{x\to2} \dfrac{\frac{1}{x+4}-\frac{1}{3x}}{2-x}$

(b) $\lim\limits_{x\to4^+} \dfrac{|4-x|}{3x^2-17x+20}$

(c) $\lim\limits_{x\to1} \dfrac{x-1}{\sqrt{x+3}-1}$

(d) $\lim\limits_{x\to-2^+} \dfrac{x-4}{x^2+7x+10}$

(e) $\lim\limits_{x\to\infty} \dfrac{2x(3x-1)(x+2)}{(5x^2+1)(4x+3)}$

3. (5 points) Given the function below, find the $x-$value(s) where the function is discontinuous. Justify your answers using the definition of continuity.

$$f(x) = \begin{cases} \dfrac{3x^2-5x-2}{x^2-7x+10} & \text{if} \quad x<3 \\ 1-\dfrac{2}{3}x^2 & \text{if} \quad 3\le x\le 5 \\ \dfrac{x}{x-6} & \text{if} \quad x>5 \end{cases}$$

4. (2 points) State if the following is True or False and explain your answer, using a graph to aid your explanation: *It is always true that, if* $\lim\limits_{x\to a} f(x) = 8$*, then* $f(a) = 8$.

5. (4 points) Find the value(s) of k so that the following function is continuous on $\mathbb{R}$. Justify your answer.

$$f(x) = \begin{cases} 4k+5x & \text{if} \quad x<2 \\ 2kx^2+k^2x+4 & \text{if} \quad x\ge 2 \end{cases}$$

6. (4 points) Use the limit definition of the derivative to find the derivative of $f(x) = \sqrt{1-2x}$.

7. (18 points) For each of the following, find y'. Do not simplify your answers.

(a) $y = 2^{x^3-3x} + e^{2\pi} - 4\dfrac{2}{\sqrt[3]{x}}$

(b) $y = \log_3(2x^2-1) + \csc^3(x)$

(c) $y = \cos(\ln(\tan(x)))$

(d) $y = (5\sin x)^{2x^2+2}$

(e) $\cot(xy) = x^2 + y^2$

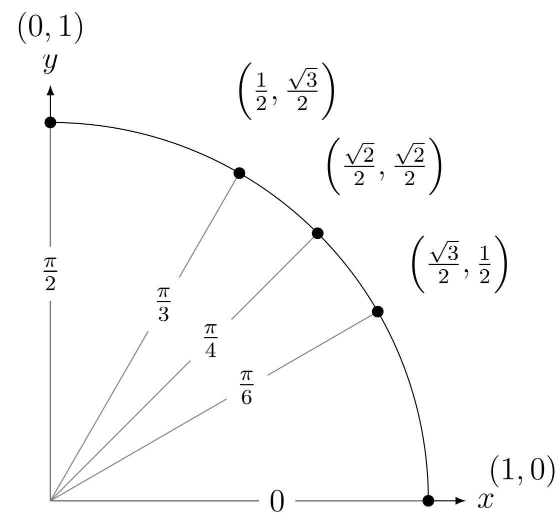
8. (4 points) Given $f(x) = (2x+1)^3 e^x$, find all the values of x where the function has a horizontal tangent.

9. (4 points) Use logarithmic differentiation to find the derivative of $y = \dfrac{\sec^4(x)\ (4x^2+3)^6}{3^{2x-5}\ \sqrt[5]{x+1}}$.
Make sure to simplify as much as possible using the laws of logarithms before differentiating.

10. (3 points) Given $f(x) = \sin(3x-5) + 2^{5x-4} + x^7$, find the 99^{th} derivative.

11. (3 points) Given the curve $(x+y)^4 - e^{xy+1} = 2x - 3$, find the slope of the tangent line to the graph at the point $(1, -1)$.

12. (4 points) Given $f(x) = 2\csc(x) + \tan\left(\dfrac{x}{2}\right)$, find an equation of the tangent line to the curve $y = f(x)$ at $x = \dfrac{\pi}{3}$. No decimals, give exact values.
This may be useful:



13. (4 points) Find the absolute extrema of $f(x) = \dfrac{x+1}{x^2+3}$ on the interval $[0, 3]$.

14. (10 points) Given $f(x) = \dfrac{8x}{(x+1)^2}$, with $f'(x) = \dfrac{8(1-x)}{(x+1)^3}$ and $f''(x) = \dfrac{16(x-2)}{(x+1)^4}$, determine the following, then neatly sketch the graph of f.

(a) The domain of f.

(b) The $x-$ and $y-$intercepts, if any.

(c) All vertical and horizontal asymptotes, if any.

(d) The intervals on which f is increasing and decreasing.

(e) All local maximums and minimums of f, if any.

(f) The intervals on which f is concave up and concave down.

(g) The inflection points of f, if any.

(h) Sketch the graph of f, choosing an appropriate scale, clearly labeling all important points and asymptotes.

15. (6 points) Joe Schmoe's lawn mowing business has determined that they have a price function of $p(x) = -\frac{0.01}{3}x^2 - 0.3x + 72 + \frac{100}{x}$, where x is the number of lawns mown per month $(0 < x \leq 100)$.

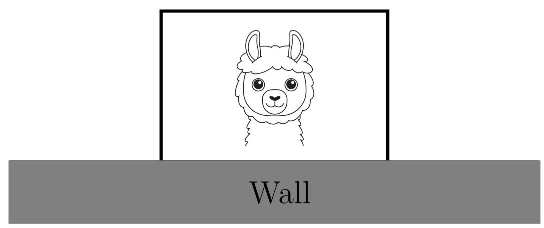
(a) Find the revenue function.

(b) What is the marginal revenue when $x = 40$ lawns mown? Carefully interpret in words your answer.

(c) How many lawns should Joe Schmoe mow per month in order to maximize revenue?

(d) What is the maximum monthly revenue?

16. (4 points) Oh no! Zariska's llamas keep getting out because their enclosure is too small. She has all the space she needs to build a new rectangular enclosure along a wall, using electric fencing, but she only has 200 metres of the fencing on hand. The side along the wall does not require fencing. What dimensions should her enclosure be in order to maximize area? What is the maximum area? Justify your answer.



17. (5 points) A ticket for the brand new Mathematical Museum of Horrors has the following demand function

$$x = (205 - 2p)^2 \qquad \text{for } 0 \leqslant p \leqslant 100.$$

(a) Calculate the price elasticity of demand when the price is $p = \$50$ per ticket. Is the demand elastic or inelastic?

(b) What would be the approximate percent change in demand if we decide to increase the ticket price by 2% ?

(c) Find the ticket price that would maximize revenue.

ANSWERS

1. (a) $-\infty$

(b) 2

(c) 1

(d) DNE

(e) 1

(f) 2

(g) ∞

(h) 0

(i) -1

(j) -1

(k) $x = -3, 0, 4, 6$

(l) $x = 1, 2$

2. (a) $-1/18$

(b) $1/7$

(c) 0

(d) $-\infty$

(e) $3/10$

3. $x = 2$ ($f(2)$ DNE), $x = 5$ ($\lim_{x\to 5} f(x)$ DNE), $x = 6$ ($f(6)$ DNE)

4. False (f might be undefined at $x = a$ or the value of the function might differ from the value of the limit)

5. $k = -3, 1$

6.
$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\sqrt{1-2(x+h)} - \sqrt{1-2x}}{h} \\ &= \lim_{h\to 0} \frac{\left(\sqrt{1-2(x+h)}\right)^2 - \left(\sqrt{1-2x}\right)^2}{h(\sqrt{1-2(x+h)} + \sqrt{1-2x})} \\ &= \lim_{h\to 0} \frac{1-2x-2h-1+2x}{h(\sqrt{1-2(x+h)} + \sqrt{1-2x})} \\ &= \lim_{h\to 0} \frac{-2h}{h(\sqrt{1-2(x+h)} + \sqrt{1-2x})} \\ &= \frac{-2}{2\sqrt{1-2x}} = \frac{-1}{\sqrt{1-2x}} \end{aligned}$$

7. (a) $y' = 2^{x^3-3x}\ln(2)(3x^2-3) + \dfrac{8}{3\sqrt[3]{x^4}}$

(b) $y' = \dfrac{4x}{(2x^2-1)\ln 3} - 3\csc^3(x)\cot(x)$

(c) $y' = -\sin\left(\ln(\tan(x))\right) \cdot \dfrac{1}{\tan x} \cdot \sec^2 x$

(d) $y' = (5\sin x)^{2x^2+2} \cdot (4x\ln(5\sin x) + (2x^2+2)\cot x)$

(e) $y' = \dfrac{2x + y\csc^2(xy)}{-x\csc^2(xy) - 2y}$

8. $x = -1/2, -7/2$

9. $\ln(y) = 4\ln(\sec x) + 6\ln(4x^2+3) - (2x-5)\ln 3 - \dfrac{1}{5}\ln(x+1)$ giving $y' = \left(\dfrac{\sec^4(x)\ (4x^2+3)^6}{(2x-3)^3\ \sqrt[5]{x+1}}\right) \cdot \left(4\tan x + \dfrac{48x}{4x^2+3} - 2\ln 3 - \dfrac{1}{5(x+1)}\right)$

10. $f^{(99)}(x) = -3^{99}\cos(3x-5) + 2^{5x-4}(5\ln 2)^{99}$

11. -1

12. $y = -\dfrac{2}{3}x + \dfrac{15\sqrt{3} + 2\pi}{9}$

13. abs min $(0, \frac{1}{3})$ and $(3, \frac{1}{3})$, abs max $(1, \frac{1}{2})$

14. (a) $(-\infty, -1) \cup (-1, \infty)$

(b) $(0, 0)$

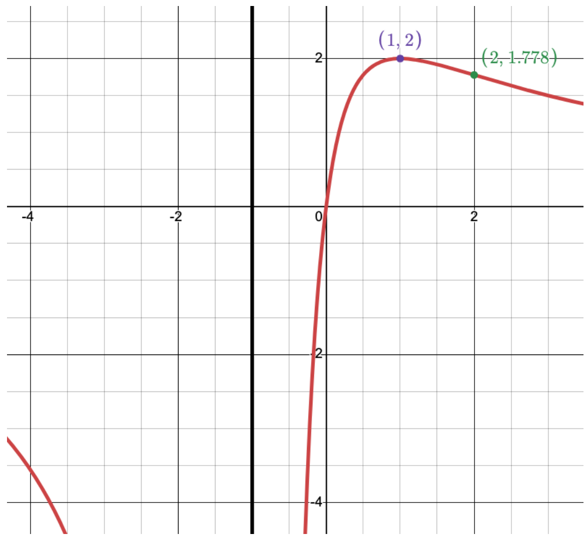
(c) VA $x = -1$, HA $y = 0$ on both sides

(d) increasing on $(-1, 1)$, decreasing on $(-\infty, -1) \cup (1, \infty)$

(e) local max $(1, 2)$

(f) CD on $(-\infty, -1) \cup (-1, 2)$, CU on $(2, \infty)$

(g) IP $(2, 1.78)$

(h) 

15. (a) $R(x) = -\dfrac{0.01}{3}x^3 - 0.3x^2 + 72x + 100$

(b) $R'(40) = 32$. The revenue from the 41st lawn mown is approximately \$32.

(c) 60

(d) \$2620

16. dimensions 50 by 100 metres, area 5000 m^2

17. (a) ≈ 1.90, demand is elastic

(b) demand would decrease by approximately 3.80%

(c) \$34.17