1. (15 points) Evaluate each of the following limits.

(a) $\lim\limits_{x\to 1} \dfrac{x-1}{\sqrt{4x+5}-3}$

(b) $\lim\limits_{x\to 4^+} \dfrac{|4-x|}{x^2-7x+12}$

(c) $\lim\limits_{x\to 0} \left(\dfrac{\sin x}{x} + \dfrac{\sin(x+\frac{\pi}{2})}{x+\frac{\pi}{2}}\right)$

(d) $\lim\limits_{x\to\infty} 2\arctan(3x)$

(e) $\lim\limits_{x\to\infty} \dfrac{e^x+\cos x}{e^x}$

2. (4 points) Find both horizontal asymptotes of the function $f(x) = \dfrac{4x-3}{\sqrt{25x^2+4x}}$.

3. (5 points) Use the limit definition of the derivative to find $f'(x)$, given that $f(x) = \dfrac{2}{3-5x}$

4. (4 points) The function $f(x) = \dfrac{|x-1|(x+1)}{x^2-1}$ has two discontinuities. Find them and identify their type (infinite, jump, or removable.)

5. (15 points) Find $\frac{dy}{dx}$ of each of the following. Do not simplify.

(a) $y = x^5 + 5^x + 5^5$

(b) $y = \dfrac{\sec x}{x^2+1}$

(c) $y = \ln\left(\sin(x^2)\right)$

(d) $y = \arctan\left(\sqrt{x}\right)$

(e) $y = (x+4)^{x+\cos x}$

6. (4 points) You are given the curve $y = \dfrac{x+1}{1-2x}$.

(a) Find the equation of the tangent line to this curve at $x = 1$.

(b) How many points are there on the curve that have a horizontal tangent line?

7. (4 points) Find both points on the curve $x^2 + 2xy + 4y^2 = 12$ that have tangent lines with slope equal to $-\frac{1}{2}$.

8. (12 points) You are given the following function f and its first two derivatives f' and f''.

$f(x) = \dfrac{(x+1)^2}{(x-1)^2}$ $\qquad$ $f'(x) = \dfrac{-4(x+1)}{(x-1)^3}$ $\qquad$ $f''(x) = \dfrac{8(x+2)}{(x-1)^4}$

(a) Find each of the following for f.

1. Domain
2. Intercepts, if any
3. Asymptotes, if any
4. Intervals of increase and decrease
5. Local extrema, if any
6. Intervals of upward and downward concavity
7. Points of inflection, if any

(b) Sketch the curve f, identifying any points or asymptotes listed in your answers above.

9. (5 points) A cylinder made of soft clay has a height of 9 cm and a radius of 2 cm. It is sitting on a potter's wheel. As the potter presses down on the clay, the cylinder's height decreases by 1 cm/s, but the total volume of clay is unchanging. What is the rate of change of the radius of the cylinder when it is 4 cm tall? (*Note: The volume of a cylinder is given by* $V = \pi r^2 h$.)

10. (5 points) You are told that a right triangle whose legs (shorter sides) are a and b must have a hypotenuse equal to 1. Find a and b so that the quantity $a^2 b$ is maximized.

11. (5 points) Use the limit of a Riemann sum to evaluate the following definite integral.

$$\int_0^2 \left(3x^2 + x\right)\, dx$$

Summation formulas are provided as follows:

$$\sum_{i=1}^{n} 1 = n, \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+2)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

12. (4 points) It is given that $\int_1^5 (1 + f(x))\, dx = 11$ and $\int_7^5 2f(x)dx = 8$. Find $\int_1^7 f(x)dx$

13. (12 points) Evaluate the following integrals

(a) $\displaystyle\int \frac{x^2 + 6x^{-2}}{x^2} dx$

(b) $\displaystyle\int \frac{\cos^2 x + \sin^2 x}{\cos^2 x} dx$

(c) $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1 - x^2}} dx$

(d) $\displaystyle\int_1^{e^2} \frac{3}{x} dx$

14. (3 points) Let $f'(x) = 30x^9 - 8x^3 + 7$. Given that $f(1) = 2$, find $f(x)$.

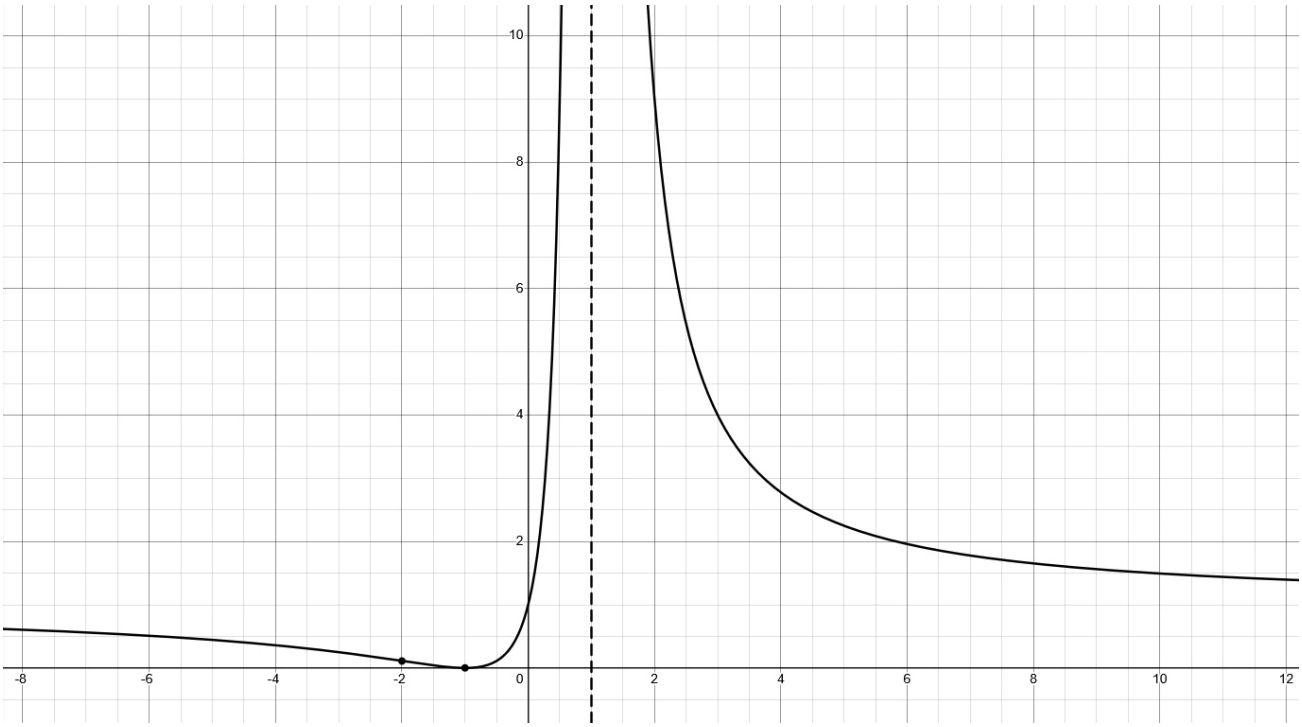
15. (3 points) Let $f(x) = \displaystyle\int_1^x \sqrt{t^2 + 5}\, dt$.

(a) Find $f(1)$.

(b) Find $f'(2)$.

Answers

1. (a) $\frac{3}{2}$; (b) 1; (c) $1+\frac{2}{\pi}$; (d) π; (e) 1
2. $y=\frac{4}{5}$, $y=-\frac{4}{5}$
3. $f'(x)=\frac{10}{(3-5x)^2}$
4. Removable discontinuity at $x=-1$; jump discontinuity at $x=1$.
5. (a) $5x^4+5^x\ln 5$; (b) $\frac{(x^2+1)\sec x\tan x-2x\sec x}{(x^2+1)^2}$; (c) $\frac{2x\cos(x^2)}{\sin(x^2)}$; (d) $\frac{1}{1+\sqrt{x}^2}\cdot\frac{1}{2\sqrt{x}}$;
   (e) $(x+4)^{x+\cos x}\left[(1-\sin x)\ln(x+4)+(x+\cos x)\frac{1}{x+4}\right]$
6. (a) $y=3x-5$; (b) None
7. $(2,1)$ and $(-2,-1)$
8. (a) 1. $\mathbb{R}\backslash\{1\}$; 2. $(-1,0)$ and $(0,1)$; 3. $x=1$ and $y=1$; 4. Increasing on $(-1,1)$ and decreasing on $(-\infty,-1)\cup(1,\infty)$; local minimum at $(-1,0)$; Concave up on $(-2,1)\cup(1,\infty)$ and concave down on $(-\infty,-2)$; Point of inflection at $(-2,\frac{1}{9})$; (b) Sketch as follows:



9. $\frac{3}{8}$ cm/s
10. $a=\sqrt{\frac{2}{3}}$, $b=\sqrt{\frac{1}{3}}$
11. 10
12. 3

13. (a) $x - 2x^{-3} + C$; (b) $\tan x$ +C; (c) $\frac{\pi}{6}$; (d) 6

14. $3x^{10} - 2x^4 + 7x - 6$

15. (a) 0; (b) 3