1. Evaluate the following integrals.

(6) (a) $\displaystyle\int \frac{x^2 - 8x - 21}{(x-3)(x^2+9)}\,dx$

(6) (b) $\displaystyle\int_{\pi/4}^{\pi/2} \csc^3 x \cot x\,dx$

(6) (c) $\displaystyle\int \frac{x^2}{\sqrt{16-x^2}}\,dx$

(6) (d) $\displaystyle\int \frac{e^x}{e^{2x}-4}\,dx$

(6) (e) $\displaystyle\int_0^1 x \arctan(x^2)\,dx$

2. Evaluate the following limits.

(3) (a) $\displaystyle\lim_{x\to\pi/2} \frac{\cos^2(5x)}{1+\sin(3x)}$

(3) (b) $\displaystyle\lim_{x\to\pi^-} (x-\pi)\csc x$

(3) (c) $\displaystyle\lim_{x\to 0^+} (1+5x)^{\frac{1}{2x}}$

(8) **3.** Determine whether the following improper integrals converge or diverge. If an integral converges, give its exact value.

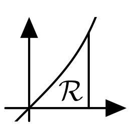
(a) $\displaystyle\int_0^1 \frac{1}{2-3x}\,dx$

(b) $\displaystyle\int_0^{\infty} \sin\theta\, e^{-\cos\theta}\,d\theta$

(4) **4.** Solve the differential equation: $(1+\cos x)\,e^y \dfrac{dy}{dx} = (e^y+1)\sin x$, with initial condition $y(0)=0$.

(5) **5.** The number $N(t)$ of people in a community who are exposed to a particular advertisement is governed by the logistic equation. Initially $N(0)=50$, and it is observed that $N(1)=200$. Solve for $N(t)$ if it is predicted that the maximum number of people in the community who will see the advertisement is 300.

(6) **6.** Given $\mathcal{R}$, the region bounded by $y=\tan(x)$, $x=\pi/4$ and $y=0$.



Set up, **but do not evaluate**, the integrals needed to find the volume of the solids of revolution obtained by revolving $\mathcal{R}$ about:

(a) the x-axis.

(b) the line $x=-2$.

(c) the line $y=3$.

(4) **7.** (a) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{(2n-1)!}$ converge? Justify your answer.

(b) Does the corresponding sequence $\left\{\dfrac{2^n}{(2n-1)!}\right\}$ converge? Justify your answer.

8. Determine whether the following series converge or diverge. State the test you are using and display a proper solution.

(4) (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n\sin^2 n}{1+n^3}$

(4) (b) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{3n+1}\right)$

(4) (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n-1}$

(4) **9.** Find the sum of the series $\displaystyle\sum_{n=1}^{\infty}\left(\cos\left(\frac{1}{n+1}\right)-\cos\left(\frac{1}{n}\right)\right)$.

10. Determine whether the following series are absolutely convergent, conditionally convergent or divergent. Justify your answer.

(4) (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \arctan(n+1)$

(4) (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-4)^{2n+1}}{n^2\,7^n}$

(5) **11.** Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n\,(x-2)^n}{\sqrt{n+3}}$.)

(5) **12.** Find the Maclaurin series expansion of $f(x)=\ln(4-x)$, and express the series using the appropriate sigma notation.

Answers:

1a. $-2\ln|x-3|+3/2\ln(x^2+9)+1/3\arctan(x/3)+C$
1b. $-1/3+2\sqrt{2}/3$
1c. $8\arcsin(x/4)-x\sqrt{16-x^2}/2+C$
1d. $1/4\ln|(e^x-2)/(e^x+2)|+C$
1e. $\pi/8-(\ln 2)/4$
2a. $50/9$
2b. -1
2c. $e^{5/2}$
3a. ∞, divergent
3b. DNE, divergent
4. $y=\ln|4/(1+\cos x)-1|$
5. $N=300\cdot 10^t/(5+10^t)$
6a. $V=\pi\int_0^{\pi/4}\tan^2 x\,dx$
6b. $V=\pi\int_0^1((2+\pi/4)^2-(2+\arctan y))\,dy$
6c. $V=\pi\int_0^{\pi/4}(3^2-(3-\tan x)^2)\,dx$
7a. ratio test, AC
7b. conv. to 0
8a. comparison test, conv.
8b. nTT, divergent
8c. ratio test, AC
9. telesc. series, conv. to $1-\cos 1$
10a. nTT, div.
10b. ratio test, div.
11. $I=(3/2,5/2],\ R=1/2$
12. $f(x)=\ln 4+\sum_{n=1}^{\infty}(-x^n/(4^n\cdot n))$