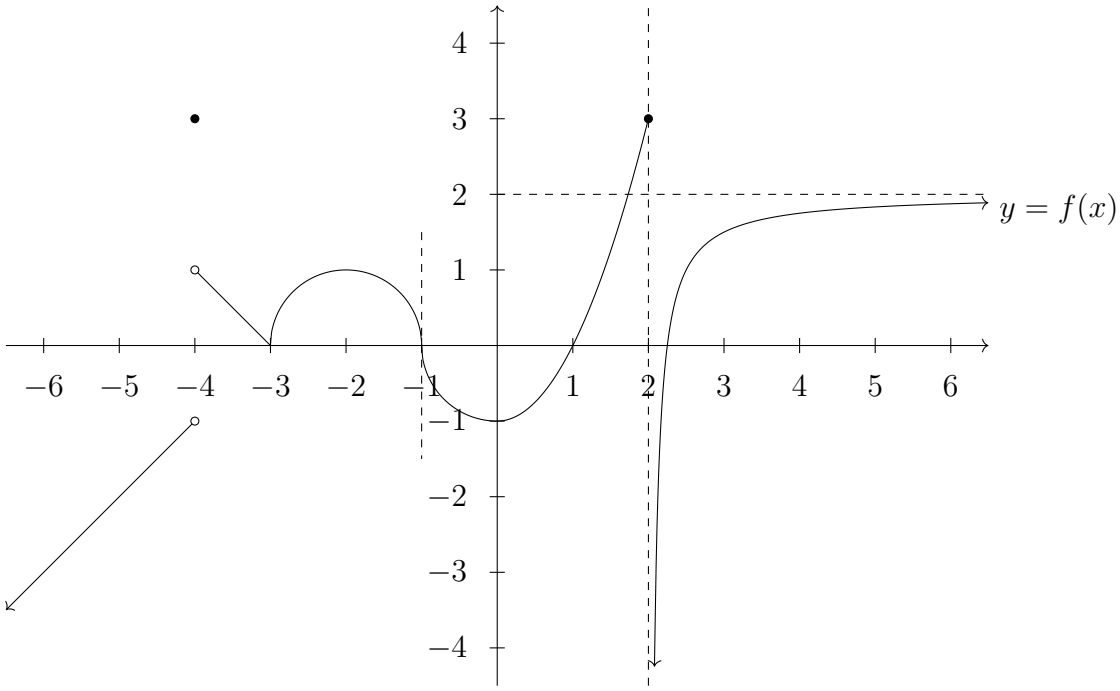
Note: this exam was shorter and covered less material

Question 1: (*9 pts*) Given the graph of the function f below, evaluate the following limits. Use $+\infty$, $-\infty$ or DNE (Does Not Exist) where appropriate.



a) $\lim_{x\to-\infty} f(x)$ **b)** $\lim_{x\to-4} f(x)$ **c)** $\lim_{x\to-3} f(x)$ **d)** $\lim_{x\to2^-} f(x)$ **e)** $\lim_{x\to2^+} f(x)$ **f)** $\lim_{x\to+\infty} f(x)$

g) Find all the x-values at which the function is not differentiable. For each value, explain why the derivative does not exist.

h) Find all the x-values at which $f'(x) = 0$.

Question 2: (*15 pts*) Evaluate the following limits. Use $+\infty$, $-\infty$ or DNE (Does Not Exist) where appropriate.

a) $\lim_{x\to2} \dfrac{\frac{1}{1+x} - \frac{1}{3}}{x^2 - 4}$ **b)** $\lim_{x\to3} \dfrac{x^2 + x - 12}{x - \sqrt{2x+3}}$ **c)** $\lim_{x\to2^+} \dfrac{3x^2 - 7x + 5}{x^2 - 9x + 14}$

d) $\lim_{x\to-2^-} \dfrac{|3x+6|}{3x^2 + 11x + 10}$ **e)** $\lim_{x\to-\infty} \dfrac{(x^2+3)(3x-1)}{2x^3 + 5x - 7}$

Question 3: (*5 pts*) Consider the function f given below. Find all the x-values at which f is discontinuous. Justify using the definition of continuity.

$$f(x) = \begin{cases} \dfrac{2x+4}{x^2 - x - 6} & \text{if} \quad x \leqslant 1 \\ 2x - 3 & \text{if} \quad 1 < x \leqslant 4 \\ 3 + 2x - x^2 & \text{if} \quad x > 4 \end{cases}$$

Question 4: (*4 pts*) Find the value(s) of a and b that would make the function f continuous on $\mathbb{R}$:

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if} \quad x < 2 \\ ax^2 - bx + 6 & \text{if} \quad 2 \leqslant x \leqslant 3 \\ 2x^2 + ax - 3b & \text{if} \quad x > 3 \end{cases}$$

Question 5: (*6 pts*) **a)** Find the derivative of $g(x) = \dfrac{2}{5x-4}$ using the **limit definition** of the derivative.

b) Find the equation of the tangent line to the graph of g at $x = 1$.

Question 6: (*20 pts*) Find the derivative of each of the following functions. Do not simplify your answers.

a) $f(x) = \dfrac{3x}{2} + \sqrt[5]{x^2} + 5x^e + 5e^x + 5e$ **b)** $f(x) = (7x+4)^4 + \sin^2(5x)$ **c)** $f(x) = \dfrac{x^3}{3x^2 + \log_9(x)}$

d) $f(x) = 5^{2x}\ \sec(3x^2)$ **e)** $y = (3x+2)^{\cos(6x)}$

Question 7: (*5 pts*) Find the 3rd derivative of $f(x) = (5x-2)\ e^{2x-1}$

Question 8: (*4 pts*) Consider the equation $e^{xy} = x^2 + 3y$. Find y'.

Question 9: (*4 pts*) Use logarithmic differentiation to find the derivative of $y = \dfrac{\sqrt[3]{5x+1}\ \tan(2x)}{e^{4x+7}}$.

Question 10: (*4 pts*) Find the absolute extrema of $f(x) = \sqrt[3]{3x^2 - 27}$ on the interval $[-1, 4]$.

Question 11: (*6 pts*) Bammer's Bang Bang has found that the demand function for their ultra deluxe Bangity Firework is given by $p = 605 - \dfrac{5}{3}\sqrt{x}$, where x is the quantity demanded per month and p is the unit price in dollars. The average cost per month is modeled by $\overline{C}(x) = 70 + \dfrac{1\,500\,000}{x}$.

a) Find a formula for the monthly cost $C(x)$. **b)** Find a formula for the monthly revenue $R(x)$.

c) Find a formula for the monthly profit $P(x)$. **d)** Find the marginal profit when $x = 22\,500$ units. Explain what that value represents.

e) What price should be set and how many units should be sold to maximize the monthly profit? What is the maximum monthly profit?

Question 12: (*12 pts*) Given: $f(x) = \dfrac{(x-1)(2x+1)}{(x+2)^2}$ $f'(x) = \dfrac{9x}{(x+2)^3}$ $f''(x) = \dfrac{-18(x-1)}{(x+2)^4}$

a) Find the domain of f. **b)** Find the x and y-intercepts of f, if any.

c) Find the vertical and horizontal asymptotes of f, if any.

d) Find the intervals of increase and decrease of f. **e)** Find the local/relative extrema of f, if any.

f) Find the intervals of concavity of f. **g)** Find the points of inflection of f, if any.

h) Use your answers from the previous parts to sketch a graph of f.

Question 13: (*6 pts*) At *North Pole Inc.*, the Grinch is making elves work in a sweat shop to produce toys. There are currently 150 elves working in the shop, and each elf produces 120 toys per day. The Grinch has noticed that when he increases the number of elves in the shop, their productivity decreases. Every time the number of elves increases by 5, each elf produces 3 fewer toys per day. How many elves should the Grinch have in his shop to maximize the number of toys produced each day? What is the maximum daily toy production?

ANSWERS:

Q1.) **a)** $-\infty$ **b)** DNE **c)** 0 **d)** 3 **e)** $-\infty$ **f)** 2

g) $x=-4$: discontinuity; $x=-3$: sharp turn; $x=-1$: vertical tangent; $x=2$: discontinuity.

h) $x=-2$ and $x=0$

Q2.) **a)** $\dfrac{-1}{36}$ **b)** $\dfrac{21}{2}$ **c)** $-\infty$ **d)** 3 **e)** $\dfrac{3}{2}$ **Q3.)** $x=-2$ and $x=4$ **Q4.)** $a=2$ and $a=5$

Q5.) **a)** $\dfrac{-10}{(5x-4)^2}$ **b)** $y=-10x+12$ **Q6.)** **a)** $\dfrac{3}{2}+\dfrac{2}{5}x^{-\frac{3}{5}}+5ex^{e-1}+5e^x$ **b)** $28(7x+4)^3+10\sin(5x)\cos(5x)$

c) $\dfrac{3x^2\left(3x^2+\log_9(x)\right)-x^3\left(6x+\dfrac{1}{x\ln(9)}\right)}{\left(3x^2+\log_9(x)\right)^2}$ **d)** $5^{2x}\sec(3x^2)\Big[2\ln(5)+6x\tan(3x^2)\Big]$

e) $(3x+2)^{\cos(6x)}\left[-6\sin(6x)\ln(3x+2)+\dfrac{3\cos(6x)}{3x+2}\right]$ **Q7.)** $(40x+44)e^{2x-1}$ **Q8.)** $\dfrac{2x-ye^{xy}}{xe^{xy}-3}$

Q9.) $\dfrac{\sqrt[3]{5x+1}\ \tan(2x)}{e^{4x+7}}\left(\dfrac{5}{3(5x+1)}+\dfrac{2\sec^2(2x)}{\tan(2x)}-4\right)$

Q10.) Abs. Min: -3, at $x=0$. Abs. Max: $\sqrt[3]{21}$, at $x=4$.

Q11.) **a)** $C(x) = 70x + 1\,500\,000$ **b)** $R(x) = 605x - \frac{5}{3}x^{3/2}$ **c)** $P(x) = -\frac{5}{3}x^{3/2} + 535x - 1\,500\,000$

d) $P'(22\,500) = \$160$/unit If production increases from 22 500 to 22 501 units, profit will increase by approximately \$160.

e) Maximum profit $P = \$6\,666\,953.33$ when $x = 45\,796$ units and $p = \$348.33$.

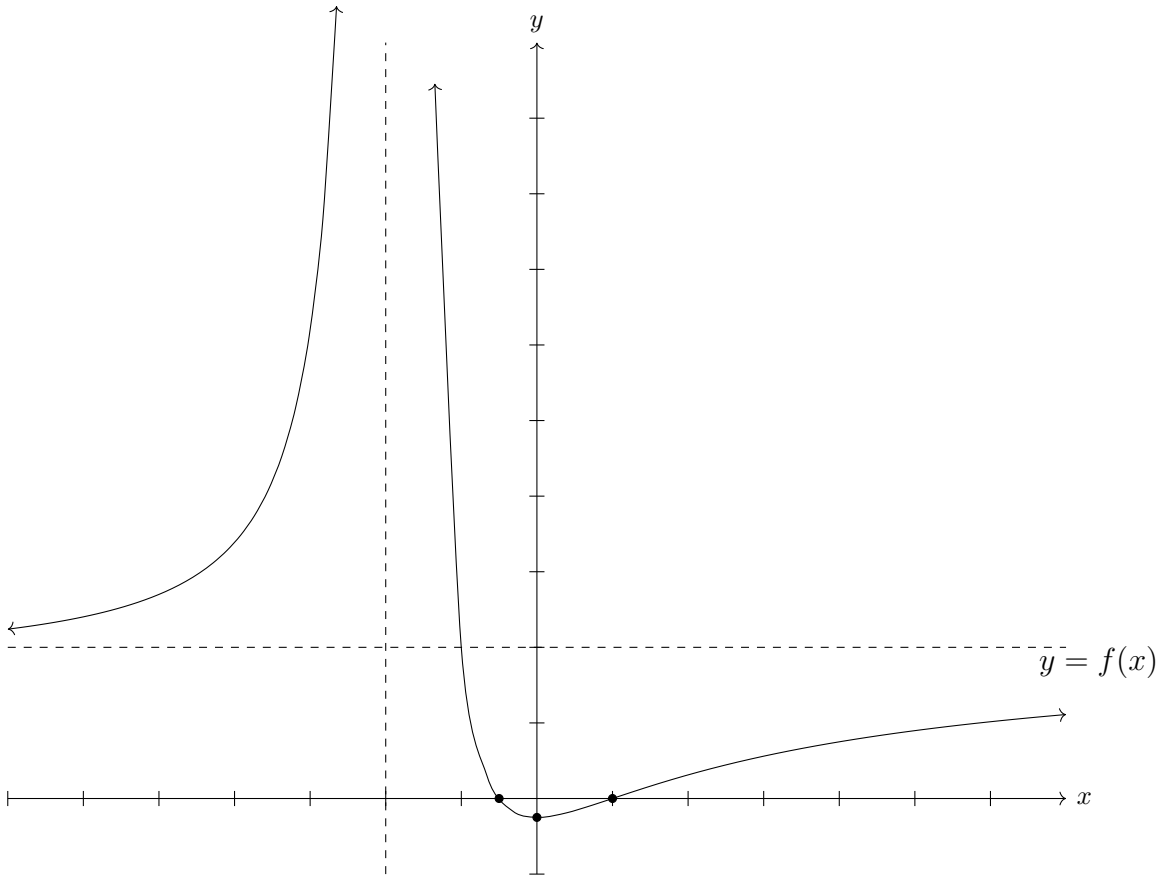
Q12.) **a)** $\mathbb{R}\backslash\{-2\}$ **b)** y-intercept: $\left(0, -\frac{1}{4}\right)$ x-intercepts: $(1, 0)$ and $\left(-\frac{1}{2}, 0\right)$.

c) V.A.: $x = -2$ H.A.: $y = 2$ **d)** Inc: $]\infty, -2[$ and $]0, +\infty[$ Dec: $]-2, 0[$

e) Local min: $\left(0, -\frac{1}{4}\right)$ Local max: none.

f) Concave up: $]-\infty, -2[$ and $]1, +\infty[$ Concave down: $]-2, 1[$ **g)** POI: $(1, 0)$

h)



Q13.) Maximum toy production: 18 375 toys per day, with 175 elves and 105 toys/elf per day.