1. (35 points) Evaluate the following integrals.

(a) $\displaystyle\int \frac{6x^3+33x^2+36x-2}{3x^2+18x+24}\,dx$

(b) $\displaystyle\int_1^4 \frac{(\sqrt{x}+1)^3}{\sqrt{x}}\,dx$

(c) $\displaystyle\int e^{\sqrt{x}}\,dx$

(d) $\displaystyle\int_0^{\pi/4} \sec^5(\theta)\tan^3(\theta)\,d\theta$

(e) $\displaystyle\int x\,\mathrm{arcsec}(x)\,dx$

(f) $\displaystyle\int \sqrt{4-x^2}\,dx$

(g) $\displaystyle\int \frac{1}{x^2-8x+41}\,dx$

2. (6 points) Evaluate the following limits. If using l'Hospital's rule, justify why it may be used.

(a) $\displaystyle\lim_{x\to 0^+} xe^{\frac{1}{x}}$

(b) $\displaystyle\lim_{x\to 0} (x+1)^{\cot x}$

3. (5 points) Find the area of the region bounded by the curves $y=x^2-8x$ and $y=7-2x$.

4. (4 points) Set up an integral to find the volume of a pyramid with height h and rectangular base with dimensions b and $3b$. Evaluate the integral.

5. (4 points) Solve the differential equation $\dfrac{dy}{dx}=\dfrac{x}{y(1+x)}$ with the initial condition $y(0)=-2$. Express y explicitly as a function of x and fully simplify your answer.

6. (5 points) A bacteria culture grows at a rate proportional to the number of bacteria present. Initially, the culture contains 1000 bacteria. After 3 hours, the population grows to 8000 bacteria.

(a) Set up a differential equation with the initial conditions describing the population growth.

(b) Find an expression for the number of bacteria as a function of time t.

(c) Find the time when the population reaches 100 000 bacteria.

7. (3 points) Find the sum of the series $\displaystyle\sum_{n=0}^{\infty}\frac{3+4^n}{7^n}$

8. (3 points) Determine whether the sequence with general term $a_n=\ln(n+1)-\ln(n)$ converges or diverges. Justify your answer.

9. (9 points) Determine whether each of the following series converges or diverges. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{2n^2+5}{n^3+3n+7}$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{(n+7)^n}{7^{n^2}}$

(c) $\displaystyle\sum_{n=1}^{\infty}\frac{2+\sin(n)}{n^4}$

10. (6 points) Determine whether each of the following series converges absolutely, converges conditionally, or diverges. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\sqrt{n}}{n+3}$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{e^n+e^{-n}}$

11. (5 points) Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n(x-5)^n}{n\,2^n}$.

12. (5 points) Find a power series representation for the function $f(x)=\arctan(x^2)$ using known series and determine the radius of convergence.

13. (4 points) Let $f(x)=\dfrac{1}{(3-x)^2}$. Write the first four nonzero terms of the Maclaurin series for $f(x)$.

14. (6 points) (a) Use a known Maclaurin series to obtain the Maclaurin series of $f(x)=e^{3x^2}$

(b) Use part (a) to evaluate $\int e^{3x^2}\,dx$ as an infinite series.

ANSWERS

1. (a) $x^2 - x + \frac{5}{3}\ln|3x+6| + \frac{1}{3}\ln|x+4| + C$

(b) $\frac{65}{2}$

(c) $2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$

(d) $\frac{12\sqrt{2}+2}{35}$

(e) $\dfrac{x^2\operatorname{arcsec}(x)}{2} - \frac{1}{2}\sqrt{x^2-1} + C$

(f) $2\arcsin(\frac{x}{2}) + \frac{x\sqrt{4-x^2}}{2} + C$

(g) $\dfrac{1}{5}\arctan\left(\dfrac{x-4}{5}\right) + C$

2. (a) ∞

(b) e

3. $\frac{256}{3}$

4. $V = \displaystyle\int_0^h 3\frac{x^2b^2}{h^2}\,dx = b^2h$

5. $y = -\sqrt{2x - 2\ln|x+1| + 4}$

6. (a) $\frac{dN}{dt} = kN,\ \ N(0) = 1000,\ \ N(3) = 8000$

(b) $N(t) = 1000(2)^t$

(c) $t = \frac{\ln 100}{\ln 2}$

7. $\frac{35}{6}$

8. $\displaystyle\lim_{n\to\infty} a_n = 0$, conv.

9. (a) div.

(b) conv.

(c) conv.

10. (a) conditionally conv.

(b) absolutely conv.

11. $R = 2$, IoC $(3, 7]$

12. $f(x) = \displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n+2}}{2n+1}$, $R = 1$

13. $\dfrac{1}{(3-x)^2} = \frac{1}{9} + \frac{2}{27}x + \frac{3}{81}x^2 + \frac{4}{243}x^3 + \dots$

14. (a) $f(x) = \displaystyle\sum_{n=0}^{\infty}\frac{3^nx^{2n}}{n!} = 1 + \frac{3x^2}{1!} + \frac{9x^4}{2!} + \frac{27x^6}{3!} + \dots$, $R = \infty$

(b) $C + \displaystyle\sum_{n=0}^{\infty}\frac{3^nx^{2n+1}}{(2n+1)\cdot n!} = C + x + \frac{3x^3}{3\cdot 1!} + \frac{9x^5}{5\cdot 2!} + \frac{27x^7}{7\cdot 3!} + \dots$, $R = \infty$