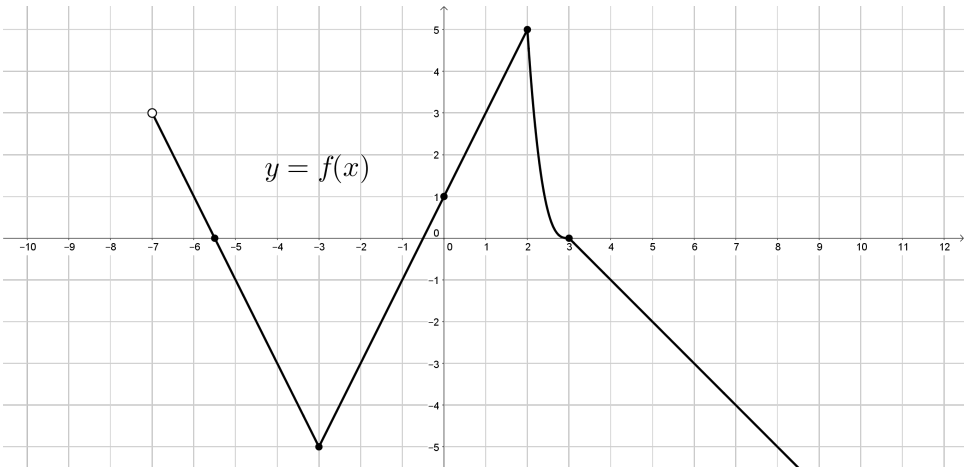
[4] **1.** Suppose $y = f(x)$ is given by the following graph:



(a) State the domain and range of $f(x)$.
(b) Evaluate: $2f(-3) + f(-6)$
(c) Evaluate: $(f \circ f)(1)$
(d) Does f have an inverse? Why or why not?

[2] **2.** Find the equation for the line with y-intercept $(0, 3)$ that is perpendicular to the line given by $2x - 3y = 4$. Give your answer in the form $y = mx + b$.

[4] **3.** Let $f(x) = \begin{cases} 2x - 1 & \text{if } 0 \leqslant x \leqslant 2 \\ x + 2 & \text{if } x > 2 \end{cases}$

(a) Sketch a graph of $y = f(x)$.
(b) State the domain and range of $y = f(x)$.
(c) Given that f has an inverse, state the domain and range of $f^{-1}(x)$.

[3] **4.** Simplify and express the answer with positive exponents only: $\dfrac{10(a^3b^{-2})^{-3}(a^{-1}b^{11})^0}{(-2a)^4}$

[2] **5.** Factor completely.
(a) $3x^2 + 8x + 5$
(b) $125x^3 - 64$

[6] **6.** Solve for x using the specified method.
(a) $3(3x - 1)^2 - 8 = 19$ (taking square roots)
(b) $(x - 3)(x + 2) = 14$ (factoring)
(c) $x^2 + 4899 = 140x$ (completing the square)

[3] **7.** Use polynomial long division to express $\dfrac{5x^3 + 3x - 5}{x + 1}$ in the form $Q(x) + \frac{R(x)}{D(x)}$, where the degree of $R(x)$ is less than degree of $D(x)$.

[4] **8.** Let $f(x) = \dfrac{x^2 - 9x - 36}{x^2 - 5x}$, let $g(x) = \dfrac{x - 12}{x}$, and let $h(x) = \dfrac{f(x)}{g(x)}$.
(a) Simplify $h(x)$.
(b) Find a formula for $g^{-1}(x)$.

[4] **9.** Solve for x: $\dfrac{x}{x - 3} + \dfrac{5}{2x - 1} = \dfrac{15}{2x^2 - 7x + 3}$

[3] **10.** Given the quadratic function $f(x) = 2x^2 - 2x + \frac{1}{2}$;

(a) Find all intercepts.
(b) Find the vertex.
(c) Sketch a graph of the function.

[3] **11.** Given the rational function $f(x) = \dfrac{x^2 - 18x + 17}{x^3 - 17x^2 + 16x}$;

(a) State the domain of $f(x)$.

(b) Give the equations of all vertical asymptotes (if any).

(c) Give the equations of all horizontal asymptotes (if any). Do **not** sketch.

[4] **12.** Given the rational function $f(x) = \dfrac{2 - 2x}{x - 2}$;

(a) Find all intercepts. (b) Find all asymptotes. (c) Sketch a graph of the function.

[2] **13.** Let $A = (1, -1)$ and $B = (-2, 5)$. Compute the distance between A and B.

[3] **14.** Reduce the radical expression: $\sqrt[3]{-8x^7y^4}\,\sqrt[3]{81x^2y^3}$

[2] **15.** Rationalize the denominator and simplify: $\dfrac{10}{2\sqrt{2} + \sqrt{3}}$

[2] **16.** Find the domain of $f(x) = \dfrac{\sqrt{5 - x}}{\sqrt{x - 3}}$.

[3] **17.** Solve: $x - 2 - \sqrt{6 - 3x} = 0$.

[2] **18.** Sweden would like to open an account in order to prepare for a catastrophic event. If the country can deposit \$3 000 000 (which is probably more than enough) at 4% interest rate compounded semi-annually, find the balance in the account after ten years. (Give your answer to the **nearest dollar**.)

[4] **19.** Given the function $y = 2^{(1-x)} - 4$;

(a) Find all intercepts.

(b) Find the equation of any asymptotes.

(c) Sketch a graph of the function.

[4] **20.** Given the function $y = -2\log_2(x + 4)$;

(a) Find all intercepts.

(b) Find the equation of any asymptotes.

(c) Sketch a graph of the function.

[3] **21.** Express in terms of the simplest possible logarithms: $\log\left(\dfrac{\sqrt{2x+1}}{100^x(x-3)^{8/3}}\right)$

[3] **22.** Compress into a single logarithm and simplify the result: $\ln(x^3y) + 2\ln(\sqrt{xz}) - 3\ln(xyz)$

[1] **23.** Evaluate to four decimal places: $\log_6(12)$

[6] **24.** Solve the following equations for x:

(a) $\log_3(x + 3) = 3 - \log_3(x + 9)$ (b) $4^{x+5} = 8 \cdot \left(\dfrac{1}{2}\right)^{7-x}$

[3] **25.** Suppose $\cos(\theta) = \frac{3}{7}$.

(a) Find the exact value of $\tan(\theta)$ if θ is acute.

(b) Find the exact value of $\tan(\theta)$ if θ is in the 4th quadrant.

[2] **26.** Find the exact value of $\csc(7\pi/6)$.

[2] **27.** Find all θ in $[0°, 360°)$ that satisfy $\sin(\theta) = -\frac{5}{6}$. (Answer in degrees to three decimal places.)

[2] **28.** Find all θ in $[0, 2\pi)$ such that $\cot(\theta)$ is undefined. (Give an exact value in radians.)

[3] **29.** A math teacher stands in front of the classroom (Say HO-214) looking directly at the back row, which is 3m away. If she turns 20° degress to the right, she will be looking at Xavier, who is in the back row. If she turns 40° to the left, she will be looking at Yannick, who is also in the back row. How far apart are Xavier and Yannick? (Answer in metres with 3 decimal places of precision.)

[3] **30.** Given the function $y = -3\cos\left(\frac{x}{2}\right)$;

(a) State the amplitude A and the period P.

(b) Sketch a graph. (At least two cycles.)

[4] **31.** A triangle has sides of length a, b, c across from angles of measure A, B, C respectively. Suppose $A = 35°$, $b = 7$ and $c = 3$. Find a, B, and C. (Give three decimal places.)

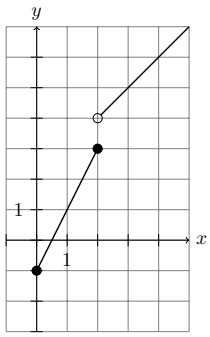
[2] **32.** Simplify: $(\csc x - \cot x)(\csc x + \cot x)$

[2] **33.** Prove the identity: $\dfrac{1}{\sin x + \cos x} = \dfrac{\sec x}{1 + \tan x}$

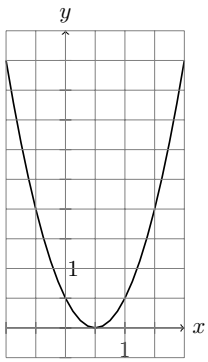
End of Exam

Answers:

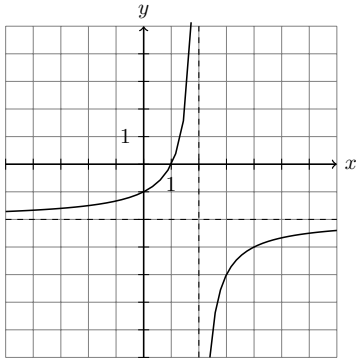
1(a) $D = (-7.\infty)$, $R = (-\infty, 5]$
1(b) -9
1(c) 0
1(d) No. It fails the horizontal line test.
2. $y = -\frac{3}{2}x + 3$
3(a)



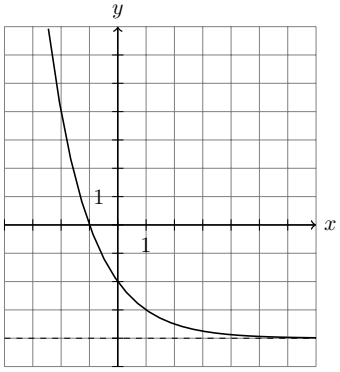
3(b) $D = [0, \infty)$, $R = [-1, 3] \cup (4, \infty)$
3(c) $D = [-1, 3] \cup (4, \infty)$, $R = [0, \infty)$
4. $5b^6/(8a^{13})$
5(a) $(3x + 5)(x + 1)$
5(b) $(5x - 4)(25x^2 + 20x + 16)$
6(a) $x = 4/3$, $x = -2/3$.
6(b) $x = 5$, $x = -4$.
6(c) $x = 69$, $x = 71$.
7. $\frac{5x^3+3x-5}{x+1} = 5x^2 - 5x + 8 + \frac{-13}{x+1}$
8(a) $h(x) = \dfrac{x+3}{x-5}$
8(b) $g^{-1}(x) = \dfrac{12}{1-x}$
9. $x = -5$
10(a) y-int: $(0, 1/2)$; x-int: $(1/2, 0)$.
10(b) Vertex: $(1/2, 0)$.
10(c)



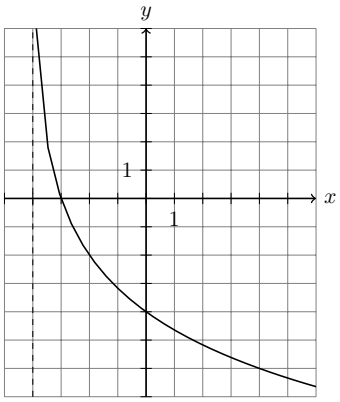
11(a). $D = \mathbb{R}\backslash\{0, 1, 16\}$.
11(b) V.A.'s at $x = 0$ and $x = 16$.
11(c) H.A. at $y = 0$.
12(a) y-int: $(0, -1)$. x-int: $(1, 0)$.
12(b) V.A. at $x = 2$. H.A. at $y = -2$.
12(c).



13. $3\sqrt{5}$
14. $-6x^3y^2\sqrt[3]{3y}$
15. $4\sqrt{2} - 2\sqrt{3}$
16. $D = (3, 5]$
17. $x = 2$
18. $\$4,457,842$
19(a) y-int: $(0, -2)$, x-int: $(-1, 0)$.
19(b) H.A. at $y = -4$. No V.A.
19(c)

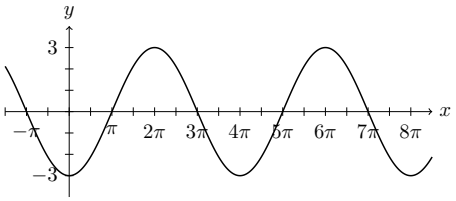


20(a) y-int: $(0, -4)$, x-int $(-3, 0)$.
20(b) V.A. at $x = -4$. No H.A.
20(c)



21. $\frac{1}{2}\log(2x + 1) - 2x - \frac{8}{3}\log(x - 3)$.
22. $\ln\left(\dfrac{x}{y^2z^2}\right)$
23. 1.3868
24(a) $x = 0$.
24(b) $x = -14$.

25(a) $2\sqrt{10}/3$
25(b) $-2\sqrt{10}/3$
26. -2
27. $\theta = 236.443°,\ \theta = 303.557°$
28. $\theta = 0,\ \theta = \pi.$
29. 3.609m
30. $A = 3, P = 4\pi$



31. $a = 4.858, C = 20.747°, B = 124.253°$
32. 1
33. Right Side$= \dfrac{\frac{1}{\cos x}}{1+\frac{\sin x}{\cos x}} = \dfrac{\frac{1}{\cos x}}{1+\frac{\sin x}{\cos x}} \cdot \dfrac{\cos x}{\cos x} = \dfrac{1}{\cos x + \sin x}$ =Left Side. ✓