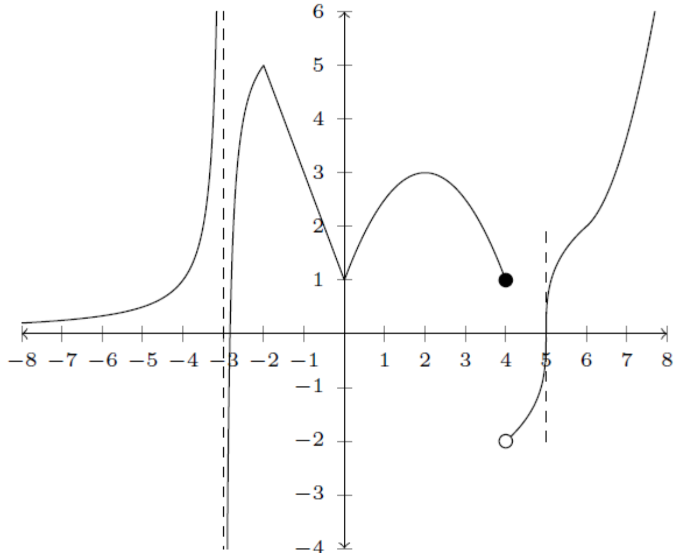
[/100]

1. Given the graph of $f(x)$ below, evaluate each of the following. Use ∞, $-\infty$ or "does not exist" where appropriate. [6]



a. $\lim\limits_{x\to-\infty} f(x)$

b. $\lim\limits_{x\to-3^-} f(x)$

c. $\lim\limits_{x\to4^+} f(x)$

d. $\lim\limits_{x\to4} f(x)$

e. $f'(2)$

f. $\lim\limits_{x\to\infty} f(x)$

g. $\lim\limits_{x\to-2} (x^2 + f(x))$

h. $\lim\limits_{h\to0} \frac{f(-1+h)-f(-1)}{h}$

i. Find the values of x at which the function shown in question 1 is NOT continuous.

j. Find all values of x at the which the function is continuous but nondifferentiable

2. Evaluate the following limits. [16]

a. $\lim\limits_{x\to5} \frac{3x^2-14x-5}{x^2-4x-5}$

b. $\lim\limits_{x\to2} \left[\frac{\frac{1}{1+x}-\frac{1}{3}}{x^2-4}\right]=$

c. $\lim\limits_{x\to-4^-} \frac{3|4+x|}{x^2+3x-4}$

d. $\lim\limits_{t\to3} \frac{\sqrt{3t}-\sqrt{t+6}}{t^2-3t}$

e. $\lim\limits_{x\to\infty} \frac{(7-2x)^2(-3x+5)^3}{2x(3x^2+1)^2}$

f. $\lim\limits_{x\to3^-} \frac{1-x}{x-3}$

3. Find the values of a that makes the following function continuous on $(-\infty,\infty)$ [4]

$$f(x) = \begin{cases} ax^2 + 2a^2x - 4 & if\ x \le 1 \\ 4ax^2 + a^2x + 6 & if\ x > 1 \end{cases}$$

4. Given the function below, find the x value(s) where the function is discontinuous. Justify your answers using the definition of continuity. [5]

$$f(x) = \begin{cases} 1 - x & x < -2 \\ \dfrac{x-1}{(x+1)(x-2)} & -2 \le x \le 2 \\ x - \dfrac{5}{3} & x > 2 \end{cases}$$

5. a. Use the limit definition of the derivative to find $f'(x)$ given that $f(x) = 2x^2 - 3x + 17$ [4]

   b. Check your answer to part a using appropriate differentiation rules.

6. For each of the following, find y′. Do not simplify your answers. [18]

   a. $y = \sqrt[7]{x^2} - \frac{7}{x^5} + \sqrt{5}\,x - \csc(x) + x^e + \pi$

   b. $y = \sqrt[5]{xe^x + 7}\ \ tan(3x)$

   c. $2(x^2 + y^2)^2 = x^2 - y^2$

   d. $y = \dfrac{3^{\sqrt{x^2+7}}}{(2x^3+5x)^4}$

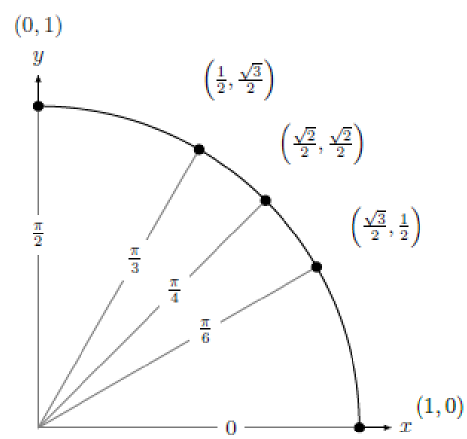
   e. $y = (x^2 + 3)^{\cos(5x)}$

7. Use logarithmic differention to find $\frac{dy}{dx}$ for $y = \dfrac{(x+3)^2\sqrt{3x+2}}{\cot^2(5x)\ \ e^{3x}}$ [4]

   (**Must** use the laws of logarithmic functions to simplify before you differentiate)

8. Find the equation of the tangent line to the graph of $y = 2\sin(x)$ at $x = \frac{2\pi}{3}$

   Give exact answer (no decimal) (you may find the graph below to be useful) [5]



9. Find the x points at which the tangent line to the graph of $f(x) = (x^2 + 4)^3(x^2 - 10)^4$ is horizontal [4]

10. Given $y = \cos(3x + 1) + e^{5x-1} + x^{11}$, find the 111^{th} derivative of y [3]

11. Find the absolute extrema of $f(x) = (x - 2)^2 e^x$ on the interval [-1,1] [4]

12. Given $f(x) = \frac{x^2+4}{4-x^2}$ $f'(x) = \frac{16x}{(4-x^2)^2}$ $f''(x) = \frac{16(1+4x^2)}{(4-x^2)^3}$ [10]

   a. Find the Domain of the function f
   b. Find $x-$ and $y-$ intercepts of f.
   c. Find any vertical and horizontal Asymptotes of f.
   d. Find the intervals of increase and decrease of f.
   e. Find any local/relative extrema of f.
   f. Find the intervals of concavity of f.
   g. Find any point of inflection of f.
   h. Use your answers from previous parts to sketch a graph of f.

13. A print shop is producing a course pack for a university course. The average production cost (in dollars per course pack) is given by the function $\bar{C}(x) = 0.0001x^2 + 4 + \frac{400}{x}$, while the selling price to students (in dollars per course pack) follows the function $p(x) = 28 - 0.0001x^2$, where x denotes the total number of packs produced. [4]

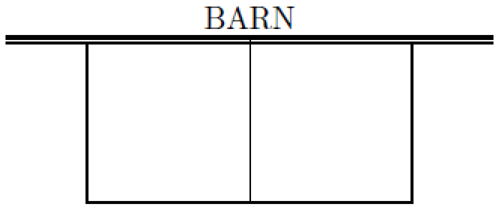
   a. Find the profit function
   b. What should be the production level to maximize the profit for the print shop?
   c. What is the maximum profit?

14. An on-line technology school charges $ 500 per child if exactly 20 kids sign up for their five-day virtual camp for programmers. However, if more than 20 kids sign up, then for each additional child the tuition fee is reduced by $ 10 per child for the entire group. [3]

   a. How many children should be enrolled in the camp to maximize the revenue?
   b. What would be the tuition per child in this case?

15. A rectangular feeding area with an interior partition is to be fenced off the side of a barn. The exterior fence on three sides costs $25 per meter. The interior partition costs $10 per meter. [5]

   a. Find the dimension of the maximum area that can be built for $1200.
   b. What is the maximum area? Remember to use a test to prove that the area is a maximum.



16. A company producing cell phone's cases with demand function $x = \frac{1}{80}(2800 - p^2)$ [5]

   a. Find the elasticity of demand function.
   b. Is the demand elastic or inelastic when the price is p = $40?
   c. When the price is p = $40, if the price is decreased by 5%, how would the demand be affected?
   d. What price would maximize revenue?

Answers

1)a. 0 b. ∞ c. -2 d. D.N.E. e. 0 f) ∞

g. 9 h. -2 i. -3, 4 j. -2, 0, 5

2) a. $\frac{8}{3}$ b. $\frac{-1}{36}$ c. $\frac{3}{5}$ d. $\frac{1}{9}$ e. -6 f. ∞

3) $a = 5 \; or - 2$

4) $f(x) is \; discontinuous \; at - 2, -1 \; and \; 2$

5) $f'(x) = 4x - 3$

6)a. $y' = \frac{2}{7}x^{-5/7} + 35x^{-6} + \sqrt{5} + \csc(x)\cot(x) + ex^{e-1}$

b. $y' = \frac{1}{5}(xe^x + 7)^{\frac{-4}{5}}(e^x + xe^x)\tan(3x) + 3\sqrt[5]{xe^x + 7}\, sec^2(3x)$

c. $y' = \frac{x(1-4x^2-4y^2)}{y(1+4x^2+4y^2)}$

d. $y' = \left[3^{\sqrt{x^2+7}}\frac{ln3}{2}(x^2+7)^{\frac{-1}{2}}2x(2x^3+5x)^4 - 3^{\sqrt{x^2+7}}4(2x^3+5x)^3(6x^2+5)\right]/(2x^3+5x)^8$

e. $y' = (x^2+3)^{\cos(5x)}\left[-5\sin(5x)ln(x^2+3) + \frac{2x}{(x^2+3)}\cos(5x)\right]$

7) $y' = \frac{(x+3)^2\sqrt{3x+2}}{\cot^2(5x)\quad e^{3x}}\left[\frac{2}{(x+3)} + \frac{3}{2(3x+2)} + \frac{10csc^2(5x)}{\cot(5x)} - 3\right]$

8) $y = -x + \frac{2\pi}{3} + \sqrt{3}$

9) $x = \pm\sqrt{2}, 0, \pm\sqrt{10}$

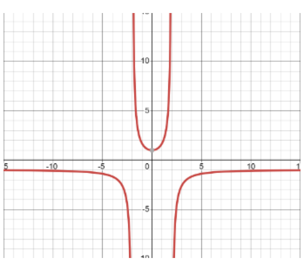
10) $\frac{d^{111}}{dx^{111}} = 3^{111}\sin(3x+1) + 5^{111}e^{5x-1}$

11) $absolute \; maximum \; at \; (0,4), absolute \; minimum \; at \; (1, e)$

12)a. $(-\infty.-2) \cup (-2,2) \cup (2,\infty)$ b. $y \; intercept \; at \; (0,1)$ c. $V.A. at \; x = \pm 2, H.A. at \; y = -1$

d. $f(x) \; dec. (-\infty,-2) \cup (-2,0), f(x) \; inc. (0,2) \cup (2,\infty)$ e. $Relative \; min. at \; (0,1)$

f. $f(x) concave \; down \; at \; (-\infty,-2) \cup (2,\infty), concave \; up \; at \; (-2,2)$ g. No P.O.I.



13)a. $P(x) = -0.0002x^3 + 24x - 400$ b. x = 200 c. $2800

14)a. 35 b. $\$350$

15)a. $10\ x\ 24\ m$ b. $240m^2$

16)a. $E = \frac{-2p^2}{2800-p^2}$ b. $E(\$40) = -2.66$ c. Demand will increase by 13.33% d. $30.55