

## 201-SH2-AB - Exercises #12 - Higher Derivatives

Evaluate the higher derivative at the given  $x$ -value.

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| <p>(1) Given <math>y = 5x^4 - 4x^5 + 6x + 10</math>, find <math>\frac{d^3y}{dx^3}</math> at <math>x = -2</math></p> <p>(2) Given <math>y = \frac{x^2 + 2}{x}</math>, find <math>\frac{d^2y}{dx^2}</math> at <math>x = 1</math></p> <p>(3) Given <math>y = (2x^3 - 5)(6x^2 + 4x)</math>, find <math>\frac{d^3y}{dx^3} \Big _{x=2}</math></p> <p>(4) Given <math>y = x^{-4} + 2\sqrt{x}</math>, find <math>y'''(1)</math></p> <p>(5) Given <math>y = \frac{3 - x^2}{x}</math>, find <math>\frac{d^2y}{dx^2}</math> at <math>x = 2</math></p> <p>(6) Given <math>y = \frac{x + 2}{(x + 1)^2}</math>, find <math>\frac{d^3y}{dx^3}</math> at <math>x = -2</math></p> <p>(7) Given <math>y = x^{-5} + 2x^3 - x^{1/5}</math>, find <math>\frac{d^3y}{dx^3}</math> at <math>x = -1</math></p> <p>(8) Given <math>y = (x + \sqrt{x})(2x + 1)</math>, find <math>y''(4)</math></p> <p>(9) Given <math>y = \frac{3x^2 - 5x}{2x^3}</math>, find <math>\frac{d^2y}{dx^2}</math> at <math>x = 3</math></p> <p>(10) Given <math>y = \frac{2x - 3\sqrt{x}}{4\sqrt{x}}</math>, find <math>\frac{d^2y}{dx^2}</math> at <math>x = 1</math></p> | <p>(11) Given <math>y = 8x^5 - 12x^6 - 5x + 10</math>, find <math>y'''(-1)</math></p> <p>(12) Given <math>y = \frac{2x + 7}{3x}</math>, find <math>y''(-1)</math></p> <p>(13) Given <math>y = (3x^2 + 4x)(x^3 - 5)</math>, find <math>\frac{d^3y}{dx^3}</math> at <math>x = 1</math></p> <p>(14) Given <math>y = \frac{4x + 5}{6x}</math>, find <math>y''(3)</math></p> <p>(15) Given <math>y = x^{-3} - 3\sqrt{x}</math>, find <math>y'''(4)</math></p> <p>(16) Given <math>y = \frac{x - 3}{(x - 1)^2}</math>, find <math>\frac{d^3y}{dx^3}</math> at <math>x = -2</math></p> <p>(17) Given <math>y = x^{-4} - 3x^2 + x^{1/3}</math>, find <math>\frac{d^3y}{dx^3}</math> at <math>x = 1</math></p> <p>(18) Given <math>y = \frac{2\sqrt{x} - 5x}{3\sqrt{x}}</math>, find <math>\frac{d^2y}{dx^2}</math> at <math>x = 1</math></p> <p>(19) Given <math>y = \frac{4x^3 + 3x}{3x^2}</math>, find <math>\frac{d^2y}{dx^2}</math> at <math>x = -3</math></p> <p>(20) Given <math>y = (3x - 1)(2x + \sqrt{x})</math>, find <math>y''(1)</math></p> |
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Find the higher derivative:

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| <p>(21) <math>y = xe^{2x}</math> find <math>\frac{d^3y}{dx^3}</math></p> <p>(24) <math>y = \frac{\ln(x)}{3x}</math> find <math>\frac{d^3y}{dx^3}</math></p> <p>(27) <math>y = \frac{\ln(\sqrt{x})}{x^2}</math> find <math>\frac{d^3y}{dx^3}</math></p> <p>(30) <math>y = \ln(\sqrt{x^3 + 3})</math> find <math>\frac{d^2y}{dx^2}</math></p> <p>(33) <math>y = \sqrt{5x + 2}</math> find <math>y'''</math></p> | <p>(22) <math>y = x \ln(x)</math> find <math>\frac{d^3y}{dx^3}</math></p> <p>(25) <math>y = (x^2 + 4)e^x</math> find <math>\frac{d^3y}{dx^3}</math></p> <p>(28) <math>y = \ln(e^x + 3)</math> find <math>\frac{d^3y}{dx^3}</math></p> <p>(31) <math>y = \frac{3\sqrt{x^3} - 2x^3e^{4x}}{2x^3}</math> find <math>y''</math></p> <p>(34) <math>g(x) = \frac{3x^5 + x^4 + x - x^3e^{3x}}{x^2}</math> find <math>g''(x)</math></p> | <p>(23) <math>y = \frac{2x}{e^x}</math> find <math>\frac{d^3y}{dx^3}</math></p> <p>(26) <math>y = (x^2 + 1)\ln(x)</math> find <math>\frac{d^3y}{dx^3}</math></p> <p>(29) <math>y = e^{\sqrt{x}+1}</math> find <math>\frac{d^2y}{dx^2}</math></p> <p>(32) <math>y = \sqrt{x^2 + 1}</math> find <math>y''</math></p> |
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- (35) Find  $\frac{d^4 y}{dx^4}$  if  $y = \cos(9 - 3x)$       (36) Find  $y'''$  if  $y = \cos\left(\frac{x}{3}\right)$       (37) Find  $\frac{d^2 y}{dx^2}$  if  $y = \ln(\cos(3x))$
- (38) Find  $y''$  if  $y = \ln(\sin^3(x))$       (39) Find  $y''$  if  $y = e^{\cos(2x)-1}$       (40) Find  $y''$  if  $y = 3x \sin(2x)$
- (41) Find  $\frac{d^2 y}{dx^2}$  if  $y = \sin(2 \ln(x))$       (42) Find  $y''$  if  $y = (2x - 1) \cos(3x)$       (43) Find  $y''$  if  $y = \frac{\sin(x) + 3}{\sin(x) + 4}$
- (44) Find  $\frac{d^2 y}{dx^2}$  if  $y = \ln(\sin(5x))$       (45) Find  $\left. \frac{d^2 y}{dx^2} \right|_{x=0}$  if  $y = \ln(\cos^2(2x))$       (46) Find  $y'''$  if  $y = \sin\left(\frac{x}{2}\right)$

Find the higher derivative.

- (47) Given  $f(x) = x^6 - x^3 + e^{5x} + 2x - 8$ , find  $f^{(4)}(x)$
- \*(48) Given  $f(x) = e^{2x} + 23x^{37} - 15x^{21} + 7x^{13} - 12x^4$ , find  $f^{(37)}(x)$
- \*(49) Given  $f(x) = 4x^{21} - 3x^{15} + 164x^7 + \frac{1}{x} + e^{-3x}$ , find  $f^{(47)}(x)$
- \*(50) Given  $f(x) = e^{4x-5} + 4x^{43} + 56x^{34} - \frac{1}{x+2}$ , find  $f^{(45)}(x)$
- (51) Given  $f(x) = 40x^{57} - 34x^{43} + x^{22} - e^{6-x} + \pi$ , find  $f^{(61)}(x)$
- \*(52) Given  $f(x) = (x+2)e^x - x^{29} + x^{15} + x^4 - x^2$ , find  $f^{(29)}(x)$
- \*(53) Given  $f(x) = 4xe^{x+4} + \frac{4}{x}$ , find  $f^{(84)}(x)$
- (54) Given  $f(x) = \sin(x)$ , find  $f^{(81)}(x)$
- (55) Given  $f(x) = \cos(2x - 5)$ , find  $f^{(54)}(x)$
- (56) Given  $f(x) = \sin(3x + 1)$ , find  $f^{(75)}(x)$
- (57) Given  $f(x) = \cos(3 - 2x)$ , find  $f^{(124)}(x)$
- (58) Given  $f(x) = \sin(1 - 6x)$ , find  $f^{(45)}(x)$
- \*(59) Given  $f(x) = x \sin(x)$ , find  $f^{(51)}(x)$
- \*(60) Given  $f(x) = x^{10} + 7x^7 - 3x^3 + 5$ , find  $f^{(10)}(x)$
- (61) Given  $f(x) = 2x^{23} + 17x^{15} - 6x^{11}$ , find  $f^{(30)}(x)$
- \*(62) Given  $f(x) = \frac{-2}{x^3}$ , find  $f^{(51)}(x)$
- (63) Given  $f(x) = e^{-5x} + x^2$ , find  $f^{(101)}(x)$
- (64) Given  $f(x) = 7^{5-2x} + ex^4 - \cos\left(-\frac{x}{2}\right)$ , find  $f^{(5)}(x)$
- (65) Given  $f(x) = 3e^{-x} - \sin 3x + x^{140} - 8$ , find  $f^{(1007)}(x)$

**ANSWERS:**

- (1)  $-1200$  (2)  $4$  (3)  $3264$  (4)  $\frac{-477}{4}$  (5)  $\frac{3}{4}$  (6)  $18$  (7)  $\frac{-24786}{125}$  (8)  $\frac{151}{32}$
- (9)  $\frac{-2}{27}$  (10)  $\frac{-1}{8}$  (11)  $1920$  (12)  $\frac{-14}{3}$  (13)  $276$  (14)  $\frac{5}{81}$  (15)  $\frac{-51}{1024}$  (16)  $\frac{-22}{81}$
- (17)  $\frac{-3230}{27}$  (18)  $\frac{5}{12}$  (19)  $\frac{-2}{27}$  (20)  $\frac{29}{2}$
- (21)  $(12 + 8x)e^{2x}$  (22)  $\frac{-1}{x^2}$  (23)  $\frac{6 - 2x}{e^x}$  (24)  $\frac{11 - 6 \ln(x)}{3x^4}$  (25)  $(x^2 + 6x + 10)e^x$
- (26)  $\frac{2x^2 + 2}{x^3}$  (27)  $\frac{13 - 12 \ln(x)}{x^5}$  (28)  $\frac{-3e^{2x} + 9e^x}{(e^x + 3)^3}$  (29)  $\frac{e^{\sqrt{x}+1}(\sqrt{x} - 1)}{4x\sqrt{x}}$  (30)  $\frac{-3x^4 + 18x}{2(x^3 + 3)^2}$
- (31)  $\frac{45}{8}x^{\frac{-7}{2}} - 4^2e^{4x}$  (32)  $\frac{1}{(x^2 + 1)^{\frac{3}{2}}}$  (33)  $\frac{375}{8(x^2 + 1)^{\frac{3}{2}}}$  (34)  $18x + 2 + 2x^{-3} - 6e^{3x} - 9xe^{3x}$
- (35)  $81 \cos(9-3x)$  (36)  $\frac{1}{27} \sin\left(\frac{x}{3}\right)$  (37)  $-9 \sec^2(3x)$  (38)  $-3 \csc^2(x)$  (39)  $[-4 \cos(2x) + 4 \sin^2(2x)] e^{\cos(2x)-1}$
- (40)  $12 \cos(2x) - 12x \sin(2x)$  (41)  $\frac{-4 \sin(2 \ln(x)) - 2 \cos(2 \ln(x))}{x^2}$  (42)  $-12 \sin(3x) - (18x - 9) \cos(3x)$
- (43)  $\frac{-\sin^2(x) - 4 \sin(x) - 2 \cos^2(x)}{(\sin(x) + 4)^3}$  (44)  $-25 \csc^2(5x)$  (45)  $-8$  (46)  $\frac{-1}{8} \cos\left(\frac{x}{2}\right)$
- (47)  $(6 \cdot 5 \cdot 4 \cdot 3)x^2 + 5^4e^{5x}$  (48)  $2^{37}e^{2x} + 23 \cdot 37!$  or  $2^{37}e^{2x} + 23(37 \cdot 36 \cdot 35 \cdots 3 \cdot 2 \cdot 1)$
- (49)  $\frac{-47!}{x^{48}} - 3^{47}e^{-3x}$  or  $\frac{-(47 \cdot 46 \cdot 45 \cdots 3 \cdot 2 \cdot 1)!}{x^{48}} - 3^{47}e^{-3x}$
- (50)  $4^{45}e^{4x-5} + \frac{45!}{(x+2)^{46}}$  or  $4^{45}e^{4x-5} + \frac{(45 \cdot 44 \cdot 43 \cdots 3 \cdot 2 \cdot 1)}{(x+2)^{46}}$  (51)  $e^{6-x}$
- (52)  $(x+31)e^x - 29!$  or  $(x+31)e^x - (29 \cdot 28 \cdot 27 \cdots 3 \cdot 2 \cdot 1)$
- (53)  $(4x+336)e^{x+4} + \frac{4 \cdot 84!}{x^{85}}$  or  $(4x+336)e^{x+4} + \frac{4 \cdot (84 \cdot 83 \cdot 82 \cdots 3 \cdot 2 \cdot 1)}{x^{85}}$  (54)  $\cos(x)$
- (55)  $-2^{54} \cos(2x - 5)$  (56)  $-3^{75} \cos(3x + 1)$  (57)  $(-2)^{124} \cos(3 - 2x)$  (58)  $(-6)^{45} \sin(1 - 6x)$
- (59)  $-51 \sin(x) - x \cos(x)$  (60)  $10!$  or  $(10 \cdot 9 \cdot 8 \cdots 3 \cdot 2 \cdot 1)$  (61)  $0$  (62)  $\frac{53!}{x^{54}}$  or  $\frac{(53 \cdot 52 \cdot 51 \cdots 3 \cdot 2 \cdot 1)}{x^{54}}$
- (63)  $-5^{101}e^{-5x}$  (64)  $-2^5 \ln^5(7)7^{5-2x} - \frac{1}{2^5} \sin\left(\frac{-x}{2}\right)$  (65)  $-3e^{-x} + 3^{1007} \cos(3x)$