1. Let $A(7,1,-1)$, $B(4,-2,-1)$, $C(3,0,-5)$, and $D(-6,-9,-5)$ be points in 3-space.
   (a) Find the vector $\overrightarrow{AB}$.
   (b) Find a vector equation for the line L that passes through C and is parallel to $\overrightarrow{AB}$.
   (c) Is the point D on the line L?
2. Let $A(5,-2,1)$, $B(0,-3,4)$, $C(1,-1,2)$, and $D(7,2,1)$ be points in 3-space.
   (a) Find *parametric equations* for the line L that passes through C and is parallel to $\overrightarrow{AB}$.
   (b) Is the point D on the line L? Justify your answer.
3. Let $A(3,0,-2)$, $B(5,1,-3)$, $C(-1,-2,0)$, and $D(-2,3,-9)$ be points.
   (a) Find *parametric equations* for the line L that passes through A and B.
   (b) Determine if C is on the line containing A and B. Justify your answer.
   (c) Find a *point-normal equation* of the plane containing the points A, B, and D.
4. Let $A(-3,6,1)$, $B(4,1,-1)$, and $C(7,-2,-3)$.
   (a) Find *parametric equations* for the line L that passes through A and is parallel to $\overrightarrow{BC}$.
   (b) Find a point on the line L other that A.
   (c) Find a *vector equation* for the plane containing the three points A, B, and C.
   (d) Find a *point-normal equation* of the plane containing the points A, B, and C.
5. Let L be the line with vector equation $(x,y,z) = (1,2,3) + t(4,5,6)$.
   For each equation below, determine if it is also a vector equation for the line L.
   (a) $(x,y,z) = (2,4,6) + t(8,10,12)$
   (b) $(x,y,z) = (-3,-3,-3) + t(8,10,12)$
   (c) $(x,y,z) = (-3,-3,-3) + t(4,5,6)$
   (d) $(x,y,z) = (1,2,3) + t(3,3,3)$
6. Find an equation of the line passing through $(1,-4)$ and $(3,7)$:
   (a) in point-normal form;
   (b) in standard form $Ax + By = C$;
   (c) in vector form;
   (d) in parametric form.
7. Find an equation of the line through $(1,-5)$ with slope$= -\frac{2}{3}$:
   (a) in point-normal form;
   (b) in standard form $Ax + By = C$;
   (c) in vector form;
   (d) in parametric form.
8. Find a vector equation of the line L which:
   (a) is parallel to $(2,-1,0)$ and passes through $P(1,-1,3)$.
   (b) passes through $P(3,-1,4)$ and $Q(1,0,-1)$.
   (c) is parallel to $(1,2,-7)$ and passes through $O(0,0,0)$.
   (d) passes through $P(1,0,-3)$ and parallel to the line
   $$\begin{cases} x = -1 + 2t \\ y = 2 - t \\ z = 3 + 3t \end{cases}$$
   (e) passes through $P(2,-1,1)$ and parallel to the line $(x,y,z) = (2,1,0) + t(-1,0,1)$
9. Find a standard equation $(ax + by = c)$ for the line in $\mathbb{R}^2$ that fits each description:
   (a) Through $(1,3)$ and perpendicular to $\langle 7,-2\rangle$. Sketch the line and the vector on the same pair of axes to confirm your result.
   (b) Through $(1,3)$ and parallel to $\langle 7,-2\rangle$. Sketch the line and the vector on the same pair of axes to confirm your result.
   (c) Through $(2,0)$ and $(0,9)$.
10. Find a standard equation $(ax + by + cz = d)$ of the plane in $\mathbb{R}^3$ that fits each description:
    (a) Through $(4,0,-1)$ and parallel to both $\langle 5,1,-1\rangle$ and $\langle -2,3,0\rangle$.
    (b) Through $(6,6,-2)$, $(1,6,1)$, and $(2,9,1)$
    (c) Through $(1,1,4)$, $(2,-3,1)$, and $(-1,5,2)$
    (d) Through $(2,1,1)$, $(3,2,3)$, and $(-2,-1,3)$
11. Determine whether each pair of planes is parallel, perpendicular, or neither.
    (a) $P_1 : x - 4y - 3z - 2 = 0$ and $P_2 : 3x - 12y - 9z - 7 = 0$
    (b) $P_1 : x - 2y + 3z = 4$ and $P_2 : -2x + 5y + 4z = -1$
    (c) $P_1 : 4x - y + 2z = 5$ and $P_2 : 7x - 3y + 4z = 8$
    (d) $P_1 : 2y = 8x - 4z + 5$ and $P_2 : x = \frac{1}{2}z + \frac{1}{4}y$
    (e) $P_1 : x + y + z = 3$ and
    $$P_2 : \begin{cases} x = -2 + s + 2t \\ y = 1 + 2t \\ z = 1 + s + 2t \end{cases}$$
    (f) $P_1 : 14(x-1) + 8(y-2) - 6z = 0$ and
    $$P_2 : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} + s \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

12. For each set of planes, find the intersection using row reduction. Describe the intersection as a plane, a line, a point, or conclude that there is none.

(a) $\begin{cases} 2x + 3y + 2z = 5 \\ x + 2y + 6z = 6 \end{cases}$

(b) $\begin{cases} 3x + 2y + 5z = 15 \\ 6x + 4y + 10z = 16 \end{cases}$

(c) $\begin{cases} x + 3y + 4z = 12 \\ 2x + 5y + 2z = 20 \\ 4x + 11y + 10z = 44 \end{cases}$

(d) $\begin{cases} x + 4y + 2z = 8 \\ 3x + 10y + 10z = 30 \\ 7x + 4y + 2z = 28 \end{cases}$

(e) $\begin{cases} -x + 4y + 2z = 5 \\ 3x - 13y + z = 6 \\ 5x - 22y + 4z = 20 \end{cases}$

(f) $\begin{cases} 3x + 6y + 9z = 3 \\ 4x + 8y + 12z = 4 \\ 2x + 4y + 6z = 2 \end{cases}$

13. Refer back to Exercises #1, problem 1. For each part, clearly state whether the solution you found represents the intersection of planes, intersection of lines, or neither. For a) - g), *h) - m), state whether the intersection is a point, a line, or a plane.

14. For each set of planes determine if the intersection is a plane, a line, a point, or the empty set $\emptyset$.
In most cases, you can do so by simple inspection!

(a) $\begin{cases} 20x + 30y - 30z = 5 \\ -16x - 24y + 24z = -4 \end{cases}$

(b) $\begin{cases} 20x + 30y - 30z = 4 \\ -16x - 24y + 24z = -5 \end{cases}$

(c) $\begin{cases} 20x + 30y - 20z = 5 \\ -16x - 24y + 24z = -4 \end{cases}$

(d) $\begin{cases} x + y = 1 \\ x + 2y + 3z = 4 \\ 4x + 3y + 2z = 1 \end{cases}$

(e) $\begin{cases} x + y + z = 1 \\ x + 2y + 3z = 4 \\ 4x + 3y + 2z = 1 \end{cases}$

(f) $\begin{cases} x + y + z = 0 \\ x + 2y + 3z = 4 \\ 4x + 3y + 2z = 1 \end{cases}$

15. Consider the system: $\begin{cases} kx + y + z = 0 \\ x + y + 2z = 0 \\ x + 2y + z = 0 \end{cases}$
For what value(s) of k do the planes...
   (a) intersect at a point?
   (b) intersect in a line?
   (c) have no points of intersection?

16. Consider the system: $\begin{cases} 2x - 4y + 6z = 2 \\ 3x - 6y + hz = k \end{cases}$
For what value(s) of h and k do the planes...
   (a) intersect in a plane?
   (b) intersect in a line?
   (c) have no points of intersection?
   (d) intersect at a right angle? (perpendicular)

ANSWERS:

1. (a) $\overrightarrow{AB} = (-3, -3, 0)$

   (b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -5 \end{bmatrix} + t \begin{bmatrix} -3 \\ -3 \\ 0 \end{bmatrix}$.

   (c) Yes. D occurs when $t = 3$ in the vector equation above.

2. (a) $\begin{cases} x = 1 - 5t \\ y = -1 - t \\ z = 2 + 3t \end{cases}$

   (b) D is not on the line as $\begin{cases} 7 = 1 - 5t \\ 2 = -1 - t \\ 1 = 2 + 3t \end{cases}$ has no solution.

3. (a) $\begin{cases} x = 3 + 2t \\ y = t \\ z = -2 - t \end{cases}$

   (b) Yes, when $t = -2$ in the equation above.

   (c) $-4(x-3) + 19y + 11(z+2) = 0$

4. (a) $\begin{cases} x = -3 + 3t \\ y = 6 - 3t \\ z = 1 - 2t \end{cases}$

   (b) In the equation above, $t = 1$ gives $(0, 3, -1)$ for example.

   (c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix} + s \begin{bmatrix} 7 \\ -5 \\ -2 \end{bmatrix} + t \begin{bmatrix} 3 \\ -3 \\ -2 \end{bmatrix}$.
   Note:
   Many answers possible, we chose one that uses $\overrightarrow{BC}$.

   (d) $2(x+3) + 4(y-6) - 3(z-1) = 0$

5. (a) No. (b) Yes. (c) Yes. (d) No.

6. (a) $11(x-1) - 2(y+4) = 0$

   (b) $11x - 2y = 19$

   (c) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix} + t \begin{bmatrix} 2 \\ 11 \end{bmatrix}$ OR $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix} + t \begin{bmatrix} 2 \\ 11 \end{bmatrix}$.

   (d) $\begin{cases} x = 1 + 2t \\ y = -4 + 11t \end{cases}$ OR $\begin{cases} x = 3 + 2t \\ y = 7 + 11t \end{cases}$

7. (a) $2(x-1) + 3(y+5) = 0$

   (b) $2x + 3y = -13$

   (c) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \end{bmatrix} + t \begin{bmatrix} 3 \\ -2 \end{bmatrix}$.

   (d) $\begin{cases} x = 1 + 3t \\ y = -5 - 2t \end{cases}$

8. (a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$.

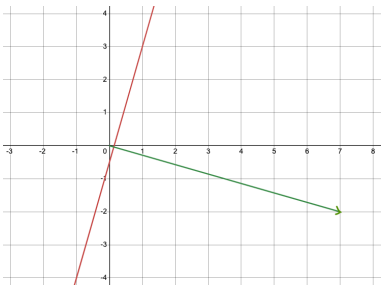
   (b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$ OR $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$.

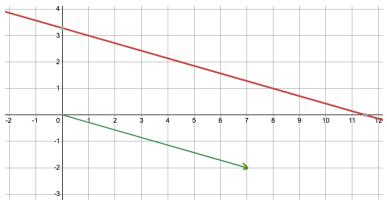
   (c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 1 \\ 2 \\ -7 \end{bmatrix}$.

   (d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$.

   (e) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

9. (a) $7x - 2y = 1$ (b) $2x + 7y = 23$ (c) $9x + 2y = 18$





10. (a) $3x + 2y + 17z = -5$ (b) $3x - y + 5z = 2$ (c) $5x + 2y - z = 3$ (d) $3x - 5y + z = 2$

11. (a) parallel
(b) perpendicular
(c) neither
(d) parallel
(e) perpendicular
(f) parallel

12. (a) LINE: $\begin{cases} x &= -8 + 14t \\ y &= 7 - 10t \\ z &= t \end{cases}$
(b) NO INTERSECTION
(c) LINE: $\begin{cases} x &= 14t \\ y &= 4 - 6t \\ z &= t \end{cases}$
(d) POINT $(\frac{10}{3}, \frac{1}{3}, \frac{5}{3})$
(e) NO INTERSECTION
(f) PLANE $\begin{cases} x &= 1 - 2s - 3t \\ y &= s \\ z &= t \end{cases}$

13. (a) The planes intersect at the point $(4, -3, 2)$.
(b) The planes intersect at the point $(-13, 26/5, 6/5)$.
(c) The planes intersect at the point $(-1/2, 0, -5/6)$.
(d) These are the same plane.
So their intersection is the plane $(6 + 4t - 3r, t, r)$
(e) The planes intersect at the point $(3, -1/2, 5/2)$.
(f) The planes intersect at the point $(-3/7, 2/7, 8/7)$.
(g) The planes intersect in the line: $(-21 - 15t, -17 - 11t, t)$
(h) Inconsistent. (No solutions.) The linear objects do not intersect.
(i) Inconsistent. The linear objects do not intersect.
(j) The linear objects intersect in the plane:
$(3 - 7s/2 + 2t, s, -1 - t/2, t)$
(k) Inconsistent. The linear objects do not intersect.
(l) The linear objects intersect in the plane: $(s/2 - 2t, s, t, 2)$
(m) The linear objects intersect at: $(-2r - s - t, r, -s - t, s, -t, t)$

14. (a) Plane.
(b) $\emptyset$.
(c) Line.
(d) Point.
(e) Line.
(f) $\emptyset$.

15. (a) $k \neq 2/3$ (b) $k = 2/3$ (c) None.

16. (a) $h = 9,\ k = 3$ (b) $h \neq 9,\ k \in \mathbb{R}$ (c) $h = 9,\ k \neq 3$ (d) $h = -5,\ k \in \mathbb{R}$