201-SH3-AB - Exercises #11: Improper Integrals

Evaluate the following improper integrals.

(1)
$$\int_{1}^{\infty} \frac{3}{x^4} dx$$

(10)
$$\int_{1}^{\infty} \frac{6x}{9+x^2} dx$$

(19)
$$\int_0^{\frac{\pi}{6}} \frac{3\cos(x)}{\left(2\sin(x)\right)^{1/3}} \ dx$$

(2)
$$\int_4^\infty \frac{2}{x^{3/2}} dx$$

(11)
$$\int_{-\infty}^{-2} \frac{2x^3 + 1}{(x^4 + 2x)^3} dx$$

(20)
$$\int_{0}^{1} \frac{e^{\sqrt{x}}}{\sqrt{x} \left(e^{\sqrt{x}} + 1\right)^{2}} dx$$

$$(3) \int_{1}^{\infty} \frac{4}{x} \, dx$$

$$(12) \int_{1}^{\infty} 6x^2 e^{1-x^3} dx$$

$$\int_{0}^{\infty} \sqrt{x} \left(e^{\sqrt{x}} + 1 \right)^{2}$$

$$(21) \int_{0}^{\infty} 2x + 1$$

$$(4) \int_0^\infty e^{-x} dx$$

(13)
$$\int_0^\infty \frac{9x^2}{(x^3+1)^{5/3}} \ dx$$

(21)
$$\int_{2}^{\infty} \frac{2x+1}{\sqrt[4]{x^2+x-2}} \ dx$$

$$(5) \int_{-\infty}^{0} e^{2x} dx$$

(14)
$$\int_0^\infty \frac{x^2 + 1}{\left(x^3 + 3x + 4\right)^{5/2}} \ dx$$

(22)
$$\int_0^\infty \frac{4x}{(x^2+1)^4} \ dx$$

$$(6) \int_0^1 \frac{dx}{\sqrt{x}}$$

$$(15) \int_0^1 \frac{2x+7}{\sqrt[3]{x^2+7x-8}} \ dx$$

$$(23) \int_{-2}^{0} \frac{6x}{\sqrt{4-x^2}} \ dx$$

(7)
$$\int_{-1}^{0} \frac{dx}{x^2}$$

(16)
$$\int_0^1 \frac{e^{2x}}{1 - e^{2x}} \ dx$$

$$(24) \int_2^\infty \frac{4}{x \left(\ln(x)\right)^2} \ dx$$

(8)
$$\int_{4}^{\infty} \frac{6}{(4-3x)^{1/3}} \ dx$$

(17)
$$\int_{1}^{2} \frac{2x^{2} + 1}{\sqrt{(2x^{3} + 3x - 5)^{3}}} dx$$

(25)
$$\int_0^{\frac{\pi}{3}} \frac{\sin(3x)}{\left(1 + \cos(3x)\right)^{2/3}} dx$$

(9)
$$\int_{-\infty}^{1} \frac{4x - 6}{\left(x^2 - 3x + 3\right)^3} dx$$

$$(18) \int_{3}^{7} \frac{e^{\sqrt{x-3}}}{\sqrt{x-3}} \ dx$$

(26)
$$\int_{-\infty}^{0} \frac{e^{3x}}{\left(3 - e^{3x}\right)^2} dx$$

ANSWERS:

(1) 1

(10) Diverges

(19) 9/4

(2) 2

(11) -1/576

 $(20) 1 - \frac{2}{e+1}$

(3) Diverges

(12) 2

(21) Diverges

(4) 1

(13) 9/2

 $(22) \ 2/3$

(5) 1/2

(14) 1/36

(23) -12

 $(6)\ 2$

(15) -6

 $(24) \ 4/\ln(2)$

(7) Diverges

(16) Diverges

(8) Diverges

(17) Diverges

 $(25) \sqrt[3]{2}$

(9) -1

 $(18) \ 2(e^2-1)$

(26) 1/18