

201-SH3-AB - Exercises #15: Applications of Differential Equations

1. A company believes that the production of N units (in hundreds) is increasing at a rate proportional to the square number of units produced at time t in years. If 200 units are produced presently, and 300 units after 1 year, what is the production in 2 years? Find a function representing units as a function of time.
2. In a country of 3,000,000 people, the prime minister suffers a heart attack which the government does not officially publicize. Initially, 50 governmental personnel know of the attack and are spreading this information as a rumor and R is the number of people who heard the rumor at time t in weeks. The spreading of the rumor is increasing at a rate proportional to the number of people who have not heard the rumor at time t . At the end of one week, 5000 people know the rumor. Find a function representing the number of people who have heard the rumor as a function of time. How many people know the rumor after two weeks?
3. A small town decides to conduct a fund-raising drive for a new park facilities for kids. The cost is \$50,000. The initial amount given by City Hall is \$8,000. The contribution to the fund F is increasing at a rate proportional to the difference between the cost of \$50,000 and the amount F at time t in months. After one month, \$36,000 is in the fund. Find a function representing the amount in the fund as a function of time. How much will be in the fund after 3 months? How long will it take for the fund to reach \$40,000?
4. The rate of increase of the population P of a village is proportional to the population size. In 1975 the population was 2500 and in 1977 it was 3000. Find a function representing population as a function of time. In what year will the population reach 4320?
5. A rumor starts, with 20 people (in a population of 8500) having heard it. The rate at which the rumor spreads at time t in weeks is proportional to the number of people N who have not heard the rumor. At the end of 2 weeks, 1000 people heard the rumor. How many weeks will it take for 4000 people to hear the rumor?
6. The sales volume S of a company (in millions of dollars) is increasing at a rate proportional to the square of the sales volume, S . The company started with a sales volume of \$100 million. Three years later, the sales volume reached \$120 million. Find a function representing sales volume at time t . When will the sales volume reach \$165 million?
7. A car is bought for \$36,000. Its value V is depreciating at a rate proportional to the present value V . After 2 years, the vehicle is worth \$18,000. What is the value of the car after 4 years?
8. A piece of machinery is worth \$1600. Its value V is depreciating at a rate proportional to the square root of its value V . The piece of machinery will be worth \$900 in two years. When will the piece of machinery be worth \$625?
9. A rumor starts in a population of 10,000. The rumor spreads at a rate proportional to the number of people who at time t have not heard the rumor. Initially, 25 people have heard the rumor; at the end of 3 weeks, 6675 people have heard it. How many people will have heard the rumor after 6 weeks?
10. A piece of furniture is worth \$2500. Its value V is depreciating at a rate proportional to the square of its value $V(t)$ at time t in years. The piece of furniture will be worth \$1500 in two years. How much will it be worth in 3 years?
11. A minivan is bought for \$32,000. The rate of depreciation of the value V (in dollars) at time t (in years) is proportional to the present value V . After 3 years, the vehicle is worth \$25,600. What is the value of the minivan 6 years later?

12. A piece of furniture is worth \$3600. The rate of depreciation of the value V (in dollars) at the time t (in years) is proportional to the square root of the value. The piece of furniture is worth \$2500 one year later. How long will it take for the piece of furniture to be worth \$900?
13. The rate of increase of the number of members N of a club is proportional to the square of time t (the number of years after 1982). In 1982, the club had 1700 members. By 1985, the membership has increased to 4400 members. What is the number of members in 1994?
14. A company has their sales volume S (in millions of dollars) at time t (in years) increasing at a rate that is proportional to the square root of the sales volume. At the present time, the sales volume is 16 millions of dollars. The company predicts that in two years, the sales volume will be 25 millions of dollars. How many years will it take for the sales volume to reach 49 millions of dollars?
15. A piece of machinery is worth \$2500. The rate of depreciation of the value V (in dollars) at time t (in years) is proportional to the square root of its value. The piece of machinery will be worth \$900 three years later. What is the value of the piece of machinery after 6 years?
16. The production N of a company during time t (in years) is increasing at a rate that is proportional to the square of the number of units. Initially, the production is 100 units, and after 2 years, the production is 200 units. Find the number of unit produced N as a function of time t .
17. The production N of a company during time t (in years) is increasing at a rate that is proportional to the square root of time t . Initially, the production is 50 units, and after 4 years, the production is 150 units. Find the number of unit produced N as a function of time t .
18. The production N of a company during time t (in years) is decreasing at a rate that is proportional to the square of the number of units. When production starts, they produce 70 units, and in the 8th year, the production is 35 units. Find the number of unit produced N as a function of time.
19. The population of a city is increasing at a rate proportional to the difference between the maximum population of 40,000 and the population P (in thousands) at time t (years after 1993). If the population in 1993 was 20,000 and in 1998 was 25,000, what will be the population in 2003? Find a function representing population as a function of time.

ANSWERS

1. $N(t) = \frac{6}{3-t}$
In 2 years, there will be 600 units produced.
2. $R(t) = 3,000,000 - 2,999,950(\frac{59900}{59999})^t$
9941 people will know after two weeks.
3. $F(t) = 50 - 42(\frac{1}{3})^t$
After three months, there is 48,444.44 in the fund.
It will take 1.3 months for the fund to reach \$40,000
4. $P(t) = 2500(\frac{6}{5})^{t/2}$
the population will reach 4320 after 6 years, in 1981.
5. About 10.32 weeks
6. $S(t) = \frac{1800}{18-t}$
Sales volume will reach 165 million in a little over 7 years
7. \$9000
8. 3 years
9. $R(t) = 9975^{-t/3} + 10000$
8891 people will have heard the rumour after 6 weeks.
10. \$1250
11. \$20,480
12. 3 years
13. 174,500 members
14. 6 years
15. \$100
16. $N(t) = \frac{400}{4-t}$
17. $N(t) = \frac{25}{2}t^{3/2} + 50$
18. $N(t) = \frac{560}{t+8}$
19. $P(t) = 40 - 20(\frac{3}{4})^{t/5}$
the population in 10 years, in 2003, will be 28,750.