

201-SH2-AB - Exercises #18 - Marginal Functions

1. The weekly demand of Pulsar DVRs is given by the demand equation

$$p = -0.02x + 300 \quad (0 \leq x \leq 15\,000)$$

where p denotes the wholesale unit price in dollars and x denotes the quantity demanded. The weekly total cost function associated with manufacturing these recorders is

$$C(x) = 0.000003x^3 - 0.04x^2 + 200x + 70\,000$$

dollars.

- (a) Find the revenue function R and the profit function P .
 - (b) Find the marginal cost function C' , the marginal revenue function R' , and the marginal profit function P' .
 - (c) Find the marginal average cost function \overline{C}' .
 - (d) Compute $C'(3000)$, $R'(3000)$, and $P'(3000)$, and interpret your results.
2. The total weekly cost (in dollars) incurred by Lincoln Records in pressing x compact discs is

$$C(x) = 2000 + 2x - 0.0001x^2 \quad (0 \leq x \leq 6000)$$

- (a) What is the actual cost incurred in producing the 1001st disc and the 2001st disc?
 - (b) What is the marginal cost when $x = 1000$ and 2000?
 - (c) Find the average cost function \overline{C} and the marginal average cost function \overline{C}' .
3. A division of Ditton Industries manufactures the Futura model microwave oven. The daily cost (in dollars) of producing these microwave ovens is

$$C(x) = 0.0002x^3 - 0.06x^2 + 120x + 5000$$

where x stands for the number of units produced.

- (a) What is the actual cost incurred in manufacturing the 101st oven? The 201st oven? The 301st oven?
 - (b) What is the marginal cost when $x = 100$, 200 and 300?
 - (c) Find the average cost function \overline{C} and the marginal average cost function \overline{C}' .
4. Custom Office makes a line of executive desks. It is estimated that the total cost for making x units of their Senior Executive model is

$$C(x) = 100x + 200\,000$$

dollars/year.

- (a) Find the average cost function \overline{C} .
- (b) Find the marginal average cost function \overline{C}' .
- (c) ★ What happens to $\overline{C}(x)$ when x is very large? Interpret your results.

5. The management of ThermoMaster Company, whose Mexican subsidiary manufactures an indoor-outdoor thermometer, has estimated that the total weekly cost (in dollars) for producing x thermometers is

$$C(x) = 5000 + 2x$$

- (a) Find the average cost function \overline{C} .
 - (b) Find the marginal average cost function \overline{C}' .
 - (c) ★ What happens to $\overline{C}(x)$ when x is very large? Interpret your results.
6. Williams Commuter Air Service realizes a monthly revenue of

$$R(x) = 8000x - 100x^2$$

dollars when the price charged per passenger is x dollars.

- (a) Find the marginal revenue R' .
 - (b) Compute $R'(39)$, $R'(40)$ and $R'(41)$.
7. The management of Acrosonic plans to market the ElectroStat, an electrostatic speaker system. The marketing department has determined that the demand function for these speakers is

$$p = -0.04x + 800 \quad (0 \leq x \leq 20\,000)$$

where p denotes the speaker's unit price (in dollars) and x denotes the quantity demanded.

- (a) Find the revenue function R .
- (b) Find the marginal revenue function R' .
- (c) Compute $R'(5000)$, and interpret your results.
- (d) Acrosonic's production department estimates that the total cost (in dollars) incurred in manufacturing x ElectroStat speaker systems in the first year of production will be

$$C(x) = 200x + 300\,000$$

Find the profit function P .

- (e) Find the marginal profit function P' .
 - (f) Compute $P'(5000)$ and $P'(8000)$.
8. Lynbrook West, an apartment complex, has 100 two-bedroom units. The monthly profit (in dollars) realized from renting x apartments is

$$P(x) = -10x^2 + 1760x - 50\,000$$

- (a) What is the actual profit realized from renting the 51st unit, assuming that 50 units have already been rented?
- (b) Compute the marginal profit when $x = 50$, and compare your result with that obtained in part (a).

9. The weekly demand for the Pulsar 25 colour LED television is

$$p = 600 - 0.05x \quad (0 \leq x \leq 12\,000)$$

where p denotes the wholesale unit price in dollars and x denotes the quantity demanded. The weekly total cost function associated with manufacturing the Pulsar 25 is given by

$$C(x) = 0.000002x^3 - 0.03x^2 + 400x + 80\,000$$

where $C(x)$ denotes the total cost incurred in producing x sets.

- (a) Find the revenue function R and the profit function P .
 - (b) Find the marginal cost function C' , the marginal revenue function R' , and the marginal profit function P' .
 - (c) Compute $C'(2000)$, $R'(2000)$, and $P'(2000)$, and interpret your results.
 - (d) Find the average cost function \overline{C} associated with the total cost function C .
 - (e) What is the marginal average cost function \overline{C}' ?
 - (f) Compute $\overline{C}'(5000)$ and $\overline{C}'(10\,000)$, and interpret your results.
10. Pulsar manufactures a series of 20-in. flat-tube LCD televisions. The quantity x of these sets demanded each week is related to the wholesale unit price p by the equation

$$p = -0.006x + 180$$

The weekly total cost incurred by Pulsar for producing x sets is

$$C(x) = 0.000002x^3 - 0.02x^2 + 120x + 60\,000$$

dollars.

- (a) Find the revenue function R and the profit function P .
 - (b) Find the marginal cost function C' , the marginal revenue function R' , and the marginal profit function P' .
 - (c) Compute $C'(2000)$, $R'(2000)$, and $P'(2000)$, and interpret your results.
 - (d) Find the average cost function \overline{C} associated with the total cost function C .
 - (e) What is the marginal average cost function \overline{C}' ?
 - (f) Compute $\overline{C}'(5000)$ and $\overline{C}'(10\,000)$, and interpret your results.
11. The quantity of Sensitech laser gaming mice demanded each month is related to the unit price by the equation

$$p = \frac{50}{0.01x^2 + 1} \quad (0 \leq x \leq 20)$$

where p is measured in thousands of dollars and x in units of a thousand.

- (a) Find the revenue function R .
- (b) Find the marginal revenue function R' .
- (c) Compute $R'(2)$, and interpret your results.

12. The total weekly cost in dollars incurred by Herald Media Corp. in producing x DVDs is given by the total cost function

$$C(x) = 2500 + 2.2x \quad (0 \leq x \leq 8000)$$

- (a) What is the marginal cost when $x = 1000$ and 2000 ?
 - (b) Find the average cost function \overline{C} and the marginal average cost function \overline{C}' .
 - (c) Using the results from part (b), show that the average cost incurred by Herald in producing a DVD approaches \$2.20/disc when the level of production is high enough.
13. The total daily cost (in dollars) incurred by Delta Electronics in producing x MP3 players is

$$C(x) = 0.0001x^3 - 0.02x^2 + 24x + 2000 \quad (0 \leq x \leq 500)$$

where x stands for the number of units produced.

- (a) What is the actual cost incurred in manufacturing the 301st MP3 player, assuming that the 300th player was manufactured?
 - (b) What is the marginal cost when $x = 300$?
14. The marketing department of Telecon has determined that the demand for their smartphones obeys the relationship

$$p = -0.02x + 600 \quad (0 \leq x \leq 30\,000)$$

where p denotes the phone's unit price (in dollars) and x denotes the quantity demanded.

- (a) Find the revenue function R .
 - (b) Find the marginal revenue function R' .
 - (c) Compute $R'(10\,000)$, and interpret your result.
15. The weekly demand for the LectroCopy photocopying machine is given by the demand equation

$$p = 2000 - 0.04x \quad (0 \leq x \leq 50\,000)$$

where p denotes the wholesale unit price in dollars and x denotes the quantity demanded. The weekly total cost function for manufacturing these copiers is given by

$$C(x) = 0.000002x^3 - 0.02x + 1000x + 120\,000$$

where $C(x)$ denotes the total cost incurred in producing x units.

- (a) Find the revenue function R , the profit function P , and the average cost function \overline{C} .
- (b) Find the marginal cost function C' , the marginal revenue function R' , the marginal profit function P' , and the marginal average cost function \overline{C}' .
- (c) Compute $C'(3000)$, $R'(3000)$, and $P'(3000)$.
- (d) Compute $\overline{C}'(5000)$ and $\overline{C}'(8000)$, and interpret your results.

16. The Custom Office makes a line of executive desks. It is estimated that the total cost for making x units of the Junior Executive model is

$$C(x) = 80x + 150\,000 \quad (0 \leq x \leq 20\,000)$$

dollars/year.

- (a) Find the average cost function \overline{C} .
- (b) Find the marginal average cost function \overline{C}' .
- (c) ★ What happens to $\overline{C}(x)$ when x is very large? Interpret your result.

Answers

1. (a) $R(x) = -0.02x^2 + 300x$ and $P(x) = 0.000003x^3 + 0.02x^2 + 100x - 70\,000$
 (b) $C'(x) = 0.000009x^2 - 0.08x + 200$
 $R'(x) = -0.04x + 300$
 $P'(x) = -0.000009x^2 + 0.04x + 100$
 (c) $C'(x) = 0.000006x - 0.04 - \frac{70\,000}{x^2}$
 (d) $C'(3000) = 0.000009(3000)^2 - 0.08(3000) + 200 = 41$. When the level of production is already 3000 recorders, the actual cost of producing one additional recorder is approximately \$41.
 $R'(3000) = -0.04(3000) + 300 = 180$. The actual revenue to be realized from selling the 3001st recorder is approximately \$180.
 $P'(3000) = -0.000009(3000)^2 + 0.04(3000) + 100 = 139$. The actual profit realized from selling the 3001st DVR is approximately \$139.
2. (a) \$1.80 and \$1.60
 (b) \$1.80 and \$1.60
 (c) $\overline{C} = \frac{2000}{x} + 2 - 0.0001x$ and $\overline{C}' = -\frac{2000}{x^2} - 0.0001$
3. (a) \$114, \$120.10 and \$138.10
 (b) \$114, \$120 and \$138
 (c) $\overline{C} = 0.0002x^2 - 0.06x + 120 + \frac{5000}{x}$ and $\overline{C}' = 0.0004x - 0.06 - \frac{5000}{x^2}$
4. (a) $\overline{C} = 100 + \frac{200\,000}{x}$
 (b) $\overline{C}' = -\frac{200\,000}{x^2}$
 (c) $\overline{C}(x)$ approaches \$100 if the production level is very high.
5. (a) $\overline{C} = \frac{5000}{x} + 2$
 (b) $\overline{C}' = -\frac{5000}{x^2}$
 (c) $\overline{C}(x)$ approaches \$2 if the production level is very high.
6. (a) $R' = 8000 - 200x$

- (b) $R'(39) = 200$, $R'(40) = 0$ and $R'(41) = -200$.
7. (a) $R = -0.04x^2 + 800x$
 (b) $R' = -0.08x + 800$
 (c) $R'(5000) = 400$. The actual revenue to be realized from selling the 5001st speaker is approximately \$400.
 (d) $P = -0.04x^2 + 600x - 300\,000$
 (e) $P' = -0.08x + 600$
 (f) $P'(5000) = 200$ and $P'(8000) = -40$.
8. (a) \$750
 (b) \$760
9. (a) $R = 600x - 0.05x^2$ and $P = -0.000002x^3 - 0.02x^2 + 200x - 80\,000$
 (b) $C' = 0.000006x^2 - 0.06x + 400$, $R' = -0.000006x^2 - 0.04x + 200$
 (c) $C'(2000) = 304$, $R'(2000) = 400$, $P'(2000) = 96$
 (d) $\bar{C} = 0.000002x^2 - 0.03x + 400 + \frac{80\,000}{x}$
 (e) $\bar{C} = 0.000004x - 0.03 - \frac{80\,000}{x^2}$
 (f) $\bar{C}'(5000) = -0.0132$ and $\bar{C}'(10\,000) = 0.0092$. The marginal average cost is negative (average cost is decreasing) when 5000 units are produced and positive (average cost is increasing) when 10 000 units are produced.
10. (a) $R = -0.006x^2 + 180x$ and $P = -0.000002x^3 + 0.014x^2 + 60x - 60\,000$
 (b) $C' = 0.000006x^2 - 0.04x + 120$, $R' = -0.012x + 180$
 (c) $C'(2000) = 64$, $R'(2000) = 156$, $P'(2000) = 92$
 (d) $\bar{C} = 0.000002x^2 - 0.02x + 120 + \frac{60\,000}{x}$
 (e) $\bar{C} = 0.000004x - 0.02 - \frac{60\,000}{x^2}$
 (f) $\bar{C}'(5000) = -0.0024$ and $\bar{C}'(10\,000) = 0.0194$. The marginal average cost is negative (average cost is decreasing) when 5000 units are produced and positive (average cost is increasing) when 10 000 units are produced.
11. (a) $R = \frac{50x}{0.01x^2 + 1}$
 (b) $R' = \frac{50 - 0.5x^2}{(0.01x^2 + 1)^2}$
 (c) $R'(2) = 44\,380$. When the level of production is 2000 units, the revenue increases at the rate of \$44,380 per additional 1000 units produced
12. (a) \$2.20 and \$2.20
 (b) $\bar{C} = \frac{2500}{x} + 2.2$ and $\bar{C}' = -\frac{2500}{x^2}$
 (c) $\lim_{x \rightarrow \infty} \left(\frac{2500}{x} + 2.2 \right) = 2.2$

13. (a) \$39.07
(b) \$39
14. (a) $R = -0.02x^2 + 600x$
(b) $R' = -0.04x + 600$
(c) $R'(10\,000) = 200$. The sale of the 10 001st phone will bring a revenue of \$200.
15. (a) $R = 2000x - 0.04x^2$
 $P = -0.000002x^3 - 0.02x^2 + 1000x - 120\,000$
 $\overline{C} = 0.000002x^2 - 0.02x + 1000 + \frac{120\,000}{x}$
 (b) $C' = 0.000006x^2 - 0.04x + 1000$
 $R' = 2000 - 0.08x$
 $P' = -0.000006x^2 - 0.04x + 1000$
 $\overline{C}' = 0.000004x - 0.02 - \frac{120000}{x^2}$
 (c) $C'(3000) = 934$, $R'(3000) = 1760$, and $P'(3000) = 826$
 (d) $\overline{C}'(5000) = -0.0048$ and $\overline{C}'(8000) = 0.010125$. At a production level of 5000, the average cost is decreasing by 0.48¢/unit. At a production level of 8000, the average cost is increasing by 1.0125¢/unit.
16. (a) $\overline{C} = 80 + \frac{150\,000}{x}$
 (b) $\overline{C}' = -\frac{150\,000}{x^2}$
 (c) If the production level is very high, then the unit cost approaches \$80/desk.

Source: Soo T. Tan. *Applied Calculus for the Managerial, Life, and Social Sciences*. 10th Edition