201-SH2-AB - Exercises #17 - Optimization

(1) The selling price in dollars/unit of a product is $p = -\frac{1}{3}x^2 - \frac{3}{2}x + 810$. How many units should be made in order to maximize the revenue and what is the maximum revenue?

(2) A company manufactures and sells x units of a product per week. The weekly average cost in dollars per unit is $\overline{C} = 8x + 331 + \frac{5000}{x}$ and the selling price in dollars per unit is $p = -\frac{1}{3}x^2 - 6x + 2160$. Find the weekly maximum profit, the production level that will realize the maximum profit and the price that the company should charge for each unit.

(3) If the average manufacturing cost (in dollars per unit) of a product is given by $\overline{C} = x^2 + 35x + 82 + \frac{11500}{x}$, where x is the number of units manufactured, and the selling price in dollars per unit is $p = -x^2 - 31x + 5500$, what production level will maximize the profit? What is the maximum profit?

(4) A travel agency organizes student trips to Cuba for the study break. At least 70 students must sign up. For 70 students, the price would be $900 per student. For each extra student, price decreases by $10. How many student should sign up to maximize the revenue? What is the maximum revenue?

(5) The student assistance fund is starting a campaign asking teachers for donations. From experience, they know that 25 teachers will donate if they ask for donations of $50. Each time they lower the requested amount by $2, they know five more teachers will make a donation. How much should they ask teachers to donate to maximize the total donations? How many teachers would donate? What would the total be?

(6) The selling price in dollars/unit of a product is $p = -\frac{1}{3}x^2 + 2x + 96$. How many units should be made in order to maximize the revenue and what is the maximum revenue?

(7) The cost in dollars to manufacture x units of a product is $C = x^2 + 3000x - 100\sqrt{x^3}$. How many units should be made in order to minimize the average cost per unit and what is the minimum average cost?

(8) A Tire Company has 20 stores on the island of Montreal, each of which has an average income of $7000 per week. After study, the company notices that for each new store opened, the average weekly income drops by $70. How many new stores should be opened to maximize income?

(9) The price to sell x units of a product is given by $p = \frac{100\,000}{250+x} - 100$. What price will maximize revenue and what is the maximum revenue?

(10) When the price is $p = 6245 - \frac{5}{6}\sqrt{x}$, x units will be sold each month. The average cost per unit is $\overline{C}(x) = 60 + \frac{700\,000}{x}$. What price will maximize the monthly profit? What is the maximum monthly profit?

(11) An new apartment building has 100 apartments. When the rent is $1200 per month, all the units will be occupied. Market analysis discovers that for each $30 increase in rent, two more units will be empty. What monthly rent will maximize the revenue?

(12) If a farmer harvests his cherries immediately, there will be an average of 48kg of fruit per tree and he can sell the cherries for $4/kg. For each additional week he waits, each tree will produce 3kg more cherries, but the price decreases by $0.20/kg. When should the farmer harvest his cherries? (Note: the farmer can wait upto 4 additional weeks.)

(13) A company manufactures and sells x units of a product per week. The weekly average cost in dollars per unit is $\overline{C} = \frac{1}{3}x^2 + 9x + 17 + \frac{1552}{x}$ and the selling price in dollars per unit is $p = -\frac{2}{3}x^2 - 12x + 7370$. Find the weekly maximum profit, the production level that will realize the maximum profit and the price that the company should charge for each unit.

(14) If the average manufacturing cost (in dollars per unit) of a product is given by $\overline{C} = 2x^2 + 7x + 56 + \frac{1597}{x}$, where x is the number of units manufactured, and the selling price in dollars per unit is $p = -x^2 - 11x + 4781$, what is the production level that will maximize the profit? What is the maximum profit?

*(15) A shop sells boxes of mixed greeting cards. Each box of cards costs 70 cents to make. When the price is 3 dollars per box, 35 boxes are sold each month . For every 20 cents the price is lowered 10 more boxes are sold per month. What price would maximize *profit*? How many boxes would be sold at this price? What would the maximum profit be? (Hint: start by finding the equations for the revenue and the cost).

*(16) An radio station has studied the listening habits of local residents from 5pm to midnight. The results for a typical week night indicate that x hours after 5pm, the percentage of the population listening to this station is given by $\frac{-1}{4}x^3 + \frac{27}{8}x^2 - \frac{27}{2}x + 30$. (a) At what time between 5pm. and midnight are there the most listeners? (b) At what time between 5pm and midnight are there the fewest listeners?

(17) A company manufactures and sells x units of a product per day. If the daily cost in dollars is $C = 3x^2 + 44x + 13\,417$ and the revenue in dollars is $R = -\frac{1}{3}x^3 - x^2 + 4517x$, find the maximum profit.

(18) An area of 4200 square meters is to be enclosed by a fence, and separated into 2 equal rectangular fields. The inner fence costs \$10 per linear meter and the outer fence costs \$30 per linear meter. Find the minimum cost of the fence.

(19) A farmer has \$3600 to fence a land next to a river. She wants to enclose a rectangular field, and the fence opposite to the river costs \$15 per linear meter while the fence of the other two sides costs \$25 per linear meter. (The side along the river does not need a fence.) Find the maximum area of the land she can enclose.

(20) A box with no top will be made from a 2-foot by 3-foot rectangular piece of material by cutting equal squares from the corner and folding up the sides. Find the maximum volume of box.
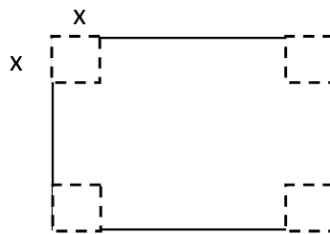

(21) A shipping company requires boxes to have a square base, and the sum of the dimension (length + width + height) must be 1.5m or less. What are the dimension of the box with the largest volume?

(22) A rancher wants to build a fence around 4500 square feet of his land, and subdivide it into 2 equal rectangular fields. The inner fence costs \$20 per linear foot and the outer fence costs \$40 per linear foot. Find the minimum cost of the fence.

(23) A rectangular prism storage container with a square base and a total volume of $60m^3$ is to be constructed (with a cover) to hold in evil mini gremlins. The four side walls are cut out of wood panels that cost $18/m^2$ while the both the square base and cover are made of steel that is currently on sale for $40/m^2$. What should the dimensions of the container be so that the cost is minimized? What is the minimal cost?

(24) A company wants to enclose a storage area with a fence, next to a wall of a building. The storage area will be 6400 square meters. The fence opposite the wall of the building costs $10 per linear meter and the fence of the other two sides costs $20 per linear meter. Find the dimensions of the storage area to minimize the cost of the fence.

(25) A park wants to create a fenced off-leash area for dogs with separate sections for big and small dogs. They have decided on the proportions seen in the diagram. If there is exactly 140m of fencing for this project, what dimensions of the outside fence maximize the area for big dogs.
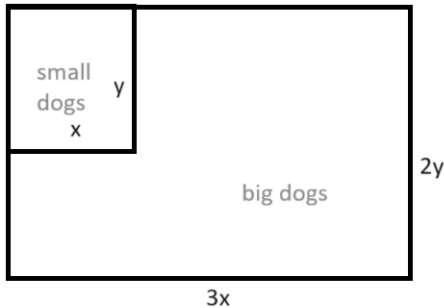
ANSWERS

(1) 27 units; $14 215.50 revenue   (2) $28 314.67 profit; 31 units; $1 653.67/unit   (3) 21 units; $54 650 profit

(4) 80 students; $64 000 revenue   (5) $30; 75 teachers; $2250 total   (6) 12 units; $864 revenue

(7) 2500 units; $500/unit.   (8) 40 new stores   (9) $100/unit; $25000 revenue

(10) $121.67/unit; $1350746.67 monthly profit   (11) $1350/month   (12) 2 weeks

(13)$196 291 profit; 43 units; $5621.33/unit   (14) 21 units; $88 934 profit

(15) Hint: $R = (3 - 0.2n)(35 + 10n)$; $C = 0.7(35 + 10n)$
.   Answer: $2.20/box; 75 boxes/month; $112.50 monthly profit

(16) (a) 5pm (b)8pm   (17) $169 157 profit   (18) $8400 cost   (19) 4320 square meters

(20) 1.056 cubic ft   (21) all dimensions are 0.5m   (22) $12 000 cost

(23) 3m by 3m by $\frac{20}{3}$m; $2160 cost   (24) 40m by 60m   (25) 30m by 20m