201-SH3-AB - Exercises #14: Differential Equations

Part I:

(1) Given $f''(x) = 30x^4 + 12x$; $f'(0) = 5$; $f(0) = -7$, find $f(x)$.

(2) Given $f''(x) = 24x^2 - 18x$; $f'(-1) = 2$; $f(1) = 4$, find $f(x)$.

(3) Given $f''(x) = 60\sqrt{x} - 48x$; $f'(1) = 25$; $f(4) = 30$, find $f(x)$.

(4) Find the cost function given $\dfrac{dC}{dx} = 5x - \dfrac{1}{x}$ and 10 units cost \$94.20.

(5) Find the cost function given $\dfrac{dC}{dx} = \dfrac{1}{x} + 2x$ and 7 units cost \$58.40.

(6) Find the demand function at $x = 90$ given $\dfrac{dR}{dx} = x^2 - 2x + 3$.

(7) Find the profit function at $x = 100$ given that $\dfrac{dP}{dx} = 2x + 20$ and profit on 20 items is \$50.

(8) Given $\dfrac{dy}{dt} = \dfrac{\sqrt{t^3} - t}{\sqrt{t^3}}$, find the function y that satisfies the condition $y(9) = 4$.

(9) Given $\dfrac{dy}{dx} = 2x^{-2} + 3x^{-1} - 1$, find the function y that satisfies the condition $y(1) = 0$.

(10) Given $f''(x) = 18x - 6x^2$, find the function $f(x)$ that satisfies the conditions $f'(1) = 20$ and $f(1) = 15$.

(11) Given $\dfrac{dy}{dt} = \dfrac{\sqrt[3]{t^2} - 4}{\sqrt[3]{t^2}}$, find the function y that satisfies the condition $y(-8) = 4$.

(12) Given $\dfrac{dy}{dx} = 4x^{-3} + 5x^{-1} + 3$, Find the function y that satisfies the condition $y(1) = 3$.

(13) Given $f''(x) = 12x^2 - 6x$, find the function $f(x)$ that satisfies the conditions $f'(1) = 8$ and $f(-1) = 5$.

(14) Find the average cost function given that the marginal cost is $0.3x^2 + 6x + 100$ and that 10 units cost \$3000.

(15) Find the demand function given that the marginal revenue is $9x^2 + 0.1x + 500$ and that the revenue from 10 units is \$8500.

(16) Find the cost function given that the marginal cost is $12x^2 + 20e^{2x}$ and that the fixed costs are \$1000.

(17) Find the demand function at $x = 16$ units given that the marginal revenue is $6\sqrt{x} + 8x + 500$.

Part II: Show that the function y is a particular solution of the given differential equation.

(18) $y = e^{x/2}$, $\quad 2y'' + 5y' - 3y = 0$

(19) $y = x^3 - \dfrac{5}{2}x$, $\quad x\dfrac{dy}{dx} - 3y = 5x$

(20) $y = 4 + 8xe^x - 3e^x$, $\quad y'' - 2y' + y - 4 = 0$

(21) $y = x^2(5 + 3\ln(x))$, $\quad xy' - 2y = 3x^2$

(22) $y = 2e^{\frac{x^4}{4}}$, $\quad \dfrac{dy}{dx} = x^3 y$

Part III: Solve the following initial value problems.

(23) $y' = y\sin(x)$, $\quad y(0) = 1$

(24) $y' = y^2\cos(x)$, $\quad y(0) = 1$

(25) $\dfrac{dy}{dx} = 3x^2 y$, $\quad y(0) = 4$

(26) $y' = y^2(2x + 1)$, $\quad y(-1) = \frac{1}{5}$

(27) $\dfrac{dy}{dx} = e^{x+2} \cdot y^2$, $\quad y(-2) = -\frac{1}{2}$

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(28) $y' = 6x^2(y-2), \quad y(2) = 3$

(29) $y' = 3e^{x-y}, \quad y(0) = 2$

(30) $y' = 4xy, \quad y(2) = 1, \quad y > 0$

(31) $y' = \dfrac{2x^2}{y}, \quad y(1) = 2, \quad y > 0$

(32) $y' = \dfrac{3x^2}{\sqrt{y}}, \quad y(1) = 9$

(33) $xy' = \dfrac{4x^2}{y}, \quad y(1) = 2, \quad y > 0$

(34) $y' = \dfrac{y}{\sqrt{x}}, \quad y(4) = 1, \quad y > 0$

(35) $y' = 2\sqrt{y}e^{3x}, \quad y(0) = \dfrac{4}{9}$

(36) $y' = 3xy - 2x, \quad y(0) = 1, \quad y > 0$

(37) $y' = 2xy + 3x^2y, \quad y(2) = 1, \quad y > 0$

(38) $y' = 2x^2y + 2x^2, \quad y(0) = 0, \quad y > -1$

(39) $y' = \dfrac{xy}{x^2+1}, \quad y(0) = 3, \quad y > 0$

ANSWERS:

(1) $f(x) = x^6 + 2x^3 + 5x - 7$

(2) $f(x) = 2x^4 - 3x^3 + 19x - 14$

(3) $f(x) = 16x^{5/2} - 8x^3 + 9x - 6$

(4) $C = \frac{5}{2}x^2 - \ln|x| - 153.50$

(5) $C = \ln|x| + x^2 + 7.45$

(6) $p(90) = \$2613$

(7) $P(100) = \$11250$

(8) $y = t - 2\sqrt{t} + 1$

(9) $y = 3\ln|x| - x - \frac{2}{x} + 3$

(10) $f(x) = -\frac{1}{2}x^4 + 3x^3 + 13x - \frac{1}{2}$

(11) $y = t - 12\sqrt[3]{t} - 12$

(12) $y = 5\ln|x| + 3x - \frac{2}{x^2} + 2$

(13) $f(x) = x^4 - x^3 + 7x + 10$

(14) $\overline{C} = 0.1x^2 + 3x + 100 + \frac{1600}{x}$

(15) $p = 3x^2 + 0.05x + 500 + \frac{495}{x}$

(16) $C = 4x^3 + 10e^{2x} + 990$

(17) $p(16) = \$580$

(23) $y = e^{1-\cos(x)}$

(24) $y = \dfrac{1}{1 - \sin(x)}$

(25) $y = 4e^{x^3}$

(26) $y = \dfrac{-1}{x^2 + x - 5}$

(27) $y = \dfrac{-1}{e^{x+2} + 1}$

(28) $y = e^{2x^3 - 16} + 2$

(29) $y = \ln\left(3e^x + e^2 - 3\right)$

(30) $y = e^{2x^2 - 8}$

(31) $y = \sqrt{\frac{4}{3}x^3 + \frac{8}{3}}$

(32) $y = \left(\frac{3}{2}x^3 + \frac{51}{2}\right)^{2/3}$

(33) $y = 2|x|$

(34) $y = e^{2\sqrt{x} - 4}$

(35) $y = \left(\frac{1}{3}e^{3x} + \frac{1}{3}\right)^2$

(36) $y = \frac{2}{3} + \frac{1}{3}e^{3x^2/2}$

(37) $y = e^{x^3 + x^2 - 12}$

(38) $y = e^{(2/3)x^3} - 1$

(39) $y = 3\sqrt{x^2 + 1}$