201-SH2-AB - Exercises #5 - Continuity

1. Match each part below to the graph which best illustrates the condition at $x = 2$.
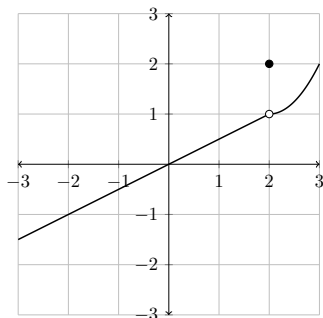
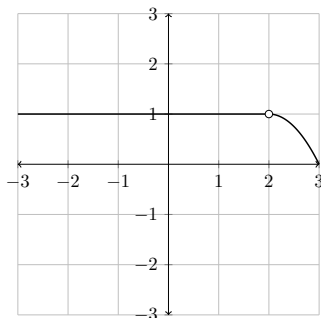   (a) $f(2)$ is undefined.   (b) $\lim\limits_{x \to 2} f(x)$ DNE   (c) $\lim\limits_{x \to 2} f(x) \neq f(2)$ (d) f is continuous at $x = 2$
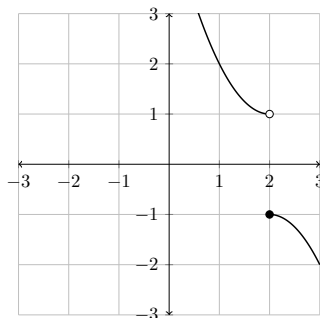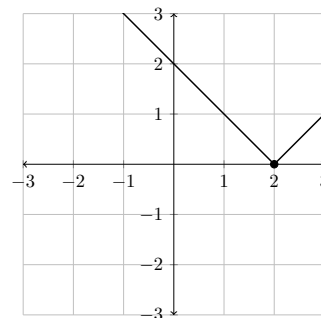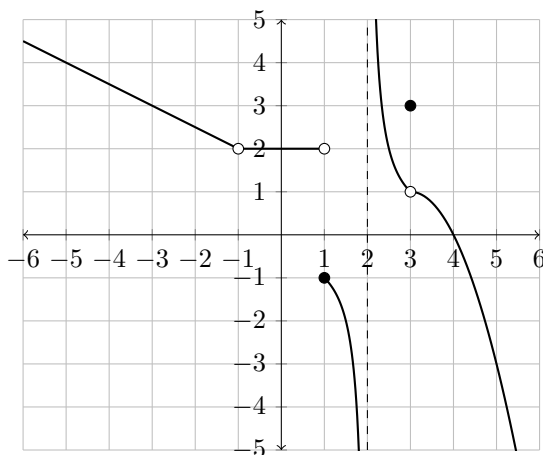
GRAPH #1:   GRAPH #2:   GRAPH #3:   GRAPH #4:

2. Given the graph of f, fill in the table that follows.

Point of Discontinuity	Criteria of Continuity NOT Satisfied (choose all that apply)		
$x =$	$\Box\, f(a)$ undefined	$\Box\, \lim\limits_{x \to a} f(x)$ DNE	$\Box\, \lim\limits_{x \to a} f(x) \neq f(a)$
$x =$	$\Box\, f(a)$ undefined	$\Box\, \lim\limits_{x \to a} f(x)$ DNE	$\Box\, \lim\limits_{x \to a} f(x) \neq f(a)$
$x =$	$\Box\, f(a)$ undefined	$\Box\, \lim\limits_{x \to a} f(x)$ DNE	$\Box\, \lim\limits_{x \to a} f(x) \neq f(a)$
$x =$	$\Box\, f(a)$ undefined	$\Box\, \lim\limits_{x \to a} f(x)$ DNE	$\Box\, \lim\limits_{x \to a} f(x) \neq f(a)$
	* Which of the points of discontinuity above are removable?		

3. Consider the function $f(x) = \dfrac{(x-4)(x+1)}{x-a}$.

    (a) Explain why $f(x)$ will always have a discontinuity at $x = a$, regardless of the value of a.

    (b) For what value(s) of a will $\lim\limits_{x \to a} f(x)$ exist?

4. * Show that $f(x) = \dfrac{x^2 - 5x + 6}{x^2 - 4}$ has a removable discontinuity at $x = 2$.

Determine the values of x at which the given function is discontinuous. Justify your answers.
*Describe the type of discontinuity.

5. $f(x) = \dfrac{x^2 + 2x - 3}{x^2 - 1}$

6. $f(x) = \dfrac{x + 2}{x^2 + x - 2}$

7. $f(x) = \dfrac{3x^2}{6x + x^2}$

8. $f(x) = \dfrac{x - 3}{x^2 - 3x}$

9. $f(x) = \dfrac{x^2}{4x - x^2}$

10. $f(x) = \dfrac{4 - x}{x^2 - 7x + 12}$

11. $f(x) = \begin{cases} \dfrac{x - 2}{(x - 3)(x + 1)} & x < 4 \\ \dfrac{3}{x - 5} & x \geq 4 \end{cases}$

12. $f(x) = \begin{cases} \dfrac{4x - 1}{x^2 - 5x - 6} & x \leq 5 \\ 3x + 2 & x > 5 \end{cases}$

13. $f(x) = \begin{cases} \dfrac{x - 3}{(x - 6)(x - 2)} & x < 4 \\ \dfrac{x + 1}{5 - x} & x \geq 4 \end{cases}$

14. $f(x) = \begin{cases} 2x - 1 & x \leq 3 \\ \dfrac{5x - 1}{x^2 - 6x + 8} & x > 3 \end{cases}$

15. $f(x) = \begin{cases} \dfrac{7}{x^2 + 2x - 15} & x < 2 \\ 5 & x = 2 \\ x^2 - 5 & x > 2 \end{cases}$

16. $f(x) = \begin{cases} \dfrac{5x + 10}{(x + 2)(x - 5)} & x \leq 0 \\ 2x^2 - 3x - 1 & x > 0 \end{cases}$

17. $f(x) = \begin{cases} \dfrac{5}{x + 1} & x \leq 0 \\ \dfrac{-3x^2 + 5x}{x} & 0 < x < 1 \\ 2x - 1 & x \geq 1 \end{cases}$

18. $f(x) = \begin{cases} \dfrac{30}{(x + 5)(x - 3)} & x < 0 \\ -3 & x = 0 \\ 5x^2 + 3x - 2 & x > 0 \end{cases}$

19. $f(x) = \begin{cases} \dfrac{(x + 1)(x - 4)}{(x - 2)(x + 1)} & x < 1 \\ \dfrac{3}{2 - x} & x \geq 1 \end{cases}$

20. $f(x) = \begin{cases} \dfrac{3x + 12}{x^2 + 2x - 8} & x \leq -2 \\ x^2 & -2 < x < 2 \\ \dfrac{4}{3 - x} & x \geq 2 \end{cases}$

21. $f(x) = \begin{cases} \dfrac{x^2 + 2x}{x^2 - 4} & x \leq 0 \\ x & 0 < x < 3 \\ \dfrac{x}{x - 5} & x \geq 3 \end{cases}$

22. $f(x) = \begin{cases} \dfrac{x - 1}{x^2 - 2x} & x < 1 \\ \dfrac{x^2 - 3x + 2}{x^2 - x - 2} & 1 \leq x \leq 3 \\ \dfrac{1}{x + 1} & 3 < x \end{cases}$

23. $f(x) = \begin{cases} \dfrac{1}{x + 4} & x \leq 1 \\ \dfrac{x^2 - 1}{x^2 + 8x - 9} & 1 < x \leq 2 \\ \dfrac{3}{x^2 + 4} & 2 < x \end{cases}$

24. $f(x) = \begin{cases} \dfrac{x - 2}{x^2 + x - 6} & x < 2 \\ \dfrac{1}{x + 3} & 2 \leq x \leq 4 \\ \dfrac{1}{x - 5} & 4 < x \end{cases}$

25. $f(x) = \begin{cases} \dfrac{x + 9}{(x - 2)(x + 4)} & x \leq 1 \\ -2 & 1 < x < 8 \\ \sqrt{3x + 1} & 8 \leq x \end{cases}$

26. $f(x) = \begin{cases} \dfrac{-2x - 1}{x + 2} & x \leq -1 \\ \sqrt{x + 1} & -1 < x \leq 3 \\ 2x - 4 & x > 3 \end{cases}$

27. $f(x) = \begin{cases} \dfrac{1}{x^2 - 4} & x \leq -4 \\ \dfrac{x^2 + 5x + 6}{x^2 - x - 12} & -4 < x \leq 0 \\ \dfrac{3}{x - 6} & x > 0 \end{cases}$

Find the value(s) of the appropriate constant(s) such that the given function is continuous everywhere.

28. $f(x) = \begin{cases} \dfrac{x^2 + 2x - 3}{x - 1} & x \neq 1 \\ k^2 & x = 1 \end{cases}$

29. $f(x) = \begin{cases} k^2 - \dfrac{12k}{x} & x \leq -4 \\ 2k - 5x & x > -4 \end{cases}$

30. $f(x) = \begin{cases} -x^2 - 7k & x < 3 \\ k^2 - \dfrac{81}{x} & x \geq 3 \end{cases}$

31. $f(x) = \begin{cases} x^2 + k^2 x - 4k & x \leq 1 \\ 7x + k & x > 1 \end{cases}$

32. $f(x) = \begin{cases} \dfrac{x^2 - 64}{x - 8} & x \neq 8 \\ k & x = 8 \end{cases}$

33. $f(x) = \begin{cases} 5 & x \leq 0 \\ ax - b & 0 < x < 8 \\ 3 & x \geq 8 \end{cases}$

34. $f(x) = \begin{cases} k^2 x^2 + 3kx - 8 & x < 2 \\ kx & x \geq 2 \end{cases}$

35. $f(x) = \begin{cases} kx + k^2 & x \neq -1 \\ 2 & x = -1 \end{cases}$

36. $f(x) = \begin{cases} kx & x \leq 2 \\ \dfrac{x^2 + 9x + 8}{x + k} & x > 2 \end{cases}$

37. $f(x) = \begin{cases} \dfrac{3}{x - k} & x < 2 \\ \dfrac{k}{1 - x} & x \geq 2 \end{cases}$

38. $f(x) = \begin{cases} ae^{-x} + be^x & x \leq 0 \\ ax^2 + b - 1 & 0 < x \leq 1 \\ \ln x & x > 1 \end{cases}$

39. $f(x) = \begin{cases} k^2 + 2x & x < -1 \\ -kx & x \geq -1 \end{cases}$

40. $f(x) = \begin{cases} k^2 + 3x & x < 1 \\ 3 + 5kx & x \geq 1 \end{cases}$

41. $f(x) = \begin{cases} ax + 5 & x \leq -3 \\ 1 - x & -3 < x \end{cases}$

42. $f(x) = \begin{cases} 2k^2 + 3x & x \leq 2 \\ x(9 - k) & x > 2 \end{cases}$

43. $f(x) = \begin{cases} x^2 + k^2 x & x \leq 1 \\ 5k + 7x & x > 1 \end{cases}$

44. $f(x) = \begin{cases} a^2 x^2 - 9 & x < -1 \\ \dfrac{2ax + 6}{2x + 3} & x \geq -1 \end{cases}$

45. $f(x) = \begin{cases} k + 10x & x < 2 \\ 3k^2 - 4kx - 5x & x \geq 2 \end{cases}$

ANSWERS

1. (a) #2   (b) #3   (c) #1   (d) #4

2. $x = -1$; $f(-1)$ undefined, $\lim\limits_{x \to -1} f(x) \neq f(-1)$
   $x = 1$; $\lim\limits_{x \to 1} f(x)$ DNE, $\lim\limits_{x \to 1} f(x) \neq f(1)$
   $x = 2$ $f(2)$ undefined, $\lim\limits_{x \to 2} f(x)$ DNE, $\lim\limits_{x \to 2} f(x) \neq f(2)$
   $x = 3$ $\lim\limits_{x \to 3} f(x) \neq f(3)$
   $x = 3$ is a removable point of discontinuity.

3. (a) $f(a)$ will be undefined regardless of the value of a.
   (b) $a = 4$ and $a = -1$.

4. Hint: Show that $\lim\limits_{x \to 2} f(x)$ exists, but that $f(2)$ does not.

5. $x = -1$: *infinite, $x = 1$: *removable

6. $x = -2$: *removable, $x = 1$: *infinite

7. $x = -6$: *infinite, $x = 0$: *removable

8. $x = 0$: *infinite, $x = 3$: *removable

9. $x = 0$: *removable, $x = 4$: *infinite

10. $x = 3$: *infinite, $x = 4$: *removable

11. $x = -1$: *infinite, $x = 3$: *infinite,
    $x = 4$: *jump, $x = 5$: *infinite

12. $x = -1$: *infinite, $x = 5$: *jump

13. $x = 2$: *infinite, $x = 4$: *jump, $x = 5$: *infinite

14. $x = 3$: *jump, $x = 4$: *infinite

15. $x = -5$: *infinite, $x = 2$: *removable

16. $x = -2$: *removable

17. $x = -1$: *infinite, $x = 1$: *jump

18. $x = -5$: *infinite, $x = 0$: *removable

19. $x = -1$: *removable, $x = 2$: *infinite

20. $x = -4$: *removable, $x = -2$: *jump, $x = 3$: *infinite

21. $x = -2$: *removable, $x = 3$: *jump, $x = 5$: *infinite

22. $x = 0$: *infinite, $x = 2$: *removable, $x = 3$: *jump

23. $x = -4$: *infinite, $x = 2$: *jump

24. $x = -3$: *infinite, $x = 4$: *jump, $x = 5$: *infinite

25. $x = -4$: *infinite, $x = 8$: *jump

26. $x = -2$: *infinite, $x = -1$: *jump

27. $x = -4$: *jump, $x = -3$: *removable, $x = 6$: *infinite

28. $k = \pm 2$

29. $k = -9, 2$

30. $k = -5, 4$

31. $k = -1, 6$

32. $k = 16$

33. $a = 1, b = -5$

34. $k = -2, 1$

35. $k = -1, 2$

36. $k = 3$ (not $k = -5$)

37. $k = 3$ (not $k = -1$)

38. $a = -1, b = 2$

39. $k = -1, 2$

40. $k = 0, 5$

41. $a = 1/3$

42. $k = -3, 2$

43. $k = -1, 6$

44. $a = -5, 3$

45. $k = -2, 5$