201-SH2-AB - Exercises #16 - Curve Sketching

Sketch the graph of each function f, using the derivatives provided. Preliminary steps usually include:

a) domain,

b) intercepts,

c) asymptotes,

d) intervals of increase/decrease and local extrema

e) intervals of concavity and points of inflection

(1) $f(x) = \dfrac{(x-2)(2x-1)}{(x+1)^2}$ with $f'(x) = \dfrac{9(x-1)}{(x+1)^3}$ and $f''(x) = \dfrac{18\,(2-x)}{(x+1)^4}$

(2) $f(x) = \left(\dfrac{x+2}{x-2}\right)^2$ with $f'(x) = \dfrac{-8(x+2)}{(x-2)^3}$ and $f''(x) = \dfrac{16\,(x+4)}{(x-2)^4}$

(3) $f(x) = \dfrac{6x^2}{4-x^2}$ with $f'(x) = \dfrac{48x}{(4-x^2)^2}$ and $f''(x) = \dfrac{48\,(3x^2+4)}{(4-x^2)^3}$

(4) $f(x) = \dfrac{6}{x^2+4x}$ with $f'(x) = \dfrac{-12(x+2)}{(x^2+4x)^2}$ and $f''(x) = \dfrac{12\,(3x^2+12x+16)}{(x^2+4x)^3}$

(5) $f(x) = \dfrac{1}{5}x^5 - \dfrac{2}{3}x^3 + x$ with $f'(x) = (x+1)^2\,(x-1)^2$ and $f''(x) = 4x\,\,(x+1)\,(x-1)$

(6) $f(x) = x^3 + 9x^2 + 120$ with $f'(x) = 3x(x+6)$ and $f''(x) = 6x + 18$

(7) $f(x) = (x-1)^4\,(3x+2)$ with $f'(x) = 5\,(x-1)^3\,(3x+1)$ and $f''(x) = 60x\,(x-1)^2$

(8) $f(x) = x + \dfrac{1}{x+2}$ with $f'(x) = \dfrac{(x+1)(x+3)}{(x+2)^2}$ and $f''(x) = \dfrac{2}{(x+2)^3}$

(9) $f(x) = \dfrac{x^2}{x-1}$ with $f'(x) = \dfrac{x(x-2)}{(x-1)^2}$ and $f''(x) = \dfrac{2}{(x-1)^3}$

(10) $f(x) = \dfrac{(x-2)(3x+1)}{(x-1)^2}$ with $f'(x) = \dfrac{9-x}{(x-1)^3}$ and $f''(x) = \dfrac{2\,(x-13)}{(x-1)^4}$

(11) $f(x) = \left(\dfrac{x+3}{x+1}\right)^2$ with $f'(x) = \dfrac{-4(x+3)}{(x+1)^3}$ and $f''(x) = \dfrac{8\,(x+4)}{(x+1)^4}$

(12) $f(x) = \dfrac{4}{x^2-4x}$ with $f'(x) = \dfrac{8(2-x)}{(x^2-4x)^2}$ and $f''(x) = \dfrac{8(3x^2-12x+16)}{(x^2-4x)^3}$

(13) $f(x) = \dfrac{1}{5}x^5 - \dfrac{8}{3}x^3 - 9x$ with $f'(x) = \left(x^2-9\right)\left(x^2+1\right)$ and $f''(x) = 4x\left(x^2-4\right)$

(14) $f(x) = (x+2)^4\,(4-3x)$ with $f'(x) = 5\,(x+2)^3\,(2-3x)$ and $f''(x) = -60x\,(x+2)^2$

(15) $f(x) = \dfrac{x^2}{x+2}$ with $f'(x) = \dfrac{x(x+4)}{(x+2)^2}$ and $f''(x) = \dfrac{8}{(x+2)^3}$

(16) $f(x) = \dfrac{(x-4)(x+1)}{x^2-4}$ with $f'(x) = \dfrac{3x^2+12}{(x^2-4)^2}$ and $f''(x) = \dfrac{-6x(x^2+12)}{(x^2-4)^3}$

*(17) $f(x) = \sqrt[3]{2x(x-3)^2}$ with $f'(x) = \dfrac{2(x-1)}{\sqrt[3]{4x^2(x-3)}}$ and $f''(x) = \dfrac{-4}{\sqrt[3]{4x^5(x-3)^4}}$

*(18) $f(x) = -(x+1)(x-4)^{2/3}$ with $f'(x) = \dfrac{5(2-x)}{3(x-4)^{1/3}}$ and $f''(x) = \dfrac{10(5-x)}{9(x-4)^{4/3}}$

Sketch the graph of a function f satisfying the following requirements.

*(19) Points at $(-3,2)$, $(-2,0)$, $(0,-2)$, $(1,0)$, $\lim\limits_{x\to+\infty} f(x) = 1$

for $x < -3$: $f'(x) < 0$; $f''(x) < 0$
for $-3 < x < 0$: $f'(x) < 0$; $f''(x) > 0$
for $x > 0$: $f'(x) > 0$; $f''(x) < 0$

*(20) Points at $(-3,0)$, $(-2,1)$, $(-1,0)$, $(0,-0.5)$, $(1,-2)$, $\lim\limits_{x\to+\infty} f(x) = 0$

$f'(x) < 0$ for $-2 < x < 1$; $f'(x) > 0$ for $x < -2$ or $x > 1$
$f''(x) < 0$ for $x < -2$ or $x > -1$; $f''(x) > 0$ for $-2 < x < -1$

*(21) Points at $(-2,0)$, $(-1,-1)$, $(0,0)$ vertical asymptote at $x = 1$ and $\lim\limits_{x\to+\infty} f(x) = 2$

for $x < -1$: $f'(x) < 0$; $f''(x) < 0$
for $-1 < x < 0$: $f'(x) > 0$; $f''(x) < 0$
for $0 < x < 1$: $f'(x) > 0$; $f''(x) > 0$
for $x > 1$: $f'(x) < 0$; $f''(x) > 0$

*(22) Points at $(-3,0)$, $(-1,-1)$, $(0,-2)$, $(1,-1)$ vertical asymptote at $x = -2$

$\lim\limits_{x\to-\infty} f(x) = 1$ $\lim\limits_{x\to+\infty} f(x) = 0$
$f'(x) < 0$ for $x < -2$ or $-1 < x < 0$; $f'(x) > 0$ for $-2 < x < -1$ or $0 < x < 1$ or $x > 1$
$f''(x) < 0$ for $x < -2$ or $-2 < x < -1$ or $x > 1$; $f''(x) > 0$ for $-1 < x < 1$

*(23) Points at $(-2,2)$, $(0,1)$, $(2,2)$ and $\lim\limits_{x\to-\infty} f(x) = 0$

for $x < -2$: $f'(x) > 0$; $f''(x) > 0$
for $-2 < x < 0$: $f'(x) < 0$; $f''(x) < 0$
for $0 < x < 2$: $f'(x) > 0$; $f''(x) < 0$
for $x > 2$: $f'(x) > 0$; $f''(x) > 0$

*(24) Points at $(-2,1)$, $(0,-1)$, $(2,0)$ and $\lim\limits_{x\to+\infty} f(x) = 2$

$f'(x) < 0$ for $x < 0$; $f'(x) > 0$ for $x > 0$
$f''(x) < 0$ for $-2 < x < 0$ or $x > 2$; $f''(x) > 0$ for $x < -2$ or $0 < x < 2$

*(25) Points at $(-2,0)$, $(0,0)$ vertical asymptote at $x = -1$ and $\lim\limits_{x\to+\infty} f(x) = 1$

for $x < -2$: $f'(x) > 0$; $f''(x) < 0$
for $-2 < x < -1$: $f'(x) < 0$; $f''(x) < 0$
for $-1 < x < 0$: $f'(x) < 0$; $f''(x) > 0$
for $x > 0$: $f'(x) > 0$; $f''(x) < 0$

*(26) Domain: $-3 < x \leqslant 4$; Points at $(-1,0)$, $(0,-1)$, $(1,0)$, $(4,2)$

$f'(x) < 0$ for $-3 < x < 0$; $f'(x) > 0$ for $0 < x < 4$

$f''(x) < 0$ for $-3 < x < -1$ or $1 < x < 4$; $f''(x) > 0$ for $-1 < x < 1$

*(27) Domain: $-2 \leqslant x < 4$; Points at $(-2,-1)$, $(0,0)$, $(2,2)$

$f'(x) < 0$ for $2 < x < 4$; $f'(x) > 0$ for $-2 < x < 2$

$f''(x) < 0$ for $-2 < x < 0$ or $2 < x < 4$; $f''(x) > 0$ for $0 < x < 2$

*(28) Domain: $-4 < x \leqslant 3$; Points at $(0,1)$, $(1,0)$, $(3,2)$; vertical asymptote at $x = -2$

for $-4 < x < -2$: $f'(x) > 0$; $f''(x) > 0$

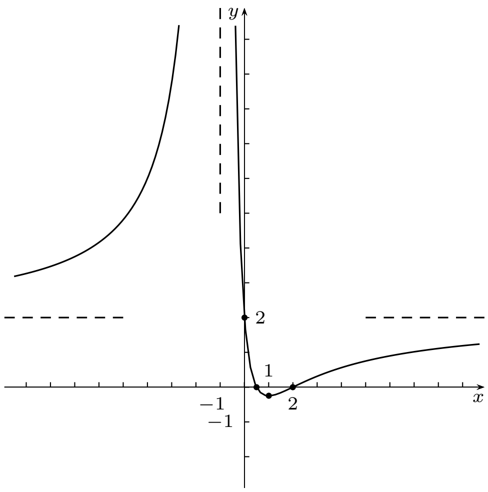
for $-2 < x < 0$: $f'(x) < 0$; $f''(x) > 0$

for $0 < x < 1$: $f'(x) < 0$; $f''(x) < 0$

for $1 < x < 3$: $f'(x) > 0$; $f''(x) < 0$

ANSWERS:

(1)



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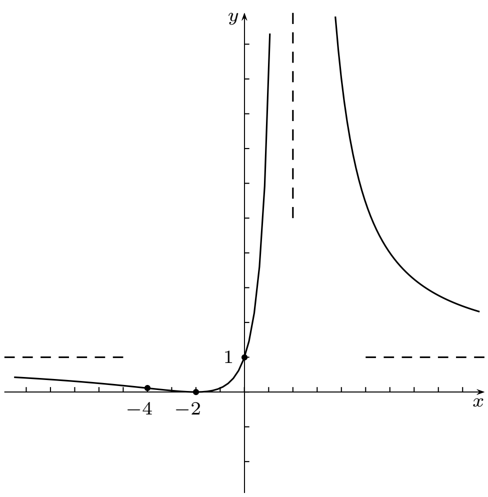

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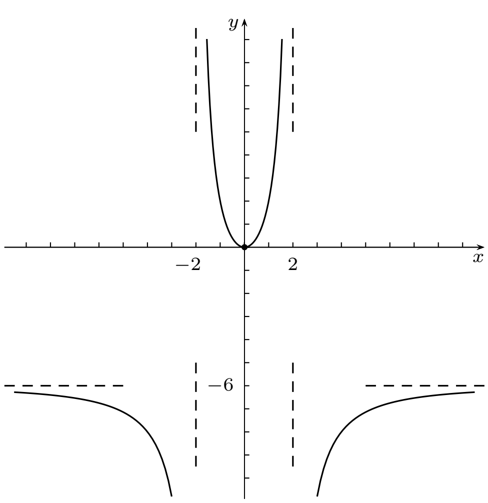

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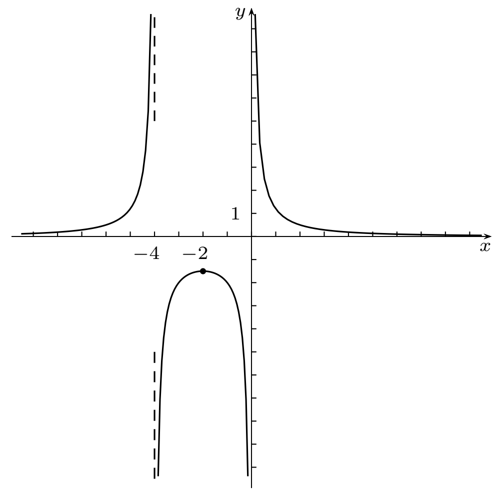

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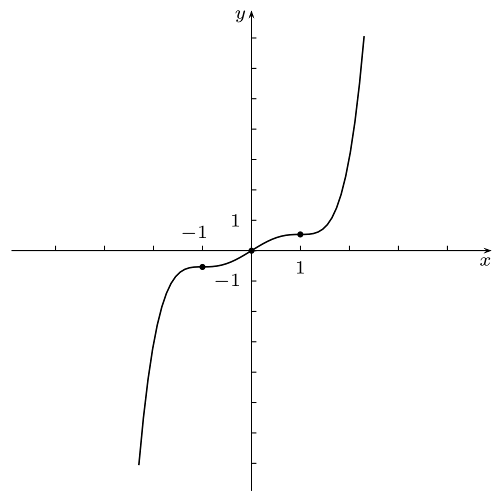

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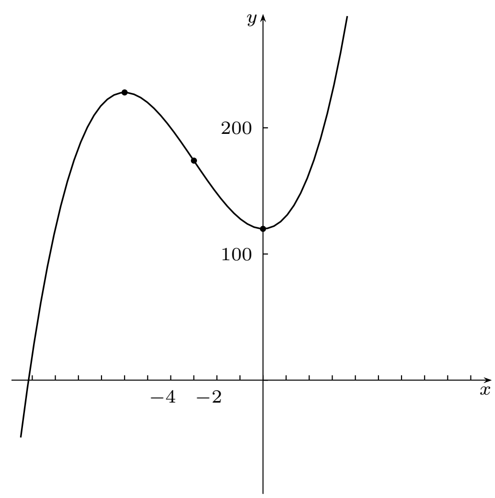

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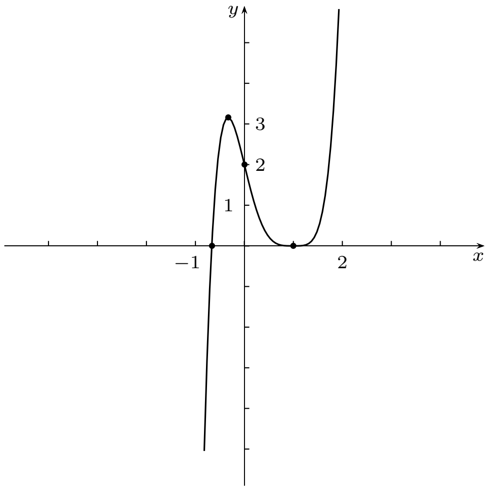

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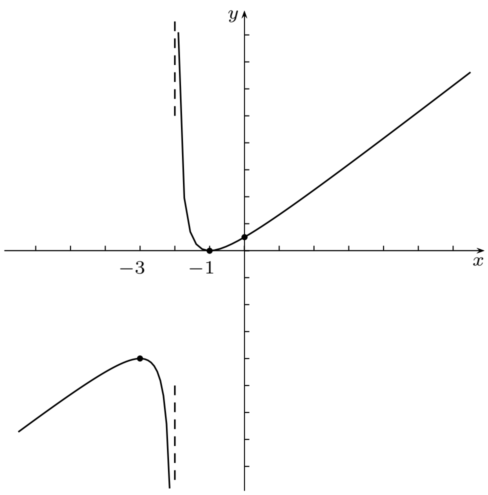

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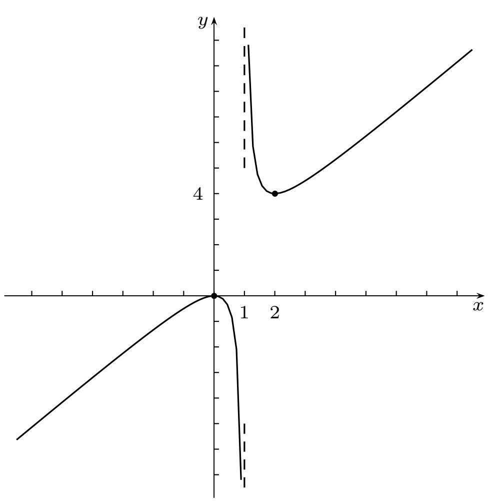

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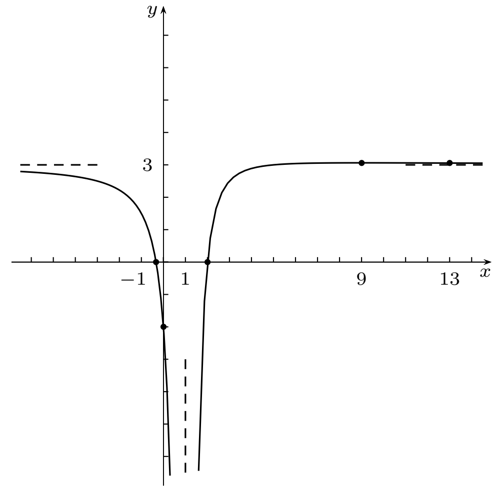

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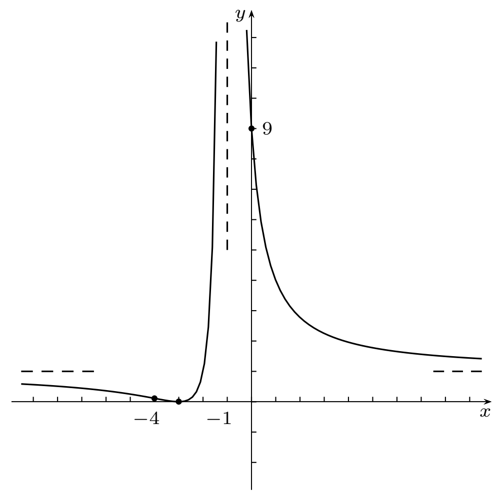

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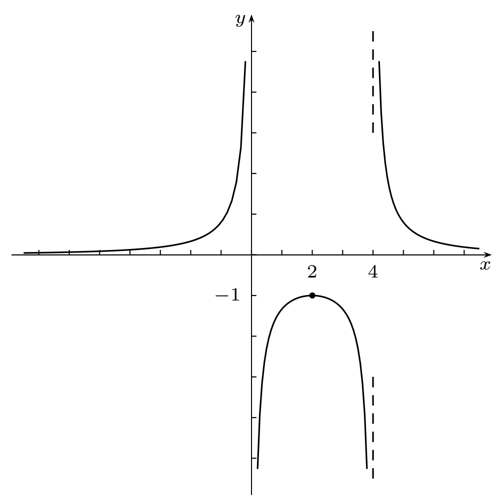

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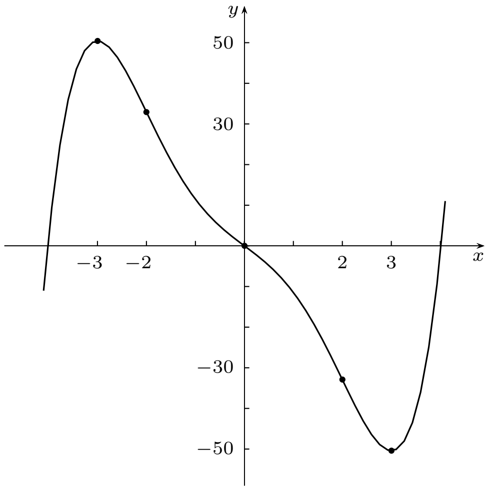

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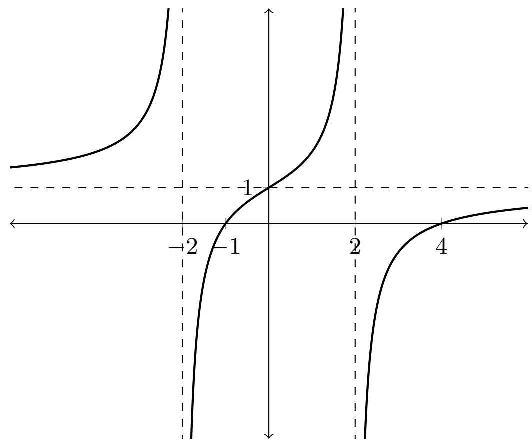

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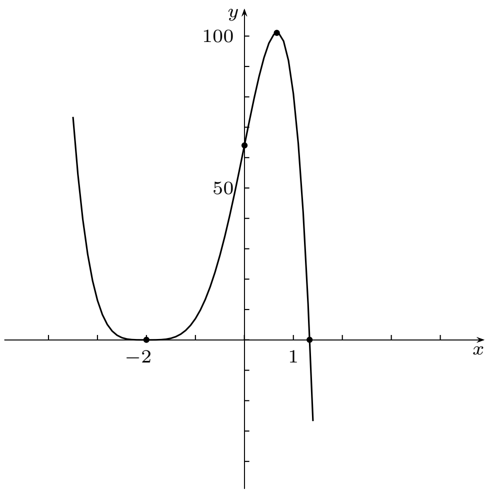

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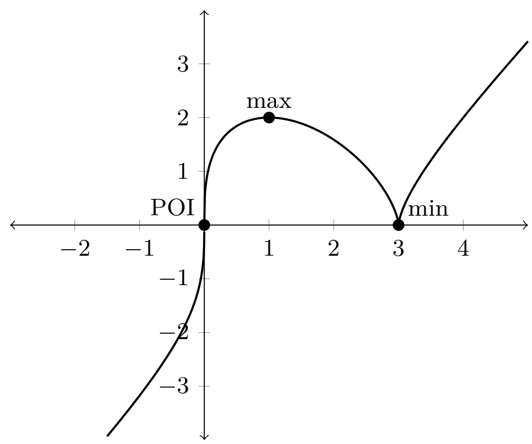

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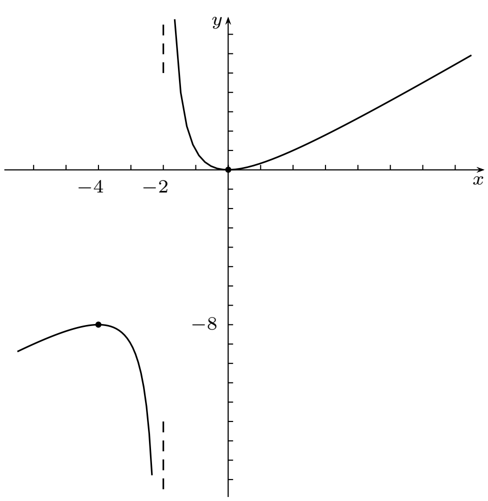

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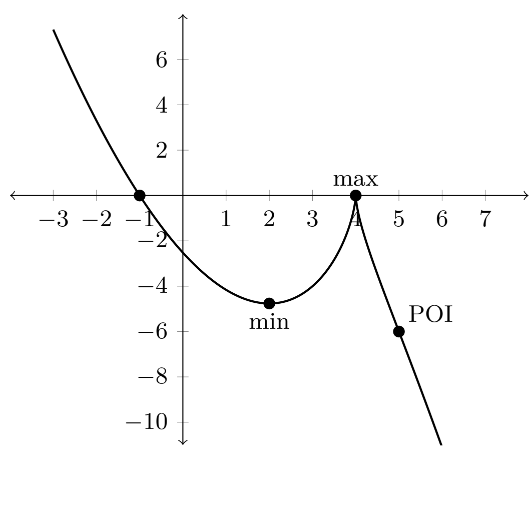

* For questions (19)-(28), answers are not unique and multiple graphs are possible.
Check with your teacher to verify your answers.