201-SH2-AB - Exercises #9 - Implicit Differentiation

1. Find $y' = \dfrac{dy}{dx}$.

   (a) $x^2 + y^2 = 16$

   (b) $2x^2 + y^2 = 16$

   (c) $x^2 - 2y^2 = 16$

   (d) $x^3 + y^3 + y - 4 = 0$

   (e) $2xy^2 - 3x^2y + 8 = 0$

   (f) $2x^{3/2} - y^{3/2} + 3x + 3 = 0$

   (g) $3xy^2 + x^3y + 3x^2 = 10$

   (h) $x + y^2 + \ln(2x - y) = 2$

   (i) $\ln(y^2) + x^3 = \ln(3x^2 - 2)$

   (j) $e^{y^2-1} + e^{x-2y} = 2(x - y)$

   (k) $\cos(3x - y) + 4y + 2x = 15$

   (l) $y\cos x + x^2 \cos y = \pi^2$

   (m) $4\cos y + 3x = 6$

   (n) $xe^x = x - y$

   (o) $\sin x + \cos y = 2x - 3y$

   (p) $e^x \sin y = x + y$

   (q) $\sin(x + y) = \cos x + \cos y$

   (r) $\tan(x - y) = 2xy^3 + 1$

   (s) $y\cos x = x^2 + y^2$

   (t) $\sin(xy) = \cos(x + y)$

   (u) $2xe^y + ye^x = 3$

   (v) $\sin x \cos y = x^2 - 5y$

   (w) $\sqrt{x + y} = x^4 + y^4$

   (x) $xy = \sqrt{x^2 + y^2}$

   (y) $e^{x/y} = x - y$

   (z) $\cos(x^2 + y^2) = xe^y$

2. Find an equation of the tangent line to the graph at the given point.

   (a) $4x^2 + 9y^2 = 36$ at $(0, 2)$

   (b) $y^2 - x^2 = 16$ at $(2, 2\sqrt{5})$

   (c) $x^2y^3 - y^2 + xy - 1 = 0$ at $(1, 1)$

   (d) $(x - y - 1)^3 = x$ at $(1, -1)$

   (e) $4x^2y - 3xy^2 + 45 = 0$ at $(3, -1)$

   (f) $x^{4/3} + 2y^{4/3} + 2y = 5$ at $(1, 1)$

   (g) $2x^2y - 4xy^3 + 2x = 24$ at $(-3, 1)$

   (h) $e^{x+y^3} = y\ln y - xy$ at $(-1, 1)$

   (i) $\sin(x + 2y) + y^3 = xy$ at $(\pi/2, 0)$

   (j) $(x^2 + y^2 - 2x)^2 = 4(x^2 + y^2)$ at $(0, 2)$

   (k) $y^{2/3} + x^{2/3} = 20$ at $(8, 64)$

   (l) $2xy = -8e^{4x+y}$ at $(1, -4)$

   (m) $ye^{\sin x} = x\cos y$ at $(0, 0)$

   (n) $\tan(x + y) + \sec(x - y) = 2$ at $(\pi/8, \pi/8)$

   (o) $y^2(6 - x) = x^3$ at $(2, \sqrt{2})$

   (p) $x^2 - xy - y^2 = 1$ at $(2, 1)$

   (q) $x^2 + 2xy + 4y^2 = 12$ at $(2, 1)$

   (r) $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ at $(0, \frac{1}{2})$

   (s) $x^2y^2 = (y + 1)^2(4 - y^2)$ at $(2\sqrt{3}, 1)$

   (t) $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ at $(3, 1)$

   (u) $y^2(y^2 - 4) = x^2(x^2 - 5)$ at $(0, -2)$

Answers

1. (a) $-\dfrac{x}{y}$

   (b) $-\dfrac{2x}{y}$

   (c) $-\dfrac{x}{2y}$

   (d) $-\dfrac{3x^2}{3y^2+1}$

   (e) $\dfrac{6xy-2y^2}{4xy-3x^2}$

   (f) $\dfrac{6\sqrt{6}+6}{3\sqrt{y}}$

   (g) $\dfrac{-3y^2-3x^2y-6x}{6xy+x^3}$

   (h) $\dfrac{y-2x-2}{4xy-2y^2-1}$

   (i) $\dfrac{3xy}{3x^2-2}-\dfrac{3x^2y}{2}$

   (j) $\dfrac{2-e^{x-2y}}{2ye^{y^2-1}-2e^{x-2y}+2}$

   (k) $\dfrac{3\sin(3x-y)-2}{\sin(3x-y)+4}$

   (l) $\dfrac{y\sin x-2x\cos y}{\cos x-x^2\sin y}$

   (m) $\dfrac{3}{4\sin y}$

   (n) $\dfrac{1-e^y}{xe^y+1}$

   (o) $\dfrac{2-\cos x}{3-\sin y}$

   (p) $\dfrac{1-e^x\sin y}{e^x\cos y-1}$

   (q) $-\dfrac{\cos(x+y)+\sin x}{\cos(x+y)+\sin y}$

   (r) $\dfrac{\sec^2(x-y)-2y^3}{6xy^2+\sec^2(x-y)}$

   (s) $\dfrac{2x+y\sin x}{\cos x-2y}$

   (t) $-\dfrac{\sin(x+y)+y\cos(xy)}{\sin(x+y)+x\cos(xy)}$

   (u) $-\dfrac{2e^y+ye^x}{2xe^y+e^x}$

   (v) $\dfrac{\cos x\cos y-2x}{\sin x\sin y-5}$

   (w) $\dfrac{1-8x^3\sqrt{x+y}}{8y^3\sqrt{x+y}-1}$

   (x) $\dfrac{x-y\sqrt{x^2+y^2}}{x\sqrt{x^2+y^2}-y}$

   (y) $\dfrac{y(y-e^{x/y})}{y^2-xe^{x/y}}$

   (z) $-\dfrac{2x\sin(x^2+y^2)+e^y}{2y\sin(x^2+y^2)+xe^y}$

2. (a) $y=2$

   (b) $y=\dfrac{1}{\sqrt{5}}x+\dfrac{8}{\sqrt{5}}$

   (c) $y=-\dfrac{3}{2}x+\dfrac{5}{2}$

   (d) $y=\dfrac{2}{3}x-\dfrac{5}{3}$

   (e) $y=\dfrac{1}{2}x-\dfrac{5}{2}$

   (f) $y=-\dfrac{2}{7}x+\dfrac{9}{7}$

   (g) $y=\dfrac{7}{27}x+\dfrac{16}{9}$

   (h) $y=-2x-1$

   (i) $y=0$

   (j) $y=x+2$

   (k) $y=-2x+80$

   (l) $y=-\dfrac{12}{5}x-\dfrac{8}{5}$

   (m) $y=x$

   (n) $y=-x+\dfrac{\pi}{4}$

   (o) $y=\dfrac{7}{4\sqrt{2}}x-\dfrac{3}{2\sqrt{2}}$

   (p) $y=\dfrac{3}{4}x-\dfrac{1}{2}$

   (q) $y=-\dfrac{1}{2}x+2$

   (r) $y=x+\dfrac{1}{2}$

   (s) $y=-\dfrac{\sqrt{3}}{5}x+\dfrac{11}{5}$

   (t) $y=-\dfrac{9}{13}x+\dfrac{40}{13}$

   (u) $y=-2$