

201-SH2-AB - Exercises #3 - Evaluating Limits Algebraically

Part 1: Factoring Quadratic Polynomials Using a Known Factor

Factor each polynomial given its factor or root (Recall that a is a root of $f(x)$ if $f(a) = 0$)

1. * $x - 4$ is a factor of $2x^2 - 5x - 12$
2. * $x + 7$ is a factor of $7x^2 + 55x + 42$
3. * $x + 3$ is a factor of $6x^2 + 19x + 3$
4. * $x - \frac{1}{2}$ is a factor of $4x^2x - 2$
5. * $x + \frac{1}{3}$ is a factor of $2x^2 + \frac{1}{6}x - \frac{1}{6}$
6. * $\frac{4}{15}$ is a root of $150x^2 + 95x - 36$
7. * $\frac{3}{2}$ is a root of $4x^2 - 9$
8. * $\frac{7}{9}$ is a root of $72x^2 + 232x - 224$

Part 2: Limits

Evaluate the following limits:

1. $\lim_{x \rightarrow 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{3x-3}$
2. $\lim_{x \rightarrow 0} \frac{x^2-4}{x+2}$
3. $\lim_{x \rightarrow 7} \frac{\frac{2}{x-1} - \frac{1}{3}}{7-x}$
4. $\lim_{x \rightarrow 3} \frac{4x^2-7x-15}{3x^2-5x-12}$
5. $\lim_{x \rightarrow 3} \sqrt{3x^2-2}$
6. $\lim_{x \rightarrow 3} \frac{\frac{5}{x^2+1} - \frac{1}{x-1}}{2x^2-3x-9}$
7. $\lim_{x \rightarrow 4} \frac{\sqrt{20-x}-4}{2x^2-5x-12}$
8. $\lim_{x \rightarrow -1} \frac{x^2-3x-4}{x^2+2x-6}$
9. $\lim_{x \rightarrow 1/2} \frac{6x^2-15x+6}{1-2x}$
10. $\lim_{x \rightarrow 2} 2x^3-4x+7$
11. $\lim_{x \rightarrow 2} \frac{x^2+8x-20}{2x^3-4x^2+4x-8}$
12. $\lim_{x \rightarrow 2} \frac{3x^2-5x-2}{\frac{x+1}{9} - \frac{2}{8-x}}$
13. $\lim_{x \rightarrow -1} \frac{\sqrt{5+x}+3x+1}{2x^2+3x+1}$
14. $\lim_{x \rightarrow -2} \frac{|5x+10|}{x^3+2x^2+5x+10}$
15. $\lim_{x \rightarrow -7^+} \frac{x^2+14x+49}{|-4x-28|}$
16. $\lim_{x \rightarrow 4^+} \frac{2x^2-5x-12}{|5x-20|}$
17. $\lim_{x \rightarrow 8} \frac{|x|-8}{x^3-512}$
18. $\lim_{x \rightarrow 4} \frac{|x+2|}{2x+5}$
19. $\lim_{x \rightarrow -3^+} \frac{-2x^2-8x-6}{|-x-3|}$
20. $\lim_{x \rightarrow 2} \frac{3|2-x|}{x-2}$
21. $\lim_{x \rightarrow 21} \frac{2x^2-37x-105}{7x^2-132x-315}$
22. $\lim_{z \rightarrow 3} \frac{z^2-z-6}{3z-z^2}$
23. $\lim_{x \rightarrow -4} \frac{1-\sqrt{x+5}}{-x-4}$
24. $\lim_{x \rightarrow 1} \frac{\frac{1}{x+2} + \frac{1}{x-4}}{3-3x}$
25. $\lim_{z \rightarrow 1} \frac{z^2-3z+2}{z-z^2}$
26. $\lim_{x \rightarrow 3} \frac{7x^2-20x-3}{4-\sqrt{x+13}}$

Part 3: Infinite Limits

Calculate the following limits. Use \inf , $-\infty$, and ∞ when appropriate. Note that not all problems will have one of these as the answer.

1. $\lim_{x \rightarrow 4^+} \frac{2x + 3}{x^2(x - 4)}$

6. $\lim_{x \rightarrow -2} \frac{5}{x - 2}$

2. $\lim_{x \rightarrow -3^-} \frac{5x(3 - x)}{(x + 3)^3}$

7. $\lim_{x \rightarrow 6^+} \frac{1 - x}{x - 6}$

3. $\lim_{x \rightarrow 5} \frac{2x^2 - 9x - 5}{2x - 5}$

8. $\lim_{x \rightarrow 1} \frac{x - 3}{x^2 - 2x + 1}$

4. $\lim_{x \rightarrow 9} \frac{3x - 5}{x^2 - 18x + 81}$

9. $\lim_{x \rightarrow 7^-} \frac{2x + 1}{x^2 - 49}$

5. $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 8}{x^2 + x - 12}$

10. $\lim_{x \rightarrow 14} \frac{14 - x}{14 + x}$

Part 4: Limits at Infinity

Evaluate $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$ for each of the following functions.

1. $f(x) = \frac{(x^2 - 3x)^4(1 - 2x)^3}{(x^2 - 4x)^5(x + 1)}$

2. $f(x) = \frac{1 - x^2 - 3x^3 + 4x^4}{2x^4 - 32}$

3. $f(x) = \frac{(1 - 3x)^3(x^2 + 8x + 2)}{x^3 + 6x^2 - 5x}$

4. $f(x) = \frac{(2x + 1)^2(3x - 4)}{(9x - 1)^2(3 - 4x)^3(x^2 + 2x + 4)^2}$

5. $f(x) = \frac{(x^2 - x - 20)^2(x^4 + 2x + 1)^2}{(x^3 + 125)^2(x^3 - 125)^2}$

6. $f(x) = \frac{1 - 4x^2}{x^4 - 3x^3}$

7. $f(x) = \frac{4x^2 - 2x + 1}{8x^2 - 72}$

8. $f(x) = \frac{(x^3 + 2x^2 + x + 2)(1 - x)^3}{(x^2 + 5x + 6)(2x + 3)^3}$

Answers

Part 1: Factoring Quadratic Polynomials Using a Known Factor

- (1) $2x^2 - 5x - 12 = (x - 4)(2x + 3)$
- (2) $7x^2 + 55x + 42 = (x + 7)(7x + 6)$
- (3) $6x^2 + 19x + 3 = (6x + 1)(x + 3)$
- (4) $4x^2 + 2x - 2 = (x - \frac{1}{2})(4x + 4)$
- (5) $2x^2 + \frac{4}{3}x - \frac{1}{6} = (x + \frac{1}{3})(2x - \frac{1}{2})$
- (6) $150x^2 + 95x - 36 = (x - \frac{4}{15})(150x + 135)$
- (7) $4x^2 - 9 = (x - \frac{3}{2})(4x + 6)$
- (8) $72x^2 + 232x - 224 = (x - \frac{7}{9})(72x + 288)$

Part 2: Limits

- | | |
|----------------------|--|
| (1) $\frac{1}{6}$ | (15) 0 |
| (2) -2 | (16) $\frac{11}{5}$ |
| (3) $\frac{1}{18}$ | (17) $\frac{1}{192}$ |
| (4) $\frac{17}{13}$ | (18) $\frac{6}{13}$ |
| (5) 5 | (19) 4 |
| (6) $\frac{-1}{180}$ | (20) dne because $\lim_{x \rightarrow 2^-} f(x) = -3 \neq$ |
| (7) $\frac{-1}{88}$ | $\lim_{x \rightarrow 2^+} f(x) = 3$ |
| (8) 0 | (21) $\frac{47}{162}$ |
| (9) $\frac{9}{2}$ | (22) $\frac{-5}{3}$ |
| (10) 15 | (23) $\frac{1}{2}$ |
| (11) 1 | (24) $\frac{2}{27}$ |
| (12) 126 | (25) 1 |
| (13) $\frac{-13}{4}$ | (26) -176 |
| (14) $\frac{-5}{9}$ | |

Part 3: Infinite Limits

- | | |
|--------------|--------------------|
| (1) ∞ | (6) $\frac{-5}{4}$ |
| (2) ∞ | (7) $-\infty$ |
| (3) 0 | (8) $-\infty$ |
| (4) ∞ | (9) $-\infty$ |
| (5) dne | (10) 0 |

Part 4: Limits at Infinity

$$(1) \lim_{x \rightarrow \infty} f(x) = -8, \lim_{x \rightarrow -\infty} f(x) = -8$$

$$(2) \lim_{x \rightarrow \infty} f(x) = 2, \lim_{x \rightarrow -\infty} f(x) = 2$$

$$(3) \lim_{x \rightarrow \infty} f(x) = -\infty, \lim_{x \rightarrow -\infty} f(x) = -\infty$$

$$(4) \lim_{x \rightarrow \infty} f(x) = 0, \lim_{x \rightarrow -\infty} f(x) = 0$$

$$(5) \lim_{x \rightarrow \infty} f(x) = 1, \lim_{x \rightarrow -\infty} f(x) = 1$$

$$(6) \lim_{x \rightarrow \infty} f(x) = 0, \lim_{x \rightarrow -\infty} f(x) = 0$$

$$(7) \lim_{x \rightarrow \infty} f(x) = \frac{1}{2}, \lim_{x \rightarrow -\infty} f(x) = \frac{1}{2}$$

$$(8) \lim_{x \rightarrow \infty} f(x) = -\infty, \lim_{x \rightarrow -\infty} f(x) = \infty$$