Part 1: Factoring Quadratic Polynomials Using a Known Factor

Factor each polynomial given its factor or root (Recall that a is a root of $f(x)$ if $f(a) = 0$)

1. $x - 4$ is a factor of $2x^2 - 5x - 12$

2. $x + 7$ is a factor of $7x^2 + 55x + 42$

3. $x + 3$ is a factor of $6x^2 + 19x + 3$

4. $x - \frac{1}{2}$ is a factor of $4x^2x - 2$

5. $x + \frac{1}{3}$ is a factor of $2x^2 + \frac{1}{6}x - \frac{1}{6}$

6. $\frac{4}{15}$ is a root of $150x^2 + 95x - 36$

7. $\frac{3}{2}$ is a root of $4x^2 - 9$

8. $\frac{7}{9}$ is a root of $72x^2 + 232x - 224$

Part 2: Limits

Evaluate the following limits:

1. $\displaystyle\lim_{x\to 1} \frac{\sqrt{3+x}-\sqrt{5-x}}{3x-3}$

2. $\displaystyle\lim_{x\to 0} \frac{x^2-4}{x+2}$

3. $\displaystyle\lim_{x\to 7} \frac{\frac{2}{x-1}-\frac{1}{3}}{7-x}$

4. $\displaystyle\lim_{x\to 3} \frac{4x^2-7x-15}{3x^2-5x-12}$

5. $\displaystyle\lim_{x\to 3} \sqrt{3x^2-2}$

6. $\displaystyle\lim_{x\to 3} \frac{\frac{5}{x^2+1}-\frac{1}{x-1}}{2x^2-3x-9}$

7. $\displaystyle\lim_{x\to 4} \frac{\sqrt{20-x}-4}{2x^2-5x-12}$

8. $\displaystyle\lim_{x\to -1} \frac{x^2-3x-4}{x^2+2x-6}$

9. $\displaystyle\lim_{x\to 1/2} \frac{6x^2-15x+6}{1-2x}$

10. $\displaystyle\lim_{x\to 2} 2x^3-4x+7$

11. $\displaystyle\lim_{x\to 2} \frac{x^2+8x-20}{2x^3-4x^2+4x-8}$

12. $\displaystyle\lim_{x\to 2} \frac{3x^2-5x-2}{\frac{x+1}{9}-\frac{2}{8-x}}$

13. * $\displaystyle\lim_{x\to -1} \frac{\sqrt{5+x}+3x+1}{2x^2+3x+1}$

14. $\displaystyle\lim_{x\to -2^-} \frac{|5x+10|}{x^3+2x^2+5x+10}$

15. $\displaystyle\lim_{x\to -7^+} \frac{x^2+14x+49}{|-4x-28|}$

16. $\displaystyle\lim_{x\to 4^+} \frac{2x^2-5x-12}{|5x-20|}$

17. *$\displaystyle\lim_{x\to 8} \frac{|x|-8}{x^3-512}$

18. $\displaystyle\lim_{x\to 4} \frac{|x+2|}{2x+5}$

19. $\displaystyle\lim_{x\to -3^+} \frac{-2x^2-8x-6}{|-x-3|}$

20. $\displaystyle\lim_{x\to 2} \frac{3|2-x|}{x-2}$

21. $\displaystyle\lim_{x\to 21} \frac{2x^2-37x-105}{7x^2-132x-315}$

22. $\displaystyle\lim_{z\to 3} \frac{z^2-z-6}{3z-z^2}$

23. $\displaystyle\lim_{x\to -4} \frac{1-\sqrt{x+5}}{-x-4}$

24. $\displaystyle\lim_{x\to 1} \frac{\frac{1}{x+2}+\frac{1}{x-4}}{3-3x}$

25. $\displaystyle\lim_{z\to 1} \frac{z^2-3z+2}{z-z^2}$

26. $\displaystyle\lim_{x\to 3} \frac{7x^2-20x-3}{4-\sqrt{x+13}}$

Part 3: Infinite Limits

Calculate the following limits. Use inf, $-$ inf, and dne when appropriate. Note that not all problems will have one of these as the answer.

1. $\lim\limits_{x \to 4^+} \dfrac{2x+3}{x^2(x-4)}$

2. $\lim\limits_{x \to -3^-} \dfrac{5x(3-x)}{(x+3)^3}$

3. $\lim\limits_{x \to 5} \dfrac{2x^2 - 9x - 5}{2x - 5}$

4. $\lim\limits_{x \to 9} \dfrac{3x - 5}{x^2 - 18x + 81}$

5. $\lim\limits_{x \to 3} \dfrac{x^2 - 2x - 8}{x^2 + x - 12}$

6. $\lim\limits_{x \to -2} \dfrac{5}{x - 2}$

7. $\lim\limits_{x \to 6^+} \dfrac{1 - x}{x - 6}$

8. $\lim\limits_{x \to 1} \dfrac{x - 3}{x^2 - 2x + 1}$

9. $\lim\limits_{x \to 7^+} \dfrac{2x + 1}{x^2 - 49}$

10. $\lim\limits_{x \to 14} \dfrac{14 - x}{14 + x}$

Part 4: Limits at Infinity

Evaluate $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$ for each of the following functions.

1. $f(x) = \dfrac{(x^2 - 3x)^4(1 - 2x)^3}{(x^2 - 4x)^5(x + 1)}$

2. $f(x) = \dfrac{1 - x^2 - 3x^3 + 4x^4}{2x^4 - 32}$

3. $f(x) = \dfrac{(1 - 3x)^3(x^2 + 8x + 2)}{x^3 + 6x^2 - 5x}$

4. $f(x) = \dfrac{(2x + 1)^2(3x - 4)}{(9x - 1)^2(3 - 4x)^3(x^2 + 2x + 4)^2}$

5. $f(x) = \dfrac{(x^2 - x - 20)^2(x^4 + 2x + 1)^2}{(x^3 + 125)^2(x^3 - 125)^2}$

6. $f(x) = \dfrac{1 - 4x^2}{x^4 - 3x^3}$

7. $f(x) = \dfrac{4x^2 - 2x + 1}{8x^2 - 72}$

8. $f(x) = \dfrac{(x^3 + 2x^2 + x + 2)(1 - x)^3}{(x^2 + 5x + 6)(2x + 3)^3}$

Part 1: Factoring Quadratic Polynomials Using a Known Factor

(1) $2x^2 - 5x - 12 = (x - 4)(2x + 3)$

(2) $7x^2 + 55x + 42 = (x + 7)(7x + 6)$

(3) $6x^2 + 19x + 3 = (6x + 1)(x + 3)$

(4) $4x^2 + 2x - 2 = (x - \frac{1}{2})(4x + 4)$

(5) $2x^2 + \frac{4}{3}x - \frac{1}{6} = (x + \frac{1}{3})(2x - \frac{1}{2})$

(6) $150x^2 + 95x - 36 = (x - \frac{4}{15})(150x + 135)$

(7) $4x^2 - 9 = (x - \frac{3}{2})(4x + 6)$

(8) $72x^2 + 232x - 224 = (x - \frac{7}{9})(72x + 288)$

Part 2: Limits

(1) $\frac{1}{6}$

(2) -2

(3) $\frac{1}{18}$

(4) $\frac{17}{13}$

(5) 5

(6) $\frac{-1}{180}$

(7) $\frac{-1}{88}$

(8) 0

(9) $\frac{9}{2}$

(10) 15

(11) 1

(12) 126

(13) $\frac{-13}{4}$

(14) $\frac{-5}{9}$

(15) 0

(16) $\frac{11}{5}$

(17) $\frac{1}{192}$

(18) $\frac{6}{13}$

(19) 4

(20) dne   because $\lim\limits_{x \to 2^-} f(x) = -3 \neq \lim\limits_{x \to 2^+} f(x) = 3$

(21) $\frac{47}{162}$

Part 3: Infinite Limits

(1) ∞

(2) ∞

(3) 0

(4) ∞

(5) dne

(6) $\frac{-5}{4}$

(7) $-\infty$

(8) $-\infty$

(9) $-\infty$

(10) 0

Part 4: Limits at Infinity

(1) $\lim\limits_{x\to\infty} f(x) = 0, \ \lim\limits_{x\to-\infty} f(x) = 0$

(2) $\lim\limits_{x\to\infty} f(x) = 2, \ \lim\limits_{x\to-\infty} f(x) = 2$

(3) $\lim\limits_{x\to\infty} f(x) = -\infty, \ \lim\limits_{x\to-\infty} f(x) = -\infty$

(4) $\lim\limits_{x\to\infty} f(x) = 0, \ \lim\limits_{x\to-\infty} f(x) = 0$

(5) $\lim\limits_{x\to\infty} f(x) = 1, \ \lim\limits_{x\to-\infty} f(x) = 1$

(6) $\lim\limits_{x\to\infty} f(x) = 0, \ \lim\limits_{x\to-\infty} f(x) = 0$

(7) $\lim\limits_{x\to\infty} f(x) = \dfrac{1}{2}, \ \lim\limits_{x\to-\infty} f(x) = \dfrac{1}{2}$

(8) $\lim\limits_{x\to\infty} f(x) = -\infty, \ \lim\limits_{x\to-\infty} f(x) = \infty$