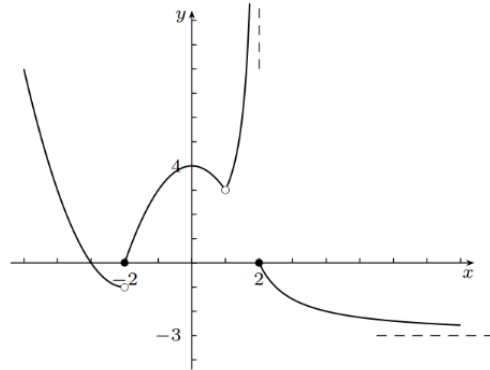


201-SH2-AB - Exercises #1 - Limits from Graphs

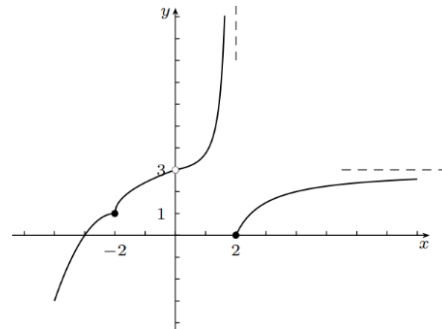
1. Given the following graph of $f(x)$, find:

- (a) $\lim_{x \rightarrow -\infty} f(x)$ (b) $\lim_{x \rightarrow -2^-} f(x)$
 (c) $\lim_{x \rightarrow 1} f(x)$ (d) $\lim_{x \rightarrow 2^+} f(x)$
 (e) $\lim_{x \rightarrow 2^-} f(x)$ (f) $\lim_{x \rightarrow 2} f(x)$
 (g) $\lim_{x \rightarrow +\infty} f(x)$

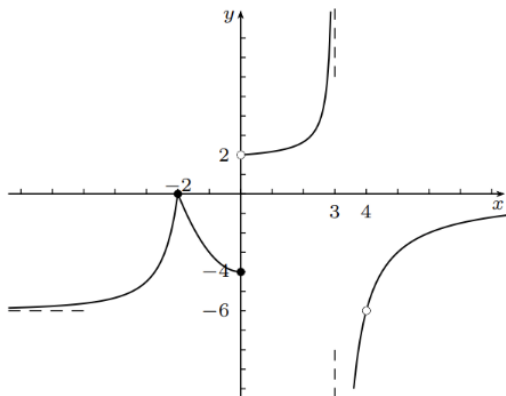


2. Given the following graph of $f(x)$, find:

- (a) $\lim_{x \rightarrow -\infty} f(x)$ (b) $\lim_{x \rightarrow -2} f(x)$
 (c) $\lim_{x \rightarrow 0} f(x)$ (d) $\lim_{x \rightarrow 2^-} f(x)$
 (e) $\lim_{x \rightarrow 2^+} f(x)$ (f) $\lim_{x \rightarrow 2} f(x)$
 (g) $\lim_{x \rightarrow +\infty} f(x)$ (h) $f(0)$

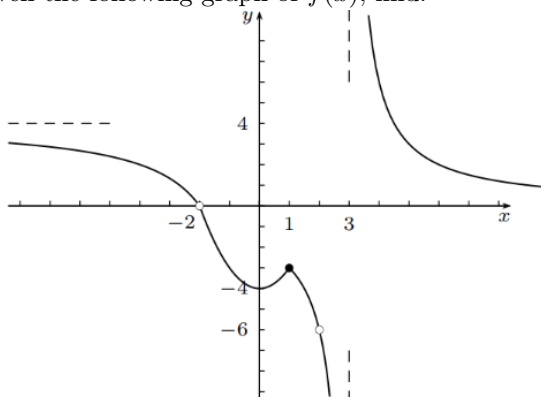


3. Given the following graph of $f(x)$, find:



- (a) $\lim_{x \rightarrow -\infty} f(x)$ (b) $\lim_{x \rightarrow -2} f(x)$
 (c) $\lim_{x \rightarrow 0^-} f(x)$ (d) $\lim_{x \rightarrow 0^+} f(x)$
 (e) $f(-2)$ (f) $\lim_{x \rightarrow 3^+} f(x)$
 (g) $\lim_{x \rightarrow 3^-} f(x)$ (h) $\lim_{x \rightarrow +\infty} f(x)$

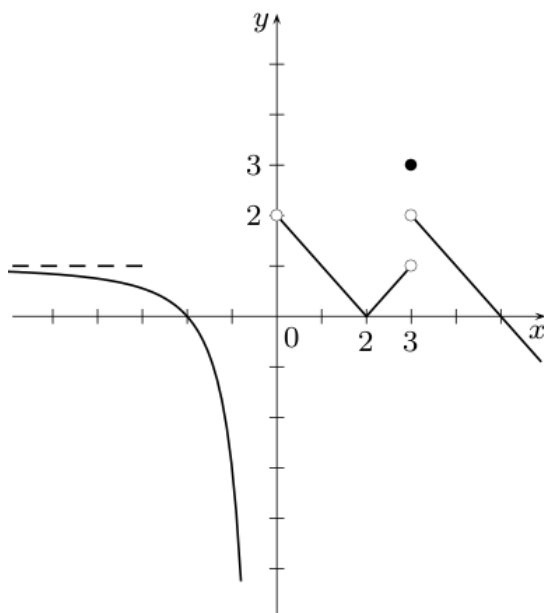
4. Given the following graph of $f(x)$, find:



- (a) $\lim_{x \rightarrow -\infty} f(x)$ (b) $\lim_{x \rightarrow 0} f(x)$
 (c) $\lim_{x \rightarrow 1} f(x)$ (d) $\lim_{x \rightarrow 3^-} f(x)$
 (e) $\lim_{x \rightarrow 3^+} f(x)$ (f) $\lim_{x \rightarrow 2} f(x)$
 (g) $\lim_{x \rightarrow +\infty} f(x)$ (h) $f(2)$

5. Use the graph of the function $f(x)$ to answer each question.

Use ∞ , $-\infty$ or DNE where appropriate.

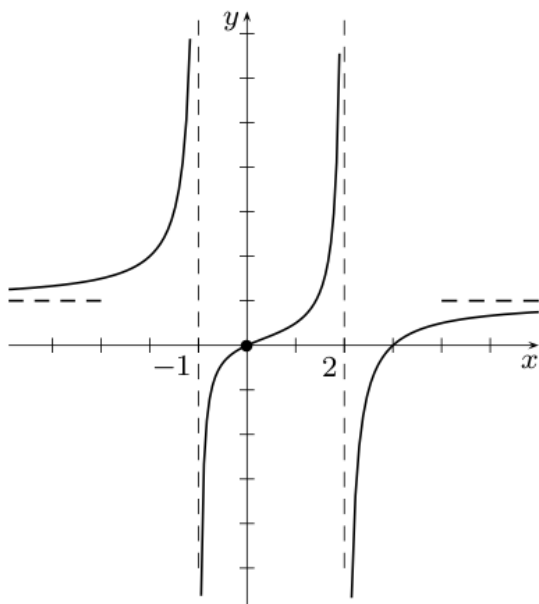


- (a) $f(0) =$
- (b) $f(2) =$
- (c) $f(3) =$
- (d) $\lim_{x \rightarrow 0^-} f(x) =$
- (e) $\lim_{x \rightarrow 0} f(x) =$
- (f) $\lim_{x \rightarrow 0^+} f(x) =$
- (g) $\lim_{x \rightarrow 3^+} f(x) =$
- (h) $\lim_{x \rightarrow 3} f(x) =$
- (i) $\lim_{x \rightarrow -\infty} f(x) =$
- (j) $\lim_{x \rightarrow \infty} f(x) =$
- (k) $\lim_{x \rightarrow 2} \frac{1-x}{f(x)}$

(l) List the value(s) where $f(x)$ is discontinuous and *state the type of discontinuity.

6. Use the graph of the function $f(x)$ to answer each question.

Use ∞ , $-\infty$ or DNE where appropriate.

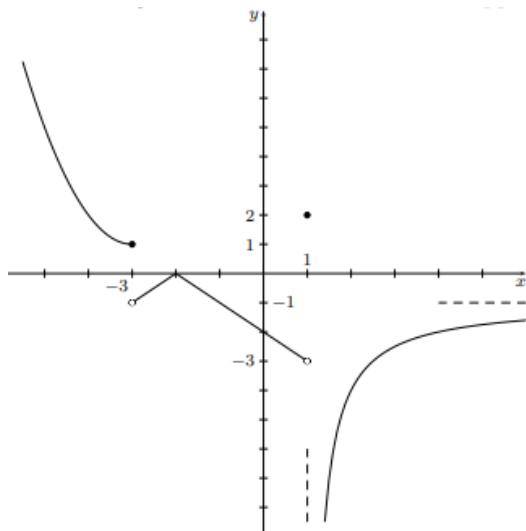


- (a) $f(0) =$
- (b) $f(2) =$
- (c) $f(3) =$
- (d) $\lim_{x \rightarrow -1} f(x) =$
- (e) $\lim_{x \rightarrow 0} f(x) =$
- (f) $\lim_{x \rightarrow 2^+} f(x) =$
- (g) $\lim_{x \rightarrow 2^-} f(x) =$
- (h) $\lim_{x \rightarrow \infty} f(x) =$
- (i) $\lim_{x \rightarrow -\infty} f(x) =$
- (j) $\lim_{x \rightarrow 0^+} \frac{3}{f(x)}$

(k) List the value(s) where $f(x)$ is discontinuous and *state the type of discontinuity.

7. Use the graph of the function $g(x)$ to answer each question.

Use ∞ , $-\infty$ or DNE where appropriate.

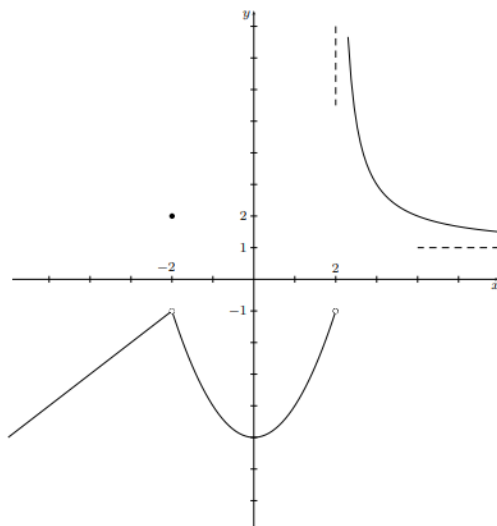


- (a) $g(-3) =$
- (b) $g(-2) =$
- (c) $g(0) =$
- (d) $\lim_{x \rightarrow -3^-} g(x) =$
- (e) $\lim_{x \rightarrow -3^+} g(x) =$
- (f) $\lim_{x \rightarrow -3} g(x) =$
- (g) $\lim_{x \rightarrow 1^-} g(x) =$
- (h) $\lim_{x \rightarrow 1^+} g(x) =$
- (i) $\lim_{x \rightarrow 1} g(x) =$
- (j) $\lim_{x \rightarrow -\infty} g(x) =$
- (k) $\lim_{x \rightarrow \infty} g(x) =$
- (l) $\lim_{x \rightarrow 0} g(x) =$
- (m) $\lim_{x \rightarrow 0^+} g(x) =$
- (n) $\lim_{x \rightarrow -2^-} \frac{3}{g(x)} =$

(0) List the value(s) where $g(x)$ is discontinuous and *state the type of discontinuity.

8. Use the graph of the function $h(x)$ to answer each question.

Use ∞ , $-\infty$ or DNE where appropriate.



- (a) $h(-2) =$
- (b) $h(0) =$
- (c) $\lim_{x \rightarrow -2^-} h(x) =$
- (d) $\lim_{x \rightarrow -2^+} h(x) =$
- (e) $\lim_{x \rightarrow -2} h(x) =$
- (f) $\lim_{x \rightarrow 0^+} h(x) =$
- (g) $\lim_{x \rightarrow 0} h(x) =$
- (h) $\lim_{x \rightarrow 2^-} h(x) =$
- (i) $\lim_{x \rightarrow 2^+} h(x) =$
- (j) $\lim_{x \rightarrow 2} h(x) =$
- (k) $\lim_{x \rightarrow -\infty} h(x) =$
- (l) $\lim_{x \rightarrow \infty} h(x) =$
- (m) $\lim_{x \rightarrow -2^+} \frac{x}{h(x) + 1} =$

(n) List the value(s) where $h(x)$ is discontinuous and *state the type of discontinuity.

9. Given the following graph of $f(x)$, find:

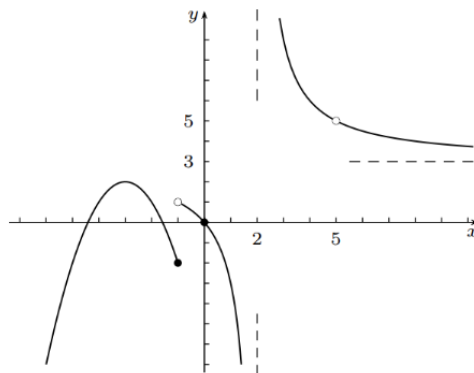
(a) $\lim_{x \rightarrow -\infty} f(x)$ (b) $\lim_{x \rightarrow -1^+} f(x)$

(c) $\lim_{x \rightarrow 5} f(x)$ (d) $\lim_{x \rightarrow 2^-} f(x)$

(e) $\lim_{x \rightarrow 2^+} f(x)$ (f) $\lim_{x \rightarrow -3} f(x)$

(g) $\lim_{x \rightarrow +\infty} f(x)$ (h) $\lim_{x \rightarrow 0^-} \frac{2x+3}{f(x)}$

(i) List the value(s) where $f(x)$ is discontinuous and *state the type of discontinuity.



10. Given the following graph of $f(x)$, find:

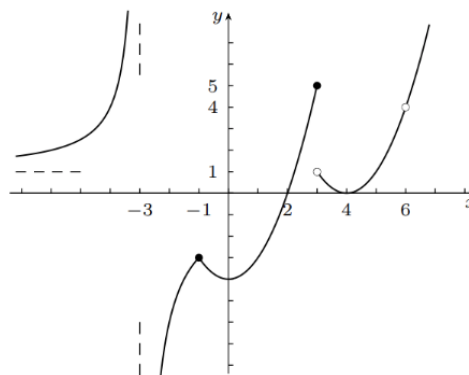
(a) $\lim_{x \rightarrow -\infty} f(x)$ (b) $\lim_{x \rightarrow -1} f(x)$

(c) $\lim_{x \rightarrow 3} f(x)$ (d) $\lim_{x \rightarrow 3^-} f(x)$

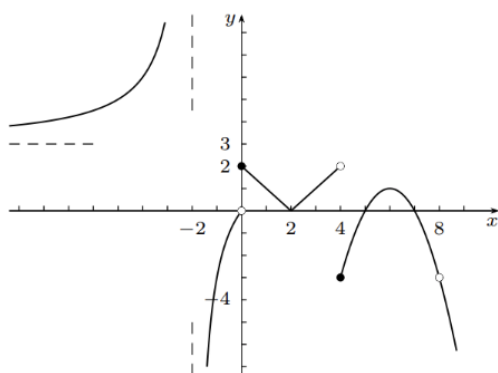
(e) $\lim_{x \rightarrow 3^+} f(x)$ (f) $\lim_{x \rightarrow 6} f(x)$

(g) $\lim_{x \rightarrow +\infty} f(x)$ (h) $f(3)$

(i) List the value(s) where $f(x)$ is discontinuous and *state the type of discontinuity.



11. Given the following graph of $f(x)$, find:



(a) $\lim_{x \rightarrow -\infty} f(x)$ (b) $\lim_{x \rightarrow -2^-} f(x)$

(c) $\lim_{x \rightarrow 0} f(x)$ (d) $\lim_{x \rightarrow 2} f(x)$

(e) $\lim_{x \rightarrow 4^+} f(x)$ (f) $f(2)$

(g) $\lim_{x \rightarrow 8} f(x)$ (h) $f(8)$

(i) List the value(s) where $f(x)$ is discontinuous and *state the type of discontinuity.

12. Given the following graph of $f(x)$, find:

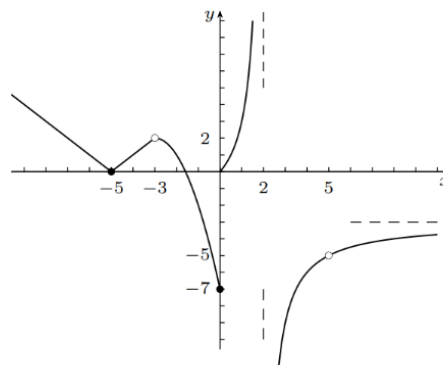
(a) $\lim_{x \rightarrow -\infty} f(x)$ (b) $\lim_{x \rightarrow -3} f(x)$

(c) $\lim_{x \rightarrow 0} f(x)$ (d) $\lim_{x \rightarrow 2^-} f(x)$

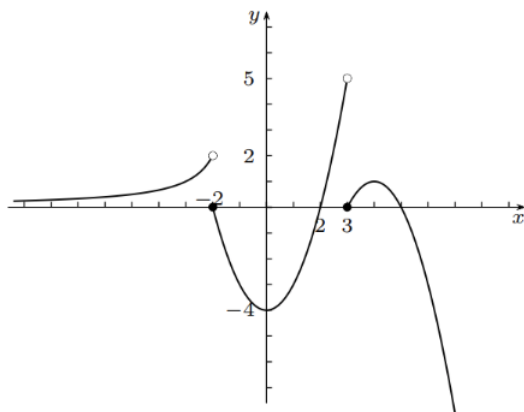
(e) $\lim_{x \rightarrow 2^+} f(x)$ (f) $\lim_{x \rightarrow 2} f(x)$

(g) $\lim_{x \rightarrow +\infty} f(x)$ (h) $f(-3)$

(i) List the value(s) where $f(x)$ is discontinuous and *state the type of discontinuity.



13. Given the following graph of $f(x)$, find:



(a) $\lim_{x \rightarrow -\infty} f(x)$ (b) $\lim_{x \rightarrow -2^-} f(x)$

(c) $\lim_{x \rightarrow -2^+} f(x)$ (d) $\lim_{x \rightarrow 3^-} f(x)$

(e) $\lim_{x \rightarrow 2} f(x)$ (f) $\lim_{x \rightarrow 0} f(x)$

(g) $\lim_{x \rightarrow +\infty} f(x)$ (h) $\lim_{x \rightarrow 2} (7f(x) - 3x + 5)$

(i) List the value(s) where $f(x)$ is discontinuous and *state the type of discontinuity.

14. Given the following graph of $f(x)$, find:

(a) $\lim_{x \rightarrow -\infty} f(x)$ (b) $\lim_{x \rightarrow -2} f(x)$

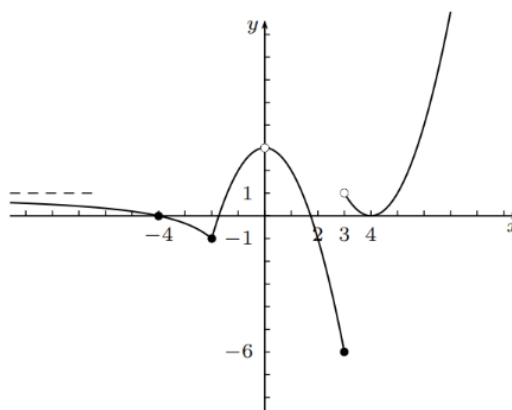
(c) $\lim_{x \rightarrow 0} f(x)$ (d) $\lim_{x \rightarrow 3^-} f(x)$

(e) $\lim_{x \rightarrow 3^+} f(x)$ (f) $\lim_{x \rightarrow 3} f(x)$

(g) $\lim_{x \rightarrow +\infty} f(x)$ (h) $f(3)$

(i) $\lim_{x \rightarrow -2} \frac{2x+1}{f(x)+1}$ (j) $\lim_{x \rightarrow 3^-} \frac{2f(x)+9}{f(x)}$

(k) List the value(s) where $f(x)$ is discontinuous and *state the type of discontinuity.



15. *Sketch ONE graph of a function that satisfies ALL of the following conditions (one graph for all the things)

a) $\lim_{x \rightarrow 4^-} f(x) = 2$

b) $\lim_{x \rightarrow 4^+} f(x) = 4$

c) $f(4) = 3$

d) $\lim_{x \rightarrow -2} f(x) = 3$

e) $f(-2) = 1$

f) there is an infinite discontinuity for some x value

16. *Sketch ONE graph of a function that satisfies ALL of the following conditions (one graph for all the things)

a) There is a removable discontinuity at $x = 3$, and $f(3)$ is defined.

b) There is a removable discontinuity at $x = 5$, but $f(5)$ is not defined

c) There is an infinite discontinuity at $x = -1$, $\lim_{x \rightarrow -1^+} = 5$, and $f(-1) = 2$.

17. (a) *Sketch ONE graph of a function that satisfies ALL of the following conditions (one graph for all the things)

(i) $\lim_{x \rightarrow 3} f(x) = 8$

(ii) $f(3) = 2$

(iii) $\lim_{x \rightarrow \infty} f(x) = 8$

(iv) There is only one horizontal asymptote.

(v) $\lim_{x \rightarrow 4^-} f(x) = 5$

(vi) $\lim_{x \rightarrow 4^+} f(x) = -3$

(vii) $\lim_{x \rightarrow -1^-} f(x) = \infty$

(viii) $\lim_{x \rightarrow -1^+} f(x) = 7$

(ix) $f(-1) = 0$

(b) List all of the discontinuities of the graph you have sketched and state the type.

Answers

- | | | |
|--|--|---|
| 1. (a) ∞
(b) -1
(c) 3 | (d) 0
(e) ∞
(f) dne | (g) -3 |
| 2. (a) $-\infty$
(b) 1
(c) 3 | (d) ∞
(e) 0
(f) dne | (g) 3
(h) dne |
| 3. (a) -6
(b) 0
(c) -4 | (d) 2
(e) 0
(f) $-\infty$ | (g) ∞
(h) 0 |
| 4. (a) 4
(b) -4
(c) -3 | (d) $-\infty$
(e) ∞
(f) -6 | (g) 0
(h) dne |
| 5. (a) dne
(b) 0
(c) 3
(d) $-\infty$
(e) dne | (f) 2
(g) 2
(h) dne
(i) 1
(j) $-\infty$ | (k) $-\infty$
(l) infinite discontinuity: $x = 0$
jump discontinuity: $x = 3$ |
| 6. (a) 0
(b) dne
(c) 0
(d) dne | (e) 0
(f) $-\infty$
(g) ∞
(h) 1 | (i) 1
(j) ∞
(k) infinite discontinuity: $x = -1, 2$ |
| 7. (a) 1
(b) 0
(c) -2
(d) 1
(e) -1
(f) dne | (g) -3
(h) $-\infty$
(i) dne
(j) ∞
(k) -1
(l) -2 | (m) -2
(n) $-\infty$
(o) infinite discontinuity: $x = 1$
jump discontinuity: $x = -3$ |
| 8. (a) 2
(b) -5
(c) -1
(d) -1
(e) -1
(f) -5 | (g) -5
(h) -1
(i) ∞
(j) dne
(k) $-\infty$
(l) 1 | (m) ∞
(n) infinite discontinuity: $x = 2$
removable discontinuity: $x = -2$ |
| 9. (a) $-\infty$
(b) -2
(c) 5
(d) $-\infty$ | (e) ∞
(f) 2
(g) 3
(h) ∞ | (i) jump discontinuity: $x = -1$
infinite discontinuity: $x = 2$
removable discontinuity: $x = 5$ |
| 10. (a) 1
(b) -3
(c) dne
(d) 5 | (e) 1
(f) 4
(g) ∞
(h) 5 | (i) infinite discontinuity: $x = -3$
jump discontinuity: $x = 3$
removable discontinuity: $x = 6$ |

- | | | |
|------------------|---------------|--|
| 11. (a) 3 | (e) 2 | (i) infinite discontinuity: $x = -2$ |
| (b) ∞ | (f) 0 | jump discontinuity: $x = 0, x = 4$ |
| (c) dne | (g) -3 | removable discontinuity: $x = 8$ |
| (d) 0 | (h) dne | |
| 12. (a) ∞ | (e) $-\infty$ | (i) removable discontinuity: $x = -3, x = 5$ |
| (b) 2 | (f) dne | jump discontinuity: $x = 0$ |
| (c) dne | (g) -3 | infinite discontinuity: $x = 2$ |
| (d) ∞ | (h) dne | |
| 13. (a) 0 | (d) 5 | (g) $-\infty$ |
| (b) 2 | (e) 0 | (h) $7(0) - (3)(2) + 5 = -1$ |
| (c) 0 | (f) -4 | (i) jump discontinuity: $x = -2, 3$ |
| 14. (a) 1 | (e) 1 | (i) $-\infty$ |
| (b) -1 | (f) dne | (j) $\frac{2(-6)+9}{-6} = \frac{1}{2}$ |
| (c) 3 | (g) ∞ | |
| (d) -6 | (h) -6 | |

15. Many possible options. Check with your teacher.

16. Many possible options. Check with your teacher.

17. (a) Many possible options. Check with your teacher.

(b) removable discontinuity: $x = 3$

jump discontinuity: $x = 4$

infinite discontinuity: $x = -1$

Plus any other discontinuities that you may have chosen to draw in.