The Leontief Closed Model

NOTE: In the closed model, there is no outside demand. Everything produced by the industries is also consumed by them. So each of the consumption vectors must represent the production of \$1 of output.

1. A closed economy has two industries: iron and steel. To produce \$1 of iron requires 30¢ of iron and 70¢ of steel. To produce \$1 of steel requires 90¢ of iron and 10¢ of steel.
   How much does each industry need to produce relative to each other in order to function?

2. A closed economy has two industries: services and manufacturing. To produce \$1 of services requires 20¢ of services and 80¢ of manufacturing. To produce \$1 of manufacturing requires 60¢ of services and 40¢ of manufacturing.
   (a) How much does each industry need to produce relative to each other in order to function?
   (b) If there was $\$14\,000$ to allocate to the industries equitably, how much should each industry get?

3. Suppose that a closed economy consists of three industries: food, shelter, and clothing. For each \$1 of food that is provided, 40¢ is spent on food, 40¢ on shelter and 20¢ on clothing. For each \$1 of shelter provided, 40¢ is spent on food, 30¢ on shelter, and 30¢ on clothing. Each \$1 in clothing takes 30¢ in food, 20¢ in shelter, and 50¢ in clothing.
   (a) Compute $\mathrm{Nul}(I - C)$.
   (b) If there is a budget of $\$158\,000$, how should it be allocated to the industries to manage production?

Leontief Open Model

4. An open economy has two industries: energy and material.
   To produce \$1 of energy requires 90¢ of energy and 30¢ of material.
   To produce \$1 of material requires 20¢ of energy and 30¢ of material.
   (a) Given an external demand for \$2000 of energy and \$1000 of material, how much of each industry should be produced to meet it?
   (b) Is the economy productive? Justify your answer.
   (c) Find the internal consumption when demand is met.

5. An open economy has two industries: iron and steel. To produce \$1 of iron requires 20¢ of iron and 10¢ of steel. To produce \$1 of steel requires 70¢ of iron and 40¢ of steel.
   (a) Given an external demand for \$8200 of Iron and \$4100 of Steel, how much of each industry should be produced to meet it?
   (b) Which industries are profitable?

6. An open economy has two industries: goods and services. To produce \$1 of goods requires 60¢ of goods and 50¢ of services. To produce \$1 of services requires 30¢ of goods and 10¢ of services.
   (a) If there is an external demand for \$6300 of goods and \$8400 of services, how much of each industry should be produced to meet it?
   (b) Is the economy productive? Justify your answer.

7. An open economy has two industries: services and manufacturing. To produce \$1 of services requires 20¢ of services and 40¢ of manufacturing. To produce \$1 of manufacturing requires 30¢ of services and 10¢ of manufacturing.
   (a) If there is an external demand for \$900 of services and \$1500 of manufacturing, how much of each industry should be produced to meet it?
   (b) Find the internal consumption when demand is met.
   (c) Which industries, if any, are profitable.

8. For each of the consumption matrices below, determine which industries are profitable and whether the economy is productive.

(a) $C = \begin{bmatrix} .8 & .3 \\ .1 & .6 \end{bmatrix}$ (b) $C = \begin{bmatrix} .8 & .1 \\ .3 & .6 \end{bmatrix}$ (c) $C = \begin{bmatrix} .8 & .1 \\ .9 & .6 \end{bmatrix}$

9. Suppose that an economy consists of three industries: a computing service, a statistical service, and an engineering service. For each \$1 of computing that is provided, 30¢ is spent on computing, 10¢ on statistical services and 30¢ on engineering. For each \$1 on statistical service, 20¢ is spent on computing, 40¢ on statistics, and 20¢ on engineering. Each \$1 in engineering takes 30¢ in computing, 10¢ in statistical services, and 30¢ in engineering. Suppose there is an external demand for \$1000 in computing, \$1500 in statistical services, and \$1800 in engineering.
   (a) Compute $\det(I - C)$.
   (b) Compute $(I - C)^{-1}$.
   (c) How much should each industry produce to meet the demand?

ANSWERS:

1. Iron production must be $9/7$ of steel production, $(x, y) = (9t/7, t)$.
2. (a) For every dollar of manufacturing that's produced, 75¢of services need to be produced, $(x, y) = (3t/4, t)$.
   (b) \$6000 should go to services with \$8000 going to manufacturing.
3. (a) $(x, y, z) = (29t/26, 12t/13, t)$
   (b) \$58 000 should go to food, \$48 000 to shelter, and \$52 000 should go to clothing.
4. (a) \$160,000 of energy and \$70,000 of material should be produced.
   (b) Yes, the economy is productive since $(I - C)^{-1} \geq 0$.
   (c) The economy consumes \$158,000 of energy and \$69,000 of material.
5. (a) \$19,000 of Iron and \$10,000 of Steel should be produced.
   (b) Only iron is profitable.
6. (a) The economy should produce \$39000 in goods and \$31000 in services.
   (b) Yes, the economy is productive since $(I - C)^{-1} \geq 0$.
7. (a) \$2100 in services and \$2600 in manufacturing should be produced.
   (b) \$1200 in services and \$1100 in manufacturing is consumed internally.
   (c) Both industries are profitable.
8. (a) Both industries are profitable, and the economy is productive.
   (b) Only the second industry is profitable, and the economy is productive.
   (c) Only the second industry is profitable, and the economy is NOT productive.
9. (a) $\det(I - C) = .2$
   (b) $(I - C)^{-1} = \frac{1}{.2} \begin{bmatrix} .4 & .2 & .2 \\ .1 & .4 & .1 \\ .2 & .2 & .4 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ .5 & 2 & .5 \\ 1 & 1 & 2 \end{bmatrix}$
   (c) \$5300 in computing, \$4400 in statistical services and \$6100 in engineering should be produced.