

201-SH2-AB - Exercises #14 - Analysis of First and Second Derivatives

Find the intervals of increase and decrease, and local extrema of the following functions.

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| (1) $f(x) = -\frac{x^3}{3} + x^2 + 3x + 4$ | (11) $f(x) = \frac{-5x^2 + 2x + 8}{x^2}$ |
| (2) $f(x) = \frac{5x^2 + 5}{x}$ | (12) $f(x) = \frac{1}{4}x^4 - \frac{5}{3}x^3 + 2x^2 + 3$ |
| (3) $f(x) = \frac{-3x^2 - 12}{x}$ | (13) $f(x) = \frac{-2x^2 + 3x - 8}{x}$ |
| (4) $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2 + 4$ | (14) $f(x) = \frac{x^2 - x + 4}{x - 1}$ |
| (5) $f(x) = \frac{3x^2 - 5x + 27}{x}$ | (15) $f(x) = \frac{3}{4}x^4 - 3x^3 + 4$ |
| (6) $f(x) = \frac{x^2 - 2x + 9}{2 - x}$ | (16) $f(x) = \frac{2x^2 + 7x + 8}{x + 2}$ |
| (7) $f(x) = \frac{1}{2}x^4 + 2x^3 + 2$ | (17) $f(x) = \frac{3x^3 + 6}{x}$ |
| (8) $f(x) = \frac{4x^2 + 9x + 9}{x + 1}$ | (18) $f(x) = \frac{x^3}{x + 2}$ |
| (9) $f(x) = \frac{2x^3 - 4}{x}$ | (19) $f(x) = x^2 e^{2x}$ |
| (10) $f(x) = -\frac{6x^2 + 24}{x}$ | (20) $f(x) = 30x^{2/3} - 6x^{5/3}$ |

Find the intervals of concavity, and any points of inflection of the following functions.

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| (21) $f(x) = \frac{3x^3 + 10x - 24}{2x}$ | (26) $f(x) = \frac{3}{2}x^2 + \frac{12}{x - 1}$ |
| (22) $f(x) = -x^4 + 2x^3 + 5x$ | (27) $f(x) = x^4 + 2x^3 - 12x^2$ |
| (23) $f(x) = \frac{x^3 - x^2 - 8}{x - 1}$ | (28) $f(x) = 4x^5 + 5x^4 - 80x^3$ |
| (24) $f(x) = \frac{x^3 + 4x + 27}{x}$ | (29) $f(x) = (x^3 - 2x^2 + 2x)e^x$ |
| (25) $f(x) = x^4 + 4x^3 - 5x$ | (30) $f(x) = 30x^{2/3} - 6x^{5/3}$ |

Analyze both the first and second derivatives of the following functions.

(intervals of increase and decrease, local extrema, intervals of concavity, points of inflection)

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| (31) $f(x) = -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 6x + 3$ | (34) $f(x) = \frac{1 - x}{x^2}$ |
| (32) $f(x) = 2x^3 - 15x^2 + 36x + 5$ | (35) $f(x) = \frac{x}{x^2 - 9}$ |
| (33) $f(x) = (x + 1)e^{-2x}$ | (36) $f(x) = \sqrt[3]{x^2 - 4x}$ |

ANSWERS:

Analysis of first derivative:

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| (1) Inc: $(-1, 3)$
Dec: $(-\infty, -1), (3, \infty)$
Local max: $(3, 13)$
Local min: $(-1, 7/3)$ | (11) Inc: $(-8, 0)$
Dec: $(-\infty, -8), (0, \infty)$
Local max: none
Local min: $(-8, -41/8)$ |
| (2) Inc: $(-\infty, -1), (1, \infty)$
Dec: $(-1, 0), (0, 1)$
Local max: $(-1, -10)$
Local min: $(1, 10)$ | (12) Inc: $(0, 1), (4, \infty)$
Dec: $(-\infty, 0), (1, 4)$
Local max: $(1, 43/12)$
Local min: $(0, 3), (4, -23/3)$ |
| (3) Inc: $(-2, 0), (0, 2)$
Dec: $(-\infty, -2), (2, \infty)$
Local max: $(2, -12)$
Local min: $(-2, 12)$ | (13) Inc: $(-2, 0), (0, 2)$
Dec: $(-\infty, -2), (2, \infty)$
Local max: $(2, -5)$
Local min: $(-2, 11)$ |
| (4) Inc: $(-2, 0), (1, \infty)$
Dec: $(-\infty, -2), (0, 1)$
Local max: $(0, 4)$
Local min: $(-2, 4/3), (1, 43/12)$ | (14) Inc: $(-\infty, -1), (3, \infty)$
Dec: $(-1, 1), (1, 3)$
Local max: $(-1, -3)$
Local min: $(3, 5)$ |
| (5) Inc: $(-\infty, -3), (3, \infty)$
Dec: $(-3, 0), (0, 3)$
Local max: $(-3, -23)$
Local min: $(3, 13)$ | (15) Inc: $(3, \infty)$
Dec: $(-\infty, 3)$
Local max: none
Local min: $(3, -65/4)$ |
| (6) Inc: $(-1, 2), (2, 5)$
Dec: $(-\infty, -1), (5, \infty)$
Local max: $(5, -8)$
Local min: $(-1, 4)$ | (16) Inc: $(-\infty, -3), (-1, \infty)$
Dec: $(-3, -2), (-2, -1)$
Local max: $(-3, -5)$
Local min: $(-1, 3)$ |
| (7) Inc: $(-3, \infty)$
Dec: $(-\infty, -3)$
Local max: none
Local min: $(-3, -23/2)$ | (17) Inc: $(1, \infty)$
Dec: $(-\infty, 0), (0, 1)$
Local max: none
Local min: $(1, 9)$ |
| (8) Inc: $(-\infty, -2), (0, \infty)$
Dec: $(-2, -1), (-1, 0)$
Local max: $(-2, -7)$
Local min: $(0, 9)$ | (18) Inc: $(-3, -2), (-2, \infty)$
Dec: $(-\infty, -3)$
Local max: none
Local min: $(-3, 27)$ |
| (9) Inc: $(-1, 0), (0, \infty)$
Dec: $(-\infty, -1)$
Local max: none
Local min: $(-1, 6)$ | (19) Inc: $(-\infty, -1), (0, \infty)$
Dec: $(-1, 0)$
Local max: $(-1, e^{-2})$
Local min: $(0, 0)$ |
| (10) Inc: $(-2, 0), (0, 2)$
Dec: $(-\infty, -2), (2, \infty)$
Local max: $(2, -24)$
Local min: $(-2, 24)$ | (20) Inc: $(0, 2)$
Dec: $(-\infty, 0), (2, \infty)$
Local max: $(2, 28.57)$
Local min: $(0, 0)$ |

Analysis of second derivative:

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| <p>(21) CU: $(-\infty, 0), (2, \infty)$
 CD: $(0, 2)$
 POI: $(2, 5)$</p> <p>(22) CU: $(0, 1)$
 CD: $(-\infty, 0), (1, \infty)$
 POI: $(0, 0), (1, 6)$</p> <p>(23) CU: $(-\infty, 1), (3, \infty)$
 CD: $(1, 3)$
 POI: $(3, 5)$</p> <p>(24) CU: $(-\infty, -3), (0, \infty)$
 CD: $(-3, 0)$
 POI: $(-3, 4)$</p> <p>(25) CU: $(-2, 0)$
 CD: $(-\infty, -2), (0, \infty)$
 POI: $(0, 0), (-2, -6)$</p> | <p>(26) CU: $(-\infty, -1), (1, \infty)$
 CD: $(-1, 1)$
 POI: $(-1, -9/2)$</p> <p>(27) CU: $(-\infty, -2), (1, \infty)$
 CD: $(-2, 1)$
 POI: $(-2, -48), (1, -9)$</p> <p>(28) CU: $(-2.86, 0), (2.10, \infty)$
 CD: $(-\infty, -2.86), (0, 2.10)$
 POI: $(-2.86, 1433.00), (0, 0), (2.10, -481.74)$</p> <p>(29) CU: $(-4, 0), (0, \infty)$
 CD: $(-\infty, -4)$
 POI: $(-4, -1.90)$</p> <p>(30) CU: $(-\infty, -1)$
 CD: $(-1, 0), (0, \infty)$
 POI: $(-1, 36)$</p> |
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Analysis of first and second derivatives:

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| <p>(31) Inc: $(-3, 2)$
 Dec: $(-\infty, -3), (2, \infty)$
 Local max: $(2, 31/3)$
 Local min: $(-3, -21/2)$
 CU: $(-\infty, -1/2)$
 CD: $(-1/2, \infty)$
 POI: $(-1/2, -1/12)$</p> <p>(32) Inc: $(-\infty, 2), (3, \infty)$
 Dec: $(2, 3)$
 Local max: $(2, 33)$
 Local min: $(3, 32)$
 CU: $(5/2, \infty)$
 CD: $(-\infty, 5/2)$
 POI: $(5/2, 65/2)$</p> <p>(33) Inc: $(-\infty, -1/2)$
 Dec: $(-1/2, \infty)$
 Local max: $(-1/2, e/2)$
 Local min: none
 CU: $(0, \infty)$
 CD: $(-\infty, 0)$
 POI: $(0, 1)$</p> | <p>(34) Inc: $(-\infty, 0), (2, \infty)$
 Dec: $(0, 2)$
 Local max: none
 Local min: $(2, -1/4)$
 CU: $(-\infty, 0), (0, 3)$
 CD: $(3, \infty)$
 POI: $(3, -2/9)$</p> <p>(35) Inc: nowhere
 Dec: $(-\infty, -3), (-3, 3), (3, \infty)$
 Local max: none
 Local min: none
 CU: $(-3, 0), (3, \infty)$
 CD: $(-\infty, -3), (0, 3)$
 POI: $(0, 0)$</p> <p>(36) Inc: $(2, 4), (4, \infty)$
 Dec: $(-\infty, 0), (0, 2)$
 Local max: none
 Local min: $(2, -\sqrt[3]{4})$
 CU: $(0, 4)$
 CD: $(-\infty, 0), (4, \infty)$
 POI: $(0, 0), (4, 0)$</p> |
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