1. Consider the linear program:
   Maximize $z = x + 3y$
   subject to
   $$2x + 3y \leq 24$$
   $$x - y \leq 7$$
   $$y \leq 6$$
   $$x \geq 0, \quad y \geq 0$$

   (a) Solve the linear program by graphing the feasibility region.

   (b) Solve using the simplex method.

   (c) In the previous part, label your simplex tables A, B, C etc. (A is the initial table, B is the second table, and so on.) Using the same labels, indicate the points on your graph that correspond to the basic feasible solution for each table.

2. Consider the linear program:
   Maximize $z = x + y$
   subject to
   $$x + 3y \leq 15$$
   $$2x + y \leq 10$$
   $$x \leq 4$$
   $$x \geq 0, \quad y \geq 0$$

   (a) Solve the linear program by graphing the feasibility region.

   (b) Solve using the simplex method.

   (c) In the previous part, label your simplex tables A, B, C etc. (A is the initial table, B is the second table, and so on.) Using the same labels, indicate the points on your graph that correspond to the basic feasible solution for each table.

3. Solve using the simplex algorithm.
   Maximize $z = 3x_1 - 4x_2 + 5x_3$
   subject to
   $$2x_1 - 3x_2 + 5x_3 \leq 30$$
   $$-2x_1 + x_2 + x_3 \leq 20$$
   $$x_1 + 7x_2 \leq 10$$
   $$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0$$

4. Solve using the simplex algorithm.
   Maximize $z = 20x_1 + 5x_2 - 13x_3$
   subject to
   $$x_1 + 4x_2 - x_3 \leq 20$$
   $$2x_1 + 3x_2 - 2x_3 \leq 30$$
   $$-4x_1 - 9x_2 + 18x_3 \leq 80$$
   $$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0$$

5. Solve using the simplex algorithm.
   Minimize $z = -5x_1 + 4x_2 - 3x_3$
   subject to
   $$x_1 + x_2 - 2x_3 \leq 10$$
   $$7x_2 + x_3 \leq 5$$
   $$5x_1 - 3x_2 + 2x_3 \leq 15$$
   $$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0$$

6. Solve using the simplex algorithm.
   Minimize $z = -4x_1 + 3x_2 - 2x_3$
   subject to
   $$4x_1 + 5x_2 \leq 2$$
   $$8x_1 - 2x_2 + 3x_3 \leq 8$$
   $$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0$$

7. Consider the optimization problem below.
   Maximize $z = -2x_1 + 4x_2 + x_3$
   subject to
   $$5x_1 + x_2 - 3x_3 \leq 3$$
   $$6x_1 - 2x_2 + 4x_3 \leq 7$$
   $$x_1 \geq 0, \quad x_2 \geq 0 \quad x_3 \geq 0$$

   (a) Use the simplex algorithm to show that the linear program is unbounded (has no maximum) by identifying an unbounded variable.

   (b) * Support your answer to part (a) by finding a feasible solution where $z \geq 1300$.

8. Consider the optimization problem below.
   Minimize $z = 3x_1 - 6x_2 + 2x_3$
   subject to
   $$x_1 + 2x_2 - 5x_3 \leq 3$$
   $$2x_2 - x_3 \leq 2$$
   $$-5x_1 - 2x_2 + 2x_3 \leq 4$$
   $$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0$$

   (a) Use the simplex algorithm to show that the linear program is unbounded (has no minimum) by identifying an unbounded variable.

   (b) * Support your answer to part (a) by finding a feasible solution where $z \leq -2012$

9. Consider the optimization problem below.
   Minimize $z = -3x_1 + 4x_2 + 2x_3$
   subject to
   $$-4x_1 + x_2 + x_3 \leq 3$$
   $$2x_1 - 5x_2 + x_3 \leq 4$$
   $$-2x_1 + x_3 \leq 1$$
   $$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0$$

   (a) Use the simplex algorithm to show that the linear program is unbounded (has no minimum) by identifying an unbounded variable.

   (b) * Support your answer to part (a) by finding a feasible solution where $z \leq -7006$.

10. Consider the optimization problem below.
    Maximize $z = -3x_1 + 4x_2 + 3x_3$
    subject to
    $$-4x_1 + 2x_2 + x_3 \leq 8$$
    $$-2x_1 + x_2 - x_3 \leq 3$$
    $$x_1 - 3x_2 \leq 6$$
    $$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0$$

    (a) Use the simplex algorithm to show that the linear program is unbounded (has no maximum) by identifying an unbounded variable.

    (b) * Support your answer to part (a) by finding a feasible solution where $z \geq 5012$.

11. Consider the optimization problem below.

$$\textbf{Maximize} \quad z = \quad x_1 \quad + \quad 5x_2 \quad + \quad 6x_3$$

subject to
$$
\begin{aligned}
-2x_1 &+ 9x_2 + 3x_3 \leq 5 \\
-x_1 &+ 4x_2 + 2x_3 \leq 2 \\
x_1 &+ x_2 - 4x_3 \leq 3 \\
x_1 \geq 0, \quad & x_2 \geq 0, \quad x_3 \geq 0
\end{aligned}
$$

(a) Use the simplex algorithm to show that the linear program is unbounded (has no maximum) by identifying an unbounded variable.

(b) * Find a solution (x_1, x_2, x_3) such that $z = 12006$.

12. You are selling martinis at an event to raise funds for charity. A dry martini is $5/3$ oz. of Gin and $1/3$ oz. of Vermouth. A medium martini is $3/2$ oz. Gin and $1/2$ oz. Vermouth. You have available 60 oz. of Gin and 16 oz. of Vermouth. You can sell dry martinis for $8 each and medium martinis for $10 each, but you only have 39 glasses for serving all of the drinks sold. How many of each kind of martini should be sold to maximize revenue? Name your variables, set up a linear program, and solve by using the simplex algorithm.

13. The Simple Machine Company makes Widgets, Gadgets, and Gizmos out of pulleys, wedges, and levers. Each Widget requires 3 pulleys, 2 wedges, and 4 levers. Each Gadget requires 7 pulleys and 1 lever. Each Gizmo requires 3 pulleys, 2 wedges, and 5 levers. The company has 56 pulleys, 16 wedges, and 33 levers available. Suppose the company sells Widgets for $2 each, Gadgets for $4 each, and Gizmos for $1 each. Set-up and use the simplex algorithm to solve a linear program to determine how many Widgets, Gadgets, and Gizmos should be made to maximize revenue.

    (a) What is the maximum revenue?

    (b) When revenue is maximized, how many pulleys, wedges, and levers are left over?

14. The Furniture Factory makes beds, chairs, and couches from the raw materials labor, lumber, and cloth. The company makes a profit of $60 per bed, $10 per chair, and $40 per couch. Each bed requires 1 hour of labor, 3 metres of lumber, and 3 metres of cloth; each chair requires 1 hour of labor, 5 meters of lumber and 7 meters of cloth; and each couch requires 2 hours of labor, 1 meter of lumber, and 1 meter of cloth. If there are 100 hours of labor, 90 meters of lumber, and 120 meters of cloth available, you would like to determine how many beds, chairs, and couches the company should make to maximize profit.

    (a) Name variables and set up a linear program that represents this situation.

    (b) What is the maximum profit?

    (c) How many beds, chairs, and couches should be made to maximize profit?

    (d) When profit is maximized, how much labor, lumber, and cloth will go unused?

15. Squeeky Cleaners makes three cleaning products: Ocean Fresh, Summer Breeze, and Lemon Zest, which sell for $1, $3, and $2 respectiviely. The products are made using three processes: separating, blending, and mixing. One batch of Ocean Fresh requires 2 hours of separating, 1 hour of blending, and 4 hours of mixing. One batch of Summer Breeze requires 1 hour of separating and 2 hours of mixing. One batch of Lemon Zest requires 1 hour of blending and 1 hour of mixing. In a week it is possible to do 25 hours of separating, 30 hours of blending, and 40 hours of mixing. The company would like to determine how many batches of each product should be made to maximize weekly revenue.

    (a) Name variables and set-up a linear program that represents this situation.

    (b) Solve the problem using the simplex algorithm.

    (c) When revenue is maximized, how many hours of each process go unused?

16. Alice, Bob, and Cathy together make Xylophones, Yoyos, and Zippers. To make a Xylophone takes 4 hours of Alice's time, 2 hours of Bob's time, and 10 hours of Cathy's time. (So together it takes them 16 hours.) Similarly to make a Yoyo takes 9 hours for Alice, 3 hours for Bob, and 5 hours for Cathy; and to make a Zipper takes 3 hours, 1 hour, and 6 hours of their time respectively. Meanwhile, Alice has 120 hours available, Bob has 50 hours available, and Cathy has 400 hours available. Finally, Xylophones sell for $12 each, Yoyos sell for $8 each and Zippers sell for $7 each.

    (a) Name variables and set-up a linear program to maximize revenue under these conditions.

    (b) Solve the program to determine how many Xylophones, Yoyos, and Zippers are made when revenue is maximized.

    (c) How many hours do Alice, Bob, and Cathy each have left over when revenue is maximized?

17. A popular cleaning brand produces two housecleaners: EnviroShine and Nature's Scent. Both cleaners contain the same basic ingredients, but in different proportions. To protect their trade secret, we will refer to the three ingredients as I_1, I_2, and I_3. These cleaners are sold in 1 L containers. The production of 1 L of EnviroShine requires 0.4 L of I_1, 0.3 L of I_2, and 0.3 L of I_3. The production of 1 L of Natrue's Scent requires 0.5 L of I_1, 0.2 L of I_2, and 0.3 L of I_3. The company's suppliers can guarantee a weekly supply of 94 L of I_1, 51 L of I_2, and 60 L of I_3. The profit margin is $1.50 for every EnviroShine bottle, and $1.20 for every Nature's Scent bottle.

    (a) How many litres of each housecleaner does the company need to produce weekly to maximize its profit?

    (b) What quantities of the 3 ingredients should the company purchase weekly to avoid needing to store surplus?

    (c) The distribution service informs the director of production that they are unable to sell all the EnviroShine stock. The reports for the last months show that the distributers cannot sell more than 70 bottles of this product per week. Given this infomation, establish a new weely production.

    (d) Does the revised work plan require that the weekly puchasing of ingredients be modified? If so, what are the new quantities of each ingredient that should be purchased weekly?

1. (a) Max: $z = 21$ at $(3, 6)$.
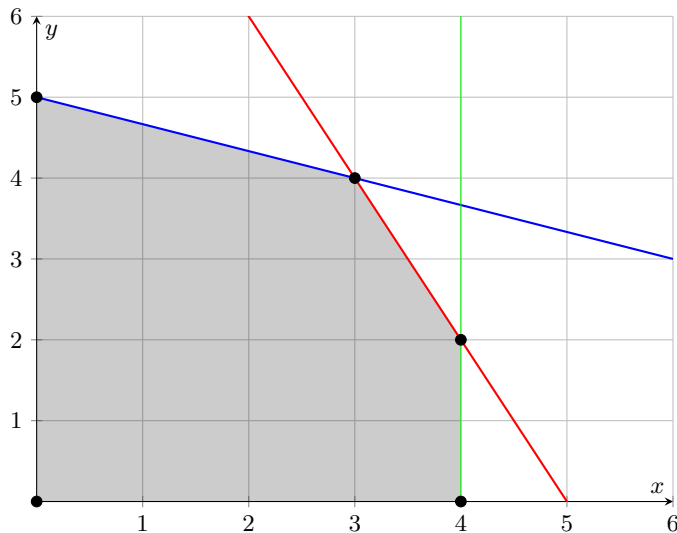
   Other corner points: $(0, 0), (0, 6), (9, 2), (7, 0)$
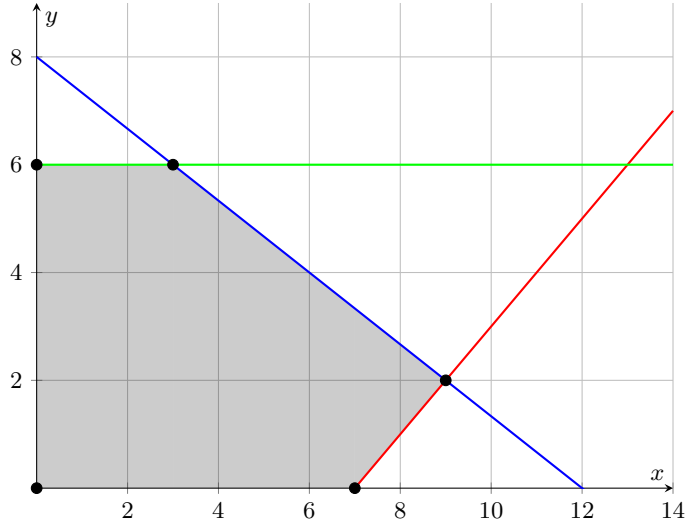
   (b) Max: $z = 21$ at $(3, 6, 0, 10, 0)$

   (c) A $\to$ $(0, 0)$; B $\to$ $(0, 6)$; C $\to$ $(3, 6)$

2. (a) Max: $z = 7$ at $(3, 4)$.

   Other corner points: $(0, 0), (0, 5), (4, 2), (4, 0)$

   (b) Max: $z = 7$ at $(3, 4, 0, 0, 1)$

   (c) There are 2 possible paths, depending on which column you start with:

   A $\to$ $(0, 0)$; B $\to$ $(4, 0)$; C $\to$ $(4, 2)$; D $\to$ $(3, 4)$

   A $\to$ $(0, 0)$; B $\to$ $(0, 5)$; C $\to$ $(3, 4)$

3. Ans: Max $z = 40$ at $(10, 0, 2, 0, 38, 0)$.

4. Ans: Max $z = 370$ at $(25, 0, 10, 5, 0, 0)$

5. Ans: Max $z = -20$ at $(1, 0, 5, 19, 0, 0)$.

6. Ans: Min $z = -16/3$ at $(0, 0, 8/3, 2, 0)$.

   Note: this is an example where choosing the column whose top entry is furthest from zero does not give the shortest path to the answer.

7. Ans: $z = 1312$ at $(0, 303, 100, 0, 213)$ for example.

8. Ans: $z = -2012$ at $(1000, 2504, 5006, 19025, 0, 0)$.

9. Ans: $z = -7006$ at $(5002, 2000, 0, 18011, 0, 10005)$.

10. Ans: z=5012 at (1000, 2003, 0, 2, 0, 5015)

11. Ans: z=12006 at (3000, 0, 1501, 1502, 0, 3007) is one possible solution

12. Ans: Maximum Revenue is $344 when 18 dry martinis and 20 medium martinis are sold.

13. Ans: Let x_1 =number of Widgets made; let x_2 =number of Gadgets made; and let x_3=number of Gizmos made.

$$
\begin{array}{lrcrcrcl}
\textbf{Maximize} & z = & 2x_1 & + & 4x_2 & + & x_3 & \\
\text{subject to} & & 3x_1 & + & 7x_2 & + & 3x_3 & \leq 56 \\
& & 2x_1 & & & + & 2x_3 & \leq 16 \\
& & 4x_1 & + & x_2 & + & 5x_3 & \leq 33 \\
& & x_1 \geq 0, & & x_2 \geq 0, & & x_3 \geq 0 &
\end{array}
$$

(a) The maximum revenue is $34 when 7 widgets, 5 gadgets, and 0 gizmos are made.

(b) When revenue is maximized there will be 0 pulleys, 2 wedges, and 0 levers remaining.

14. Ans: (a) Let x_1 be the number of beds, let x_2 be the number of chairs, and let x_3 be the number of couches made.

$$
\begin{array}{lrcrcrcl}
\textbf{Maximize} & z = 60x_1 & + & 10x_2 & + & 40x_3 & \\
\text{subject to} & x_1 & + & x_2 & + & 2x_3 & \leq 100 \\
& 3x_1 & + & 5x_2 & + & x_3 & \leq 90 \\
& 3x_1 & + & 7x_2 & + & x_3 & \leq 120 \\
& x_1 \geq 0, & & x_2 \geq 0, & & x_3 \geq 0 &
\end{array}
$$

(b) Max profit = $2640.

(c) Profit is maximized when 16 beds, 0 chairs, and 42 couches are made.

(d) When profit is maximized, there are 0 hours of labor, 0 meters of lumber, and 30 meters of cloth unused.

15. Ans: (a) Let x_1 =number of batches of Ocean Fresh; let x_2 =number of batches of Summer Breeze; and let x_3=number of batches of Lemon Zest.

$$
\begin{array}{lrcrcrcl}
\textbf{Maximize} & z = & x_1 & + & 3x_2 & + & 2x_3 & \\
\text{subject to} & & 2x_1 & + & x_2 & & & \leq 25 \\
& & x_1 & & & + & x_3 & \leq 30 \\
& & 4x_1 & + & 2x_2 & + & x_3 & \leq 40 \\
& & x_1 \geq 0, & & x_2 \geq 0, & & x_3 \geq 0 &
\end{array}
$$

(b) The maximum revenue is $75 when 0 batches of Ocean Fresh, 5 batches of Summer Breeze, and 30 batches of Lemon Zest are made.

(c) When revenue is maximized there will be 20 hours of separating, 0 hours of blending, and 0 hours of mixing remaining.

16. (a) Let x_1 =Xylophones; let x_2 =Yoyos; and let x_3 =Zippers.

$$
\begin{array}{lrcrcrcl}
\textbf{Maximize} & z = & 12x_1 & + & 8x_2 & + & 7x_3 & \\
\textbf{subject to} & & 4x_1 & + & 9x_2 & + & 3x_3 & \leq 120 \\
& & 2x_1 & + & 3x_2 & + & x_3 & \leq 50 \\
& & 10x_1 & + & 5x_2 & + & 6x_3 & \leq 400 \\
& & x_1 \geq 0, & & x_2 \geq 0, & & x_3 \geq 0 &
\end{array}
$$

(b) Maximum revenue of $320 occurs when 15 Xylophones, 0 Yoyos, and 20 Zippers are made.

(c) When revenue is maximized Alice and Bob have 0 hours left over, while Cathy has 130 hours left over.

17. (a) Let x be the number of litres of EnviroShine, and let y be the number of litres of Nature's Scent cleaners.

$$
\begin{array}{lrcrcl}
\textbf{Maximize} & z = & 1.5x & + & 1.2y & \\
\textbf{subject to} & & 0.4x & + & 0.5y & \leq 94 \\
& & 0.3x & + & 0.2y & \leq 51 \\
& & 0.3x & + & 0.3y & \leq 60 \\
& & x \geq 0, & & y \geq 0 &
\end{array}
$$

Maximum profit = $273 occurs when 110 L of EnviroShine and 90 L of Nature's Scent are made

(b) 89 L of I_1, 51 L of I_2 and 60 L of I_3.

(c) New constraint: $x \leq 70$. New production: 70 L of EnviroShine and 130 L of Nature's Scent, with a profit of $261.

(d) The weekly purchasing must be modified to 93 L of I_1, 47 L of I_2, and 60 L of I_3.