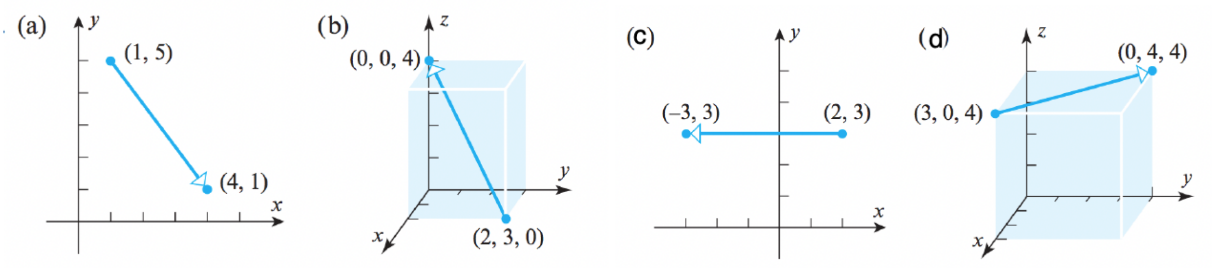
1. Find the components of the vector.



2. Find the components of the vector $\overrightarrow{AB}$.

   (a) $A(3,5), B(2,8)$ (b) $A(-6,2), B(-4,-1)$ (c) $A(-5,-2,1), B(2,4,2)$

3. Find the terminal point of the vector that is equivalent to $\overrightarrow{u} = \langle 1,2\rangle$ and whose initial point is $A(1,1)$.

4. Find the initial point of the vector that is equivalent to $\overrightarrow{u} = \langle 1,1,3\rangle$ and whose terminal point is $B(-1,-1,2)$.

5. Given point $B(3,0,-5)$, find the point A such that

   (a) $\overrightarrow{AB} = \langle 4,-2,1\rangle$ (b) $\overrightarrow{BA} = \langle 4,-2,1\rangle$

6. Let $\overrightarrow{u} = \langle 4,-1\rangle$, $\overrightarrow{v} = \langle 0,5\rangle$, and $\overrightarrow{w} = \langle -3,-3\rangle$. Find the components of

   (a) $\overrightarrow{u} + \overrightarrow{w}$ (b) $\overrightarrow{v} - 3\overrightarrow{u}$ (c) $2(\overrightarrow{u} - 5\overrightarrow{w})$ (d) $3\overrightarrow{v} - 2(\overrightarrow{u} + 2\overrightarrow{w})$ (e) $-3(\overrightarrow{w} - 2\overrightarrow{u} + \overrightarrow{v})$

7. Let $\overrightarrow{u} = \langle -3,2,1,0\rangle$, $\overrightarrow{v} = \langle 4,7,-3,2\rangle$, and $\overrightarrow{w} = \langle 5,-2,8,1\rangle$. Find the components of

   (a) $\overrightarrow{v} - \overrightarrow{w}$ (b) $2\overrightarrow{u} + 7\overrightarrow{v}$ (c) $-\overrightarrow{u} + (\overrightarrow{v} - 4\overrightarrow{w})$ (d) $6(\overrightarrow{u} - 3\overrightarrow{v})$ (e) $(6\overrightarrow{v} - \overrightarrow{w}) - (4\overrightarrow{u} + \overrightarrow{v})$

8. Let $\overrightarrow{u} = \langle -3,1,2,4,4\rangle$, $\overrightarrow{v} = \langle 4,0,-8,1,2\rangle$, and $\overrightarrow{w} = \langle 6,-1,-4,3,-5\rangle$.
   Find the components of the vector $\overrightarrow{x}$ that satisfies the equation: $2\overrightarrow{u} - \overrightarrow{v} + \overrightarrow{x} = 7\overrightarrow{x} + \overrightarrow{w}$.

9. For each vector $\overrightarrow{v}$, find:
   (i) its magnitude, (ii) a unit vector that has the same direction as $\overrightarrow{v}$, and (iii) a unit vector that is the oppositely directed to $\overrightarrow{v}$.

   (a) $\overrightarrow{v} = \langle 4,-3\rangle$ (b) $\overrightarrow{v} = \langle 2,2,2\rangle$ (c) $\overrightarrow{v} = \langle 1,0,2,1,3\rangle$

10. Evaluate the given expression with $\overrightarrow{u} = \langle 2,-2,3\rangle$, $\overrightarrow{v} = \langle 1,-3,4\rangle$, and $\overrightarrow{w} = \langle 3,6,-4\rangle$.

    (a) $\|\overrightarrow{u} + \overrightarrow{v}\|$ (b) $\|\overrightarrow{u}\| + \|\overrightarrow{v}\|$ (c) $\|-2\overrightarrow{u} + 2\overrightarrow{v}\|$ (d) $\|3\overrightarrow{u} - 5\overrightarrow{v} + \overrightarrow{w}\|$

11. Evaluate the given expression with $\overrightarrow{u} = \langle -2,-1,4,5\rangle$, $\overrightarrow{v} = \langle 3,1,-5,7\rangle$, and $\overrightarrow{w} = \langle -6,2,1,1\rangle$.

    (a) $\|3\overrightarrow{u} - 5\overrightarrow{v} + \overrightarrow{w}\|$ (b) $\|3\overrightarrow{u}\| - 5\|\overrightarrow{v}\| + \|\overrightarrow{w}\|$ (c) * $\|-\|\overrightarrow{u}\|\overrightarrow{v}\|$

12. Let $\overrightarrow{v} = \langle -2,3,0,6\rangle$. Find all scalars k such that $\|k\overrightarrow{v}\| = 5$.

13. Find: (i) $\vec{u} \cdot \vec{v}$, (ii) $\vec{u} \cdot \vec{u}$, and (iii) $\vec{v} \cdot \vec{v}$

(a) $\vec{u} = \langle 3, 1, 4\rangle$, $\vec{v} = \langle 2, 2, -4\rangle$

(b) $\vec{u} = \langle 1, 1, 4, 6\rangle$, $\vec{v} = \langle 2, -2, 3, -2\rangle$

14. Determine whether $\vec{u}$ and $\vec{v}$ are orthogonal vectors.

(a) $\vec{u} = \langle 6, 1, 4\rangle$, $\vec{v} = \langle 2, 0, -3\rangle$

(b) $\vec{u} = \langle 0, 0, -1\rangle$, $\vec{v} = \langle 1, 1, 1\rangle$

(c) $\vec{u} = \langle -6, 0, 4\rangle$, $\vec{v} = \langle 3, 1, 6\rangle$

(d) $\vec{u} = \langle 2, 4, -8\rangle$, $\vec{v} = \langle 5, 3, 7\rangle$

15. Determine whether the vectors form an orthogonal set.

(a) $\vec{u} = \langle 2, 3\rangle$, $\vec{v} = \langle 3, 2\rangle$

(b) $\vec{u} = \langle -1, 1\rangle$, $\vec{v} = \langle 1, 1\rangle$

(c) $\vec{u} = \langle -2, 1, 1\rangle$, $\vec{v} = \langle 1, 0, 2\rangle$,$\vec{w} = \langle -2, -5, 1\rangle$

(d) $\vec{u} = \langle -3, 4, -1\rangle$, $\vec{v} = \langle 1, 2, 5\rangle$,$\vec{w} = \langle 4, -3, 0\rangle$

16. Do the points $A(1, 1, 1)$, $B(-2, 0, 3)$, and $C(-3, -1, 1)$ form the vertices of a right triangle? If so, at which point is the right angle?

17. Let $\vec{u} = \langle 3, 2, -1\rangle$, $\vec{v} = \langle 0, 2, -3\rangle$, and $\vec{w} = \langle 2, 6, 7\rangle$. Compute the indicated vectors.

(a) $\vec{v} \times \vec{w}$

(b) $\vec{u} \times (\vec{v} \times \vec{w})$

(c) $(\vec{u} \times \vec{v}) \times \vec{w}$

18. Use the cross product to find a vector that is orthogonal to both $\vec{u}$ and $\vec{v}$.

(a) $\vec{u} = \langle -6, 4, 2\rangle$, $\vec{v} = \langle 3, 1, 5\rangle$

(b) $\vec{u} = \langle -2, 1, 5\rangle$, $\vec{v} = \langle 3, 0, -3\rangle$

19. Find a unit vector that is orthogonal to both Let $\vec{u} = \langle 1, 0, 1\rangle$, $\vec{v} = \langle 0, 1, 1\rangle$.

20. If possible, write $\vec{v}$ as a linear combination of $\vec{u_1}$ and $\vec{u_2}$ (and of $\vec{u_3}$ where given).

(a) $\vec{v} = \langle 2, 10\rangle$, $\vec{u_1} = \langle -1, 2\rangle$, $\vec{u_2} = \langle 2, 3\rangle$

(b) $\vec{v} = \langle 2, 4\rangle$, $\vec{u_1} = \langle -1, 4\rangle$, $\vec{u_2} = \langle 3, -6\rangle$

(c) $\vec{v} = \langle 6, -4, 4\rangle$, $\vec{u_1} = \langle -2, 2, 6\rangle$, $\vec{u_2} = \langle 4, -3, -8\rangle$

(d) $\vec{v} = \langle 3, -4, 5\rangle$, $\vec{u_1} = \langle -3, 6, -3\rangle$, $\vec{u_2} = \langle 6, -11, 7\rangle$

(e) $\vec{v} = \langle -1, 1, 19\rangle$, $\vec{u_1} = \langle 1, -1, 0\rangle$,
$\vec{u_2} = \langle 3, 2, 1\rangle$, $\vec{u_3} = \langle 0, 1, 4\rangle$

(f) $\vec{v} = \langle 1, -4, 9, 18\rangle$, $\vec{u_1} = \langle 1, -1, 3, 5\rangle$, $\vec{u_2} = \langle 2, 1, 0, -3\rangle$

21. Terra Quarries mines fine sand, coarse sand, and gravel at their three quarries; Eastwood, Westlane, and Northfolk. The daily outputs for the quarries are as follows: At Eastwood, 20 tons of fine sand, 40 tons of coarse sand, and 20 tons of gravel; At Westlane, 60 tons of fine sand, 60 tons of coarse sand, and 30 tons of gravel; At Northfolk, 30 tons of fine sand, 40 tons of coarse sand, and 20 tons of gravel. None of these locations are mined everyday and management wants to reduce fixed costs by closing one of them, specifically Northfolk. Is there some combination of the outputs from the other two mines that will equal the output of the Northfolk mine?

22. Sawdust Paper Company produces three types of paper; copy paper, cardstock, and newsprint at their Springfield, Summerhill, and Fallbrook mills. The daily outputs for the three mills are as follows: 3 tons copy paper, 3 tons cardstock, and 6 tons newsprint at Springfield, 6 tons copy paper, 8 tons cardstock, and 10 tons newsprint at Summerhill, and 9 tons copy paper, 10 tons cardstock, and 17 tons newsprint at Fallbrook. The economy is in recession and the company wants to cut expenses by shutting down the Fallbrook mill. Is there some combination of the outputs from the other two mills that will equal the output of the Fallbrook mill?

23. Earthworks is a glass recycling company that manufactures two bottle types from mixtures of recycled white, brown, and green glass. The first bottle type is made from 74% white glass, 22% brown glass, and 4% green glass; the second type is made from 81% white glass, 13% brown glass, and 6% green glass. Two regions of their city are the sources of the recycled glass. On the North End, the glass bottles collected are consistently 92% white glass, 6% brown glass, and 2% green glass. The West Side has a consistent glass bottle collection that is 70% white glass, 20% brown glass, and 10% green glass.
Determine if each bottle type can be blended from these glass sources.

ANSWERS:

1. (a) $\langle 3, -4\rangle$

   (b) $\langle -2, -3, 4\rangle$

   (c) $\langle -5, 0\rangle$

   (d) $\langle -3, 4, 0\rangle$

2. (a) $\langle -1, 3\rangle$ (b) $\langle 2, -3\rangle$ (c) $\langle 7, 6, 1\rangle$

3. $B(2, 3)$

4. $A(-2, -2, -1)$

5. (a) $A(-1, 2, -6)$ (b) $A(7, -2, -4)$

6. (a) $\langle 1, -4\rangle$ (b) $\langle -12, 8\rangle$ (c) $\langle 38, 28\rangle$ (d) $\langle 4, 29\rangle$ (e) $\langle 33, -12\rangle$

7. (a) $\langle -1, 9, -11, 1\rangle$ (b) $\langle 22, 53, -19, 14\rangle$ (c) $\langle -13, 13, -36, -2\rangle$ (d) $\langle -90, -114, 60, -36\rangle$ (e) $\langle 27, 29, -27, 9\rangle$

8. $\overrightarrow{x} = \langle -\frac{8}{3}, \frac{1}{2}, \frac{8}{3}, \frac{2}{3}, \frac{11}{6}\rangle$

9. (a) (i) $\|\overrightarrow{v}\| = 5$, (ii) $\dfrac{\overrightarrow{v}}{\|\overrightarrow{v}\|} = \langle \frac{4}{5}, -\frac{3}{5}\rangle$, (iii) $-\dfrac{\overrightarrow{v}}{\|\overrightarrow{v}\|} = \langle -\frac{4}{5}, \frac{3}{5}\rangle$

   (b) (i) $\|\overrightarrow{v}\| = 2\sqrt{3}$, (ii) $\dfrac{\overrightarrow{v}}{\|\overrightarrow{v}\|} = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\rangle$, (iii) $-\dfrac{\overrightarrow{v}}{\|\overrightarrow{v}\|} = \langle -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\rangle$

   (c) (i) $\|\overrightarrow{v}\| = \sqrt{15}$, (ii) $\dfrac{\overrightarrow{v}}{\|\overrightarrow{v}\|} = \dfrac{1}{\sqrt{15}}\langle 1, 0, 2, 1, 3\rangle$, (iii) $-\dfrac{\overrightarrow{v}}{\|\overrightarrow{v}\|} = -\dfrac{1}{\sqrt{15}}\langle 1, 0, 2, 1, 3\rangle$,

10. (a) $\sqrt{83}$ (b) $\sqrt{17} + \sqrt{26}$ (c) $2\sqrt{3}$ (d) $\sqrt{466}$

11. (a) $\sqrt{2570}$ (b) $3\sqrt{46} - 10\sqrt{21} + \sqrt{42}$ (c) $2\sqrt{966}$

12. $k = \pm\dfrac{5}{7}$

13. (a) (i) -8 (ii) 26 (iii) 24 (b) (i) 0 (ii) 54 (iii) 21

14. (a) Orthogonal (b) Not orthogonal (c) Not orthogonal (d) Not orthogonal

15. (a) Not an orthogonal set (b) Orthogonal set (c) Orthogonal set (d) Not an orthogonal set

16. Yes, at point B.

17. (a) $\langle 32, -6, -4\rangle$ (b) $\langle -14, -20, -82\rangle$ (c) $\langle 27, 40, -42\rangle$

18. (a) $\langle 18, 36, -18\rangle$ (b) $\langle -3, 9, -3\rangle$

19. $\pm\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\rangle$

20. (a) Yes. $\overrightarrow{v} = 2\overrightarrow{u_1} + 2\overrightarrow{u_2}$ (c) Not possible (e) Yes. $\overrightarrow{v} = 2\overrightarrow{u_1} - \overrightarrow{u_2} + 5\overrightarrow{u_3}$

    (b) Yes. $\overrightarrow{v} = 4\overrightarrow{u_1} + 2\overrightarrow{u_2}$ (d) Yes. $\overrightarrow{v} = 3\overrightarrow{u_1} + 2\overrightarrow{u_2}$ (f) Yes. $\overrightarrow{v} = 3\overrightarrow{u_1} - \overrightarrow{u_2}$

21. Yes. If $\overrightarrow{u_1}$=the daily output of Eastwood, $\overrightarrow{u_2}$ = the daily output at Westlane, and $\overrightarrow{v}$=the daily output at Northfolk, then $\overrightarrow{v} = \frac{1}{2}\overrightarrow{u_1} + \frac{1}{3}\overrightarrow{u_2}$.

22. Yes. One day's output at Fallbrook can be replaced by 2 day's output at Springfield plus 1/2 day's output at Summerhill.

23. If $\overrightarrow{u_1} = \langle 92, 6, 2\rangle$= the glass input from the North End, and $\overrightarrow{u_2} = \langle 70, 20, 10\rangle$ = the glass input from the West Side, then the first bottle type, $\overrightarrow{v_1} = \langle 74, 22, 4\rangle$ cannot be manufactured. However the second bottle type, $\overrightarrow{v_2} = \langle 81, 13, 6\rangle = \frac{1}{2}\overrightarrow{u_1} + \frac{1}{2}\overrightarrow{u_2}$.