

201-SH2-AB - Exercises #4 - Mixed Limits

1. Evaluate the following limits. Use ∞ , $-\infty$, and dne as appropriate.

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| (1) $\lim_{x \rightarrow 2} \frac{\sqrt{2x^2 + 1} + x - 1}{x + 2}$ | (12) $\lim_{x \rightarrow 2} \frac{x^3 - 8}{ x - 2 }$ | (23) $\lim_{x \rightarrow 2^-} \frac{x^2 - 1}{x - 2}$ |
| (2) $\lim_{x \rightarrow -4} \frac{2x^2 + 8x}{x^2 + 8x + 16}$ | (13) $\lim_{x \rightarrow 2^+} \frac{\frac{1}{x+1} - \frac{2}{8-x}}{ x - 2 }$ | (24) $\lim_{x \rightarrow -3} \frac{x + 3}{3 - \sqrt{x + 12}}$ |
| (3) $\lim_{x \rightarrow 3^-} \frac{2x - 7}{x^2 + 2x - 15}$ | (14) $\lim_{x \rightarrow 2^-} \frac{\frac{1}{x-2} + 3}{x + 1}$ | (25) $\lim_{x \rightarrow 4^+} \frac{9 - x^2}{(x - 4)^3}$ |
| (4) $\lim_{x \rightarrow -5} \frac{2x - 1}{x(x + 5)^2}$ | (15) $\lim_{x \rightarrow 5} \frac{\frac{x}{x+1} - \frac{x+5}{2x+2}}{(2x^2 - 20x + 50)}$ | (26) $\lim_{x \rightarrow -2} \frac{2 - \frac{8}{x+6}}{x^3 + 8}$ |
| (5) $\lim_{x \rightarrow 0} \frac{2x - 1}{x(x + 5)^2}$ | (16) $\lim_{x \rightarrow \infty} \frac{2 + 2x - 4x^2}{2x^2 + x + 5}$ | (27) $\lim_{x \rightarrow 3^-} \frac{2x}{2x - 6}$ |
| (6) $\lim_{x \rightarrow 1^-} \frac{ x^2 - 1 }{x^2 + x - 2}$ | (17) $\lim_{x \rightarrow -2} \frac{\frac{1}{x+3} + \frac{1}{x+1}}{x^2 - 4}$ | (28) $\lim_{x \rightarrow -1^+} \frac{ x^2 + 2x }{x^2 - 1}$ |
| (7) $\lim_{x \rightarrow 3} \frac{2x^2 - x - 15}{x^2 - 5x + 6}$ | (18) $\lim_{x \rightarrow 1^+} \frac{3 - x}{x - 1}$ | (29) $\lim_{x \rightarrow 2^-} \frac{3x + 4}{x - 2}$ |
| (8) $\lim_{x \rightarrow 4} \frac{\sqrt{x + 12} - 4}{x - 4}$ | (19) $\lim_{x \rightarrow 4} \frac{\sqrt{x + 21} - 5}{x^2 - 8x + 16}$ | (30) $\lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x^3 - 3x^2 - 9x + 27}$ |
| (9) $\lim_{x \rightarrow -2^-} \frac{\frac{1}{x+6} - \frac{1}{4}}{x^2 + 4x + 4}$ | (20) $\lim_{x \rightarrow 3^-} \frac{6x - x^2}{x - 3}$ | (31) $\lim_{x \rightarrow \infty} \frac{(2 - x)(x + 3)}{(3x - 1)(x + 1)}$ |
| (10) $\lim_{x \rightarrow -1} \frac{x^2 - 3x - 4}{ x - 3 }$ | (21) $\lim_{x \rightarrow -1} \frac{3 - \frac{12}{x+5}}{x^3 + x^2 + 4x + 4}$ | (32) $\lim_{x \rightarrow 1} \frac{19x^2 - 17x - 2}{\frac{1}{4x} - \frac{1}{x+3}}$ |
| (11) $\lim_{h \rightarrow 0} \frac{\sqrt{x + h} - \sqrt{x}}{h}$ | (22) $\lim_{x \rightarrow 2^+} \frac{4x}{2 - x}$ | (33) $\lim_{x \rightarrow -2^-} \frac{ x + 2 }{\sqrt{x + 3} - 1}$ |

$$(34) \lim_{x \rightarrow 4^+} \frac{5 - \sqrt{5x + 5}}{x^3 - 4x^2 - 16x + 64}$$

$$(35) \lim_{x \rightarrow 5} \frac{\frac{1}{4x+7} - \frac{1}{x^2+2}}{x^4 - 4x^3 - 125x + 500}$$

$$(36) \lim_{x \rightarrow -\infty} \frac{(6 - x)^3(2x + 9)^2}{(3x + 5)^2(x - 6)^2}$$

$$(37) \lim_{x \rightarrow \infty} \frac{(x^2 + 9x + 7)^5(x^3 - 2)^2}{(x^2 + 3x + 1)^3(5x^4 - 9x + 9)(2 - x^2)^4}$$

2. Answer the following questions for the piecewise function $f(x)$ described on the right hand side.

(a) $\lim_{x \rightarrow -3^-} f(x)$

(b) $\lim_{x \rightarrow -3^+} f(x)$

(c) $\lim_{x \rightarrow -3} f(x)$

(d) $f(-3)$

$$f(x) = \begin{cases} \sqrt{1-x} & \text{for } x \leq -3, \\ \frac{2-x}{x+3} & \text{for } x > -3. \end{cases}$$

- (e) Is $f(x)$ continuous at $x = -3$? If not, *state the type of discontinuity.

3. Answer the following questions for the piecewise function $h(t)$ described on the right hand side.

(a) $\lim_{t \rightarrow 0^-} h(t)$

(b) $\lim_{t \rightarrow 0^+} h(t)$

(c) $\lim_{t \rightarrow 0} h(t)$

(d) $h(0)$

(e) $\lim_{t \rightarrow 5^-} h(t)$

(f) $\lim_{t \rightarrow 5^+} h(t)$

(g) $\lim_{t \rightarrow 5} h(t)$

(h) $h(5)$

$$h(t) = \begin{cases} t^2 & \text{for } t < 0 \\ t^2 - t & \text{for } 0 < t \leq 5 \\ \frac{2t-2}{t-5} & \text{for } t > 5 \end{cases}$$

- (i) Is $h(t)$ continuous at $t = 0$? If not, *state the type of discontinuity.

- (j) Is $h(t)$ continuous at $t = 5$? If not, *state the type of discontinuity.

Answers

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|----------------------------|-------------------------|
| 1. (1) 1 | (20) $-\infty$ |
| (2) dne | (21) $\frac{3}{20}$ |
| (3) ∞ | (22) $-\infty$ |
| (4) ∞ | (23) $-\infty$ |
| (5) dne | (24) -9 |
| (6) $-\frac{2}{3}$ | (25) ∞ |
| (7) 11 | (26) $\frac{1}{24}$ |
| (8) $\frac{1}{8}$ | (27) $-\infty$ |
| (9) ∞ | (28) $-\infty$ |
| (10) 0 | (29) $-\infty$ |
| (11) $\frac{1}{2\sqrt{x}}$ | (30) dne |
| (12) 12 | (31) $-\frac{1}{3}$ |
| (13) $-\frac{1}{6}$ | (32) -112 |
| (14) $-\infty$ | (33) -2 |
| (15) dne | (34) $-\infty$ |
| (16) -2 | (35) $\frac{2}{18,225}$ |
| (17) $\frac{1}{2}$ | (36) ∞ |
| (18) ∞ | (37) 0 |
| (19) dne | |
2. (a) 2
 (b) ∞
 (c) dne
 (d) 2
 (e) $f(x)$ has an infinite discontinuity at $x = 3$
3. (a) 0
 (b) 0
 (c) 0
 (d) dne
 (e) 20
 (f) ∞
 (g) dne
 (h) 20
 (i) $h(t)$ has a removable discontinuity at $t = 0$
 (j) $h(t)$ has an infinite discontinuity at $t = 5$