201-SH2-AB - Exercises #8 - Derivatives - Trigonometric, Exponential, Logarithmic Functions

1. For each of the following functions, find y' (trigonometric functions):

(a) $y = 3\sec(x) - 5\cot(x)$

(b) $y = 4x^5 \tan(x)$

(c) $y = 9\sin(x) + \sqrt{x}\cos(x)$

(d) $y = 2\sin(x)\cos(x)$

(e) $y = \dfrac{\csc(x) - 9e^x}{x^3 - 2x}$

(f) $y = \sec(3x+5)$

(g) $y = \sqrt{\sin(x)}$

(h) $y = \sin(\cos(x))$

(i) $y = 3x^2 \sin(8+2x)$

(j) $y = (5x-2)\cos(5x)$

(k) $y = 6x - \dfrac{5x}{\tan(4x)}$

(l) $y = x^2\cot(2x) - 4x$

(m) $y = \dfrac{3x-1}{\csc(3x)}$

(n) $y = (2+\sin(2x))(\sec(2x)+4)$

(o) $y = \dfrac{\sin(x)+x^2}{4x-\cos(x)}$

(p) $y = 12x^2 + \sec(3-x)$

(q) $y = \sin\left[(3x-x^2)^2\right]$

(r) $y = \cos^2(6-2x) + x^3$

(s) $y = \sin\left(\dfrac{x+1}{2x}\right)$

2. Find an equation of the tangent line to $y = x\cos(x)$ at $x = 0$.

3. Find an equation of the tangent line to $y = \sin(x)$ at $x = \dfrac{4\pi}{3}$.

4. Find an equation of the tangent line to $y = \cos(x)$ at $x = 5\pi$.

5. Find an equation of the tangent line to $y = \sin(x)\tan(x)$ at $x = \dfrac{11\pi}{6}$.

6. Find an equation of the tangent line to $y = \sec(x)$ at $x = \dfrac{\pi}{4}$.

7. Find an equation of the tangent line to $y = x + \tan(x)$ at $x = -\pi$.

8. Find an equation of the tangent line to $y = \csc(x)$ at $x = -\dfrac{2\pi}{3}$.

9. Find an equation of the tangent line to $y = \cos(x)\cot(x)$ at $x = -\dfrac{\pi}{4}$.

10. Find an equation of the tangent line to $y = 3x + 6\cos x$ at $x = \dfrac{\pi}{3}$.

11. Find an equation of the tangent line to $y = e^x\cos x + \sin x$ at the point $(0, 1)$.

12. Find an equation of the tangent line to $y = \sin(\sin x)$ at the point $(\pi, 0)$.

13. Find an equation of the tangent line to $y = 3\sin x + 7\cos x$ at $x = \dfrac{3\pi}{2}$.

14. Find an equation of the tangent line to $y = \tan x \sec x$ at $x = 0$.

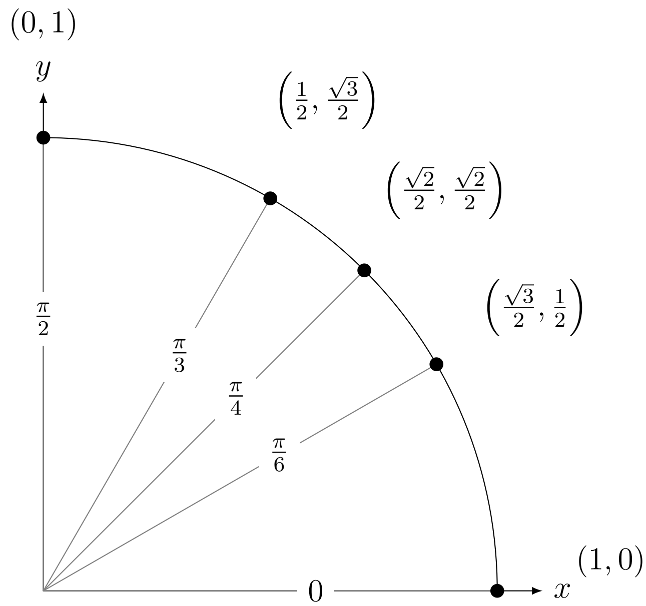
15. Given $f(x) = \cot(3x) + \sec(2x)$, find an equation of the tangent line to the curve $y = f(x)$ at the point $\left(\frac{\pi}{6}, 2\right)$.

16. Given $f(x) = \csc(x/2)\tan(x/3)$, find an equation of the tangent line to the curve $y = f(x)$ at the point $\left(3\pi, 0\right)$.

17. (*) Find the $x-$values where the tangent line to $f(x) = x + 2\sin x$ is horizontal.

18. (*) Find the $x-$values where the tangent line to $f(x) = e^x \cos x$ is horizontal.

The first quadrant of the unit circle is provided on Final Examinations:



19. For each of the following, find y' (mixed trigonometric, exponential, logarithmic functions):

(a) $y = \sqrt[3]{\sin(3x) + \cos(3x) + 2}$

(b) $y = 4^{2x} \sin(3x)$

(c) $y = 7^{4\sin(x)+x^2}$

(d) $y = \tan\left(3^{3x} - 1\right)$

(e) $y = 2^{x+3} \sin(\pi x)$

(f) $y = \dfrac{\sin(4x)}{e^{3x}}$

(g) $y = 13^{\sin(2x-6)}$

(h) $y = \sin\left(\log_5(x) + \pi x\right)$

(i) $y = \dfrac{\cos(x)}{\log_{13}(2x+1) + 3}$

(j) $y = \log_2\left(4\sin(x) + e^{3x}\right)$

(k) $y = (4 - 3x)\cot(3x)$

(l) $y = 5x - \dfrac{4x}{\sec(2x)}$

(m) $y = 3x\cos(2x) - 2x^2$

(n) $y = \dfrac{5x - 3}{\cos(4x)}$

(o) $y = (3 - \cos(3x))(\tan(3x) + 6)$

(p) $y = \dfrac{3x - \sin(x)}{x^2 + \cos(x)}$

(q) $y = 5x^2 - \sin(1 - x)$

(r) $y = \cos\left[\left(x^3 - 4x\right)^2\right]$

(s) $y = \sqrt[4]{2\cos(2x) - \sin(2x) - 1}$

(t) $y = \tan\left(\dfrac{3x}{x-2}\right)$

(u) $y = e^{-x}\cos(2x)$

(v) $y = e^{\sin(x)+x}$

20. For each of the following, find y' (mixed trigonometric, exponential, logarithmic functions):

(a) $y = 6^{\sin^3 x} \log_6(3x + \tan x)$

(b) $y = (\sec^3 x + 5x^2 + 7^x)(\csc x + 5)^6$

(c) $y = \dfrac{\cot^2 x}{4x^2 + e^x + \pi}$

(d) $y = e^{\cos x \csc x}$

(e) $y = \ln\left(\sin^2 x - 7e^x \sec x\right)$

(f) $y = \log_4(4 + \cos(2^x))$

(g) $y = 2^{\cot(x^2+e^x)}$

(h) $y = \ln(\cos^5(3x^4) + e^{x^2})$

(i) $y = \sec^4(6x^2 + \log_2(2x+1))$

(j) $y = \sqrt{\sin(7x + \ln(5x))}$

(k) $y = \ln(\ln(\ln(\sec(x))))$

(l) $y = \tan^3(\sqrt{\cot(7x)})$

(m) $y = \dfrac{\log_3(x^4)}{\sqrt{x} - \sec(x^3 + 7)}$

Answers

1. (a) $y' = 3\sec(x)\tan(x) - 5\csc^2(x)$

   (b) $y' = 20x^4\tan(x) + 4x^5\sec^2(x)$

   (c) $y' = 9\cos(x) + \frac{1}{2}x^{-1/2}\cos(x) - \sqrt{x}\sin(x)$

   (d) $y' = 2\cos^2(x) - 2\sin^2(x)$

   (e) $y' = \frac{(-\csc(x)\cot(x)-9e^x)(x^3-2x)-(\csc(x)-9e^x)(3x^2-2)}{(x^3-2x)^2}$

   (f) $y' = 3\sec(3x+5)\tan(3x+5)$

   (g) $y' = \frac{1}{2}(\sin(x))^{-1/2}\cos(x)$

   (h) $y' = \cos(\cos(x)) \cdot -\sin(x)$

   (i) $y' = 6x\sin(8+2x) + 6x^2\cos(8+2x)$

   (j) $y' = 5\cos(5x) - (25x-10)\sin(5x)$

   (k) $y' = 6 - \dfrac{5\tan(4x) - 20x\sec^2(4x)}{\tan^2(4x)}$

   (l) $y' = 2x\cot(2x) - 2x^2\csc^2(2x) - 4$

   (m) $y' = \dfrac{3\csc(3x) + (9x-3)\csc(3x)\cot(3x)}{\csc^2(3x)}$

   (n) $y' = 2\cos(2x)[\sec(2x) + 4] + 2\sec(2x)\tan(2x)[2+\sin(2x)]$

   (o) $y' = \frac{(\cos(x)+2x)(4x-\cos(x))-(\sin(x)+x^2)(4+\sin(x))}{(4x-\cos(x))^2}$

   (p) $y' = 24x - \sec(3-x)\tan(3-x)$

   (q) $y' = \cos\left[(3x-x^2)^2\right] \cdot 2(3x-x^2)(3-2x)$

   (r) $y' = 4\cos(6-2x) \cdot \sin(6-2x) + 3x^2$

   (s) $y' = \cos\left(\dfrac{x+1}{2x}\right)\dfrac{-1}{4x^2}$

2. $y = x$

3. $y = -\frac{1}{2}x + \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

4. $y = -1$

5. $y = -\frac{7}{6}x + \frac{77\pi}{36} + \frac{\sqrt{3}}{6}$

6. $y = \sqrt{2}x - \frac{\sqrt{2}\pi}{4} + \sqrt{2}$

7. $y = 2x + \pi$

8. $y = \frac{2}{3}x + \frac{4\pi}{9} - \frac{2}{\sqrt{3}}$

9. $y = -\frac{3\sqrt{2}}{2}x - \frac{3\sqrt{2}\pi}{8} - \frac{\sqrt{2}}{2}$

10. $y = (3 - 3\sqrt{3})x + \left(\frac{\pi}{3}(3\sqrt{3}-3) + \pi + 3\right)$

11. $y = 2x + 1$

12. $y = -x + \pi$

13. $y = 7x - \frac{21\pi}{2} - 3$

14. $y = x$

15. $y = \left(-3 + 4\sqrt{3}\right)x + \left(2 - \frac{\pi}{2} + \frac{2\pi\sqrt{3}}{3}\right)$

16. $y = -\frac{1}{3}x + \pi$

17. $x = \frac{2\pi}{3} + 2k\pi$ and $x = \frac{4\pi}{3} + 2k\pi$ where k is any integer.

18. $x = \frac{\pi}{4} + k\pi$ where k is any integer.

19. (a) $\dfrac{3\cos(3x) - 3\sin(3x)}{3\left(\sin(3x) + \cos(3x) + 2\right)^{\frac{2}{3}}}$

    (b) $4^{2x}(2\ln(4)\sin(3x) + 3\cos(3x))$

    (c) $7^{4\sin(x)+x^2}\ln(7)(2x + 4\cos(x))$

    (d) $3\ln(3)3^{3x}\sec^2(3^{3x} - 1)$

    (e) $2^{x+3}(\ln(2)\sin(\pi x) + \pi\cos(\pi x))$

    (f) $\dfrac{4\cos(4x) - 3\sin(4x)}{e^{3x}}$

    (g) $2\ln(13)\cos(2x-6)13^{\sin(2x-6)}$

    (h) $\cos(\log_5(x) + \pi x)\left(\dfrac{1}{x\ln(5)} + \pi\right)$

(i) $\dfrac{\frac{-2\cos(x)}{(2x+1)\ln(13)} - \sin(x)\left(\log_{13}(2x+1)+3\right)}{\left(\log_{13}(2x+1)+3\right)^2}$

(j) $\dfrac{4\cos(x)+3e^{3x}}{(4\sin(x)+e^{3x})\ln(2)}$

(k) $-3\cot(3x) - 3(4-3x)\csc^2(3x)$

(l) $5 - \dfrac{4-8x\tan(2x)}{\sec(2x)}$

(m) $3\cos(2x) - 6x\sin(2x) - 4x$

(n) $\dfrac{5\cos(4x)+4(5x-3)\sin(4x)}{\cos^2(4x)}$

(o) $9\sec^2(3x) - 3\cos(3x) + 18\sin(3x)$

(p) $\frac{(3-\cos(x))(x^2+\cos(x))-(3x-\sin(x))(2x-\sin(x))}{(x^2+\cos(x))^2}$

(q) $10x + \cos(1-x)$

(r) $-2(x^3-4x)(3x^2-4)\sin\left[(x^3-4x)^2\right]$

(s) $\dfrac{-4\sin(2x)-2\cos(2x)}{4\left(2\cos(2x)-\sin(2x)-1\right)^{\frac{3}{4}}}$

(t) $\sec^2\left(\dfrac{3x}{x-2}\right)\dfrac{-6}{(x-2)^2}$

(u) $-e^{-x}(\cos(2x)+2\sin(2x))$

(v) $e^{\sin(x)+x}(\cos(x)+1)$

20. (a) $6^{\sin^3 x}\ln 6 \cdot 3\sin^2(x)\cos(x)\log_6(3x+\tan x) + 6^{\sin^3 x}\cdot \frac{1}{(3x+\tan x)\ln 6}\cdot(3+\sec^2 x)$

(b) $y = (3\sec^2 x + 10x + 7^x\ln 7)(\csc x+5)^6 + (\sec^3 x + 5x^2 + 7^x)6(\csc x+5)^5(-\csc x\cot x)$

(c) $y = \frac{-2\cot x\csc^2 x(4x^2+e^x+\pi) - \cot^2 x(8x+e^x)}{(4x^2+e^x+\pi)^2}$

(d) $y = e^{\cos x\csc x}(-\sin x\csc x - \cos x\csc x\cot x)$

(e) $y = \frac{2\sin x\cos x - 7e^x\sec x - 7^x\sec x\tan x}{(\sin^2 x - 7e^x\sec x)}$

(f) $y = \dfrac{-\sin(2^x)(2^x\ln 2)}{(4+\cos(2^x))\ln 4}$

(g) $y = 2^{\cot(x^2+e^x)}\ln 2\left(-\csc^2(x^2+e^x)(2x+e^x)\right)$

(h) $y = \frac{5\cos^4(3x^4)(-\sin(3x^4)(12x^3)) + e^{x^2}(2x)}{(\cos^5(3x^4)+e^{x^2})}$

(i) $y = 4\sec^4(6x^2+\log_2(2x+1))\tan(6x^2+\log_2(2x+1))(12x+\frac{2}{(2x-1)\ln 2})$

(j) $y = \frac{1}{2}\left(\sin(7x+\ln(5x))\right)^{-1/2}\cos(7x+\ln(5x))\left(7+\frac{1}{x}\right)$

(k) $y = \dfrac{1}{\ln(\ln(\sec(x)))}\dfrac{1}{\ln(\sec(x))}\tan x$

(l) $y = 3\tan^2(\sqrt{\cot(7x)})\sec^2(\sqrt{\cot(7x)})\cdot\frac{1}{2}\left(\cot(7x)\right)^{-1/2}\left(-7\csc^2(7x)\right)$

(m) $y = \frac{\frac{4(\sqrt{x}-\sec(x^3+7))}{x\ln 3} - \log_3(x^4)(\frac{1}{2\sqrt{x}} - 3x^2\sec(x^3+7)\tan(x^3+7))}{(\sqrt{x}-\sec(x^3+7))^2}$