# 201-SH2-AB - Exercises #10 - Logarithmic Differentiation

1. Find $y' = \dfrac{dy}{dx}$. Use laws of logarithms to simplify the expression of $y$ as much as possible before differentiating.

   (a) $y = \ln\left[\dfrac{(x^3+1)^2(\tan x+2)^3}{\sqrt{\cos x+2}}\right]$

   (b) $y = \ln\left[\dfrac{\sqrt[3]{2x-\cos x}}{(\sin x+4)^3\sqrt{x+1}}\right]$

   (c) $y = \ln\left[\dfrac{\cos^2(x^2-1)}{\sqrt{x+3}\,(x^2+1)^3}\right]$

   (d) $y = \ln\left(\dfrac{\cos^3 x(3x^2-8)^7}{\sqrt{x}(2x-1)^{11}}\right)$

2. Find $y' = \dfrac{dy}{dx}$ using logarithmic differentiation.

   (a) $y = \dfrac{(9x^2-4)^7\sqrt{3x^4-7}}{e^x\ln x}$

   (b) $y = \dfrac{(x+1)^4\ln x}{e^{3x}\cos^2(5x)}$

   (c) $y = \dfrac{6e^x\sqrt[5]{x^3+6x^2}}{(2x-9)^6\ln x}$

   (d) $y = \dfrac{\log_2 x^3}{\sqrt{x}+8\sin x}$

3. Find $y' = \dfrac{dy}{dx}$ using logarithmic differentiation.

   (a) $y = x^x$

   (b) $y = 2x^{1/x}$

   (c) $y = 3x^{\sin x}$

   (d) $y = 5(\sqrt{x})^x$

   (e) $y = 7(\cos x)^x$

   (f) $y = 11(\sin x)^{\ln x}$

   (g) $y = 13x^{\ln x}$

   (h) $y = 17(\ln x)^{\cos x}$

   (i) $y = (2x^3-x)^{x-2}$

   (j) $y = (x^2-8)^{x^2}$

   (k) $y = (5-x^2)^{x+1}$

   (l) $y = (2+x^3)^{x^3}$

   (m) $y = (x+1)^{2\cos x}$

   (n) $y = (4+x)^{\sin(4-x)}$

4. Find $y' = \dfrac{dy}{dx}$ using logarithmic differentiation.

   ⋆ State whether logarithmic differentiation is necessary or simply useful.

   (a) $y = \dfrac{(6x+1)^2\sqrt[4]{2x^2+1}}{e^{1-\cos x}}$

   (b) $y = \dfrac{\sin(4x)e^{3\sin x}}{\sqrt[3]{9x+1}}$

   (c) $y = \dfrac{\sqrt{\sin(3x)+1}}{\cos^2 x\sqrt[3]{x^2+1}}$

   (d) $y = \ln\left(\dfrac{(2x+4)^3e^{4x}}{\cot^5 x}\right)$

   (e) $y = \dfrac{(x+2)\sqrt{\cos^3(x)}}{(3x+\cos(2x))^4}$

   (f) $y = \dfrac{(\sin(3x)-\cos(2x))^4}{2\sec x\,(\tan x+2)^2}$

   (g) $y = (\sin(3x))^{\frac{1}{x+1}}$

   (h) $y = (x^2+2)^{\tan x}$

   (i) $y = (\tan(2x)+3)^{\cos x}$

   (j) $y = \ln\left[\dfrac{(5x^3-7x)^3\sqrt{x^4-3x}}{(x^2-6x^5)^2}\right]$

   (k) $y = (\sin(3x)+\cos x)^{\sqrt{x+1}}$

   (l) $y = \Big(\ln(\cos x)+4\Big)^{\tan(2x)}$

   (m) $y = \dfrac{\sqrt[3]{3x+1}e^{\sin(2x)}}{(x^3+1)^3}$

   (n) $y = \dfrac{\cos(2x)\sqrt{4x+1}}{e^{\sin(3x)}}$

   (o) $y = (\tan x)^{2x}$

## Answers

1. (a) $\dfrac{6x^2}{x^3+1}+\dfrac{3\sec^2 x}{\tan x+2}+\dfrac{\sin x}{2(\cos x+2)}$

   (b) $\dfrac{2+\sin x}{3(2x-\cos x)}-\dfrac{3\cos x}{\sin x+4}-\dfrac{1}{2(x+1)}$

   (c) $-4x\tan(x^2-1)-\dfrac{1}{2(x+3)}-\dfrac{6x}{x^2+1}$

   (d) $-3\tan x+\dfrac{42x}{3x^2-8}-\dfrac{1}{2x}-\dfrac{22}{2x-1}$

2. (a) $\dfrac{(9x^2-4)^7\sqrt{3x^4-7}}{e^x\ln x}\left[\dfrac{126x}{9x^2-4}+\dfrac{6x^3}{3x^4-7}-1-\dfrac{1}{x\ln x}\right]$

   (b) $\dfrac{(x+1)^4\ln x}{e^{3x}\cos^2(5x)}\left[\dfrac{4}{x+1}+\dfrac{1}{x\ln x}-3+10\tan(5x)\right]$

   (c) $\dfrac{6e^x\sqrt[5]{x^3+6x^2}}{(2x-9)^6\ln x}\left[1+\dfrac{3x^2+12x}{5(x^3+6x^2)}-\dfrac{12}{2x-9}-\dfrac{1}{x\ln x}\right]$

   (d) $\dfrac{6\sqrt{x}(\sqrt{x}+8\sin x)-x\ln 2(\log_2(x^3))(1+16\sqrt{x}\cos x)}{2(\ln 2)x^{3/2}(\sqrt{x}+8\sin x)^2}$

3. (a) $x^x(1+\ln x)$

   (b) $2x^{1/x}\left(\dfrac{1}{x^2}-\dfrac{\ln x}{x^2}\right)$

   (c) $3x^{\sin x}\left(\dfrac{\sin x}{x}+\ln x\cos x\right)$

   (d) $5(\sqrt{x})^x\left(\dfrac{1}{2}+\dfrac{\ln x}{2}\right)$

   (e) $7(\cos x)^x\left(-x\tan x+\ln x\cos x\right)$

   (f) $11(\sin x)^{\ln x}\left(\cot x\ln x+\dfrac{\ln(\sin x)}{x}\right)$

   (g) $\dfrac{(26x^{\ln x})\ln x}{x}$

   (h) $17(\ln x)^{\cos x}\left(\dfrac{\cos x}{x\ln x}-\ln(\ln x)\sin x\right)$

   (i) $(2x^3-x)^{x-2}\left[\ln(2x^3-x)+\dfrac{(x-2)(6x^2-1)}{2x^3-x}\right]$

   (j) $(x^2-8)^{x^2}\left[2x\ln(x^2-8)+\dfrac{2x^3}{x^2-8}\right]$

   (k) $(5-x^2)^{x+1}\left[\ln(5-x^2)-\dfrac{2x^2+2x}{5-x^2}\right]$

   (l) $(2+x^3)^{x^3}\left[3x^2\ln(2+x^3)+\dfrac{3x^5}{2+x^3}\right]$

   (m) $(x+1)^{2\cos x}\left[\dfrac{2\cos x}{x+1}-2\sin x\ln(x+1)\right]$

   (n) $(4+x)^{\sin(4-x)}\left[\dfrac{\sin(4-x)}{4+x}-\cos(4-x)\ln(4+x)\right]$

4. (a) $\dfrac{(6x+1)^2\sqrt[4]{2x^2+1}}{e^{1-\cos x}}\left[\dfrac{12}{6x+1}+\dfrac{x}{2x^2+1}-\sin x\right]$ $\star$ Logarithm differentiation is useful

(b) $\dfrac{\sin(4x)e^{3\sin x}}{\sqrt[3]{9x+1}}\left[4\cot(4x)+3\cos x-\dfrac{3}{9x+1}\right]$ $\star$ Logarithm differentiation is useful

(c) $\dfrac{\sqrt{\sin(3x)+1}}{\cos^2 x\sqrt[3]{x^2+1}}\left[\dfrac{3\cos(3x)}{2(\sin(3x)+1)}+2\tan x-\dfrac{2x}{3(x^2+1)}\right]$ $\star$ Logarithm differentiation is useful

(d) $\dfrac{3}{x+2}+4+\dfrac{5\csc^2 x}{\cot x}$ $\star$ Logarithm differentiation is useful

(e) $\dfrac{(x+2)\sqrt{\cos^3(x)}}{(3x+\cos(2x))^4}\left[\dfrac{1}{x+2}-\dfrac{3}{2}\tan x-\dfrac{12-8\sin(2x)}{3x+\cos(2x)}\right]$ $\star$ Logarithm differentiation is useful

(f) $\dfrac{(\sin(3x)-\cos(2x))^4}{2\sec x\,(\tan x+2)^2}\left[\dfrac{4(3\cos(3x)+2\sin(2x))}{\sin(3x)-\cos(2x)}-\tan x-\dfrac{2\sec^2 x}{\tan x+2}\right]$ $\star$ useful

(g) $(\sin(3x))^{\frac{1}{x+1}}\left[\dfrac{3\cot(3x)}{x+1}-\dfrac{\ln(\sin(3x))}{(x+1)^2}\right]$ $\star$ Logarithm differentiation is necessary

(h) $(x^2+2)^{\tan x}\left[\dfrac{2x\tan x}{x^2+2}+\sec^2 x\ln(x^2+2)\right]$ $\star$ Logarithm differentiation is necessary

(i) $(\tan(2x)+3)^{\cos x}\left[\dfrac{2\sec^2(2x)\cos x}{\tan(2x)+3}-\sin x\ln(\tan(2x)+3)\right]$ $\star$ necessary

(j) $\dfrac{3(15x^2-7)}{5x^3-7x}+\dfrac{4x^3-3}{2(x^4-3x)}-\dfrac{2(2x-30x^4)}{x^2-6x^5}$ $\star$ Logarithm differentiation is useful

(k) $(\sin(3x)+\cos x)^{\sqrt{x+1}}\left[\dfrac{(3\cos(3x)-\sin x)\sqrt{x+1}}{\sin(3x)+\cos x}+\dfrac{\ln(\sin(3x))+\cos x}{2\sqrt{x+1}}\right]$ $\star$ necessary

(l) $\Big(\ln(\cos x)+4\Big)^{\tan(2x)}\left[2\sec^2(2x)\ln(\ln(\cos x)+4)-\dfrac{\tan x\tan(2x)}{\ln(\cos x)+4}\right]$ $\star$ necessary

(m) $\dfrac{\sqrt[3]{3x+1}e^{\sin(2x)}}{(x^3+1)^3}\left[\dfrac{1}{3x+1}+2\cos(2x)-\dfrac{9x^2}{x^3+1}\right]$ $\star$ Logarithm differentiation is useful

(n) $\dfrac{\cos(2x)\sqrt{4x+1}}{e^{\sin(3x)}}\left[-2\tan(2x)+\dfrac{2}{4x+1}-3\cos(3x)\right]$ $\star$ Logarithm differentiation is useful

(o) $(\tan x)^{2x}\left[\dfrac{2x\sec^2 x}{\tan x}+2\ln(\tan x)\right]$ $\star$ Logarithm differentiation is necessary