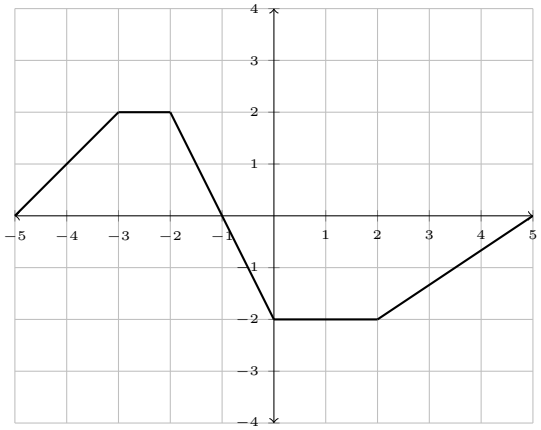
201-SH3-AB - Exercises #4: Integrals as Area

1. Given the following graph of f, find:



   (a) $\int_{-5}^{-3} f(x)\ dx$

   (b) $\int_{-5}^{-1} f(x)\ dx$

   (c) $\int_{-3}^{1} f(x)\ dx$

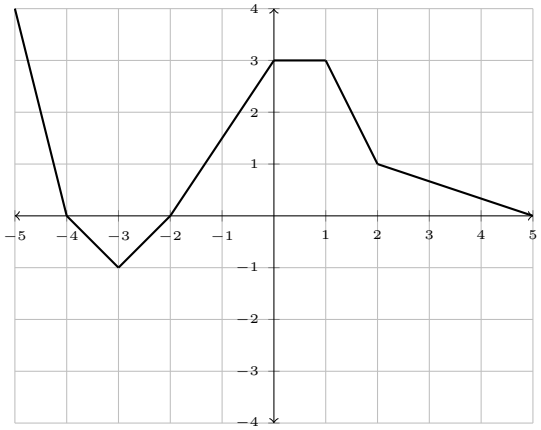
   (d) $\int_{-1}^{2} f(x)\ dx$

   (e) $\int_{-5}^{5} f(x)\ dx$

   (f) $\int_{5}^{-5} f(x)\ dx$

   (g) $\int_{-2}^{-4} f(x)\ dx$

2. Given the following graph of g, find:



   (a) $\int_{-5}^{-2} g(x)\ dx$

   (b) $\int_{-2}^{0} g(x)\ dx$

   (c) $\int_{-5}^{0} g(x)\ dx$

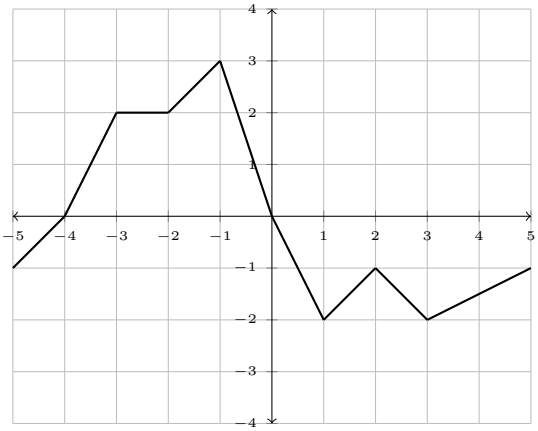
   (d) $\int_{-1}^{1} g(x)\ dx$

   (e) $\int_{-2}^{2} g(x)\ dx$

   (f) $\int_{3}^{3} g(x)\ dx$

   (g) $\int_{-5}^{5} g(x)\ dx$

3. Given the following graph of h, find:



(a) $\int_{-5}^{-2} h(x)\ dx$

(b) $\int_{0}^{-4} h(x)\ dx$

(c) $\int_{0}^{5} h(x)\ dx$

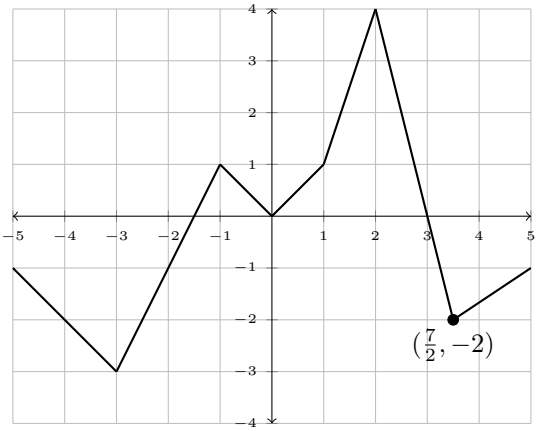
(d) $\int_{-1}^{-1} h(x)\ dx$

(e) $\int_{-1}^{1} h(x)\ dx$

(f) $\int_{-1}^{1} 2h(x)\ dx$

(g) $\int_{-1}^{1} |h(x)|\ dx$

4. Given the following graph of p, find:



(a) $\int_{-3}^{-5} p(x)\ dx$

(b) $\int_{-3}^{2} p(x)\ dx$

(c) $\int_{2}^{5} p(x)\ dx$

(d) $\int_{-5}^{5} p(x)\ dx$

(e) $\int_{-3}^{0} -p(x)\ dx$

(f) $\int_{-4}^{1} |p(x)|\ dx$

(g) $\int_{-2}^{-3} 3p(x)\ dx$

5. Use the graphs of f, g, h, and p in the questions above. Find:

(a) $\int_{-5}^{5} f(x) + g(x)\ dx$

(b) $\int_{-3}^{0} p(x) - h(x)\ dx$

(c) $\int_{0}^{1} 2f(x) + g(x) - 3h(x) - 4p(x)\ dx$

6. Evaluate the integrals:

(a) $\int_{-4}^{1} |x+2| \, dx$

(b) $\int_{-1}^{5} |3x-2| \, dx$

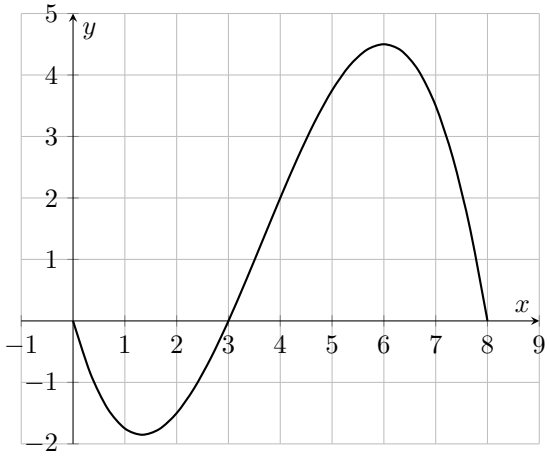
(c) $\int_{3}^{7} |5-x| \, dx$

(d) $\int_{-8}^{-2} |-x-6| \, dx$

(e) $\int_{-2}^{3} |4-4x| \, dx$

(f) $\int_{-1}^{4} |5-3x| \, dx$

7. For the function f whose graph is shown, list the following quantities in increasing order, from smallest to largest.



(A) $\int_{0}^{8} f(x) \, dx$

(B) $\int_{0}^{3} f(x) \, dx$

(C) $\int_{3}^{8} f(x) \, dx$

(D) $\int_{4}^{8} f(x) \, dx$

8. Evaluate the integral by interpreting it in terms of areas.

(a) $\int_{0}^{3} 4x \, dx$

(b) $\int_{0}^{8} (3-2x) \, dx$

(c) $\int_{-2}^{5} (10-5x) \, dx$

(d) $\int_{-2}^{5} |10-5x| \, dx$

(e) $\int_{-4}^{3} \left|\frac{1}{2}x\right| \, dx$

(f) $\int_{1}^{1} \sqrt{1+x^4} \, dx$

(g) $\int_{1}^{5} |x-3| \, dx$

(h) $\int_{-2}^{4} |2x-5| \, dx$

ANSWERS:

(1) (a) 2
(b) 5
(c) 0
(d) -5
(e) -3
(f) 3
(g) $-\frac{7}{2}$

(2) (a) 1
(b) 3
(c) 4
(d) $\frac{21}{4}$
(e) 8
(f) 0
(g) $\frac{21}{2}$

(3) (a) $\frac{5}{2}$
(b) -7
(c) -7
(d) 0
(e) $\frac{1}{2}$
(f) 1
(g) $\frac{5}{2}$

(4) (a) 4
(b) $\frac{3}{2}$
(c) $-\frac{3}{4}$
(d) $-\frac{13}{4}$
(e) $\frac{3}{2}$
(f) 6
(g) 6

(5) (a) $\frac{15}{2}$
(b) $-\frac{15}{2}$
(c) 0

(6) (a) $\frac{13}{2}$
(b) $\frac{97}{3}$
(c) 4
(d) 10
(e) 26
(f) $\frac{113}{6}$

(7) B < A < D < C

(8) (a) 18
(b) -40
(c) $\frac{35}{2}$
(d) $\frac{125}{2}$
(e) $\frac{25}{4}$
(f) 0
(g) 4
(h) $\frac{45}{2}$