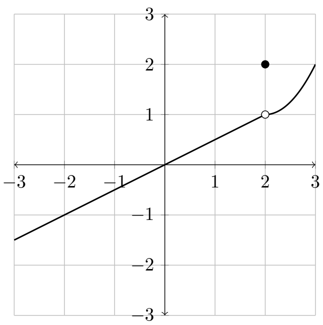
201-SH2-AB - Exercises #5 - Continuity

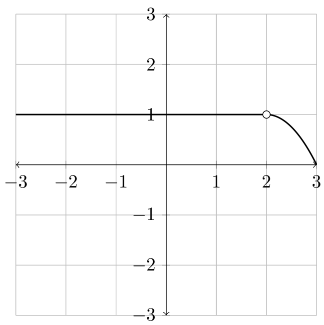
1. Match each part below to the graph which best illustrates the condition at $x = 2$.

(a) $f(2)$ is undefined. (b) $\lim_{x \to 2} f(x)$ DNE (c) $\lim_{x \to 2} f(x) \neq f(2)$ (d) f is continuous at $x = 2$

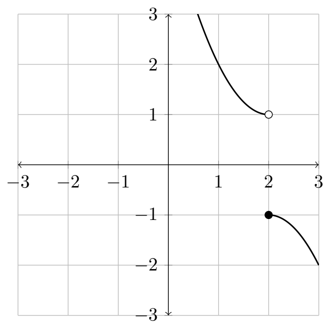
GRAPH #1:



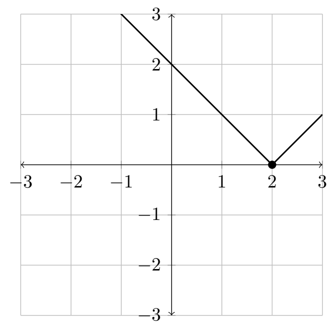
GRAPH #2:



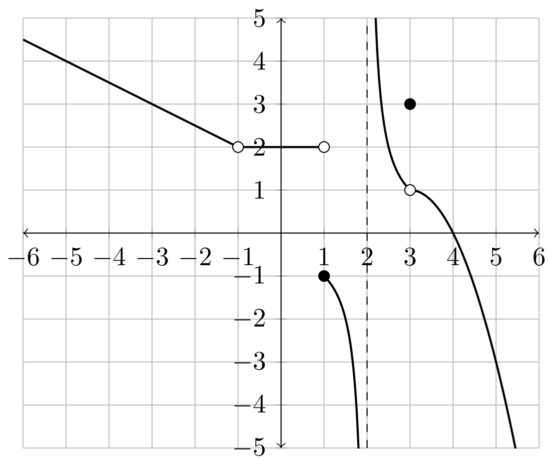
GRAPH #3:



GRAPH #4:



2. Given the graph of f, fill in the table that follows.



Point of Discontinuity	Criteria of Continuity NOT Satisfied (choose all that apply)		
$x =$	☐$f(a)$ undefined	☐ $\lim_{x \to a} f(x)$ DNE	☐ $\lim_{x \to a} f(x) \neq f(a)$
$x =$	☐$f(a)$ undefined	☐ $\lim_{x \to a} f(x)$ DNE	☐ $\lim_{x \to a} f(x) \neq f(a)$
$x =$	☐$f(a)$ undefined	☐ $\lim_{x \to a} f(x)$ DNE	☐ $\lim_{x \to a} f(x) \neq f(a)$
$x =$	☐$f(a)$ undefined	☐ $\lim_{x \to a} f(x)$ DNE	☐ $\lim_{x \to a} f(x) \neq f(a)$
	* Which of the points of discontinuity above are removable?		

3. Consider the function $f(x) = \dfrac{(x-4)(x+1)}{x-a}$.

(a) Explain why $f(x)$ will always have a discontinuity at $x = a$, regardless of the value of a.

(b) For what value(s) of a will $\lim_{x \to a} f(x)$ exist?

4. * Show that $f(x) = \dfrac{x^2 - 5x + 6}{x^2 - 4}$ has a removable discontinuity at $x = 2$.

Determine the values of x at which the given function is discontinuous. Justify your answers.
*Describe the type of discontinuity.

5. $f(x) = \dfrac{x^2+2x-3}{x^2-1}$

6. $f(x) = \dfrac{x+2}{x^2+x-2}$

7. $f(x) = \dfrac{3x^2}{6x+x^2}$

8. $f(x) = \dfrac{x-3}{x^2-3x}$

9. $f(x) = \dfrac{x^2}{4x-x^2}$

10. $f(x) = \dfrac{4-x}{x^2-7x+12}$

11. $f(x) = \begin{cases} \dfrac{x-2}{(x-3)(x+1)} & x < 4 \\ \dfrac{3}{x-5} & x \geq 4 \end{cases}$

12. $f(x) = \begin{cases} \dfrac{4x-1}{x^2-5x-6} & x \leq 5 \\ 3x+2 & x > 5 \end{cases}$

13. $f(x) = \begin{cases} \dfrac{x-3}{(x-6)(x-2)} & x < 4 \\ \dfrac{x+1}{5-x} & x \geq 4 \end{cases}$

14. $f(x) = \begin{cases} 2x-1 & x \leq 3 \\ \dfrac{5x-1}{x^2-6x+8} & x > 3 \end{cases}$

15. $f(x) = \begin{cases} \dfrac{7}{x^2+2x-15} & x < 2 \\ 5 & x = 2 \\ x^2-5 & x > 2 \end{cases}$

16. $f(x) = \begin{cases} \dfrac{5x+10}{(x+2)(x-5)} & x \leq 0 \\ 2x^2-3x-1 & x > 0 \end{cases}$

17. $f(x) = \begin{cases} \dfrac{5}{x+1} & x \leq 0 \\ \dfrac{-3x^2+5x}{x} & 0 < x < 1 \\ 2x-1 & x \geq 1 \end{cases}$

18. $f(x) = \begin{cases} \dfrac{30}{(x+5)(x-3)} & x < 0 \\ -3 & x = 0 \\ 5x^2+3x-2 & x > 0 \end{cases}$

19. $f(x) = \begin{cases} \dfrac{(x+1)(x-4)}{(x-2)(x+1)} & x < 1 \\ \dfrac{3}{2-x} & x \geq 1 \end{cases}$

20. $f(x) = \begin{cases} \dfrac{3x+12}{x^2+2x-8} & x \leq -2 \\ x^2 & -2 < x < 2 \\ \dfrac{4}{3-x} & x \geq 2 \end{cases}$

21. $f(x) = \begin{cases} \dfrac{x^2+2x}{x^2-4} & x \leq 0 \\ x & 0 < x < 3 \\ \dfrac{x}{x-5} & x \geq 3 \end{cases}$

22. $f(x) = \begin{cases} \dfrac{x-1}{x^2-2x} & x < 1 \\ \dfrac{x^2-3x+2}{x^2-x-2} & 1 \leq x \leq 3 \\ \dfrac{1}{x+1} & 3 < x \end{cases}$

23. $f(x) = \begin{cases} \dfrac{1}{x+4} & x \leq 1 \\ \dfrac{x^2-1}{x^2+8x-9} & 1 < x \leq 2 \\ \dfrac{3}{x^2+4} & 2 < x \end{cases}$

24. $f(x) = \begin{cases} \dfrac{x-2}{x^2+x-6} & x < 2 \\ \dfrac{1}{x+3} & 2 \leq x \leq 4 \\ \dfrac{1}{x-5} & 4 < x \end{cases}$

25. $f(x) = \begin{cases} \dfrac{x+9}{(x-2)(x+4)} & x \leq 1 \\ -2 & 1 < x < 8 \\ \sqrt{3x+1} & 8 \leq x \end{cases}$

26. $f(x) = \begin{cases} \dfrac{-2x-1}{x+2} & x \leq -1 \\ \sqrt{x+1} & -1 < x \leq 3 \\ 2x-4 & x > 3 \end{cases}$

27. $f(x) = \begin{cases} \dfrac{1}{x^2-4} & x \leq -4 \\ \dfrac{x^2+5x+6}{x^2-x-12} & -4 < x \leq 0 \\ \dfrac{3}{x-6} & x > 0 \end{cases}$

Find the value(s) of the appropriate constant(s) such that the given function is continuous everywhere.

28. $f(x) = \begin{cases} \dfrac{x^2+2x-3}{x-1} & x \neq 1 \\ k^2 & x = 1 \end{cases}$

29. $f(x) = \begin{cases} k^2 - \dfrac{12k}{x} & x \leq -4 \\ 2k - 5x & x > -4 \end{cases}$

30. $f(x) = \begin{cases} -x^2 - 7k & x < 3 \\ k^2 - \dfrac{81}{x} & x \geq 3 \end{cases}$

31. $f(x) = \begin{cases} x^2 + k^2 x - 4k & x \leq 1 \\ 7x + k & x > 1 \end{cases}$

32. $f(x) = \begin{cases} \dfrac{x^2-64}{x-8} & x \neq 8 \\ k & x = 8 \end{cases}$

33. $f(x) = \begin{cases} 5 & x \leq 0 \\ ax - b & 0 < x < 8 \\ 3 & x \geq 8 \end{cases}$

34. $f(x) = \begin{cases} k^2x^2 + 3kx - 8 & x < 2 \\ kx & x \geq 2 \end{cases}$

35. $f(x) = \begin{cases} kx + k^2 & x \neq -1 \\ 2 & x = -1 \end{cases}$

36. $f(x) = \begin{cases} kx & x \leq 2 \\ \dfrac{x^2+9x+8}{x+k} & x > 2 \end{cases}$

37. $f(x) = \begin{cases} \dfrac{3}{x-k} & x < 2 \\ \dfrac{k}{1-x} & x \geq 2 \end{cases}$

38. $f(x) = \begin{cases} ae^{-x} + be^{x} & x \leq 0 \\ ax^2 + b - 1 & 0 < x \leq 1 \\ \ln x & x > 1 \end{cases}$

39. $f(x) = \begin{cases} k^2 + 2x & x < -1 \\ -kx & x \geq -1 \end{cases}$

40. $f(x) = \begin{cases} k^2 + 3x & x < 1 \\ 3 + 5kx & x \geq 1 \end{cases}$

41. $f(x) = \begin{cases} ax + 5 & x \leq -3 \\ 1 - x & -3 < x \end{cases}$

42. $f(x) = \begin{cases} 2k^2 + 3x & x \leq 2 \\ x(9-k) & x > 2 \end{cases}$

43. $f(x) = \begin{cases} x^2 + k^2 x & x \leq 1 \\ 5k + 7x & x > 1 \end{cases}$

44. $f(x) = \begin{cases} a^2x^2 - 9 & x < -1 \\ \dfrac{2ax+6}{2x+3} & x \geq -1 \end{cases}$

45. $f(x) = \begin{cases} k + 10x & x < 2 \\ 3k^2 - 4kx - 5x & x \geq 2 \end{cases}$

ANSWERS

1. (a) #2 (b) #3 (c) #1 (d) #4

2. $x=-1$; $f(-1)$ undefined, $\lim_{x\to -1} f(x) \neq f(-1)$
$x=1$; $\lim_{x\to 1} f(x)$ DNE, $\lim_{x\to 1} f(x) \neq f(1)$
$x=2$ $f(2)$ undefined, $\lim_{x\to 2} f(x)$ DNE, $\lim_{x\to 2} f(x) \neq f(2)$
$x=3$ $\lim_{x\to 3} f(x) \neq f(3)$
$x=-1, x=3$ are removable points of discontinuity.

3. (a) $f(a)$ will be undefined regardless of the value of a.
(b) $a=4$ and $a=-1$.

4. Hint: Show that $\lim_{x\to 2} f(x)$ exists, but that $f(2)$ does not.

5. $x=-1$: *infinite, $x=1$: *removable
6. $x=-2$: *removable, $x=1$: *infinite
7. $x=-6$: *infinite, $x=0$: *removable
8. $x=0$: *infinite, $x=3$: *removable
9. $x=0$: *removable, $x=4$: *infinite
10. $x=3$: *infinite, $x=4$: *removable
11. $x=-1$: *infinite, $x=3$: *infinite,
$x=4$: *jump, $x=5$: *infinite
12. $x=-1$: *infinite, $x=5$: *jump
13. $x=2$: *infinite, $x=4$: *jump, $x=5$: *infinite
14. $x=3$: *jump, $x=4$: *infinite
15. $x=-5$: *infinite, $x=2$: *removable
16. $x=-2$: *removable
17. $x=-1$: *infinite, $x=1$: *jump
18. $x=-5$: *infinite, $x=0$: *removable
19. $x=-1$: *removable, $x=2$: *infinite
20. $x=-4$: *removable, $x=-2$: *jump, $x=3$: *infinite
21. $x=-2$: *removable, $x=3$: *jump, $x=5$: *infinite
22. $x=0$: *infinite,$x=2$: *removable, $x=3$: *jump
23. $x=-4$: *infinite, $x=2$: *jump
24. $x=-3$: *infinite, $x=4$: *jump, $x=5$: *infinite
25. $x=-4$: *infinite, $x=8$: *jump
26. $x=-2$: *infinite, $x=-1$: *jump
27. $x=-4$: *jump, $x=-3$: *removable, $x=6$: *infinite
28. $k=\pm 2$
29. $k=-5,4$
30. $k=-5,4$
31. $k=-1,6$
32. $k=16$
33. $a=1, b=-5$
34. $k=-2,1$
35. $k=-1,2$
36. $k=3$ (not $k=-5$)
37. $k=3$ (not $k=-1$)
38. $a=-1, b=2$
39. $k=-1,2$
40. $k=0,5$
41. $a=1/3$
42. $k=-3,2$
43. $k=-1,6$
44. $a=-5,3$
45. $k=-2,5$