201-SH2-AB - Exercises #13 - Critical Numbers - Absolute Extrema

Find the critical numbers for each of the following functions.

(1) $f(x) = x^4 - 32x + 28$

(2) $f(x) = 3x^4 + 8x^3 - 18x^2 + 7$

(3) $f(x) = \frac{2}{3}x^3 + \frac{1}{2}x^2 - 6x + 4$

(4) $f(x) = \frac{3x-1}{2x+9}$

(5) $f(x) = \frac{x^2 - 2x + 9}{2 - x}$

(6) $f(x) = x^4 e^{-2x}$

(7) $f(x) = (x^2 - 3)^4 (x^2 - 9)^8$

(8) $f(x) = \frac{(x+3)^2}{(x-2)^3}$

(9) $f(x) = x^{1/3}(x - 8)$

(10) $f(x) = 2x - \sqrt{x}$

(11) $f(x) = 9\sqrt[9]{x}\, e^x$

(12) $f(x) = x^{2/3}\,(x-3)^2$

(13) $f(x) = (x^2 - 3)\, e^{4x}$

Find the absolute extrema of the function on the given interval.

(14) $f(x) = \frac{1}{2}x^4 - 4x^2 + 5;\ [1, 3]$

(15) $f(x) = \frac{-x^3 - 4}{x^2};\ [1, 4]$

(16) $f(x) = \frac{5}{2}x^4 - \frac{20}{3}x^3 + 6;\ [-1, 3]$

(17) $f(x) = \frac{3}{2}x^4 - 4x^3 + 4;\ [0, 3]$

(18) $f(x) = 2x^4 - 36x^2 + 20;\ [-4, -1]$

(19) $f(x) = \frac{2x^3 + 27}{2x^2};\ [2, 5]$

(20) $f(x) = \frac{40}{3}x^3 - 2x^4 + 10;\ [-1, 6]$

(21) $f(x) = -\frac{4}{5}x^5 + \frac{1}{2}x^4 + 8;\ [-2, 1]$

(22) $f(x) = \frac{x^2 + 25}{4x};\ [2, 6]$

(23) $f(x) = x^4 - 8x^2;\ [-2, 3]$

(24) $f(x) = \frac{2x}{x^2 + 4};\ [0, 4]$

(25) $f(x) = e^x(x^2 - 2x - 7);\ [-4, 4]$

(26) $f(x) = x\sqrt{x+1};\ [0, 3]$

(27) $f(x) = (x^2 - 25)^{2/3};\ [-4, 6]$

(28) $f(x) = e^{2x}(x^2 - 2);\ [-1, 3]$

(29) $f(x) = x^{4/3} - 32x^{1/3};\ [-1, 27]$

ANSWERS:

Critical numbers:

(1) $x = 2$

(2) $x = -3,\ 0,\ 1$

(3) $x = -2,\ \dfrac{3}{2}$

(4) none

(5) $x = -1,\ 5$

(6) $x = 0,\ 2$

(7) $x = 0,\ \pm\sqrt{3},\ \pm\sqrt{5},\ \pm 3$

(8) $x = -13,\ -3$

(9) $x = 0,\ 2$

(10) $x = 0,\ \dfrac{1}{16}$

(11) $x = -\dfrac{1}{9},\ 0$

(12) $x = 0,\ \dfrac{3}{4},\ 3$

(13) $x = -2,\ \dfrac{3}{2}$

Absolute extrema:

(14) Abs. min: $(2, -3)$
Abs. max: $(3, 19/2)$

(15) Abs. min: $(1, -5)$
Abs. max: $(2, -3)$

(16) Abs. min: $(2, -22/3)$
Abs. max: $(3, 57/2)$

(17) Abs. min: $(2, -4)$
Abs. max: $(3, 35/2)$

(18) Abs. min: $(-3. - 142)$
Abs. max: $(-1, -14)$

(19) Abs. min: $(3, 9/2)$
Abs. max: $(5, 277/50)$

(20) Abs. min:$(-1, -16/3)$
Abs. max: $(5, 1280/3)$

(21) Abs. min: $(1, 77/10)$
Abs. max: $(-2, 208/5)$

(22) Abs. min: $(5, 5/2)$
Abs. max: $(2, 29/8)$

(23) Abs. min: $(-2, -16),\ (2, -16)$
Abs. max: $(3, 9)$

(24) Abs. min: $(0, 0)$
Abs. max: $(2, 1/2)$

(25) Abs. min: $(3, -4e^3)$
Abs. max: $(4, e^4)$

(26) Abs. min: $(0, 0)$
Abs. max: $(3, 6)$

(27) Abs. min: $(5, 0)$
Abs. max: $(0, \sqrt[3]{625})$

(28) Abs. min: $(1, -e^2)$
Abs. max: $(3, 7e^6)$

(29) Abs. min: $(8, -48)$
Abs. max: $(-1, 33)$