1. Solve the system or show that it is inconsistent. If the system has infinitely many solutions, express them parametrically.

(a) $\begin{cases} 4x + y - 5z = 3 \\ 3x + y - 2z = 5 \\ 2x + 2y + z = 4 \end{cases}$

(b) $\begin{cases} x + 2y + 3z = 1 \\ 2x + 5y = 0 \\ 3x + 6y + 9z = 3 \\ 4x + 9y + 11z = 8 \end{cases}$

(c) $\begin{cases} 3x + y + 3z = -4 \\ 3x + 2y + 9z = -9 \\ -9x - 4y - 3z = 7 \end{cases}$

(d) $\begin{cases} 2x - 8y + 6z = 12 \\ x - 4y + 3z = 6 \\ 5x - 20y + 15z = 30 \end{cases}$

(e) $\begin{cases} 3x + 7y - z = 3 \\ -5x - 11y + 3z = -2 \\ 3x + 9y + z = 7 \end{cases}$

(f) $\begin{cases} 3x + 4y + z = 1 \\ 2x + 3y = 0 \\ 4x + 3y - z = -2 \end{cases}$

(g) $\begin{cases} -2x + 3y + 3z = -9 \\ 3x - 4y + z = 5 \\ -5x + 7y + 2z = -14 \end{cases}$

(h) $\begin{cases} x_2 + 4x_3 - x_4 = -5 \\ x_1 + 3x_2 + 5x_3 = -2 \\ 3x_1 + 7x_2 + 7x_3 + 2x_4 = 6 \end{cases}$

(i) $\begin{cases} 2x_1 + x_2 - 3x_3 + 5x_4 = 7 \\ 6x_1 + 3x_2 + x_4 = 4 \\ 8x_1 + 4x_2 - 3x_3 + 6x_4 = 10 \end{cases}$

(j) $\begin{cases} 2x_1 + 7x_2 + 2x_3 - 3x_4 = 4 \\ 6x_1 + 21x_2 + 10x_3 - 7x_4 = 8 \\ 4x_1 + 14x_2 + 8x_3 - 4x_4 = 4 \end{cases}$

(k) $\begin{cases} x_2 + 4x_3 - x_4 = -5 \\ x_1 + 3x_2 + 5x_3 = 2 \\ 3x_1 + 7x_2 + 7x_3 + 2x_4 = 6 \end{cases}$

(l) $\begin{cases} 4x_1 - 2x_2 + 8x_3 + x_4 = 2 \\ 6x_1 - 3x_2 + 12x_3 + 2x_4 = 4 \end{cases}$

(m) $\begin{cases} 4x_1 + 8x_2 + x_3 + 5x_4 + x_5 + 6x_6 = 0 \\ 2x_1 + 4x_2 + 2x_3 + 4x_4 + 2x_5 + 6x_6 = 0 \\ 3x_1 + 6x_2 + 2x_3 + 5x_4 + 3x_5 + 8x_6 = 0 \end{cases}$

2. For each of the following row-reduced matrices, state if the resulting system of equations has infinite solutions, one solution, or no solution. If infinite solutions, write out the infinite solutions (finish the row reduction if necessary).

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 1 & 9 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & 0 & -6 & 5 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 0 & 84 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & 0 & 19 & 4 \\ 0 & 1 & -3 & 24 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(d) $\left[\begin{array}{ccc|c} 2 & 9 & 8 & 13 \\ 0 & 5 & 10 & 0 \end{array}\right]$

(e) $\left[\begin{array}{cc|c} 3 & 5 & 6 \\ 0 & 5 & -11 \\ 0 & 0 & 0 \end{array}\right]$

(f) $\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 8 & 0 & 4 \\ 0 & 0 & 1 & 9 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{array}\right]$

(g) $\left[\begin{array}{ccc|c} -7 & 6 & 1 & 1 \\ 0 & 5 & 10 & 0 \\ 0 & 0 & 12 & 15 \end{array}\right]$

(h) $\left[\begin{array}{ccc|c} 1 & 2 & 0 & -14 \\ 0 & 0 & 1 & 0 \end{array}\right]$

(i) $\left[\begin{array}{ccc|c} 1 & 2 & 0 & -14 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(j) $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 & 48 \\ 0 & 0 & 1 & 0 & -13 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$

(k) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -14 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

3. Find all values of k for which the system is consistent.
$$\begin{cases} x \;+\; 3y \;+\; 3z = -2 \\ \quad\;\; 2y \;+\; 2z = 3 \\ 2x \;+\; 5y \;+\; 5z = k-5 \end{cases}$$

4. Find a condition on a, b, and c so that the system is consistent.

(a) $$\begin{cases} x + 2y - 2z = a \\ 3x \quad\;\; + 2z = b \\ \quad\; + 3y - 4z = c \end{cases}$$

(b) $$\begin{cases} 6x + 12y + 2z = a \\ 4x + \;\;6y + 2z = b \\ 3x + 12y - \;\;z = c \end{cases}$$

5. Find a condition on a, b, and c so that the system has a unique solution.
$$\begin{cases} 2x + \;\;y + az = 3 \\ 5x - 2y + bz = 3 \\ -4x + \;\;y + cz = -3 \end{cases}$$

6. Find all values of k such that the system
$$\begin{cases} x + 5y + 8z = k-1 \\ \quad (k-3)y + 4z = 2k-8 \\ \quad\quad (3k-15)z = k \end{cases}$$

(a) has a unique solution;

(b) has infinitely many solutions;

(c) has no solutions.

7. Find all values of k such that the system
$$\begin{cases} -x - ky = 5 \\ kx + 4y = 10 \end{cases}$$

(a) has a unique solution;

(b) has infinitely many solutions;

(c) has no solutions.

8. Find all values of k such that the system
$$\begin{cases} x - \;\; y = 1 \\ kx + k^2y = k^2 \end{cases}$$

(a) has a unique solution;

(b) has infinitely many solutions;

(c) has no solutions.

9. Find all values of k such that the system
$$\begin{cases} 3x + \;\;y + \quad k^2z = k^2 \\ -6x + \;\;3y + 2(k-k^2)z = 10-2k^2 \\ -3x + 24y + \quad 16z = -14 \end{cases}$$

(a) has a unique solution;

(b) has infinitely many solutions;

(c) has no solutions.

10. Find all values of k such that the augmented matrix
$$\left[\begin{array}{cc|c} (k-1) & (k-2) & (k-3) \\ 0 & (k-4) & (k-4) \end{array}\right]$$
represents a system with:

(a) a unique solution;

(b) infinitely many solutions;

(c) no solutions.

11. Find all the value(s) for k such that the system $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 4-k \\ 0 & k & 2k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$ will have

(a) no solution

(b) infinite solutions

(c) one solution

12. Find the values of h and k such that the system
$$\begin{cases} x + hy = k \\ 2x + 4y = 7 \end{cases}$$

(a) has a unique solution;

(b) has infinitely many solutions;

(c) has no solutions.

13. * Find all value of h such that the augmented matrix
$$\left[\begin{array}{cc|c} (h+1) & (h+2) & 0 \\ 0 & (h-2) & (h-2) \\ 0 & 0 & (h^2-1) \end{array}\right]$$
represents a system with:

(a) no solutions;

(b) a unique solution;

(c) infinitely many solutions.

14. A company manufactures three products A, B, and C. Each product requires labour and material. Product A requires 4 hours of labour and 6 units of material. Product B requires 2 hours of labour and 2 units of material. Product C requires 1 hour of labour and 2 units of material. If there are 22 hours of labour and 40 units of material available, how many of each product can be made so that all the labour and material is used? Name variables, set-up a system of equations, solve it, and give all possible realistic answers.

15. A furniture company makes kitchen tables, coffee tables, and desks. Each product requires material and labour. One kitchen table requires 10 units of material and 5 hours of labour; one coffee table requires 5 units of material and 5 hours of labour; and one desk requires 7 units of material and 3 hours of labour. If there are 150 units of material and 70 hours of labour available, how many of each product can be made so that all the material and labour is used? Name variables, set-up a system of equations, solve it, and give all possible realistic answers.

16. The Simple Machine Company makes Widgets, Gadgets, and Gizmos out of pulleys, wedges, and levers. Each Widget requires 2 pulleys, 1 wedges, and 3 levers. Each Gadget requires 2 pulleys, 3 wedges, and 1 lever. Each Gizmo requires 1 pulley and 2 levers. If the company has 20 pulleys, 6 wedges, and 34 levers available, how many Widgets, Gadgets, and Gizmos can be made to use up all of the resources? Name variables, set-up a system of equations, solve it, and give all possible realistic answers.

17. The Toy Company manufactures three kinds of toy dolls–Woody, Buzz, and Jessie–out of blocks, screws, and springs. The number of parts in one toy of each type is summarized in the following table:

	blocks	screws	springs
Woody	10	5	20
Buzz	15	10	25
Jesse	1	1	1

In addition, the company has 50 blocks, 45 screws, and 60 springs available. They would like to determine how many of each type of doll can be made to use up all of the parts. Name variables, set-up a system of equations, solve it, and give all possible realistic answers.

18. The Phone Store sells three kinds of smart-phones: The E-phone sells for \$210, the BrownBerry sells for \$300, and the Robophone sells for \$150. One day the store sells fourteen phones and has a revenue of \$2400. How many of each type of phone were sold on that day? Give all possible realistic solutions.

ANSWERS ON NEXT PAGE.

ANSWERS:

1. (a) $(4,-3,2)$
   (b) $(-13,26/5,6/5)$
   (c) $(-1/2,0,-5/6)$
   (d) $(6+4t-3r,t,r)$
   (e) $(3,-1/2,5/2)$
   (f) $(-3/7,2/7,8/7)$
   (g) $(-21-15t,-17-11t,t)$
   (h) Inconsistent.
   (i) Inconsistent.
   (j) $(3-7s/2+2t,s,-1-t/2,t)$
   (k) Inconsistent.
   (l) $(s/2-2t,s,t,2)$
   (m) $(-2r-s-t,r,-s-t,s,-t,t)$
2. (a) one solution
   (b) no solution
   (c) infinite solutions: $(4-19t,24+3t,t)$
   (d) infinite solutions: $(\frac{13}{2}+5t,-2t,t)$
   (e) one solution
   (f) no solution
   (g) one solution
   (h) infinite solutions: $(-14-2t,t,0)$
   (i) infinite solutions: $(-14-2t,t,0)$
   (j) one solution
   (k) one solution
3. $k=-1/2$
4. (a) $-3a+b+2c=0$
   (b) $-5a+6b+2c=0$
5. $a+2b+3c\neq 0$
6. (a) $k\neq 3,\neq 5$
   (b) $k=3$
   (c) $k=5$
7. (a) $k\neq\pm 2$
   (b) $k=-2$
   (c) $k=2$
8. (a) $k\neq 0,\ k\neq -1$
   (b) $k=0$
   (c) $k=-1$
9. (a) $k\neq 8,\ k\neq 2$
   (b) $k=8$
   (c) $k=2$
10. (a) $k\neq 1,\ k\neq 4$
    (b) $k=4$
    (c) $k=1$
11. (a) $k=0$
    (b) $k=2$
    (c) $k\neq 0, k\neq 2$
12. (a) $h\neq 2$, k doesn't matter
    (b) $h=2, k=\frac{7}{2}$
    (c) $h=2, k\neq\frac{7}{2}$
13. (a) $h\neq 1$
    (b) $h=1$
    (c) Impossible.
14. Let x=# of product A,
    y=# of product B,
    z=# of product C.
    $$\begin{cases} 4x+2y+z=22 \\ 6x+2y+2z=40 \end{cases}$$
    Possible realistic solutions: $(2,0,14)$, $(1,1,16)$, $(0,2,18)$.
15. Let x=# of kitchen tables,
    y=# of coffee tables,
    z=# of desks.
    $$\begin{cases} 10x+5y+7z=150 \\ 5x+5y+3z=70 \end{cases}$$
    Possible realistic solutions: $(8,0,10)$, $(4,1,15)$, $(0,2,20)$.
16. Let x=# of Widgets, y=# of Gadgets, z=# of Gizmos.
    $$\begin{cases} 2x+2y+z=20 \\ x+3y=6 \\ 3x+y+2z=34 \end{cases}$$
    Possible realistic solutions: $(6,0,8)$, $(3,1,12)$, $(0,2,16)$.
17. Let x_1=# of Woody dolls,
    x_2=# of Buzz Dolls,
    x_3=# of Jesse Dolls.
    $$\begin{cases} 10x_1+15x_2+x_3=50 \\ 5x_1+10x_2+x_3=45 \\ 20x_1+25x_2+x_3=60 \end{cases}$$
    Two realistic solutions: $(0,1,35)$, $(1,0,40)$.
18. Let x_1=# of E-phones,
    x_2=# of Robophones,
    x_3=# of Brownberries.
    $$\begin{cases} x_1+x_2+x_3=14 \\ 210x_1+300x_2+150x_3=2400 \end{cases}$$
    Two realistic solutions: $(5,0,9)$, $(0,2,12)$.