## 201-SH3-AB - Exercises #5: Riemann Sums

1. Approximate $\int_{-6}^{4} (x-1)\ dx$ using Riemann sums with five subintervals, taking the sample points to be the right endpoints. Draw a diagram explaining what the Riemann sum represents.

2. A table of values of an increasing function $f$ is shown:

| $x$ | 10 | 14 | 18 | 22 | 26 | 30 |
|---|---|---|---|---|---|---|
| $f(x)$ | $-12$ | $-6$ | $-2$ | 1 | 3 | 8 |

   Use the table to find an estimate for $\int_{10}^{30} f(x)\ dx$.

3. Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using:
   (a) Three rectangles and the right endpoints.
   (b) Improve your estimate by using 6 rectangles.
   (c) Sketch the curve and the approximating rectangles.

4. Estimate the area under the graph of $f(x) = x^3 + 4$ from $x = 0$ to $x = 4$ using:
   (a) Four rectangles and the right endpoints.
   (b) Eight rectangles and the right endpoints.
   (c) Which estimate is better?

5. Estimate the area under the graph of $f(x) = (x+3)^2$ from $x = 1$ to $x = 4$ using:
   (a) Three rectangles and the right endpoints.
   (b) Six rectangles and the right endpoints.
   (c) Which estimate is better?

6. Approximate the following integrals using the right endpoint method with the given $n$.
   Round your answers to four decimals.

   (a) $\int_{1}^{4} \frac{2}{4x^2+9}\ dx \qquad n = 6$

   (b) $\int_{1}^{3} (\ln(x)+3)^2\ dx \qquad n = 4$

   (c) $\int_{-2}^{2} \left(x^3+6\right)^{2/3}\ dx \qquad n = 4$

   (d) $\int_{2}^{6} \frac{10}{\sqrt{x^2+4}}\ dx \qquad n = 4$

   (e) $\int_{2}^{7} \frac{e^{3-x}}{\ln(x)}\ dx \qquad n = 4$

   (f) $\int_{0}^{3} \sqrt{x^2+2x}\ dx \qquad n = 6$

   (g) $\int_{0}^{8} \cos\left(x^2+x\right)\ dx \qquad n = 4$

   (h) $\int_{1}^{13} \frac{x^2+1}{x^3+1}\ dx \qquad n = 4$

   (i) $\int_{0}^{4} e^{\sin(x)}\ dx \qquad n = 4$

7. Use Riemann sums to evaluate the integral. Recall that:

$$\sum_{k=1}^{n} c = c \cdot n \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

(a) $\displaystyle\int_0^2 3x\ dx$

(b) $\displaystyle\int_0^3 (5x+2)\ dx$

(c) $\displaystyle\int_0^3 4x\ dx$

(d) $\displaystyle\int_0^4 \left(2 - \frac{1}{2}x\right)\ dx$

(e) $\displaystyle\int_0^8 (3-2x^3)\ dx$

(f) $\displaystyle\int_0^9 (2x^2-3)\ dx$

(g) $\displaystyle\int_0^5 (x^2+2x-5)\ dx$

(h) $\displaystyle\int_0^3 (3x^2+4x)\ dx$

(i) $\displaystyle\int_0^2 (4x^3-5x)\ dx$

(j) $\displaystyle\int_0^4 (x-x^2+x^3)\ dx.$

(k) $\displaystyle\int_0^3 x^2\ dx.$

(l) $\displaystyle\int_0^4 (6-x^2)\ dx$

(m) $\displaystyle\int_0^5 (3x^2+7x)\ dx$

(n) $\displaystyle\int_0^2 (4x^2+x+2)\ dx$

(o) $\displaystyle\int_0^4 (x^2-3x^3)\ dx$

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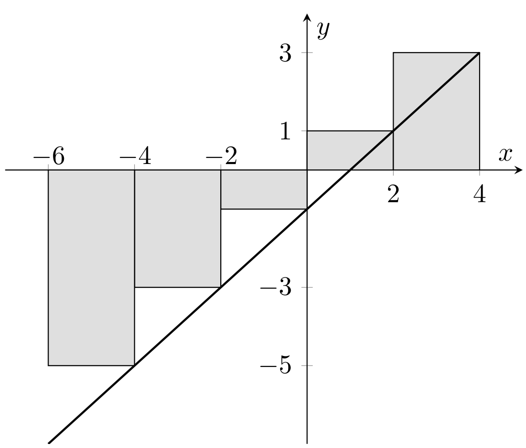
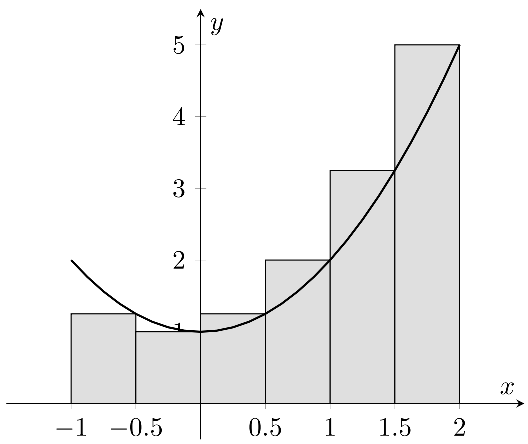
**ANSWERS:**

1. The Riemann sum equals $-10$ and it represents the sum of the areas of the two rectangles above the $x$-axis minus the sum of the areas of the three rectangles below the $x$-axis.




2. Estimate using left endpoints: $-64$.
   Estimate using right endpoints: 16

3. (a) 8
   (b) 6.875
   (c) Here is the curve and the approximating rectangles:




4. (a) 116
   (b) 97
   (c) 97 (eight rectangles)

5. (a) 110
   (b) 101.375
   (c) 101.375 (six rectangles)

6. (a) 0.1781
   (b) 28.6862
   (c) 15.6940
   (d) 8.4477
   (e) 1.0688
   (f) 7.5984
   (g) 0.0020
   (h) 1.7554
   (i) 6.4231

7. (a) 6
   (b) $\dfrac{57}{2}$
   (c) 18
   (d) 4
   (e) $-2024$
   (f) 459
   (g) $\dfrac{125}{3}$
   (h) 45
   (i) 6
   (j) $\dfrac{152}{3}$
   (k) 9
   (l) $\dfrac{8}{3}$
   (m) $\dfrac{425}{2}$
   (n) $\dfrac{50}{3}$
   (o) $-\dfrac{512}{3}$