201-SH2-AB - Exercises #7 - Derivatives - Basic Differentiation Rules

1. Find the derivative (constant, sum, power, product, quotient rules).

(a) $f(x) = 7x^2 - 2\sqrt{x} + \dfrac{16}{x} - 12$

(b) $g(x) = 6x^3 - 4\sqrt[3]{x} - \dfrac{3}{x^2} + 3$

(c) $g(x) = \dfrac{4}{\sqrt[3]{x}} - 12x^2 - 6\sqrt{x} + 7$

(d) $g(x) = 18x^4 + 6\sqrt[4]{x^3} - \dfrac{5}{x^2} + 10$

(e) $g(x) = 6x - \dfrac{15}{x^4} + 4x^2 + 13$

(f) $g(x) = 7x + \dfrac{12}{x^3} - 16x^2 + 25$

(g) $g(x) = \dfrac{5x^2 + 2x - 3}{x}$

(h) $g(x) = \dfrac{4x^5 - 5x^{3/2} + 2\sqrt[3]{x}}{\sqrt{x}}$

(i) $f(x) = (x^2 + 1)(1 - x^3)$

(j) $f(x) = (3x^2 + x + 2)(1 + 3x)$

(k) $f(x) = (x^2 - 2x)(5 - x + 2x^2)$

(l) $f(x) = (5 + x - x^2)(2x^2 - 3x + 1)$

(m) $f(x) = (2x + \sqrt{x})(10\sqrt{x} - 3x^2)$

(n) $f(x) = (3x - 2\sqrt{x})(x^3 - x)$

(o) $f(x) = \dfrac{2x}{x + 3}$

(p) $f(x) = \dfrac{1 - 3x}{x - 1}$

(q) $f(x) = \dfrac{2 + \sqrt{x}}{2 - \sqrt{x}}$

(r) $f(x) = \dfrac{4\sqrt{x}}{x - 3}$

(s) $f(x) = \dfrac{2x + 3}{4 + \sqrt{x}}$

(t) $f(x) = \dfrac{3x}{x - 4}$

2. Find the derivative (now with chain rule).

(a) $g(x) = (x^4 - 2x^2 - 9)^4$

(b) $g(x) = \dfrac{6}{(2 + x - x^2)^3}$

(c) $g(x) = \sqrt[3]{(4 - 4x - x^2)^4}$

(d) $g(x) = \dfrac{9}{\sqrt{2x^2 - 4x + 3}}$

(e) $g(x) = \sqrt[5]{x^3 - 3x + 3}$

(f) $g(x) = \dfrac{4}{(3 + 2x + x^2)^2}$

(g) $f(x) = \left(\frac{3x^2 - 1}{3x^2 + 3}\right)^2$

(h) $f(x) = (x + 3)^2(2x - 7)^2$

(i) $g(x) = \dfrac{(3x + 2)^3}{(2x - 9)^4}$

(j) $g(x) = (9 - 7x)^5(2x + 1)^{1/3}$

(k) $f(x) = x^2\sqrt{1 - x^2}$

(l) $g(x) = ((x + 2)(3x^2 + 4))^3$

(m) $h(x) = \left(\dfrac{2x^3 + x^2}{3x^4 - 2}\right)^2$

(n) $f(x) = \left(\dfrac{6x^3 - 7x + 2}{4x^5 - 7x}\right)^4$

3. Find an equation of the tangent line to the graph of the function at the given x-value.

(a) $f(x) = \dfrac{x^2}{x - 2} \quad at \quad x = 3$

(b) $f(x) = (x^3 - 2x^2 + 3x - 1)^{3/2}\ at\ x = 1$

(c) $f(x) = (4x - x^2)(x^3 + 4)$ *at* $x = 1$

(d) $f(x) = \dfrac{24}{\sqrt{x}} + \dfrac{16}{x^2} + 3x$ *at* $x = 4$

(e) $f(x) = \dfrac{3\sqrt{x}}{2 - x}$ *at* $x = 1$

(f) $f(x) = \dfrac{5x - 1}{2\sqrt{x} - 3}$ *at* $x = 4$

4. Given $f(x) = x^3 + 6x^2 - 15x + 4$, find the x-value(s) such that the tangent line to the curve of $f(x)$ is horizontal.

5. Find the point(s) on the curve of $f(x) = 2x^3 + 15x^2 - 140x + 10$ such that the slope of the tangent line is 4.

6. Given $f(x) = \dfrac{x^2}{x + 4}$, find the point(s) such that the tangent line to the curve of $f(x)$ is horizontal.

7. Given $f(x) = \dfrac{\sqrt{x}}{x^2 + 3}$, find the point(s) such that the tangent line to the curve of $f(x)$ is horizontal.

8. Find the x value(s) where the tangent line to the function $f(x) = \frac{1}{4}x^4 + x^3 - 2x^2 - 12x + 15$ is horizontal.

9. Find the equation(s) for the tangent line(s) to $f(x) = 2x^3 - 10x + 2$ that are parallel to $y = 14x + 13$.

10. Find the x-value(s) where the tangent line to $f(x) = \dfrac{x^2}{x - 4}$ is horizontal.

11. Find the x-values where the tangent line to the function $f(x) = \frac{1}{2}x^4 + 2x^2 + 4x$ is parallel to the tangent line to $g(x) = \frac{1}{4}x^4 + 2x^2 - 4x$.

12. Find x-values where the tangent line to the function $f(x) = 6x^3 + 6x^2 + 8x + 13$ is parallel to $y = -2x + 3$.

13. * If $f(1) = 5$, $f'(1) = -2$ and $g(x) = x^3 \cdot f(x)$, then find $g'(1)$.

14. * If $h(2) = 4$, $h'(2) = -3$ and $f(x) = \dfrac{2\,h(x)}{x^2}$, then find $f'(2)$.

15. * If $g(-1) = -4$, $g'(1) = 7$ and $f(x) = g(x^2)$, then find $f'(-1)$.

16. For each problem below, find the x-value(s), if any, at which the graph of f has a horizontal tangent.

    (a) $f(x) = (x^2 + 2)^4(2x + 2)^2$

    (b) $f(x) = \dfrac{(3x - 4)^2}{(x + 1)^3}$

    (c) $f(x) = (7x + 1)^3 \cdot \sqrt{2x + 4}$

    (d) $f(x) = (x^2 - 9)^9(1 - x^2)^3$

    (e) $f(x) = \dfrac{(9x - 6)^3}{\sqrt[3]{x + 1}}$

17. Find the derivative (exponential base e, product, quotient, chain rules).

(a) $f(x) = 3x^2e^x$

(b) $f(x) = \dfrac{e^x - 3x}{5x^3 - 2\sqrt{x}}$

(c) $f(x) = 5e^x(\pi x - \sqrt[3]{x})$

(d) $f(x) = \dfrac{2x^6 + e^2 - x\ln 2}{5x^2 - 3e^x}$

(e) $f(x) = e^{5-3x}$

(f) $f(x) = 4e^{-x}x^3$

18. Find the derivative (exponential and logarithmic rules, no product, quotient or chain rule).

(a) $f(x) = x^e + ex + e^x + e + 7^x + x^7$

(b) $f(x) = 5\sqrt[4]{x^7} + 4\ln(x) - 2\log_3(x)$

(c) $g(x) = \dfrac{x^4 4^x + x^4\log_8 x + \sqrt[4]{x^3} + 8x^9}{x^4}$

(d) $h(x) = \dfrac{9x^2}{\sqrt{x}} + x + \pi x + \dfrac{1}{x}$

(e) $f(x) = \dfrac{8}{7x} + \dfrac{8}{7\sqrt{x}} + \dfrac{x8^x}{x} + \dfrac{9x\log_9 x}{5x}$

(f) $g(x) = x^{6/7} + x^{7/6} + 7^x + 6^x + \log_6 x$

(g) $h(x) = \dfrac{x^2e^x + 5e^x + 9e^x\log_9 x}{e^x}$

(h) $g(x) = \pi x^6 - \dfrac{\pi}{x} + \sqrt[5]{x^3} + \ln 4$

(i) $f(x) = \sqrt[4]{x^9} - \dfrac{4}{x^9} + \ln 8 - \sqrt{8}x + \sqrt{4x}$

(j) $f(x) = e^2 + \log_2 5 + \pi + e$

19. Find the derivative (exponential and logarithmic rules, with product, quotient and chain rule).

(a) $f(x) = \left(8x^2 + 2^x - \dfrac{3}{x^2}\right)^4$

(b) $g(x) = \dfrac{5^x + 1}{(3x^2 - 6x + 7)^3}$

(c) $p(x) = e^{x^2+8x-e}$

(d) $p(x) = 2^{8\ln x - 7x^3 + 5}$

(e) $f(x) = e^{\sqrt{e^x - 4x}}$

(f) $f(x) = (2x^2 + e^{x^2-6x} + 5)^4$

(g) $p(x) = \sqrt[5]{(\log_5(x) + 7x^2 + 3x^e)^6}$

(h) $f(x) = \sqrt{e^{x^3+x} + \sqrt[3]{x^2+1}}$

(i) $f(x) = (3x - 4)^5(e^x - 6x^2)^4$

(j) $f(x) = ((e^{x^4-6e} + 4)(2x^2 + 1))^5$

(k) $f(x) = \dfrac{(e^{x^2+2} - 4x + 1)^3}{(8x - 4x^3 + 1)^5}$

20. Find the derivative (exponential and logarithmic rules, with product, quotient and chain rule).

(a) $y = \ln(3 - \sqrt{x})$

(b) $y = \ln\left(\dfrac{1+2x}{1-3x}\right)$

(c) $y = \sqrt{\ln(x) + 4x}$

(d) $y = (x^2 + 2x)\ln(x^2)$

(e) $y = 3x\Big(\ln(2x - x^2)\Big)^2$

(f) $y = \sqrt[3]{x + \ln(x)}$

(g) $y = \dfrac{x-2}{x + \ln(x)}$

(h) $y = \dfrac{3}{1 + 2e^{3x}}$

(i) $y = \sqrt{2x + e^x}$

(j) $y = x^2e^{1-x}$

(k) $y = e^{x^2-2x}$

(l) $y = \ln\left(2(x+1)e^{3x}\right)$

(m) $y = \ln\left(\dfrac{e^{3-x}}{x-2}\right)$

(n) $y = \log_5(\sqrt{x}-1)$

(o) $y = \log_2\left(\dfrac{4x-1}{2x-1}\right)$

(p) $y = \sqrt{5^x + \log_3(x) - 1}$

(q) $y = (3^{2x+4} - x^2)\ln(3-2x)$

(r) $y = 2^{\sqrt{x}}\Big(\log_7(x^3+2)\Big)^3$

(s) $y = \sqrt[5]{x - 2\log_4(x)}$

(t) $y = \dfrac{4^e - 4^x}{x^2 + \log_4(x)}$

(u) $y = \dfrac{4}{1-3^{2x}}$

(v) $y = \sqrt{2x^2 + \pi^x}$

(w) $y = (x^2+1)\,e^{2-x}$

(x) $y = e^{x^3-3}$

(y) $y = \ln\Big((x^2+1)\,e^{2-x}\Big)$

(z) $y = \ln\left(\dfrac{x+4}{e^{x+2}}\right)$

21. Find an equation of the tangent line for the following functions at the given x value.

(a) $f(x) = (x^2+2)e^x$ at $x = 0$

(b) $f(x) = \ln x(2x+1)$ at $x = e$

(c) $f(x) = \dfrac{x^3+4}{2x-1}$ at $x = 1$

(d) $f(x) = \dfrac{\sqrt{x-4}+1}{x-5}$ at $x = 8$

22. For each function below, find the x-coordinate(s), if any, for the points at which the graph of f has a horizontal tangent.

(a) $f(x) = x^2e^{6x}$

(b) $f(x) = 3xe^{1-8x^2}$

(c) $f(x) = e^x(x-6)^5$

(d) $f(x) = e^{x^2}(2x+6)^4$

(e) $f(x) = e^x\sqrt{2x+9}$

Answers

1. (a) $f'(x) = 14x - \dfrac{1}{\sqrt{x}} - \dfrac{16}{x^2}$
   (b) $g'(x) = 18x^2 - \dfrac{4}{3x^{2/3}} + \dfrac{6}{x^3}$
   (c) $g'(x) = -\dfrac{4}{3x^{4/3}} - 24x - \frac{3}{\sqrt{x}}$
   (d) $g'(x) = 72x^3 + \dfrac{9}{2x^{1/4}} + \dfrac{10}{x^3}$
   (e) $g'(x) = 6 + \dfrac{60}{x^5} + 8x$
   (f) $g'(x) = 7 - \dfrac{36}{x^4} - 32x$
   (g) $g'(x) = 5 + \dfrac{3}{x^2}$
   (h) $g'(x) = 18x^{7/2} - 5 - \dfrac{1}{3x^{7/6}}$
   (i) $f'(x) = 2x(1-x^3) - 3x^2(x^2+1)$
   (j) $f'(x) = (6x+1)(1+3x) + 3(3x^2+x+2)$
   (k) $f'(x) = (2x-2)(5-x+2x^2)+(x^2-2x)(4x-1)$
   (l) $f'(x) = (1-2x)(2x^2-3x+1) + (5+x-x^2)(4x-3)$
   (m) $f'(x) = \left(2 + \dfrac{1}{2\sqrt{x}}\right)(10\sqrt{x} - 3x^2) + (2x + \sqrt{x})\left(\dfrac{5}{\sqrt{x}} - 6x\right)$
   (n) $f'(x) = \left(3 - \dfrac{1}{\sqrt{x}}\right)(x^3 - x) + (3x - 2\sqrt{x})(3x^2-1)$
   (o) $f'(x) = \dfrac{2(x+3) - 2x}{(x+3)^2}$
   (p) $f'(x) = \dfrac{-3(x-1) - (1-3x)}{(x-1)^2}$
   (q) $f'(x) = \dfrac{\frac{1}{2\sqrt{x}}(2-\sqrt{x}) - (2+\sqrt{x})\left(-\frac{1}{2\sqrt{x}}\right)}{(2-\sqrt{x})^2}$
   (r) $f'(x) = \dfrac{\frac{2}{\sqrt{x}}(x-3) - 4\sqrt{x}}{(x-3)^2}$
   (s) $f'(x) = \dfrac{2(4+\sqrt{x}) - (2x+3)\frac{1}{2\sqrt{x}}}{(4+\sqrt{x})^2}$
   (t) $f'(x) = \dfrac{3(x-4) - 3x}{(x-4)^2}$

2. (a) $g'(x) = 4\left(x^4 - 2x^2 - 9\right)^3 (4x^3 - 4x)$
   (b) $g'(x) = -18\left(2 + x - x^2\right)^{-4}(1-2x)$
   (c) $g'(x) = \dfrac{4}{3}\left(4 - 4x - x^2\right)^{1/3}(-4-2x)$
   (d) $g'(x) = -\dfrac{9}{2}\left(2x^2 - 4x + 3\right)^{-3/2}(4x-4)$
   (e) $g'(x) = \dfrac{1}{5}(x^3 - 3x + 3)^{-4/5}(3x^2 - 3)$
   (f) $g'(x) = -8\left(3 + 2x + x^2\right)^{-3}(2+2x)$
   (g) $f'(x) = 2\left(\dfrac{3x^2-1}{3x^2+3}\right)\dfrac{6x(3x^2+3) - (3x^2-1)6x}{(3x^2+3)^2}$
   (h) $f'(x) = 2(x+3)(2x-7)^2 + (x+3)^2 \cdot 4(2x-7)$
   (i) $g'(x) = \dfrac{9(3x+2)(2x-9)^4 - (3x+2)^3 \cdot 8(2x-9)^3}{(2x-9)^8}$
   (j) $g'(x) = -35(9-7x)^4(2x+1)^{1/3} + (9-7x)^5 \cdot \dfrac{2}{3}(2x+1)^{-2/3}$
   (k) $f'(x) = 2x\sqrt{1-x^2} - x^3(1-x^2)^{-1/2}$

(l) $g'(x) = 3\left((x+2)(3x^2+4)\right)^2\left((3x^2+4)+6x(x+2)\right)$

(m) $h'(x) = 2\left(\dfrac{2x^3+x^2}{3x^4-2}\right)\left(\dfrac{(6x^2+2x)(3x^4-2)-12x^3(2x^3+x^2)}{(3x^4-2)^2}\right)$

(n) $f'(x) = 4\left(\frac{6x^3-7x+2}{4x^5-7x}\right)^3\left(\dfrac{(18x^2-7)(4x^5-7x)-(6x^3-7x+2)(20x^4-7)}{(4x^5-7x)^2}\right)$

3. (a) $y = -3x + 18$

   (b) $y = 3x - 2$

   (c) $y = 19x - 4$

   (d) $y = x + 21$

   (e) $y = \dfrac{9}{2}x - \dfrac{3}{2}$

   (f) $y = \dfrac{-9}{2}x + 37$

4. $x = -5, 1$

5. $(3, -221)$ and $(-8, 1066)$

6. $(0, 0)$ and $(-8, -16)$

7. $\left(1, \dfrac{1}{4}\right)$

8. $x = -3, -2, 2$

9. $y = 14x - 30$ and $y = 14x + 34$

10. $x = 0, 8$

11. $x = -2$

12. none

13. 13

14. $\dfrac{-7}{2}$

15. -14

16. (a) $x = -1$

    (b) $x = 6, \quad x = 4/3$

    (c) $x = -85/49, \quad x = -1/7$

    (d) $x = 0, \quad x = -3, \quad x = 3, \quad x = -1, \quad x = 1, \quad x = -\sqrt{3}, \quad x = \sqrt{3}$

    (e) $x = -2/3, \quad x = -25/24$

17. (a) $f'(x) = 6xe^x + 3x^2e^x$

    (b) $f'(x) = \frac{(e^x-3)(5x^3-2\sqrt{x})-(e^x-3x)\left(15x^2-\frac{1}{\sqrt{x}}\right)}{(5x^3-2\sqrt{x})^2}$

    (c) $f'(x) = 5e^x(\pi x - \sqrt[3]{x}) + 5e^x(\pi - \frac{1}{3}x^{-2/3})$

    (d) $f'(x) = \frac{(12x^5-\ln 2)(5x^2-3e^x)-(2x^6+e^2-x\ln 2)(10x-3e^x)}{(5x^2-3e^x)^2}$

    (e) $f'(x) = -3e^{5-3x}$

    (f) $f'(x) = -4e^{-x}x^3 + 4e^{-x}\cdot 3x^2$

18. (a) $f'(x) = ex^{e-1} + e + e^x + 7^x\ln 7 + 7x^6$

    (b) $f'(x) = \dfrac{35}{4}x^{3/4} + \dfrac{4}{x} - \dfrac{2}{x\ln 3}$

    (c) $g'(x) = 4^x\ln 4 + \frac{1}{x\ln 8} - \frac{13}{4}x^{-17/4} + 40x^4$

    (d) $h'(x) = \frac{27}{2}\sqrt{x} + 1 + \pi - \frac{1}{x^2}$

    (e) $f'(x) = -\frac{8}{7}x^{-2} - \frac{4}{7}x^{-3/2} + 8^x\ln 8 + \frac{9}{5x\ln 9}$

    (f) $g'(x) = \frac{6}{7}x^{-1/7} + \frac{7}{6}x^{1/6} + 7^x\ln 7 + 6^x\ln 6 + \frac{1}{x\ln 6}$

    (g) $h'(x) = 2x + \frac{9}{x\ln 9}$

    (h) $g'(x) = 6\pi x^5 + \dfrac{\pi}{x^2} + \dfrac{3}{5}x^{-2/5}$

    (i) $f'(x) = \dfrac{9}{4}\sqrt[4]{x^5} + \dfrac{36}{x^{10}} - \sqrt{8} + x^{-1/2}$

    (j) $f'(x) = 0$

19. (a) $f'(x) = 4\left(8x^2 + 2^x - \dfrac{3}{x^2}\right)^3 (16x + 2^x \ln 2 + 6x^{-3})$

(b) $g'(x) = \dfrac{(5^x \ln 5)(3x^2 - 6x + 7)^3 - (5^x + 1) \cdot 3(3x^2 - 6x + 7)^2(6x - 6)}{(3x^2 - 6x + 7)^6}$

(c) $p'(x) = (2x + 8)e^{x^2+8x-e}$

(d) $p'(x) = 2^{8\ln x - 7x^3 + 5} \cdot \ln 2 \cdot \left(\frac{8}{x} - 21x^2\right)$

(e) $f'(x) = e^{\sqrt{e^x - 4x}} \cdot \frac{1}{2}(e^x - 4x)^{-1/2}(e^x - 4)$

(f) $f'(x) = 4(2x^2 + e^{x^2-6x} + 5)^3 \left(4x + e^{x^2-6x}(2x - 6)\right)$

(g) $p'(x) = \dfrac{6}{5}(\log_5(x) + 7x^2 + 3x^e)^{1/5} \left(\frac{1}{x \ln 5} + 14x + 3ex^{e-1}\right)$

(h) $f'(x) = \frac{1}{2}\left(e^{x^3+x} + \sqrt[3]{x^2 + 1}\right)^{-1/2} \left(e^{x^3+x}(3x^2 + 1) + \frac{2x}{3}(x^2 + 1)^{-2/3}\right)$

(i) $f'(x) = 15(3x - 4)^4(e^x - 6x^2)^4 + (3x - 4)^5 \cdot 4(e^x - 6x^2)^3(e^x - 12x)$

(j) $f'(x) = 5((e^{x^4-6e} + 4)(2x^2 + 1))^4 \left((4x^3e^{x^4-6e}(2x^2 + 1) + (e^{x^4-6e} + 4) \cdot 4x\right)$

(k) $f'(x) = \frac{3(e^{x^2+2}-4x+1)^2(2xe^{x^2+2}-4)(8x-4x^3+1)^5-(e^{x^2+2}-4x+1)^3 \cdot 5(8x-4x^3+1)^4(8-12x^2)}{(8x-4x^3+1)^{10}}$

20. (a) $\dfrac{-1}{2\sqrt{x}(3 - \sqrt{x})}$

(b) $\dfrac{2}{1 + 2x} + \dfrac{3}{1 - 3x}$

(c) $\dfrac{1 + 4x}{2x\sqrt{\ln(x) + 4x}}$

(d) $(2x + 2)\ln(x^2) + 2x + 4$

(e) $3\Big(\ln(2x - x^2)\Big)^2 + \dfrac{6x(2 - 2x)\ln(2x - x^2)}{2x - x^2}$

(f) $\dfrac{x + 1}{3x\,(x + \ln(x))^{2/3}}$

(g) $\dfrac{\ln(x) + 1 + \frac{2}{x}}{(x + \ln(x))^2}$

(h) $\dfrac{-18e^{3x}}{(1 + 2e^{3x})^2}$

(i) $\dfrac{2 + e^x}{2\sqrt{2x + e^x}}$

(j) $(2x - x^2)\, e^{1-x}$

(k) $(2x - 2)e^{x^2-2x}$

(l) $\dfrac{1}{x + 1} + 3$

(m) $-1 - \dfrac{1}{x - 2}$

(n) $\dfrac{1}{2\sqrt{x}\,(\sqrt{x} - 1)}$

(o) $\dfrac{4}{4x - 1} - \dfrac{2}{2x - 1}$

(p) $\dfrac{5x + 1}{2x\sqrt{5x + \ln(x) - 1}}$

(q) $(3 - 2x)\ln(3 - 2x) - \dfrac{2\,(3x - x^2)}{3 - 2x}$

(r) $2\Big(\ln(x^3 + 2)\Big)^3 + \dfrac{18x^3\Big(\ln(x^3 + 2)\Big)^2}{x^3 + 2}$

(s) $\dfrac{x - 2}{5x\,(x - 2\ln(x))^{4/5}}$

(t) $\dfrac{x^2 - \ln(x) - 8x - \frac{4}{x} + 1}{(x^2 + \ln(x))^2}$

(u) $\dfrac{24e^{2x}}{(1 - 3e^{2x})^2}$

(v) $\dfrac{4x + e^x}{2\sqrt{2x^2 + e^x}}$

(w) $(-x^2 + 2x - 1)\, e^{2-x}$

(x) $3x^2e^{x^3-3}$

(y) $\dfrac{2x}{x^2+1}-1$

(z) $\dfrac{1}{x+4}-1$

21. (a) $y = 2x + 2$

    (b) $y = \left(4 + \frac{1}{e}\right) x - 2e$

    (c) $y = -7x + 12$

    (d) $y = \dfrac{-1}{4}x + 3$

22. (a) $x = 0, -1/3$

    (b) $x = \pm 1/4$

    (c) $x = 1, 6$

    (d) $x = -3, -2, -1$

    (e) no solution