201-SH3-AB - Exercises #2: Infinite Sequences

1. Write out the first 5 terms of the following sequences.

   a) $a_n = (-1)^{n-1}\dfrac{n^2}{n+2}$

   b) $a_n = 2 + (-1)^n$

2. Find a formula for the general term of the following sequences.

   a) $\{5,\ 7,\ 9,\ 11,\ \cdots\}$

   b) $\left\{3,\ \dfrac{3}{2},\ \dfrac{3}{4},\ \dfrac{3}{8},\ \cdots\right\}$

   c) $\left\{\frac{3}{-4}, \frac{15}{-1}, \frac{75}{2}, \frac{375}{5}, \frac{1875}{8}, ...\right\}$

   d) $\{384, -192, 96, -48, 24, ...\}$

   e) $\left\{\dfrac{7}{4}, \dfrac{5}{7}, \dfrac{3}{10}, \dfrac{1}{13}, ...\right\}$

   f) $\left\{\dfrac{4}{7}, \dfrac{12}{11}, \dfrac{36}{15}, \dfrac{108}{19}, ...\right\}$

3. Determine whether the following sequences converge or diverge. If a sequence converges, find its limit.

   a) $\left\{\dfrac{\sqrt{9n^2+2}}{2n+5}\right\}$

   b) $a_n = (-1)^n\left(\dfrac{3n}{n+2} - \dfrac{6n-4}{n}\right)$

   c) $\left\{4\cdot\left(\dfrac{1}{3}\right)^{n-1}\right\}$

   d) $\left\{\dfrac{4n+5}{2^{n-1}}\right\}$

   e) $\left\{\dfrac{(-2)^n}{(-3)^{n-1}}\right\}$

   f) $\left\{\dfrac{5+n}{\sqrt{3+4n^2}}\right\}$

   g) $\left\{\left(\dfrac{-4}{3}\right)^{n-2}\right\}$

   h) $a_n = \dfrac{3n}{\sqrt{n^2-1}}$

   i) $a_n = (-1)^n\left(\frac{1}{3}\right)^n$

   j) $\left\{\dfrac{1}{2}\cdot 5^{n+1}\right\}$

   k) $a_n = \left(\frac{9}{4}\right)^{n+1}$

   l) $a_n = 3^{n+2}(2n-6)$

4. Determine whether the following sequences converge or diverge (if necessary, find the general term). If it converges, find the limit.

   a) $\left\{\dfrac{64}{9}, \dfrac{32}{27}, \dfrac{16}{81}, \dfrac{8}{243}, ...\right\}$

   b) $\left\{2, 1+\dfrac{1}{2}, 1+\dfrac{1}{4}, 1+\dfrac{1}{8}, 1+\dfrac{1}{16}, ...\right\}$

   c) $\left\{\dfrac{1}{\sqrt{2}}, \dfrac{\sqrt{2}}{\sqrt{3}}, \dfrac{\sqrt{3}}{2}, \dfrac{2}{\sqrt{5}}, \dfrac{\sqrt{5}}{\sqrt{6}}, ...\right\}$

   d) $\left\{\dfrac{9}{2}, \dfrac{27}{4}, \dfrac{81}{8}, \dfrac{243}{16}, \dfrac{729}{32}, ...\right\}$

5. An amphitheatre has 50 rows of seats with 30 seats in the first row, 32 in the second, 34 in the third, and so on. Find the total number of seats.

6. Telephone poles are being stored in a pile with 25 poles in the first layer, 24 in the second layer, and so on. If there are 12 layers, how many poles does the pile contain?

7. A man gets a job with a salary of $\$28,000$. He is promised a 2400 raise each subsequent year. Find his total earnings for a 10 year period.

8. An architect designs a theatre with 15 seats in the first row, 18 in the second, 21 in the third, and so on. If the seating capacity for the theatre is 870 people, how many rows are there?

ANSWERS

1. number 1
   (a) $\frac{1}{3}.\frac{-4}{4}, \frac{9}{5}, \frac{-16}{6}, \frac{25}{7}, ...$
   (b) $1, 3, 1, 3, 1, ...$
2. number 2
   a) $a_n = 2n + 3$
   b) $a_n = \dfrac{3}{2^{n-1}}$
   c) $a_n = \dfrac{3(5)^{n-1}}{3n - 7}$
   d) $a_n = 384\left(\dfrac{-1}{2}\right)^{n-1}$
   e) $a_n = \dfrac{-2n + 9}{3n + 1}$
   f) $a_n = \dfrac{4(3)^{n-1}}{4n + 3}$
3. number 3
   a) converges to $3/2$
   b) Diverges
   c) converges to 0
   d) converges to 0
   e) converges to 0
   f) converges to $1/2$
   g) Diverges
   h) converges to 3
   i) converges to 0
   j) diverges
   k) diverges
   l) diverges
4. number 4
   a) $a_n = \dfrac{64(\frac{1}{2})^{n-1}}{(9)3^{n-1}}$ and converges to 0
   b) $a_n = 1 + (\frac{1}{2})^{n-1}$ and converges to 1
   c) $a_n = \dfrac{\sqrt{n}}{\sqrt{n+1}}$ and convertes to 1
   d) $a_n = \dfrac{(9)3^{n-1}}{(2)2^{n-1}}$ and diverges
5. 3950 seats
6. 234 poles
7. $\$388,000$
8. 20 rows