201-SH2-AB - Exercises #18 - Marginal Functions

1. The weekly demand of Pulsar DVRs is given by the demand equation

$$p = -0.02x + 300 \qquad (0 \leq x \leq 15\,000)$$

where p denotes the wholesale unit price in dollars and x denotes the quantity demanded. The weekly total cost function associated with manufacturing these recorders is

$$C(x) = 0.000003x^3 - 0.04x^2 + 200x + 70\,000$$

dolars.

   (a) Find the revenue function R and the profit function P.

   (b) Find the marginal cost function C', the marginal revenue function R', and the marginal profit function P'.

   (c) Find the marginal average cost function $\overline{C}'$.

   (d) Compute $C'(3000)$, $R'(3000)$, and $P'(3000)$, and interpret your results.

2. The total weekly cost (in dollars) incurred by Lincoln Records in pressing x compact discs is

$$C(x) = 2000 + 2x - 0.0001x^2 \qquad (0 \leq x \leq 6000)$$

   (a) What is the actual cost incurred in producing the 1001st disc and the 2001st disc?

   (b) What is the marginal cost when $x = 1000 and 2000$?

   (c) Find the average cost function $\overline{C}$ and the marginal average cost function $\overline{C}'$.

3. A division of Ditton Industries manufactures the Futura model microwave oven. The daily cost (in dollars) of producing these microwave ovens is

$$C(x) = 0.0002x^3 - 0.06x^2 + 120x + 5000$$

where x stands for the number of units produced.

   (a) What is the actual cost incurred in manufacturing the 101st oven? The 201st oven? The 301st oven?

   (b) What is the marginal cost when $x = 100, 200 and 300$?

   (c) Find the average cost function $\overline{C}$ and the marginal average cost function $\overline{C}'$.

4. Custom Office makes a line of executive desks. It is estimated that the total cost for making x units of their Senior Executive model is

$$C(x) = 100x + 200\,000$$

dollars/year.

   (a) Find the average cost function $\overline{C}$.

   (b) Find the marginal average cost function $\overline{C}'$.

   (c) $\star$ What happens to $\overline{C}(x)$ when x is very large? Interpret your results.

5. The management of ThermoMaster Company, whose Mexican subsidiary manufactures an indoor-outdoor thermometer, has estimated that the total weekly cost (in dollars) for producing x thermometers is

$$C(x) = 5000 + 2x$$

   (a) Find the average cost function $\overline{C}$.

   (b) Find the marginal average cost function $\overline{C}'$.

   (c) $\star$ What happens to $\overline{C}(x)$ when x is very large? Interpret your results.

6. Williams Commuter Air Service realizes a monthly revenue of

$$R(x) = 8000x - 100x^2$$

   dollars when the price charged per passenger is x dollars.

   (a) Find the marginal revenue R'.

   (b) Compute $R'(39)$, $R'(40)$ and $R'(41)$.

7. The management of Acrosonic plans to market the ElectroStat, an electrostatic speaker system. The marketing department has determined that the demand function for these speakers is

$$p = -0.04x + 800 \qquad (0 \leq x \leq 20\,000)$$

   where p denotes the speaker's unit price (in dollars) and x denotes the quantity demanded.

   (a) Find the revenue function R.

   (b) Find the marginal revenue function R'.

   (c) Compute $R'(5000)$, and interpret your results.

8. Refer to Exercise **??**. Acrosonic's production department estimates that the total cost (in dollars) incurred in manufacturing x ElectroStat speaker systems in the first year of production will be

$$C(x) = 200x + 300\,000$$

   (a) Find the profit function P.

   (b) Find the marginal profit function P'.

   (c) Compute $P'(5000)$ and $P'(8000)$.

9. Lynbrook West, an apartment complex, has 100 two-bedroom units. The monthly profit (in dollars) realized from renting x apartments is

$$P(x) = -10x^2 + 1760x - 50\,000$$

   (a) What is the actual profit realized from renting the 51st unit, assuming that 50 units have already been rented?

   (b) Compute the marginal profit when $x = 50$, and compare your result with that obtained in part (a).

10. The weekly demand for the Pulsar 25 colour LED television is

$$p = 600 - 0.05x \qquad (0 \leq x \leq 12\,000)$$

where p denotes the wholesale unit price in dollars and x denotes the quantity demanded. The weekly total cost unction associated with manufacturing the Pulsar 25 is given by

$$C(x) = 0.000002x^3 - 0.03x^2 + 400x + 80\,000$$

where $C(x)$ denotes the total cost incurred in producing x sets.

(a) Find the revenue function R and the profit function P.

(b) Find the marginal cost function C', the marginal revenue function R', and the marginal profit function P'.

(c) Compute $C'(2000)$, $R'(2000)$, and $P'(2000)$, and interpret your results.

(d) Find the average cost function $\overline{C}$ associated with the total cost function C.

(e) What is the marginal average cost function $\overline{C}'$?

(f) Compute $\overline{C}'(5000)$ and $\overline{C}'(10\,000)$, and interpret your results.

11. Pulsar manufactures a series of 20-in. flat-tube LCD televisions. The quantity x of these sets demanded each week is related to the wholesale unit price p by the equation

$$p = -0.006x + 180$$

The weekly total cost incurred by Pulsar for producing x sets is

$$C(x) = 0.000002x^3 - 0.02x^2 + 120x + 60\,000$$

dollars.

(a) Find the revenue function R and the profit function P.

(b) Find the marginal cost function C', the marginal revenue function R', and the marginal profit function P'.

(c) Compute $C'(2000)$, $R'(2000)$, and $P'(2000)$, and interpret your results.

(d) Find the average cost function $\overline{C}$ associated with the total cost function C.

(e) What is the marginal average cost function $\overline{C}'$?

(f) Compute $\overline{C}'(5000)$ and $\overline{C}'(10\,000)$, and interpret your results.

12. The quantity of Sensitech laser gaming mice demanded each month is related to the unit price by the equation

$$p = \frac{50}{0.01x^2 + 1} \qquad (0 \leq x \leq 20)$$

where p is measured in thousands of dollars and x in units of a thousand.

(a) Find the revenue function R.

(b) Find the marginal revenue function R'.

(c) Compute $R'(2)$, and interpret your results.

13. The total weekly cost in dollars incurred by Herald Media Corp. in producing x DVDs is given by the total cost function

$$C(x) = 2500 + 2.2x \qquad (0 \leq x \leq 8000)$$

(a) What is the marginal cost when $x = 1000 and 2000$?

(b) Find the average cost function $\overline{C}$ and the marginal average cost function $\overline{C}'$.

(c) Using the results from part (b), show that the average cost incurred by Herald in producing a DVD approaches \$2.20/disc when the level of production is high enough.

14. The total daily cost (in dollars) incurred by Delta Electronics in producing x MP3 players is

$$C(x) = 0.0001x^3 - 0.02x^2 + 24x + 2000 \qquad (0 \leq x \leq 500)$$

where x stands for the number of units produced.

(a) What is the actual cost incurred in manufacturing the 301st MP3 player, assuming that the 300th player was manufactured?

(b) What is the marginal cost when $x = 300$?

15. The marketing department of Telecon has determined that the demand for their smartphones obeys the relationship

$$p = -0.02x + 600 \qquad (0 \leq x \leq 30\,000)$$

where p denotes the phone's unit price (in dollars) and x denotes the quantity demanded.

(a) Find the revenue function R.

(b) Find the marginal revenue function R'.

(c) Compute $R'(10\,000)$, and interpret your result.

16. The weekly demand for the LectroCopy photocopying machine is given by the demand equation

$$p = 2000 - 0.04x \qquad (0 \leq x \leq 50\,000)$$

where p denotes the wholesale unit price in dollars and x denotes the quantity demanded. The weekly total cost function for manufacturing these copiers is given by

$$C(x) = 0.000002x^3 - 0.02x + 1000x + 120\,000$$

where $C(x)$ denotes the total cost incurred in producing x units.

(a) Find the revenue function R, the profit function P, and the average cost function $\overline{C}$.

(b) Find the marginal cost function C', the marginal revenue function R', the marginal profit function P', and the marginal average cost function $\overline{C}'$.

(c) Compute $C'(3000)$, $R'(3000)$, and $P'(3000)$.

(d) Compute $\overline{C}'(5000)$ and $\overline{C}'(8000)$, and interpret your results.

17. The Custom Office makes a line of executive desks. It is estimated that the total cost for making x units of the Junior Executive model is

$$C(x) = 80x + 150\,000 \qquad (0 \le x \le 20\,000)$$

dollars/year.

(a) Find the average cost function $\overline{C}$.

(b) Find the marginal average cost function $\overline{C}'$.

(c) $\star$ What happens to $\overline{C}(x)$ when x is very large? Interpret your result.

Answers

1. (a) $R(x) = -0.02x^2 + 300x$ and $P(x) = 0.000003x^3 + 0.02x^2 + 100x - 70\,000$

   (b) $C'(x) = 0.000009x^2 - 0.08x + 200$
   $R'(x) = -0.04x + 300$
   $P'(x) = -0.000009x^2 + 0.04x + 100$

   (c) $C'(x) = 0.000006x - 0.04 - \dfrac{70\,000}{x^2}$

   (d) $C'(3000) = 0.000009(3000)^2 - 0.08(3000) + 200 = 41$. When the level of production is already 3000 recorders, the actual cost of producing one additional recorder is approximately \$41.
   $R'(3000) = -0.04(3000) + 300 = 180$. The actual revenue to be realized from selling the 3001st recorder is approximately \$180.
   $P'(3000) = -0.000009(3000)^2 + 0.04(3000) + 100 = 139$. The actual profit realized from selling the 3001st DVR is approximately \$139.

2. (a) \$1.80 and \$1.60

   (b) \$1.80 and \$1.60

   (c) $\overline{C} = \dfrac{2000}{x} + 2 - 0.0001x$ and $\overline{C}' = -\dfrac{2000}{x^2} - 0.0001$

3. (a) \$114, \$120.10 and \$138.10

   (b) \$114, \$120 and \$138

   (c) $\overline{C} = 0.0002x^2 - 0.06x + 120 + \dfrac{5000}{x}$ and $\overline{C}' = 0.0004x - 0.06 - \dfrac{5000}{x^2}$

4. (a) $\overline{C} = 100 + \dfrac{200\,000}{x}$

   (b) $\overline{C}' = -\dfrac{200\,000}{x^2}$

   (c) $\overline{C}(x)$ approaches \$100 if the production level is very high.

5. (a) $\overline{C} = \dfrac{5000}{x} + 2$

   (b) $\overline{C}' = -\dfrac{5000}{x^2}$

   (c) $\overline{C}(x)$ approaches \$2 if the production level is very high.

6. (a) $R' = 8000 - 200x$

(b) $R'(39) = 200, \quad R'(40) = 0$ and $R'(41) = -200$.

7. (a) $R = -0.04x^2 + 800x$

   (b) $R' = -0.08x + 800$

   (c) $R'(5000) = 400$. The actual revenue to be realized from selling the 5001st speaker is approximately \$400.

8. (a) $P = -0.04x^2 + 600x - 300\,000$

   (b) $P' = -0.08x + 600$

   (c) $P'(5000) = 200$ and $P'(8000) = -40$.

9. (a) \$750

   (b) \$760

10. (a) $R = 600x - 0.05x^2$ and $P = -0.000002x^3 - 0.02x^2 + 200x - 80\,000$

    (b) $C' = 0.000006x^2 - 0.06x + 400, \quad R' = -0.000006x^2 - 0.04x + 200$

    (c) $C'(2000) = 304, \quad R'(2000) = 400, \quad P'(2000) = 96$

    (d) $\overline{C} = 0.000002x^2 - 0.03x + 400 + \dfrac{80\,000}{x}$

    (e) $\overline{C} = 0.000004x - 0.03 - \dfrac{80\,000}{x^2}$

    (f) $\overline{C}'(5000) = -0.0132$ and $\overline{C}'(10\,000) = 0.0092$. The marginal average cost is negative (average cost is decreasing) when 5000 units are produced and positive (average cost is increasing) when 10 000 units are produced.

11. (a) $R = -0.006x^2 + 180x$ and $P = -0.000002x^3 + 0.014x^2 + 60x - 60\,000$

    (b) $C' = 0.000006x^2 - 0.04x + 120, \quad R' = -0.012x + 180$

    (c) $C'(2000) = 64, \quad R'(2000) = 156, \quad P'(2000) = 92$

    (d) $\overline{C} = 0.000002x^2 - 0.02x + 120 + \dfrac{60\,000}{x}$

    (e) $\overline{C} = 0.000004x - 0.02 - \dfrac{60\,000}{x^2}$

    (f) $\overline{C}'(5000) = -0.0024$ and $\overline{C}'(10\,000) = 0.0194$. The marginal average cost is negative (average cost is decreasing) when 5000 units are produced and positive (average cost is increasing) when 10 000 units are produced.

12. (a) $R = \dfrac{50x}{0.01x^2 + 1}$

    (b) $R' = \dfrac{50 - 0.5x^2}{(0.01x^2 + 1)^2}$

    (c) $R'(2) = 44\,380$. When the level of production is 2000 units, the revenue increases at the rate of \$44,380 per additional 1000 units produced

13. (a) \$2.20 and \$2.20

    (b) $\overline{C} = \dfrac{2500}{x} + 2.2$ and $\overline{C}' = -\dfrac{2500}{x^2}$

    (c) $\lim\limits_{x \to \infty} \left(\dfrac{2500}{x} + 2.2 \right) = 2.2$

14. (a) \$39.07

    (b) \$39

15. (a) $R = -0.02x^2 + 600x$

    (b) $R' = -0.04x + 600$

    (c) $R'(10\,000) = 200$. The sale of the 10 001st phone will bring a revenue of \$200.

16. (a) $R = 2000x - 0.04x^2$

    $P = -0.000002x^3 - 0.02x^2 + 1000x - 120\,000$

    $\overline{C} = 0.000002x^2 - 0.02x + 1000 + \dfrac{120\,000}{x}$

    (b) $C' = 0.000006x^2 - 0.04x + 1000$

    $R' = 2000 - 0.08x$

    $P' = -0.000006x^2 - 0.04x + 1000$

    $\overline{C}' = 0.000004x - 0.02 - \dfrac{120000}{x^2}$

    (c) $C'(3000) = 934$, $\quad R'(3000) = 1760$, and $P'(3000) = 826$

    (d) $\overline{C}'(5000) = -0.0048$ and $\overline{C}'(8000) = 0.010125$. At a production level of 5000, the average cost is decreasing by 0.48¢/unit. At a production level of 8000, the average cost is increasing by 1.0125¢/unit.

17. (a) $\overline{C} = 80 + \dfrac{150\,000}{x}$

    (b) $\overline{C}' = -\dfrac{150\,000}{x^2}$

    (c) If the production level is very high, then the unit cost approaches \$80/desk.

Source: Soo T. Tan. *Applied Calculus for the Managerial, Life, and Social Sciences.* 10th Edition