201-SH2-AB - Exercises #6 - Derivatives - Limit Definition, Graphs

1. Use the **limit definition** of the derivative to find **the slope** of the tangent line to the graph of the function at the given point.

(a) * $f(x) = 2$ at $x = 3$

(b) * $f(x) = x + 4$ at $x = -2$

(c) * $f(x) = 5x - 7$ at $x = -3$

(d) * $f(x) = 6 - 8x$ at $x = 1$

(e) $f(x) = 6 + x - 3x^2$ at $x = 2$

(f) $f(x) = 7x^2 - 6x + 3$ at $x = -1$

2. Use the **limit definition** of the derivative to find **an equation** of the tangent line to the graph of the function at the given point.

(a) * $f(x) = 5$ at $x = 2$

(b) * $f(x) = 6 - 9x$ at $x = 3$

(c) $f(x) = 5x^2 + 6$ at $x = -3$

(d) $f(x) = 4 - 2x - x^2$ at $x = 1$

(e) $f(x) = 2x^2 + 4x + 1$ at $x = -2$

(f) $f(x) = -4x^2 + 5x + 6$ at $x = -1$

(g) $f(x) = \dfrac{3}{2 - x}$ at $x = 1$

(h) $f(x) = \dfrac{4}{x + 1}$ at $x = 3$

(i) $f(x) = 6\sqrt{x + 2}$ at $x = 2$

(j) $f(x) = \dfrac{2}{x^2 - 3}$ at $x = -2$

(k) $f(x) = \sqrt{x^3 - 4}$ at $x = 2$

3. Use the **limit definition** of the derivative to find **the derivative** of the function at any x-value (find $f'(x)$).

(a) *$f(x) = -4$

(b) *$f(x) = x + 2$

(c) *$f(x) = 9x + 4$

(d) *$f(x) = 3 - 5x$

(e) *$f(x) = 5x^2$

(f) $f(x) = 2x^2 - 7x - 4$

(g) $f(x) = 2x^2 + 4x$

(h) $f(x) = \sqrt{5 - x}$

(i) $f(x) = 3x - 2x^2$

(j) $f(x) = \dfrac{1}{3 - 2x}$

(k) $f(x) = \sqrt{3 - 2x}$

(l) $f(x) = \sqrt{2x - 4}$

(m) $f(x) = 5x^2 - 3x$

(n) $f(x) = \sqrt{5 - 3x}$

(o) $f(x) = (x - 4)^2$

(p) $f(x) = \sqrt{2x - 1}$

(q) $f(x) = \dfrac{3}{2x - 1}$

(r) $f(x) = \dfrac{5}{x + 1}$

(s) $f(x) = 3x - x^2$

(t) $f(x) = \sqrt{5x - 2}$

4. Given the following graph,

   (a) Find the value of $f'(2)$

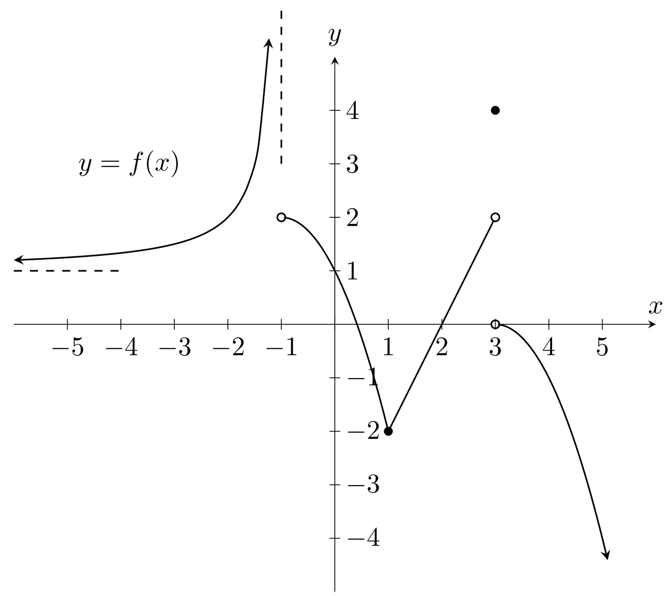
   (b) Is $f'(-2)$ positive or negative?

   (c) Is $f'(4)$ positive or negative?

   (d) *Give the interval(s) where $f'(x)$ is negative.

   (e) Locate the x-value(s) where f is <u>not</u> differentiable.

   (f) Locate the x-value(s) where f is continuous but <u>not</u> differentiable.



5. Given the following graph,

   (a) Locate the x-value(s) where $f'(x) = 0$.

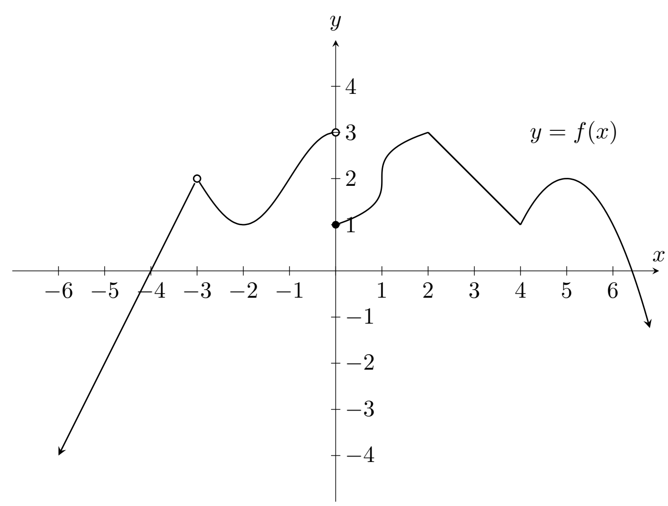
   (b) Locate the x-value(s) where f is <u>not</u> differentiable.

   (c) Locate the x-value(s) where f is continuous but <u>not</u> differentiable.

   (d) Find the value of $f'(-4)$

   (e) Find the value of $\lim_{h\to 0} \dfrac{f(3+h)-f(3)}{h}$

   (f) *Give the interval(s) where $f'(x)$ is positive.



6. Given the following graph,

(a) Locate the x-value(s) where $f'(x) = 0$.

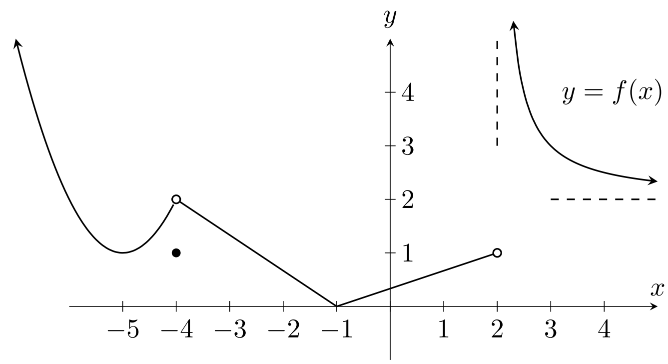
(b) Find the value of $f'(-3)$

(c) Find the value of $\lim_{h \to 0} \dfrac{f(1+h) - f(1)}{h}$

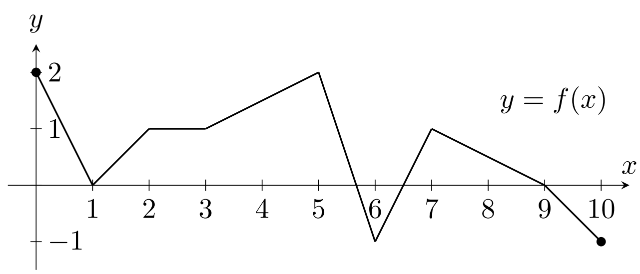
(d) *Give the interval(s) where $f'(x)$ is negative.

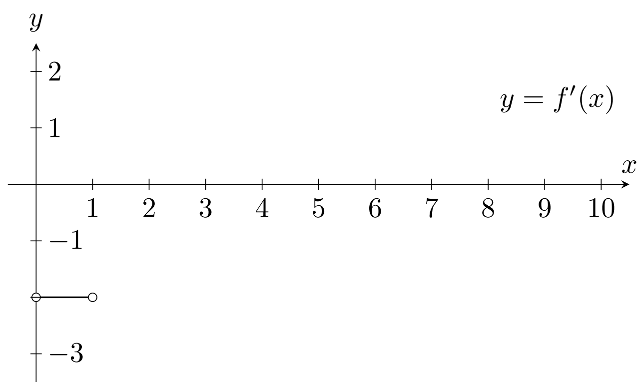
(e) Locate the x-value(s) where f is not differentiable.

(f) Locate the x-value(s) where f is continuous but not differentiable.



7. *Given the graph of f, sketch the graph of its derivative below.





Answers

1. (a) $f'(3) = 0$
   (b) $f'(-2) = 1$
   (c) $f'(-3) = 5$
   (d) $f'(1) = -8$
   (e) $f'(2) = -11$
   (f) $f'(-1) = -20$

2. (a) $y = 5$
   (b) $y = -9x + 6$
   (c) $y = -30x - 39$
   (d) $y = -4x + 5$
   (e) $y = -4x - 7$
   (f) $y = 13x + 10$
   (g) $y = 3x$
   (h) $y = \dfrac{-1}{4}x + \dfrac{7}{4}$
   (i) $y = \dfrac{3}{2}x + 9$
   (j) $y = 8x + 18$
   (k) $y = 3x - 4$

3. (a) $f'(x) = 0$
   (b) $f'(x) = 1$
   (c) $f'(x) = 9$
   (d) $f'(x) = -5$
   (e) $f'(x) = 10x$
   (f) $f'(x) = 4x - 7$
   (g) $f'(x) = 4x + 4$
   (h) $f'(x) = \dfrac{-1}{2\sqrt{5-x}}$
   (i) $f'(x) = 3 - 4x$
   (j) $f'(x) = \dfrac{2}{(3-2x)^2}$
   (k) $f'(x) = \dfrac{-1}{\sqrt{3-2x}}$
   (l) $f'(x) = \dfrac{1}{\sqrt{2x-4}}$
   (m) $f'(x) = 10x - 3$
   (n) $f'(x) = \dfrac{-3}{2\sqrt{5-3x}}$
   (o) $f'(x) = 2(x-4)$
   (p) $f'(x) = \dfrac{1}{\sqrt{2x-1}}$
   (q) $f'(x) = \dfrac{-6}{(2x-1)^2}$
   (r) $f'(x) = \dfrac{-5}{(x+1)^2}$
   (s) $f'(x) = 3 - 2x$
   (t) $f'(x) = \dfrac{-5}{2\sqrt{5x-2}}$

4. (a) 2
   (b) positive
   (c) negative
   (d) $(-1, 1) \cup (3, \infty)$
   (e) $x = -1, 1, 3$
   (f) $x = 1$

5. (a) $x = -2, 5$
   (b) $x = -3, 0, 1, 2, 4$
   (c) $x = 1, 2, 4$
   (d) 2
   (e) -1
   (f) $(-\infty, -3) \cup (-2, 0) \cup (0, 1) \cup (1, 2) \cup (4, 5)$

6. (a) $x = -5$
   (b) $-2/3$
   (c) $1/3$
   (d) $(-\infty, -5) \cup (-4, -1) \cup (2, \infty)$
   (e) $x = -4, -1, 2$
   (f) $x = -1$

7.