

## General Information.

*Discipline:* Mathematics

*Course code:* 201-SH4-AB

*Ponderation:* 2-2-2      *Credits:* 2 $\frac{2}{3}$

*Prerequisite:* 201-SH2-AB

*Objective:*

- OPU4: Analyze problems studied in the social sciences by using linear algebra and vector geometry.

Your teacher will give you their schedule and availability.  
*Students are strongly advised to seek help from their instructor as soon as they encounter difficulties in the course.*

**Introduction.** Linear Algebra is the third Mathematics course in the Social Science Program. It is generally taken in the third semester. Linear Algebra introduces the student to matrices and vector spaces with applications to Business, Commerce and Computer Science.

The primary purpose of the course is the attainment of Objective OPU4 ("Analyze problems studied in the social sciences by using linear algebra and vector geometry."). To achieve this goal, the course must help the student understand the following basic concepts: systems of linear equations, matrices, determinants, two-dimensional vectors from both an algebraic and a geometric perspective, three-dimensional vectors from an algebraic perspective, and business applications which incorporate these concepts.

Emphasis is placed on clarity in the presentation of concepts and on problem solving. The students will learn to solve various problems using tools available in linear algebra. Some abstract work is required but the emphasis is on problem solving and applications including contexts related to the field of Social Science such as production problems (systems of linear equations and linear combinations), Leontief Input-Output Model (systems of linear equations and the inverse of a matrix) and the optimization of (economic) functions (Linear Programming and the Simplex method). In this way, the basic concepts are illustrated by applying them to various problems where their application helps arrive at a solution. Consequently, the course encourages the student to apply learning in one context to problems arising in another.

**Reference Text.** The textbook for this course, which is optional, is *Elementary Linear Algebra, Tenth Edition Abridged*, by Rorres & Anton - (Wiley); it is available from the college bookstore for about \$50. Please consult with your teacher before buying the textbook.

**Calculator Policy.** Only calculators which have previously been inspected and approved via sticker by the instructor will be permitted for use on quizzes, tests or the final examination. The only calculators that will be approved begin with the model number **SHARP EL-531**. An acceptable calculator model is available for purchase at the bookstore.

## Course Costs.

The approved model of scientific calculator costs about \$25.

**Teaching Methods.** This course will be 60 hours, meeting three times a week for a total of 4 hours a week. Most teachers of this course rely mainly on the lecture method, although most also employ at least one of the following techniques as well: question-and-answer sessions, labs, problem solving periods and class discussions. Generally, each class session starts with a question period of previous topics, then new material is introduced, followed by worked examples. No marks are deducted for absenteeism (however, see below). Failure to keep pace with the lectures results in a cumulative inability to cope with the material, and a possible failure in the course. A student will generally succeed or fail depending on how many problems have been attempted and solved successfully. It is entirely the student's responsibility to complete suggested homework assignments as soon as possible following the lecture, as the material will be fresher in his/her mind. This also allows the student the maximum benefit from any

discussion of the homework (which usually occurs in the following class). The answers to a selected number of problems can be found in the back of the text. Each teacher will provide supplementary notes and problems as he/she sees fit.

**Evaluation Plan.** The Final Grade is a combination of the Class Mark and the mark on the Final Exam. The Class Mark will include results from three or more tests (worth at least 75% of the Class Mark), homework, quizzes, or other assignments. The specifics of the Class Mark will be given by each instructor during the first week of classes in an appendix to this outline. Every effort is made to ensure equivalence between the various sections of this course. The Final Exam is set by the Course Committee (which consists of all instructors currently teaching this course), and is marked by each individual instructor.

The Final Grade will be the better of:

50% Class Mark and 50% Final Exam Mark

or

25% Class Mark and 75% Final Exam Mark

A student *choosing not to write* the Final Exam will receive a failing grade of 50% or their Class Mark, whichever is less. **Students must be available until the end of the final examination period to write exams.**

## Other Resources.

*Math Website.*

<http://departments.johnabbott.qc.ca/departments/mathematics>

*Math Study Area.* Located in H-200A and H-200B; the common area is usually open from 8:30 to 17:30 on weekdays as a quiet study space. Computers and printers are available for math-related assignments. It is also possible to borrow course materials when the attendant is present.

*Math Help Centre.* Located in H-216; teachers are on duty from 8:30 until 15:30 to give math help on a drop-in basis.

*Peer Tutoring.* Starting on the fifth week of each semester, first year students can be paired with a fellow finishing student for a weekly appointment of tutoring. Ask your teacher for details.

*Academic Success Centre.* The Academic Success Centre, located in H-139, offers study skills workshops and individual tutoring.

## College Policies.

*Policy No. 7 - IPESA, Institutional Policy on the Evaluation of Student Achievement:* <https://www.johnabbott.qc.ca/wp-content/uploads/2021/05/Policy-No.-7-IPESA-FINAL.pdf>.

*Religious Holidays (Article 3.2.13 and 4.1.6).* Students who wish to miss classes in order to observe religious holidays must inform their teacher of their intent in writing within the first two weeks of the semester.

*Student Rights and Responsibilities: (Article 3.2.18).* It is the responsibility of students to keep all assessed material returned to them and/or all digital work submitted to the teacher in the event of a grade review. (The deadline for a Grade Review is 4 weeks after the start of the next regular semester.)

*Student Rights and Responsibilities: (Article 3.3.6).* Students have the right to receive graded evaluations, for regular day division courses, within two weeks after the due date or exam/test date, except in extenuating circumstances. A maximum of three (3) weeks may apply in certain circumstances (ex. major essays) if approved by the department and stated on the course outline. For evaluations at the end of the semester/course, the results must be given to the student by the grade submission deadline (see current Academic Calendar). For intensive courses (i.e.: intersession, abridged courses) and AEC courses, timely feedback must be adjusted accordingly.

*Academic Procedure: Academic Integrity, Cheating and Plagiarism (Article 9.1 and 9.2).* Cheating and plagiarism are unacceptable at John Abbott College. They represent infractions against academic integrity. Students are expected to conduct themselves accordingly and must be responsible for all of their actions.

*College definition of Cheating:* Cheating means any dishonest or deceptive practice relative to examinations, tests, quizzes, lab assignments, research papers or other forms of evaluation tasks. Cheating includes, but is not restricted to, making use of or being in possession of unauthorized material or devices and/or obtaining or providing unauthorized assistance in writing examinations, papers or any other evaluation task and submitting the same work in more than one course without the teacher's permission. It is incumbent upon the department through the teacher to ensure students are forewarned about unauthorized material, devices or practices that are not permitted.

*College definition of Plagiarism:* Plagiarism is a form of cheating. It includes copying or paraphrasing (expressing the ideas of someone else in one's own words), of another person's work or the use of another person's

work or ideas without acknowledgement of its source. Plagiarism can be from any source including books, magazines, electronic or photographic media or another student's paper or work.

**Course Content.** The exercises listed should help you practice and learn the material taught in this course; they form a good basis for homework. Your teacher may supplement this list during the semester. Regular work done as the course progresses should make it easier for you to master the course.

- Exercises #1 - Linear Systems
- Exercises #2 - Matrix Operations
- Exercises #3 - Determinants
- Exercises #4 - Vectors
- Exercises #5 - Lines and Planes
- Exercises #6 - Markov Chains
- Exercises #7 - Leontief Models
- Exercises #8 - Linear Programming
- Exercises #9 – Cryptography

OBJECTIVES	STANDARDS
<p><b>Statement of the competency</b></p> <p>Analyze problems studied in the social sciences by using linear algebra and vector geometry. (OPU4)</p>	<p><b>General Performance Criteria</b></p> <ul style="list-style-type: none"> <li>• Accurate recognition of the characteristics of the matrices pertaining to current human realities.</li> <li>• Correct use of appropriate matrix operations.</li> <li>• Appropriate use of matrices to represent a current human reality.</li> <li>• Appropriate use of a system of linear equations to represent a current human reality</li> <li>• Correct calculation of <math>2 \times 2</math> and <math>3 \times 3</math> determinants</li> <li>• Relevant application of the methods for solving systems of linear equations</li> <li>• Correct application of Leontief's method to open and closed economy models</li> <li>• Accurate recognition of the characteristics of vectors, lines, and planes</li> <li>• Appropriate application of vector operations and products</li> <li>• Establishment of relevant relationships between the geometric loci and constraints specific to a current human reality</li> <li>• Appropriate use of a system of linear inequalities to represent a current human reality</li> <li>• Correct solution of systems of linear inequalities</li> <li>• Complete solution of a linear programming problem specific to a current human reality</li> <li>• Correct application of the simplex method</li> <li>• Appropriate use of Markov chains to represent a current human reality</li> <li>• Appropriate representation of a transition diagram</li> <li>• Accurate interpretation of how a current human reality manifests itself over the long term</li> </ul>
<p><b>Elements of the Competency</b></p> <ol style="list-style-type: none"> <li>1. To use matrices to apply mathematical models to current human realities.</li> <li>2. To use matrices to solve an economic production problem.</li> <li>3. To apply methods of linear algebra and vector geometry to model situations that are specific to current human realities and that involve two or more variables.</li> <li>4. To solve optimization problems using methods of solving systems of linear inequalities.</li> <li>5. To solve problems specific to current human realities by using Markov processes.</li> </ol>	<p><b>Specific Performance Criteria</b></p> <p><i>[Specific performance criteria for each of these elements of the competency are shown below with the corresponding intermediate learning objectives. For the items in the list of learning objectives, it is understood that each is preceded by: "The student is expected to ...". ]</i></p>

Specific Performance Criteria	Intermediate Learning Objectives
<p>1. <i>The Development of Linear Algebra</i></p> <p>1.1 The history of Linear Algebra</p> <p>2. <i>Systems of Linear Equations</i></p> <p>2.1 Use of an augmented matrix and row operations to solve systems of linear equations</p> <p>3. <i>Vectors</i></p> <p>3.1 Defining a vector geometrically.</p> <p>3.2 Performing operations on vectors.</p> <p>3.3 Use the concept of linear combinations.</p> <p>4. <i>Applications of Linear Equations</i></p> <p>4.1 Use of linear systems of equations to solve applied problems.</p> <p>5. <i>Matrices and Inverses</i></p> <p>5.1 Performing operations on matrices.</p> <p>5.2 Performing operations on matrices involving the inverse of a matrix</p> <p>6. <i>Determinants</i></p> <p>6.1 Use determinant properties to evaluate the determinant of a square matrix.</p> <p>6.2 Use of determinants to find the inverse of a <math>2 \times 2</math> and <math>3 \times 3</math> matrix and to solve systems of linear equations.</p>	<p>1.1.1. Place linear algebra in a historical context by an investigation made by Gauss.</p> <p>1.1.2. Examine the historical context of linear algebra applications in today's society.</p> <p>2.1.1. Write the augmented matrix for a system of linear equations.</p> <p>2.1.2. Define elementary row operations.</p> <p>2.1.3. Solve systems of linear equations using Gaussian elimination and Gauss–Jordan Elimination (inconsistent systems or consistent systems with one or infinitely many solutions).</p> <p>2.1.4. Understand the solutions to systems of equations as the intersection of planes and lines.</p> <p>2.1.5. Determine whether the solution/intersection represents no intersection, a point, a line, a plane, or none of these.</p> <p>2.1.6. Determine consistency conditions related to the solution of systems of linear equations.</p> <p>3.1.1. State the geometric definition of a vector in <math>\mathbb{R}^2</math> and <math>\mathbb{R}^3</math>.</p> <p>3.1.2. Define the equality of two vectors algebraically and geometrically.</p> <p>3.2.1. Define vector operations (addition, subtraction, scalar multiplication, dot product, and cross product).</p> <p>3.2.2. State the properties of those operations both algebraically and geometrically.</p> <p>3.2.3. Calculate a vector that is a result of a series of vector operations both algebraically and geometrically.</p> <p>3.2.4. Find the magnitude of a vector.</p> <p>3.2.5. Normalize a vector.</p> <p>3.2.6. Determine whether or not two vectors are (i) parallel and (ii) perpendicular.</p> <p>3.2.7. Determine whether or not a set of vectors forms an orthogonal set.</p> <p>3.2.8. Find a vector that is orthogonal to two other vectors.</p> <p>3.3.1. State the definition of a linear combination of vectors and determine whether a given vector is a linear combination of a set of vectors.</p> <p>4.1.1. Set up and solve a system of linear equations in a variety of related problems. For example, – set up and solve a system of equations related to production problems.</p> <p>5.1.1. Give the definition of a matrix.</p> <p>5.1.2. Determine whether two matrices are equal.</p> <p>5.1.3. Define matrix operations (addition, subtraction, scalar multiplication, matrix multiplication, transpose of a matrix).</p> <p>5.1.4. State and demonstrate properties of those operations.</p> <p>5.1.5. Calculate a matrix that is the result of a series of matrix operations.</p> <p>5.1.6. Find a matrix by solving a matrix equation.</p> <p>5.1.7. Identify a square matrix, a zero matrix, an identity matrix, and the inverse of a matrix.</p> <p>5.2.1. Find the inverse of a matrix by row reduction.</p> <p>5.2.2. State the properties of the inverse of a matrix.</p> <p>5.2.3. Use matrix inverse and transpose properties to solve matrix equations.</p> <p>5.2.4. Write a system of equations as a matrix equation.</p> <p>5.2.5. Solve systems of equations by finding the inverse of the coefficient matrix.</p> <p>5.2.6. Relate the existence of the inverse of a matrix to the reduced form of <math>A</math>, and the solutions to the systems <math>A\vec{x} = \vec{b}</math> and <math>A\vec{x} = \vec{0}</math>.</p> <p>6.1.1. Define the determinant of a square matrix for <math>2 \times 2</math> and <math>3 \times 3</math> matrices.</p> <p>6.1.2. State the properties of the determinant.</p> <p>6.2.1. Use the determinant to determine whether a matrix has an inverse.</p> <p>6.2.2. Find the adjoint of a matrix.</p> <p>6.2.3. Find the inverse of a matrix using the adjoint and the determinant of a matrix.</p>

Specific Performance Criteria	Intermediate Learning Objectives
<p><i>7. Lines and Planes</i></p> <p>7.1 Determination of equations of lines and planes.</p> <p>7.2 Use of Cartesian axis systems to sketch lines.</p> <p>7.3 Determination of the intersection of two or more planes.</p> <p><i>8. Markov Chains</i></p> <p>8.1 Transition Diagrams</p> <p>8.2 Transition Matrices</p> <p>8.3 Steady State Matrices and Vectors</p> <p><i>9. Leontief Input-Output Model</i></p> <p>9.1 Solve economics problems using the Leontief Input-Output Model</p> <p><i>10. Optimization</i></p> <p>10.1 Solution of problems using the Simplex method.</p> <p>10.2 Use of Linear Programming to solve concrete problems.</p>	<p>7.1.1. Find the equation of a line in <math>\mathbb{R}^2</math> in (i) standard form, (ii) parametric form, (iii) vector form, and (iv) point-normal form.</p> <p>7.1.2. Find the equation of a plane in <math>\mathbb{R}^3</math> in (i) standard form, (ii) parametric form, (iii) vector form, and (iv) point-normal form.</p> <p>7.1.3. Determine whether or not two planes are (i) parallel or (ii) perpendicular.</p> <p>7.2.1. Plot points, lines, and vectors in the Cartesian plane for <math>\mathbb{R}^2</math>.</p> <p>7.3.1. Use an augmented matrix to determine the intersection of two or more planes.</p> <p>8.1.1. Apply the transition diagram to real world situations.</p> <p>8.2.1. Define and identify Markov (stochastic) matrices.</p> <p>8.2.2. Interpret transition matrices to determine probabilities of events.</p> <p>8.3.1. Define and identify regular matrices.</p> <p>8.3.2. Calculate steady state matrices and vectors corresponding to regular transition matrices.</p> <p>9.1.1. Set up a system of equations related to the Leontief Input-Output Model for both Closed and Open Economies.</p> <p>9.1.2. Use the inverse method or Gaussian Elimination to solve the system.</p> <p>10.1.1. Find the maximum (minimum) of a linear function using the Simplex algorithm.</p> <p>10.1.2. Determine whether or not a maximum (minimum) may be obtained for a given specific linear function and a list of inequalities.</p> <p>10.1.3. Relate the graphical method of optimization to the Simplex Method.</p> <p>10.2.1. Use linear programming to solve problems in minimizing production cost and/or maximizing revenue or profit and interpret the meaning of the slack variables.</p>