General Information.
Discipline: Mathematics *Course code*: 201-105-RE
Ponderation: 3-2-3 *Credits*: $2\frac{2}{3}$ *Prerequisite*: 201-103-RE
Objectives:

- 022Z: To apply methods of linear algebra and vector geometry to the study of various phenomena of human activity
- 022R: To thoroughly analyze a human phenomenon
- 022S: To apply concepts related to Social Science disciplines to the understanding of the human phenomena in concrete situations

Students are strongly advised to seek help from their instructor as soon as they encounter difficulties in the course.

Introduction. Linear Algebra is the third Mathematics course in the Social Science Program. It is generally taken in the third semester. Linear Algebra introduces the student to matrices and vector spaces with applications to Business, Commerce and Computer Science.

The primary purpose of the course is the attainment of Objectives 022Z, 022R, 022S ("To apply methods of linear algebra and vector geometry to the study of various phenomena of human activity. To thoroughly analyze a human phenomenon. To apply concepts related to Social Science disciplines to the understanding of the human phenomena in concrete situations"). To achieve this goal, the course must help the student understand the following basic concepts: systems of linear equations, matrices, determinants, two-dimensional and three-dimensional vectors from both an algebraic and a geometric perspective, n-dimensional vector spaces, and business applications which incorporate these concepts.

Emphasis is placed on clarity in the presentation of concepts and on problem solving. The students will learn to solve various problems using tools available in linear algebra. Some abstract work is required but the emphasis is on problem solving and applications including contexts related to the field of Social Science such as production problems (systems of linear equations and linear combinations), Leontief Input-Output Model (systems of linear equations and the inverse of a matrix) and the optimization of (economic) functions. In this way, the basic concepts are illustrated by applying them to various problems where their application helps arrive at a solution. Consequently, the course encourages the student to apply learning in one context to problems arising in another.

Only calculators which have previously been inspected and approved via sticker by the instructor will be permitted for use on quizzes, tests or the final examination. The only calculators that will be approved begin with the model number **SHARP EL-531**. An acceptable calculator model is available for purchase at the bookstore.

Students will have access to computers where suitable mathematical software programs, including MAPLE, are available for student use. The course uses a standard college level Linear Algebra textbook, chosen by the Linear Algebra course committee.

Textbook. Your teacher may require

> *Elementary Linear Algebra, Twelfth Edition,*
> by Rorres & Anton - (***custom e-book***)

You can purchase the textbook for about $45 here:

> https://jac.bookware3000.ca/Item?item=9781119855415

Course Costs.
In addition to the cost of the text listed above. A scientific calculator (about $25) is necessary; see above for the list of permitted calculators.

Teaching Methods. This course will be 75 hours, meeting three times a week for a total of 5 hours a week. Most teachers of this course rely mainly on the lecture method, although most also employ at least one of the following techniques as well: question-and-answer sessions, labs, problem solving periods and class discussions. Generally, each class session starts with a question period of previous topics, then new material is introduced, followed by worked examples. No marks are deducted for absenteeism (however, see below). Failure to keep pace with the lectures results in a cumulative inability to cope with the material, and a possible failure in the course. A student will generally succeed or fail depending on how many problems have been attempted and solved successfully. It is entirely the student's responsibility to complete suggested homework assignments as soon as possible following the lecture, as the material will be fresher in his/her mind. This also allows the student the maximum benefit from any discussion of the homework (which usually occurs in the following class). The answers to a selected number of problems can be found in the back of the text. Each teacher will provide supplementary notes and problems as he/she sees fit.

Evaluation Plan. The Final Evaluation in this course consists of the Final Exam, which covers all elements of the competency. The Final Grade is a combination of the Class Mark and the mark on the Final Exam. The Class Mark will include results from three or more in-class written tests (worth at least 75% of the Class Mark), homework, quizzes, or other assignments. The specifics of the Class Mark will be given by each instructor during the first week of classes in an appendix to this outline. Every effort is made to ensure equivalence between the various sections of this course. The Final Exam is set by the Course Committee (which consists of all instructors currently teaching this course), and is marked by each individual instructor.

The Final Grade will be the better of:

50% Class Mark and 50% Final Exam Mark
or
25% Class Mark and 75% Final Exam Mark

A student *choosing not to write* the Final Exam will receive a failing grade of 50% or their Class Mark, whichever is less.

Students must be available until the end of the final examination period to write exams.

Note that in the event of unexpected changes to the academic calendar, the evaluation plan may be modified.

Other Resources.
Math Website.
http://departments.johnabbott.qc.ca/departments/mathematics

Math Study Area. Located in H-200A and H-200B; the common area is usually open from 8:30 to 17:30 on weekdays as a quiet study space. Computers and printers are available for math-related assignments. It is also possible to borrow course materials when the attendant is present.

Math Help Centre. Located in H-216; teachers are on duty from 8:30 until 15:30 to give math help on a drop-in basis.

Academic Success Centre. The Academic Success Centre, located in H-139, offers study skills workshops and individual tutoring.

College Policies.
Policy No. 7 - IPESA, Institutional Policy on the Evaluation of Student Achievement: https://www.johnabbott.qc.ca/wp-content/uploads/2021/05/Policy-No.-7-IPESA-FINAL.pdf.

Religious Holidays (Article 3.2.13 and 4.1.6). Students who wish to miss classes in order to observe religious holidays must inform their teacher of their intent in writing within the first two weeks of the semester.

Student Rights and Responsibilities: (Article 3.2.18). It is the responsibility of students to keep all assessed material returned to them and/or all digital work submitted to the teacher in the event of a grade review. (The deadline for a Grade Review is 4 weeks after the start of the next regular semester.)

Student Rights and Responsibilities: (Article 3.3.6). Students have the right to receive graded evaluations, for regular day division courses, within two weeks after the due date or exam/test date, except in extenuating circumstances. A maximum of three (3) weeks may apply in certain circumstances (ex. major essays) if approved by the department and stated on the course outline. For evaluations at the end of the semester/course, the results must be given to the student by the grade submission deadline (see current Academic Calendar). For intensive courses (i.e.: intersession, abridged courses) and AEC courses, timely feedback must be adjusted accordingly.

Academic Procedure: Academic Integrity, Cheating and Plagiarism (Article 9.1 and 9.2). Cheating and plagiarism are unacceptable at John Abbott College. They represent infractions against academic integrity. Students are expected to conduct themselves accordingly and must be responsible for all of their actions.

College definition of Cheating: Cheating means any dishonest or deceptive practice relative to examinations, tests, quizzes, lab assignments, research papers or other forms of evaluation tasks. Cheating includes, but is not restricted to, making use of or being in possession of unauthorized material or devices and/or obtaining or providing unauthorized assistance in writing examinations, papers or any other evaluation task and submitting the same work in more than one course without the teacher's permission. It is incumbent upon the department through the teacher to ensure students are forewarned about unauthorized material, devices or practices that are not permitted.

College definition of Plagiarism: Plagiarism is a form of cheating. It includes copying or paraphrasing (expressing the ideas of someone else in one's own words), of another person's work or the use of another person's work or ideas without acknowledgement of its source. Plagiarism can be from any source including books, magazines, electronic or photographic media or another student's paper or work.

Course Content (with selected exercises)**.** The exercises listed should help you practice and learn the material taught in this course; they form a good basis for homework. Your teacher may supplement this list during the semester. Regular work done as the course progresses should make it easier for you to master the course.

Systems of Linear Equations and Matrices.

- 1.1 Introduction: #1 to 10, 13, 17, 18
- 1.2 Gaussian Elimination: #5 to 8, 13 to 30, 32
- Ⓢ Supplement: 105A.pdf
- 1.3 Matrices and Matrix Operations: #1, 3 & 5(a to h), 6, 11 to 15 odd, 23, 24, 26, 30 to 32
- 1.4 Inverses; Properties of Matrices: #1 to 8, 10 to 19, 39, 40, 43, 46a, 50
- 1.5 A Method for Finding A^{-1}: #9 to 18 odd, 19 to 22
- 1.6 More on Invertible Matrices: #1 to 19 odd
- 1.7 Diagonal, Triangular, and Symmetric Matrices: #1, 2, 7, 9, 17 to 22, 25, 26, 35
- Ⓢ Supplement: 105B.pdf #1-10

Determinants.

- 2.1 Determinants by Cofactor Expansion: #1 to 31 odd
- 2.2 Evaluating Determinants by Row Reduction: #1, 3, 9 to 21 odd, 24, 25 to 29 odd
- 2.3 Properties of Determinants; Cramer's Rule: #1 to 35 odd
- Ⓢ Supplement: 105B.pdf #11-23

Euclidean Vector Spaces.

- 3.1 Vectors in 2-Space, 3-Space, and n-Space: #1 to 5, 7 to 13 odd, 17 to 23 odd, 24
- 3.2 Norm, Dot Product, and Distance in $\mathbb{R}^n$: #1 to 9 odd
- 3.3 Orthogonality: #1 to 11 odd, 29, 31
- 3.4 The Geometry of Linear Systems: #1 to 13 odd
- 3.5 Cross Product #1, 7, 8
- Ⓢ Supplement: 105C.pdf

General Vector Spaces.

- 4.2 Subspaces: #1, 2, 18, 19
- 4.3 Spanning Sets: #1, 2, 7, 8
- 4.4 Linear Independence: #1 to 3, 7 to 10, 12
- 4.5 Coordinates and Basis: #1, 2, 7
- 4.6 Dimension: #1 to 7
- 4.8 Row, Column and Null Space: #3 to 9, 11, 14 to 17, 30 (omit row space questions)
- 4.9 Rank and Nullity: #1 to 10, 28, 29
- Ⓢ Supplement: 105D.pdf

Applications in Economics.

- 1.11 Leontief Open Model: #1 to 7
- 2.6 Linear Programming: Simplex Algorithm #31 to 42
  Chapter found on LEA
- 2.7 Simplex Algorithm: Additional Considerations (Optional) #11 to 16, 25 to 28
  Chapter found on LEA
- 4.7 Generalized Simplex Algorithm #1 to 6
  Chapter found on LEA
- 10.4 Markov Chains: #1 to 4, 7, 8
- 10.13 Cryptography #1 to 3
- Ⓢ Supplement: 105E.pdf

OBJECTIVES	STANDARDS
Statement of the competency To apply methods of linear algebra and vector geometry to the study of various phenomena of human activity (022Z). To thoroughly analyze a human phenomenon (022R). To apply concepts related to Social Science disciplines to the understanding of the human phenomena in concrete situations (022S)	**General Performance Criteria** • Basic knowledge of the historical context of the development of linear algebra and vector geometry. • Appropriate use of concepts • Satisfactory representation of situations using matrices, vectors, and systems of equations and inequalities. • Satisfactory graph linear systems. • Algebraic operations in conformity with rules • Correct selection and application of methods of solving systems of linear equations. • Correct application of algorithms. • Accuracy of calculations • Explanation of steps in the problem solving procedure. • Correct interpretation of results • Use of appropriate terminology
Elements of the Competency 1. To place the development of linear algebra and vector geometry in historical context. 2. To apply different methods of solving systems of linear equations. 3. To use matrices to solve concrete problems. 4. To use vector/matrix operations to solve concrete problems. 5. To establish connections between vector geometry and linear algebra. 6. To apply the methods of linear algebra and vector geometry to the study of line and plane geometry. 7. To solve optimization problems using methods of solving systems of linear inequalities with two or more variables.	**Specific Performance Criteria** *[Specific performance criteria for each of these elements of the competency are shown below with the corresponding intermediate learning objectives. For the items in the list of learning objectives, it is understood that each is preceded by: "The student is expected to . . . ".]*

Specific Performance Criteria	Intermediate Learning Objectives
1. *The Development of Linear Algebra*	
1.1 The history of Linear Algebra	1.1.1. Place linear algebra in a historical context by an investigation made by Gauss. 1.1.2. Examine the historical context of linear algebra applications in today's society.
2. *Systems of Linear Equations*	
2.1 Use of an augmented matrix and row operations to solve systems of linear equations	2.1.1. Write the augmented matrix for a system of linear equations. 2.1.2. Define elementary row operations. 2.1.3. Solve systems of linear equations using Gaussian elimination and Gauss–Jordan Elimination (inconsistent systems or consistent systems with one or infinitely many solutions). 2.1.4. Determine consistency conditions related to the solution of systems of linear equations.
3. *Applications of Linear Equations*	
3.1 Use of linear systems of equations to solve applied problems.	3.1.1. Set up and solve a system of linear equations in a variety of related problems. For example, – set up and solve a system of equations related to production problems.
4. *Matrices, Inverses and Determinants*	
4.1 Performing operations on matrices.	4.1.1. Give the definition of a matrix. 4.1.2. Determine whether two matrices are equal. 4.1.3. Define matrix operations (addition, subtraction, scalar multiplication, matrix multiplication, transpose of a matrix). 4.1.4. State and demonstrate properties of those operations. 4.1.5. Calculate a matrix that is the result of a series of matrix operations. 4.1.6. Find a matrix by solving a matrix equation. 4.1.7. Identify a square matrix, a zero matrix, an identity matrix, the inverse of a matrix, a diagonal matrix, a symmetric matrix. 4.1.8. Find unknown elements in a symmetric matrix.
4.2 Performing operations on matrices involving the inverse of a matrix.	4.2.1. Find the inverse of a matrix by row reduction. 4.2.2. State the properties of the inverse of a matrix. 4.2.3. Use matrix inverse and transpose properties to solve matrix equations. 4.2.4. Write a system of equations as a matrix equation. 4.2.5. Solve systems of equations by finding the inverse of the coefficient matrix. 4.2.6. Relate the existence of the inverse of a matrix to the reduced form of A, and the solutions to the systems $A\vec{x} = \vec{b}$ and $A\vec{x} = \vec{0}$. 4.2.7. Set up a system of equations related to the Leontief Input-Output Model. Use the inverse method or Gaussian Elimination to solve the system.
4.3 Use of cofactor expansion and determinant properties to evaluate the determinant of a square matrix.	4.3.1. Define the determinant of a square matrix. 4.3.2. State the properties of the determinant. 4.3.3. Calculate the determinant of a square matrix by cofactor expansion. 4.3.4. Calculate the determinant of a square matrix using a combination of row reduction and cofactor expansion.
4.4 Use of determinants to find the inverse of a matrix and to solve systems of linear equations.	4.4.1. Use the determinant to determine whether a matrix has an inverse. 4.4.2. Find the adjoint of a matrix. 4.4.3. Find the inverse of a matrix using the adjoint and the determinant of a matrix. 4.4.4. Solve systems of equations using Cramer's Rule.
5. *Vector Spaces*	
5.1 Defining a vector geometrically.	5.1.1. State the geometric definition of a vector in $\Re^2$ and $\Re^3$. 5.1.2. Define the equality of two vectors algebraically and geometrically.
5.2 Performing operations on vectors.	5.2.1. Define vector operations (addition, subtraction, scalar multiplication, dot product, and cross product). 5.2.2. State the properties of those operations both algebraically and geometrically. 5.2.3. Calculate a vector that is a result of a series of vector operations both algebraically and geometrically. 5.2.4. Find the magnitude of a vector. 5.2.5. Normalize a vector. 5.2.6. Determine whether or not two vectors are (i) parallel and (ii) perpendicular. 5.2.7. Determine whether or not a set of vectors forms an orthogonal set. 5.2.8. Find a vector that is orthogonal to two other vectors.
5.3 Subspaces in $\Re^2$ and $\Re^3$	5.3.1. Define a subspace of a vector space and determine whether or not a given subset of a vector space is a subspace.
5.4 Use of the concepts of linear combinations, linear independence (dependence) and spanning in $\Re^2$ and $\Re^3$.	5.4.1. State the definition of a linear combination of vectors and determine whether a given vector is a linear combination of a set of vectors. 5.4.2. State the definition of linear independence and dependence and determine whether a set of vectors is linearly independent or linearly dependent. 5.4.3. State the definition of a span of a set of vectors and determine the span of a set of vectors. 5.4.4. Interpret geometrically: linear combinations, linear independence (dependence) and spanning.
5.5 Determination of a basis for a vector space or a subspace.	5.5.1. State the definition of a basis for a vector space or a subspace and determine a basis for a given vector space or subspace. 5.5.2. Define the dimension of a subspace. Determine the dimension of a given subspace.
5.6 Determination of the null space and column space of a matrix A.	5.6.1. Define the null space and column space of a matrix A. Find a basis for each of these spaces. 5.6.2. Define the rank and nullity of a matrix. Determine the rank and nullity of a matrix. 5.6.3. State the Dimension Theorem and use it to determine the rank or nullity of a matrix or its transpose. 5.6.4. Express a general solution of a nonhomogeneous system as a particular solution plus a general solution of the associated homogeneous system.

Specific Performance Criteria	Intermediate Learning Objectives
6. *Lines and Planes*	
6.1 Determination of equations of lines and planes.	6.1.1. Find the equation of a line in $\Re^2$ in (i) standard form, (ii) parametric form, (iii) vector form, and (iv) point-normal form. 6.1.2. Find the equation of a plane in $\Re^3$ in (i) standard form, (ii) parametric form, (iii) vector form, and (iv) point-normal form. 6.1.3. Determine whether or not two planes are (i) parallel or (ii) perpendicular.
6.2 Use of Cartesian axis systems to sketch lines and planes.	6.2.1. Develop the Cartesian axis systems for $\Re^3$. 6.2.2. Plot points and vectors in a Cartesian axis system for $\Re^3$. 6.2.3. Sketch lines in $\Re^2$ and in $\Re^3$. 6.2.4. Sketch planes in $\Re^3$.
6.3 Determination of the intersection of two or more planes.	6.3.1. Use an augmented matrix to determine the intersection of two or more planes.
7. *Optimization*	
7.1 Solution of problems using the Simplex method.	7.1.1. Find the maximum (minimum) of a linear function using the Simplex algorithm. 7.1.2. Determine whether or not a maximum (minimum) may be obtained for a given specific linear function and a list of inequalities. (Optional)
7.2 Use the Simplex method to solve concrete linear programming problems.	7.2.1. Use the Simplex method to solve problems in minimizing production cost and/or maximizing revenue or profit.
8. *Markov Chains*	
8.1 Transition Matrices	8.1.1. Define and identify Markov (stochastic) matrices. 8.1.2. Interpret transition matrices to determine probabilities of events.
8.2 Steady State Matrices and Vectors	8.2.1. Define and identify regular matrices. 8.2.2. Calculate steady state matrices and vectors corresponding to regular transition matrices.
9. *Cryptography*	
9.1 Modular Arithmetic	9.1.1. Encode messages into numbers and vice versa. 9.1.2. Determining invertibility and finding inverses of numbers modulo n. 9.1.3. Compute determinants and inverses of matrices modulo n.
9.2 Encryption and Decryption	9.2.1. Use matrix multiplication and the Hill cipher to encrypt messages. 9.2.2. Decrypt encrypted messages using matrix inverses.