Calculus III

General Information.
Discipline: Mathematics *Course code*: 201-DDB-05
Ponderation: 3-2-3 *Credits*: $2\frac{2}{3}$
Prerequisite: 201-NYB-05 (grade$>65\%$)
Objectives:

- OOUV: To apply a scientific or technological approach to a field in the natural sciences
- OOUU: To apply knowledge and skills that have already been acquired to one or more topics in the natural sciences

Students are strongly advised to seek help from their instructor as soon as they encounter difficulties in the course.

Introduction. Calculus III is the last course in the Calculus sequence. It is offered as an option course in the Science Program for students intending to pursue university studies primarily in Engineering, Physics, Mathematics or Chemistry. It is normally taken in the fourth semester.

Calculus III extends the basic concepts of Calculus I and Calculus II (limits, continuity, derivatives and integrals) to vector-valued functions and to functions of two or more variables. It reinforces the student's skills in techniques of sketching graphs in two and three dimensions using parametric equations, polar, cylindrical and spherical coordinates. Geometric concepts from Linear Algebra are widely used. Calculus III also advances the treatment of power series begun in Calculus II. Emphasis will be placed on clarity and rigour in reasoning and in the application of methods. Some of the applied problems solved in this course will be taken from other disciplines in the program, primarily Physics.

Students are encouraged to use a scientific graphing calculator and suitable mathematical software programs (such as Maple) that are available for their use in the Math Study Area. However, only non-graphical calculators are permitted on tests and exams (more details under "Course Costs"). The course uses a standard college-level Calculus textbook, chosen in collaboration with the Calculus I and II course committees.

Methodology. This course will be 75 hours, meeting three times a week for a total of five hours. The main technique used is the lecture approach. Other methods that may be used are: problem-solving sessions, class discussions, assigned reading for independent study, and computer lab projects. Regular homework involving a minimum of five hours per week should be expected. Students are responsible for all problems and exercises in the text relevant to material covered in class.

The Mathematics Department (H-200A and H-200B) functions as a quiet study area and as a centre where students may seek help with their mathematics courses. There are also several computers equipped with the Maple software program available for student use.

Textbook. Your teacher may require

Calculus, Multivariable (Eighth or Ninth Edition),
by James Stewart.

It is available at the bookstore for about $156.

Course Costs. In addition to the cost of the text listed above, a scientific calculator ($10 - $25) is required. Any basic scientific (non-graphical) calculator without numerical differentiation or integration capabilities will be acceptable (check with your teacher!). A graphics calculator ($100 - $150) could also be useful, but will not be allowed for tests or the final exam.

Evaluation. The Final Evaluation in this course consists of the Final Exam, which covers all elements of the competency. A student's Final Grade is a combination of the Class Mark and the mark on the Final Exam. The Class mark will include three to four in-class written tests worth 75%, and other homework and assignments will make up the balance.

The Final Exam is set by the Course Committee (which consists of the instructors currently teaching this course).

Students must be available until the end of the final examination period to write exams.

The Final Grade will be the better of:

- 50% Class Mark and 50% Final Exam Mark, OR
- 25% Class Mark and 75% Final Exam Mark

Students choosing not to write the final examination will receive a failing grade of 50% or their class mark, whichever is less.

Note that in the event of unexpected changes to the academic calendar, the evaluation plan may be modified.

Other Resources.
Math Website.
http://departments.johnabbott.qc.ca/departments/mathematics

Math Study Area. Located in H-200A and H-200B; the common area is usually open from 8:30 to 17:30 on weekdays as a quiet study space. Computers and printers are available for math-related assignments. It is also possible to borrow course materials when the attendant is present.

Math Help Centre. Located in H-216; teachers are on duty from 8:30 until 15:30 to give math help on a drop-in basis.

Academic Success Centre. The Academic Success Centre, located in H-139, offers study skills workshops and individual tutoring.

Course Content (with selected exercises). The exercises listed below (referring to the eighth edition of the textbook) should help you practice and learn the material taught in this course; they form a good basis for homework. Your teacher may supplement this list during the semester. Regular work done as the course progresses should make it easier for you to master the course.

Chapter 11: Infinite Series.

11.8 Power Series (1-30, 41, 42)
11.9 Representations of Functions as Power Series (1-20, 25-32, 39, 40)
11.10 Taylor and Maclaurin Series (31-44, 49-80)
11.11 Applications of Taylor Polynomials (13ab, 14ab, 15ab, 18ab)

Chapter 10: Parametric Equations and Polar Curves.

10.1 Curves Defined by Parametric Equations (1-22, 34)
10.2 Calculus with Parametric Curves (1-16, 25, 31-35, 37-40, 51)
10.3 Polar Coordinates (1-26, 29-42, 49, 61-64)
10.4 Areas and Lengths in Polar Coordinates (1, 5-12, 17-21, 23-35, 37-41, 45-47)

Chapter 12: Vectors and Geometry of Space.

12.1-12.5 Vector Geometry (Review)
12.6 Cylinders and Quadric Surfaces (1, 3-9, 11, 13, 15, 17, 31-38, 43, 44)

Chapter 13: Vector Functions.

13.1 Vector Functions and Space Curves (1-14, 21-27)
13.2 Derivatives and Integrals of Vector Functions (3-27, 35-40, 47-49)
13.3 Arc Length and Curvature (1-6, 13a, 14a, 17-23, 42-45, 60)
13.4 Velocity and Acceleration (3-16, 22, 37-40)

Chapter 14: Partial Derivatives.

14.1 Functions of Several Variables (1, 13-31, 45-54, 67-70)
14.2 Limits and Continuity (5-22, 30-37, 39, 40)
14.3 Partial Derivatives (5-8, 15-65, 78, 79)
14.4 Tangent Planes and Linear Approximations (1, 3, 11, 13, 17-19, 21, 25-34)
14.5 The Chain Rule (1-12, 15, 17, 21-34, 38, 45, 47, 49, 53)
14.6 Directional Derivatives and the Gradient Vector (1, 4-13, 21-25, 33, 41-43)
14.7 Maximum and Minimum Values (1, 5, 7, 9, 11, 13, 15, 17, 19, 31-38, 41, 45, 49, 53)
14.8 Lagrange Multipliers (3-14, 21, 23)

Chapter 15: Multiple Integrals.

15.1 Double Integrals Over a Rectangles (8, 15-35, 39, 42)
15.2 Double Integrals Over General Regions (1-31, 45-56, 58)
15.3 Double Integrals in Polar Coordinates (1-4, 6-23, 39)
15.6 Triple Integrals (3-8, 13, 15, 17, 19-22, 23a, 27, 28, 33, 35)
15.7 Triple Integrals in Cylindrical Coordinates (1-12, 15-19, 22, 29, 30)
15.8 Triple Integrals in Spherical Coordinates (1-15, 17, 18, 21, 23, 29, 30, 41-43)
15.9 Change of Variables in Multiple Integrals (1-6, 15, 17, 19, 25, 27)

College Policies.

Policy No. 7 - IPESA, Institutional Policy on the Evaluation of Student Achievement: https://www.johnabbott.qc.ca/wp-content/uploads/2021/05/Policy-No.-7-IPESA-FINAL.pdf.

Religious Holidays (Article 3.2.13 and 4.1.6). Students who wish to miss classes in order to observe religious holidays must inform their teacher of their intent in writing within the first two weeks of the semester.

Student Rights and Responsibilities: (Article 3.2.18). It is the responsibility of students to keep all assessed material returned to them and/or all digital work submitted to the teacher in the event of a grade review. (The deadline for a Grade Review is 4 weeks after the start of the next regular semester.)

Student Rights and Responsibilities: (Article 3.3.6). Students have the right to receive graded evaluations, for regular day division courses, within two weeks after the due date or exam/test date, except in extenuating circumstances. A maximum of three (3) weeks may apply in certain circumstances (ex. major essays) if approved by the department and stated on the course outline. For evaluations at the end of the semester/course, the results must be given to the student by the grade submission deadline (see current Academic Calendar). For intensive courses (i.e.: intersession, abridged courses) and AEC courses, timely feedback must be adjusted accordingly.

Academic Procedure: Academic Integrity, Cheating and Plagiarism (Article 9.1 and 9.2). Cheating and plagiarism are unacceptable at John Abbott College. They represent infractions against academic integrity. Students are expected to conduct themselves accordingly and must be responsible for all of their actions.

College definition of Cheating: Cheating means any dishonest or deceptive practice relative to examinations, tests, quizzes, lab assignments, research papers or other forms of evaluation tasks. Cheating includes, but is not restricted to, making use of or being in possession of unauthorized material or devices and/or obtaining or providing unauthorized assistance in writing examinations, papers or any other evaluation task and submitting the same work in more than one course without the teacher's permission. It is incumbent upon the department through the teacher to ensure students are forewarned about unauthorized material, devices or practices that are not permitted.

College definition of Plagiarism: Plagiarism is a form of cheating. It includes copying or paraphrasing (expressing the ideas of someone else in one's own words), of another person's work or the use of another person's work or ideas without acknowledgement of its source. Plagiarism can be from any source including books, magazines, electronic or photographic media or another student's paper or work.

OBJECTIVES	STANDARDS
Statement of the competency To apply the methods of multivariable calculus to the study of functions and problem solving.	**General Performance Criteria** • Appropriate choice of concepts, laws and principles. • Rigorous application of concepts, laws and principles. • Appropriate use of terminology. • Adequate graphical or mathematical representation. • Consistency and rigour in problem solving and justification of the approach used. • Observance of the scientific method and, where applicable, experimental procedure. • Justification of the approach used. • Assessment of the plausibility of results. • Correct algebraic operations. • Accuracy of calculations.
Elements of the Competency 1. To analyze the convergence of power series. 2. To describe plane curves using parametric equations and polar coordinates. 3. To apply calculus to vector-valued functions. 4. To make three dimensional drawings. 5. To study calculus of functions of several variables. 6. To solve optimization problems. 7. To calculate and apply double and triple integrals.	**Specific Performance Criteria** *[Specific performance criteria for each of these elements of the competency are listed on the following pages.]*

OBJECTIVES	STANDARDS
Specific Performance Criteria	**Intermediate Learning Objectives**
1. *Power series*	
1.1 Use of Taylor's Theorem to define functions	1.1.1. Write Taylor polynomials of a function centred at c. 1.1.2. Write Maclaurin polynomials of a function. 1.1.3. Write the Lagrange form of remainder. 1.1.4. Calculate bounds on errors. 1.1.5. Determine the accuracy of an approximation.
1.2 Analysis of convergence of power series	1.2.1. Define a power series centred at c. 1.2.2. Determine the radius and interval of convergence of a power series. 1.2.3. Determine convergence at the endpoints. 1.2.4. Differentiate power series. 1.2.5. Integrate power series.
1.3 Representing functions by power series	1.3.1. Recognize and use geometric power series. 1.3.2. Add power series. 1.3.3. Multiply power series. 1.3.4. Divide power series. 1.3.5. Substitute kx or x^n for the argument of a power series.
1.4 Use of Taylor/Maclaurin series to define a function	1.4.1. Define a Taylor/Maclaurin series. 1.4.2. Determine when a function equals its Taylor/Maclaurin series. 1.4.3. Recognize and use binomial series.
2. *Representation of plane curves*	
2.1 Use of parametric equations to analyze curves	2.1.1. Parametrize a variety of curves. 2.1.2. Recognize the orientation of a parametrized curve. 2.1.3. Determine when a parametrized curve is smooth or piecewise-smooth. 2.1.4. Determine dy/dx and d^2y/dx^2 for a parametrized curve.
2.2 Applications of parametric equations	2.2.1. Determine the arc length of a parametrized curve. 2.2.2. Determine the area of a surface of revolution generated by revolving a parametrized curve about the x or y axes.
2.3 Use of polar coordinates to analyze curves	2.3.1. Convert from rectangular to polar coordinates and vice versa. 2.3.2. Graph curves given in polar coordinates. 2.3.3. Find dy/dx on the graph of an equation in polar coordinates. 2.3.4. Find horizontal and vertical tangents to graphs in polar coordinates. 2.3.5. Find tangents at the pole. 2.3.6. Find the points of intersection of graphs in polar coordinates.
2.4 Applications of polar coordinates	2.4.1. Find the area of a region in polar coordinates. 2.4.2. Find the arc length of a curve in polar coordinates. 2.4.3. Find the surface area of a surface of revolution generated by revolving a curve given in polar coordinates.
3. *Vector valued functions*	
3.1 Investigation of vectors in the plane and in space	3.1.1. Perform vector addition and scalar multiplication. 3.1.2. Recognize the axioms of a vector space. 3.1.3. Find the length and direction of a vector. 3.1.4. Be conversant with two different vector notations: row vectors and the i,j,k notation. 3.1.5. Compute the dot product of two vectors. 3.1.6. Give a geometric interpretation of the dot product. 3.1.7. State the properties of the dot product. 3.1.8. Find the angle between two vectors. 3.1.9. Determine when two vectors are orthogonal. 3.1.10. Compute the direction cosines of a vector. 3.1.11. Find the orthogonal projection of one vector onto another. 3.1.12. Write a vector as an orthogonal sum of two vectors. 3.1.13. Compute the cross product of two vectors. 3.1.14. Give a geometric interpretation of the cross product. 3.1.15. State the properties of the cross product. 3.1.16. Give a geometric interpretation of the triple scalar product.
3.2 Description of lines and planes in space	3.2.1. Write the parametric equations of a line. 3.2.2. Write the symmetric equations of a line. 3.2.3. Write the equation of a plane. 3.2.4. Find the angle between two intersecting planes or lines.3 3.2.5. Find the line common to two intersecting planes. 3.2.6. Find the distance between a point and a plane. 3.2.7. Find the distance between a point and a line.

OBJECTIVES	STANDARDS
Specific Performance Criteria	**Intermediate Learning Objectives**
3. *Vector valued functions (continued)*	
3.3 Study of vector calculus	3.3.1. Find the domain of a vector-valued function. 3.3.2. Sketch the graph of a vector-valued function as a curve in $\mathbb{R}^2$ and $\mathbb{R}^3$. 3.3.3. Define the derivative of a vector-valued function. 3.3.4. Differentiate and integrate vector-valued functions. 3.3.5. Apply the properties of the derivative: linearity, product rules, and the chain rule.
3.4 Applications of vector calculus	3.4.1. Define velocity, acceleration, and speed. 3.4.2. Find the unit tangent vector and the principal unit normal vector to a curve. 3.4.3. Find the tangential and normal components of acceleration. 3.4.4. Find the arc length of a space curve given by a vector-valued function. 3.4.5. Use the arc length parameter to parametrize the curve. 3.4.6. Define curvature. 3.4.7. Use various formulas for curvature.
4. *Three dimensional drawings*	
4.1 Analyzing surfaces in space	4.1.1. Recognize when an equation describes a cylindrical surface. 4.1.2. Recognize the shape of a quadric surface from a second-degree equation. 4.1.3. Write the equation for a surface of revolution. 4.1.4. Transform coordinates between rectangular, cylindrical, and spherical coordinate systems. 4.1.5. Sketch surfaces in cylindrical and spherical coordinates.
5. *Multivariable calculus*	
5.1 Study of functions of more than one variable	5.1.1. Find the domain of a function of two or more variables. 5.1.2. Sketch a function of two variables as a surface. 5.1.3. Describe the level curves of a function of two variables. 5.1.4. Describe the level surfaces of a function of three variables. 5.1.5. Investigate limits and continuity for functions of two variables (intuitively). 5.1.6. Define Partial Derivatives. 5.1.7. Find the first and second partial derivatives. 5.1.8. Find the total differential. 5.1.9. Apply the appropriate chain rule in differentiation. 5.1.10. Find partial derivatives of an implicitly-defined function. 5.1.11. Find and interpret directional derivatives and the gradient. 5.1.12. State and prove the properties of the gradient. 5.1.13. Find the equation of the tangent plane and the normal line to a surface at a point.
6. *Optimization*	
6.1 Solutions of optimization problems	6.1.1. Determine critical points. 6.1.2. Find relative extrema for functions of two variables. 6.1.3. Apply the second partials test to determine extrema. 6.1.4. Use the method of Lagrange Multipliers to determine extrema.
7. *Double and triple integrals*	
7.1 Evaluating double and triple integrals	7.1.1. Evaluate iterated integrals. 7.1.2. Define the double and triple integrals as Riemann sums. 7.1.3. State the properties of the double and triple integrals. 7.1.4. Apply Fubini's Theorem in rectangular coordinates. 7.1.5. Transform a double integral from rectangular to polar coordinates. 7.1.6. Transform a triple integral from rectangular to cylindrical and spherical coordinates. 7.1.7. Define the Jacobian. 7.1.8. Use the Jacobian to change variables in evaluating a double integral.
7.2 Applications of the double and triple integrals	7.2.1. Find areas, volumes, mass, centre of mass and surface area using the double integral. 7.2.2. Find volumes using the triple integral.