Calculus I

General Information.
Discipline: Mathematics — *Course code*: 201-SH2-AB
Ponderation: 3-2-3 — *Credits*: $2\frac{2}{3}$
Prerequisite: Secondary V Math TS5 or SN5 (or equivalent)
Objective: 0PU2

> Your teacher will give you their schedule and availability.
> *Students are strongly advised to seek help promptly from their teacher if they encounter difficulties in the course.*

Introduction. Calculus I is the first of the mathematics courses in the Social Science program. It is usually taken in the first semester and it introduces the student to the limit processes that are so vital to the development of calculus. Since differential calculus is a basic tool in business, economics, psychology and sociology, some of the applications will be related to problems in these fields.

The primary purpose of the course is the attainment of objective 0PU2 (see below). To achieve this goal, the course will help the student understand the following basic concepts: limits, continuity and derivatives involving real-valued functions of one variable (including algebraic, trigonometric, exponential, and logarithmic functions).

Emphasis will be placed on clarity and rigour in reasoning and in the application of methods. The student will learn to interpret the derivative both as a mathematical tool and as a rate of change. The derivative will be used in various Social Science contexts such as financial mathematics, curve sketching, optimization and cost-benefit analysis (cost, revenue, and profit). The basic concepts are illustrated by applying them to various problems where their application helps arrive at a solution. In this way the course encourages the student to apply learning acquired in one context to problems arising in another.

Textbook. There is no required textbook for this course. Sets of exercises will be provided by your teacher. A good reference for the course material is *Applied Calculus for the Managerial, Life, and Social Sciences, 10th edition*, by Soo T. Tan. Note that this book may not be available for purchase at the bookstore, but reference copies are available in the Math Study Area and at the Library.

Calculator Policy. Only calculators which have previously been inspected and approved via sticker by the instructor will be permitted for use on quizzes, tests or the final examination. The only calculators that will be approved begin with the model number **SHARP EL-531**. An acceptable calculator model is available for purchase at the bookstore.

Course Costs. The approved model of scientific calculator costs about \$25.

Teaching Methods. This course will be 75 hours, meeting three times a week for a total of five hours a week. It relies mainly on the lecture method, although some of the following techniques are also used: question-and-answer sessions, labs, problem-solving periods, and class discussions. In general, each class begins with a question period on previous topics, then new material is introduced, followed by worked examples. No marks are deducted for absenteeism (however, see below). Failure to keep pace with the lectures results in a cumulative inability to cope with the material and a failure in the course. A student will generally succeed or fail depending on how many problems have been attempted and solved successfully. It is entirely the student's responsibility to complete suggested homework assignments as soon as possible following the lecture, as the material will be fresher in their mind. This also allows the student the maximum benefit from any discussion of the homework (which usually occurs in the following class).

Evaluation Plan. The Final Evaluation in this course consists of the Final Exam, which covers all elements of the competency. A student's Final Grade is a combination of the Class Mark and the mark on the Final Exam. The Class Mark will be 75% (three or more in-class written tests) and 25% at your teacher's discretion (more tests, quizzes or assignments). The specifics of the Class Mark are included in an appendix that is distributed to students along with this course outline. The Final Exam is set by the course committee (which consists of all instructors currently teaching this course), and is marked by each individual instructor. Every effort is made to ensure equivalence between the various sections of the course.

The Final Grade will be the better of:

50% Class Mark and 50% Final Exam Mark
or
25% Class Mark and 75% Final Exam Mark

A student *choosing not to write* the Final Exam will receive a failing grade of 50% or their Class Mark, whichever is less.

Students must be available until the end of the final examination period to write exams.

Note that in the event of unexpected changes to the academic calendar, the evaluation plan may be modified.

Other Resources.

Math Website.
http://departments.johnabbott.qc.ca/departments/mathematics

Math Study Area. Located in H-200A and H-200B; the common area is usually open from 8:30 to 17:30 on weekdays as a quiet study space. Computers and printers are available for math-related assignments. It is also possible to borrow course materials when the attendant is present.

Math Help Centre. Located in H-216; teachers are on duty from 8:30 until 15:30 to give math help on a drop-in basis.

Peer Tutoring. Starting on the fifth week of each semester, first year students can be paired with a fellow finishing student for a weekly appointment of tutoring. Ask your teacher for details.

Academic Success Centre. The Academic Success Centre, located in H-139, offers study skills workshops and individual tutoring.

College Policies.

Policy No. 7 - IPESA, Institutional Policy on the Evaluation of Student Achievement: https://www.johnabbott.qc.ca/wp-content/uploads/2021/05/Policy-No.-7-IPESA-FINAL.pdf.

Religious Holidays (Article 3.2.13 and 4.1.6). Students who wish to miss classes in order to observe religious holidays must inform their teacher of their intent in writing within the first two weeks of the semester.

Student Rights and Responsibilities: (Article 3.2.18). It is the responsibility of students to keep all assessed material returned to them and/or all digital work submitted to the teacher in the event of a grade review. (The deadline for a Grade Review is 4 weeks after the start of the next regular semester.)

Student Rights and Responsibilities: (Article 3.3.6). Students have the right to receive graded evaluations, for regular day division courses, within two weeks after the due date or exam/test date, except in extenuating circumstances. A maximum of three (3) weeks may apply in certain circumstances (ex. major essays) if approved by the department and stated on the course outline. For evaluations at the end of the semester/course, the results must be given to the student by the grade submission deadline (see current Academic Calendar). For intensive courses (i.e.: intersession, abridged courses) and AEC courses, timely feedback must be adjusted accordingly.

Academic Procedure: Academic Integrity, Cheating and Plagiarism (Article 9.1 and 9.2). Cheating and plagiarism are unacceptable at John Abbott College. They represent infractions against academic integrity. Students are expected to conduct themselves accordingly and must be responsible for all of their actions.

College definition of Cheating: Cheating means any dishonest or deceptive practice relative to examinations, tests, quizzes, lab assignments, research papers or other forms of evaluation tasks. Cheating includes, but is not restricted to, making use of or being in possession of unauthorized material or devices and/or obtaining or providing unauthorized assistance in writing examinations, papers or any other evaluation task and submitting the same work in more than one course without the teacher's permission. It is incumbent upon the department through the teacher to ensure students are forewarned about unauthorized material, devices or practices that are not permitted.

College definition of Plagiarism: Plagiarism is a form of cheating. It includes copying or paraphrasing (expressing the ideas of someone else in one's own words), of another person's work or the use of another person's work or ideas without acknowledgement of its source. Plagiarism can be from any source including books, magazines, electronic or photographic media or another student's paper or work.

Course Content. The exercises listed should help you practice and learn the material taught in this course; they form a good basis for homework. Your teacher may supplement this list during the semester. Regular work done as the course progresses should make it easier for you to master the course.

Exercises #1 - Limits from Graphs
Exercises #2 - Limits by Direct Substitution
Exercises #3- Evaluating Limits Algebraically
Exercises #4 - Mixed Limits
Exercises #5 - Continuity
Exercises #6 - Derivatives - Limit Definition
Exercises #7 - Derivatives - Basic Differentiation Rules
Exercises #8 - Derivatives - Trig - Exp - Log
Exercises #9 - Implicit Differentiation
Exercises #10 - Logarithmic Differentiation
Exercises #11 - Mixed Derivatives
Exercises #12 - Higher Derivatives
Exercises #13 - Critical Numbers - Absolute Extrema
Exercises #14 - Analysis of First and Second Derivatives
Exercises #15 - Asymptotes
Exercises #16 - Curve Sketching
Exercises #17 - Optimization
Exercises #18 - Marginal Functions
Exercises #19 - Elasticity

OBJECTIVES	STANDARDS
Statement of the competency Analyze problems studied in the social sciences by using differential calculus. (0PU2).	**Performance criteria for the competency as a whole** • Accurate recognition of the context in which differential calculus emerged • Appropriate mathematical modelling of real-world situations studied in the social sciences • Correct use of mathematical syntax • Demonstration of rigorous mathematical reasoning • Accurate and coherent interpretation of results
Elements of the competency	**General performance criteria**
1. Apply mathematical models to current human realities.	1.1 Accurate recognition of the characteristics of the main mathematical models used with regard to the basic functions (algebraic, exponential, logarithmic and trigonometric) 1.2 Correct manipulation of equations and inequalities involving the basic functions (Tools for such manipulations include factoring, rationalization, simplification, division by a polynomial, multiplication by the conjugate, finding a common denominator, laws of exponents, logarithmic properties, trigonometric identities and the unit circle) 1.3 Appropriate graphs of the basic functions 1.4 Establishment of relevant relationships between mathematical models and current human realities
2. Examine how a current human reality manifests itself.	2.1 Accurate recognition of the concepts of continuity and limit 2.2 Accurate algebraic and graphical determination of the limits of the functions associated with the main mathematical models 2.3 Accurate determination of the continuity or discontinuity of a function 2.4 Appropriate use of calculus techniques for resolving an indeterminate form ($0/0$, $\pm\infty/\pm\infty$ only) 2.5 Accurate interpretation of how a current human reality manifests itself, and a specific point in time and over the long term
3. Carry out the marginal analysis of a current human reality.	3.1 Accurate recognition of the concept of the derivative of a function 3.2 Accurate interpretation of the derivative function related to a current human reality 3.3 Accurate use of the definition of the derivative 3.4 Correct calculation of the derivative of a function 3.5 Correct application of differentiation rules and techniques 3.6 Correct application of marginal analysis methods to a current human reality
4. Analyze the variations of a function specific to a current human reality.	4.1 Accurate recognition of the steps involved in analysing a function 4.2 Correct analysis of a function (related to its domain, zeros, initial value, signs, vertical & horizontal asymptotes, extrema and inflection points, variation table and sketch) 4.3 Correct use of the method for analysing the variations of a function specific to a current human reality
5. Solve problems specific to a current human reality.	5.1 Accurate recognition of problems requiring the use of differential calculus 5.2 Correct solution of problems involving rates of change, optimization and demographics

Specific Performance Criteria	**Intermediate Learning Objectives**
Specific performance criteria for each of these elements of the competency are shown below with the corresponding intermediate learning objectives.	*For the items in the list of learning objectives, it is understood that each is preceded by: “The student is expected to . . . ”.]*
1. *The Development of Calculus*	
1.1 The history of Calculus	1.1.1. Recognize Newton and Leibniz as the founders of calculus. 1.1.2. Examine the historical context of differential calculus by an investigation of the slope of a tangent line, a problem posed by Newton and Leibniz and its relevance in today’s society.
2. *Functions*	
2.1 Recognition of functions	2.1.1. Decide whether a given relation is a function from its graphical representation. 2.1.2. Recognize and name the following functions from their symbolic representations: $f(x) = c$ constant function $f(x) = ax + b$ linear function $f(x) = ax^2 + bx + c$ quadratic function $f(x) = \|x\|$ absolute value function $f(x) = \sqrt{x}$ square root function $f(x) = a^x$ exponential function $f(x) = \log_a x$ logarithmic function $f(x) = \sin x$ sine function $f(x) = \cos x$ cosine function $f(x) = \tan x$ tangent function $f(x) = \csc x$ cosecant function $f(x) = \sec x$ secant function $f(x) = \cot x$ cotangent function 2.1.3. Recognize and name the following function from its symbolic representation: $f(x) = \sqrt[n]{x}$ (nth root function). 2.1.4. Recognize and name the functions listed in 2.1.2 from their graphical representations.
2.2 Finding domain, range and intercepts	2.2.1. Find and state the domain of functions listed in 2.1.2 from both their graphical and their symbolic representations. 2.2.2. Find and state the range of functions listed in 2.1.2 from both their graphical and their symbolic representations. 2.2.3. Find and state the x- and y-intercepts, if they exist, of functions listed in 2.1.2 from both their graphical and their symbolic representations.
2.3 Graphing of functions	2.3.1. Graph the functions listed in 2.1.2. 2.3.2. Graph piecewise defined functions whose pieces are made up of the functions listed in 2.1.2. 2.3.3. Apply vertical and horizontal shifts and reflections about the horizontal and vertical axes, and any combination of these to the functions listed in 2.1.2.
2.4 Operations on functions	2.4.1. Perform addition, subtraction, multiplication, division and composition of functions. 2.4.2. Find exact solutions of equations involving the functions listed in 2.1.2. 2.4.3. Divide two polynomial functions and express the answer in the form $\dfrac{p(x)}{d(x)} = q(x) + \dfrac{r(x)}{d(x)}$. 2.4.4. Find the value of a function at a point in its domain. 2.4.5. Evaluate $\dfrac{f(x+h) - f(x)}{h}$ (the difference quotient) for a polynomial of degree 1, 2 or 3, square root and simple rational functions.
2.5 Appropriate use of functions to represent given situations	2.5.1. Given an applied problem, decide which function best represents the situation and express the relationship using appropriate notation.
3. *Limits and Continuity*	
3.1 Determination of Limits	3.1.1. Give an intuitive description of the limit of a function at a point. 3.1.2. Explain how limits are motivated by the appearance of indeterminate forms, particularly $\frac{0}{0}$ and $\frac{\pm\infty}{\pm\infty}$. 3.1.3. Evaluate a limit of a function by viewing the graph of the function. 3.1.4. Estimate a limit numerically by using successive approximations (using a table of values), particularly those arising from the indeterminate forms $\frac{0}{0}$ and $\frac{\pm\infty}{\pm\infty}$. 3.1.5. Evaluate a limit analytically by direct substitution, factoring, rationalizing or simplifying rational expressions, particularly those arising from the indeterminate forms $\frac{0}{0}$ and $\frac{\pm\infty}{\pm\infty}$. 3.1.6. Evaluate analytically limits at infinity. 3.1.7. Evaluate one-sided limits. 3.1.8. Recognize and evaluate infinite limits.
3.2 Determination of whether a function is continuous at a point or on an interval	3.2.1. Define continuity of a function at a point; that is, state the three conditions which must be satisfied in order that a function be continuous at a point. 3.2.2. Use the definition of continuity to determine if a function is continuous at a specific point. 3.2.3. Determine on which interval(s) a function is continuous.
4. *The Derivative of a Function*	
4.1 Use of the limit definition of the derivative	4.1.1. Define the derivative of a function as (i) the limit of a difference quotient, (ii) the slope of a tangent line, and (iii) the rate of change (marginal functions).

Specific Performance Criteria	Intermediate Learning Objectives
	4.1.2. Use the limit definition of the derivative to determine the derivative of a polynomial of degree 1, 2 or 3, square root and simple rational functions. 4.1.3. Use the limit definition of the derivative to find the slope of the tangent line to a curve at a specific point.
4.2 Use of the graph of a function to determine whether a function is differentiable at a point or on an interval	4.2.1. Determine if the derivative of a function exists at a point or on an interval by examining the graph of the function.
4.3 Recognition of the equivalence of various derivative notations	4.3.1. Recognize different notations for the derivative of y with respect to x: $y',\ f'(x),\ \frac{dy}{dx},\ \frac{d}{dx}f(x),\ D_x y$
4.4 Use of basic differentiation formulas and rules	4.4.1. Recognize when and how to use the basic differentiation formulas: $\frac{d}{dx}[c] = 0$ $\quad \frac{d}{dx}[\sin x] = \cos x$ $\frac{d}{dx}[x^n] = nx^{n-1}$ $\quad \frac{d}{dx}[\cos x] = -\sin x$ $\frac{d}{dx}[e^x] = e^x$ $\quad \frac{d}{dx}[\tan x] = \sec^2 x$ $\frac{d}{dx}[\ln x] = \frac{1}{x}$ $\quad \frac{d}{dx}[\sec x] = \sec x \tan x$ $\frac{d}{dx}[a^x] = a^x \ln a$ $\quad \frac{d}{dx}[\csc x] = -\csc x \cot x$ $\frac{d}{dx}[\log_a x] = \frac{1}{x \ln a}$ $\quad \frac{d}{dx}[\cot x] = -\csc^2 x$ 4.4.2. Recognize when and how to use the following differentiation formulas derived from the chain rule: $\frac{d}{dx}[u^n] = nu^{n-1}u'$ $\quad \frac{d}{dx}[\sin u] = \cos u\, u'$ $\frac{d}{dx}[e^u] = e^u u'$ $\quad \frac{d}{dx}[\cos u] = -\sin u\, u'$ $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ $\quad \frac{d}{dx}[\tan u] = \sec^2 u\, u'$ $\frac{d}{dx}[a^u] = a^u(\ln a)u'$ $\quad \frac{d}{dx}[\sec u] = \sec u \tan u\, u'$ $\frac{d}{dx}[\log_a u] = \frac{u'}{u \ln a}$ $\quad \frac{d}{dx}[\csc u] = -\csc u \cot u\, u'$ $\frac{d}{dx}[\cot u] = -\csc^2 u\, u'$ 4.4.3. Recognize when and how to use the following rules: constant rule, power rule, constant multiple rule, sum and difference rule. 4.4.4. Recognize when and how to use the product, quotient and chain rules.
4.5 Use of differentiation rules to perform implicit and logarithmic differentiation	4.5.1. Recognize when and how to use implicit differentiation. 4.5.2. Recognize when and how to use logarithmic differentiation.
4.6 Evaluation and application of higher order derivatives	4.6.1. Find derivatives of order 2 or higher.
4.7 Use of derivatives to find the slope of a tangent line to a curve at a point	4.7.1. Use the differentiation rules listed in 4.4.1 and 4.4.2 to find the slope of the tangent line to a curve at a point. 4.7.2. Use the differentiation rules listed in 4.4.1 and 4.4.2 to find the equation of the tangent line to a curve at a point.
5. *Curve Sketching*	
5.1 Use of the derivative and related concepts to analyze the variations of a function and to sketch a graph of the function	5.1.1. Find critical numbers. 5.1.2. Find intervals on which a function is increasing and decreasing using the sign of the first derivative. 5.1.3. Find relative and absolute extrema. 5.1.4. Use the first derivative test to decide whether the critical points represent relative maxima or relative minima. 5.1.5. Find inflection points. 5.1.6. Find intervals on which a function is concave up or concave down using the sign of the second derivative. 5.1.7. Use limits to find all vertical and horizontal asymptotes. 5.1.8. Use 5.1.1–5.1.7 to graph polynomial and rational functions.
6. *Optimization and Rate-of-Change Problems*	
6.1 Solution of optimization problems	6.1.1. Represent an optimization word problem in functional form. 6.1.2. Determine the quantity, Q, to be maximized or minimized and identify the variables which are involved. 6.1.3. Draw a diagram, if possible, to illustrate the problem and list any other relationship(s) between the variables. 6.1.4. Express Q as a function of one variable. 6.1.5. Find the derivative of the function for Q obtained in 5.1.4. 6.1.6. Find all the possible critical values by solving the equation $Q' = 0$. 6.1.7. Test the critical value(s) and interval endpoints for absolute maximum or minimum. 6.1.8. Interpret (explain) the results found in the optimization problem.
6.2 Solution of problems involving rates of change	6.2.1. Find the marginal cost, marginal revenue and marginal profit using the derivative. 6.2.2. Find the elasticity of demand function. Evaluate and classify it at a given production level.