Calculus II

General Information.
Discipline: Mathematics — *Course code*: 201-A2S-AB
Ponderation: 2-2-2 — *Credits*: 2
Prerequisite: 201-A2S-AB
Objective: Analyze scientific problems by applying integral calculus

Your teacher will give you his/her schedule and availability.
Students are strongly advised to seek help promptly from their teacher if they encounter difficulties in the course.

Introduction. Integral Calculus is the sequel to Differential Calculus. It is generally taken in the second semester. The Arts and Sciences student at John Abbott will already be familiar with the notions of definite and indefinite integration from Differential Calculus. In Integral Calculus these notions are studied to a greater depth. In addition, the course introduces the student to the concept of infinite series, and to the representation of functions by power series.

The primary purpose of the course is the attainment of objective 7MA2 ("Analyze scientific problems by applying integral calculus"'). To achieve this goal, the course must help the student understand the following basic concepts: limits, derivatives, indefinite and definite integrals, sequences, infinite series, power series involving real-valued functions of one variable (including algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions).

Emphasis is placed on clarity and rigour in reasoning and in the application of methods. The student will learn to use the techniques of integration in several contexts, and to interpret the integral both as an antiderivative and as a limit of a sum of products. The basic concepts are illustrated by applying them to various problems where their application helps arrive at a solution. In this way, the course encourages the student to apply learning acquired in one context to problems arising in another.

Students may be permitted to use a scientific or graphing calculator in class; however, calculators (of any kind) will not be permitted on tests and the final exam.

Textbook. Your teacher may require *Single Variable Calculus: Early Transcendentals*, 9th edition, by James Stewart (Brooks/Cole). It is available from the college bookstore for about $132. Note that the 7th or 8th editions of this book are also suitable.

Course Costs. In addition to the cost of the textbook (see above), your instructor might recommend you acquire an inexpensive scientific calculator ($15–$25). *No calculators are allowed during tests or the final exam.*

Teaching Methods. This course will be 60 hours, meeting three times a week for a total of four hours a week. It relies mainly on the lecture method, although some of the following techniques are also used: question-and-answer sessions, labs, problem solving periods, class discussions, and assigned reading for independent study. No marks are deducted for absenteeism, however failure to keep pace with the lectures typically results in a cumulative inability to cope with the material and a failure in the course. A student will generally succeed or fail depending on how many problems have been attempted and solved successfully. It is entirely the student's responsibility to complete suggested homework assignments as soon as possible following the lecture, as the material will be fresher in his/her mind. This also allows the student the maximum benefit from any discussion of the homework. Answers to a selected number of problems can be found in the back of the text. Individual teachers may provide supplementary notes and problems as they see fit.

High School Topics Review. The transition from High School to CEGEP Math can sometimes be difficult, especially if you've forgotten some of the topics or techniques learned in High School. To that end, we've made videos and practice problems available to help you review a few key concepts. You can find them here: High School Review

Evaluation Plan. The Final Evaluation in this course consists of the Final Exam, which covers all elements of the competency. A student's Final Grade is a combination of the Class Mark and the mark on the Final Exam. The Class Mark will be comprised of 75% tests (three or four in-class written tests) and 25% consisting of minor assessments, including at least one piece of written feedback before each test. The specifics of the Class Mark are included in an appendix that is distributed to students along with this course outline. The Final Exam is set and marked by the individual instructor.

The Final Grade will be the better of:

50% Class Mark and 50% Final Exam Mark
or
25% Class Mark and 75% Final Exam Mark

A student *choosing not to write* the Final Exam will receive a failing grade of 50% or their Class Mark, whichever is less.

Students must be available until the end of the final examination period to write exams.

Note that in the event of unexpected changes to the academic calendar, the evaluation plan may be modified.

Other Resources.

Math Website.
http://departments.johnabbott.qc.ca/departments/mathematics

Math Study Area. Located in H-200A and H-200B; the common area is usually open from 8:30 to 17:30 on weekdays as a quiet study space. Computers and printers are available for math-related assignments. It is also possible to borrow course materials when the attendant is present.

Math Help Centre. Located near H-211; teachers are on duty from 8:30 until 15:30 to give math help on a drop-in basis.

Peer Tutoring. Starting on the fifth week of each semester, first year students can be paired with a fellow finishing student for a weekly appointment of tutoring. Ask your teacher for details.

Academic Success Centre. The Academic Success Centre, located in H-139, offers study skills workshops and individual tutoring.

College Policies.

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Policy No. 7 - IPESA, Institutional Policy on the Evaluation of Student Achievement: https://www.johnabbott.qc.ca/wp-content/uploads/2021/05/Policy-No.-7-IPESA-FINAL.pdf.

Religious Holidays (Article 3.2.13 and 4.1.6). Students who wish to miss classes in order to observe religious holidays must inform their teacher of their intent in writing within the first two weeks of the semester.

Student Rights and Responsibilities: (Article 3.2.18). It is the responsibility of students to keep all assessed material returned to them and/or all digital work submitted to the teacher in the event of a grade review. (The deadline for a Grade Review is 4 weeks after the start of the next regular semester.)

Student Rights and Responsibilities: (Article 3.3.6). Students have the right to receive graded evaluations, for regular day division courses, within two weeks after the due date or exam/test date, except in extenuating circumstances. A maximum of three (3) weeks may apply in certain circumstances (ex. major essays) if approved by the department and stated on the course outline. For evaluations at the end of the semester/course, the results must be given to the student by the grade submission deadline (see current Academic Calendar). For intensive courses (i.e.: intersession, abridged courses) and AEC courses, timely feedback must be adjusted accordingly.

Academic Procedure: Academic Integrity, Cheating and Plagiarism (Article 9.1 and 9.2). Cheating and plagiarism are unacceptable at John Abbott College. They represent infractions against academic integrity. Students are expected to conduct themselves accordingly and must be responsible for all of their actions.

College definition of Cheating: Cheating means any dishonest or deceptive practice relative to examinations, tests, quizzes, lab assignments, research papers or other forms of evaluation tasks. Cheating includes, but is not restricted to, making use of or being in possession of unauthorized material or devices and/or obtaining or providing unauthorized assistance in writing examinations, papers or any other evaluation task and submitting the same work in more than one course without the teacher's permission. It is incumbent upon the department through the teacher to ensure students are forewarned about unauthorized material, devices or practices that are not permitted.

College definition of Plagiarism: Plagiarism is a form of cheating. It includes copying or paraphrasing (expressing the ideas of someone else in one's own words), of another person's work or the use of another person's work or ideas without acknowledgement of its source. Plagiarism can be from any source including books, magazines, electronic or photographic media or another student's paper or work.

Course Content (with selected exercises)**.** The exercises listed below should help you practice and learn the material taught in this course; they form a good basis for homework but they don't set a limit on the type of question that may be asked. Your teacher may supplement this list or assign equivalent exercises during the semester. Regular work done as the course progresses should make it easier for you to master the course.

An item beginning with a decimal number (e. g., 3.5) refers to a section in *Single Variable Calculus: Early Transcendentals*, 8th edition. Answers to odd-numbered exercises can be found in the back of the text. Additional resources for the textbook may be found at

http://stewartcalculus.com/media/17_home.php

Techniques of Integration.

5.5 The Substitution Rule (7–28, 30–35, 38–48, 53–71, 79, 87–91)
7.1 Integration by Parts (3–13, 15, 17–24, 26–42)
7.3 Trigonometric Substitution (optional) (5–20, 22–30)
7.4 Integration of Rational Functions and Partial Fractions (1–36, 46–51, 53, 54)
7.5 Strategy for Integration (1–80; skip 53, 74)

Indeterminate Forms.

4.4 Indeterminate Forms and l'Hospital's Rule (13–67; skip 24, 28, 29, 38, 42, 58)

Applications of Integration.

6.1 Areas Between Curves (1–14, 17, 22, 23, 25, 27)
6.2 Volumes (1–12, 15–18)
9.3 Separable Equations (1–14, 16–20, 39, 42, 45–48)
9.4 Models for Population Growth (9, 11)

Infinite Sequences and Series.

11.1 Sequences (1–3, 13–18, 23–51)
11.2 Series (1–4, 17–47, 60–62; skip 45)
11.3 The Integral Test and Estimates of Sums (3–5, 21, 22, 29)
11.4 The Comparison Tests (1–31, 41, 44–46)
11.5 Alternating Series (2–7, 12–15)
11.6 Absolute Convergence and the Ratio and Root Tests (1–38)
11.7 Strategy for Testing Series (1–28, 30–34)
11.8 Power Series (3–21, 23–26, 29–31)
11.10 Taylor and Maclaurin Series (3–9, 11–14, 21–26)
11.11 Applications of Taylor Polynomials

To strengthen your skills with more practice, attempt any of the exercises in the sections above which are not omitted explicitly below. While doing so, it is a good idea to focus on types of problems with which you struggle.

Practice exercises: Omissions

1.5: Omit 63–76
Chap. 1 Review: omit 25c, 25d, 26d
2.5: omit 29, 32, 60
2.6: omit 35, 40
Chap. 2 Review: omit 19
3.5: omit 17, 49–64
Chap. 3 Review: omit 6, 12, 17, 31, 38, 43, 45, 47, 48
4.1: omit 42, 62
4.2: omit 34, 35
4.3: omit 56, 64
4.5: omit 41, 42, 45, 48, 51–54, 71
4.9: omit 18, 19, 22, 24, 33, 44
Chap. 4 Review: omit 7–14, 31, 33, 34, 61–64 , 66, 68, 73, 81–83
5.3: omit 38, 39, 42, 62
5.4: omit 12, 13, 30, 40, 41, 43, 48
Chap. 5 Review: omit 8, 14, 17–24, 26–38, 41, 42, 44, 56, 63, 65, 66, 71

OBJECTIVE	STANDARD
Statement of the Competency Analyze scientific problems by applying integral calculus (7MA2)	**Performance Criteria for the Competency as a Whole** • Accurate recognition of the context in which integral calculus emerged • Correct use of mathematical terminology an syntax. • Algebraic manipulations in accordance with established rules. • Appropriate use of required technological tools. • Demonstration of rigorous mathematical reasoning through the use of concepts, properties and theorems.
Elements of the Competency	**Performance Criteria**
1. Evaluate the limit of a function yielding an indeterminate form.	• Correct recognition of all indeterminate forms. • Correct manipulation of indeterminate forms. • Accurate determination of a limit by using l'Hopital's rule.
2. . Determine the indefinite integral of a function.	• Correct use of the basic derivative rules and formulas in order to determine the anti-derivative. • Correct use of the substitution rule (change of variables) • Relevant application of the rules, formulas and some common integration techniques
3. Determine the definite integral of a function over an interval.	• Correct use of the definition and properties of the definite integral. • Correct use of the Fundamental Theorem of Calculus.
4. Expand functions into power series	• Accurate determination of the general term of a series. • Proper determination of the convergence or divergence of series in $\mathbb{R}$. • Accurate determination of the interval of convergence of a power series. • Accurate determination of the Maclaurin series expansion of a function.
5. Use integral calculus methods in mathematical applications.	• Appropriate graphical representation of a bounded region. • Accurate determination of the area of a bounded region. • Accurate determination of the volume of a solid of revolution. • Accurate determination of an improper integral. • Accurate determination of an integral using a Maclaurin series expansion.
6. Carry out the analysis of problems related to the sciences	• Rigorous use of integral calculus methods • Correct problem solving through the use of definite and indefinite series and integrals. • Correct problem solving by using differential equations with separable variables. • Accurate interpretation of results.

Specific Performance Criteria	Intermediate Learning Objectives
1. *Limits of indeterminate forms*	
1.1 Use of l'Hôpital's rule to determine limits of indeterminate forms.	1.1.1. State l'Hôpital's rule and the conditions under which it is valid. 1.1.2. Calculate limits of the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ using l'Hôpital's rule. 1.1.3. For the indeterminate forms $0 \cdot \infty$, $\infty - \infty$, 1^{∞}, 0^{0}, ∞^{0}, use the appropriate transformation to determine the limit using l'Hôpital's rule.
2. *Indefinite integrals*	
2.1 Use of basic substitutions to determine simple indefinite integrals.	2.1.1. Express Calculus I differentiation rules as antidifferentiation rules. 2.1.2. Use these antidifferentiation rules and appropriate substitutions to calculate indefinite integrals.
2.2 Use of more advanced techniques to determine more complex indefinite integrals.	2.2.1. Use identities to prepare indefinite integrals for solution by substitution. 2.2.2. Evaluate an indefinite integral using integration by parts. 2.2.3. Evaluate an indefinite integral using simple trigonometric identities. 2.2.4. Evaluate an indefinite integral by trigonometric substitution 2.2.5. Evaluate an indefinite integral by selecting an appropriate technique. 2.2.6. Evaluate an indefinite integral by using a combination of techniques.
3. *Definite integrals*	
3.1 Use of the Fundamental Theorem of Calculus to evaluate a definite integral.	3.1.1. Use the Fundamental Theorem of Calculus to calculate definite integrals.
4. *Infinite series*	
4.1 Determination of the convergence or divergence of a sequence.	4.1.1. State the definition of the limit of a sequence. 4.1.2. Determine whether a sequence converges, and calculate its limit if it does, using: properties of the limit of a sequence; l'Hôpital's Rule; the Squeeze Theorem.
4.2 Determination of the convergence or divergence of an infinite series of positive terms.	4.2.1. State the definition of convergence for an infinite series. 4.2.2. State the test for divergence of an infinite series. 4.2.3. Use 6.2.1 to determine if a telescoping series converges, and if so, calculate the sum. 4.2.4. State the criterion for the convergence of an infinite geometric series. 4.2.5. Calculate the sum of a convergent geometric series (4.2.4); use this to solve appropriate problems (*e.g.*, the distance travelled by a bouncing ball). 4.2.6. State the p-series, (direct) comparison, limit comparison, ratio and (n^{th}) root tests for convergence of an infinite series. 4.2.7. Determine whether an infinite series converges or diverges by choosing (and using) correct methods among (4.2.1–4.2.6)
4.3 Determination of the convergence, conditional or absolute, or divergence of an infinite series.	4.3.1. State the definitions of absolute and conditional convergence of an infinite series. 4.3.2. State the definition of an alternating series. 4.3.3. State the criterion for the (conditional) convergence of an alternating series. 4.3.4. Determine if an infinite series is absolutely convergent, conditionally convergent, or divergent, using the methods of (4.2.1–4.2.7, 4.3.1–4.3.3).
4.4 Expression of functions as power series	4.4.1. Use the methods of (4.2, 4.3) to find the radius and interval of convergence for a power series. 4.4.2. State the definitions of the Maclaurin polynomials of degree n for a function f. 4.4.3. State the definitions of the Maclaurin series for a function f. 4.4.4. Develop power series representations for functions by modifying known series for $\frac{1}{1-x}$, e^{x}, $\sin(x)$ and $\cos(x)$. 4.4.5. Use 4.4.4 to develop power series representations for indefinite integrals.
5. *Areas, volumes, and lengths*	
5.1 Use of differentials to set up definite integrals.	5.1.1. Analyze a quantity A as a sum $\sum \Delta A$ over an interval $[\,a, b\,]$; approximate ΔA by a product $f(x)\,dx$; conclude that A is the definite integral $\int_a^b f(x)\,dx$.
5.2 Calculation of areas of planar regions.	5.2.1. Use 5.1.1 to set up a definite integral to calculate an area. 5.2.2. Sketch the area between two functions ($y = f(x)$, $y = g(x)$) and use 5.2.1 to calculate the area.
5.3 Calculation of volumes	5.3.1. Use 5.1.1 to set up a definite integral to calculate the volume of a solid by cross-sections.
6. *Differential equations*	
6.1 Use the language of differential equations to express physical problems.	6.1.1. Translate a physical problem into the language of differential equations.
6.2 Use of antidifferentiation to obtain general solutions to simple differential equations.	6.2.1. Express a simple differential equation in the language of integration, and obtain the general solution.
6.3 Use of antidifferentiation to obtain particular solutions to simple initial value problems.	6.3.1. Express a simple initial value problem in the language of integration, and obtain the particular solution.