Differential Calculus

General Information.
Discipline: Mathematics — *Course code*: 201-A1S-AB
Ponderation: 3-2-3 — *Credits*: $2\frac{2}{3}$
Prerequisite: Secondary V mathematics, Science option or Technical and Scientific option (SN5 or TS5)
Objective: 7MA1 (Analyze scientific problems by applying differential calculus)

Your teacher will give you his/her schedule and availability.
Students are strongly advised to seek help promptly from their teacher if they encounter difficulties in the course.

Introduction. Differential Calculus is the first of the required mathematics courses in the Arts and Sciences program. It is usually taken in the first semester and it introduces the student to the limit processes that are so vital to the development of calculus. Since differential calculus is a basic tool in physics, some of the applications will be related to problems in physics. Differential calculus can also be applied to problems in chemistry and biology.

The main purpose of the course is the attainment of objective 7MA1 ("Analyze scientific problems by applying differential calculus"). The course will help the student understand the following basic concepts: limits, continuity and derivatives involving real-valued functions of one variable (including algebraic, trigonometric, exponential, logarithmic and inverse trigonometric functions).

Emphasis will be placed on clarity and rigour in reasoning and in the application of methods. The student will learn to interpret the derivative both as a mathematical tool and as a rate of change. The derivative will be used in various contexts including velocity, acceleration, curve sketching, optimization and related rates. The basic concepts are illustrated by applying them to various problems where their application helps arrive at a solution. In this way, the course encourages the student to apply learning acquired in one context to problems arising in another. Towards the end of the course, the student will be introduced to antiderivatives in order to help with the transition from Differential Calculus to Integral Calculus.

Textbook. Your teacher may require *Single Variable Calculus: Early Transcendentals*, 9th edition, by James Stewart (Brooks/Cole). It is available from the college bookstore for about $132. Note that the 7th or 8th editions of this book are also suitable.

Course Costs. In addition to the cost of the textbook (see above), your instructor might recommend you acquire an inexpensive scientific calculator ($15–$25). *No calculators are allowed during tests or the final exam.*

Teaching Methods. This course will be 75 hours, meeting three times a week for a total of five hours a week. It relies mainly on the lecture method, although some of the following techniques are also used: question-and-answer sessions, labs, problem solving periods, class discussions, and assigned reading for independent study. No marks are deducted for absenteeism, however failure to keep pace with the lectures typically results in a cumulative inability to cope with the material and a failure in the course. A student will generally succeed or fail depending on how many problems have been attempted and solved successfully. It is entirely the student's responsibility to complete suggested homework assignments as soon as possible following the lecture, as the material will be fresher in his/her mind. This also allows the student the maximum benefit from any discussion of the homework. Answers to a selected number of problems can be found in the back of the text. Individual teachers may provide supplementary notes and problems as they see fit.

High School Topics Review. The transition from High School to CEGEP Math can sometimes be difficult, especially if you've forgotten some of the topics or techniques learned in High School. To that end, we've made videos and practice problems available to help you review a few key concepts. You can find them here: High School Review

Evaluation Plan. The Final Evaluation in this course consists of the Final Exam, which covers all elements of the competency. A student's Final Grade is a combination of the Class Mark and the mark on the Final Exam.

The Class Mark will be 75% (three to five in-class written tests) and 25% consisting of minor assessments, including a least one piece of written feedback before each test. The specifics of the Class Mark are included in an appendix that is distributed to students along with this course outline.

The Final Grade will be the better of:

50% Class Mark and 50% Final Exam Mark
or
25% Class Mark and 75% Final Exam Mark

A student *choosing not to write* the Final Exam will receive a failing grade of 50% or their Class Mark, whichever is less.

Students must be available until the end of the final examination period to write exams.

Note that in the event of unexpected changes to the academic calendar, the evaluation plan may be modified.

Other Resources.

Math Website.
http://departments.johnabbott.qc.ca/departments/mathematics

Math Study Area. Located in H-200A and H-200B; the common area is usually open from 8:30 to 17:30 on weekdays as a quiet study space. Computers and printers are available for math-related assignments. It is also possible to borrow course materials when the attendant is present.

Math Help Centre. Located near H-211; teachers are on duty from 8:30 until 15:30 to give math help on a drop-in basis.

Peer Tutoring. Starting on the fifth week of each semester, first year students can be paired with a fellow finishing student for a weekly appointment of tutoring. Ask your teacher for details.

Academic Success Centre. The Academic Success Centre, located in H-139, offers study skills workshops and individual tutoring.

College Policies.

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Policy No. 7 - IPESA, Institutional Policy on the Evaluation of Student Achievement: https://www.johnabbott.qc.ca/wp-content/uploads/2021/05/Policy-No.-7-IPESA-FINAL.pdf.

Religious Holidays (Article 3.2.13 and 4.1.6). Students who wish to miss classes in order to observe religious holidays must inform their teacher of their intent in writing within the first two weeks of the semester.

Student Rights and Responsibilities: (Article 3.2.18). It is the responsibility of students to keep all assessed material returned to them and/or all digital work submitted to the teacher in the event of a grade review. (The deadline for a Grade Review is 4 weeks after the start of the next regular semester.)

Student Rights and Responsibilities: (Article 3.3.6). Students have the right to receive graded evaluations, for regular day division courses, within two weeks after the due date or exam/test date, except in extenuating circumstances. A maximum of three (3) weeks may apply in certain circumstances (ex. major essays) if approved by the department and stated on the course outline. For evaluations at the end of the semester/course, the results must be given to the student by the grade submission deadline (see current Academic Calendar). For intensive courses (i.e.: intersession, abridged courses) and AEC courses, timely feedback must be adjusted accordingly.

Academic Procedure: Academic Integrity, Cheating and Plagiarism (Article 9.1 and 9.2). Cheating and plagiarism are unacceptable at John Abbott College. They represent infractions against academic integrity. Students are expected to conduct themselves accordingly and must be responsible for all of their actions.

College definition of Cheating: Cheating means any dishonest or deceptive practice relative to examinations, tests, quizzes, lab assignments, research papers or other forms of evaluation tasks. Cheating includes, but is not restricted to, making use of or being in possession of unauthorized material or devices and/or obtaining or providing unauthorized assistance in writing examinations, papers or any other evaluation task and submitting the same work in more than one course without the teacher's permission. It is incumbent upon the department through the teacher to ensure students are forewarned about unauthorized material, devices or practices that are not permitted.

College definition of Plagiarism: Plagiarism is a form of cheating. It includes copying or paraphrasing (expressing the ideas of someone else in one's own words), of another person's work or the use of another person's work or ideas without acknowledgement of its source. Plagiarism can be from any source including books, magazines, electronic or photographic media or another student's paper or work.

Course Content (with selected exercises)**.** This is a *minimal* list of exercises which you should attempt, assuming you are also doing regular homework (*e.g.*, WEBWORK) assigned by your instructor. **These problems can be found in the 8th edition of the textbook - a PDF file will be provided.**

1.1: 1, 2, 7, 9, 14, 49, 55, 63, 72, 73
1.2: 1, 3, 8, 10, 11, 17
1.3: 3, 6, 12, 24, 30, 31, 37, 41, 45, 53, 61, 63, 65
1.4: 12, 14, 21, 23, 30, 37
1.5: 18, 21, 37, 41, 50, 53, 56, 61
Chap. 1 Review (p. 69): 2, 10, 17, 18, 23, 24, 25a, 25b
2.1: [instructor's discretion]
2.2: 6, 9, 11, 17, 31, 35, 40
2.3: 2, 9, 15, 19, 23, 29, 37, 40, 43, 45, 59, 64, 65
2.4: [instructor's discretion]
2.5: 4, 7, 21, 33, 35, 41, 46, 47, 52, 69, 71
2.6: 4, 9, 21, 24, 29, 32, 33, 39, 45, 49, 52, 54, 59, 63, 67
2.7: 3, 8, 10, 11, 17, 22, 23, 28, 35, 37, 39, 59, 60
2.8: 3, 25, 28, 30, 43, 51, 57, 63, 67
Chap. 2 Review (p. 166): 1, 2, 8, 12, 15, 17, 20, 22, 23, 29, 33, 36, 43, 47, 54
3.1: 3, 9, 23, 24, 27, 31, 37, 55, 60, 62, 66, 69, 81, 83, 86
3.2: 9, 15, 20, 23, 25, 26, 27, 32, 41, 44, 48, 49, 52, 53, 62
3.3: 5, 8, 13, 16, 20, 22, 32, 34, 37, 43, 45, 50, 52, 54, 58
3.4: 19, 25, 38, 40, 41, 44, 45, 48, 53, 59, 65, 74, 76, 78, 84
3.5: 7, 16, 19, 20, 21, 23, 29, 38, 39, 43, 46, 75, 77, 79, 80
3.6: 9, 12, 13, 21, 22, 29, 34, 40, 42, 45, 49, 50, 52, 56
3.7: 1, 5, 10
3.8: [instructor's discretion]
3.9: 12, 18, 23, 27, 29, 30, 33, 42, 44, 47, 50
Chap. 3 Review (p. 267): 28, 37, 41, 50, 53, 59, 60, 66, 81, 85, 89, 95, 98, 109, 112
4.1: 5, 7, 10, 13, 27, 39, 43, 44, 51, 56, 57, 63, 67, 72, 77
4.2: 9, 12, 18, 19, 22, 24, 25, 37, 38
4.3: 8, 11, 12, 16, 18, 19, 27, 30, 35, 45, 47, 55, 60, 75, 79, 82
4.5: 7, 13, 15, 24, 27, 30, 34, 39, 40, 43, 46, 50
4.7: 13, 16, 22, 33, 34, 36, 39, 50, 58, 68, 71, 73, 74
4.9: 5, 12, 15, 16, 29, 37, 38, 47, 50, 53, 55, 61, 63, 69, 77
Chap. 4 Review (p. 359): 5, 6, 17, 24, 29, 32, 47, 54, 72, 74, 78, 79, 84, 85
5.1: 5, 7, 17, 21, 25, 27
5.2: 4, 7, 17, 25, 30, 33, 37, 49, 53
5.3: 3, 7, 18, 26, 29, 33, 37, 41, 43, 63, 65, 73, 75, 83
5.4: 1, 4, 9, 14, 15, 16, 18, 29, 31, 37, 38, 46, 49, 61, 71
Chap. 5 Review (p. 422): 2, 5, 7, 12, 25, 40, 49, 58, 62, 67, 72

To strengthen your skills with more practice, attempt any of the exercises in the sections above which not omitted explicitly below. While doing so, it is a good idea to focus on types of problems with which you struggle.

Practice exercises: Omissions

1.5: Omit 63–76
Chap. 1 Review: omit 25c, 25d, 26d
2.5: omit 29, 32, 60
2.6: omit 35, 40
Chap. 2 Review: omit 19
3.5: omit 17, 49–64
Chap. 3 Review: omit 6, 12, 17, 31, 38, 43, 45, 47, 48
4.1: omit 42, 62
4.2: omit 34, 35
4.3: omit 56, 64
4.5: omit 41, 42, 45, 48, 51–54, 71
4.9: omit 18, 19, 22, 24, 33, 44
Chap. 4 Review: omit 7–14, 31, 33, 34, 61–64 , 66, 68, 73, 81–83
5.3: omit 38, 39, 42, 62
5.4: omit 12, 13, 30, 40, 41, 43, 48
Chap. 5 Review: omit 8, 14, 17–24, 26–38, 41, 42, 44, 56, 63, 65, 66, 71

OBJECTIVE	STANDARD
Statement of the Competency Analyze scientific problems by applying differential calculus (7MA1).	**Performance Criteria for the Competency as a Whole** • Accurate recognition of the context in which differential calculus emerged • Proper use of language and concepts in the application of differential calculus • Correct use of mathematical terminology and syntax • Algebraic manipulations in accordance with established rules • Demonstration of rigorous mathematical reasoning through the use of concepts, properties and theorems
Elements of the Competency	**Performance Criteria**
1. To recognize and describe the characteristics of a function expressed in symbolic or graphical form.	• Recognition of functions • Finding domain, range and intercepts • Graphing of basic functions • Performing operations on functions
2. Determine the limit of a function.	• Accurate algebraic and graphic determination of the limit of a function • Accurate determination of infinite limits and limits at infinity • Correct use of algebraic manipulation for evaluating an indeterminate form • Accurate determination of the continuity of a function at a point and on an interval
3. Determine the derivative function.	• Correct distinction between average and instantaneous rates of change • Correct use of the definition of the derivative • Exact calculation of the derivative function • Accurate interpretation of the derivative function • Relevant application of derivative rules and formulas
4. Use the methods of differential calculus in mathematical applications.	• Accurate determination of the equation of the tangent line to a function at a point • Relevant application of the methods of differential calculus to analyze a function
5. Carry out the analysis of problems related to the natural sciences.	• Application of appropriate methods of differential calculus • Correct resolution of problems involving rates of change • Correct resolution of optimization problems • Accurate interpretation of results
6. Apply basic rules and techniques of integration	• Evaluation of the indefinite integral • Evaluation of the definite integral • Solution of simple differential equations

Specific Performance Criteria	**Intermediate Learning Objectives**
1. *Functions*	
1.1 Recognition of functions	1.1.1. Decide whether a given relation is a function from its graphical representation. 1.1.2. Recognize and name the following functions from their symbolic representations: $f(x)=c$ constant; $f(x)=\tan x$ tangent $f(x)=ax+b$ linear; $f(x)=\cot x$ cotangent $f(x)=ax^2+bx+c$ quadratic; $f(x)=\sec x$ secant $f(x)=\lvert x\rvert$ absolute value; $f(x)=\csc x$ cosecant $f(x)=\sqrt{x}$ square root; $f(x)=\arcsin(x)$ inverse sine $f(x)=\sqrt[n]{x}$ nth root function; $f(x)=\arccos(x)$ inverse cosine $f(x)=a^x$ exponential; $f(x)=\arctan(x)$ inverse tangent $f(x)=\log_a x$ logarithmic; $f(x)=\operatorname{arccot}(x)$ inverse cotangent $f(x)=\sin x$ sine; $f(x)=\operatorname{arcsec}(x)$ inverse secant $f(x)=\cos x$ cosine; $f(x)=\operatorname{arccsc}(x)$ inverse secant 1.1.3. Recognize and name the functions listed in 1.1.2 from their graphical representations.
1.2 Finding domain, range and intercepts	1.2.1. Find and state the domain of functions listed in 1.1.2 from both their graphical and their symbolic representations. 1.2.2. Find and state the range of functions listed in 1.1.2 and 1.1.3 from both their graphical and their symbolic representations. 1.2.3. Find and state the x- and y-intercepts, if they exist, of functions listed in 1.1.2 from both their graphical and their symbolic representations.
1.3 Graphing of functions	1.3.1. Graph the functions listed in 1.1.2. 1.3.2. Graph piecewise defined functions whose pieces are made up of the functions listed in 1.1.2. 1.3.3. Find the value of a function at a point in its domain. 1.3.4. Evaluate $\dfrac{f(x+h)-f(x)}{h}$ (the difference quotient) for linear, quadratic and simple rational functions.
1.4 Appropriate use of functions to represent given situations	1.4.1. Given an applied problem, decide which function best represents the situation and express the relationship using appropriate notation.
2. *Limits, Continuity and Derivatives*	
2.1 Determination of Limits	2.1.1. Give an intuitive description of the limit of a function at a point. 2.1.2. Evaluate a limit of a function by viewing the graph of the function. 2.1.3. Evaluate a limit analytically by direct substitution, factoring, rationalizing, simplifying rational expressions. 2.1.4. Evaluate analytically limits at infinity. 2.1.5. Evaluate one-sided limits. 2.1.6. Recognize and evaluate infinite limits.
2.2 Determination of whether a function is continuous at a point or on an interval	2.2.1. Define continuity of a function at a point; that is, state the conditions which must be satisfied in order that a function be continuous at a point. 2.2.2. Use the definition of continuity to determine if a function is continuous at a specific point. 2.2.3. Determine on which interval(s) a function is continuous.
2.3 Use of the limit definition of the derivative	2.3.1. Define the derivative of a function as (i) the limit of a difference quotient, (ii) the slope of a tangent line, and (iii) the rate of change (in particular the velocity function associated with a position function). 2.3.2. Use the limit definition of the derivative to determine the derivative of a polynomial of degree 1, 2 or 3, square root and simple rational functions. 2.3.3. Use the limit definition of the derivative to determine the numerical value of the derivative at a given point. 2.3.4. Use the limit definition of the derivative to determine the slope of the tangent line to a curve at a specific point. 2.3.5. Use the limit definition of the derivative to determine the equation of the tangent line to a curve at a specific point.
2.4 Use of the graph of a function to determine whether a function is differentiable at a point or on an interval	2.4.1. Determine if the derivative of a function exists at a point or on an interval by examining the graph of the function.
3. *Rules and Techniques of Differentiation*	
3.1 Recognition of the equivalence of various derivative notations	3.1.1. Recognize different notations for the derivative of y with respect to x: y', $f'(x)$, $\dfrac{dy}{dx}$, $\dfrac{d}{dx}f(x)$
3.2 Use of basic differentiation formulas and rules and proof of simple propositions	3.2.1. Recognize when and how to use the basic differentiation formulas: $\dfrac{d}{dx}[c]=0$; $\dfrac{d}{dx}[\sin x]=\cos x$; $\dfrac{d}{dx}[\arcsin(x)]=\dfrac{1}{\sqrt{1-x^2}}$ $\dfrac{d}{dx}[x^n]=nx^{n-1}$; $\dfrac{d}{dx}[\cos x]=-\sin x$; $\dfrac{d}{dx}[\arccos(x)]=\dfrac{-1}{\sqrt{1-x^2}}$ $\dfrac{d}{dx}[e^x]=e^x$; $\dfrac{d}{dx}[\tan x]=\sec^2 x$; $\dfrac{d}{dx}[\arctan(x)]=\dfrac{1}{x^2+1}$ $\dfrac{d}{dx}[\ln x]=\dfrac{1}{x}$; $\dfrac{d}{dx}[\cot x]=-\csc^2 x$; $\dfrac{d}{dx}[\operatorname{arccot}(x)]=\dfrac{-1}{x^2+1}$ $\dfrac{d}{dx}[a^x]=a^x\ln a$; $\dfrac{d}{dx}[\sec x]=\sec x\tan x$; $\dfrac{d}{dx}[\operatorname{arcsec}(x)]=\dfrac{1}{x\sqrt{x^2-1}}$ $\dfrac{d}{dx}[\log_a x]=\dfrac{1}{x\ln a}$; $\dfrac{d}{dx}[\csc x]=-\csc x\cot x$; $\dfrac{d}{dx}[\operatorname{arccsc}(x)]=\dfrac{-1}{x\sqrt{x^2-1}}$ 3.2.2. Recognize when and how to use the following rules: constant rule, power rule, constant multiple rule, sum and difference rule. 3.2.3. Recognize when and how to use the product, quotient and chain rules. 3.2.4. Prove a selection of the rules in 3.2.1 using the limit definition of the derivative.
3.3 Determination of whether a function is differentiable at a point or on an interval	3.3.1. Determine whether a function is differentiable at a specified point or on an interval using graphical methods.
3.4 Use of differentiation rules to perform implicit and logarithmic differentiation	3.4.1. Recognize when and how to use implicit differentiation to compute first derivatives. 3.4.2. Recognize when and how to use logarithmic differentiation.
3.5 Evaluation and application of higher order derivatives	3.5.1. Find higher order derivatives. 3.5.2. Use higher-order derivatives to solve position, velocity and acceleration problems.
3.6 Use of derivatives to find the slope of a tangent (normal) line to a curve at a point	3.6.1. Use the differentiation rules listed in 3.2.1 and 3.2.2 to find the slope of the tangent line to a curve at a point. 3.6.2. Use the differentiation rules listed in 3.2.1 and 3.2.2 to find the equation of the tangent line to a curve at a point. 3.6.3. Use the differentiation rules listed in 3.2.1 and 3.2.2 to find the equation of the normal line to a curve at a point.

Specific Performance Criteria	**Intermediate Learning Objectives**
4. *Graphing of Functions*	
4.1 Use of the derivative and related concepts to analyze the variations of a function and to sketch a graph of the function	4.1.1. Find critical numbers. 4.1.2. Find intervals on which a function is increasing and decreasing using the sign of the first derivative. 4.1.3. Find relative and absolute extrema. 4.1.4. Use the first or second derivative test to decide whether the critical points represent relative maxima or relative minima. 4.1.5. Find inflection points. 4.1.6. Find intervals on which a function is concave up or concave down using the sign of the second derivative. 4.1.7. Use limits to find all vertical and horizontal asymptotes. 4.1.8. Use 4.1.1–4.1.7 to graph polynomial, rational, trigonometric, logarithmic and exponential functions.
5. *Optimization Problems*	
5.1 Solution of optimization problems	5.1.1. Represent an optimization word problem in functional form. 5.1.2. Determine the quantity, P, to be maximized or minimized and identify the variables which are involved. 5.1.3. Draw a diagram, if possible, to illustrate the problem and list any other relationship(s) between the variables. 5.1.4. Express P as a function of one variable. 5.1.5. Find absolute maxima or minima for P using methods from 4.1 5.1.6. Interpret (explain) the results found in the optimization problem.
6. *Integration*	
6.1 Evaluation of the indefinite integral	6.1.1. Give the definition of the indefinite integral as an antiderivative. 6.1.2. Express the basic differentiation formulas listed in 3.2.1 as antidifferentiation formulas. 6.1.3. Recognize when and how to use the constant multiple rule and the sum and difference rule in the evaluation of integrals. 6.1.4. Use the antidifferentiation formulas from 6.1.2 and the rules in 6.1.3 to evaluate indefinite integrals.
6.2 Evaluation of the definite integral	6.2.1. State the definition of the definite integral. 6.2.2. State the Fundamental Theorem of Calculus. 6.2.3. Find the definite integral of functions described in 6.1. 6.2.4. Use the Fundamental Theorem of Calculus to find the area of a region under a curve on a closed interval.
6.3 Solution of simple differential equations	6.3.1. Find the general solution to a differential equation of the form $y' = f(x)$. 6.3.2. Find the particular solution to a differential equation of the form $y' = f(x)$ given an initial condition $y(a) = b$.