UNIVERSITY of BRADFORD

Faculty of Engineering and Informatics

ENM7005-B
MODELLING AND OPTIMISATION

MODELLING COURSEWORK

Catapult Experiments on a Virtual Model

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1. INTRODUCTION

Design of experiments (DoE) can be defined as a method of systematically applying statistics to experimentations. These experimentations may be virtual or laboratory based. In most real-life engineering applications or designs, many factors or input variables exist. These factors interact with each other and in turn affect the response or output of the system. The input variables are usually independent variables and good examples are temperature, speed, force etc. The output variables are usually dependent on the input variables and good examples are efficiency, power, wear rate, etc.

DoE allows multiple input factors to be manipulated and their effect on the desired output is determined from the results. This enables engineers to identify which factors have the maximum impact on the output or which interactions have a huge impact on the response. There are many methods used for DoE approach – Classical, Shainin, Taguchi, Full Factorial and Half-factorial are just some of the design of experiments approaches (Rafidah et al., 2014). In engineering practices, Full Factorial approach is most preferred due to its low mean square error, efficiency and simplicity.

The other reason as to why engineers prefer using DoE is to reduce the cost of fabricating prototypes. For instance, if one is investigating input variables which require 20 experimental runs, one might be required to have about 20 fabricated prototypes. However, by using DoE, time and cost of optimisation will be greatly reduced. In this report, the virtual catapult will be used to show how DoE can be applied in the design process.

1.1 Terms and Concepts Used In Design of Experiment

As mentioned, a DoE methodology consists of controllable and uncontrollable input factors and responses. Figure 1 shows a simple flow chart of a DoE setup.

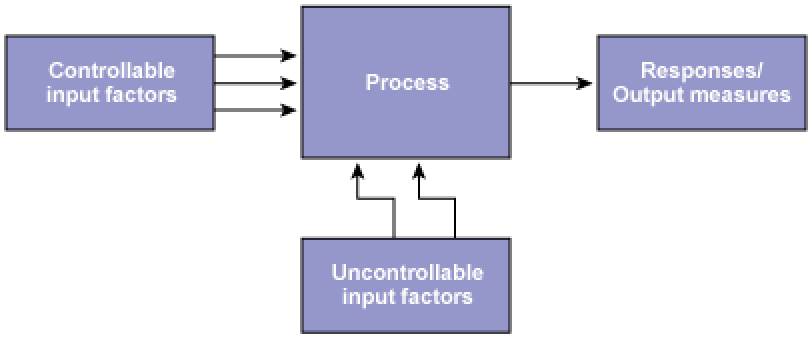


Figure 1 Input factors and output measures

- Controllable Input Factors – These are input parameters which can be modified in the experiment. For instance, in cooking, the amount of water and quality of food can be modified.
- Uncontrollable Input factors- These are parameters which cannot be changed. A good example is room temperature.
- Reponses – These are parameters which gauge the desired effect. This can be efficiency of a machine, taste and texture of food, etc.
- Levels – These are the lower and upper limits of the factors as shown in Table 1

Table 1 Demonstration of DoE Limits

Run	Lower Limit	Upper Limit
1	250	400
2	-157	200
3	-1	+25
4	+10	+10

- Centre point – The stability or variability of the process is measured using this feature. The curvature of the response surface is also investigated using the centre point.
- Response surface – The mean response at any level of the factors in the design space is depicted using the response surface.

1.2 Types of Experiments

There are two main considerations which distinguish experiment designs:

i. The number of design variables included
ii. The complexity of the model provided.

In most cases, a design contains many variables which give rise to many types of experimental designs. Of all the experimental designs, the extreme designs that stand out from all the others are screening experiments and response surface experiments.

Screening experiments are applicable where the most important design variables are to be identified from a large number of variables (Mathews, 2010). Some of the screening experiments may identify the critical variables with very few experimental or simulation runs. The only limitation of screening experiments is that they only use two levels of each design and cannot analyse the interactions between two variables. This makes such designs quite risky.

Response surface experiments, on the other hand, are more complex and difficult to demonstrate than screening experiments. They involve two to five variable. In a response surface experiment, each variable must have at least three levels and must be quantitative. The main advantage of response surface experiments is that they provide complex models which include – two factor interactions, the main effects and terms to measure the curvature induced in the response by each variable in the design.

1.3 Objectives

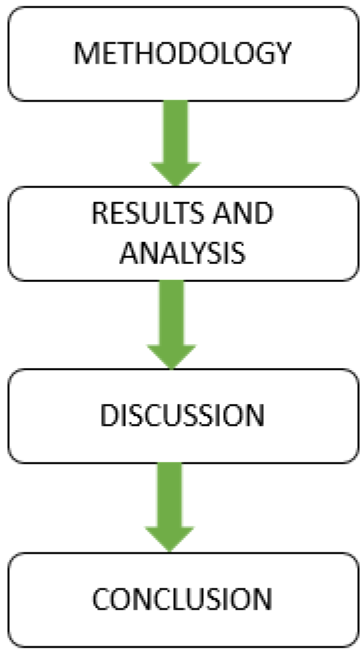
The main objective of this report is to demonstrate a critical understanding of design of experiments and response surface in theory and practice as applied to engineering problems. To achieve this, the following specific objectives are set:

- To conduct an analysis on the catapult experiment using design of experiments.
- To employ the usage of Response Surface Methodology (RSM) statistical analysis in optimisation.
- To develop a prediction model using MINITAB software.
- To draw conclusions from the data interpretation.

1.4 Scope and Limitations or Constraints

The scope of this optimisation assignment has fully covered what has been learned both in class and reference material. The only limitation is the unavailability of experiments for the DoE exercise due to the COVID-19 crisis. However, a virtual catapult found at https://sigmazone.com/catapult/ will be used for the optimisation and modelling exercises.

1.5 Outline of the Assignment



2 METHODOLOGY

For statistical analysis, the following tools in MINITAB were used

Tool	Purpose
Interaction Plot	To evaluate how various combinations of the input factors affect the response/output.
Main effects plot	This shows how individual factors affect the response.
Pareto Plots	To evaluate the absolute value of the standardised effect on the response.

Response surface plot	A 3D surface plot which shows the impact of two variables on the response.
Response surface regression analysis	This was used to test hypothesis, inspect errors, reduce the model size and predict model equation through parameter estimations.

2.1 Determining the variables and the levels

In this assignments, the response was the distance travelled by the ball when thrown using a virtual catapult. Table 2 shows the input variables and the respective levels.

Table 2 Input variables and levels used in DoE optimisation

Input Variable	Lower Level	Upper Level
Pin Elevation	120	180
Bungee Position	120	180
Cup Elevation	220	280
Release Angle	140	180

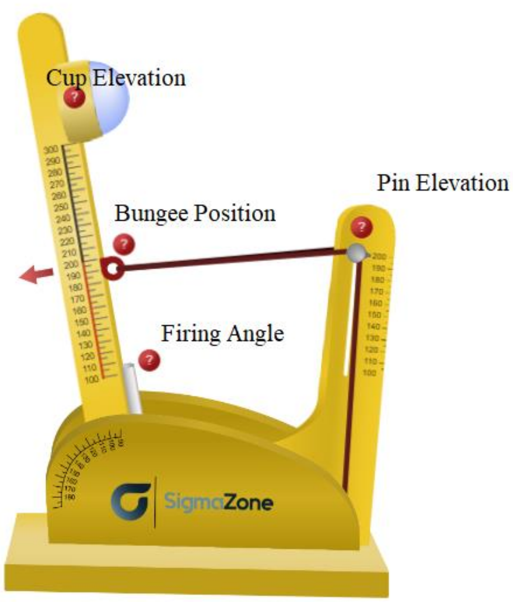


Figure 2 Virtual catapult used showing the input variables

In Table 2, there are a total of four variables with two levels. Therefore, the total number of runs or simulations were determined as follows.

$$X^{y} = \text{Total number of Simulations}$$

Where, X is the number of levels and y is the number of variables. Hence,

$$2^{4} = 16 \text{ Runs}$$

2.2 Full Factorial Design

The full factorial design was crucial in determining the important parameters. The important factors were found to have the highest impact on the firing distance. However, while using the full factorial design, it was not possible to obtain a 2nd or higher order polynomial (Minitab 18 Support - Minitab, 2020) . Therefore, use of central composite was employed and a quadratic model was obtained which in turn generated a more accurate curved response. For the full factorial design, the following steps were taken:

i. After MINITAB software was launched, the following navigation was done *Stat>DOE>Factorial>Create Factorial Design.*
ii. A prompt appeared in which the number of factors were to be put. It was noted that some of the ‘buttons’ were inactive as of this step. In this assignment, four factors were used and hence the number of factors was put to 4.
iii. After selecting the number of factors, the type of experimental design was selected by clicking *Designs* in the same prompt. *Full Factorial Design* was selected.
iv. Now, all the other tabs were active, the levels of the factors can now be put. The *Factors* tab was clicked and a prompt in which the factor name and levels appeared.
v. After keying in the factors, the *Options* tabs was clicked and *Randomize Runs* was disabled. If the factors in this assignment were time based, the *Randomize Runs* option would have been left active.

Once all the options were keyed in, *OK* was clicked and a table showing the number of runs appeared as shown below.

Table 3 Total number of runs for DOE- Full Factorial

C1	C2	C3	C4	C5	C6	C7	C8	C9
StdOrder	RunOrder	CenterPt	Blocks	Pin Elevation	Bungee Position	Cup Elevation	Release Angle	
1	1	1	1	120	120	220	140	
2	2	1	1	180	120	220	140	
3	3	1	1	120	180	220	140	
4	4	1	1	180	180	220	140	
5	5	1	1	120	120	280	140	
6	6	1	1	180	120	280	140	
7	7	1	1	120	180	280	140	
8	8	1	1	180	180	280	140	
9	9	1	1	120	120	220	180	
10	10	1	1	180	120	220	180	
11	11	1	1	120	180	220	180	
12	12	1	1	180	180	220	180	
13	13	1	1	120	120	280	180	
14	14	1	1	180	120	280	180	
15	15	1	1	120	180	280	180	
16	16	1	1	180	180	280	180	

Table 3 does not contain the response. The values of response were put after simulations were conducted. There are a total of 16 runs and this agrees with the calculation for number of runs.

2.3 Central Composite Design

In a central composite design, a mid-level value will be added between the upper and lower bounds. This will make it possible for MINITAB to develop a higher order polynomial. It will increase the number of runs as new interactions will be created. Table 4 shows the number of runs arising from central composite design.

Table 4 Central composite runs

C1	C2	C3	C4	C5	C6	C7	C8	C9
StdOrder	RunOrder	PtType	Blocks	Pin Elevation	Bungee Position	Cup ELevation	Release Angle	
1	1	1	1	120	120	220	140	
2	2	1	1	180	120	220	140	
3	3	1	1	120	180	220	140	
4	4	1	1	180	180	220	140	
5	5	1	1	120	120	280	140	
6	6	1	1	180	120	280	140	
7	7	1	1	120	180	280	140	
8	8	1	1	180	180	280	140	
9	9	1	1	120	120	220	180	
10	10	1	1	180	120	220	180	
11	11	1	1	120	180	220	180	
12	12	1	1	180	180	220	180	
13	13	1	1	120	120	280	180	
14	14	1	1	180	120	280	180	
15	15	1	1	120	180	280	180	
16	16	1	1	180	180	280	180	
17	17	-1	1	120	150	250	160	
18	18	-1	1	180	150	250	160	
19	19	-1	1	150	120	250	160	
20	20	-1	1	150	180	250	160	
21	21	-1	1	150	150	220	160	
22	22	-1	1	150	150	280	160	
23	23	-1	1	150	150	250	140	
24	24	-1	1	150	150	250	180	
25	25	0	1	150	150	250	160	
26	26	0	1	150	150	250	160	
27	27	0	1	150	150	250	160	
28	28	0	1	150	150	250	160	
29	29	0	1	150	150	250	160	
30	30	0	1	150	150	250	160	
31	31	0	1	150	150	250	160	

3 RESULTS

The simulations were run using the virtual catapult as guided by Table 3 and Table 4. The firing angle was held constant at 110°. Table 5 shows the ball distance for various input variable settings for the full factorial.

Table 5 Results for full factorial design

C1	C2	C3	C4	C5	C6	C7	C8	C9
StdOrder	RunOrder	CenterPt	Blocks	Pin Elevation	Bungee Position	Cup Elevation	Release Angle	Ball Distance
1	1	1	1	120	120	220	140	90
2	2	1	1	180	120	220	140	123
3	3	1	1	120	180	220	140	118
4	4	1	1	180	180	220	140	159
5	5	1	1	120	120	280	140	123
6	6	1	1	180	120	280	140	168
7	7	1	1	120	180	280	140	172
8	8	1	1	180	180	280	140	223
9	9	1	1	120	120	220	180	83
10	10	1	1	180	120	220	180	245
11	11	1	1	120	180	220	180	232
12	12	1	1	180	180	220	180	323
13	13	1	1	120	120	280	180	240
14	14	1	1	180	120	280	180	334
15	15	1	1	120	180	280	180	320
16	16	1	1	180	180	280	180	450

The analysis of the results shown in Table 5 were conducted as follows.

3.1 Part A: Screening Analysis

Screening of the results was crucial as it identified which factors had the maximum impact on ball distance (response).

3.1.1 Pareto Chart Effect

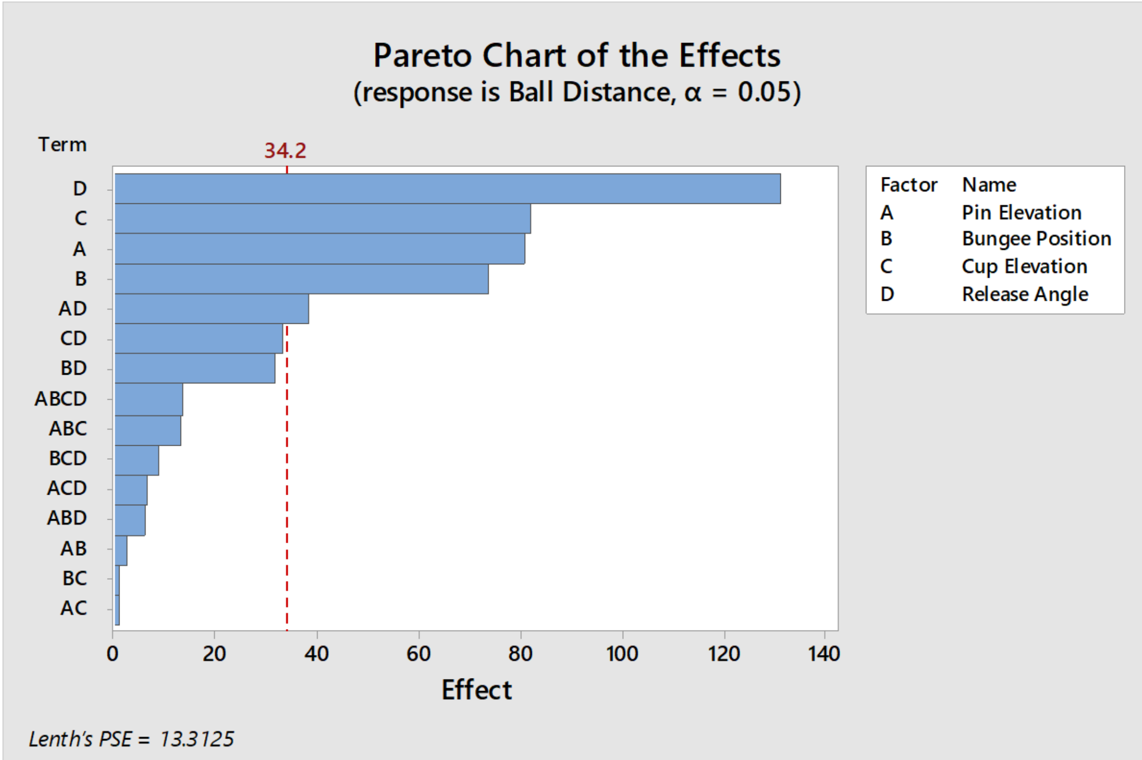


Figure 3 Pareto chart of effects

Figure 3 shows how each factor or a combination of factors affect the ball distance. The release angle has the highest impact on ball distance while the bungee position has the minimum effect on ball distance. It was also observed that the combination of pin elevation and release angle had a higher impact on ball distance than all other combinations. The combination between pin elevation and cup elevation had almost 'zero' impact on the ball distance.

In the Pareto chart of effects, there is a red dotted line with a value of 34.2. Any values above the dotted line are considered to be critical to the response while those below the line are not critical. The error of margin is represented by the value of α which is 0.05. This implies that there is a percentage error of 5% which is acceptable. Even though Figure 3 shows which factors have the highest impact on the response, it does not depict how these parameters interact with each other.

3.1.2 Main Effects Plots

In this section, the impact of individual input parameters was analysed. In Figure 4, the minimum and maximum values of input variables are connected by a straight line. The higher the gradient of the line, the more the impact the input variable has on the response. The release angle was observed to have the highest gradient hence it has the maximum impact on the distance travelled by the ball.

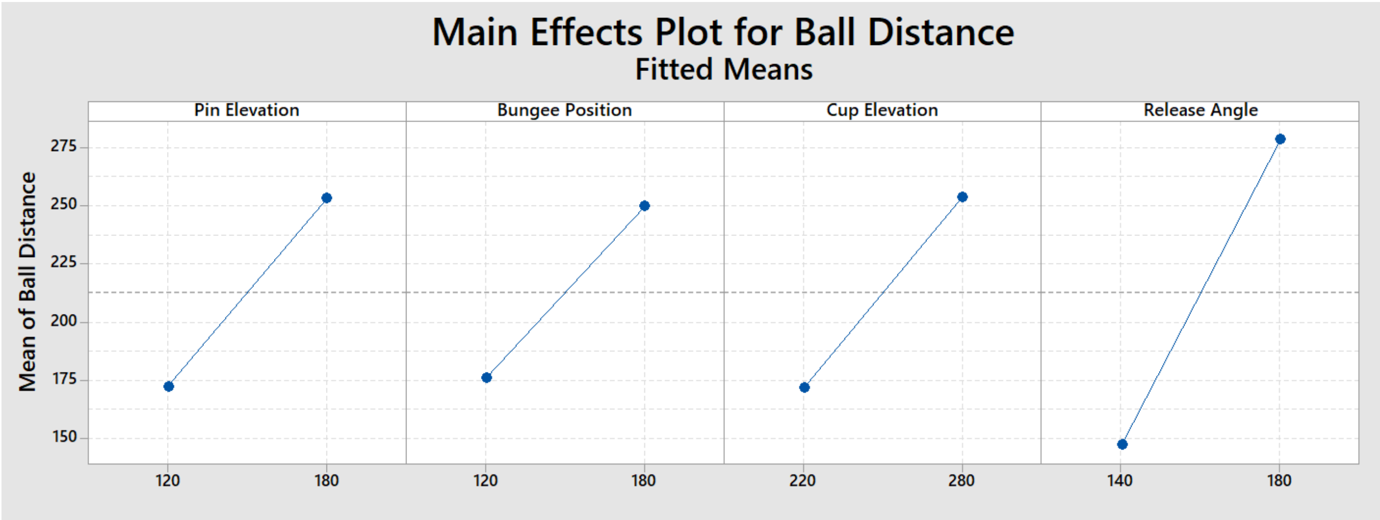


Figure 4 Main effects plots for mean of ball distance

3.1.3 Interaction Plots

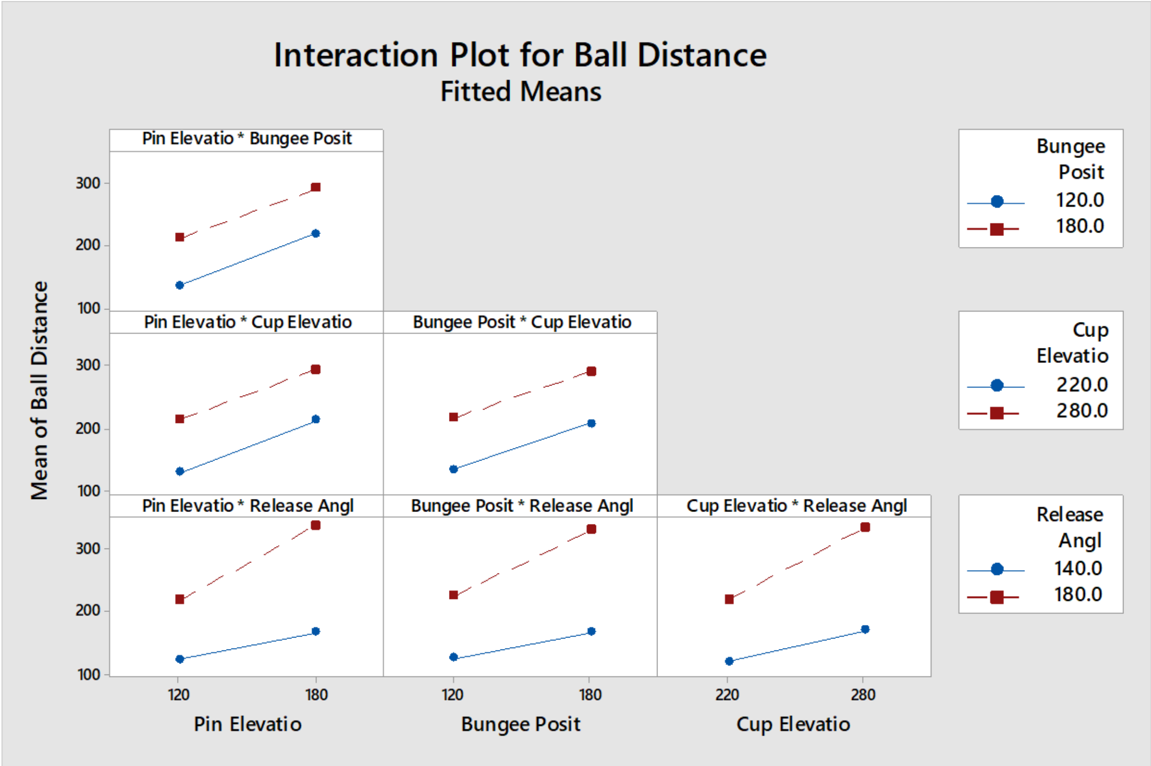


Figure 5 Interaction plots for ball mean of ball distance

Unlike main effects plots, interaction plots show which input variables interact more with each other. The interpretation of Figure 5 is that lines which are almost parallel or parallel to each other do not interact with each other while non-parallel line have more interaction with each other.

3.1.4 Response Surface Plots and Half-Normal Plots

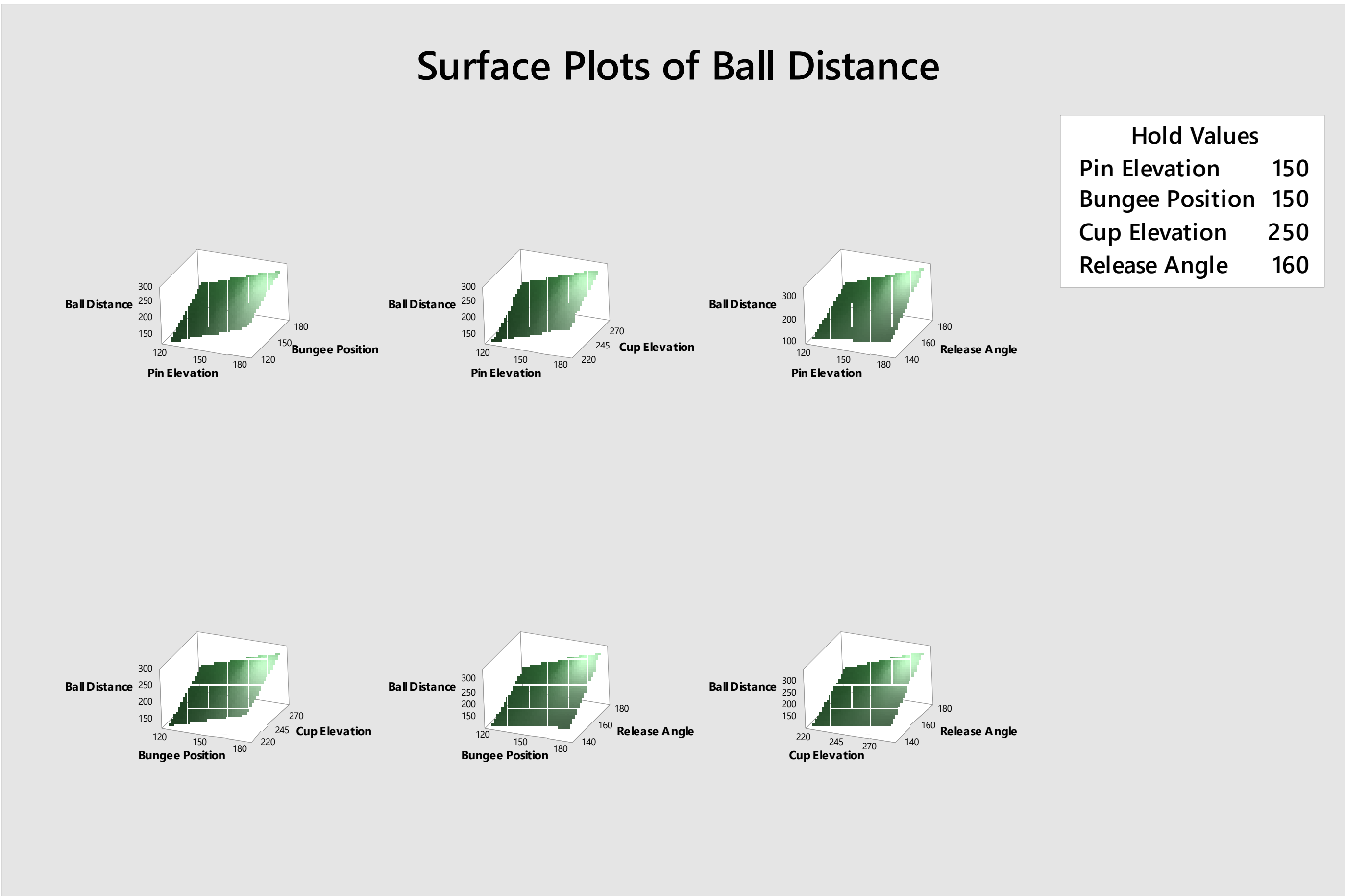


Figure 6 Surface plots for full factorial design

Figure 6 shows the surface plots for the factors used in full factorial design. Here, two variables were held constant at values indicated in the Hold Values box while the other two were varied. The outcome is not appealing since the plots are linear. Therefore, this analysis will be done using Central Composite Design and results compared.

The half normal plots are shown in Figure 7. These plots indicate the significant and non-significant input variables. Just like the Pareto chart of effects, *Release angle*, *cup elevation*, *pin* *elevation*, *bungee position* and *combination of pin elevation and release angle* are significant to the ball distance. The non-significant variables are represented by the blue dots.

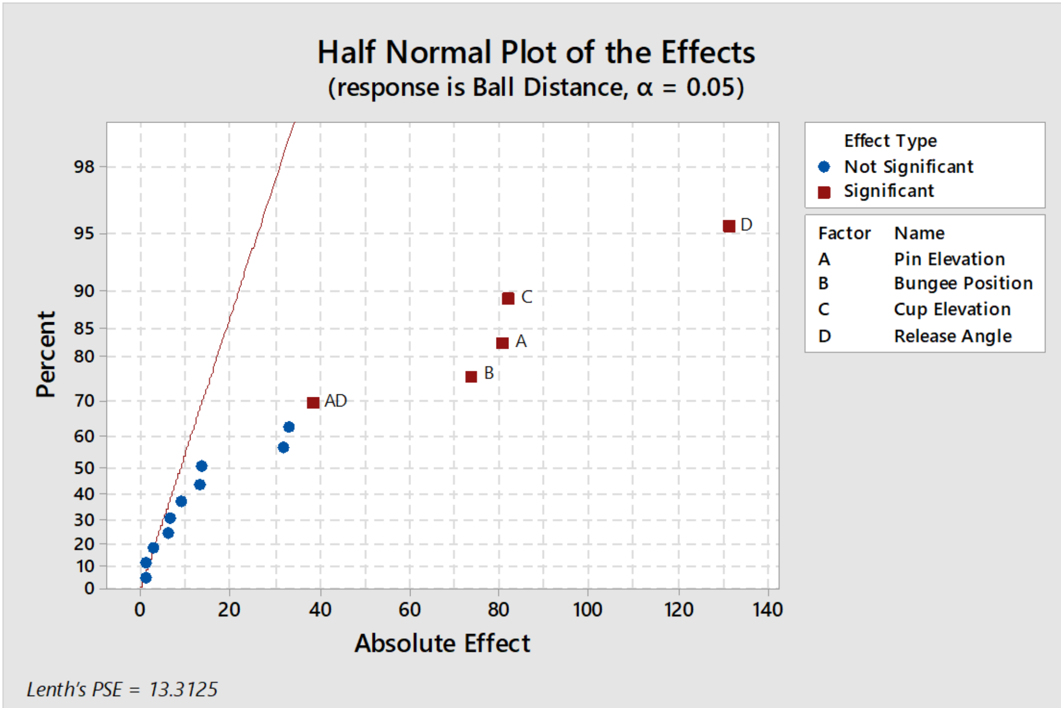


Figure 7 Half normal plots for input variables

3.2 Part B: Response Surface Model Analysis – Central Composite Design

In central composite design, a total of 31 simulations were conducted in which mid-level values were introduced. These values were pre-determined by MINITAB software. The firing angle was held constant at 110°.

Table 6 Ball distance for simulations guided by central composite design

C1	C2	C3	C4	C5	C6	C7	C8	C9
StdOrder	RunOrder	PtType	Blocks	Pin Elevation	Bungee Position	Cup ELevation	Release Angle	Ball Distance CCD
1	1	1	1	120	120	220	140	86
2	2	1	1	180	120	220	140	123
3	3	1	1	120	180	220	140	121
4	4	1	1	180	180	220	140	164
5	5	1	1	120	120	280	140	128
6	6	1	1	180	120	280	140	160
7	7	1	1	120	180	280	140	169
8	8	1	1	180	180	280	140	219
9	9	1	1	120	120	220	180	174
10	10	1	1	180	120	220	180	241
11	11	1	1	120	180	220	180	232
12	12	1	1	180	180	220	180	322
13	13	1	1	120	120	280	180	243
14	14	1	1	180	120	280	180	339
15	15	1	1	120	180	280	180	326
16	16	1	1	180	180	280	180	447
17	17	-1	1	120	150	250	160	191
18	18	-1	1	180	150	250	160	264
19	19	-1	1	150	120	250	160	197
20	20	-1	1	150	180	250	160	261
21	21	-1	1	150	150	220	160	191
22	22	-1	1	150	150	280	160	268
23	23	-1	1	150	150	250	140	149
24	24	-1	1	150	150	250	180	298
25	25	0	1	150	150	250	160	231
26	26	0	1	150	150	250	160	227
27	27	0	1	150	150	250	160	228
28	28	0	1	150	150	250	160	232
29	29	0	1	150	150	250	160	228
30	30	0	1	150	150	250	160	233
31	31	0	1	150	150	250	160	230

3.2.1 Pareto Chart of Effects

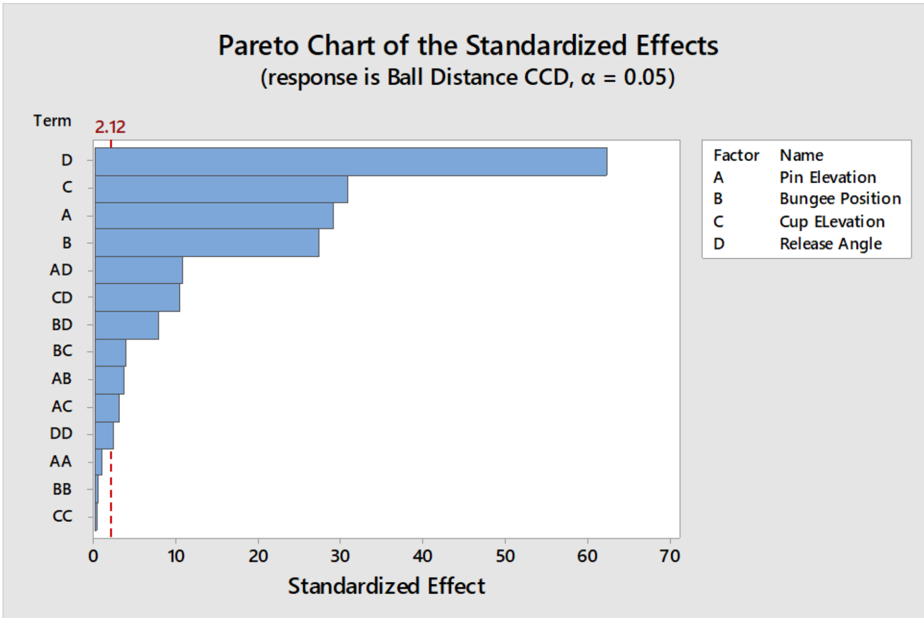


Figure 8 Response surface methodology - Pareto chart of effects

Figure 9 shows that most of the input variables are critical to the response of the system. The quadratic terms of pin elevation, bungee position and cup elevation are the non-critical values. In addition, it is observed that the *Release angle* has the highest impact on the ball distance.

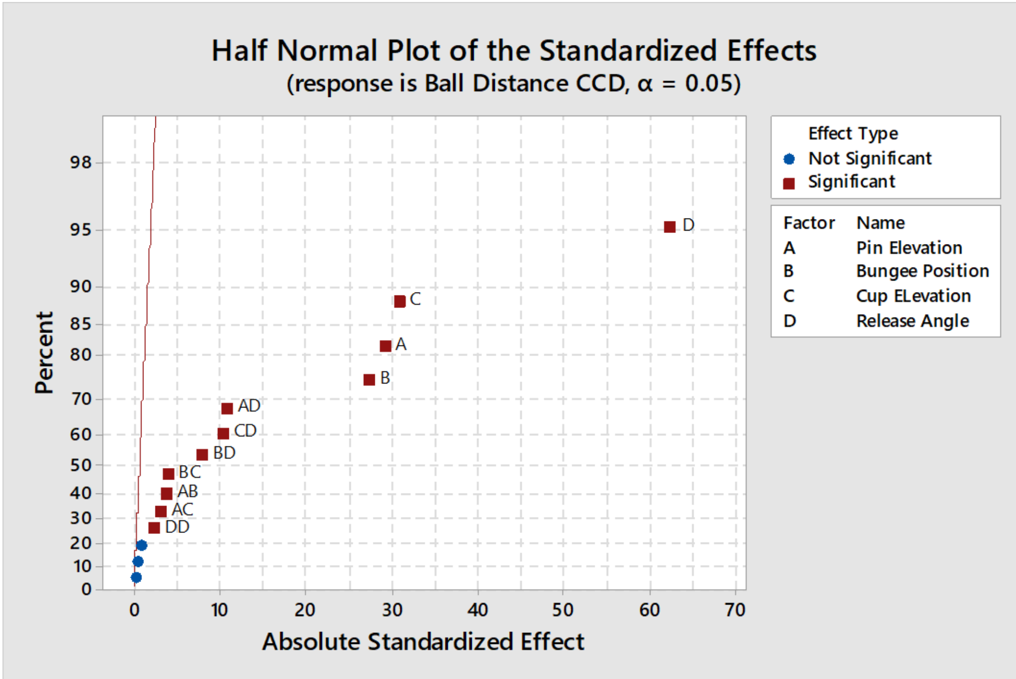


Figure 9 Half normal Plots for RSM

In Figure 10, the normal plot of the standardised effects is represented. Only three input variables are non-significant to the output.

3.2.2 Normal Probability and Deleted Residual Plots

This analysis was conducted so as to investigate whether the model is acceptable for design.

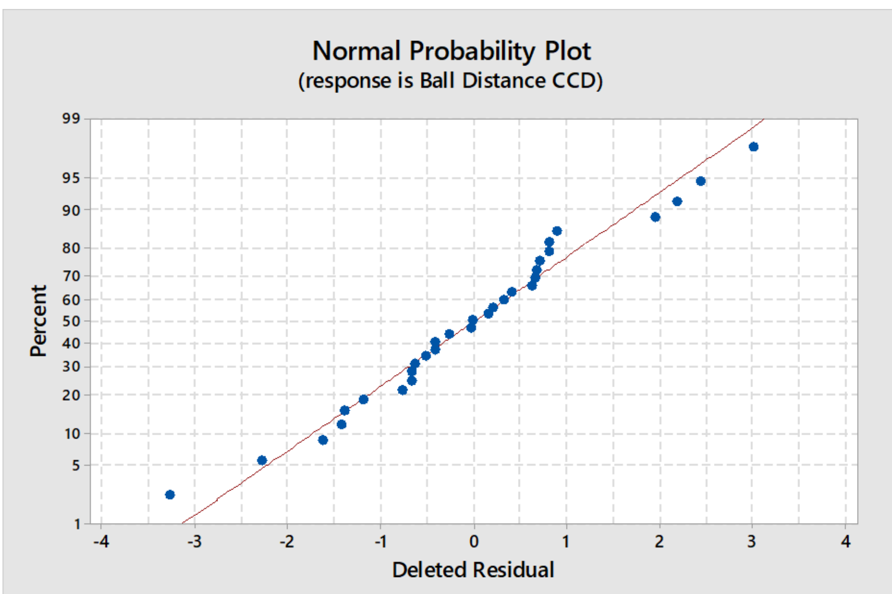


Figure 10 Normal probability plot for deleted residual

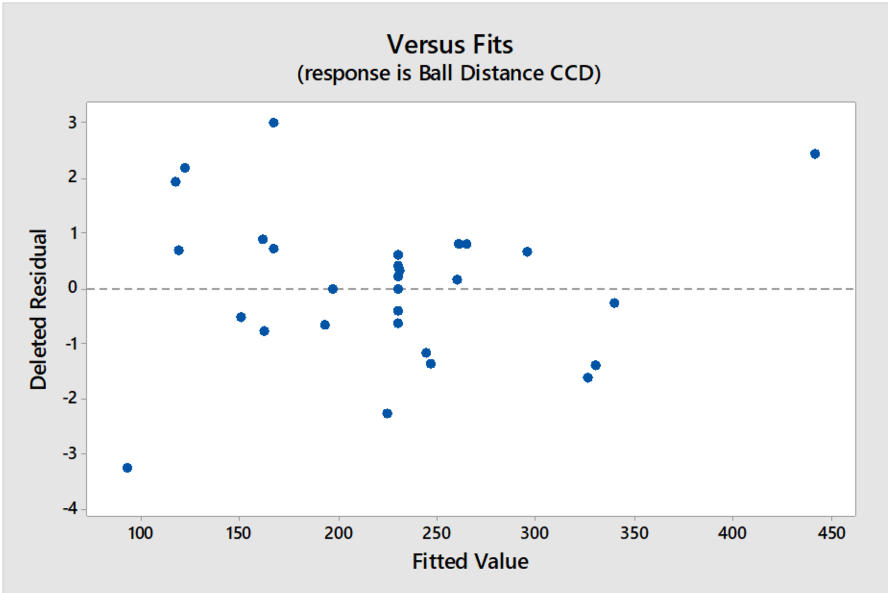


Figure 11 Plot of deleted residual vs fitted value

In Figure 11, Most of the points are closer to the trendline hence the design model may be acceptable. However, looking at Figure 12, some of the points are way further from the *x-x* axis and hence the model can neither be accepted nor rejected. To understand the error of margin of these points, the *P-values* have to be investigated.

3.2.3 Regression Analysis for P-values

The model summary is

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
4.92193	99.77%	99.56%	98.24%

This R-squared value is 99.77% before analysis of the coded coefficients.

Coded Coefficients

Table 7 Table for coded coefficients - CCD

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	230.06	1.46	157.56	0.000	
Pin Elevation	33.83	1.16	29.16	0.000	1.00
Bungee Position	31.67	1.16	27.30	0.000	1.00
Cup ELevation	35.83	1.16	30.89	0.000	1.00
Release Angle	72.39	1.16	62.40	0.000	1.00
Pin Elevation*Pin Elevation	-2.79	3.06	-0.91	0.375	2.91
Bungee Position*Bungee Position	-1.29	3.06	-0.42	0.679	2.91
Cup ELevation*Cup ELevation	-0.79	3.06	-0.26	0.800	2.91
Release Angle*Release Angle	-6.79	3.06	-2.22	0.041	2.91
Pin Elevation*Bungee Position	4.50	1.23	3.66	0.002	1.00
Pin Elevation*Cup ELevation	3.88	1.23	3.15	0.006	1.00
Pin Elevation*Release Angle	13.25	1.23	10.77	0.000	1.00
Bungee Position*Cup ELevation	4.75	1.23	3.86	0.001	1.00
Bungee Position*Release Angle	9.62	1.23	7.82	0.000	1.00
Cup ELevation*Release Angle	12.75	1.23	10.36	0.000	1.00

The P-value of a statistical design should not be greater than 0.05. Therefore, all the variables in Table 6 with a value of greater than 0.05 must be removed from the Response Surface Analysis.

3.2.4 Second Analysis of Regression Model

The non-significant variables in Table 6 were deleted from the analysis and the following results were obtained.

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
4.82587	99.73%	99.58%	98.28%

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	229.31	1.34	171.32	0.000	
Pin Elevation	33.83	1.14	29.74	0.000	1.00
Bungee Position	31.67	1.14	27.84	0.000	1.00
Cup ELevation	35.83	1.14	31.50	0.000	1.00
Release Angle	72.39	1.14	63.64	0.000	1.00
Release Angle*Release Angle	-10.36	1.76	-5.90	0.000	1.00
Pin Elevation*Bungee Position	4.50	1.21	3.73	0.001	1.00
Pin Elevation*Cup ELevation	3.87	1.21	3.21	0.005	1.00
Pin Elevation*Release Angle	13.25	1.21	10.98	0.000	1.00
Bungee Position*Cup ELevation	4.75	1.21	3.94	0.001	1.00
Bungee Position*Release Angle	9.62	1.21	7.98	0.000	1.00
Cup ELevation*Release Angle	12.75	1.21	10.57	0.000	1.00

In the model summary, the R-squared value reduced from 99.77% to 99.73%. Therefore, the first regression model is accepted since the linearity has not been affected by deleting the terms with high P-values.

The residual plots for ball distance or response analysis using central composite design are shown in Figure 13

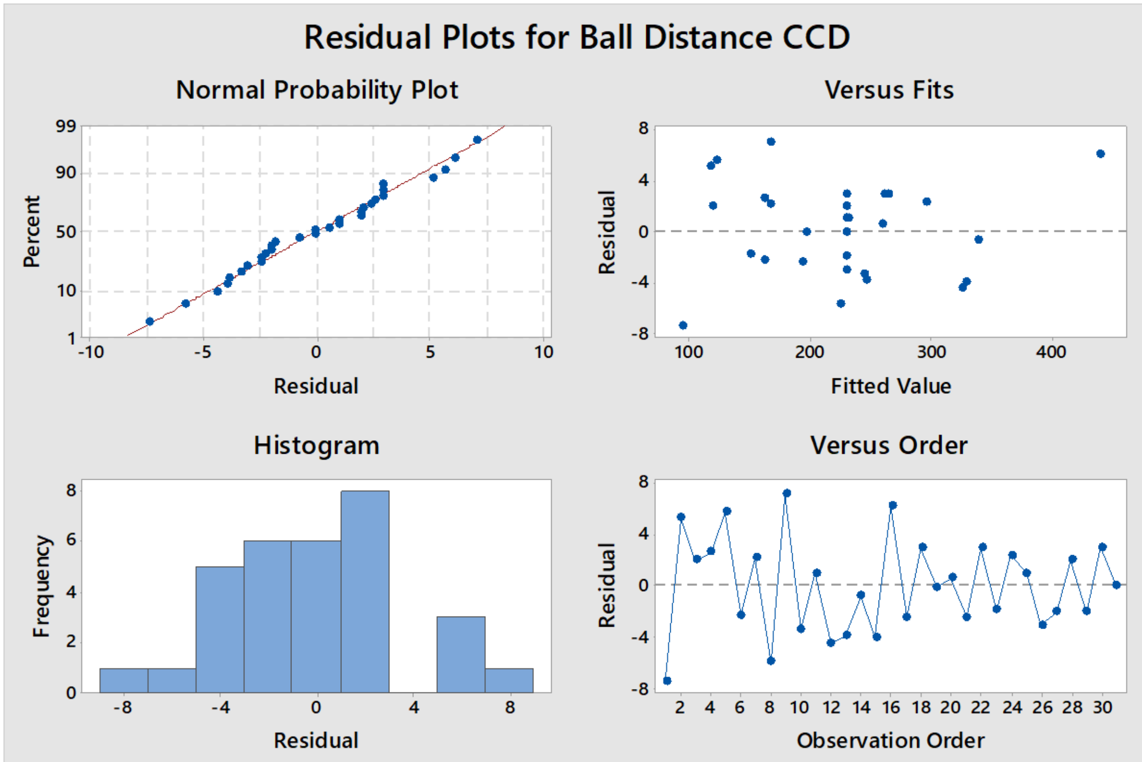


Figure 12 Residual plots for ball distance

3.2.5 Surface Response Plots

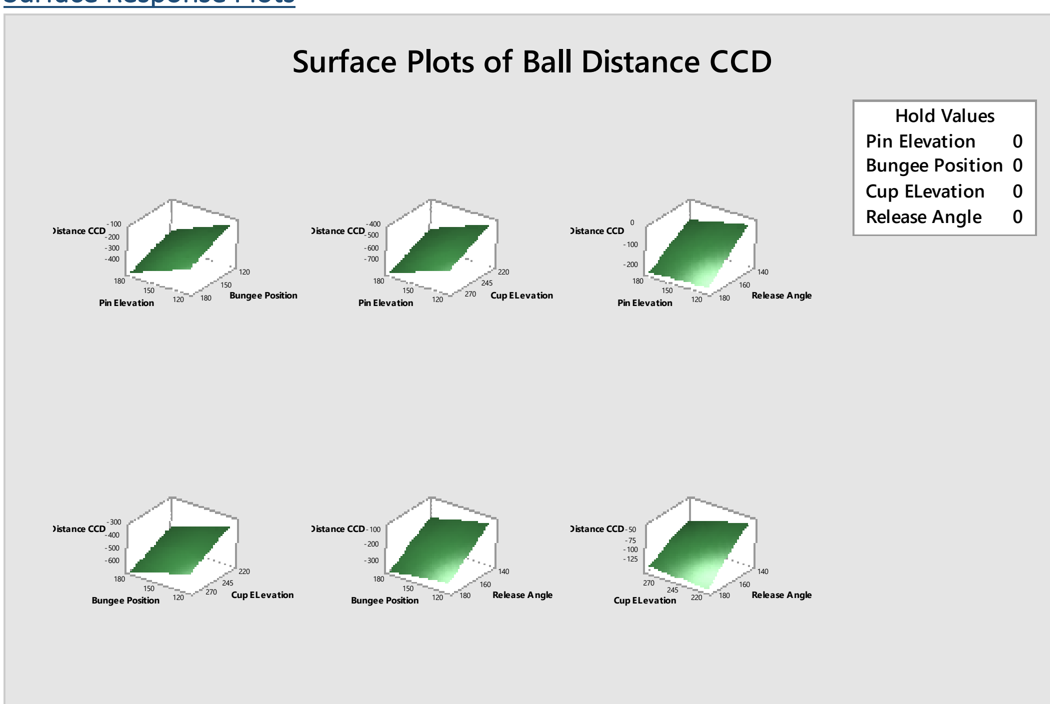


Figure 13 Surface plots for response surface methodology

Figure 14 shows that the graphs are parabolic in nature hence acceptable for design. The central composite design has superior qualities than the basic full factorial design. However, the critical input variables do not change as shown in Figure 15 and Figure 16 below.

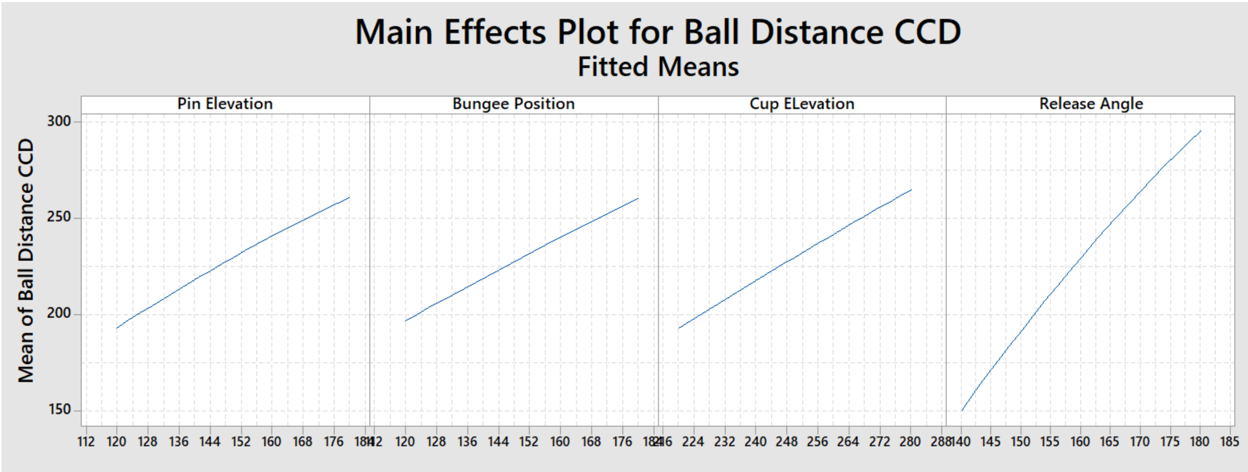


Figure 14 Main effects plot for ball distance -RSM

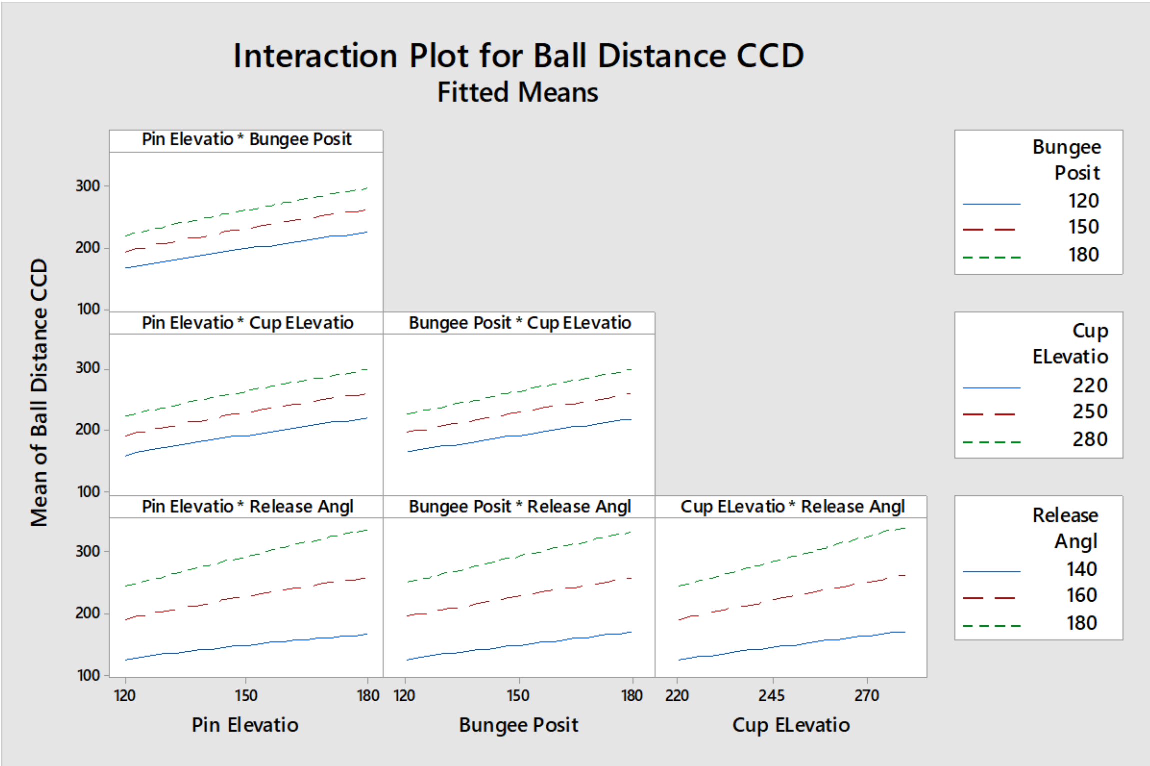


Figure 15 Interaction plot for ball distance using RSM

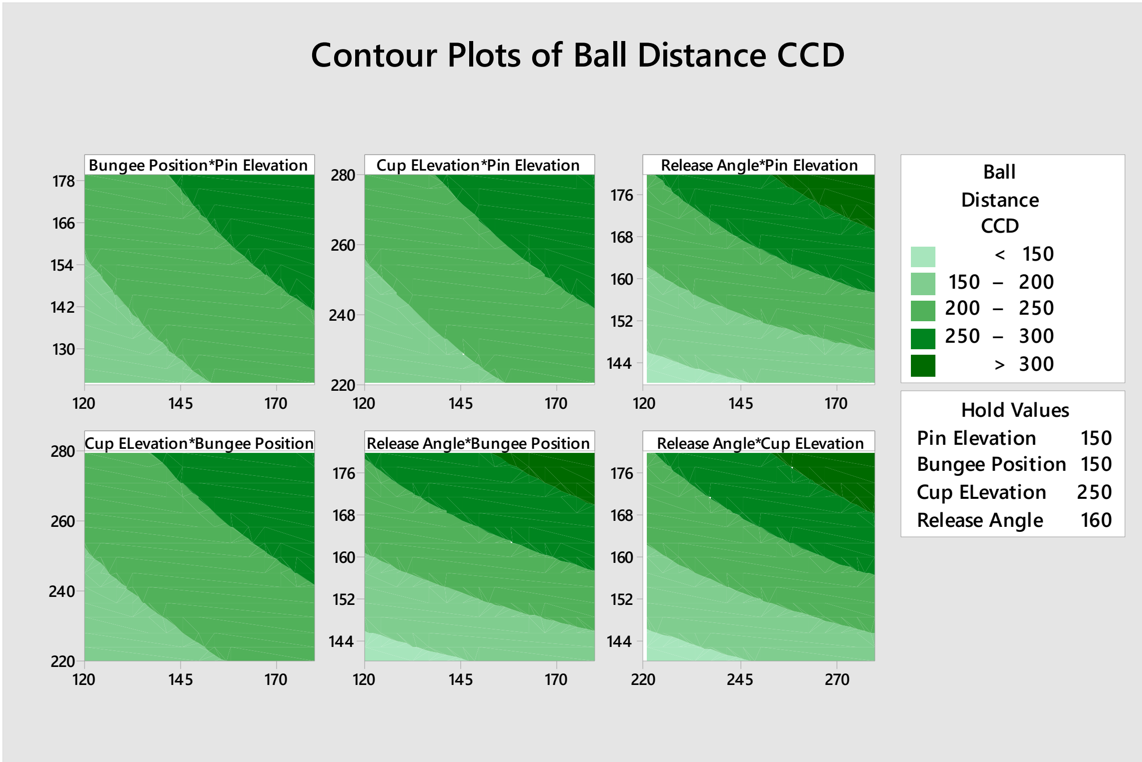


Figure 16 Contour plots for RSM model

Figure 16 and Figure 17 show the interaction between the input variable parameters. It is evident that increasing the interaction between the Cup Elevation and Pin Elevation had a maximum impact on the output response.

3.3 Validation and Accuracy Comparison

To validate the models, a target value was set in MINITAB and the proposed input variables used in virtual catapult. The accuracy was also calculated and compared.

3.3.1 Full Factorial Method

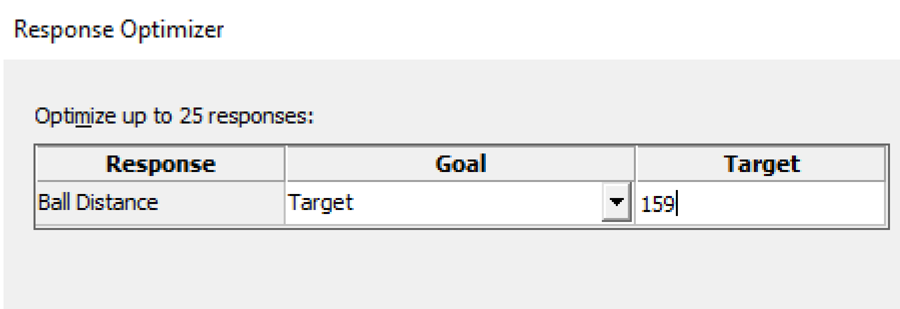


Figure 17 Target value input

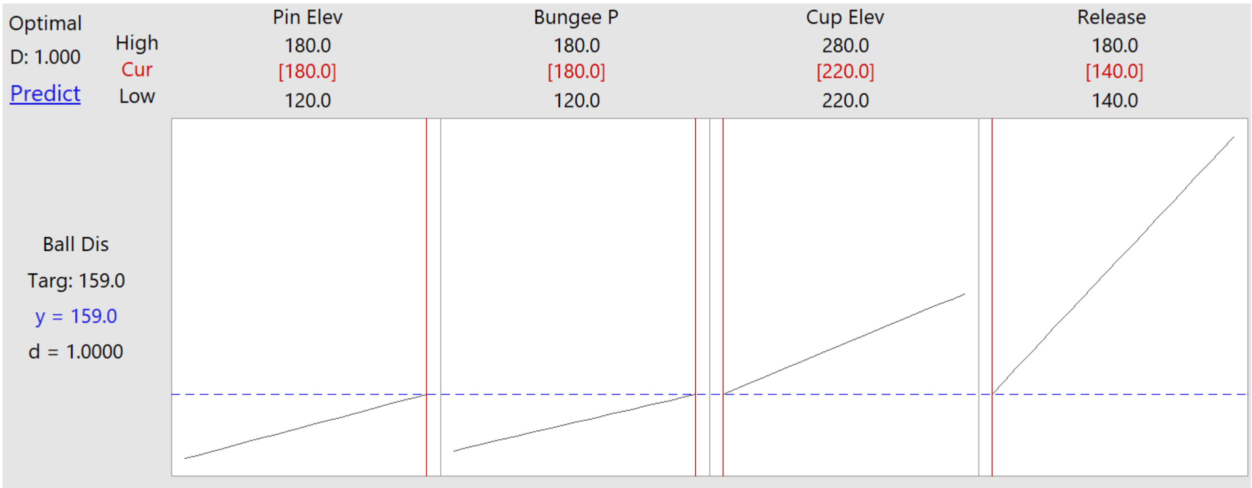


Figure 18 Predicted input values

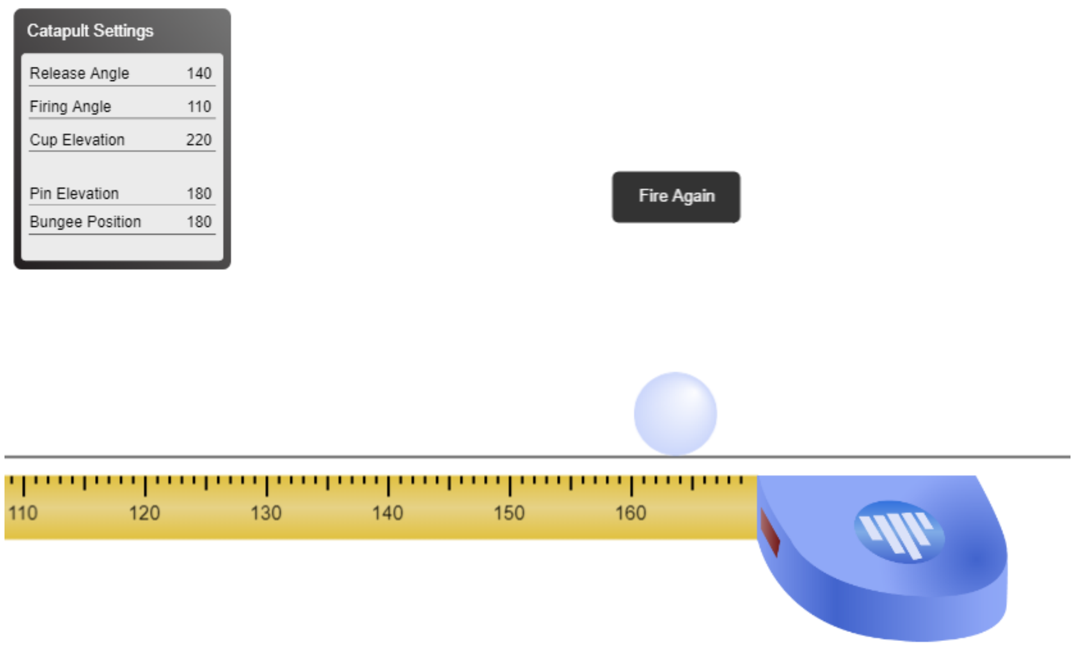


Figure 19 Ball distance from predicted values

Figure 20 shows that the ball distance is 164 units. The predicted value was 159 units. Therefore,

$$\%\ Error = \frac{164 - 159}{159} \times 100 = 3.145\ \%$$

3.3.2 Response Surface Methodology – Central Composite Design

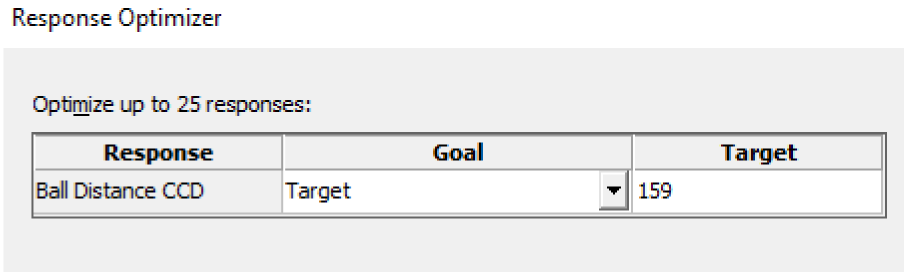


Figure 20 Target value input

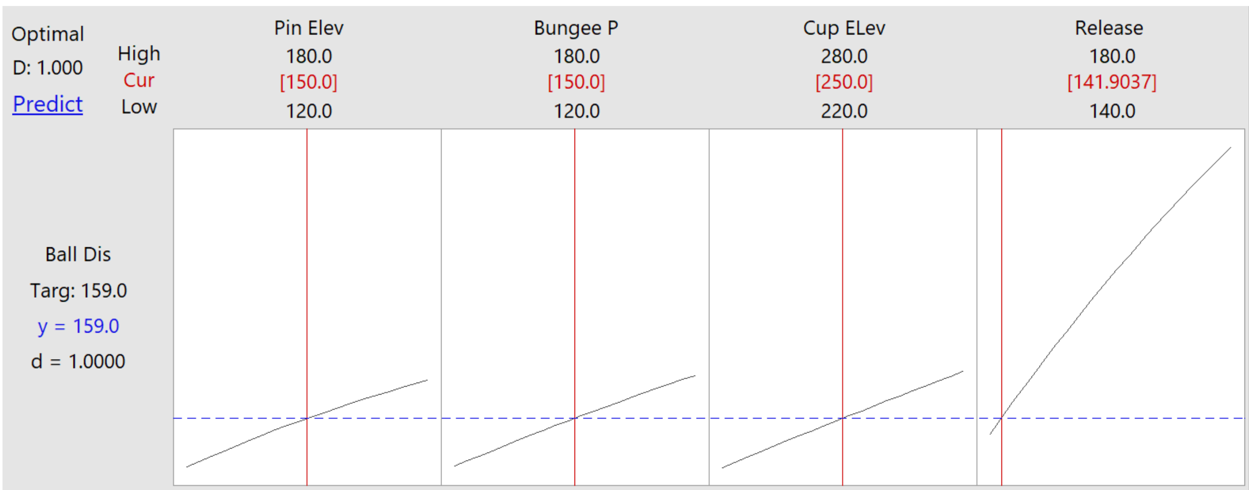


Figure 21 Predicted values from RSM

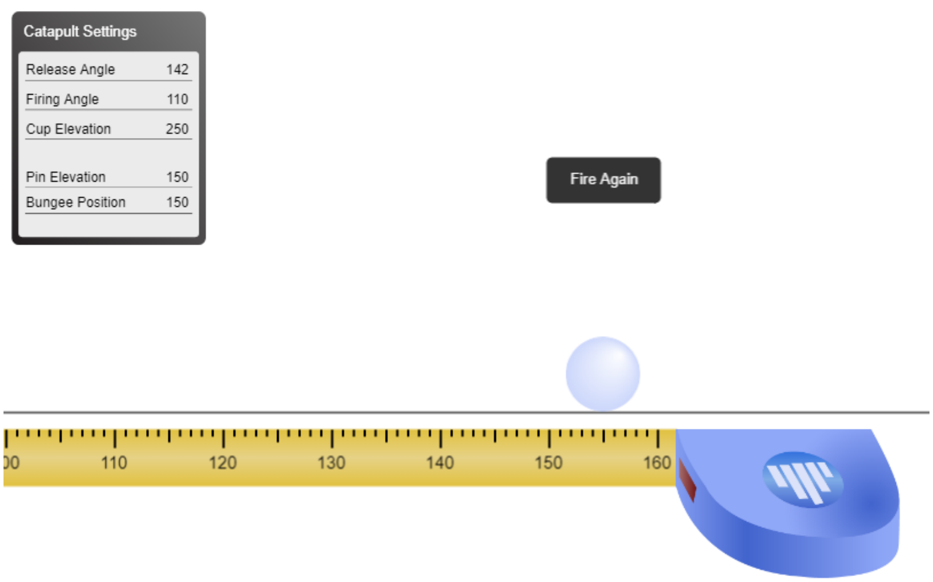


Figure 22 Ball distance from predicted values using RSM

The actual ball distance arising from RSM is 156 units while the predicted value was 159 units. Therefore,

$$\%Error = \frac{159 - 156}{159} \times 100 = 1.887\%$$

4 DISCUSSION

The main objective of this assignment was to demonstrate a critical understanding of experiments and response surface methodology in engineering practices and theory. However, due to the constraints present, a virtual catapult was used to conduct the Design of Experiments statistical method.

The input variables were Bungee Position, Release Angle, Pin Elevation and Cup Elevation. These variables were varied according to Table 2. The Firing Angle was kept at a constant value of 110°. Two DoE techniques were employed in this assignment – Full Factorial design and Response Surface Methodology.

In Full factorial design, 16 simulations were conducted and analysis of how the input variables affect the response was conducted. The Release Angle was found to have the maximum effect on the Ball Distance while the Bungee Position had the minimum effect on response. It was also observed that interaction between the input variables affected the response of the system.

In Response Surface Methodology, 31 simulations were done. The number of simulations increased due to the employment of Central Composite Design. Using RSM was proposed so as to increase the accuracy of the model. In addition, RSM developed a polynomial regression model unlike Full Factorial design in which only linear models were possible.

The accuracy of the models was also investigated. Full factorial design had a percentage error of 3.145% while RSM had a percentage error of 1.887%.This proved that RSM is a superior tool for conducting Design of Experiments.

4.1 Comparison between Virtual Catapult and Real Catapult

As mentioned at the beginning of the assignment, a virtual catapult was selected for analysis due to the limitations present. However, in a virtual catapult, the conditions are ideal *i.e.* there is no drag, friction and human errors in a virtual world. This makes the findings from a DoE methodology conducted via simulations not valid until validated using either an experiment or existing data in literature. In most cases, there is usually a slight margin of error between the simulation data and experimental data. For instance, if the margin of error is about 10%, the results are not correct but are acceptable in a catapult experiment. Some simulations will require a lower percentage error for the results to be acceptable. The differences in outcomes of simulations in real and virtual world are due to the errors present in real world and also factors such as friction, drag, elasticity, etc.

5 CONCLUSION

Design of Experiments is a powerful method for engineering design applications. Both full factorial and RSM techniques have proved to have low margins of error which in most engineering applications are acceptable. Predicting the response of a system in design stage saves on total cost of production and also time. For example, if you are designing a machine in which energy consumption is to be minimised, developing a prediction model might be the best method to settle on. The RSM technique was also found to be superior to Full Factorial method due to its low percentage error.

6 REFERENCES

1. Mathews, P., 2010. Design Of Experiments With MINITAB. New Dehli: New Age, pp.99-105.
2. Rafidah, A., Nurulhuda, A., Azrina, A., Suhaila, Y., Anwar, I. and Syafiq, R., 2014. Comparison Design of Experiment (DOE): Taguchi Method and Full Factorial Design in Surface Roughness. Applied Mechanics and Materials, 660, pp.275-279.
3. Support.minitab.com. 2020. Minitab 18 Support - Minitab. [online] Available at: <https://support.minitab.com/en-us/minitab/18/> [Accessed 24 July 2020].
4. Coursework and class work notes.