

## Sigmoid Function

In logistic regression

$$\rightarrow 0 \leq P(Y = \frac{1}{x}) \leq 1$$

Linear function :-  $b_0 + b_1 x$

$\rightarrow$  can give any value

$\Rightarrow (-\infty, >1)$  or anything

$\rightarrow$  We need a function

$\Rightarrow$  takes input  $(-\infty, +\infty)$

$\Rightarrow$  Output  $= [0, 1]$

$\rightarrow$  Squeezing function  $\Rightarrow$  Sigmoid (S-shaped)

Odds  $\nearrow$  If a ratio  $\rightarrow$  involves a reciprocal

Probability =  $p$

Odds = chances of success

## chances of failure

$$\Rightarrow \text{Odds} = \frac{p}{1-p}$$

$$\text{logit}(p) = \ln\left(\frac{p}{1-p}\right)$$

$\rightarrow$  log because gives value from  $(-\infty, \infty)$

We want linear relationship with inputs

$$\ln\left(\frac{p}{1-p}\right) = b_0 + b_1x_1 + b_2x_2 + \dots$$

Invert eqn to derive sigmoid

$$\Rightarrow \ln\left(\frac{p}{1-p}\right) = z \quad \text{---} \quad b_0 + b_1x_1 + \dots$$

$$\frac{p}{1-p} = e^2$$

$$\Rightarrow p = e^2(1-p)$$

$$\Rightarrow p = e^2 - pe^2$$

$$\Rightarrow p + pe^2 = e^2$$

$$p(1+e^2) = e^2$$

$$\Rightarrow p = \frac{e^2}{1+e^2}$$

$$\Rightarrow \boxed{p = \frac{1}{1+e^{-2}}}$$