

Sigmoid Function

In logistic regression

$$\rightarrow 0 \leq P(Y = \frac{1}{x}) \leq 1$$

Linear function :- $b_0 + b_1 x$

\rightarrow can give any value

$\Rightarrow (-\infty, \infty)$ or anything

\rightarrow we need a function

\Rightarrow takes input $(-\infty, \infty)$

\Rightarrow Output $= [0, 1]$

\rightarrow Squeezing function \Rightarrow Sigmoid (S-shaped)

Odds \rightarrow tip a ratio \rightarrow involves a reciprocal

Probability = p

Odds = chances of success

chances of failure

$$\Rightarrow \text{Odds} = \frac{P}{1-P}$$

$$\text{logit}(P) = \ln\left(\frac{P}{1-P}\right)$$

\rightarrow log because gives value from $(-\infty, \infty)$

We want linear relationship with inputs

$$\ln\left(\frac{P}{1-P}\right) = b_0 + b_1 x_1 + b_2 x_2 + \dots$$

Invert eqn to derive sigmoid

$$\Rightarrow \ln\left(\frac{P}{1-P}\right) = Z = b_0 + b_1 z_1 + \dots$$

$$\frac{p}{1-p} = e^z$$

$$\Rightarrow p = e^z(1-p)$$

$$\Rightarrow p = e^z - pe^z$$

$$\Rightarrow p + pe^z = e^z$$

$$p(1+e^z) = e^z$$

$$\Rightarrow p = \frac{e^z}{1+e^z}$$

$$\Rightarrow p = \boxed{p = \frac{1}{1+e^{-z}}}$$