19/8/24

Assignment - 1

1. a) $|x^2(t)|$ is neither even, nor odd as $|x(t)|^2 \neq |x(-t)|^2$ ~~always~~

b) $x(t^2)$ is neither even nor odd, as $t^2 < 0$ is not possible.

c) Assume a $y(t) = x(t) + x(-t)$

Then,

$$y(-t) = x(-t) + x(-(-t))$$
$$= x(-t) + x(t)$$
$$= y(t)$$

$$\Rightarrow \boxed{y(-t) = y(t)}$$

$\therefore$ The given signal is even.

d) $e^{x(t)}$ is always positive $\Rightarrow$ $e^{x(t)}$ cannot be odd

Assume $e^{x(t)} = e^{x(-t)}$ $\Rightarrow$ Condition for even

Then $x(t) = x(-t)$, which is not true.

$\therefore e^{x(t)}$ is not even.

$\therefore e^{x(t)}$ is neither even nor odd.

e) Assume $y(t) = x(t) - x(-t)$

Then,

$$y(-t) = x(-t) - x(-(-t))$$
$$= x(-t) - x(t)$$
$$= -(x(t) - x(-t))$$
$$= -y(t)$$

$\Rightarrow \boxed{y(-t) = -y(t)}$ $\therefore$ The given signal is odd.

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From parts (c) and (e), we can see that

$x(t) + x(-t)$ is always even
$x(t) - x(-t)$ is always odd.

Assume, $x(t) + x(-t) = x_e(t)$, where $x_e(t)$ is an even signal.
$x(t) - x(-t) = x_o(t)$, where $x_o(t)$ is an odd signal.

$$x_e(t) + x_o(t) = x(t) + x(-t) + x(t) - x(-t)$$

$$= \quad x_e(t) + x_o(t) = 2x(t)$$

$$\Rightarrow \boxed{x(t) = x'_e(t) + x'_o(t)} \Rightarrow \left(x'_e(t) = \frac{1}{2} x_e(t),\ x'_o(t) = \frac{1}{2} x_o(t) \right)$$

$x'_e(t)$ and $x'_o(t)$ are still even and odd respectively as multiplying a signal by a constant does not change its parity (even/odd).

$\therefore$ Any arbitrary signal can be expressed in the form of a sum of an even and an odd signal.

2. $x(t) = \underbrace{0.5\cos(2\pi f_1 t + \phi_1)}_{①} + \underbrace{e^{j(2\omega_2 t + \phi_2)}}_{②}$

$x_1(t) = 0.5\cos(2\pi f_1 t + \phi_1)$

Period of x_1: $T_1 = 1/f_1$

$x_2(t) = e^{j(2\omega_2 t + \phi_2)} = e^{j(4\pi f_2 t + \phi_2)}$

Period of x_2: $T_2 = 1/f_2$

Assume a $T = LCM(T_1, T_2)$

$\Rightarrow x_1(t+T) = x_1(t)$ & $x_2(t+T) = x_2(t)$ (Since $T = mT_1 = nT_2$ where $m, n \in \mathbb{Z}$)

$x(t) = x_1(t) + x_2(t)$

$x(t+T) = x_1(t+T) + x_2(t+T)$

$\Rightarrow x(t+T) = x_1(t) + x_2(t) = x(t)$

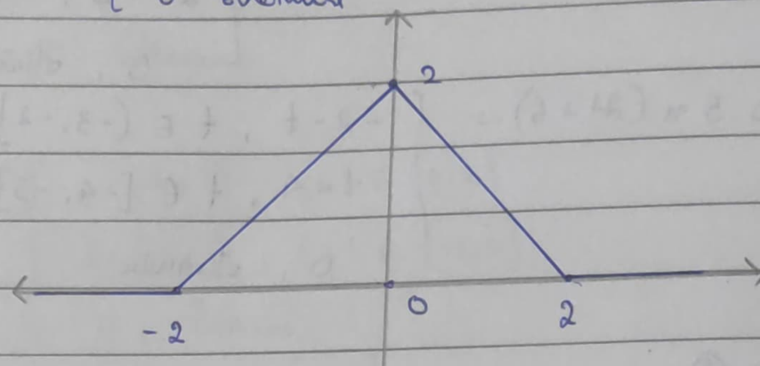
$\Rightarrow \boxed{x(t+T) = x(t)}$

$\therefore$ The given signal is periodic, ~~whose~~ with a fundamental period of $T = LCM(T_1, T_2)$

Similar Proof can be applied to any number of sinusoidal signals. The period of the resulting composite signal will be the LCM of the periods of the constituent periodic signals.

ϕ_1 and ϕ_2's values do not influence the resulting signals fundamental period. However, they may change the shape of the signal.

3. a) $y(t) = x(t) = \begin{cases} 2-t & \text{if } t \in (0,2] \\ 2+t & \text{if } t \in [-2,0] \\ 0 & \text{otherwise} \end{cases}$



b) $y(t) = x(\underbrace{3t+5}) = \begin{cases} 2-(3t+5), & 3t+5 \in (0,2] \\ 2+(3t+5), & 3t+5 \in [-2,0] \\ 0 & \text{otherwise} \end{cases}$

$3t+5 = 0$

$t = \frac{-5}{3}$

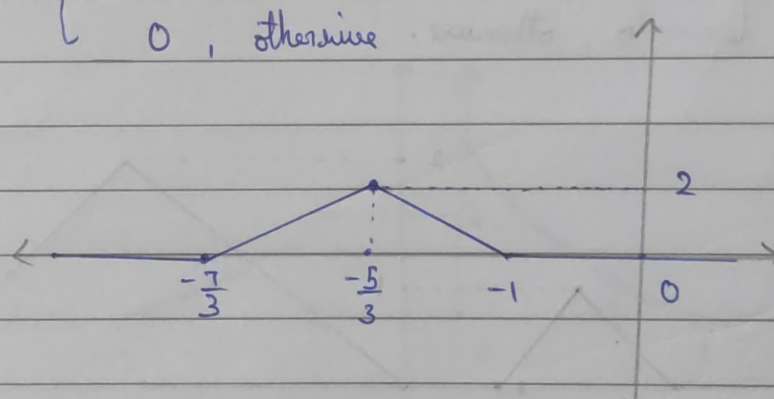
$= \begin{cases} -3t-3, & t \in (\frac{-5}{3}, -1] \\ 3t+7, & t \in [\frac{-7}{3}, \frac{-5}{3}] \\ 0, & \text{otherwise} \end{cases}$

$3t+5 = 2$

$t = -1$

$3t+5 = -2$

$t = \frac{-7}{3}$



c) $y(t) = \underset{①}{x(2-0.5t)} + \underset{②}{0.5x(2(t+3))}$

$2-0.5t = -2$

$0.5t = 4$

$t = 8$

①

$x(2-0.5t) = \begin{cases} 2-(2-0.5t), & 2-0.5t \in (0,2] \\ 2+(2-0.5t), & 2-0.5t \in [-2,0] \\ 0, & \text{otherwise} \end{cases}$

$= \begin{cases} 0.5t, & t \in [0,4) \\ 4-0.5t, & t \in [4,8] \\ 0, & \text{otherwise} \end{cases}$

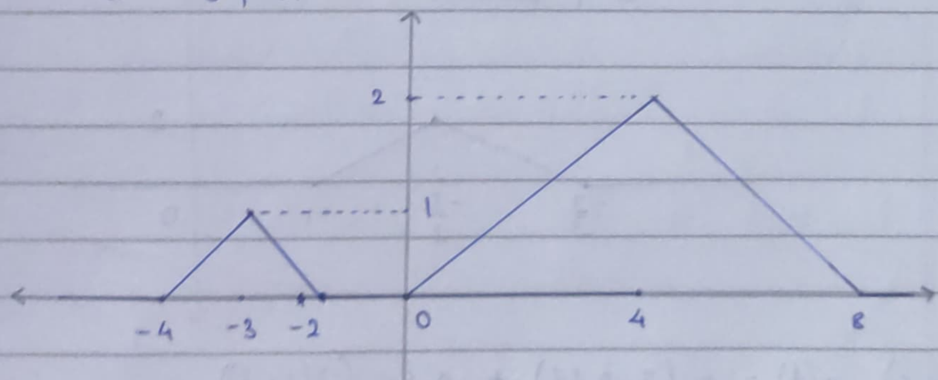
② $x(2(t+3)) = x(2t+6) = \begin{cases} 2-(2t+6), & 2t+6 \in (0,2] \\ 2+(2t+6), & 2t+6 \in [-2,0] \\ 0, & \text{otherwise} \end{cases}$

$= \begin{cases} -4-2t, & t \in (-3,-2] \\ 2t+8, & t \in [-4,-3] \\ 0, & \text{otherwise} \end{cases}$

$0.5\,x(2t+6) = \begin{cases} -2-t, & t \in (-3,-2] \\ t+4, & t \in [-4,-3] \\ 0, & \text{otherwise} \end{cases}$

① + ②,

$y(t) = \begin{cases} t+4, & t \in [-4,-3] \\ -2-t, & t \in (-3,-2] \\ 0.5t, & t \in [0,4) \\ 4-0.5t, & t \in [4,8] \\ 0, & \text{otherwise.} \end{cases}$



d) $y(t) = \underbrace{x(t)}_{①}\ \underbrace{x(t+1)}_{②}$

①,

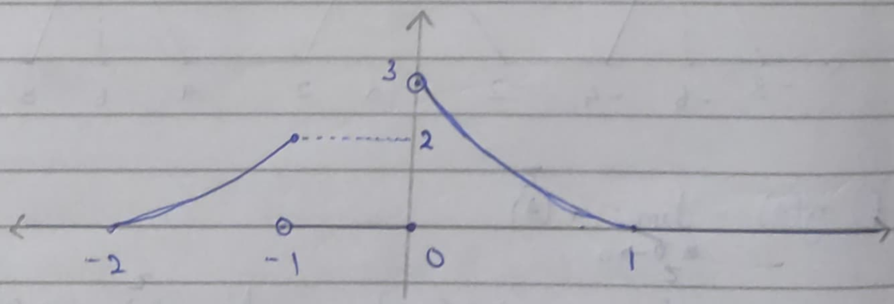
$$x(t) = \begin{cases} 2-t, & t \in (0,2] \\ 2+t, & t \in [-2,0] \\ 0, & \text{otherwise} \end{cases}$$

②,

$$x(t+1) = \begin{cases} 2-(t+1), & t+1 \in (0,2] \\ 2+(t+1), & t+1 \in [-2,0] \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} 1-t, & t \in (-1,1] \\ 3+t, & t \in [-3,-1] \\ 0, & \text{otherwise} \end{cases}$$

$$y(t) = x(t)\cdot x(t+1) = \begin{cases} (1-t)(2-t), & t \in (0,1] \\ (3+t)(2+t), & t \in [-2,-1] \\ 0, & \text{otherwise} \end{cases}$$



e) $y(t) = \sum_{k \in \mathbb{Z}} x(t-6k)$

$$x(t-6k) = \begin{cases} 2-(t-6k), & t-6k \in (0,2] \\ 2+(t-6k), & t-6k \in [-2,0] \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} 2-t+6k, & t \in (6k, 6k+2) \\ 2+t-6k, & t \in (-2+6k, 6k) \\ 0, & \text{otherwise} \end{cases}$$

$$x(t-6k) = \begin{cases} 2-t+6k, & t \in (6k, 6k+2] \\ 2+t-6k, & t \in [6k-2, 6k] \\ 0, & \text{otherwise} \end{cases}$$

$k=0$,

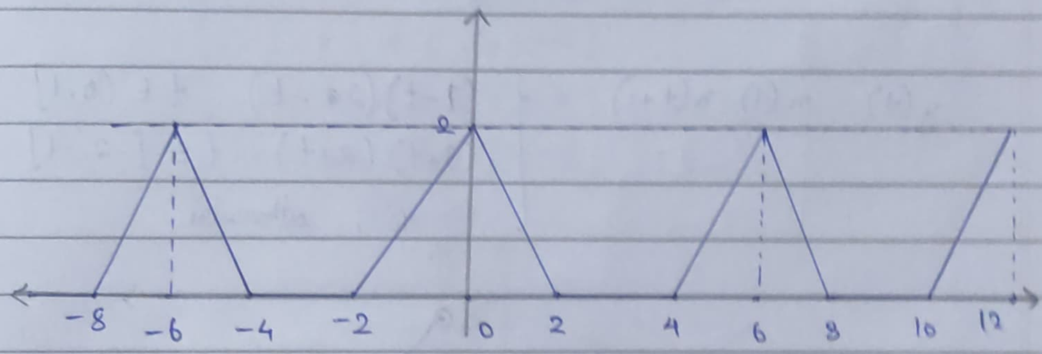
$$x(t) = \begin{cases} 2-t, & t \in (0,2] \\ 2+t, & t \in [-2,0] \\ 0, & \text{otherwise} \end{cases}$$

$k=1$,

$$x(t) = \begin{cases} 8-t, & t \in (6,8] \\ t-4, & t \in [4,6] \\ 0, & \text{otherwise} \end{cases}$$

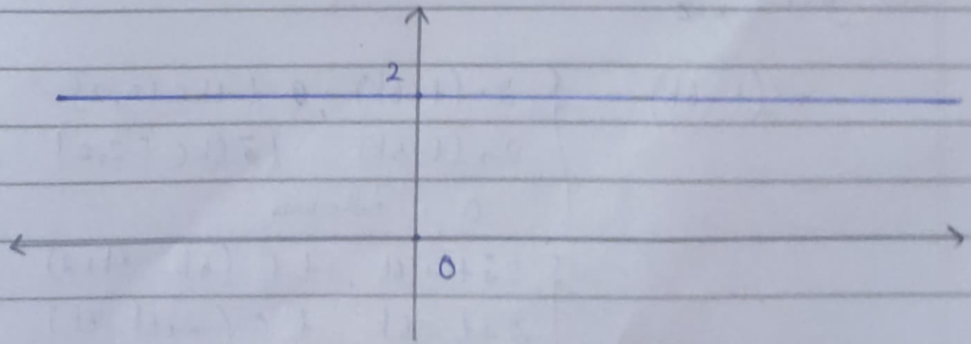
$k=-1$

$$x(t) = \begin{cases} -4-t, & t \in (-6,-4] \\ 8+t, & t \in [-8,-6] \\ 0, & \text{otherwise} \end{cases}$$



f) $y(t) = \lim_{\tau \to 0} x_\tau(t)$

$$x(t) = \begin{cases} 2-t, & t \in (0,2] \\ 2+t, & t \in [-2,0] \\ 0, & \text{otherwise} \end{cases}, \quad \lim_{\tau \to 0} x_\tau(t) = x(0) = \underline{\underline{2}}$$



3. Calculation of Energy & Power.

a) $y(t) = x(t)$

$$\Rightarrow y(t) = \begin{cases} 2-t, & \text{if } t \in (0,2] \\ 2+t, & \text{if } t \in [-2,0] \\ 0, & \text{otherwise} \end{cases}$$

$$E_\infty = \int_{-\infty}^{\infty} |y(t)|^2 dt$$

$$= \int_{-2}^{0} |2+t|^2 dt + \int_{0}^{2} |2-t|^2 dt$$

$$= \left[\frac{(2+t)^3}{3}\right]_{-2}^{0} + \left[\frac{-(2-t)^3}{3}\right]_{0}^{2}$$

$$= \left[\frac{2^3}{3} - 0\right] + \left[-0 + \frac{2^3}{3}\right]$$

$$= \frac{8}{3} + \frac{8}{3} = \underline{\underline{\frac{16}{3}}}$$

Since E_∞ is finite, $\underline{P_\infty = 0}$.

b) $y(t) = x(3t+5)$

$$\Rightarrow y(t) = \begin{cases} -3t-3, & t \in \left(\frac{-5}{3}, -1\right] \\ 3t+7, & t \in \left[\frac{-7}{3}, -\frac{5}{3}\right] \\ 0, & \text{otherwise} \end{cases}$$

$$E_\infty = \int_{-\infty}^{\infty} |y(t)|^2 \, dt$$

$$= \int_{-\frac{7}{3}}^{-\frac{5}{3}} (3t+7)^2 \, dt + \int_{-\frac{5}{3}}^{-1} (-3t-3)^2 \, dt$$

$$= \int_{-\frac{7}{3}}^{-\frac{5}{3}} (3t+7)^2 \, dt + \int_{-\frac{5}{3}}^{-1} (3t+3)^2 \, dt$$

$$= \left[\frac{(3t+7)^3}{3.3} \right]_{-\frac{7}{3}}^{-\frac{5}{3}} + \left[\frac{(3t+3)^3}{3.3} \right]_{-\frac{5}{3}}^{-1}$$

$$= \left[\frac{2^3}{3.3} - 0 \right] + \left[0 - \frac{(-2)^3}{3.3} \right]$$

$$= \frac{8}{9} + \frac{8}{9} = \frac{16}{9}$$

Since E_∞ is finite, $P_\infty = 0$.

e) $y(t) = \sum_{k \in \mathbb{Z}} x(t-6k)$

Since the signal is periodic, $E_\infty = \infty$.

$$P_{avg} = \frac{1}{T_0} \int_0^{T_0} |y(t)|^2 \, dt.$$

$T_0 = 6$.

$$\Rightarrow P_{avg} = \frac{1}{6} \int_0^6 |y(t)|^2 \, dt$$

$$= \frac{1}{6} \int_0^6 |x(t-6k)|^2 \, dt =$$

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$$= \frac{1}{6}\left(\int_0^2 (2-t)^2 dt + \int_4^6 (t-4)^2 \cdot dt\right)$$

$$= \frac{1}{6}\left(\left[-\frac{(2-t)^3}{3}\right]_0^2 + \left[\frac{(t-4)^3}{3}\right]_4^6\right)$$

$$= \frac{1}{6}\left(\left[-0-\left(\frac{-8}{3}\right)\right] + \left[\frac{8}{3}-0\right]\right)$$

$$= \frac{1}{6}\left(\frac{16}{3}\right)$$

$$= \frac{16}{18} = \frac{8}{9} \text{ units}$$

4. a) $y(t) = e^t x^2(t)$

i) LTI:

To prove linearity,

Assume y_1 is output of x_1, y_2 is output of x_2

$\Rightarrow y_1(t) = e^t x_1^2(t)$, $y_2(t) = e^t x_2^2(t)$

Output of $\alpha x_1(t) + \beta x_2(t) = \alpha y_1(t) + \beta y_2(t)$

$= e^t(\alpha x_1(t) + \beta x_2(t))^2 = \alpha e^t x_1^2(t) + \beta e^t x_2^2(t)$

Clearly, they are not equal

$\therefore$ System is <u>not linear</u> $\Rightarrow$ System is <u>not ~~an~~ LTI</u>.

ii) <u>Causal:</u>

~~To prove~~

For any t, $y(t)$ clearly depends on $x(\tau)$, $\forall \tau \leq t$. Therefore system is <u>causal</u>

iii) <u>Bounded:</u>

If we assume $x(t)$ is bounded $\forall t \in \mathbb{R}$, the value of e^t is definitely not bounded.

Therefore $y(t)$ may not always be bounded

$\therefore$ System is <u>not Bounded.</u>

b) $y(t) = t - x(t)$

i) LTI:

To prove linearity,

Assume y_1 is the output of x_1, y_2 is the output of x_2

$\Rightarrow y_1(t) = t - x_1(t)$, $y_2(t) = t - x_2(t)$.

Output of $\alpha x_1(t) + \beta x_2(t) = \alpha y_1(t) + \beta y_2(t)$

$$\Rightarrow t - (\alpha x_1(t) + \beta x_2(t)) = \alpha(t - x_1(t)) + \beta(t - x_2(t))$$

$$= t - \alpha x_1(t) - \beta x_2(t) = \alpha t - \alpha x_1(t) + \beta(t) - \beta x_2(t)$$

Clearly the statement is false

$\therefore$ System is not linear $\Rightarrow$ System is not LTI.

ii) Causal:

For any t, $y(t)$ depends on $x(\tau)$ only $\forall \tau \leq t$.

$\therefore$ System is causal.

iii) Bounded:

Assume $x(t)$ is bounded.

The value of t is not bounded $\forall t \in R$

So $y(t)$ will not be bounded $\forall t \in R$

$\therefore$ System is not Bounded.

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c) $y(t) = \frac{e^{2x(t)}}{t}$

i) LTI:

To prove linearity,

Assume y_1 is the output of x_1, y_2 is the output of x_2

$\Rightarrow y_1(t) = \frac{e^{2x_1(t)}}{t}, \quad y_2(t) = \frac{e^{2x_2(t)}}{t}$

Output of $\alpha x_1(t) + \beta x_2(t) = \alpha y_1(t) + \beta y_2(t)$

$\Rightarrow \frac{e^{2(\alpha x_1(t) + \beta x_2(t))}}{t} = \frac{\alpha e^{2x_1(t)}}{t} + \frac{\beta e^{2x_2(t)}}{t}$

$= \frac{e^{2\alpha x_1(t)} \, e^{2\beta x_2(t)}}{t} = \frac{\alpha e^{2x_1(t)}}{t} + \frac{\beta e^{2x_2(t)}}{t}$

Clearly the above statement is false

$\therefore$ System is not linear $\Rightarrow$ System is not LTI

ii) Causal:

For any t, the value of $y(t)$ depends ~~only~~ on $x(\tau)$ $\forall \tau \leq t$.

Hence, system is causal.

iii) Bounded:

Assume $x(t)$ is bounded $\Rightarrow e^{2x(t)}$ is ~~bounded~~ bounded.

But, the value of $1/t$ is not bounded and diverges as t approaches zero.

$\therefore$ $y(t)$ is not bounded $\Rightarrow$ System is not bounded

d) $y(t) = \int_{-\infty}^{t/2} h(t-\tau)\, x(\tau)\, d\tau$

i) LTI,

To prove linearity,

Assume $y_1(t)$ is the output of $x_1(t)$, $y_2(t)$ is the output of $x_2(t)$

$\Rightarrow y_1(t) = \int_{-\infty}^{t/2} h(t-\tau)\, x_1(\tau)\, d\tau \qquad y_2(t) = \int_{-\infty}^{t/2} h(t-\tau)\, x_2(\tau)\, d\tau.$

Output of $\alpha x_1(t) + \beta x_2(t) = \alpha y_1(t) + \beta y_2(t)$

$$= \int_{-\infty}^{t/2} h(t-\tau)\left(\alpha x_1(\tau) + \beta x_2(\tau)\right) d\tau = \alpha \int_{-\infty}^{t/2} h(t-\tau)\, x_1(\tau)\, d\tau + \beta \int_{-\infty}^{t/2} h(t-\tau)\, x_2(\tau)\, d\tau$$

$$= \alpha \int_{-\infty}^{t/2} h(t-\tau)\, x_1(\tau)\, d\tau + \beta \int_{-\infty}^{t/2} h(t-\tau)\, x_2(\tau)\, d\tau$$

$$= \alpha \int_{-\infty}^{t/2} h(t-\tau)\, x_1(\tau)\, d\tau + \beta \int_{-\infty}^{t/2} h(t-\tau)\, x_2(\tau)\, d\tau.$$

$\therefore$ System is linear.

To prove time invariance,

$$y(t) = \int_{-\infty}^{\frac{t}{2}} h(t-\tau)\, x(\tau)\, d\tau$$

Let $\int_{-\infty}^{\frac{t}{2}} h(t-\tau)\, x(\tau)\, d\tau = w(\tau)$

$$\Rightarrow \quad y(t) = \left[w(\tau) \right]_{-\infty}^{\frac{t}{2}}$$

$$\Rightarrow \quad y(t) = w\left(\frac{t}{2}\right) - w(-\infty)$$

Delaying the input by t_0

$$\Rightarrow \quad y'(t) = w\left(\frac{t}{2} - t_0\right) - w(-\infty) \quad \text{—— (1)}$$

For $y(t-t_0)$,

$$y(t-t_0) = w\left(\frac{t-t_0}{2}\right) - w(-\infty) \quad \text{—— (2)}$$

Clearly (1) $\neq$ (2).

$\Rightarrow$ Output of $x(t-t_0) \neq y(t-t_0)$

$\Rightarrow$ System is <u>time variant</u>.

$\therefore$ System is <u>not a LTI</u>.

ii) Causal.

$$y(t) = \int_{-\infty}^{\frac{t}{2}} h(t-\tau)\, x(\tau)\, d\tau$$

For any t, the value of $y(t)$ depends on the values of $x(\phi)$ and $h(\phi)$ only for values of $\phi \leq t$.

$\therefore$ System is <u>Causal</u>.

iii) ~~B~~ Bounded.

$$y(t) = \int_{-\infty}^{\frac{t}{2}} h(t-\tau)\, x(\tau)\, d\tau$$

By Cauchy-Schwarz inequality

$$\int_{-\infty}^{\frac{t}{2}} h(t-\tau)\, x(\tau)\, d\tau \leq \int_{-\infty}^{\frac{t}{2}} h(t-\tau)\, d\tau \int_{-\infty}^{\frac{t}{2}} x(\tau)\, d\tau$$

Since $h(t)$ is a finite energy signal, it follows that $\int_{-\infty}^{\frac{t}{2}} h(t-\tau)\,d\tau$ is finite.

But $\int_{-\infty}^{\frac{t}{2}} x(\tau)\, d\tau$ may not be bounded even if $x(t)$ is bounded.
(ex: $x(t) = k$, where k is some const)

$\therefore$ $y(t)$ is unbounded $\forall\, t \in \mathbb{R}$

$\Rightarrow$ System is not bounded

e) $y(t) = \frac{d}{dt} x(t+t_0)$.

i) <u>LTI</u>:

To prove linearity,

Let $y_1(t) = \frac{d}{dt} x_1(t+t_0)$, $y_2(t) = \frac{d}{dt} x_2(t+t_0)$

~~Then output of $\alpha x_1(t+t_0) + \beta y_1(t)$~~

Then output of $\alpha x_1(t) + \beta x_2(t) = \alpha y_1(t) + \beta y_2(t)$

$$\Rightarrow \frac{d}{dt}(\alpha x_1(t+t_0)) + \frac{d}{dt}(\beta x_2(t+t_0))$$

$$= \alpha \frac{d}{dt} x_1(t+t_0) + \beta \frac{d}{dt} x_2(t+t_0)$$

Clearly the statement holds true.

$\therefore$ System is linear.

To prove time invariance,

$$y(t) = \frac{d}{dt} x(t+t_0)$$

Shifting the input,

$$y'(t) = \frac{d}{dt} x(t+t_0-t_0') \quad \text{--- (1)}$$

For $y(t-t_0')$,

$$\Rightarrow y(t-t_0') = \frac{d}{dt}(x - t_0' + t_0) \quad \text{--- (2)}$$

(1) = (2) $\therefore$ Output of $x(t-t_0)$ is $y(t-t_0)$

$\Rightarrow$ Behavior of system is consistent with time.

$\therefore$ System is time-invariant.

$\therefore$ System is <u>LTI</u>.

ii) Causal.

Case 1: $t_0 > 0$

The value of $y(t)$ depends on some value of $x(\tau)$ where $\tau > t$.
$\therefore$ System is not causal.

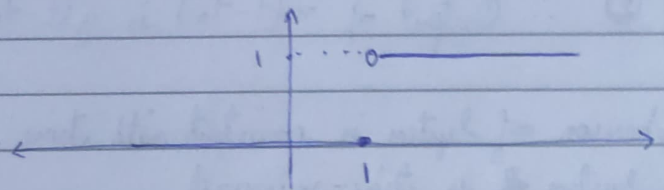
Case 2: $t_0 \leq 0$

The value of $y(t)$ (for all t) depends on the value of $x(\tau)$ $\forall\ \tau \leq t$.
$\therefore$ System is causal

$\therefore$ System is causal only if $t_0 \leq 0$.

iii) Bounded:

Assume $x(t) = \begin{cases} 0, & t \leq 1 \\ 1, & t > 1 \end{cases}$



$|x(t)| \leq 1\ \forall\ t \in \mathbb{R}$ $\therefore$ $x(t)$ is bounded

But, $\frac{d}{dt} x(t+t_0)$ is not defined when $(t+t_0) = 1$.

$\therefore$ $y(t)$ is not bounded $\forall\ t \in \mathbb{R}$
$\therefore$ System is not bounded.

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4. d) (Contenuation)

iii) Bounded : Proof that $\int_{-\infty}^{\frac{t}{2}} x(\tau)\, d\tau$ ~~may~~ is not bounded

Assume $x(t) = 5$

$|x(t)| \leq 5 \;\; \forall\, t \in \mathbb{R}$

$\therefore$ $x(t)$ is bounded.

But, $\int_{-\infty}^{\frac{t}{2}} 5\, d\tau = [5\tau]_{-\infty}^{\frac{t}{2}}$

$= \frac{5t}{2} - 5(-\infty) \leftarrow$ Clearly not bounded.

$\therefore \int_{-\infty}^{\frac{t}{2}} x(\tau)\, d\tau$ may not be bounded even if $x(t)$ is bounded.