Digital Systems & Microcontrollers

Tutorial Quiz Week - 1 Solutions

19th - 21st August, 2025

19th August (Tuesday)

Set-1: Convert $(101110.011)_2$ into decimal, octal, and hexadecimal.

Solution:

(a) Decimal Conversion:

$$\begin{aligned}(101110.011)_2 &= 1\cdot 2^5 + 0\cdot 2^4 + 1\cdot 2^3 + 1\cdot 2^2 + 1\cdot 2^1 + 0\cdot 2^0 + 0\cdot 2^{-1} + 1\cdot 2^{-2} + 1\cdot 2^{-3}\\ &= 32 + 0 + 8 + 4 + 2 + 0 + 0 + 0.25 + 0.125\\ &= 46.375_{10}\end{aligned}$$

(b) Octal Conversion:
For binary to octal conversion, *group bits in threes* from the decimal point. Here, bits can be readily be grouped in three so no need for zero padding.

$$\begin{array}{ccccc} \underbrace{101}_{5} & \underbrace{110}_{6} & . & \underbrace{011}_{3} \\ \downarrow & \downarrow & & \downarrow \\ 5 & 6 & . & 3 \end{array}$$

Therefore:

$$(101110.011)_2 = (56.3)_8$$

(c) Hexadecimal Conversion:
For binary to hexadecimal conversion, *group bits in fours* from the decimal point. For the purpose of grouping, adding zeros to make complete groups of four:

$$(10\ 1110\ .\ 011)_2 = (0010\ 1110\ .\ 0110)_2$$

$$\underbrace{0010}_{2}\underbrace{1110}_{E}.\underbrace{0110}_{6}$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$2 \quad E \; . \; 6$$

Therefore:

$$(101110.011)_2 = (2E.6)_{16}$$

Common Mistakes:

- Incorrect grouping of bits when converting
- Mistakes in handling negative powers in decimal conversion
- Forgetting padding with zeros

Set-2: Convert $(16.51)_{10}$ to octal and hexadecimal (upto 2 decimals).

Solution: To convert a decimal number to another base, we convert the integer and fractional parts separately.

(a) Converting $(16.51)_{10}$ to octal:

Step 1: Convert the integer part (16) to octal

Division	Remainder
$16 \div 8 = 2$	0
$2 \div 8 = 0$	2

Reading the remainders from bottom to top: $(16)_{10} = (20)_8$

Step 2: Convert the fractional part (0.51) to octal

Multiplication	Integer Part
$0.51 \times 8 = 4.08$	4
$0.08 \times 8 = 0.64$	0
$0.64 \times 8 = 5.12$	5

Reading the integer parts from top to bottom: $(0.51)_{10} = (0.405...)_8$

Since we need only 2 decimal places: $(0.51)_{10} \approx (0.41)_8$

Step 3: Combine the results

$$(16.51)_{10} = (20.41)_8$$

(b) Converting $(16.51)_{10}$ to hexadecimal:

Step 1: Convert the integer part (16) to hexadecimal

Division	Remainder
$16 \div 16 = 1$	0
$1 \div 16 = 0$	1

Reading the remainders from bottom to top: $(16)_{10} = (10)_{16}$

Step 2: Convert the fractional part (0.51) to hexadecimal

Multiplication	Integer Part
$0.51 \times 16 = 8.16$	8
$0.16 \times 16 = 2.56$	2
$0.56 \times 16 = 8.96$	8

Reading the integer parts from top to bottom: $(0.51)_{10} = (0.828...)_{16}$

Since we need only 2 decimal places: $(0.51)_{10} \approx (0.83)_{16}$

Step 3: Combine the results

$$(16.51)_{10} = (10.83)_{16}$$

Final Answer: $(16.51)_{10} = (20.41)_8 = (10.83)_{16}$

Common Mistakes:

- Incorrectly reading remainders/integer parts: for integer division read remainders from bottom to top, for fractional multiplication read integer parts from top to bottom
- Rounding errors in the final decimal places

20th August (Wednesday)

Set-1: Using 4-bit 2's complement representation, is it possible to add +5 and +3? Show the addition and justify the answer with reasoning.

Solution: For 4-bit 2's complement representation: the Most-Significant Bit (MSB) is the sign bit. Both 5 and 3 are positive, so their MSBs are 0 and will be represented as:

$$+5 = 0101, \quad +3 = 0011$$

Adding these two binary numbers:

$$\begin{array}{r} 0\ 1\ 0\ 1 \\ +\ 0\ 0\ 1\ 1 \\ \hline 1\ 0\ 0\ 0 \end{array}$$

In 4-bit 2's complement, 1000 means -8 !!, NOT +8 (actual right addition)

Conclusion: Overflow occurs; 8 cannot be represented in 4-bit 2's complement. To solve this problem, we need to use more bits to represent the numbers.

Common Mistakes:

- Students did right addition (got 1000) but couldn't identify that addition output is not correct.
- Forget about the sign bit when interpreting the result.

Set-2: What is the maximum number that can be represented by 4-bit BCD and 4-bit Gray code? What is the advantage of Gray code over BCD?

Hint: Gray code only change one bit at a time between successive numbers.

Solution:

Using four bits, we can represent we can represent $2^4 = 16$ different values (0-15). So, it seems like both 4-bit BCD and Gray code can represent numbers from 0 to 15. BUT, here is the catch:

- **Binary Coded Decimal (BCD)**: Explicitly designed to represent only decimal digits 0-9, with each digit encoded in 4 bits. The bit combinations for values 10-15 (1010 to 1111) are considered invalid in standard BCD.
- **Gray code**: Can represent all 16 values (0-15) and is designed so that adjacent numbers differ by exactly one bit.

$$\text{Max in BCD (4-bit)} = 9, \quad \text{Max in Gray (4-bit)} = 15$$

Advantage of Gray code:

- Only one bit changes between successive numbers, reducing transition errors and glitches in digital systems (this is its primary purpose)
- Memory utilization can be mentioned as another advantage, as Gray code can represent a larger range of values within the same bit-width in comparison of BCD but this is not its main purpose.

Common Mistakes:

- Confusing the total possible combinations (16 or 10) with the maximum representable value (15 for Gray code, 9 for BCD)
- Not recognizing that BCD deliberately leaves bit patterns 1010 (10) through 1111 (15) unused to maintain decimal digit correspondence

21st August (Thursday) Slot-1

Set-1: Show that $E = F_1 + F_2$ contains the minterms of both F_1 and F_2.

Solution:

Let F_1 and F_2 be Boolean functions in the same n variables with sum of minterms

$$F_1 = \sum_{a \in \mathcal{M}(F_1)} \mathrm{m}_a, \qquad F_2 = \sum_{b \in \mathcal{M}(F_2)} \mathrm{m}_b \,. \tag{1}$$

Then

$$\begin{aligned} E &= F_1 + F_2 \\ &= \Bigg(\sum_{a \in \mathcal{M}(F_1)} \mathrm{m}_a \Bigg) + \Bigg(\sum_{b \in \mathcal{M}(F_2)} \mathrm{m}_b \Bigg) \\ &= \sum_{c \in \mathcal{M}(F_1) \cup \mathcal{M}(F_2)} \mathrm{m}_c \,, \end{aligned}$$

where the last step uses the lemma: if a minterm appears in both sums (i.e., $c \in \mathcal{M}(F_1) \cap \mathcal{M}(F_2)$), then $\mathrm{m}_c + \mathrm{m}_c = \mathrm{m}_c$; distinct minterms simply list together. Therefore, the expression for E is

$$\mathcal{M}(E) = \mathcal{M}(F_1) \cup \mathcal{M}(F_2),$$

so E *contains the sum of the minterms of* F_1 *and* F_2 (i.e., every minterm present in either F_1 or F_2 appears in E).

Alternative viewpoint (truth-set union). Interpreting a minterm as the indicator of a single assignment, $F_1 + F_2$ evaluates to 1 on exactly those assignments where $F_1 = 1$ or $F_2 = 1$, i.e., the union of their set of minterms. The sum therefore lists exactly the minterms for assignments in $\mathcal{M}(F_1) \cup \mathcal{M}(F_2)$.

Set-2: Show that $G = F_1 F_2$ contains only the common minterms of F_1 and F_2.

Solution: Using (1), compute the product:

$$\begin{aligned} G &= F_1 F_2 \\ &= \Big(\sum_{a \in \mathcal{M}(F_1)} \mathrm{m}_a\Big)\Big(\sum_{b \in \mathcal{M}(F_2)} \mathrm{m}_b\Big) \\ &= \sum_{a \in \mathcal{M}(F_1)} \sum_{b \in \mathcal{M}(F_2)} \mathrm{m}_a \, \mathrm{m}_b \\ &= \sum_{c \in \mathcal{M}(F_1) \cap \mathcal{M}(F_2)} \mathrm{m}_c \,. \end{aligned}$$

As $\mathrm{m}_a \, \mathrm{m}_b = 0$ for $a \neq b$ and $\mathrm{m}_c \, \mathrm{m}_c = \mathrm{m}_c$, so all cross terms vanish except those where the minterm index matches in both sums. Consequently,

$$\mathcal{M}(G) = \mathcal{M}(F_1) \cap \mathcal{M}(F_2),$$

meaning that G *contains only* the minterms that are common to F_1 and F_2.

Alternative viewpoint (truth-set intersection). $F_1 F_2$ evaluates to 1 exactly on assignments where $F_1 = 1$ *and* $F_2 = 1$, i.e., the intersection of their set of minterms; hence the product lists precisely the common minterms.

21st August (Thursday) Slot-2

Set-1: Find the maxterm expression:

$$F(A, B, C) = \prod(0, 2, 4, 7)$$

Solution:

$$\begin{aligned} F(A, B, C) &= \prod(0, 2, 4, 7) \\ &= M_0 M_2 M_4 M_7 \\ &= (A + B + C)(A + \overline{B} + C)(\overline{A} + B + C)(\overline{A} + \overline{B} + \overline{C}) \end{aligned}$$

Set-2: Find the minterm expression:

$$F(A, B, C) = \sum(1, 4, 6)$$

Solution:

$$\begin{aligned} F(A, B, C) &= \sum(1, 4, 6) \\ &= m_1 + m_4 + m_6 \\ &= A\overline{B}\overline{C} + \overline{A}B\overline{C} + \overline{A}\overline{B}C \end{aligned}$$