$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cdots}}}$$

term 0 $\quad a_0 \Rightarrow \dfrac{a_0}{1}$

term 1 $\quad a_0 + \dfrac{1}{a_1} = \dfrac{a_0 a_1 + 1}{a_1}$

term 2 $\quad a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2}} = a_0 + \cfrac{1}{\cfrac{a_1 a_2 + 1}{a_2}}$

$$= a_0 + \frac{a_2}{a_1 a_2 + 1}$$

$$= \frac{a_0(a_1 a_2 + 1) + a_2}{a_1 a_2 + 1}$$

$$= \frac{a_0 a_1 a_2 + a_0 + a_2}{a_1 a_2 + 1}$$

$$\Rightarrow \frac{a_2(a_0 a_1 + 1) + a_0}{a_1 a_2 + 1}$$

$$\cdots\, a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3}}$$

$$\Rightarrow \quad a_0 + \cfrac{1}{a_1 + \cfrac{1}{\cfrac{a_2a_3+1}{a_3}}}$$

$$= \quad a_0 + \cfrac{1}{a_1 + \cfrac{a_3}{a_2a_3+1}}$$

$$\Rightarrow \quad a_0 + \cfrac{1}{\cfrac{a_1(a_2a_3+1)+a_3}{a_2a_3+1}}$$

$$\Rightarrow \quad a_0 + \cfrac{1}{\cfrac{a_3(a_1a_2+1)+a_1}{a_2a_3+1}}$$

$$\Rightarrow \quad \frac{a_0(a_2a_3+1)+1}{a_3(a_1a_2+1)+a_1}$$

$$\Rightarrow \quad a_0a_2a_3 + a_0 + 1$$

$$\Rightarrow a_0 + \cfrac{1}{\cfrac{a_3(a_1a_2+1)+a_1}{a_2a_3+1}}$$

$$= a_0 + \frac{a_2a_3+1}{a_3(a_1a_2+1)+a_1}$$

$$\Rightarrow \frac{a_0(a_3(a_1a_2+1)+a_1)+a_2a_3+1}{a_3(a_1a_2+1)+a_1}$$

$$\Rightarrow \frac{a_3a_0(a_1a_2+1)+a_2a_3+(a_0a_1+1)}{a_3(a_1a_2+1)+a_1}$$

$$\Rightarrow \frac{a_3(a_2(a_0a_1+1))+(a_0a_1+1)}{a_3(a_1a_2+1)+a_1}$$

Numerator Sequence

$$a_1 a_0 + 1$$

$$a_0 \;;\; a_0 a_1 + 1 \;;\; a_2(a_0 a_1 + 1) + a_0 \;;$$

$$a_3(a_2(a_0 a_1 + 1)) + (a_0 a_1 + 1)$$

$$\boxed{h_n = a_n h_{n-1} + h_{n-2}}$$

$n \Rightarrow ? \rightarrow 0, 1, 2, 3 \dots$

$$\boxed{\begin{array}{l} h_{-1} = 1 \\ h_{-2} = 0 \end{array}} \qquad \cancel{\begin{array}{l} K_{n-1} = 1 \\ K_{n-2} = 2 \end{array}}$$

Similarly $\boxed{K_n = a_n K_{n-1} + K_{n-2}}$

~~Test~~ $\cancel{n \neq 0}$

$\boxed{K_0 = 1}$? $K_1 = a_1$

$$\boxed{K_{-1} = 0 \;;\; K_{-2} = 1}$$

$$K_0 = \quad K_0 = a_0 K_{-1} + K_{-2}$$

$$= a_0(0) + 1 = 1 \checkmark$$

$$K_1 = a_1 K_0 + K_{-1} = a_1(1) + 0$$

$$= a_1 \checkmark$$

$$K_2 = a_2 K_1 + K_0 = a_2 a_1 + 1$$

Numerator

$$\boxed{h_n = a_n h_{n-1} + h_{n-2}}$$

Base Case

$$\boxed{\begin{aligned} h_{-1} &= 1 \\ h_{-2} &= 0 \end{aligned}}$$

Denominator

$$\boxed{K_n = a_n K_{n-1} + K_{n-2}}$$

Base Case

$$\boxed{\begin{aligned} K_{-1} &= 0 \\ K_{-2} &= 1 \end{aligned}}$$

$$\sqrt{2} \Rightarrow 1 + \sqrt{2} - 1$$

$$\Rightarrow 1 + \frac{(\sqrt{2}-1)(\sqrt{2}+1)}{(\sqrt{2}+1)}$$

$$= 1 + \frac{(2-1)}{1+\sqrt{2}} \Rightarrow 1 + \frac{1}{1+\sqrt{2}}$$

$$\therefore \boxed{\sqrt{2} = 1 + \frac{1}{1+\sqrt{2}}}$$

$$\cancel{\sqrt{2} = \frac{1}{1+}}$$

$$\sqrt{2} = 1 + \frac{1}{1+\sqrt{2}}$$

$$= 1 + \frac{1}{1 + 1 + \frac{1}{1+\sqrt{2}}}$$

$$= 1 + \frac{1}{2 + \frac{1}{1+\sqrt{2}}}$$

$$= 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{1+\sqrt{2}}}}$$

$$\Rightarrow \boxed{\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \cdots}}}}$$

$$\therefore \sqrt{2} = [1; 2, 2, 2, 2 \cdots]$$

$$\therefore a_0 = 1 ; \quad a_{n \neq 0} = 2$$