



```
In [1]: import numpy as np
def gradient_descent(f,initial_point,learning_rate,iterations):
    point=initial_point.copy()
    history=[]

    for _ in range(iterations):
        gradient=np.array([2*point[0],2*point[1]])
        point-=learning_rate*gradient
        value=f(point[0],point[1])
        history.append((point.copy(),value))
        print("Values of points and objective function value are: {} and {}".format(point, value))
    return point,history

def objective_function(x,y):
    return x**2 + y**2 +4
initial_point=np.array([1.0,1.0])
learning_rate=0.1
iterations=100

minimum_point,history=gradient_descent(objective_function,initial_point,learning_rate,iterations)
print("Minimum point(x,y):", minimum_point)
print("Minimum value of the function:",objective_function(*minimum_point))
```

Values of points and objective function value are: [0.8 0.8] and 5.28  
Values of points and objective function value are: [0.64 0.64] and 4.8192  
Values of points and objective function value are: [0.512 0.512] and 4.524  
288  
Values of points and objective function value are: [0.4096 0.4096] and 4.3  
3554432  
Values of points and objective function value are: [0.32768 0.32768] and  
4.2147483648  
Values of points and objective function value are: [0.262144 0.262144] and  
4.137438953472  
Values of points and objective function value are: [0.2097152 0.2097152] a  
nd 4.08796093022208  
Values of points and objective function value are: [0.16777216 0.16777216]  
and 4.056294995342131  
Values of points and objective function value are: [0.13421773 0.13421773]  
and 4.036028797018964  
Values of points and objective function value are: [0.10737418 0.10737418]  
and 4.023058430092137  
Values of points and objective function value are: [0.08589935 0.08589935]  
and 4.014757395258967  
Values of points and objective function value are: [0.06871948 0.06871948]  
and 4.009444732965739  
Values of points and objective function value are: [0.05497558 0.05497558]  
and 4.006044629098073  
Values of points and objective function value are: [0.04398047 0.04398047]  
and 4.003868562622767  
Values of points and objective function value are: [0.03518437 0.03518437]  
and 4.0024758800785705  
Values of points and objective function value are: [0.0281475 0.0281475] a  
nd 4.001584563250285  
Values of points and objective function value are: [0.022518 0.022518] and  
4.001014120480183  
Values of points and objective function value are: [0.0180144 0.0180144] a  
nd 4.0006490371073165  
Values of points and objective function value are: [0.01441152 0.01441152]  
and 4.000415383748683  
Values of points and objective function value are: [0.01152922 0.01152922]  
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Values of points and objective function value are: [0.00922337 0.00922337]  
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Values of points and objective function value are: [0.0073787 0.0073787] a  
nd 4.0001088903574145  
Values of points and objective function value are: [0.00590296 0.00590296]  
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Values of points and objective function value are: [0.00472237 0.00472237]  
and 4.000044601490397  
Values of points and objective function value are: [0.00377789 0.00377789]  
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Values of points and objective function value are: [0.00302231 0.00302231]  
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Values of points and objective function value are: [0.00241785 0.00241785]  
and 4.000011692013099  
Values of points and objective function value are: [0.00193428 0.00193428]  
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Values of points and objective function value are: [0.00154743 0.00154743]  
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Values of points and objective function value are: [0.00123794 0.00123794]  
and 4.000003064991081  
Values of points and objective function value are: [0.00099035 0.00099035]  
and 4.000001961594292  
Values of points and objective function value are: [0.00079228 0.00079228]

and 4.000001255420347  
Values of points and objective function value are: [0.00063383 0.00063383]  
and 4.000000803469022  
Values of points and objective function value are: [0.00050706 0.00050706]  
and 4.000000514220174  
Values of points and objective function value are: [0.00040565 0.00040565]  
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Values of points and objective function value are: [0.00032452 0.00032452]  
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Values of points and objective function value are: [0.00025961 0.00025961]  
and 4.000000134799733  
Values of points and objective function value are: [0.00020769 0.00020769]  
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Values of points and objective function value are: [0.00016615 0.00016615]  
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Values of points and objective function value are: [0.00013292 0.00013292]  
and 4.000000035336941  
Values of points and objective function value are: [0.00010634 0.00010634]  
and 4.000000022615643  
Values of points and objective function value are: [8.50705917e-05 8.50705  
917e-05] and 4.000000014474011  
Values of points and objective function value are: [6.80564734e-05 6.80564  
734e-05] and 4.000000009263367  
Values of points and objective function value are: [5.44451787e-05 5.44451  
787e-05] and 4.0000000059285545  
Values of points and objective function value are: [4.3556143e-05 4.355614  
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Values of points and objective function value are: [2.78759315e-05 2.78759  
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Values of points and objective function value are: [2.23007452e-05 2.23007  
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Values of points and objective function value are: [1.42724769e-05 1.42724  
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Values of points and objective function value are: [1.14179815e-05 1.14179  
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Values of points and objective function value are: [7.30750819e-06 7.30750  
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Values of points and objective function value are: [5.84600655e-06 5.84600  
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Values of points and objective function value are: [4.67680524e-06 4.67680  
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Values of points and objective function value are: [3.74144419e-06 3.74144  
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Values of points and objective function value are: [2.99315535e-06 2.99315  
535e-06] and 4.000000000017918  
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Values of points and objective function value are: [1.91561943e-06 1.91561  
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Values of points and objective function value are: [1.53249554e-06 1.53249  
554e-06] and 4.000000000004697  
Values of points and objective function value are: [1.22599643e-06 1.22599  
643e-06] and 4.0000000000030065  
Values of points and objective function value are: [9.80797146e-07 9.80797  
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Values of points and objective function value are: [7.84637717e-07 7.84637717e-07] and 4.0000000000001231

Values of points and objective function value are: [6.27710174e-07 6.27710174e-07] and 4.000000000000788

Values of points and objective function value are: [5.02168139e-07 5.02168139e-07] and 4.0000000000005045

Values of points and objective function value are: [4.01734511e-07 4.01734511e-07] and 4.000000000000322

Values of points and objective function value are: [3.21387609e-07 3.21387609e-07] and 4.000000000000207

Values of points and objective function value are: [2.57110087e-07 2.57110087e-07] and 4.000000000000132

Values of points and objective function value are: [2.0568807e-07 2.0568807e-07] and 4.000000000000084

Values of points and objective function value are: [1.64550456e-07 1.64550456e-07] and 4.000000000000054

Values of points and objective function value are: [1.31640365e-07 1.31640365e-07] and 4.000000000000035

Values of points and objective function value are: [1.05312292e-07 1.05312292e-07] and 4.000000000000022

Values of points and objective function value are: [8.42498333e-08 8.42498333e-08] and 4.000000000000014

Values of points and objective function value are: [6.73998667e-08 6.73998667e-08] and 4.000000000000009

Values of points and objective function value are: [5.39198933e-08 5.39198933e-08] and 4.000000000000006

Values of points and objective function value are: [4.31359147e-08 4.31359147e-08] and 4.000000000000036

Values of points and objective function value are: [3.45087317e-08 3.45087317e-08] and 4.000000000000003

Values of points and objective function value are: [2.76069854e-08 2.76069854e-08] and 4.000000000000002

Values of points and objective function value are: [2.20855883e-08 2.20855883e-08] and 4.000000000000001

Values of points and objective function value are: [1.76684706e-08 1.76684706e-08] and 4.000000000000001

Values of points and objective function value are: [1.41347765e-08 1.41347765e-08] and 4.0

Values of points and objective function value are: [1.13078212e-08 1.13078212e-08] and 4.0

Values of points and objective function value are: [9.04625697e-09 9.04625697e-09] and 4.0

Values of points and objective function value are: [7.23700558e-09 7.23700558e-09] and 4.0

Values of points and objective function value are: [5.78960446e-09 5.78960446e-09] and 4.0

Values of points and objective function value are: [4.63168357e-09 4.63168357e-09] and 4.0

Values of points and objective function value are: [3.70534686e-09 3.70534686e-09] and 4.0

Values of points and objective function value are: [2.96427748e-09 2.96427748e-09] and 4.0

Values of points and objective function value are: [2.37142199e-09 2.37142199e-09] and 4.0

Values of points and objective function value are: [1.89713759e-09 1.89713759e-09] and 4.0

Values of points and objective function value are: [1.51771007e-09 1.51771007e-09] and 4.0

Values of points and objective function value are: [1.21416806e-09 1.21416806e-09] and 4.0

Values of points and objective function value are: [9.71334446e-10 9.71334446e-10] and 4.0

446e-10] and 4.0

Values of points and objective function value are: [7.77067557e-10 7.77067557e-10] and 4.0

Values of points and objective function value are: [6.21654046e-10 6.21654046e-10] and 4.0

Values of points and objective function value are: [4.97323236e-10 4.97323236e-10] and 4.0

Values of points and objective function value are: [3.97858589e-10 3.97858589e-10] and 4.0

Values of points and objective function value are: [3.18286871e-10 3.18286871e-10] and 4.0

Values of points and objective function value are: [2.54629497e-10 2.54629497e-10] and 4.0

Values of points and objective function value are: [2.03703598e-10 2.03703598e-10] and 4.0

Minimum point(x,y): [2.03703598e-10 2.03703598e-10]

Minimum value of the function: 4.0

In [ ]: