Model

Parameters

卡车相关参数

- α **:** fixed cost per day per truck
- β: transportation cost per package per unit distance
- s : a sequence of index of different capacities
- C_s: capacity of sth type of truck
- L **:** max number of legs allowed to be traveled by a truck
- D **:** max distance allowed to be traveled by a truck
- $Speed$ **:** average speed of trucks, if necessary it can be truck specific
- $DrivingTimePerDay$ **:** driving time per day allowed for trucks
- $b_{ij}^{\tau} = 1$ if τ contains arc(i,j)
- L_{so} **:** the number of trucks available starting from origin o with capacity of C_s

节点相关参数

- q^p: quantity of pickup and delivery demand p
- $l_{i,j}$: distance of arc(i,j)

Auxiliary graph $G'(V', A')$

- V' **:** for each $u \in V$, associate T vertices: $u_1, u_2, \ldots, u_T$
- $A_T = \{(u_t, u_{(t+1) mod T}) | u \in V, t \in \{1, 2, \ldots, T\}\}$
- $\tilde{A} = \{(u_t, (w_{(t+t(u,w)) mod T})) | (u, w) \in A, t \in \{1, 2, \ldots, T\}\}$
- $A' = A_T \cup \tilde{A}$
- cost: $A_T = 0 \qquad \tilde{A} = l(u, w)$

Decision variables

- z_τ: the number of trucks choose to run in cycle τ
- $x_{i,j}^p$: a split of demand q^p shipped on arc $(i,j) \in \tilde{A} \cup A_T$

Sets

- $\boldsymbol{V}$: set of nodes
- $\boldsymbol{A}$: set of arcs
- $\boldsymbol{P}$: set of demand O-D pairs
- $\boldsymbol{S}$: set of index of different capacities of trucks

Indices

- $\boldsymbol{i, j}$: index of nodes
- $\boldsymbol{(i, j)}$: index of arcs
- $\boldsymbol{p}$: index of O-D pairs

Const

$$b_i^p = \begin{cases} q^p & i = o(p) \\ -q^p & i = d(p) \\ 0 & \text{otherwise} \end{cases}$$

Minimize

$$\sum_{\tau \in \theta} \sum_{(i,j) \in \tilde{A} \cup A_T} \frac{\alpha l_{ij} b_{ij}^{\tau} z_{\tau}}{Speed * DrivingTimePerDay} + \sum_{s \in S} \sum_{\tau \in \theta_s} \gamma_s z_{\tau} + \sum_{p \in P} \sum_{(i,j) \in \tilde{A} \cup A_T} \beta l_{ij} x_{ij}^p$$

Subject to:

$$\sum_{(i,j) \in \delta^+(i)} x_{ij}^p - \sum_{(j,i) \in \delta^-(i)} x_{ji}^p = b_i^p \qquad \forall p \in P, i \in V' \qquad (1)$$

$$\sum_{p \in P} x_{ij}^p \leqslant \sum_{s \in S} \sum_{\tau \in \theta_s} C_s b_{ij}^{\tau} z_{\tau} \qquad \forall (i,j) \in \tilde{A} \qquad (2)$$

$$\sum_{\tau \in \theta} b_{ij}^{\tau} z_{\tau} \leqslant 1 \qquad \forall (i,j) \in \tilde{A} \qquad (3)$$

$$\sum_{\tau \in \theta_{so}} z_{\tau} \leqslant L_{so} \qquad \forall s \in S, \forall o \in O \qquad (4)$$

$$x_{ij}^p \geqslant 0 \qquad \forall p \in P, (i,j) \in \tilde{A} \cup A_T \qquad (5)$$

$$z_{\tau} \in Z \qquad \forall \tau \in \theta \qquad (6)$$