Model

Parameters

卡车相关参数

- α : fixed cost per day per truck
- β: transportation cost per package per unit distance
- s : a sequence of index of different capacities
- C_s: capacity of sth type of truck
- L : max number of legs allowed to be traveled by a truck
- D : max distance allowed to be traveled by a truck
- $Speed$: average speed of trucks, if necessary it can be truck specific
- $DrivingTimePerDay$: driving time per day allowed for trucks
- $b_{ij}^{\tau} = 1$ if τ contains arc(i,j)
- L_{so} : the number of trucks available starting from origin o with capacity of C_s

节点相关参数

- q^p: quantity of pickup and delivery demand p
- $l_{i,j}$: distance of arc(i, j)

Auxiliary graph $G^{'}(V^{'}, A^{'})$

- $V^{'}$: for each $u \in V$, associate T vertices: $u_1, u_2, \ldots, u_T$
- $A_T = \{(u_t, u_{(t+1)modT})|u \in V, t \in \{1, 2, \ldots, T\}\}$
- $\tilde{A} = \{(u_t, (w_{(t+t(u,w))modT}))|(u, w) \in A, t \in \{1, 2, \ldots, T\}\}$
- $A^{'} = A_T \cup \tilde{A}$
- cost: $A_T = 0 \qquad \tilde{A} = l(u, w)$

Decision variables

- z_τ: the number of trucks choose to run in cycle τ
- $x_{i,j}^p$: a split of demand q^p shipped on arc $(i, j) \in \tilde{A} \cup A_T$

Sets

- V: set of nodes
- A: set of arcs
- P: set of demand O-D pairs
- S: set of index of different capacities of trucks

Indices

- i, j: index of nodes
- (i, j): index of arcs
- p: index of O-D pairs

Const

$$b_i^p = \begin{cases} q^p & i = o(p) \\ -q^p & i = d(p) \\ 0 & \text{otherwise} \end{cases}$$

Minimize

$$\sum_{\tau \in \theta} \sum_{(i,j) \in \tilde{A} \cup A_T} \frac{\alpha l_{ij} b_{ij}^\tau z_\tau}{Speed * DrivingTimePerDay} + \sum_{s \in S} \sum_{o \in O} \sum_{\tau \in \theta_{so}} \gamma_{so} z_\tau + \sum_{p \in P} \sum_{(i,j) \in \tilde{A} \cup A_T} \beta l_{ij} x_{ij}^p$$

Subject to:

$$\sum_{(i,j) \in \delta^+(i)} x_{ij}^p - \sum_{(j,i) \in \delta^-(i)} x_{ji}^p = b_i^p \qquad \forall p \in P, i \in V' \qquad (1)$$

$$\sum_{p \in P} x_{ij}^p \leqslant \sum_{s \in S} \sum_{\tau \in \theta_s} C_s b_{ij}^\tau z_\tau \qquad \forall (i,j) \in \tilde{A} \qquad (2)$$

$$\sum_{\tau \in \theta_{so}} z_\tau + q_{so} = L_{so} \qquad \forall s \in S, \forall o \in O \qquad (3)$$

$$x_{ij}^p \geqslant 0 \qquad \forall p \in P, (i,j) \in \tilde{A} \cup A_T \qquad (4)$$
$$z_\tau \in Z \qquad \forall \tau \in \theta \qquad (5)$$
$$q_{so} \geqslant 0 \qquad \forall s \in S, o \in O \qquad (6)$$

$$\sum_{\tau \in \theta} b_{ij}^\tau z_\tau \leqslant 1 \qquad \forall (i,j) \in \tilde{A}$$