

Model

Parameters

卡车相关参数

- α : fixed cost per day per truck
- β : transportation cost per package per unit distance
- C_k : capacity of truck k
- L : max number of legs allowed to be traveled by a truck
- D : max distance allowed to be traveled by a truck
- $Speed$: average speed of trucks, if necessary it can be truck specific
- $DrivingTimePerDay$: driving time per day allowed for trucks

节点相关参数

- a_i : arrival time lower bound of node i
- b_i : departure time upper bound of node i
- HL_i : open time of node i
- HU_i : close time of node i
- P_i : processing time at node i
- $d_{s,t}$: demand quantity from node s to node t
- $l_{i,j}$: distance of arc (i, j)
- \overline{M} : a sufficiently large value

Decision variables

- $X_{i,j,k}$: =1 if arc (i, j) belongs to the route of truck k , otherwise 0
- $y_{i,j}^{k,s,t}$: a split of demand $d_{s,t}$ shipped on arc (i, j) by truck k
- $x_{i,j}^{k,s,t}$: =1 if arc (i, j) is part of the route to ship demand from s to t by truck k
- $t_{k,i}^a$: arrival time at node i by truck k
- $t_{k,i}^d$: departure time at node i by truck k

Sets

- N : set of nodes
- A : set of arcs
- K : set of trucks
- $DemandODs$: set of demand O-D pairs

Indices

- i, j : index of nodes
- (i, j) : index of arcs
- (s, t) : index of O-D pairs
- k : index of trucks

Minimize

$$\sum_{j \in N: (j,i) \in A} \sum_{k \in K} \frac{\alpha_{ij} X_{ijk}}{\text{Speed} * \text{DrivingTimePerDay}} + \sum_{k \in K} \sum_{(s,t) \in \text{demandODs}} \sum_{(i,j) \in A} \beta_{ij} y_{ij}^{kst}$$

Subject to:

$$\sum_{(j,i) \in A} X_{jik} = \sum_{(j,i) \in A} X_{ijk} \quad \forall i \in N, k \in K \quad (1)$$

$$\sum_{(i,j) \in A} X_{ijk} \leq 1 \quad \forall i \in N : i = \text{StartNode}_k, k \in K \quad (2)$$

$$\sum_{(i,j) \in A} X_{ijk} \leq L \quad \forall k \in K \quad (3)$$

$$\sum_{(i,j) \in A} l_{ij} X_{ijk} \leq D \quad \forall k \in K \quad (4)$$

$$t_{ki}^d \leq b_i + \bar{M}(1 - \sum_{(j,i) \in A} X_{jik}) \quad \forall k \in K, i \in N \quad (5)$$

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$$t_{ki}^a \geq a_i - \bar{M}(1 - \sum_{(j,i) \in A} X_{jik}) \quad \forall k \in K, i \in N \quad (6)$$

$$t_{ki}^d \leq HU_i + \bar{M}(1 - \sum_{(j,i) \in A} X_{jik}) \quad \forall k \in K, i \in N \quad (7)$$

$$t_{ki}^a \geq HL_i - \bar{M}(1 - \sum_{(j,i) \in A} X_{jik}) \quad \forall k \in K, i \in N \quad (8)$$

$$t_{kj}^a \leq t_{ki}^d + \frac{l_{ij}}{\text{Speed}} + \bar{M}(1 - X_{ijk}) \quad \forall k \in K, (i,j) \in A \quad (9)$$

$$t_{kj}^a \geq t_{ki}^d + \frac{l_{ij}}{\text{Speed}} - \bar{M}(1 - X_{ijk}) \quad \forall k \in K, (i,j) \in A \quad (10)$$

$$t_{ki}^d \geq t_{ki}^a + P_i \quad \forall k \in K, i \in N : i \neq \text{StartNode}_k \quad (11)$$

$$t_{k1,i}^a + P_i \leq t_{k2,i}^d + \bar{M}(2 - x_{i1,i}^{k1,st} - x_{i,i2}^{k2,st}) \quad \forall k1, k2 \in K : k1 \neq k2; (i1,i), (i,i2) \in A; (s,t) \in \text{DemandODs} \quad (12)$$

$$\sum_{k \in K} \sum_{(s,j) \in A} y_{sj}^{kst} = d_{st} \quad \forall (s,t) \in \text{DemandODs} \quad (13)$$

$$\sum_{k \in K} \sum_{(j,t) \in A} y_{jt}^{kst} = d_{st} \quad \forall (s,t) \in \text{DemandODs} \quad (14)$$

$$\sum_{k \in K} \sum_{(i,j) \in A} y_{ij}^{kst} - \sum_{k \in K} \sum_{(j,i) \in A} y_{ji}^{kst} = 0 \quad \forall k \in K; (s,t) \in \text{DemandODs}; i \in N : i \neq s, i \neq t \quad (15)$$

$$y_{ij}^{kst} \leq \bar{M} x_{ij}^{kst} \quad \forall k \in K, (s,t) \in \text{DemandODs}, (i,j) \in A \quad (16)$$

$$x_{ij}^{kst} \leq X_{ijk} \quad \forall k \in K, (s,t) \in \text{DemandODs}, (i,j) \in A \quad (17)$$

$$\sum_{(s,j) \in A} x_{sj}^{kst} \leq 1 \quad \forall k \in K, (s,t) \in \text{DemandODs} \quad (18)$$

$$\sum_{(s,t) \in \text{DemandODs}} y_{ij}^{kst} \leq C_k \quad \forall k \in K, (i,j) \in A \quad (19)$$

$$\sum_{(i,j) \in A} X_{ijk} \leq M \sum_{(s,j) \in A} X_{sjk} \quad \forall k \in K : s = \text{StartNode}_k \quad (20)$$

$$\sum_{(s,j) \in A} x_{sj}^{kst} + \sum_{(i,s) \in A} x_{is}^{kst} \leq 1 \quad \forall k \in K : s = \text{StartNode}_k, \forall (s,t) \in \text{DemandODs} \quad (21)$$

$$X_{ijk} \in \{0, 1\} \quad \forall k \in K, (i,j) \in A \quad (22)$$

$$x_{ijk}^{kst} \in \{0, 1\} \quad \forall k \in K, (s,t) \in \text{DemandODs}, (i,j) \in A \quad (23)$$

