Model

Parameters

卡车相关参数

- α : fixed cost per day per truck
- β : transportation cost per package per unit distance
- C_k: capacity of truck k
- γ_k : fixed cost for the use of truck k
- L : max number of legs allowed to be traveled by a truck
- D : max distance allowed to be traveled by a truck
- $Speed$: average speed of trucks, if necessary it can be truck specific
- $DrivingTimePerDay$: driving time per day allowed for trucks

节点相关参数

- q^p: quantity of pickup and delivery demand p
- $l_{i,j}$: distance of arc(i,j)
- $\overline{M}$: a sufficiently large value

Auxiliary graph $G'(V', A')$

- V_0 : depot of all vehicles
- V_{st} : for each $u \in V \setminus V_0$, associate T+1 vertices: $u_0, u_1, \ldots, u_T$
- $A_T = \{(u_t, u_{t+1}) | u \in V \setminus V_0, t \in \{0, 1, \ldots, T-1\}\}$
- $\tilde{A} = \{(u_t, w_t + t(u,w)) | (u,w) \in A \setminus \delta(V_0), t \in \{0, 1, \ldots, T - t(u,w)\}\}$
- O, D : origin and destination of vehicles(depot)
- $A^O = \{(o_k, u_0) | u \in V \setminus V_0, k \in K\}$
- $A^D = \{(u_T, d_k) | u \in V \setminus V_0, k \in K\}$
- $V' = V_{st} \cup \{O, D\}$
- $A' = A_t \cup \tilde{A} \cup A^O \cup A^D$
- cost: $A_T = 0 \qquad \tilde{A} = l(u,w)$

Decision variables

- $X_{i,j}^k$:=1 if arc (i,j)belongs to the route of vehicle k, otherwise 0
- $y_{i,j}^p$: a split of demand q^p shipped on arc $(i,j) \in \tilde{A} \cup A_T$

Sets

- V: set of nodes
- A: set of arcs
- K: set of tracks
- P: set of demand O-D pairs

Indices

- i, j: index of nodes
- (i, j): index of arcs
- p: index of O-D pairs
- k: index of tracks

Const

$$b_{u_t}^p = \begin{cases} q^p & u = o^p, t = 0 \\ -q^p & u = d^p, t = T \\ 0 & \text{otherwise} \end{cases}$$

Minimize

$$\sum_{k \in K} \sum_{(i,j) \in \tilde{A} \cup A_T} \frac{\alpha l_{ij} X_{ij}^k}{Speed * DrivingTimePerDay} + \sum_{k \in K} \sum_{(o_k, i) \in A'} \gamma_k X_{o_k, i}^k + \sum_{(i,j) \in \tilde{A} \cup A_T} \sum_{p \in P} \beta l_{ij} y_{ij}^p$$

Subject to:

$$\sum_{(j,i) \in A'} X_{ji}^k = \sum_{(i,j) \in A'} X_{ij}^k \qquad \forall i \in V_{st}, k \in K \qquad (1)$$

$$\sum_{(o_k, i) \in A'} X_{o_k, i}^k \leqslant 1 \qquad \forall k \in K \qquad (2)$$

$$\sum_{(o_{k'}, i) \in A', k' \neq k} X_{o_{k'}, i}^k = 0 \qquad \forall k \in K \qquad (3)$$

$$\sum_{(i, d_{k'}) \in A', d_{k'} \neq d_k} X_{i, d_{k'}}^k = 0 \qquad \forall k \in K \qquad (4)$$

$$\sum_{(i,j) \in \tilde{A}} X_{ij}^k \leqslant L \qquad \forall k \in K \qquad (5)$$

$$\sum_{(i,j) \in \tilde{A}} l_{ij} X_{ij}^k \leqslant D \qquad \forall k \in K \qquad (6)$$

$$\sum_{(i,j) \in \delta^+(i) \setminus A^D} y_{ij}^p - \sum_{(j,i) \in \delta^-(i) \setminus A^O} y_{ji}^p = b_i^p \qquad \forall p \in P, i \in V_{st} \qquad (7)$$

$$\sum_{p \in P} y_{ij}^p \leqslant \sum_{k \in K} C_k X_{ij}^k \qquad \forall (i,j) \in \tilde{A} \qquad (8)$$

$$X_{ij}^{k} \in \{0,1\} \qquad \forall k \in K, (i,j) \in A^{'} \tag{9}$$

$$y_{ij}^{p} \geqslant 0 \qquad \forall p \in P, (i,j) \in \tilde{A} \cup A_{T} \tag{10}$$