Model

Parameters

卡车相关参数

- α **:** fixed cost per day per truck
- β: transportation cost per package per unit distance
- s: a sequence of index of different capacities
- C_s: capacity of sth type of truck
- L **:** max number of legs allowed to be traveled by a truck
- D **:** max distance allowed to be traveled by a truck
- $Speed$ **:** average speed of trucks, if necessary it can be truck specific
- $DrivingTimePerDay$ **:** driving time per day allowed for trucks
- $b_{ij}^{kos} = 1$ if θ_k^{os} contains arc(i,j)
- L_{os} **:** the number of trucks available starting from origin o with capacity of C_s

节点相关参数

- q^p: quantity of pickup and delivery demand p
- $l_{i,j}$: distance of arc(i,j)

Auxiliary graph $G'(V', A')$

- V_0 **:** depot of all vehicles
- V_{st} **:** for each $u \in V \setminus V_0$, associate T+1 vertices: $u_0, u_1, \ldots, u_T$
- $A_T = \{(u_t, u_{t+1}) | u \in V \setminus V_0, t \in \{0, 1, \ldots, T-1\}\}$
- $\tilde{A} = \{(u_t, w_t + t(u,w)) | (u,w) \in A \setminus \delta(V_0), t \in \{0, 1, \ldots, T - t(u,w)\}\}$
- O, D **:** origin and destination of vehicles(depot)
- $A^O = \{(o_k, u_0) | u \in V \setminus V_0, k \in K\}$
- $A^D = \{(u_T, d_k) | u \in V \setminus V_0, k \in K\}$
- $V' = V_{st} \cup \{O, D\}$
- $A' = A_t \cup \tilde{A} \cup A^O \cup A^D$
- cost: $A_T = 0 \qquad \tilde{A} = l(u,w)$

Decision variables

- θ_k^{os}: the number of trucks with capacity C_s start from depot o choses the kth route which $r_k \in \Omega_o$
- $y_{i,j}^p$: a split of demand q^p shipped on arc $(i,j) \in \tilde{A} \cup A_T$

Sets

- V: set of nodes

- A: set of arcs
- P: set of demand O-D pairs
- O: set of origins
- S: set of index of different capacities of trucks

Indices

- i, j: index of nodes
- (i, j): index of arcs
- p: index of O-D pairs

Const

$$b_{u_t}^p = \begin{cases} q^p & u = o^p, t = 0 \\ -q^p & u = d^p, t = T \\ 0 & \text{otherwise} \end{cases}$$

Minimize

$$\sum_{o \in O} \sum_{s \in S} \sum_{r_k \in \Omega_o} \sum_{(i,j) \in \tilde{A} \cup A_T} \frac{\alpha l_{ij} b_{ij}^{kos} \theta_k^{os}}{Speed * DrivingTimePerDay} + \sum_{o \in O} \sum_{s \in S} \sum_{r_k \in \Omega_o} \gamma_s \theta_k^{os} + \sum_{(i,j) \in \tilde{A} \cup A_T} \sum_{p \in P} \beta l_{ij} y_{ij}^p$$

Subject to:

$$\sum_{(i,j) \in \delta^+(i) \backslash A^D} y_{ij}^p - \sum_{(j,i) \in \delta^-(i) \backslash A^O} y_{ji}^p = b_i^p \qquad \forall p \in P, i \in V_{st} \qquad (1)$$

$$\sum_{p \in P} y_{ij}^p \leqslant \sum_{o \in O} \sum_{s \in S} \sum_{r_k \in \Omega_o} C_s b_{ij}^{kos} \theta_k^{os} \qquad \forall (i,j) \in \tilde{A} \qquad (2)$$

$$\sum_{r_k \in \Omega_o} \theta_k^{os} \leqslant L_{os} \qquad \forall o \in O, \forall s \in S \qquad (3)$$

$$y_{ij}^p \geqslant 0 \qquad \forall p \in P, (i,j) \in \tilde{A} \cup A_T \qquad (4)$$

$$\theta_k^{os} \in Z \qquad \forall o \in O, s \in S \qquad (5)$$