POLAR CODES

Lab Group 4 Sub-Group 3

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HONOUR CODE

We declare that

- The work that we are presenting is our own work.
- We have not copied the work (the code, the results, etc.) that someone else has done.
- Concepts, understanding and insights we will be describing are our own.
- We make this pledge truthfully. We know that violation of this solemn pledge can carry grave consequences.

CONTENT

OBJECTIVE

OBJECTIVE : To understand Polar Codes, implement encoder and a Polar Successive Cancellation List Decoder, evaluate its performance, and explore theoretical proofs of its capacity achieving properties

PROBLEM STATEMENT : Polar codes are the latest among modern established coding methods. They can achieve Shannon’s capacity at low complexity and are as a result, adopted for 5G NR. We intend to find ways to make them encode and decode information more efficiently and handle noisy channels better.

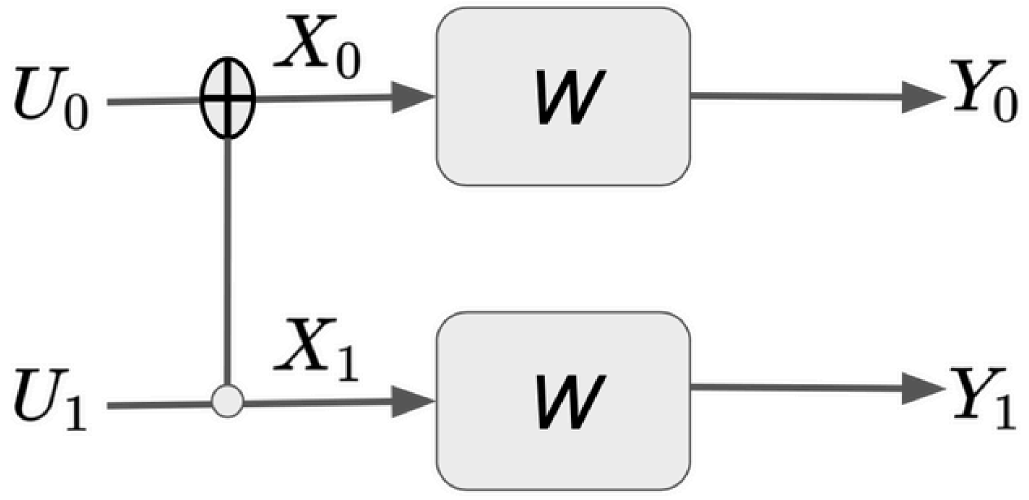
DEVELOPMENT

- Introduced by Arıkan in 2008
- First practical codes proven to achieve the channel capacity at infinite length
- Competitive against modern schemes such as turbo and LDPC codes
- Low complexity, both in encoding and decoding operations
- Achieves time complexity of O(nlogn)

WHY THEY ARE CALLED POLAR CODES

- Polar code is based on the principle of Channel Polarization.
- Polarization refers to the splitting of channels into two extreme channels :
  - Good channels
  - Poor channels
- This technique is information lossless.
- Consists of two steps :
  - Channel Combining
  - Channel Splitting
- Hence, Polar coding refers to putting the information bits only to the reliable set of channels.

CHANNEL COMBINING



Combination of two input bits

[1]

New Channels

$W_0 : U_0 \rightarrow (Y_0, Y_1)$

$W_1 : U_1 \rightarrow (Y_0, Y_1, U_0)$

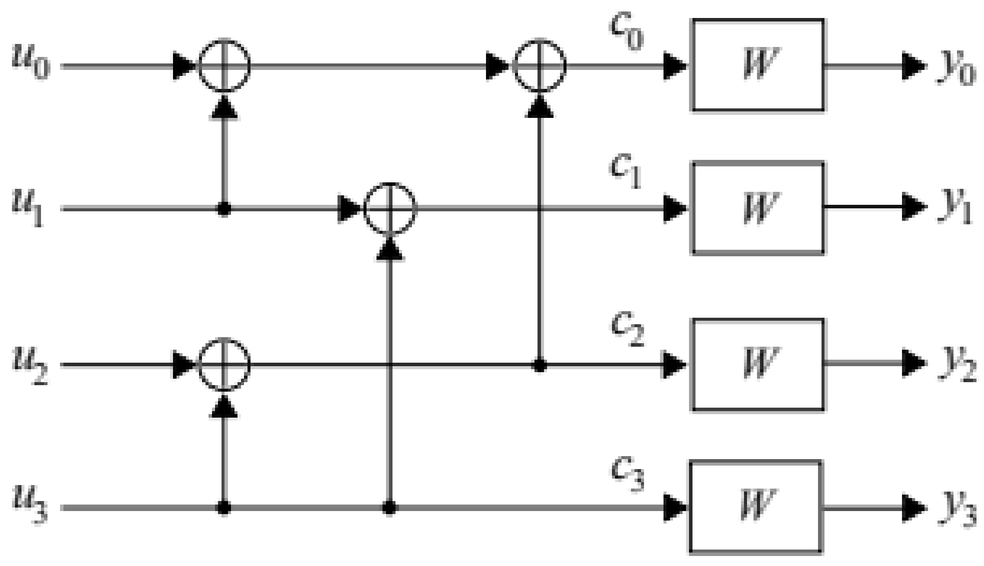
- For the given binary input channels $W : F_2 \rightarrow Y$,

  $X_0 = U_0 \oplus U_1$

  $X_1 = U_1$

  with (U_0, U_1) uniform on F_2^2
- $W^- : U_0 \rightarrow Y_0, Y_1$ --> Poor channel
- $W^+ : U_1 \rightarrow Y_0, Y_1, U_0$ --> Good channel
- As (X_0, X_1) is also uniform on F_2^2,
  - $2I(W) = I(X_0 X_1; Y_0 Y_1) = I(W^-) + I(W^+)$
  - $I(W^-) \leqslant I(W) \leqslant I(W^+)$

CHANNEL SPLITTING



Channel Splitting for N = 4

[2]

- Combined multi-input channels are split back into set of N binary-input channels.
- Channel splitting can be done recursively using kernel of length N = 2

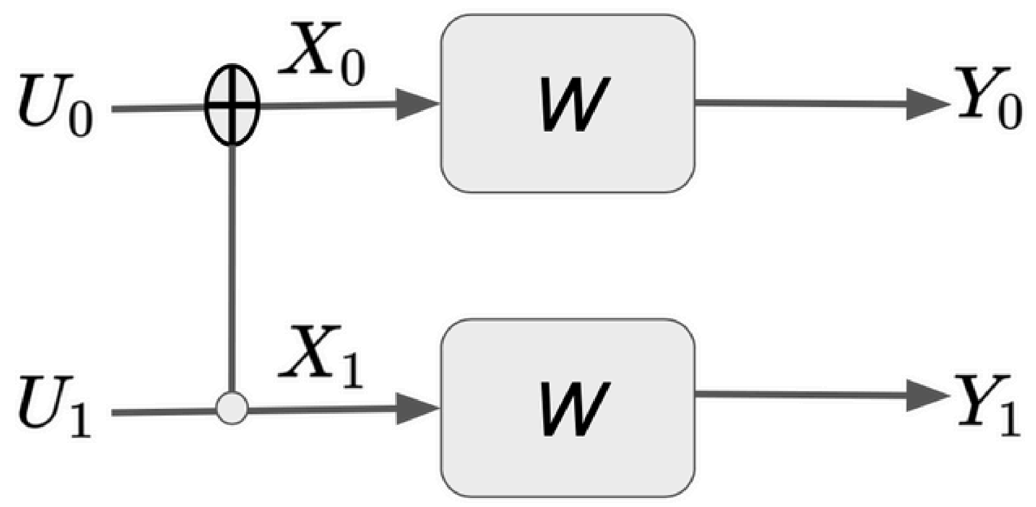
$$\mathbf{F} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

- For longer lengths, the transform is given by -

$$\mathbf{F}^{\otimes n} = \mathbf{F} \otimes \mathbf{F} \ldots \otimes \mathbf{F} \quad (n \text{ times})$$

where F⊗n is the Kronecker product of F with itself n times

CHANNEL SPLITTING



Channel Splitting for N = 2

[1]

- For a 2-bit input,

$$\begin{bmatrix} X_0 \\ X_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} U_0 \\ U_1 \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

- For longer lengths, the transform is given by -

$$\mathbf{F}^{\otimes n} = \mathbf{F} \otimes \mathbf{F} \ldots \otimes \mathbf{F} \quad (n \text{ times})$$

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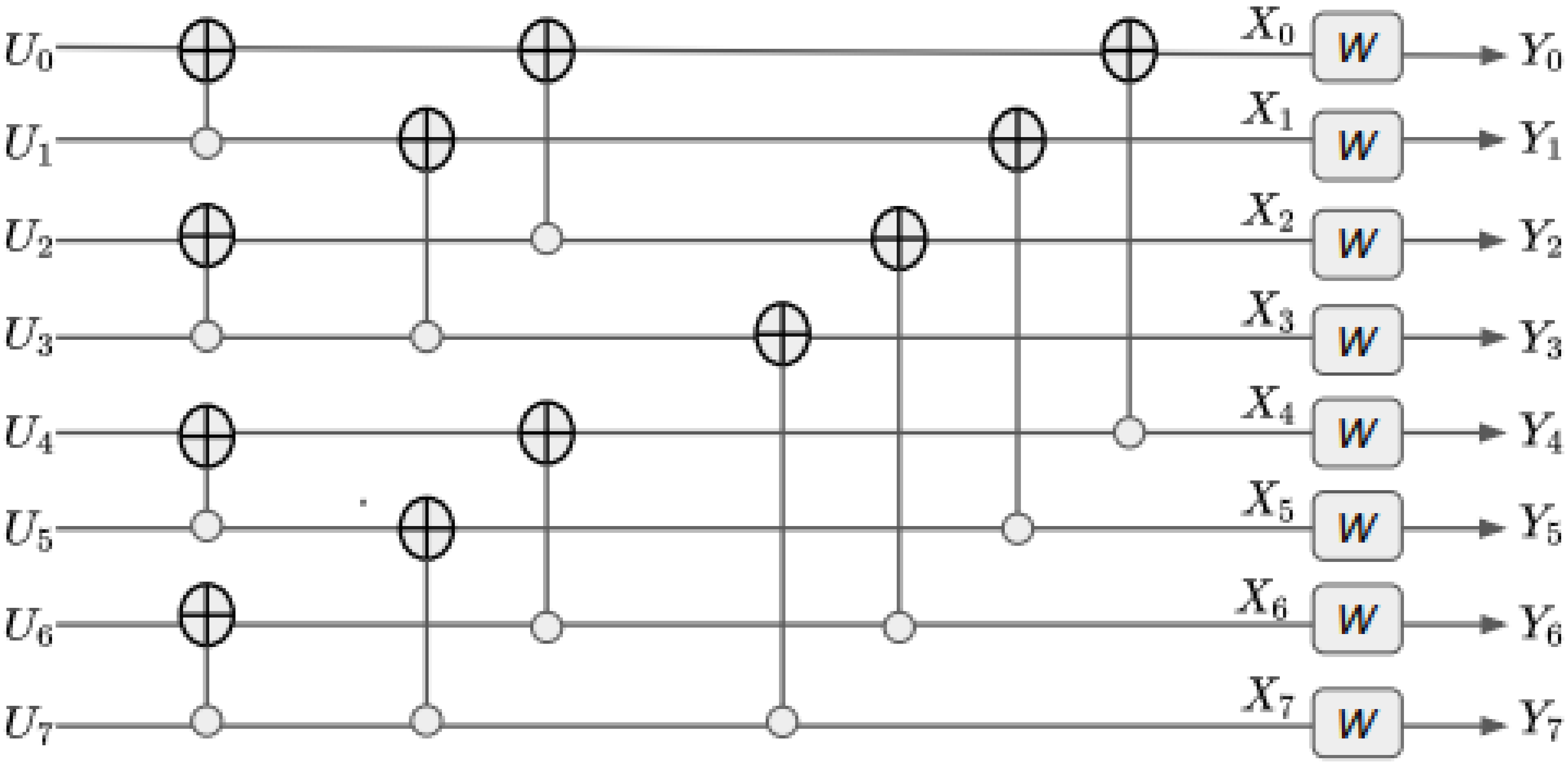
POLARIZATION CONSTRUCTION



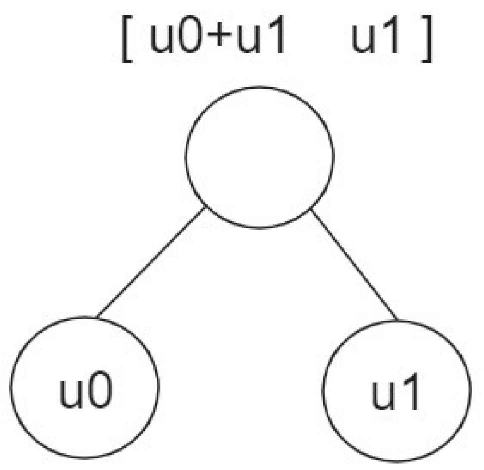
Channel Polarization : N=8 Polar code
[1]

- For the given channel, $W : X \rightarrow Y$
  - $W^{-} : U_0 \rightarrow Y_0\ Y_1$
  - $W^{+} : U_1 \rightarrow Y_0\ Y_1\ U_0$

- By Combining $W^{+}$and W^{-}:
  - $W^{--}: V_0 \rightarrow\ Y_0\ Y_1\ Y_2\ Y_3$
  - $W^{-+}: V_1 \rightarrow\ Y_0\ Y_1\ Y_2\ Y_3\ V_0$
  - $W^{+-}: V_2 \rightarrow\ Y_0\ Y_1\ Y_2\ Y_3\ V_0\ V_1$
  - $W^{++}: V_3 \rightarrow\ Y_0\ Y_1\ Y_2\ Y_3\ V_0\ V_1\ V_2$

- On further combining the above, W^{+++}, W^{++-}, W^{+-+},W^{+--},W^{-++},W^{--+},W^{-+-} and W^{---} are obtained.

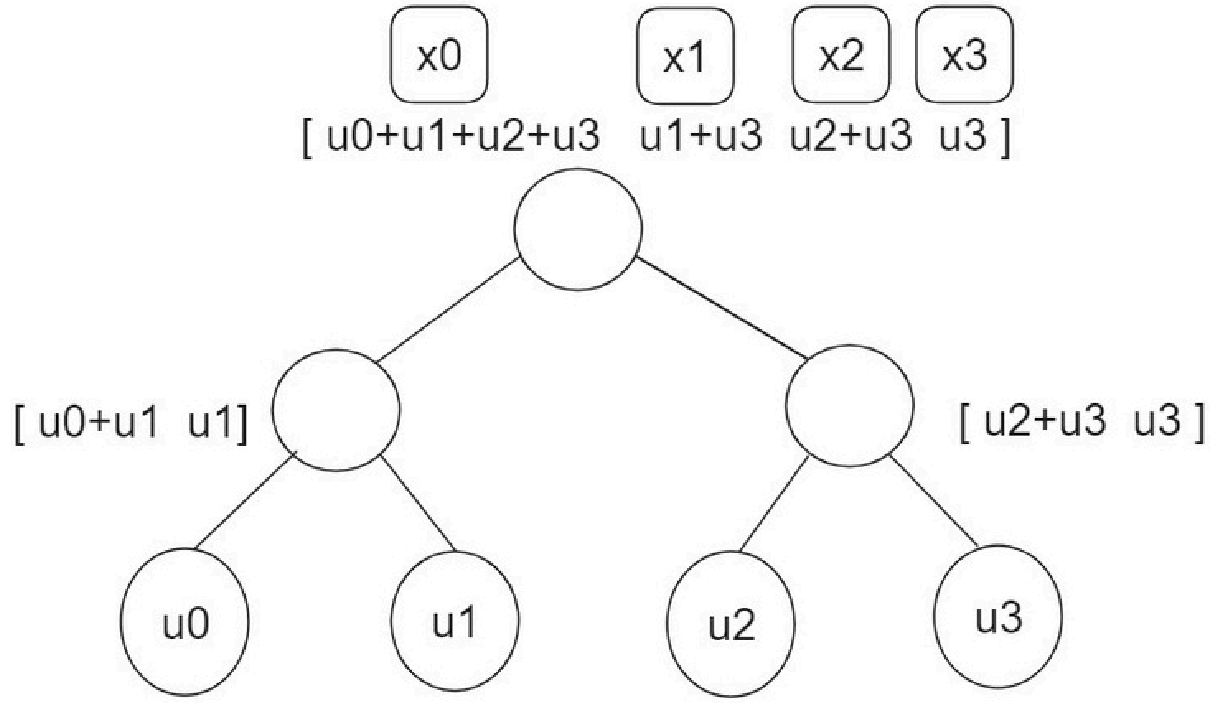
ENCODING



Polar encoder of length 2

- For a polar encoder of length 2 with input bits U_0 and U_1,
  - The left encoded bit is the modulo sum 2 of the first and second input bits and the right encoded bit is simply the second input bit passed as it is. (Here, + indicates XOR operation)

- For the given polar encoder of length 4 with input bits U_0, U_1, U_2 and U_3, the same above procedure is followed.

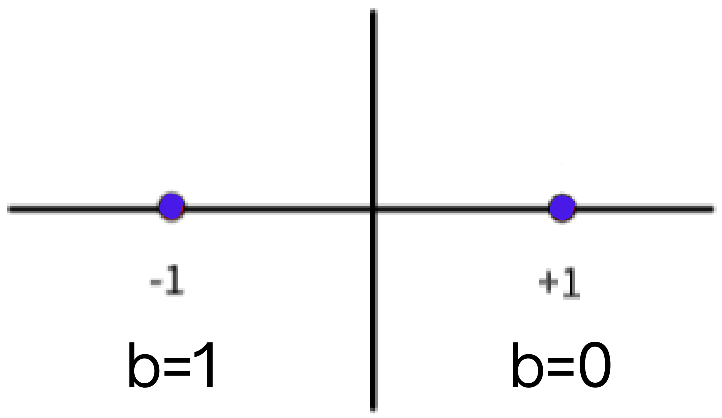


Polar encoder of length 4.

(N,K) POLAR CODE

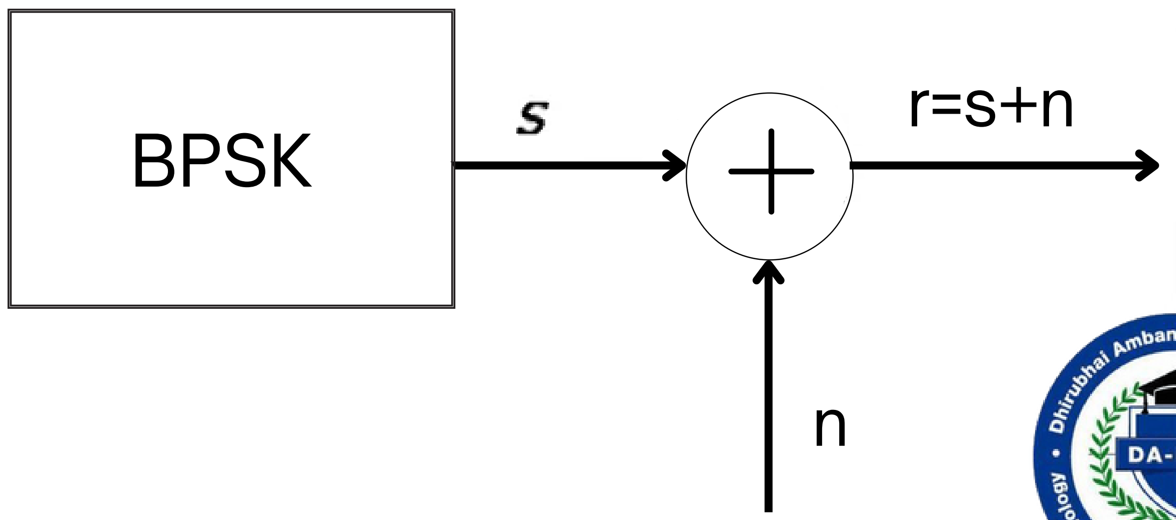
- The polar reliability sequence is a sorted order of virtual channels, starting from worst to the best.
- $N = 2^n$ (where n = 2,3,...)
- Code word U of length N
- Message of length k, where $k \leqslant N$
- Using the reliability sequence, polar code is generated by selecting k best bit channels.
- For the N-k worst channels, U_i is set as zero (Frozen positions)
- The remaining k bits of U are the message positions.
- Complexity : $n\log_2 n$
- Rate = k/N

BPSK MODULATION AND AWGN

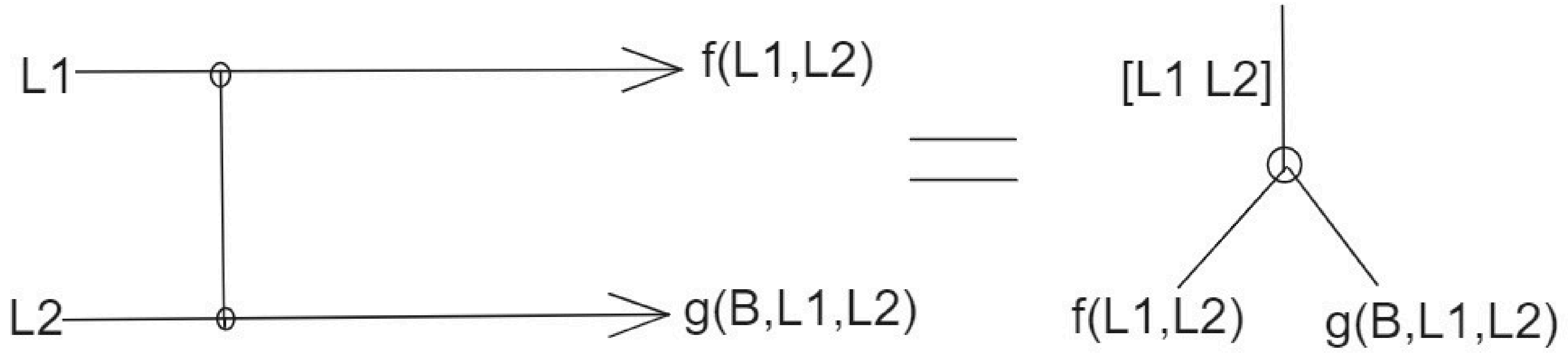


- Channel encoder gives output in the form of a vector.
- Each element of the vector is either 0 or 1
- BPSK modulator maps the bit to s, where
  - s = +1 (volts) if b = 0, else
  - s = −1 volts.

- AWGN introduced per-symbol SNR γ = ES/N0 (linear scale)
- Signal power/energy is ES = 1 Joule.
- Noise power , $\sigma_n^2 = 1/\gamma$
- The AWGN is simulated as $u = \sigma_n \times u_s$, where u_s $N(0, 1)$



DECODING



$$f(L_1, L_2) = sign(L_1)sign(L_2)min(|L_1|, |L_2|)$$

$$g(\beta, L_1, L_2) = (1 - 2\beta)L_1 + L_2$$

2-bit Decoder :

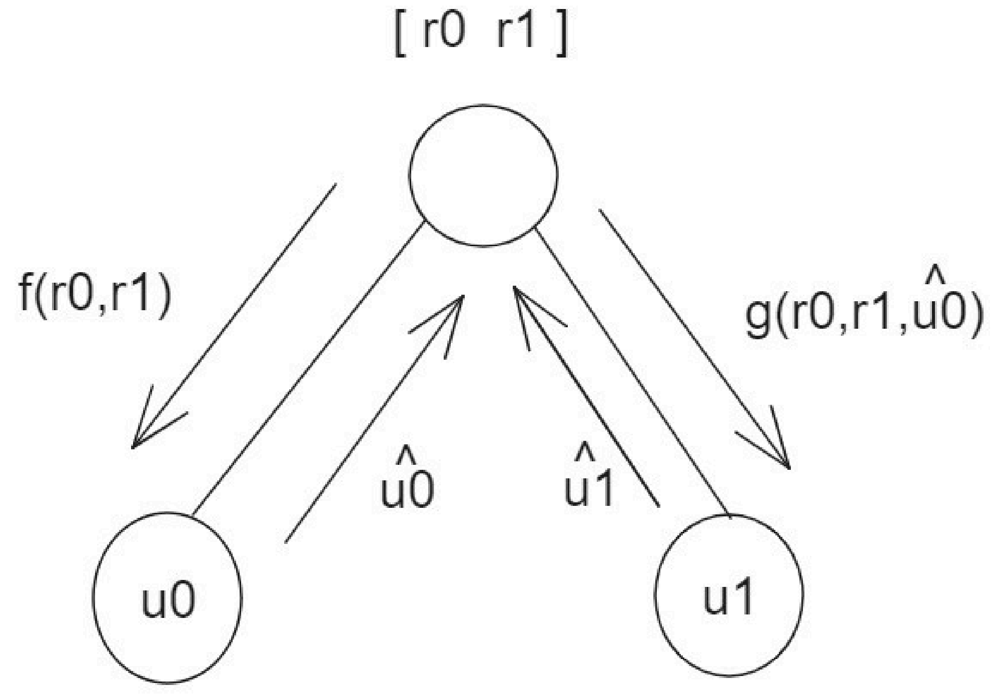
- r_0 and r_1 are the received bits.

-> SISO decode u_0 first(SPC)

- $L(u_0) = f(r_0,r_1)$
- $\hat{u}_0=0$ if $L(u_0)\geqslant 0$ else $\hat{u}_0=1$

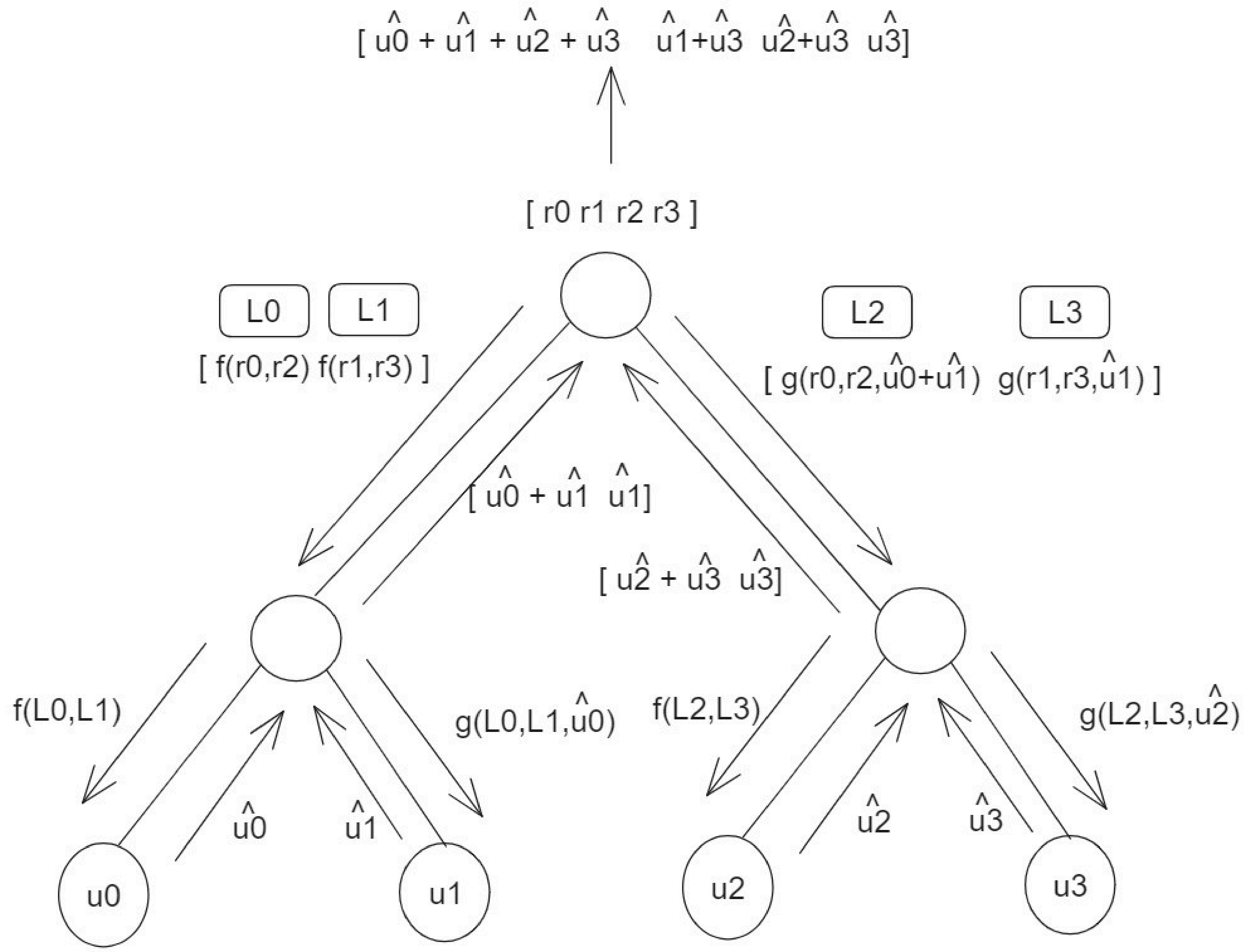
->Given $\hat{u}_0$ decode u1 (REP)

- if $\hat{u}_0 = 0$, $L(u_1) = r_0+r_1$
- if $\hat{u}_0 = 1$, $L(u_1) = r_1-r_0$



SC Decoder for N=2

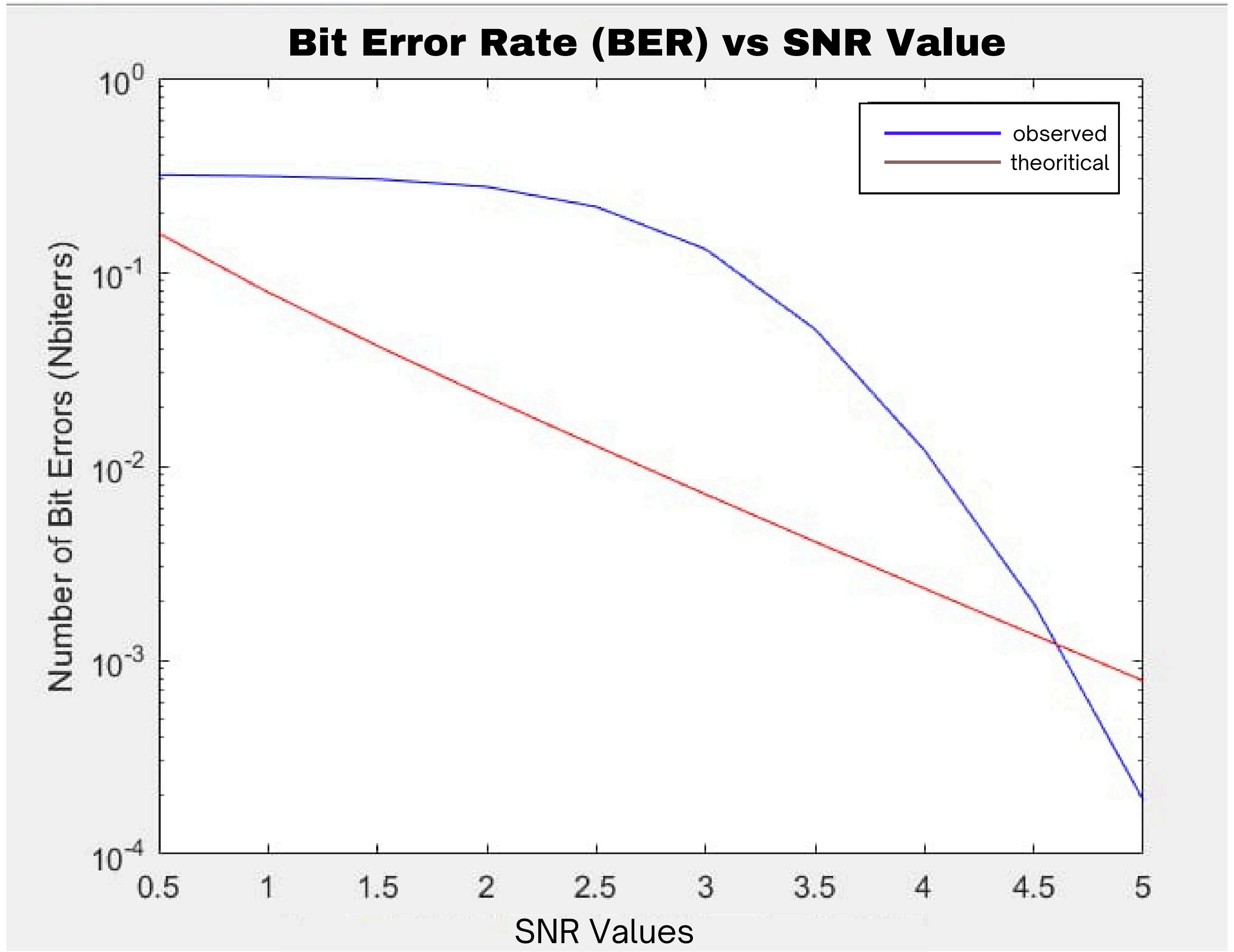
4-BIT DECODING

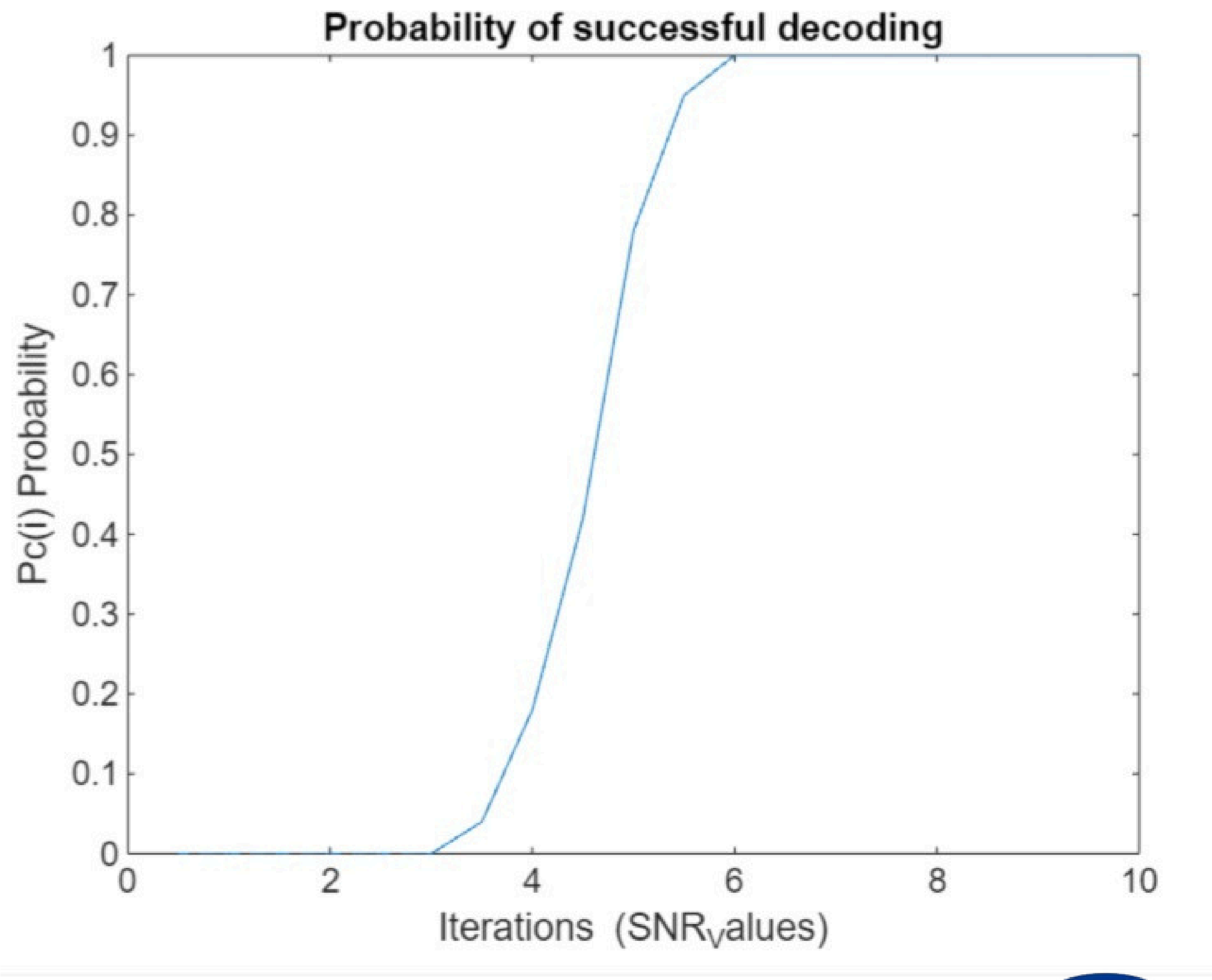


MONTE-CARLO SIMULATION

```
Command Window
Enter the value of N:
512
Enter the value of K:
340
```

SNR Value	BER
0.5	0.3175
1	0.3188
1.5	0.3034
2	0.2818
2.5	0.2194
3	0.1353
3.5	0.0476
4	0.0104
4.5	0.0019

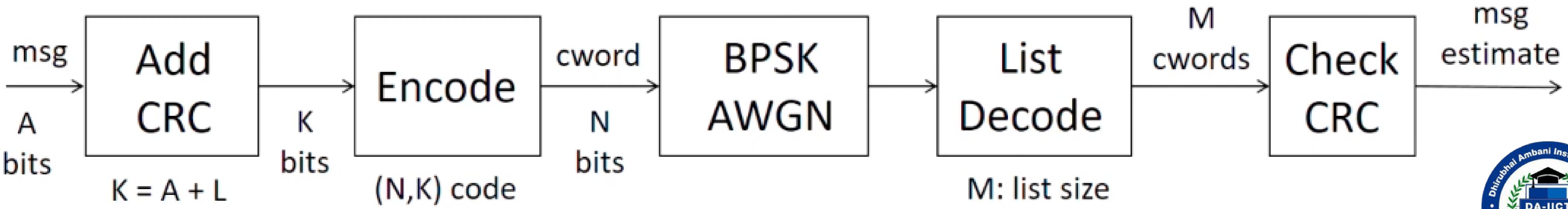




Performed C.3.2

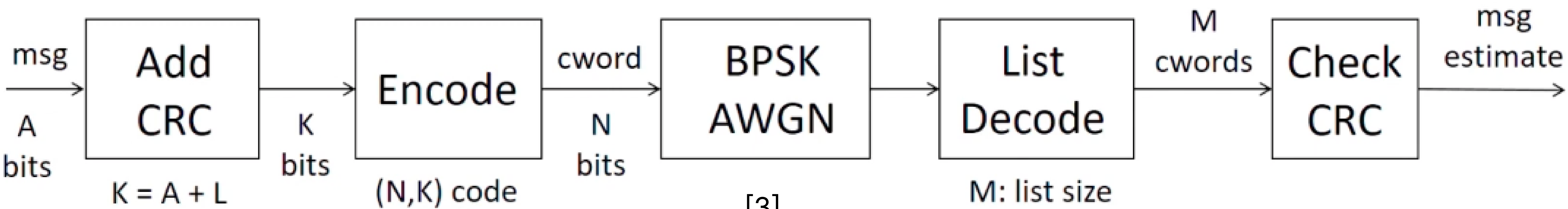
SUCCESSIVE CANCELLATION LIST DECODING

- Successive Cancellation decoding can be further improved in performed
- List Decoding produces a list of possible codewords
  - can be of length 4 or 8
- Use Cyclic Redundancy Checks (CRCs)
- Message of length A
- CRC length L
- k=A+L
- Encoding is as previously done for (N,k) polar code



[3]

SUCCESSIVE CANCELLATION LIST DECODING



[3]

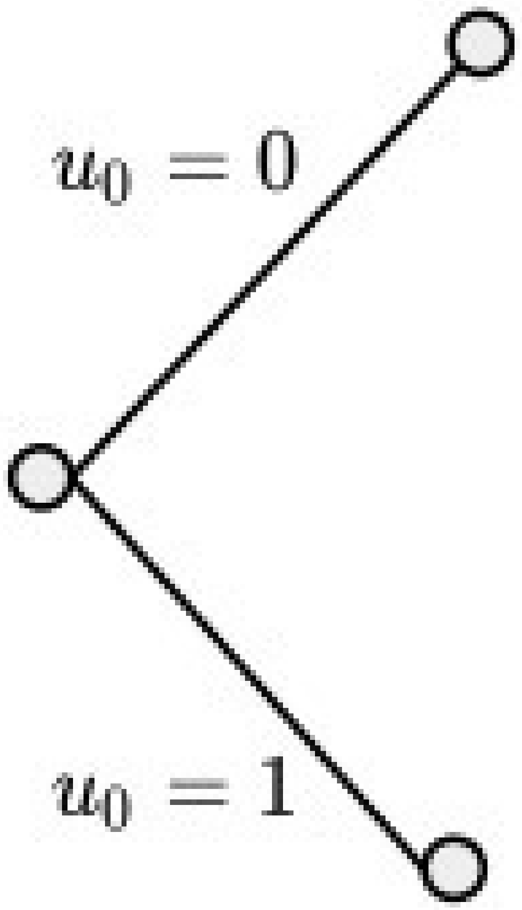
- For generating more than one codeword We consider both decision on each bit
- Assign each bit Decision Metric(DM)

  - If $L(u_i) \geqslant 0$; $\hat{u}_i = 0$ has $DM_i = 0$, $\hat{u}_i = 1$ has $DM_i = |L(u_i)|$
  - If $L(u_i) < 0$; $\hat{u}_i = 1$ has $DM_i = 0$, $\hat{u}_i = 0$ has $DM_i = |L(u_i)|$

- Here, Important thing is we have to assigned DM to frozen bit too.

- Path Metric(PM): Sum of decision Metric on a path of choices

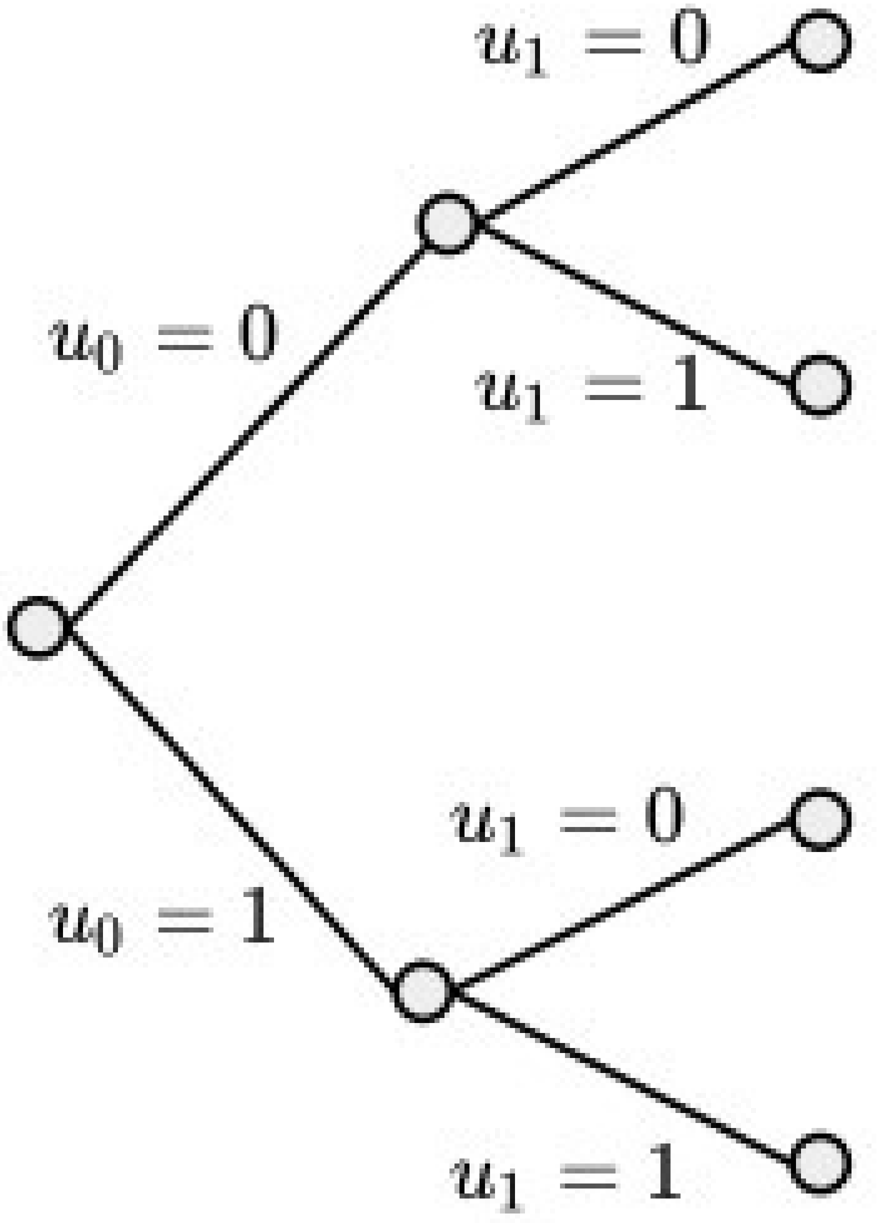
SUCCESSIVE CANCELLATION LIST DECODING



[1]

$$u_0 = \begin{cases} 0 \\ 1 \end{cases}$$

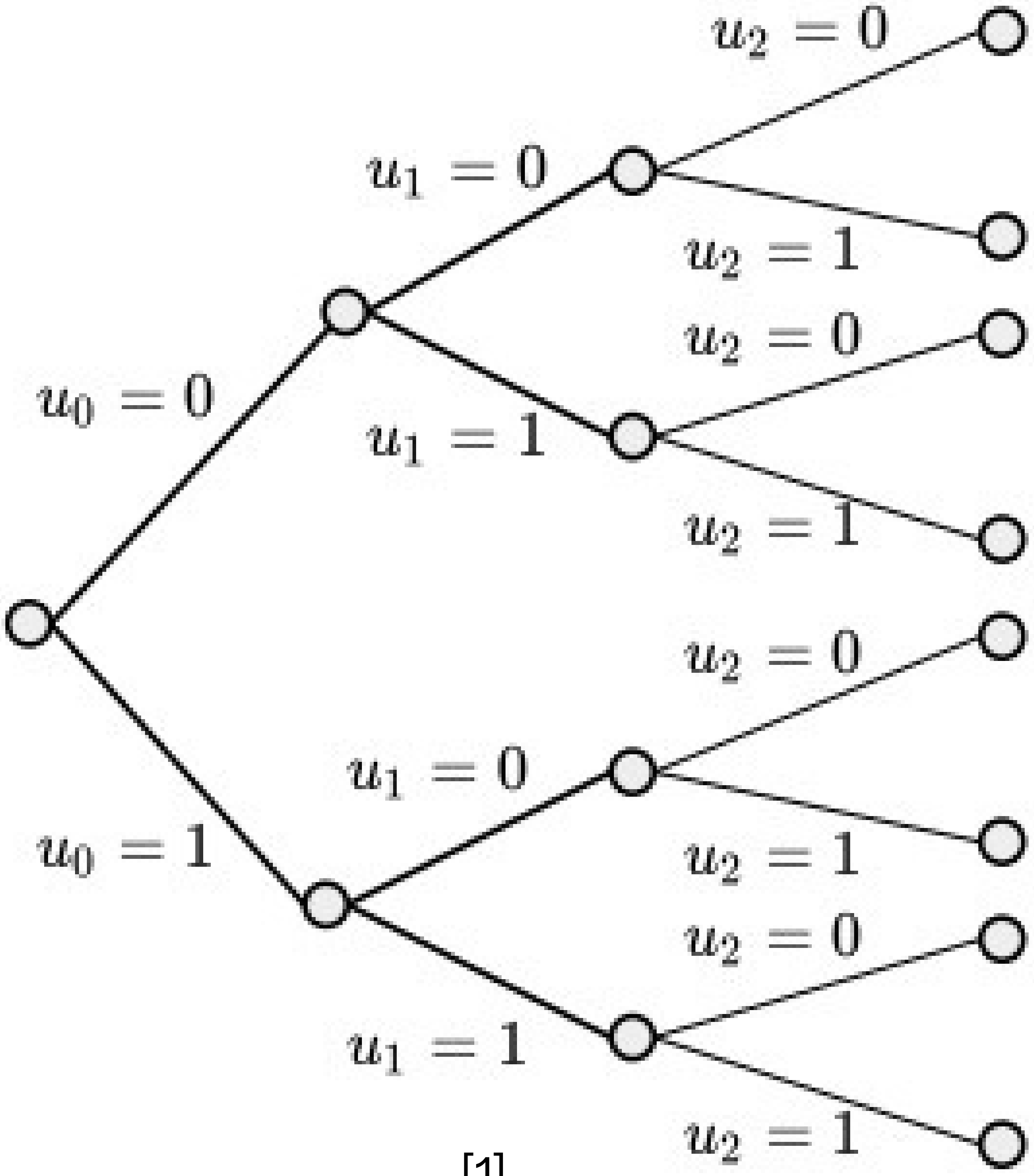
SUCCESSIVE CANCELLATION LIST DECODING



[1]

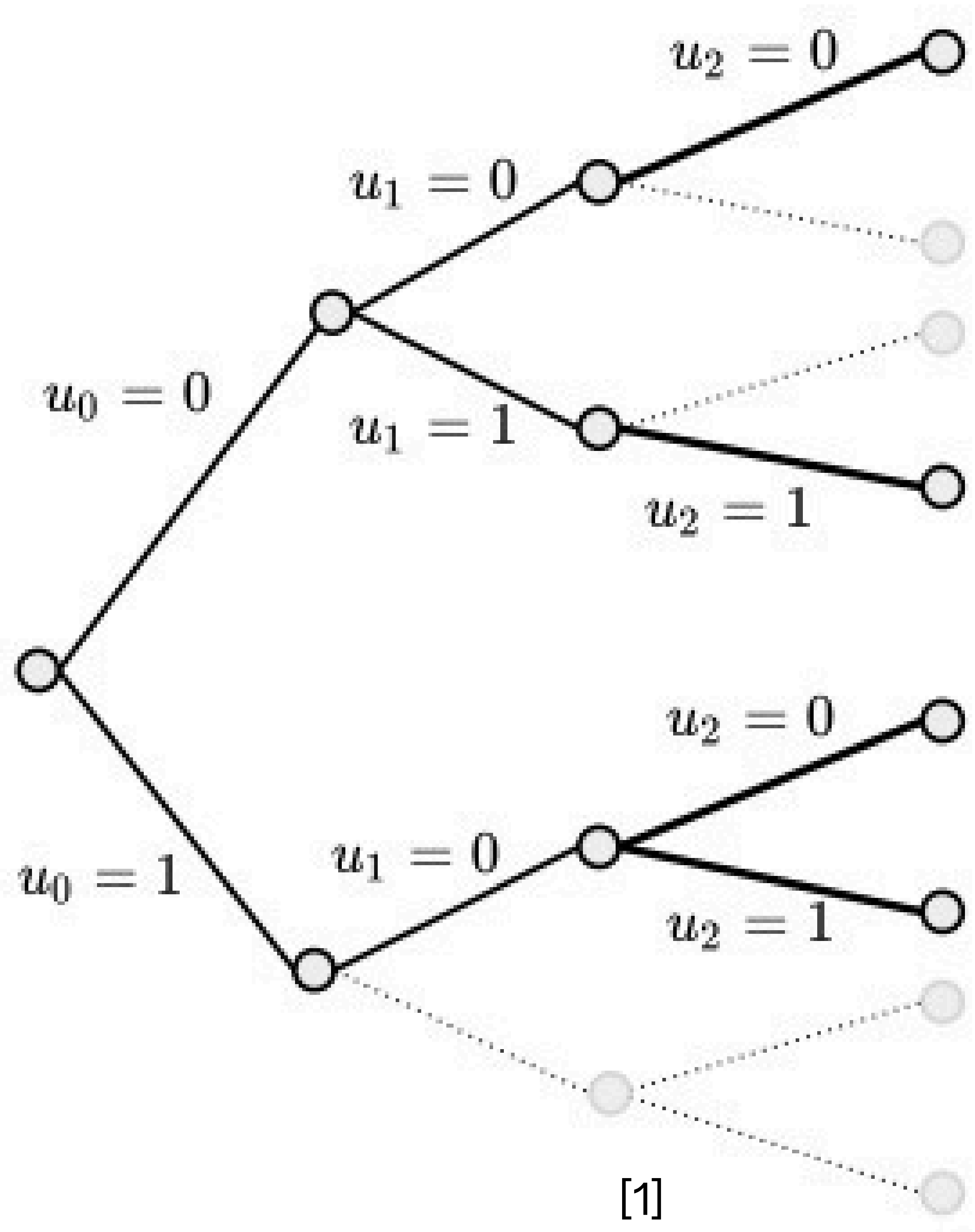
$$u_0 u_1 = \begin{cases} 00 \\ 01 \\ 10 \\ 11 \end{cases}$$

SUCCESSIVE CANCELLATION LIST DECODING



[1]

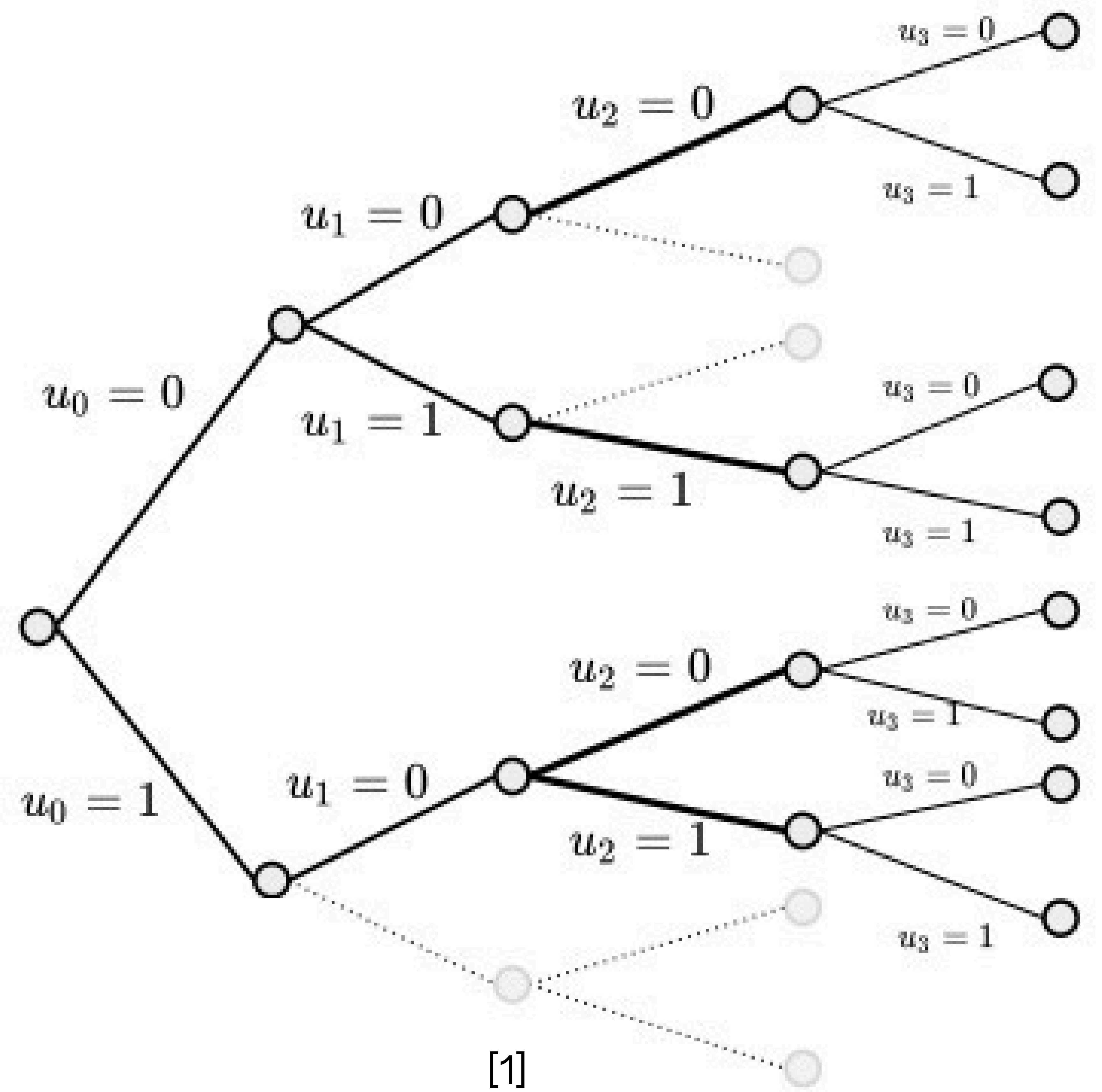
SUCCESSIVE CANCELLATION LIST DECODING



[1]

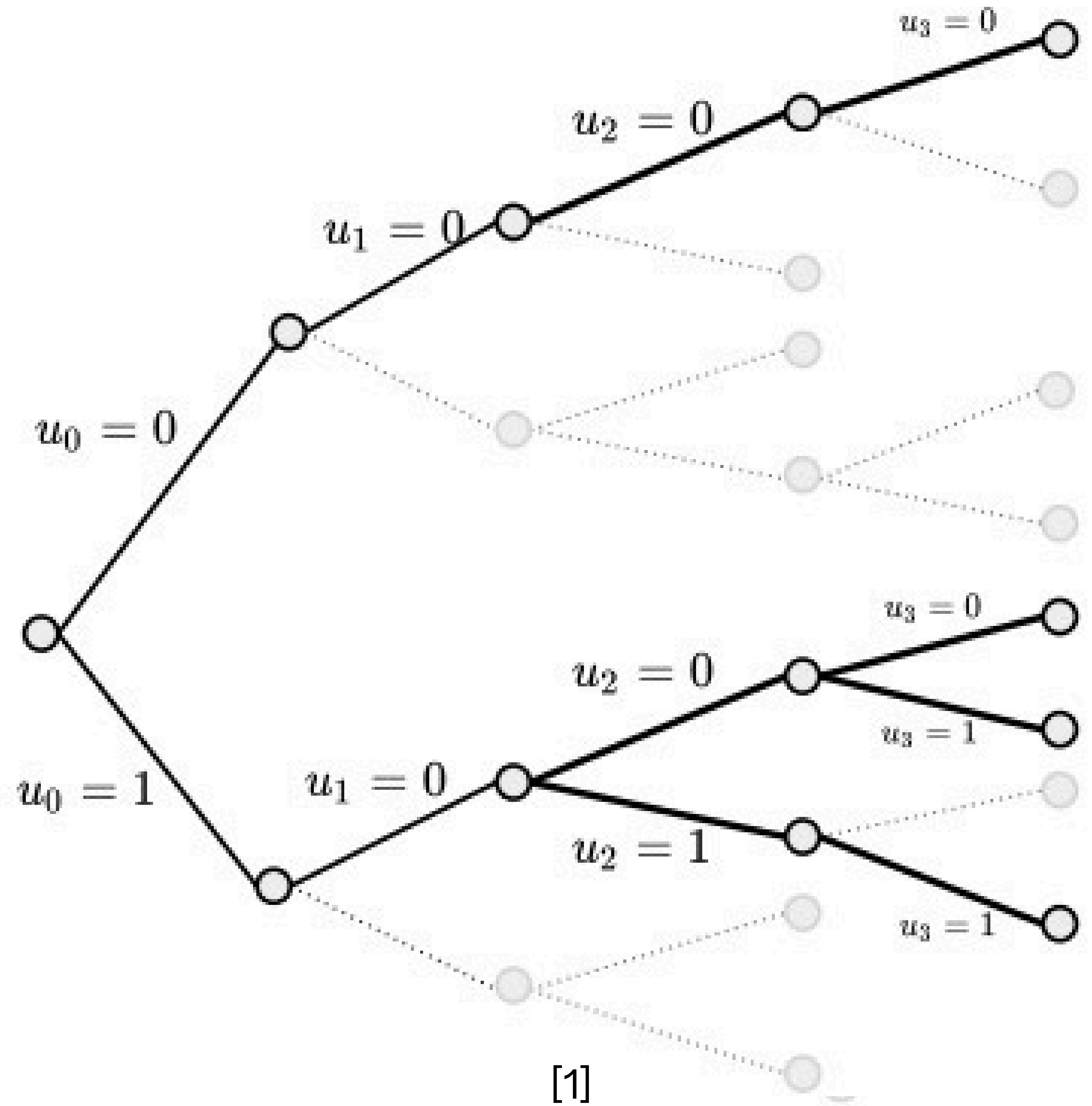
$$u_0 u_1 u_2 = \begin{cases} 000 \\ 011 \\ 100 \\ 101 \end{cases}$$

SUCCESSIVE CANCELLATION LIST DECODING



[1]

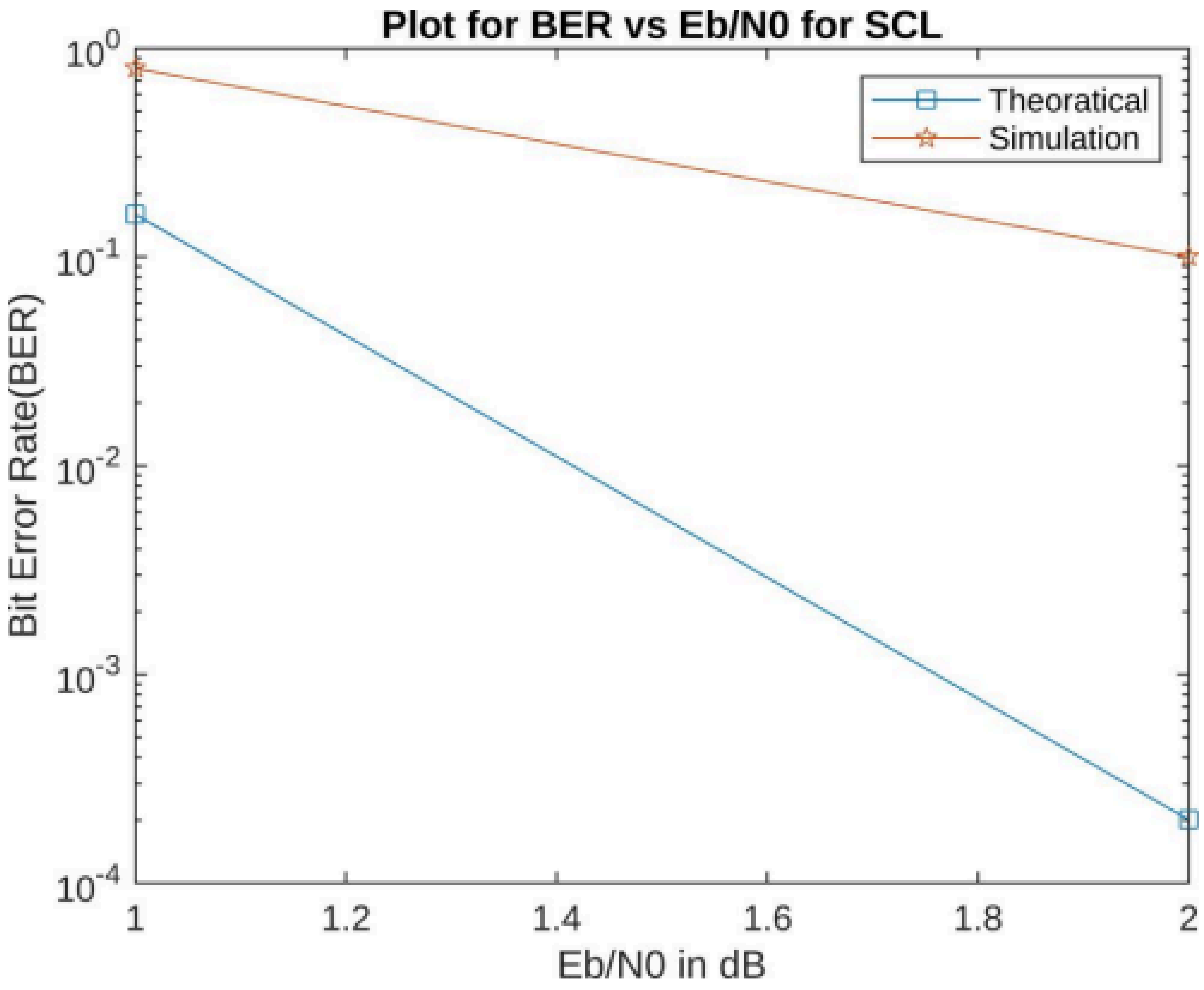
SUCCESSIVE CANCELLATION LIST DECODING



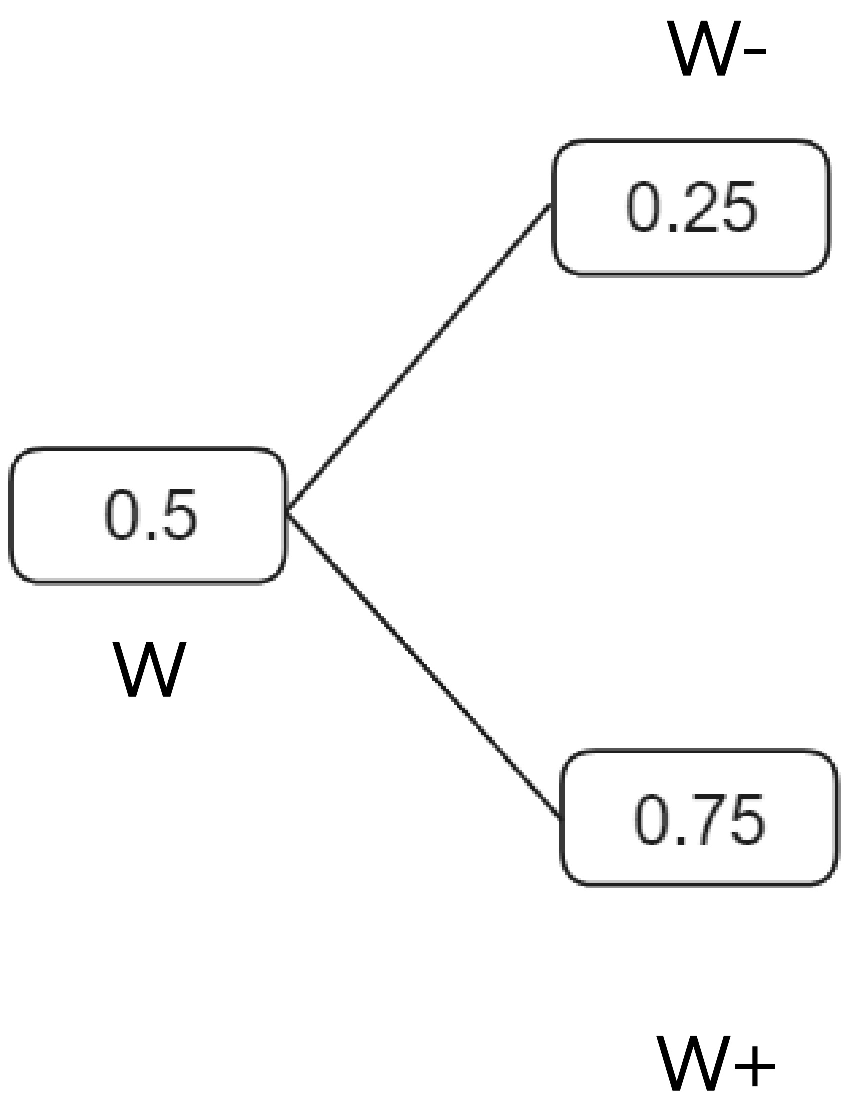
[1]

$$u_0 u_1 u_2 u_3 = \begin{cases} 0000 \\ 1000 \\ 1001 \\ 1011 \end{cases}$$

SUCCESSIVE CANCELLATION LIST DECODING



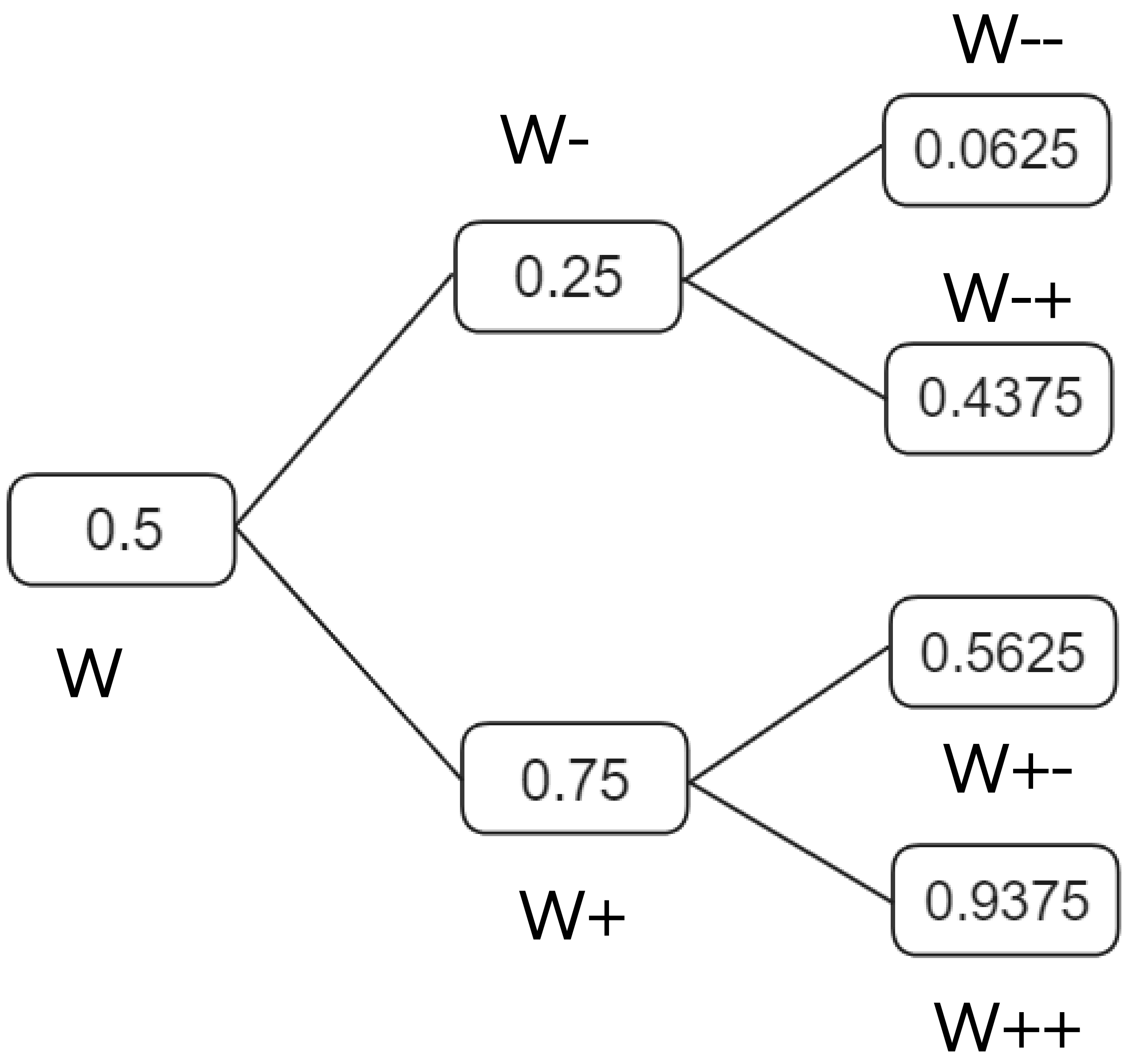
ANALYSIS



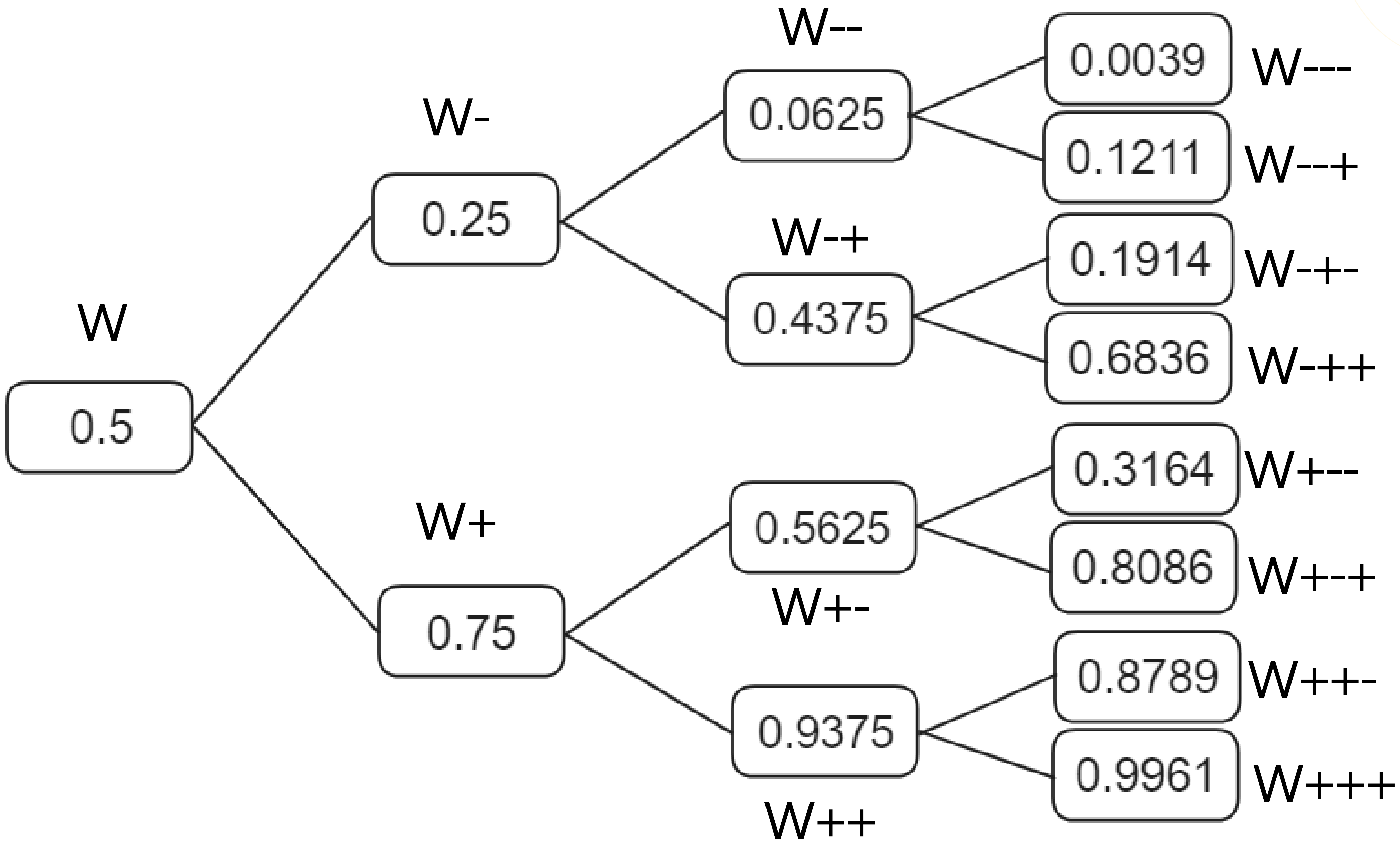
BEC(0.5) Modeled W with length N=2

- For channels with input-output symmetry, the capacity is given by $C(W) \overset{\Delta}{=} I(X; Y)$
- $0 \leqslant C(W) \leqslant 1$
- Given a polar code of length N=2 and **binary erasure channel** W with an erasure probability p = 0.5
- As channel splits,
  - $I(W-) = p^2$ (bad channel)
  - $I(W+) = 2p - p^2$ (good channel)
  - $I(W) = p$

POLAR CODES AND SHANNON CAPACITY

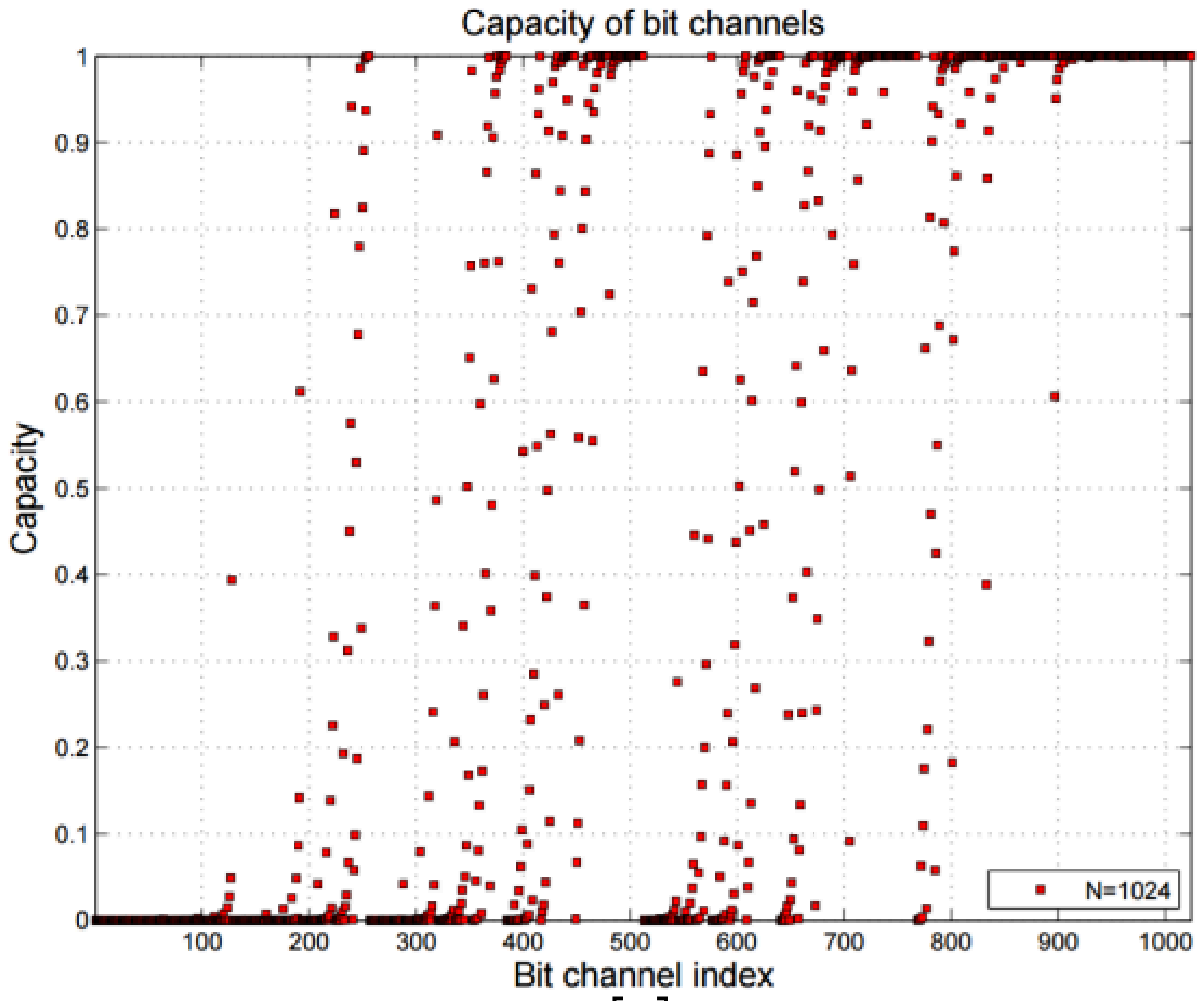


BEC(0.5) Modeled W with length N=4



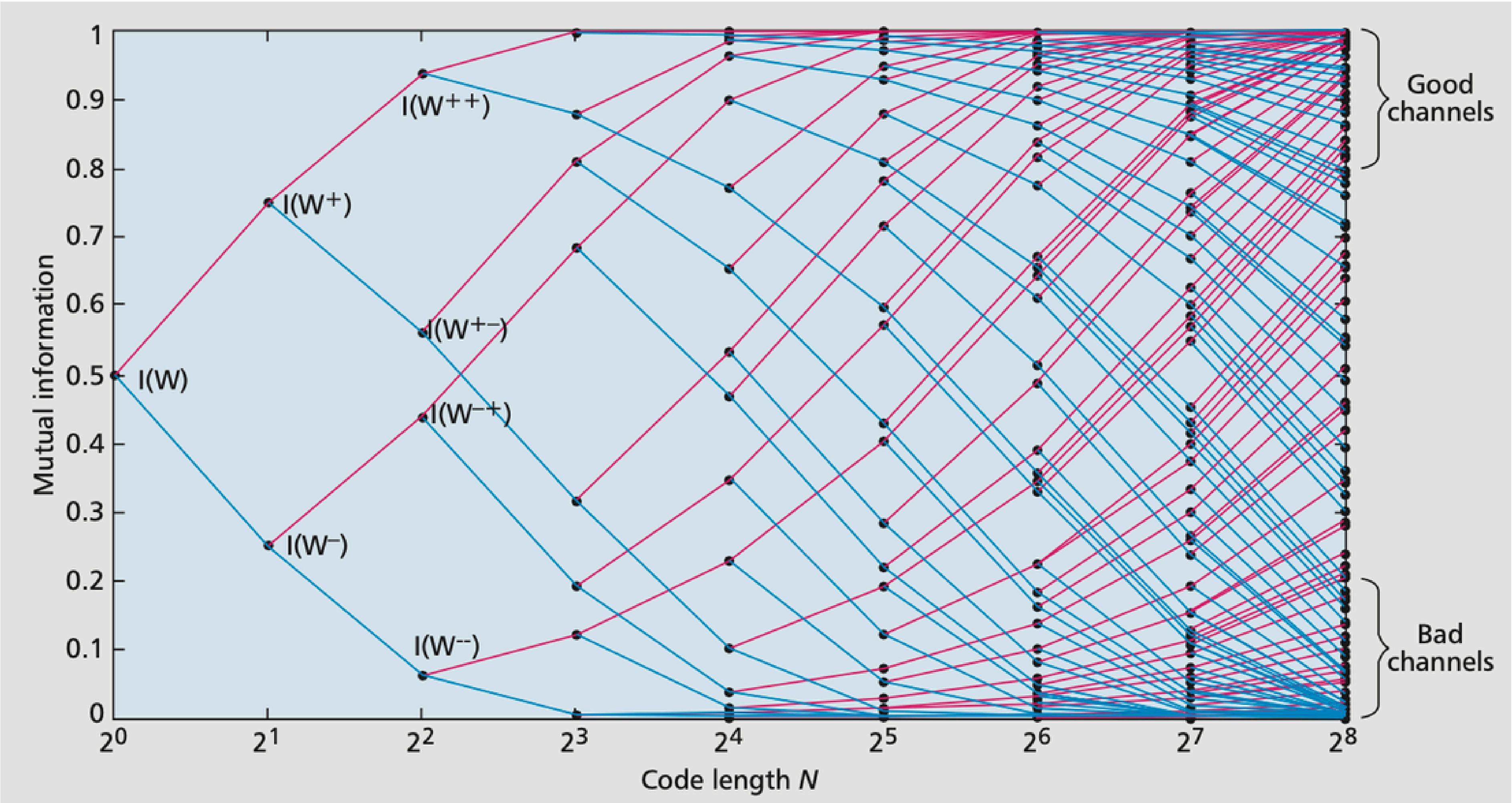
BEC(0.5) Modelled W with length N=8

EVOLUTION OF CHANNEL POLARIZATION



[4]

- Capacity of channel converges over either 0 or 1.



[5]

- On further splitting, as the number of channels grow, the bit channels converge to either a completely noiseless channel or a useless channel.

CONCLUSION AND SUMMARY

- Polar Codes that are the latest among established coding methods.
- They show exceptional efficiency, having been adopted by the 3GPP body for 5G NR Channels.
- They achieve lower encoding time complexity along with simple encoding and decoding algorithms of $O(n*\log_2 n)$and therefore, are easy to implement.
- Implementation of List decoding further improves their efficiency.
- They take advantage of polarisation to reach Shannon limits.
- This makes them attractive for application in high speed optical communications.
- While, they are efficient, their implementation is costlier for higher value of n.

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5)Niu, K., Chen, K., Lin, J., & Zhang, Q. T. (2014). Polar codes: Primary concepts and practical decoding algorithms. IEEE Communications Magazine, 52(7), 192–203.

THANK YOU