

# Transformation de Laplace inverse

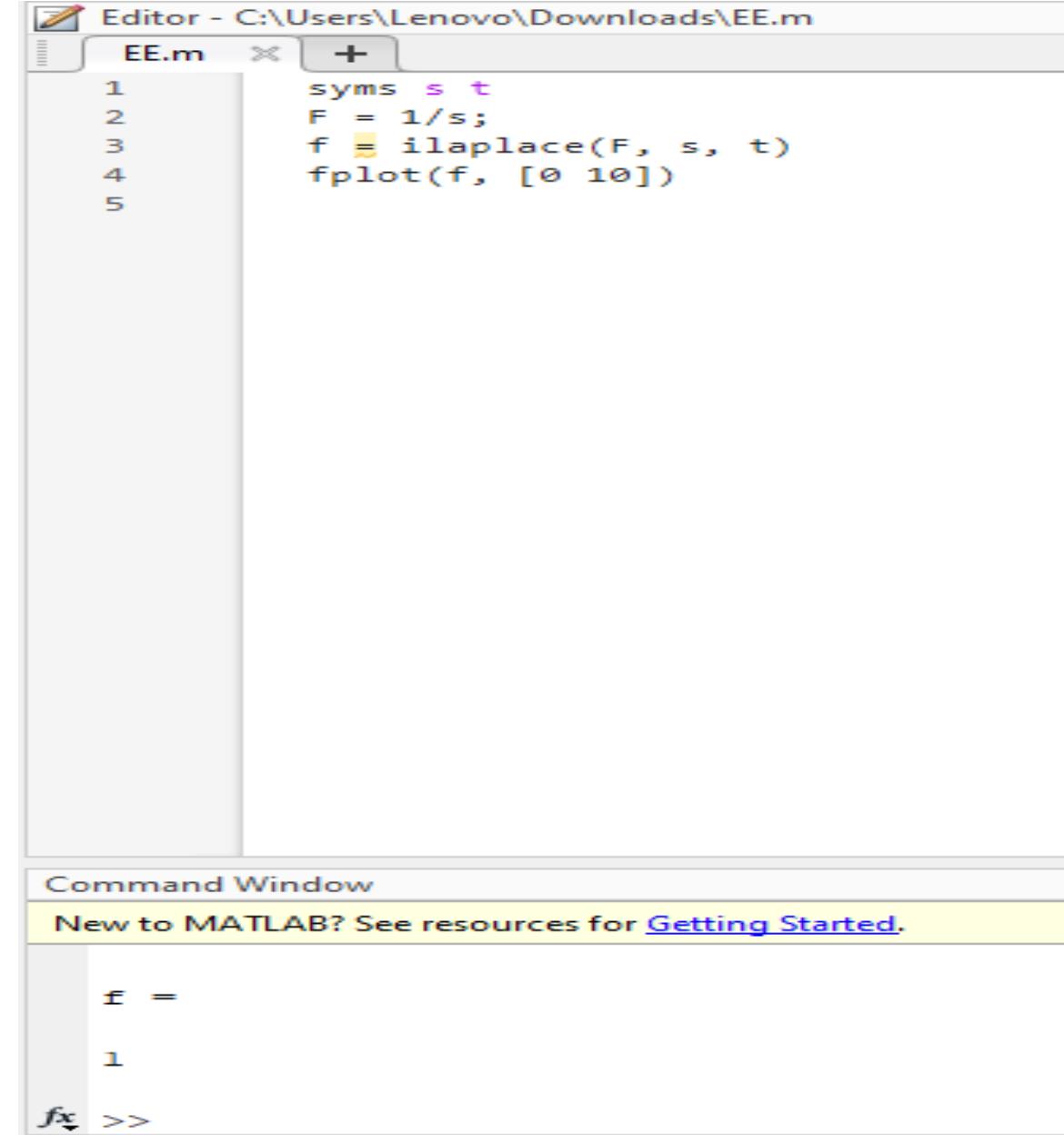
# Transformée de Laplace inverse du signal échelon $u(t)$ :

$$u(t) = \begin{cases} 0 & \text{si } t < 0 \\ 1 & \text{si } t \geq 0 \end{cases}$$

$$\mathcal{L}\{u(t)\} = \frac{1}{s} \quad \text{Pour: } s > 0$$

↓  
↓  
↓  
 $T\mathcal{L}^{-1}$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = u(t)$$



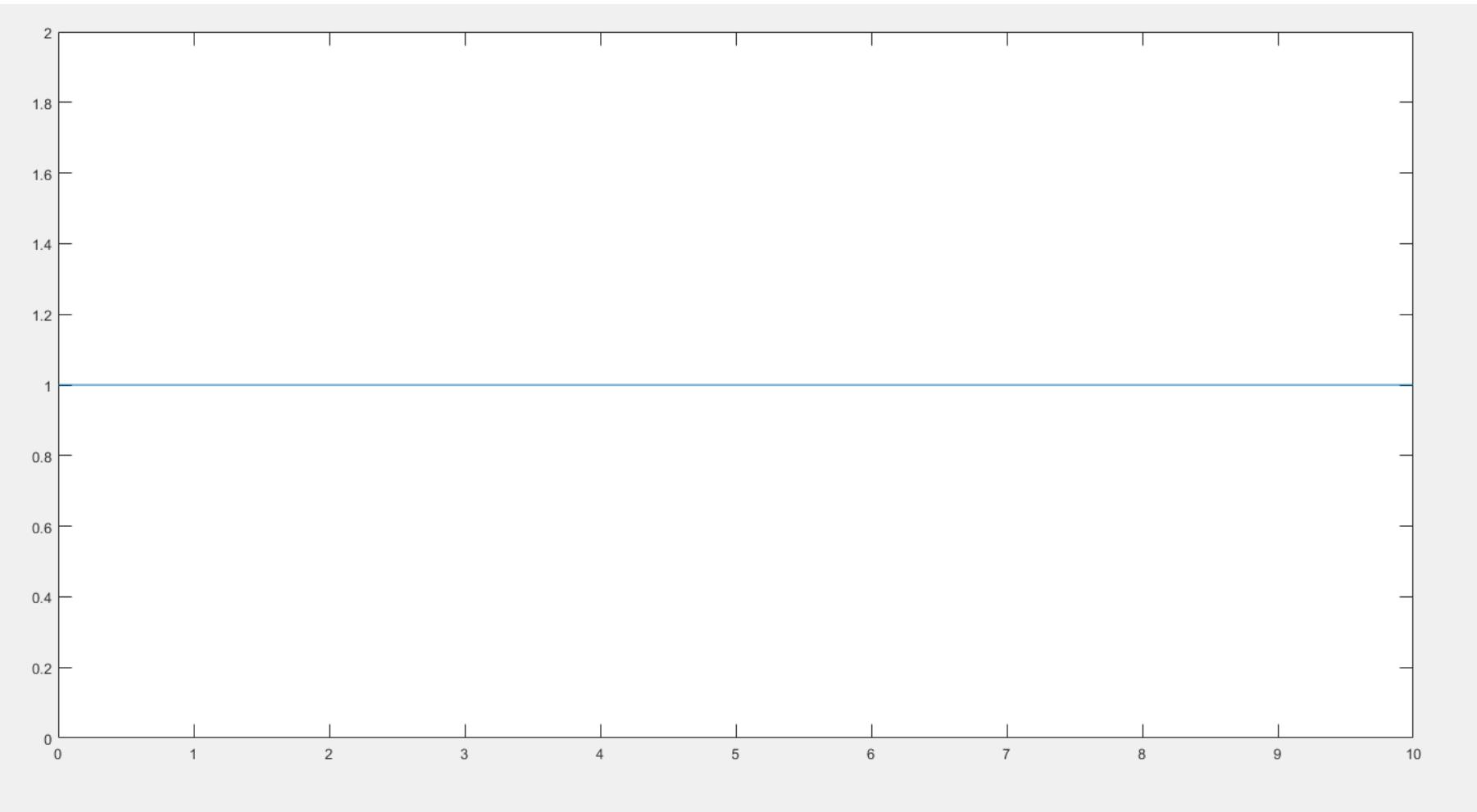
The image shows a MATLAB environment with two windows. The top window is the 'Editor' showing a script named 'EE.m' with the following code:

```
Editor - C:\Users\Lenovo\Downloads\EE.m
EE.m
1 syms s t
2 F = 1/s;
3 f = ilaplace(F, s, t)
4 fplot(f, [0 10])
5
```

The bottom window is the 'Command Window' with the following text:

```
Command Window
New to MATLAB? See resources for Getting Started.
f =
1
fplot >>
```

# Graphe Transformée de Laplace inverse du signal échelon $u(t)$



# Transformée de Laplace inverse du signal exponentiel

$$x(t) = e^{at} \cdot u(t)$$

$$\mathcal{L}\{e^{at} \cdot u(t)\} = \frac{1}{s - a} \quad \text{Pour : } s > a$$

TL<sup>-1</sup>

$$\mathcal{L}^{-1}\left\{\frac{1}{s - a}\right\} = e^{at} \cdot u(t)$$

a : est le taux de croissance ou de décroissance de l'exponentielle dans le signal.

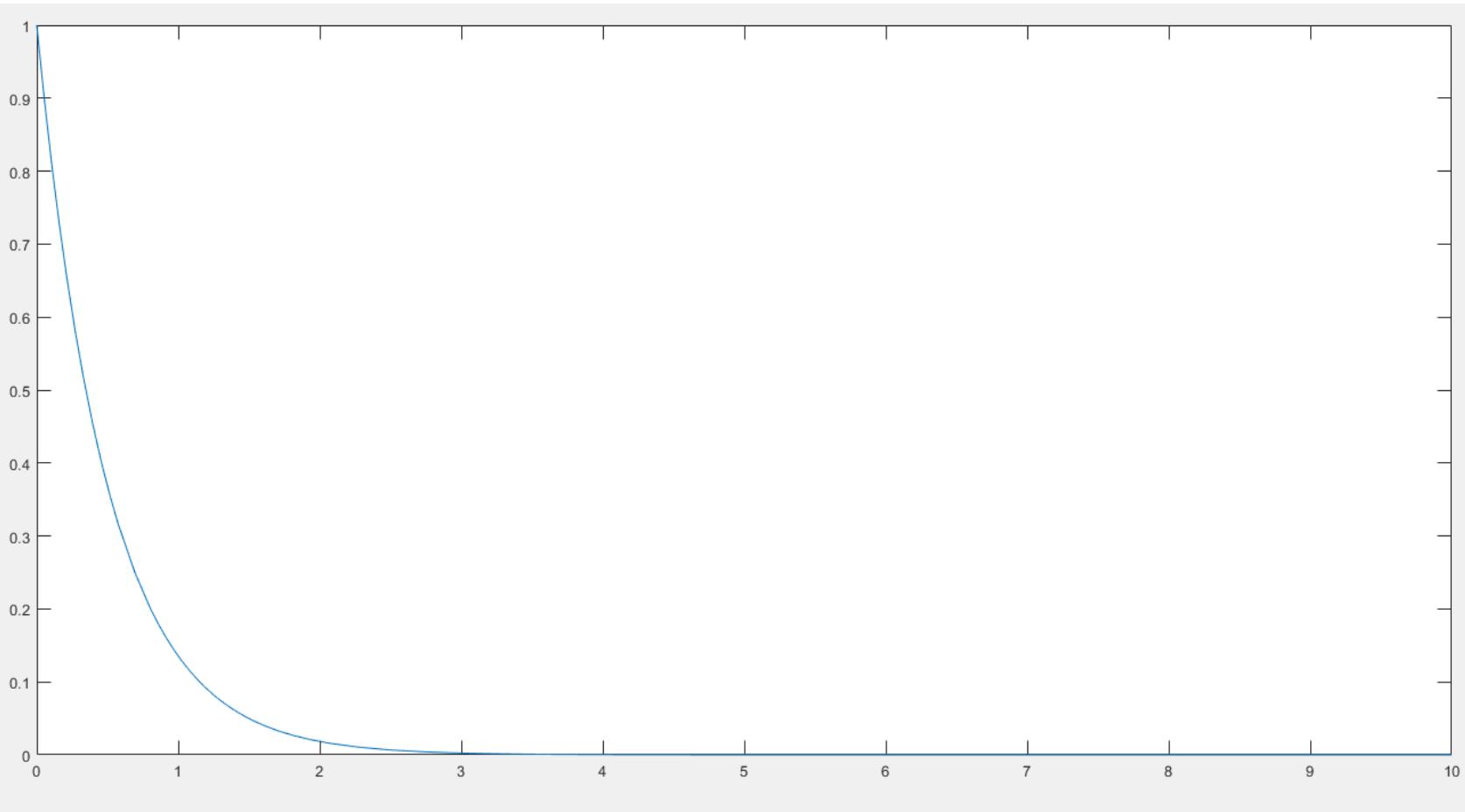
The screenshot shows the MATLAB environment. In the Editor window, a script named 'expo.m' is open with the following code:

```
EE.m expo.m +  
1 syms s t a  
2 F = 1/(s + a);  
3 f = ilaplace(F, s, t)  
4 fplot(subs(f, a, 2), [0 10])  
5
```

In the Command Window, the output is:

```
f =  
exp(-a*t)
```

# Graphe Transformée de Laplace inverse du signal exponentiel

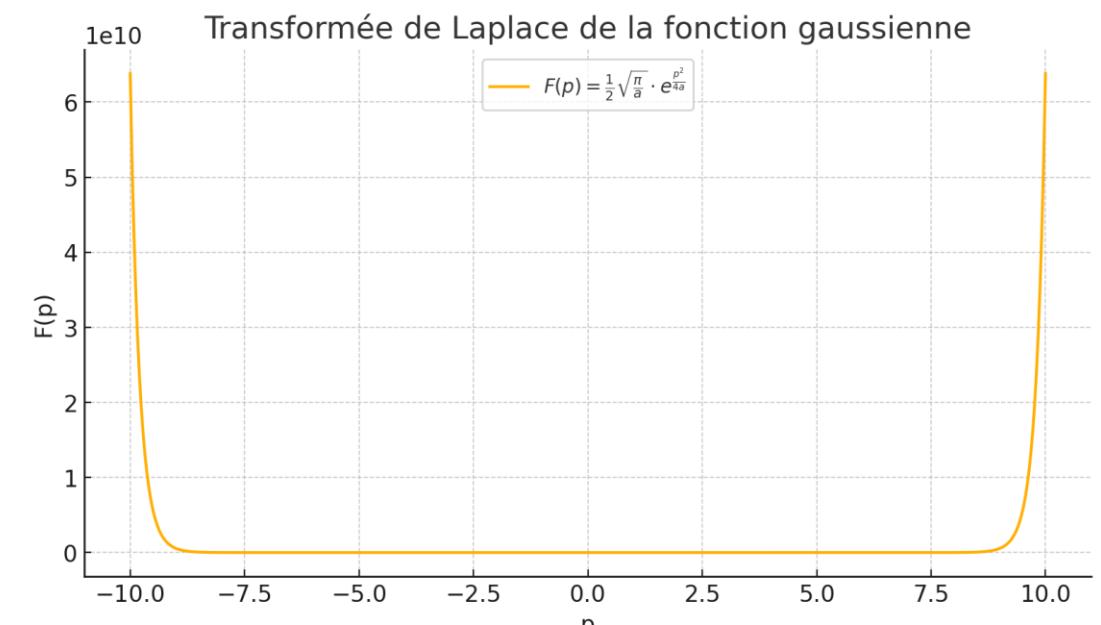


# Transformée de Laplace inverse du signal Gaussiennes

$$F(S) = \frac{1}{2} \sqrt{\frac{\pi}{a}} \cdot e^{\frac{S^2}{4a}}$$

$a$  : facteur de contrôle de la largeur de la courbe. , Pour  $a > 0$

$$TL^{-1}(F(S)) = e^{-at^2}$$

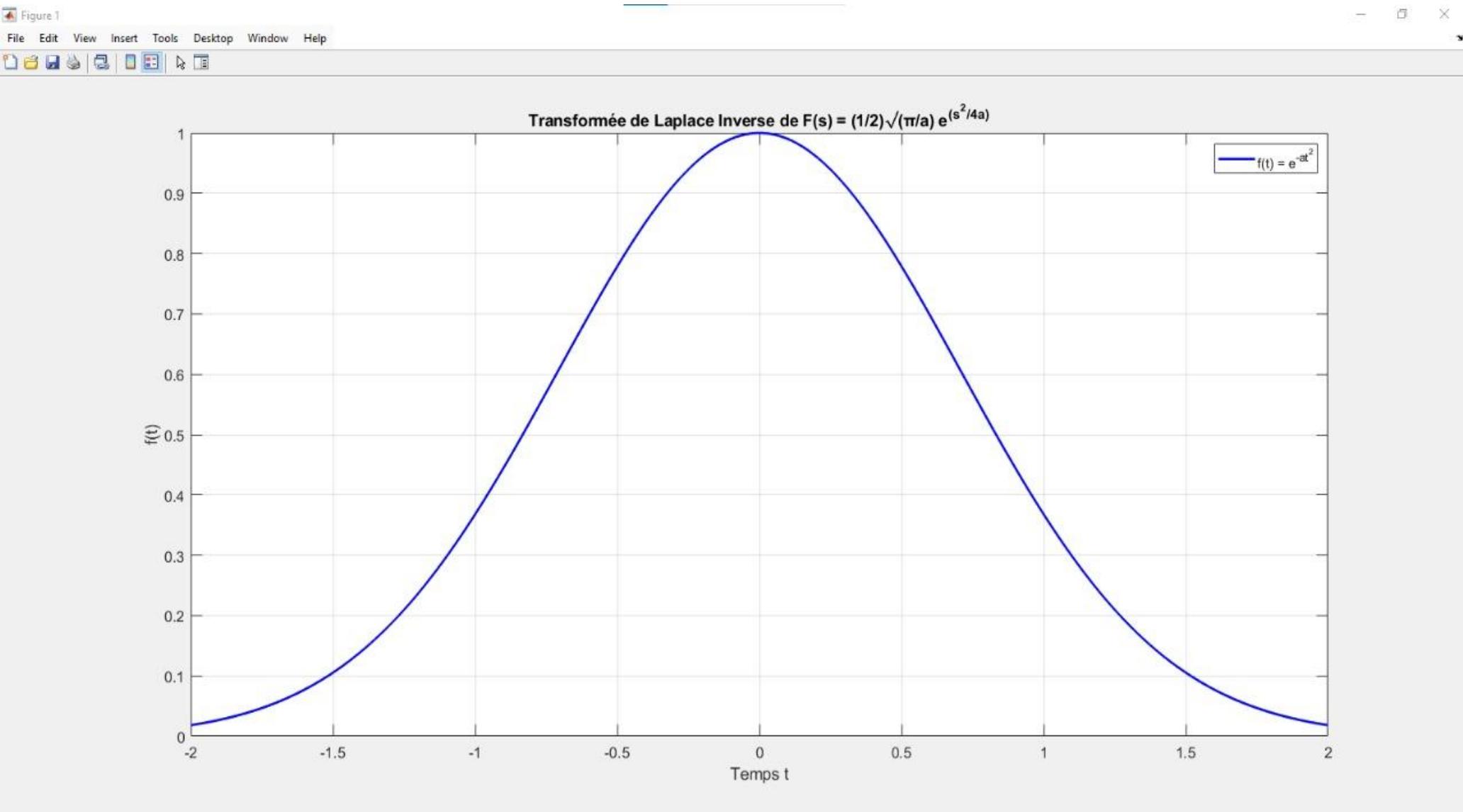


# Code MATLAB Transformée de Laplace inverse du signal Gaussiennes

```
+9 | untitled67.m ✘ | untitled555.m ✘ | untitled45.m ✘ | untitled53.m ✘ | untitled7.m ✘ Editor - C:\Users\Lenovo\Downloads\untitled72.m
+9 | untitled67.m ✘ | untitled555.m ✘ | untitled45.m ✘ | untitled53.m ✘ | untitled9999.m ✘ | untitled999999.m ✘
1 clc;
2 clear;
3 close all; % Fermer toutes les figures précédentes
4
5 syms s t a real
6
7 % Définition de la fonction F(s)
8 F = (1/2)*sqrt(pi/a) * exp(s^2 / (4*a));
9
10 % Valeur numérique pour a
11 a_val = 1;
12
13 % Substitution de la valeur de a dans F(s)
14 F_sub = subs(F, a, a_val);
15
16 % Calcul de la transformée de Laplace inverse
17 f_t = ilaplace(F_sub, s, t);
18
19 % Simplification du résultat
20 f_t = simplify(f_t);
21
22 % Vérification manuelle avec la forme connue
23 f_t_correct = exp(-a_val*t^2); % Forme théorique attendue
24
25 % Création de la fonction numérique
26 f_numeric = matlabFunction(f_t_correct, 'Vars', t);
27
28 % Génération des valeurs de t
29 t_vals = linspace(-2, 2, 1000); % Intervalle ajusté
30
31 % Évaluation de f(t)
32 f_vals = f_numeric(t_vals);
33
34 % Tracé
35
36 % Évaluation de f(t)
37 f_vals = f_numeric(t_vals);
38
39 % Tracé
40 figure;
41 plot(t_vals, f_vals, 'b', 'LineWidth', 2);
42 title('Transformée de Laplace Inverse de F(s) = (1/2)\surd(\pi/a) e^{-(s^2/4a)}');
43 xlabel('Temps t');
44 ylabel('f(t)');
45 grid on;
46 legend('f(t) = e^{-at^2}');
47 set(gca, 'FontSize', 12); % Meilleure lisibilité
48
49 % Affichage des résultats symboliques
50 disp('Résultat symbolique de la transformée inverse:');
51 disp(f_t);
52 disp('Résultat théorique attendu:');
53 disp(f_t_correct);

Command Window
New to MATLAB? See resources for Getting Started.
Résultat théorique attendu:
exp(-t^2)
fx >>
```

# Graphe Transformée de Laplace inverse du signal Gaussiennes



## Transformée de Laplace inverse du signal peigne de Dirac

$$F(s) = \frac{1}{1 - e^{-TS}}$$

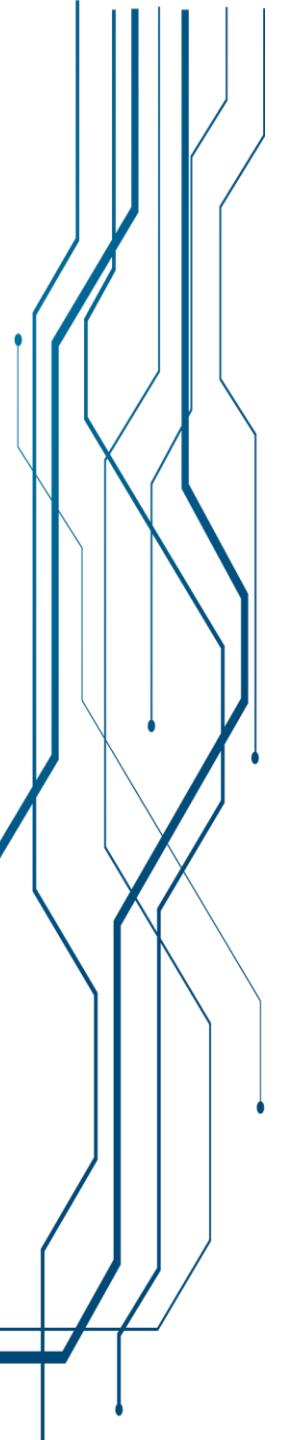
✓  $TL(\delta(t - a)) = e^{-as}$



$TL^{-1}(e^{-as}) = \delta(t - nt)$

$$TL^{-1} \left( \frac{1}{1 - e^{-TS}} \right) = \sum_{n=0}^{\infty} \delta(t - nT)$$

# Code MATLAB Transformée de Laplace inverse du peigne de Dirac



The image shows a MATLAB code editor window titled "Editor - C:\Users\Lenovo\Downloads\www.m". The code is written in French and generates a plot of a Dirac comb signal. The code uses a for loop to create narrow pulses at regular intervals, which are then plotted as a series of sharp peaks.

```
T = 1; % Période entre les impulsions
N = 10; % Nombre d'impulsions
t = 0:0.01:N*T; % Temps

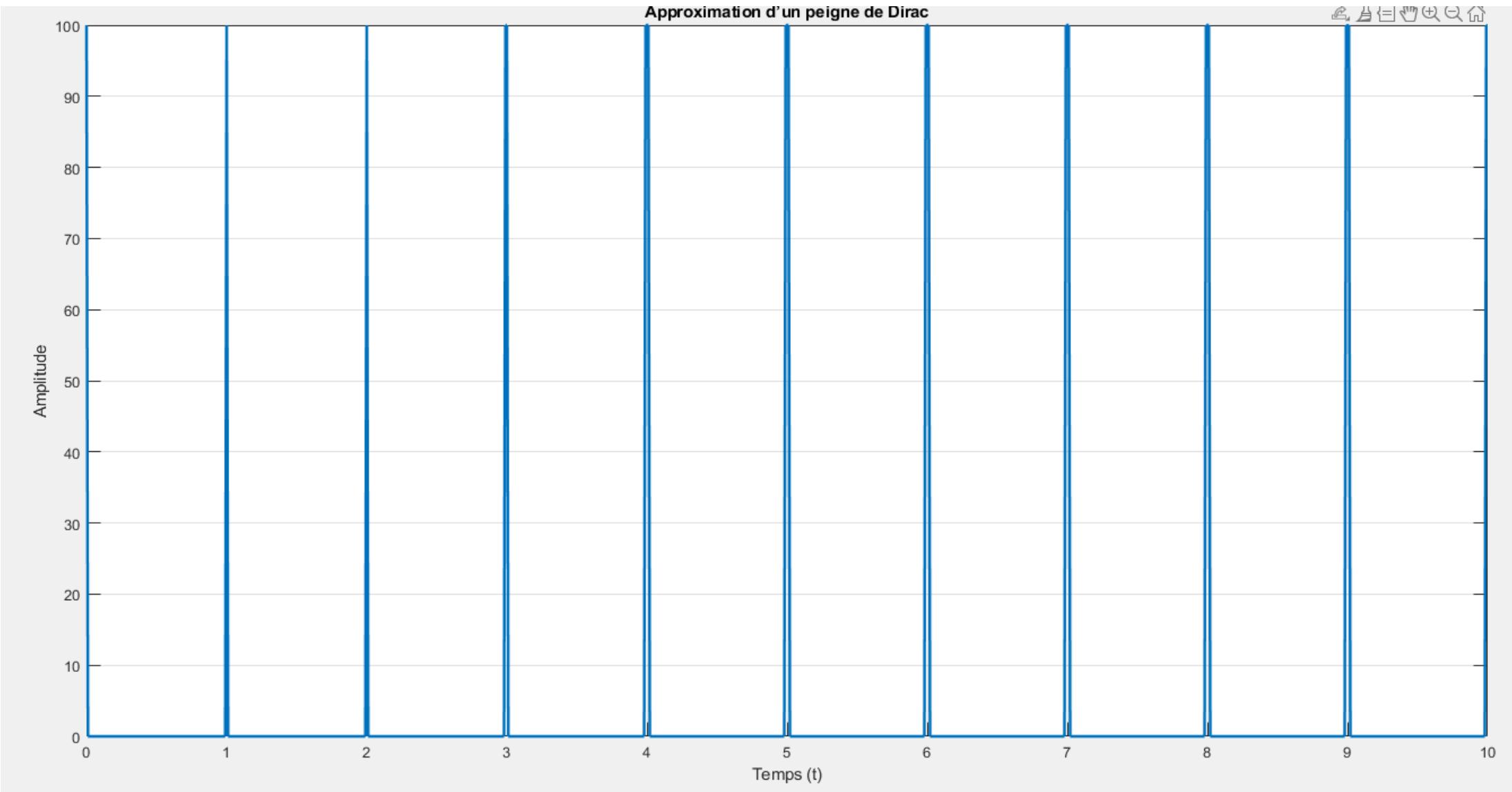
% Initialiser le signal
f = zeros(size(t));

% Générer les impulsions comme des pics très étroits
for n = 0:N
    idx = find(abs(t - n*T) < 0.01); % Approximation de Dirac
    f(idx) = 1 / 0.01; % Impulsion très fine
end

plot(t, f, 'LineWidth', 2)
xlabel('Temps (t)')
ylabel('Amplitude')
title('Approximation d'un peigne de Dirac')
grid on
```

At the bottom of the image, there is a small "Command Window" placeholder.

# Graphe Transformée de Laplace inverse du peigne de Dirac

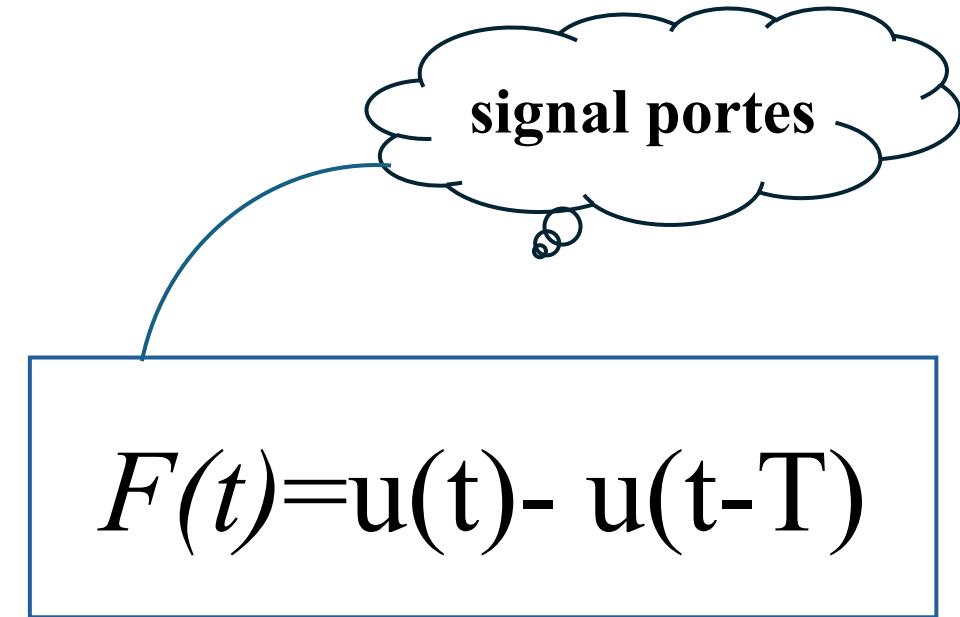


## Transformée de Laplace inverse du signal portes

$$F(s) = \frac{1 - e^{-ST}}{S} = \frac{1}{S} - \frac{e^{-ST}}{S}$$

- ✓  $TL^{-1}\left(\frac{1}{S}\right) = u(t)$
  - ✓  $TL^{-1}\left(\frac{e^{-ST}}{S}\right) = u(t - T)$
- >>>>

$$\Pi(t) = \begin{cases} 1 & \text{si } 0 \leq t \leq T \\ 0 & \text{sinon} \end{cases}$$



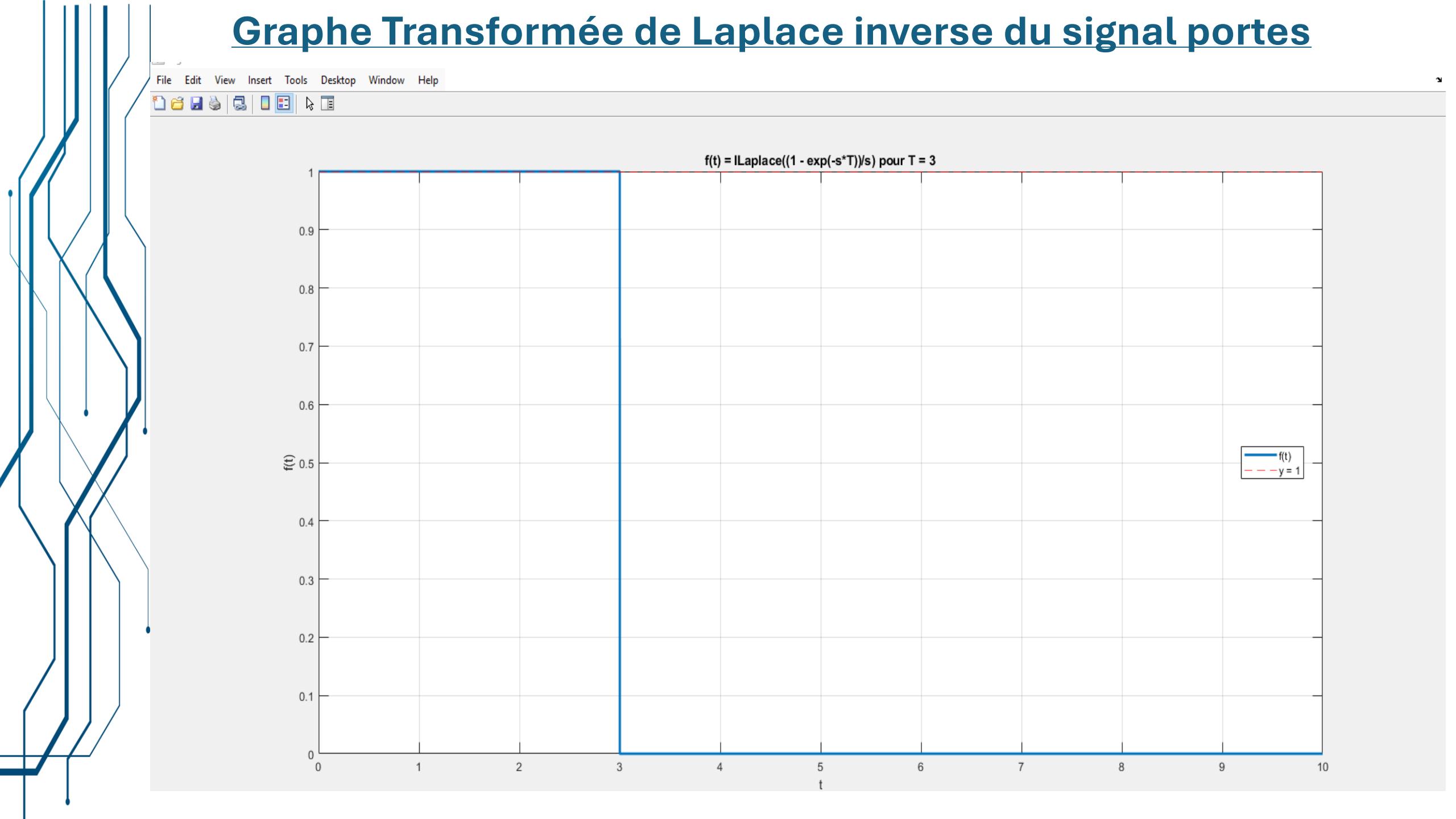
# Code MATLAB Transformée de Laplace inverse du signal portes

The image shows the MATLAB graphical user interface (GUI) with the following details:

- Toolbar:** PLOTS, APPS, EDITOR, PUBLISH, VIEW.
- FILE Menu:** Save, Print, Go To, Find, Bookmarks.
- NAVIGATE:** Refactor, CODE, ANALYZE.
- PUBLISH:** Profiler, Analyze, Run Section, Section Break, Run and Advance, Run to End.
- VIEW:** Run, Step, RUN.
- Editor Area:** Shows the current file path: C:\Users\Lenovo\Downloads\untitled23456.m. Below it, a list of open files is shown: +9, untitled67.m, untitled555.m, untitled45.m, untitled53.m, untitled9999.m, untitled999999.m, and untitled244.
- Code Editor:** Displays the following MATLAB script:

```
1 syms s t T
2 F = (1 - exp(-s*T)) / s;
3 f = ilaplace(F, s, t);
4 f_T3 = subs(f, T, 3);
5 fplot(f_T3, [0 10], 'LineWidth', 2);
6 grid on;
7 xlabel('t');
8 ylabel('f(t)');
9 title('f(t) = ILaplace((1 - exp(-s*T))/s) pour T = 3');
10 hold on;
11 yline(1, '--r');
12 legend('f(t)', 'y = 1', 'Location', 'Best');
```

# Graphe Transformée de Laplace inverse du signal portes



## Transformée de Laplace inverse du signal triangulaire

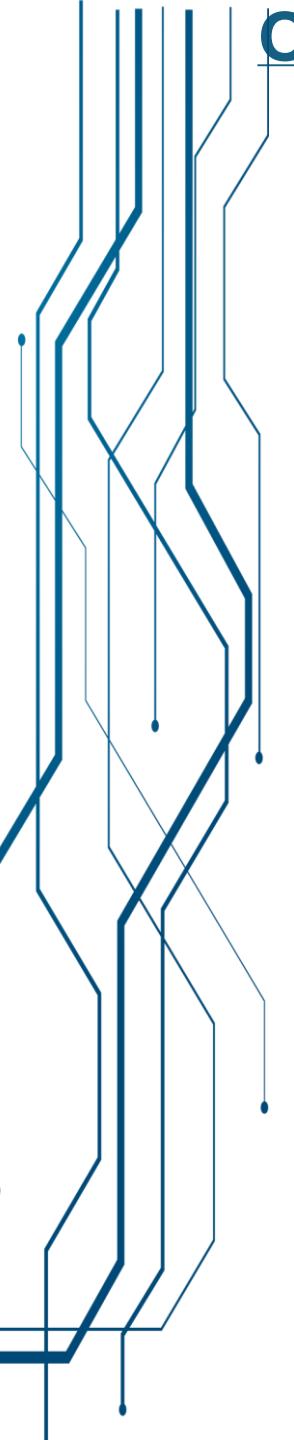
$$F(s) = \frac{1 - 2e^{-TS} + e^{-2TS}}{S^2} = \frac{1}{S^2} - \frac{2e^{-TS}}{S^2} + \frac{e^{-2TS}}{S^2}$$

Terme	Transformé de Laplace
$r(t)=tu(t)$	$\frac{1}{S^2}$
$r(t-T)=(t-T)u(t - T)$	$\frac{e^{-TS}}{S^2}$
$r(t-2T)=(t-2T)u(t - 2T)$	$\frac{e^{-2TS}}{S^2}$

$$f(t)=tu(t)-2(t-T)u(t - T)+(t-2T)u(t - 2T)$$

signal triangulaire

# Code MATLAB Transformée de Laplace inverse du signal triangulaire



The screenshot shows the MATLAB interface with the following details:

- Toolbar:** PLOTS, APPS, EDITOR, PUBLISH, VIEW.
- FILE Menu:** Save, Compare, Print, Go To, Find, Refactor, CODE, ANALYZE, Run Section, SECTION, RUN.
- NAVIGATE Menu:** Bookmarks.
- PATH:** C:\Users\Lenovo\Downloads\untitled9999.m
- Editor Content:** Untitled9999.m (selected tab)
- Code:**

```
1 syms s t T
2
3 % Définir la fonction F(s)
4 F = (1 - 2*exp(-T*s) + exp(-2*T*s)) / s^2;
5
6 % Calcul de la transformée de Laplace inverse
7 f_t = ilaplace(F, s, t);
8
9 % Affichage du résultat analytique
10 disp('Transformée de Laplace inverse :')
11 pretty(f_t)
12
13 % Tracer la fonction pour une valeur donnée de T
14 T_val = 1;
15 f_t_plot = subs(f_t, T, T_val); % Substituer T par une valeur numérique
16 fplot(f_t_plot, [0, 2*T_val], 'LineWidth', 2)
17 grid on
18 xlabel('t')
19 ylabel('f(t)')
20 title('Transformée de Laplace inverse d''un signal triangulaire')
21
```

# Graphe Transformée de Laplace inverse du signal triangulaire

