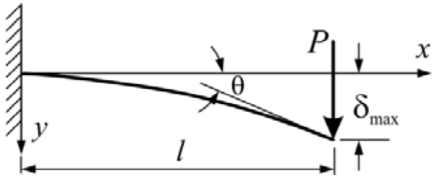
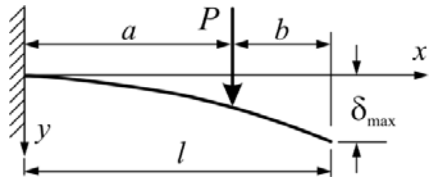
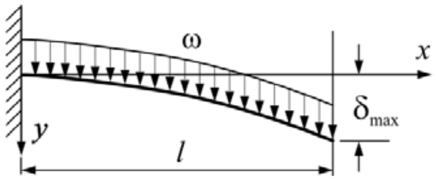
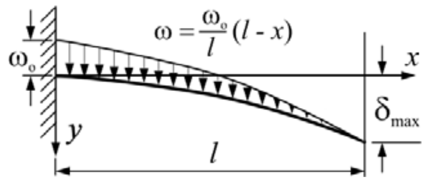
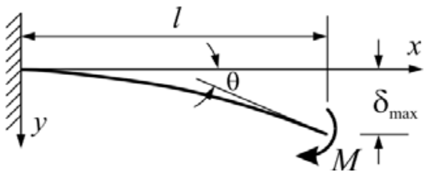
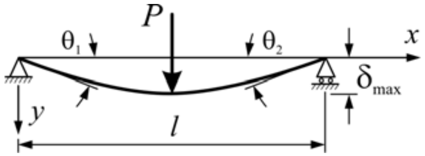
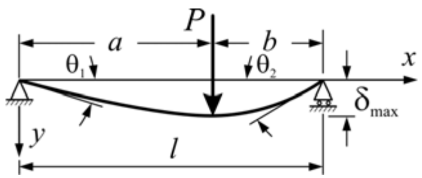
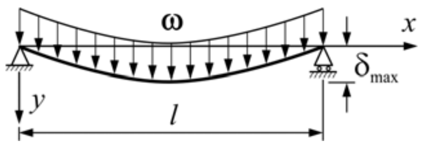
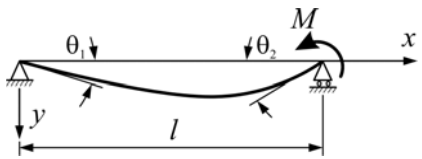
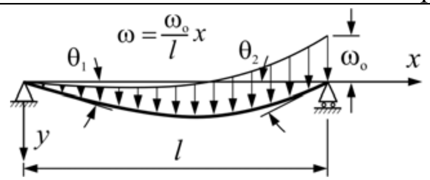
BEAM DEFLECTION FORMULAE

BEAM TYPE	SLOPE AT FREE END	DEFLECTION AT ANY SECTION IN TERMS OF *x*	MAXIMUM DEFLECTION
1. Cantilever Beam – Concentrated load *P* at the free end			
	$\theta = \frac{Pl^2}{2EI}$	$y = \frac{Px^2}{6EI}(3l - x)$	$\delta_{max} = \frac{Pl^3}{3EI}$
2. Cantilever Beam – Concentrated load *P* at any point			
	$\theta = \frac{Pa^2}{2EI}$	$y = \frac{Px^2}{6EI}(3a - x)$ for $0 < x < a$ $y = \frac{Pa^2}{6EI}(3x - a)$ for $a < x < l$	$\delta_{max} = \frac{Pa^2}{6EI}(3l - a)$
3. Cantilever Beam – Uniformly distributed load ω (N/m)			
	$\theta = \frac{\omega l^3}{6EI}$	$y = \frac{\omega x^2}{24EI}\left(x^2 + 6l^2 - 4lx\right)$	$\delta_{max} = \frac{\omega l^4}{8EI}$
4. Cantilever Beam – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta = \frac{\omega_o l^3}{24EI}$	$y = \frac{\omega_o x^2}{120lEI}\left(10l^3 - 10l^2x + 5lx^2 - x^3\right)$	$\delta_{max} = \frac{\omega_o l^4}{30EI}$
5. Cantilever Beam – Couple moment *M* at the free end			
	$\theta = \frac{Ml}{EI}$	$y = \frac{Mx^2}{2EI}$	$\delta_{max} = \frac{Ml^2}{2EI}$

BEAM DEFLECTION FORMULAS

<table>
<tr><th>BEAM TYPE</th><th>SLOPE AT ENDS</th><th>DEFLECTION AT ANY SECTION IN TERMS OF x</th><th>MAXIMUM AND CENTER DEFLECTION</th></tr>
<tr><td colspan="4">6. Beam Simply Supported at Ends – Concentrated load P at the center</td></tr>
<tr><td>
</td><td>$\theta_1 = \theta_2 = \dfrac{Pl^2}{16EI}$</td><td>$y = \dfrac{Px}{12EI}\left(\dfrac{3l^2}{4} - x^2\right)$ for $0 < x < \dfrac{l}{2}$</td><td>$\delta_{max} = \dfrac{Pl^3}{48EI}$</td></tr>
<tr><td colspan="4">7. Beam Simply Supported at Ends – Concentrated load P at any point</td></tr>
<tr><td>
</td><td>$\theta_1 = \dfrac{Pb(l^2 - b^2)}{6lEI}$
$\theta_2 = \dfrac{Pab(2l - b)}{6lEI}$</td><td>$y = \dfrac{Pbx}{6lEI}\left(l^2 - x^2 - b^2\right)$ for $0 < x < a$
$y = \dfrac{Pb}{6lEI}\left[\dfrac{l}{b}(x-a)^3 + \left(l^2 - b^2\right)x - x^3\right]$
for $a < x < l$</td><td>$\delta_{max} = \dfrac{Pb\left(l^2 - b^2\right)^{3/2}}{9\sqrt{3}\,lEI}$ at $x = \sqrt{\left(l^2 - b^2\right)/3}$
$\delta = \dfrac{Pb}{48EI}\left(3l^2 - 4b^2\right)$ at the center, if $a > b$</td></tr>
<tr><td colspan="4">8. Beam Simply Supported at Ends – Uniformly distributed load ω (N/m)</td></tr>
<tr><td>
</td><td>$\theta_1 = \theta_2 = \dfrac{\omega l^3}{24EI}$</td><td>$y = \dfrac{\omega x}{24EI}\left(l^3 - 2lx^2 + x^3\right)$</td><td>$\delta_{max} = \dfrac{5\omega l^4}{384EI}$</td></tr>
<tr><td colspan="4">9. Beam Simply Supported at Ends – Couple moment M at the right end</td></tr>
<tr><td>
</td><td>$\theta_1 = \dfrac{Ml}{6EI}$
$\theta_2 = \dfrac{Ml}{3EI}$</td><td>$y = \dfrac{Mlx}{6EI}\left(1 - \dfrac{x^2}{l^2}\right)$</td><td>$\delta_{max} = \dfrac{Ml^2}{9\sqrt{3}\,EI}$ at $x = \dfrac{l}{\sqrt{3}}$
$\delta = \dfrac{Ml^2}{16EI}$ at the center</td></tr>
<tr><td colspan="4">10. Beam Simply Supported at Ends – Uniformly varying load: Maximum intensity ω_o (N/m)</td></tr>
<tr><td>
</td><td>$\theta_1 = \dfrac{7\omega_o l^3}{360EI}$
$\theta_2 = \dfrac{\omega_o l^3}{45EI}$</td><td>$y = \dfrac{\omega_o x}{360lEI}\left(7l^4 - 10l^2x^2 + 3x^4\right)$</td><td>$\delta_{max} = 0.00652\dfrac{\omega_o l^4}{EI}$ at $x = 0.519\,l$
$\delta = 0.00651\dfrac{\omega_o l^4}{EI}$ at the center</td></tr>
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