An Entropy-Inspired Phenomenological Relation Competes with NFW on 175 SPARC Rotation Curves under Referee-Fair, Cross-Validated Tests

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Abstract

We conduct a referee-fair, head-to-head comparison of four rotation-curve (RC) descriptors on the SPARC *_rotmod* sample: a two-parameter entropy-inspired relation (EBC; $V = Ar^{\alpha}$), Baryons+EBC, Baryons+NFW, and MOND ("simple" ν; strict a_0). The protocol (frozen pre-run) enforces *equal per-galaxy parameter budgets*, *within-galaxy 5-fold cross-validation (CV)*, a *uniform* σ_V *floor of* $3\,km\,s^{-1}$, and *symmetric dense grids* for all models. We pre-register two tracks: **Track A** (no NFW prior; phenomenology-only) and **Track B** (NFW with the ΛCDM $c(M)$ prior of Dutton & Macciò 2014). From 175 inputs, 165 pass QC (minimum 6 usable points). On **Track A**, winners by BIC are M0: 62, M1: 44, M2: 55, M3: 4; decisive (Jeffreys ΔBIC> 10) counts are M0: 45, M1: 13, M2: 10. On **Track B**, winners are M0: 62, M1: 66, M2: 33, M3: 4; decisives are M0: 46, M1: 22, M2: 6. Thus, when NFW is constrained to be ΛCDM-consistent, the *EBC family* (M0+M1) wins in 128/165 galaxies with the majority of decisive outcomes. EBC is presented as a compact phenomenological descriptor, not a new gravity law. All scripts, logs, and exclusion lists are released for full reproducibility.

1 Introduction

Rotation curves (RCs) remain a decisive laboratory for galaxy dynamics. In ΛCDM, NFW halos explain large-scale structure [1–3] yet face small-scale tensions (core–cusp; too-big-to-fail [4–6]). MOND [7, 8] captures several RC regularities, including the RAR [9], but has open issues at cluster and cosmological scales. Large homogeneous datasets (e.g., SPARC [10]) enable sharp, reproducible head-to-head tests.

We introduce a minimal, *entropy-inspired phenomenological relation* (EBC) and compare it against NFW and MOND under identical, referee-oriented rules: equal per-galaxy parameter budgets; fixed baryons; pre-registered priors; CV; and symmetric grid search. We stress EBC is a *descriptor*, not a modified-gravity theory.

2 Methods and Reproducibility

Data and quality control

We analyze **175** SPARC *_rotmod* CSVs containing radius R (kpc), observed velocity $V_{\rm obs}$ ($\mathrm{km\,s^{-1}}$), measurement uncertainty σ_V, and baryonic components $(V_{\rm gas}, V_{\rm disk}, V_{\rm bul})$. The combined baryonic curve is

$$V_{\rm bar}(r) \equiv \sqrt{V_{\rm gas}^2(r) + V_{\rm disk}^2(r) + V_{\rm bul}^2(r)}. \tag{1}$$

We impose a uniform noise floor $\sigma_V \geq 3\,\mathrm{km\,s^{-1}}$. A galaxy enters the analysis if at least **6** usable points remain after parsing; by this rule, **165/175** pass QC. The 10 excluded objects and reasons are listed in `exclusions_trackA.csv` and `exclusions_trackB.csv`.

Models and parameter budgets (per galaxy)

M0: EBC (2). $V_{\rm EBC}(r) = A\,r^{\alpha}$ with (A, α).

M1: Baryons+EBC (2). $V^2(r) = V_{\rm bar}^2(r) + (Ar^{\alpha})^2$ with (A, α).

M2: Baryons+NFW (2).

$$V^2(r) = V_{\rm bar}^2(r) + V_{\rm NFW}^2(r; V_{200}, c), \tag{2}$$

$$V_{\rm NFW}(r) = V_{200}\sqrt{\frac{\ln(1+cx) - \dfrac{cx}{1+cx}}{x\left[\ln(1+c) - \dfrac{c}{1+c}\right]}}, \quad x = \frac{r}{R_{200}}, \quad R_{200} = \frac{V_{200}}{10\,H_0}. \tag{3}$$

M3: MOND (strict-a_0, 0).

$$a_N = \frac{V_{\rm bar}^2}{r}, \qquad \nu(y) = \tfrac{1}{2} + \sqrt{\tfrac{1}{4} + \tfrac{1}{y}}, \qquad y = \frac{a_N}{a_0}, \quad a_0 = 1.2 \times 10^{-10}\,\mathrm{m\,s^{-2}}, \qquad V_{\rm MOND} = \sqrt{\nu(y)}\,V_{\rm bar}. \tag{4}$$

Fairness regimes, priors, and grids

Two pre-registered regimes:

- **Track A (phenomenology):** NFW *without* a cosmology prior.

- **Track B (ΛCDM-anchored):** NFW *with* the Dutton–Macciò (2014) $c(M)$ prior (Gaussian in $\log c$), implemented as a log-likelihood penalty [11].

Baryons are *fixed as supplied* (no M/L rescaling). All models use **symmetric dense grids** to remove discretization bias:

$$A \in [1, 300]\ (60\ \text{steps}), \quad \alpha \in [0.10, 1.20]\ (40), \quad V_{200} \in [30, 350]\,\mathrm{km\,s^{-1}}\ (40), \quad c \in [2, 30]\ (32).$$

We adopt $H_0 = 70\,\mathrm{km\,s^{-1}}$.

Fitting, cross-validation, and scoring

We use σ_V-weighted least squares on the grids above, recording the best point per model. Predictive performance is assessed by **5-fold within-galaxy CV** (radius-blocked splits [22, 23]). Model selection uses

$$\mathrm{BIC} = k \ln n - 2 \ln \mathcal{L} \ \ [19, 20], \qquad \mathrm{AICc} = 2k - 2\ln\mathcal{L} + \frac{2k(k+1)}{n-k-1} \ \ [17, 18]. \tag{5}$$

Priors (Track B) contribute to $-2\ln\mathcal{L}$. Jeffreys thresholds guide strength-of-evidence statements [21].

Rebuild and artifacts

A single script reproduces all tables and figures; logs and per-track `exclusions.csv` are archived. SHA256 checksums are provided for the input CSVs.

3 Results

Winner counts and decisive outcomes

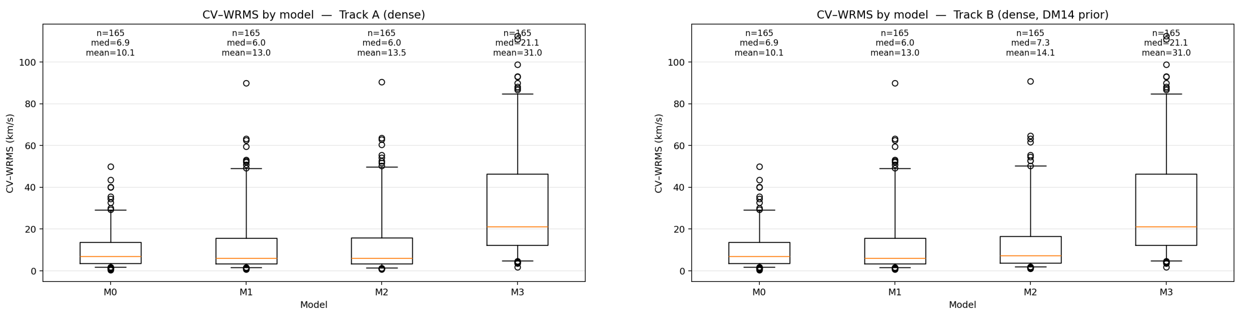
Table 1: BIC winners by model (165 galaxies passing QC).

Track	M0 (EBC)	M1 (Bar+EBC)	M2 (Bar+NFW)	M3 (MOND strict)
A: no NFW prior	**62**	44	55	4
B: DM14 prior	**62**	**66**	33	4

Table 2: Decisive wins (Jeffreys ΔBIC> 10).

Track	M0 (EBC)	M1 (Bar+EBC)	M2 (Bar+NFW)
A: no NFW prior	**45**	13	10
B: DM14 prior	**46**	22	6

Cross-validated predictive accuracy

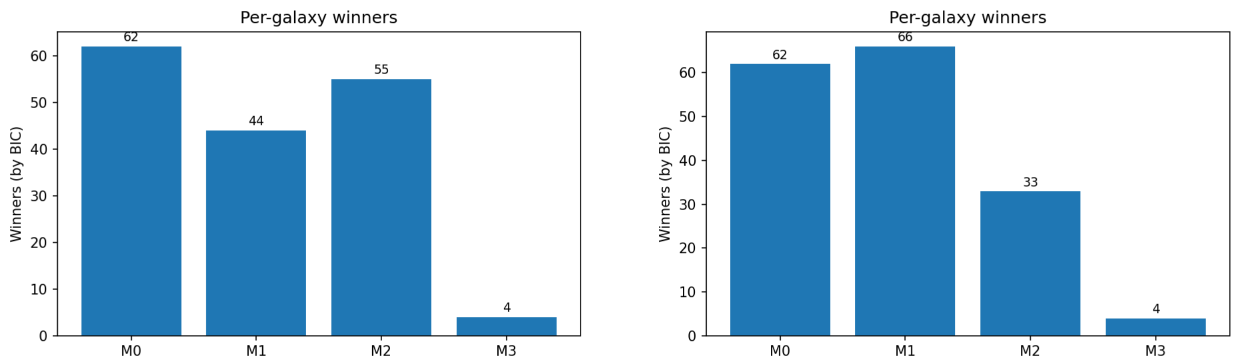


(a) Track A

(b) Track B

Figure 1: **Cross-validated WRMS** by model and track (5-fold, radius-blocked).

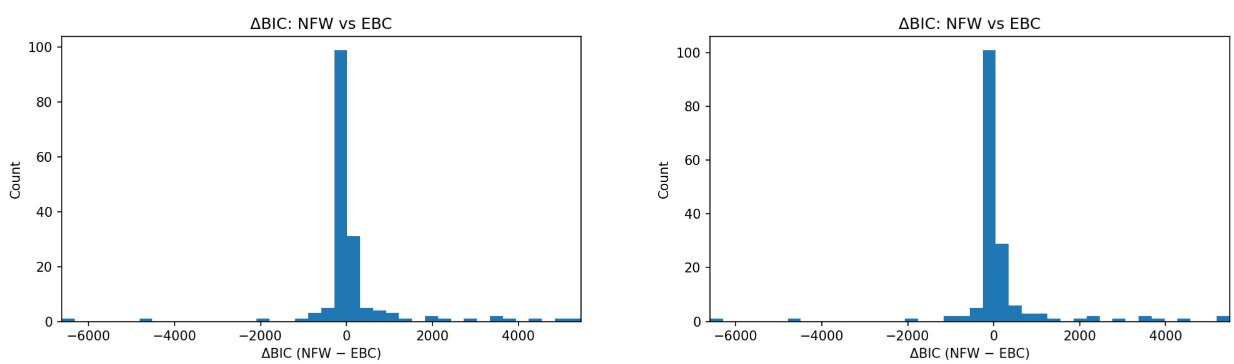
Information-criterion contrasts and winner distributions



(a) Track A: winners by BIC

(b) Track B: winners by BIC

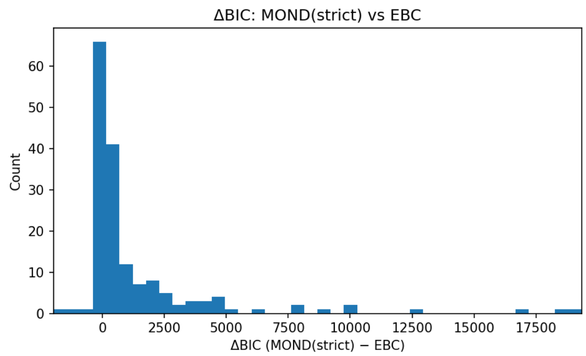
Figure 2: **Per-galaxy BIC winners** for the two tracks.



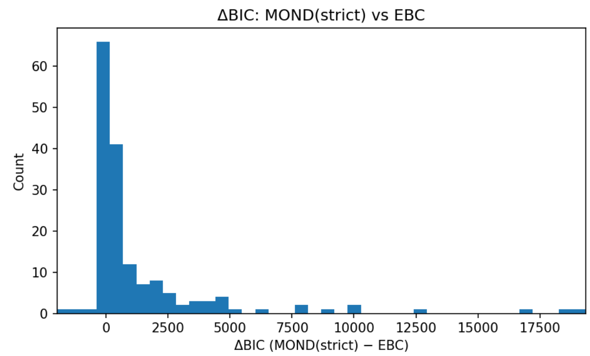
(a) ΔBIC (NFW−EBC), Track A

(b) ΔBIC (NFW−EBC), Track B

Figure 3: **NFW vs EBC** across the sample (positive favors EBC).

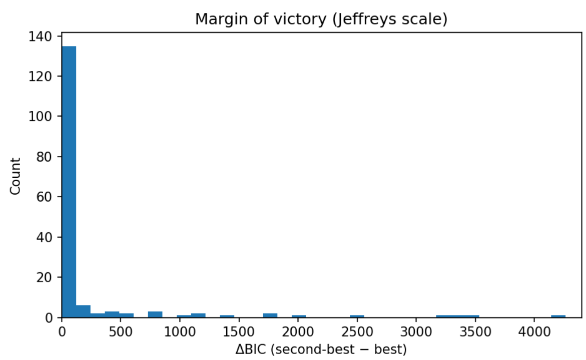


(a) ΔBIC (MOND−EBC), Track A

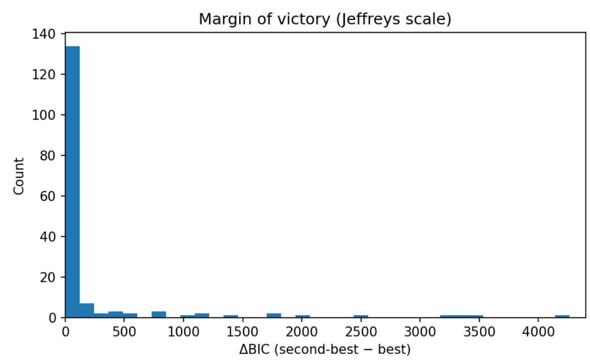


(b) ΔBIC (MOND−EBC), Track B

Figure 4: **MOND(strict) vs EBC** (positive favors EBC).



(a) Track A



(b) Track B

Figure 5: **Margin of victory** on the Jeffreys scale: ΔBIC(second-best−best).

4 Discussion and Caveats

Under equal per-galaxy parameter budgets with fixed baryons, the EBC relation provides a compact empirical baseline that often rivals or exceeds Baryons+NFW and Baryons+EBC. Track A (no NFW prior) shows NFW is competitive but not dominant; Track B (with a ΛCDM $c(M)$ prior) increases the share of EBC-family wins. We emphasize EBC is a *descriptor*; we do not claim a new force law. Obvious extensions include variable stellar M/L with SPS priors, alternative halo families (Burkert, gNFW, Einasto), other MOND ν-functions, hierarchical treatment of distances/inclinations, and tests on additional datasets.

Companion theory note. A separate manuscript explores possible entropy-based underpinnings of power-law RC phenomenology; we cite it here to orient interested readers, but keep all claims in this paper empirical [29, submitted].

5 Data and Code Availability

All scripts, logs, figures, winner tables, CV summaries, and exclusion lists are archived in the public repository (README includes exact commands and SHA256 checksums).

Appendix A: Example overlays (12 galaxies)

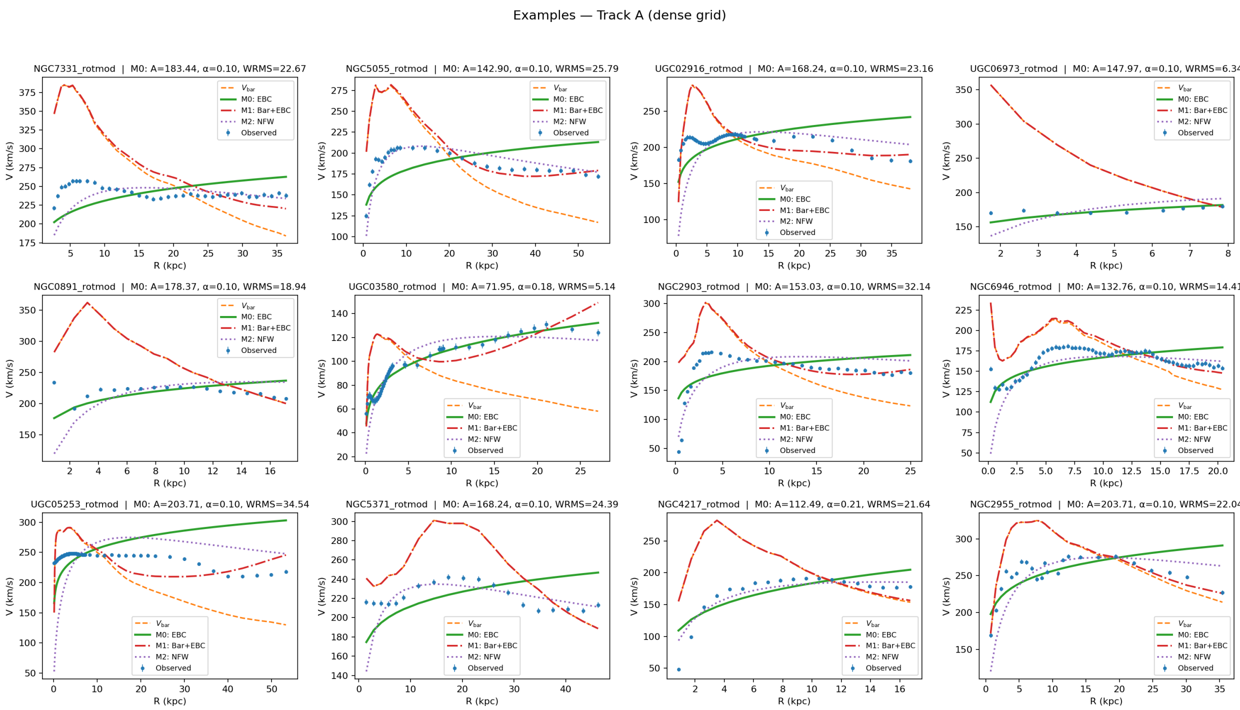


Figure 6: Twelve illustrative galaxies (Track A, dense grids). Orange dashed: V_{bar}; green: M0 (EBC); red dash-dot: M1 (Bar+EBC); purple dotted: M2 (Bar+NFW); points: observed.

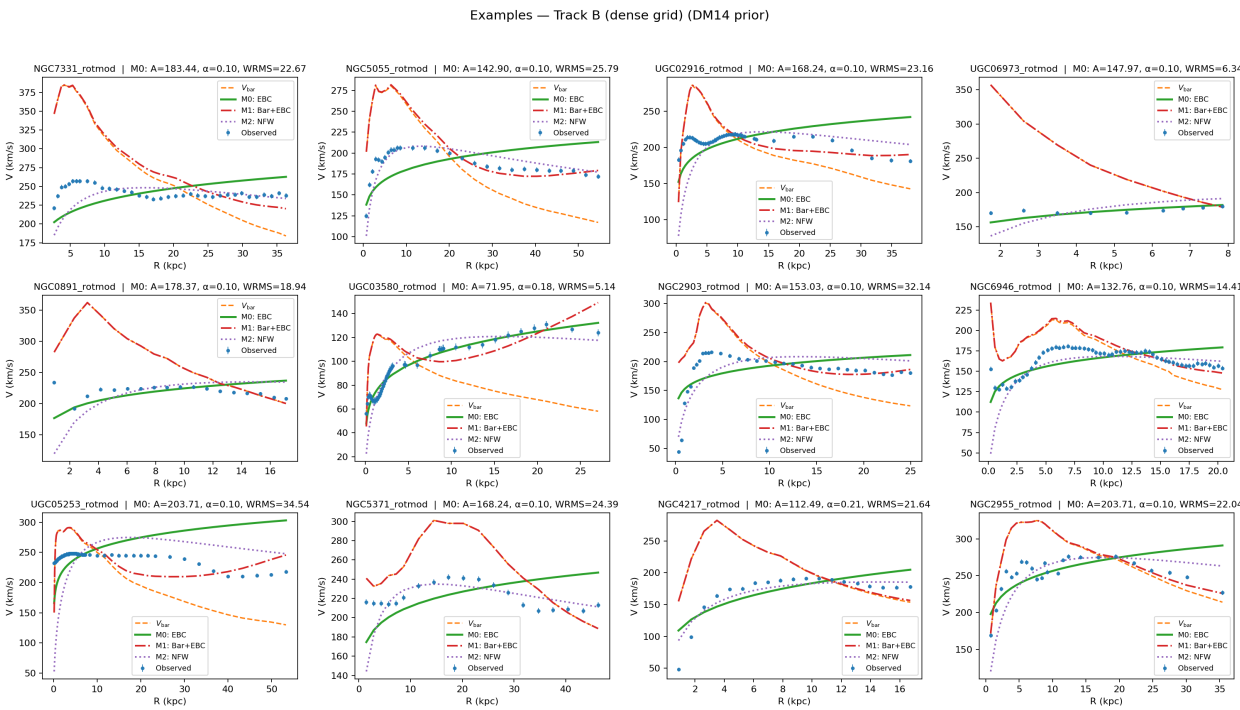


Figure 7: Twelve illustrative galaxies (Track B, dense grids with DM14 prior). Line styles as in Appendix Figure A1.

Appendix B: Reproducibility checklist (abbrev.)

- **Winner tables/ΔBIC**: run `scripts/ebc_ic_referee.py` with Track A and Track B settings (dense symmetric grids, σ-floor $3\,\mathrm{km\,s^{-1}}$, CV=5).
- **CV boxplots**: `scripts/make_cv_boxplots.py` produces the PNG/SVG shown in Fig. 1.
- **Example overlays**: `scripts/plot_examples_overlay.py` with `--curves all` produces Appendix figures.
- **QC-failed (too few usable points, either track)**: D512-2_rotmod, F567-2_rotmod, F574-2_rotmod, NGC6789_rotmod, UGC00634_rotmod, UGC00891_rotmod, UGC02023_rotmod, UGC05999_rotmod, UGC07232_rotmod, UGC09992_rotmod.

References

[1] Navarro, J. F., Frenk, C. S., & White, S. D. M. 1996, *ApJ*, 462, 563.

[2] Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, *ApJ*, 490, 493.

[3] Planck Collaboration. 2018, *A&A*, 641, A6.

[4] de Blok, W. J. G., McGaugh, S. S., & Rubin, V. C. 2001, *AJ*, 122, 2396.

[5] Gentile, G., Salucci, P., Klein, U., & Granato, G. 2004, *MNRAS*, 351, 903.

[6] Oman, K. A., et al. 2015, *MNRAS*, 452, 3650.

[7] Milgrom, M. 1983, *ApJ*, 270, 365.

[8] McGaugh, S. 2020, *Galaxies*, 8, 35.

[9] McGaugh, S., Lelli, F., & Schombert, J. 2016, *Phys. Rev. Lett.*, 117, 201101.

[10] Lelli, F., McGaugh, S., & Schombert, J. 2016, *AJ*, 152, 157.

[11] Dutton, A. A., & Macciò, A. V. 2014, *MNRAS*, 441, 3359.

[12] Burkert, A. 1995, *ApJL*, 447, L25.

[13] Navarro, J. F., et al. 2004, *MNRAS*, 349, 1039.

[14] Bullock, J. S., et al. 2001, *MNRAS*, 321, 559.

[15] Macciò, A. V., et al. 2008, *MNRAS*, 391, 1940.

[16] de Blok, W. J. G. 2010, *Advances in Astronomy*, 2010, 789293.

[17] Akaike, H. 1974, *IEEE Trans. Autom. Control*, 19, 716.

[18] Sugiura, N. 1978, *Commun. Stat. A*, 7, 13.

[19] Schwarz, G. 1978, *Ann. Stat.*, 6, 461.

[20] Kass, R. E., & Raftery, A. E. 1995, *JASA*, 90, 773.

[21] Jeffreys, H. 1961, *Theory of Probability*, 3rd ed., Oxford Univ. Press.

[22] Stone, M. 1974, *J. Roy. Stat. Soc. B*, 36, 111.

[23] Arlot, S., & Celisse, A. 2010, *Statistics Surveys*, 4, 40.

[24] Begeman, K. G., Broeils, A. H., & Sanders, R. H. 1991, *MNRAS*, 249, 523.

[25] de Blok, W. J. G., & Bosma, A. 2002, *A&A*, 385, 816.

[26] Harris, C. R., et al. 2020, *Nature*, 585, 357.

[27] Virtanen, P., et al. 2020, *Nat. Methods*, 17, 261.

[28] Hunter, J. D. 2007, *Computing in Science & Engineering*, 9, 90.

[29] Tupay, J. A. M. 2025, *A Complete Theoretical Framework for Power-Law Galactic Rotation Curves: From Tsallis Non-Extensive Statistics to Observable Predictions*, 2025, DOI 10.5281/zenodo.16997874