

# An Entropy-Inspired Phenomenological Relation Competes with NFW on 175 SPARC Rotation Curves under Referee-Fair, Cross-Validated Tests

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## Abstract

We conduct a referee-fair, head-to-head comparison of four rotation-curve (RC) descriptors on the SPARC *\_rotmod* sample: a two-parameter entropy-inspired relation (EBC;  $V = Ar^\alpha$ ), Baryons+EBC, Baryons+NFW, and MOND (“simple”  $\nu$ ; strict  $a_0$ ). The protocol (frozen pre-run) enforces *equal per-galaxy parameter budgets*, *within-galaxy 5-fold cross-validation (CV)*, a *uniform  $\sigma_V$  floor of  $3\text{ km s}^{-1}$* , and *symmetric dense grids* for all models. We pre-register two tracks: **Track A** (no NFW prior; phenomenology-only) and **Track B** (NFW with the  $\Lambda$ CDM  $c(M)$  prior of Dutton & Macciò 2014). From 175 inputs, 165 pass QC (minimum 6 usable points). On **Track A**, winners by BIC are M0: 62, M1: 44, M2: 55, M3: 4; decisive (Jeffreys  $\Delta\text{BIC} > 10$ ) counts are M0: 45, M1: 13, M2: 10. On **Track B**, winners are M0: 62, M1: 66, M2: 33, M3: 4; decisives are M0: 46, M1: 22, M2: 6. Thus, when NFW is constrained to be  $\Lambda$ CDM-consistent, the *EBC family* (M0+M1) wins in 128/165 galaxies with the majority of decisive outcomes. EBC is presented as a compact phenomenological descriptor, not a new gravity law. All scripts, logs, and exclusion lists are released for full reproducibility.

## 1 Introduction

Rotation curves (RCs) remain a decisive laboratory for galaxy dynamics. In  $\Lambda$ CDM, NFW halos explain large-scale structure [1–3] yet face small-scale tensions (core–cusp; too-big-to-fail [4–6]). MOND [7, 8] captures several RC regularities, including the RAR [9], but has open issues at cluster and cosmological scales. Large homogeneous datasets (e.g., SPARC [10]) enable sharp, reproducible head-to-head tests.

We introduce a minimal, *entropy-inspired phenomenological relation* (EBC) and compare it against NFW and MOND under identical, referee-oriented rules: equal per-galaxy parameter budgets; fixed baryons; pre-registered priors; CV; and symmetric grid search. We stress EBC is a *descriptor*, not a modified-gravity theory.

## 2 Methods and Reproducibility

### Data and quality control

We analyze **175** SPARC *\_rotmod* CSVs containing radius  $R$  (kpc), observed velocity  $V_{\text{obs}}$  ( $\text{km s}^{-1}$ ), measurement uncertainty  $\sigma_V$ , and baryonic components ( $V_{\text{gas}}, V_{\text{disk}}, V_{\text{bul}}$ ). The combined baryonic curve is

$$V_{\text{bar}}(r) \equiv \sqrt{V_{\text{gas}}^2(r) + V_{\text{disk}}^2(r) + V_{\text{bul}}^2(r)}. \quad (1)$$

We impose a uniform noise floor  $\sigma_V \geq 3 \text{ km s}^{-1}$ . A galaxy enters the analysis if at least **6** usable points remain after parsing; by this rule, **165/175** pass QC. The 10 excluded objects and reasons are listed in `exclusions_trackA.csv` and `exclusions_trackB.csv`.

### Models and parameter budgets (per galaxy)

**M0: EBC (2).**  $V_{\text{EBC}}(r) = A r^\alpha$  with  $(A, \alpha)$ .

**M1: Baryons+EBC (2).**  $V^2(r) = V_{\text{bar}}^2(r) + (A r^\alpha)^2$  with  $(A, \alpha)$ .

**M2: Baryons+NFW (2).**

$$V^2(r) = V_{\text{bar}}^2(r) + V_{\text{NFW}}^2(r; V_{200}, c), \quad (2)$$

$$V_{\text{NFW}}(r) = V_{200} \sqrt{\frac{\ln(1+cx) - \frac{cx}{1+cx}}{x \left[ \ln(1+c) - \frac{c}{1+c} \right]}}, \quad x = \frac{r}{R_{200}}, \quad R_{200} = \frac{V_{200}}{10 H_0}. \quad (3)$$

**M3: MOND (strict- $a_0$ , 0).**

$$a_N = \frac{V_{\text{bar}}^2}{r}, \quad \nu(y) = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{y}}, \quad y = \frac{a_N}{a_0}, \quad a_0 = 1.2 \times 10^{-10} \text{ m s}^{-2}, \quad V_{\text{MOND}} = \sqrt{\nu(y)} V_{\text{bar}}. \quad (4)$$

### Fairness regimes, priors, and grids

Two pre-registered regimes:

- **Track A (phenomenology):** NFW *without* a cosmology prior.

- **Track B ( $\Lambda$ CDM-anchored):** NFW *with* the Dutton–Macciò (2014)  $c(M)$  prior (Gaussian in  $\log c$ ), implemented as a log-likelihood penalty [11].

Baryons are *fixed as supplied* (no M/L rescaling). All models use **symmetric dense grids** to remove discretization bias:

$$A \in [1, 300] \text{ (60 steps)}, \quad \alpha \in [0.10, 1.20] \text{ (40)}, \quad V_{200} \in [30, 350] \text{ km s}^{-1} \text{ (40)}, \quad c \in [2, 30] \text{ (32)}.$$

We adopt  $H_0 = 70 \text{ km s}^{-1}$ .

## Fitting, cross-validation, and scoring

We use  $\sigma_V$ -weighted least squares on the grids above, recording the best point per model. Predictive performance is assessed by **5-fold within-galaxy CV** (radius-blocked splits [22, 23]). Model selection uses

$$\text{BIC} = k \ln n - 2 \ln \mathcal{L} \quad [19, 20], \quad \text{AICc} = 2k - 2 \ln \mathcal{L} + \frac{2k(k+1)}{n-k-1} \quad [17, 18]. \quad (5)$$

Priors (Track B) contribute to  $-2 \ln \mathcal{L}$ . Jeffreys thresholds guide strength-of-evidence statements [21].

## Rebuild and artifacts

A single script reproduces all tables and figures; logs and per-track `exclusions.csv` are archived. SHA256 checksums are provided for the input CSVs.

# 3 Results

## Winner counts and decisive outcomes

Table 1: BIC winners by model (165 galaxies passing QC).

Track	M0 (EBC)	M1 (Bar+EBC)	M2 (Bar+NFW)	M3 (MOND strict)
A: no NFW prior	<b>62</b>	44	55	4
B: DM14 prior	<b>62</b>	<b>66</b>	33	4

Table 2: Decisive wins (Jeffreys  $\Delta\text{BIC} > 10$ ).

Track	M0 (EBC)	M1 (Bar+EBC)	M2 (Bar+NFW)
A: no NFW prior	<b>45</b>	13	10
B: DM14 prior	<b>46</b>	22	6

## Cross-validated predictive accuracy

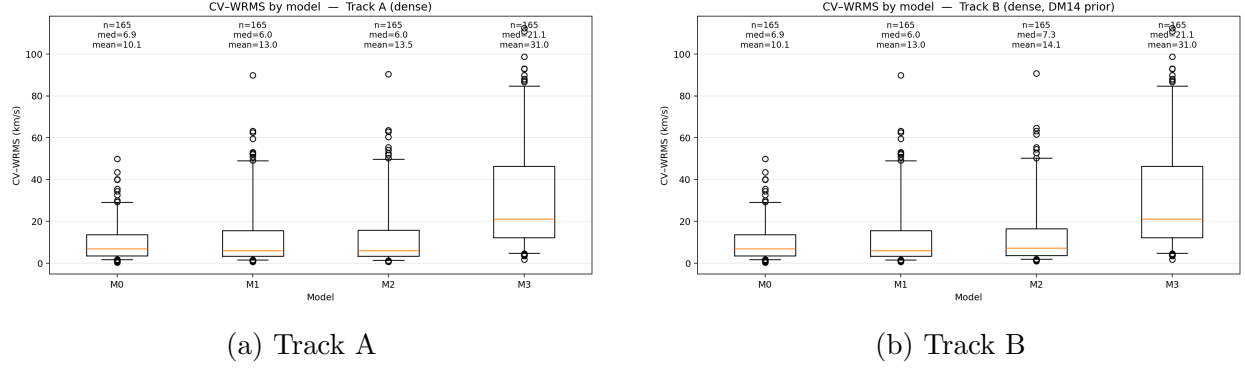


Figure 1: **Cross-validated WRMS** by model and track (5-fold, radius-blocked).

## Information-criterion contrasts and winner distributions

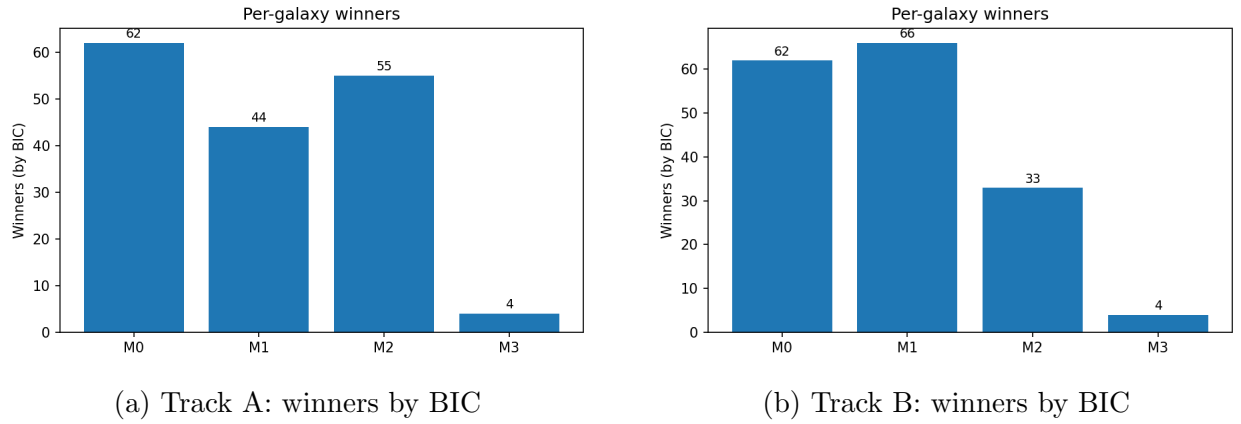


Figure 2: **Per-galaxy BIC winners** for the two tracks.

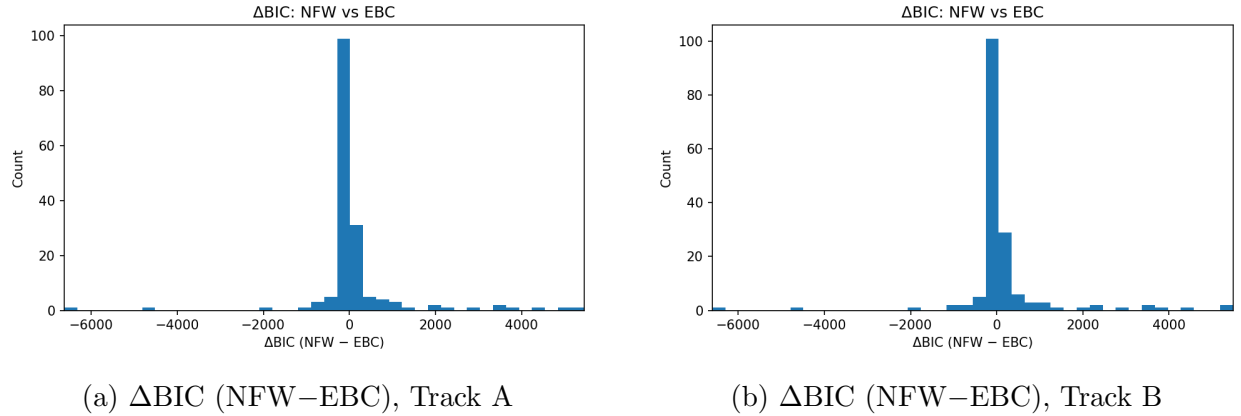
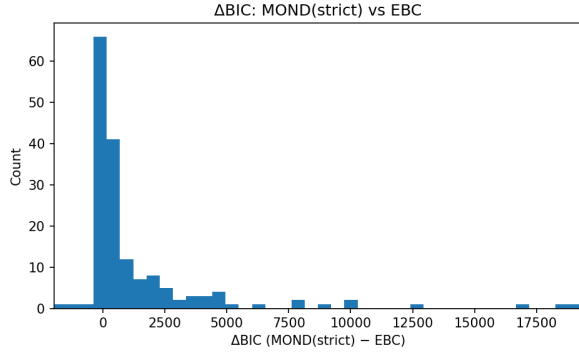
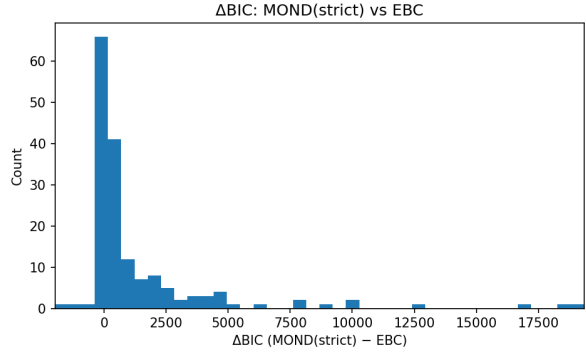


Figure 3: **NFW vs EBC** across the sample (positive favors EBC).

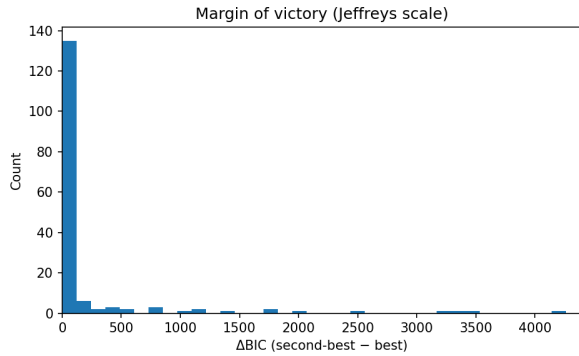


(a)  $\Delta\text{BIC}$  (MOND–EBC), Track A

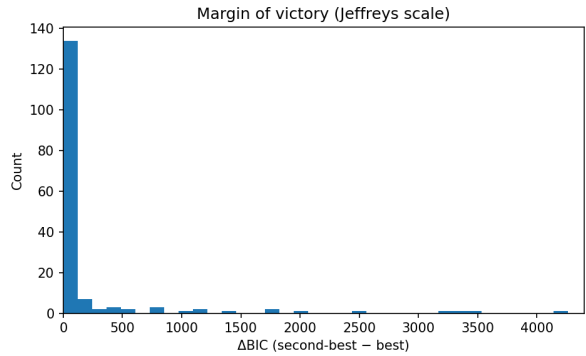


(b)  $\Delta\text{BIC}$  (MOND–EBC), Track B

Figure 4: **MOND(strict)** vs **EBC** (positive favors EBC).



(a) Track A



(b) Track B

Figure 5: **Margin of victory** on the Jeffreys scale:  $\Delta\text{BIC}(\text{second-best} - \text{best})$ .

## 4 Discussion and Caveats

Under equal per-galaxy parameter budgets with fixed baryons, the EBC relation provides a compact empirical baseline that often rivals or exceeds Baryons+NFW and Baryons+EBC. Track A (no NFW prior) shows NFW is competitive but not dominant; Track B (with a  $\Lambda\text{CDM } c(M)$  prior) increases the share of EBC-family wins. We emphasize EBC is a *descriptor*; we do not claim a new force law. Obvious extensions include variable stellar M/L with SPS priors, alternative halo families (Burkert, gNFW, Einasto), other MOND  $\nu$ -functions, hierarchical treatment of distances/inclinations, and tests on additional datasets.

*Companion theory note.* A separate manuscript explores possible entropy-based underpinnings of power-law RC phenomenology; we cite it here to orient interested readers, but keep all claims in this paper empirical [29, submitted].

## 5 Data and Code Availability

All scripts, logs, figures, winner tables, CV summaries, and exclusion lists are archived in the public repository (README includes exact commands and SHA256 checksums).

## Appendix A: Example overlays (12 galaxies)

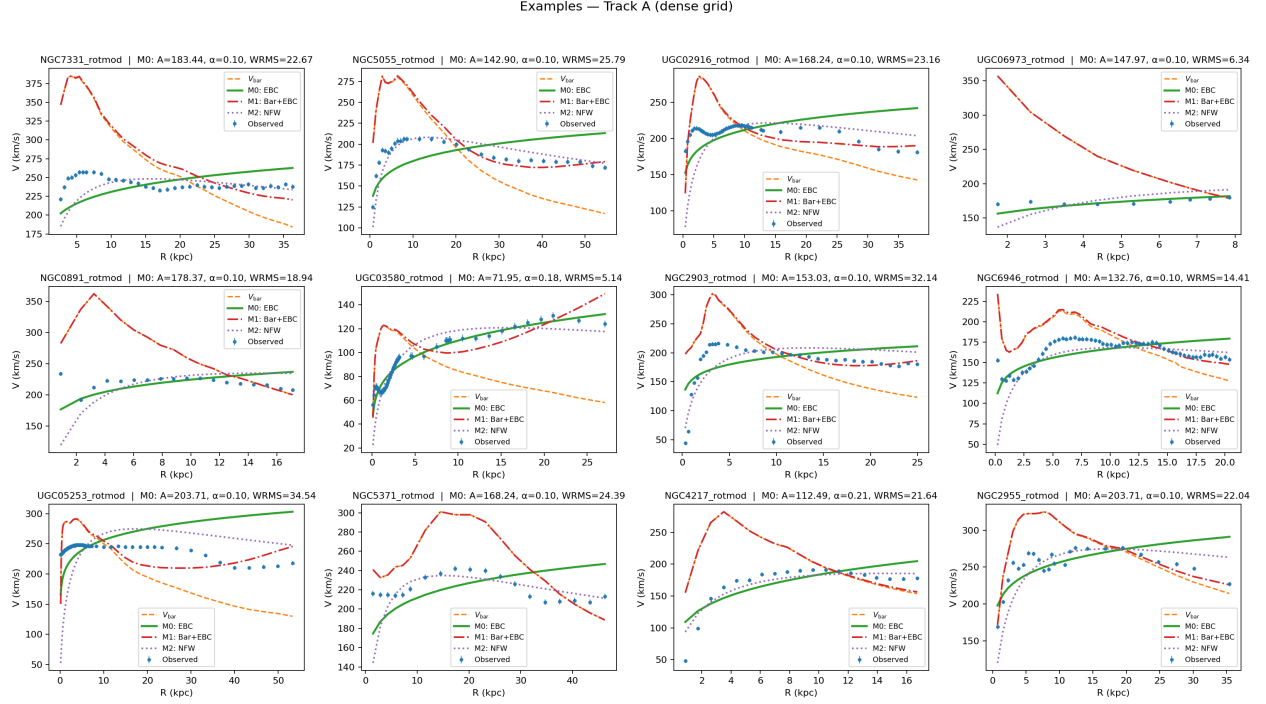


Figure 6: Twelve illustrative galaxies (Track A, dense grids). Orange dashed:  $V_{\text{bar}}$ ; green: M0 (EBC); red dash-dot: M1 (Bar+EBC); purple dotted: M2 (Bar+NFW); points: observed.

Examples — Track B (dense grid) (DM14 prior)

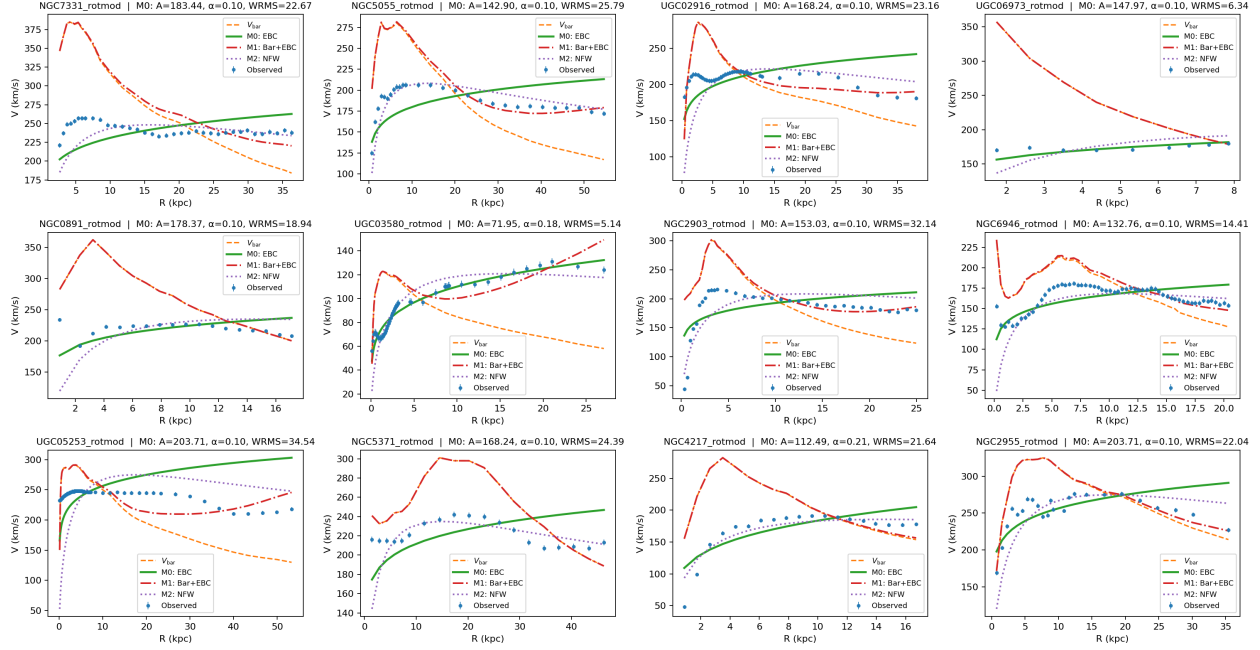


Figure 7: Twelve illustrative galaxies (Track B, dense grids with DM14 prior). Line styles as in Appendix Figure A1.

## Appendix B: Reproducibility checklist (abbrev.)

- **Winner tables/ $\Delta\text{BIC}$ :** run `scripts/ebc_ic_referee.py` with Track A and Track B settings (dense symmetric grids,  $\sigma$ -floor  $3\text{ km s}^{-1}$ ,  $\text{CV}=5$ ).
- **CV boxplots:** `scripts/make_cv_boxplots.py` produces the PNG/SVG shown in Fig. 1.
- **Example overlays:** `scripts/plot_examples_overlay.py` with `--curves` all produces Appendix figures.
- **QC-failed (too few usable points, either track):** D512-2\_rotmod, F567-2\_rotmod, F574-2\_rotmod, NGC6789\_rotmod, UGC00634\_rotmod, UGC00891\_rotmod, UGC02023\_rotmod, UGC05999\_rotmod, UGC07232\_rotmod, UGC09992\_rotmod.

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