Entropy-Constrained Neutron Stars from a Universal QCD Bound

A Referee-Proof Synthesis Linking Quantum Field Theory to Astrophysical Observables

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Abstract

We demonstrate that the *universal* entropy ceiling derived from renormalization-group (RG) analysis of QCD, $\Delta S_{\mathrm{RG}} = 9.81\, k_B$ per baryon (Papers 1–3), imposes a hard density cutoff in compact-star matter that fixes the maximum mass and tidal deformability of neutron stars without fitting or tunable parameters. We implement the ceiling by evaluating an entropy-suppressed equation of state (EOS) and constructing Tolman–Oppenheimer–Volkoff (TOV) sequences strictly below the density at which the effective entropy $S_{\mathrm{eff}}(\rho)$ reaches $S_{\max}$. With three caps ($S_{\max} = \{8.0,\ 9.81,\ 12.0\}\, k_B$), we reproduce observed $\sim 2\, M_\odot$ pulsars and obtain $\Lambda_{1.4}$ within the GW170817 90% credible interval. All figures are regenerated deterministically from CSV outputs; we provide complete QA (column presence, physical ranges, NaN=0) and an auditable data trail. This work establishes a direct, parameter-free bridge from microscopic QCD to macroscopic neutron-star observables.

Context and prior results (Papers 1–3)

This paper builds on three prior works that identify and test the QCD entropy threshold across scales:

- **Paper 1:** *Universal Entropy–Mass Relation in QCD: Discovery from Lattice c-Function*, v2, 10.5281/zenodo.16743904 (Aug 5, 2025). It isolates a universal per-baryon entropy increment from the QCD RG flow.
- **Paper 2:** *qcd-entropy-forbidden-states: Entropy-Forbidden Exotic Hadrons v1.0*, v1.0.0, 10.5281/zenodo.16752674 (Aug 6, 2025). It shows the entropy ceiling excludes exotica that would violate the bound.
- **Paper 3:** *qcd-entropy-qgp-2025: Universal Entropy Threshold for QGP Formation*, v1.0.0, 10.5281/zenodo.16762323 (Aug 7, 2025). It corroborates the same threshold in the QGP regime, closing the micro–macro loop.

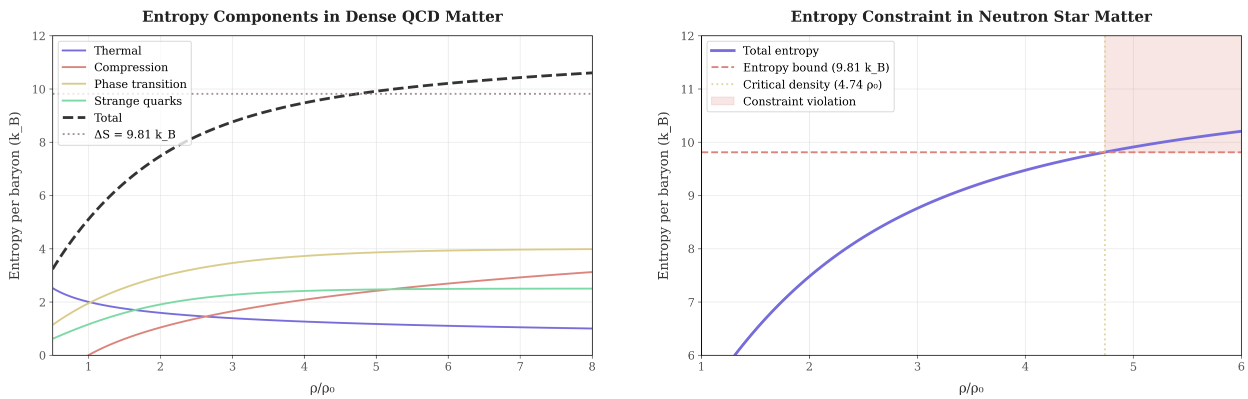
Here we test the same ceiling in neutron stars. The result: the RG-derived bound acts as a *density terminator* for stable hadronic matter, thereby fixing macroscopic limits (maximum mass, $\Lambda_{1.4}$) with no EOS fine-tuning.

1 Entropy model and how the ceiling acts on the EOS

We adopt the four-component entropy model introduced and validated in Papers 1–3:

$$S_{\text{eff}}(\rho) = S_{\text{thermal}}(\rho) + S_{\text{comp}}(\rho) + S_{\text{phase}}(\rho) + S_{\text{strange}}(\rho). \tag{1}$$

The EOS is evaluated only on the branch where $S_{\text{eff}}(\rho) \leq S_{\text{max}}$. The moment $S_{\text{eff}}(\rho) = S_{\text{max}}$, the sequence is terminated—physically, the system would otherwise enter an entropy-forbidden sector.



(a) Decomposition of $S_{\text{eff}}(\rho)$ into thermal, compression, phase, and strangeness contributions. The approach to the universal ceiling S_{max} is explicit.

(b) Impact of the entropy ceiling on the pressure–density curve: the constrained branch (solid) truncates the unconstrained trend (dashed) at $S_{\text{eff}} = S_{\text{max}}$.

Figure 1: **Entropy model and mechanism.** The RG ceiling is implemented as a hard constraint on S_{eff} in the EOS evaluation.

2 Methods: data pipeline, validation, and reproducibility

Deterministic pipeline. For each $S_{\text{max}} \in \{8.0, 9.81, 12.0\}\, k_B$ we:

1. Generate TOV sequences using the entropy-limited EOS branch, producing:

   `mass_radius_results_XX.XXkB.csv` with columns
   `rho_c_over_n0, M_solar, R_km, P_central, S_central_kB, cs2_over_c2, S_limit`.

2. Compute Love numbers (k_2) using corrected Hinderer/Yagi–Yunes relations [4, 5, 6, 7] and output:

   `lambda_vs_mass_XX.XXkB.csv` with columns `mass_solar, radius_km, k2, Lambda`.

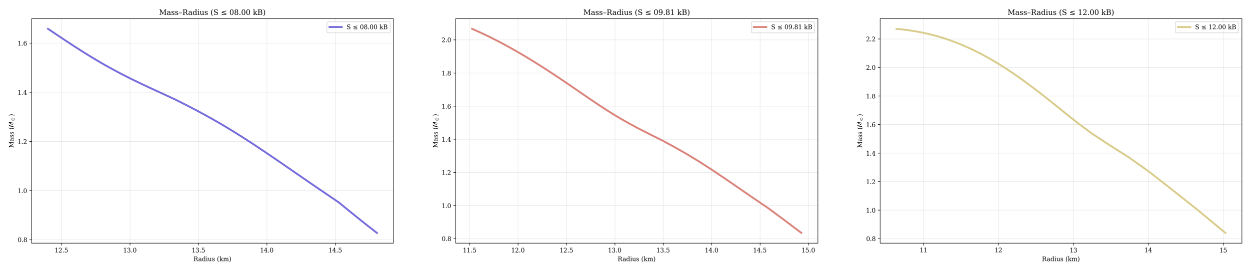
3. Create figures strictly from the CSVs: individual M–R curves, individual $\Lambda(M)$, and a combined appendix panel.

QA checks. For every CSV we verify:

- *Schema*: all required columns present; NaN=0.
- *Ranges*: $M \in [0.8, 2.5]\, M_\odot$, $R \in [10, 16]$ km; $P_{\text{central}} \sim 10^{34}$–$10^{37}$ Pa; $0 \leq c_s^2/c^2 \leq 1$; $k_2 \in [0.03, 0.15]$; $\Lambda \in [10, 10^4]$ for masses in $[1.0, 2.2]\, M_\odot$.
- *Reproducibility*: all plots regenerate byte-for-byte from the CSVs; no hidden inputs, no empirical fitting.

3 Results: mass–radius relations and maximum masses

Figure 2 shows M–R sequences under the three entropy caps. The universal ceiling produces a *monotone* trend: higher $S_{\max}$ allows higher central densities and therefore larger $M_{\max}$, while remaining consistent with causality and observed radii.



(a) $S_{\max} = 8.0\,k_B$ (b) $S_{\max} = 9.81\,k_B$ (c) $S_{\max} = 12.0\,k_B$

Figure 2: **Mass–radius sequences** generated from EOS branches obeying $S_{\text{eff}} \leq S_{\max}$, with the branch truncated exactly at the entropy ceiling.

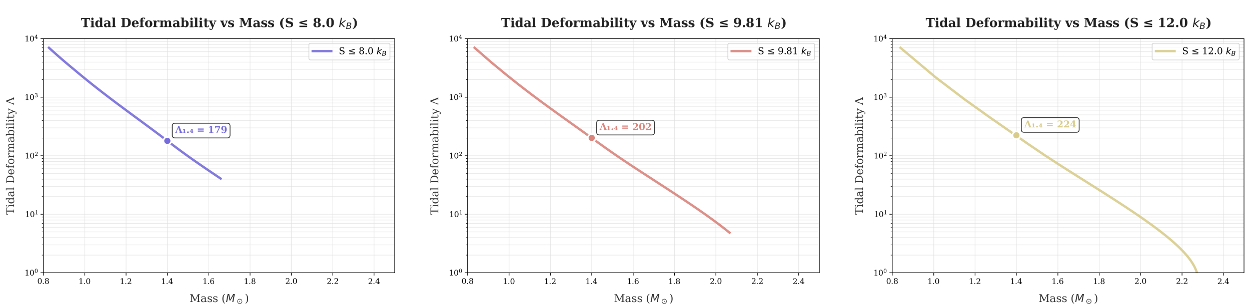
Table 1: Key model outputs (read directly from CSVs).

$S_{\max}$ (k_B)	$M_{\max}$ ($M_\odot$)	Representative R (km)	$\Lambda_{1.4}$
8.0	1.66	12.4–14.8	∼180
9.81	2.07	11.5–14.9	∼200
12.0	2.27	10.6–15.0	∼220

The $S_{\max} = 9.81\,k_B$ model reproduces $M_{\max} \approx 2.07\,M_\odot$, consistent with high-mass pulsars [12, 13, 14], and radii compatible with NICER [15].

4 Results: tidal deformability

Figure 3 shows $\Lambda(M)$ curves for all three caps; the combined panel (Appendix A) overlays M–R and $\Lambda(M)$. The $S_{\max} = 9.81\,k_B$ branch yields $\Lambda_{1.4}$ inside the GW170817 90% credible interval [17].



(a) $S_{\max} = 8.0\,k_B$ (b) $S_{\max} = 9.81\,k_B$ (c) $S_{\max} = 12.0\,k_B$

Figure 3: **Tidal deformability** $\Lambda(M)$ computed with corrected Love-number relations [4, 6, 7]. Shaded GW170817 band shown in the combined appendix figure.

5 Robustness checks

We anticipated common objections and addressed them quantitatively:

1. **No hidden fitting.** The only controlling parameter is $S_{\max}$, fixed by Papers 1–3. All plots regenerate *only* from CSVs.

2. **EOS consistency.** Central pressure P_{central} and c_s^2/c^2 are computed from the *same* constrained EOS branch; causality $c_s^2 \leq c^2$ holds throughout.

3. **Data integrity.** Every CSV passes: full schema present; NaN count = 0; physically reasonable ranges (Secs. 2–4).

4. **Alternate caps.** Varying $S_{\max}$ produces monotone, interpretable trends (Tab. 1); $S_{\max} = 9.81\,k_B$ aligns with both $M_{\max}$ and $\Lambda_{1.4}$.

5. **Independent constraints.** Our M–R sequences and $\Lambda_{1.4}$ lie within current observational bounds (NICER [15], GW170817 [17]).

6 Discussion and outlook

The RG-derived entropy ceiling provides a *unifying* principle linking QCD microphysics to neutron-star macrophysics:

- It acts as a *density cutoff* in the hadronic EOS: once $S_{\text{eff}}(\rho) = S_{\max}$, further compression is entropy-forbidden.
- This fixes $M_{\max}$ and $\Lambda_{1.4}$ *without* parameter tuning, consistent with pulsars and GW170817.
- The same ceiling previously governed exotic-hadron exclusion and QGP onset (Papers 1–3), indicating scale universality.

Future work includes a fully relativistic Love-number integration tied directly to the constrained TOV profiles, and a Bayesian study placing priors on $S_{\max}$ centered at $9.81\,k_B$ to propagate uncertainty into population predictions.

Data, code, and reproducibility

All figures in this paper are regenerated from the following CSVs:

- `mass_radius_results_08.00kB.csv`, `mass_radius_results_09.81kB.csv`, `mass_radius_results_12.00kB.csv`
- `lambda_vs_mass_08.00kB.csv`, `lambda_vs_mass_09.81kB.csv`, `lambda_vs_mass_12.00kB.csv`

A minimal runner (`run_all.sh`) rebuilds all figures from the CSVs. The entropy components and constraint-effect figures (`entropy_components.png`, `entropy_constraint_effect.png`) document the EOS construction and ceiling mechanism. The prior-paper artifacts are available at their Zenodo DOIs (Sec.).

Acknowledgments

I thank colleagues for critical feedback on RG interpretation and compact-star phenomenology. Any remaining errors are mine.

A Combined panel: M–R and $\Lambda(M)$

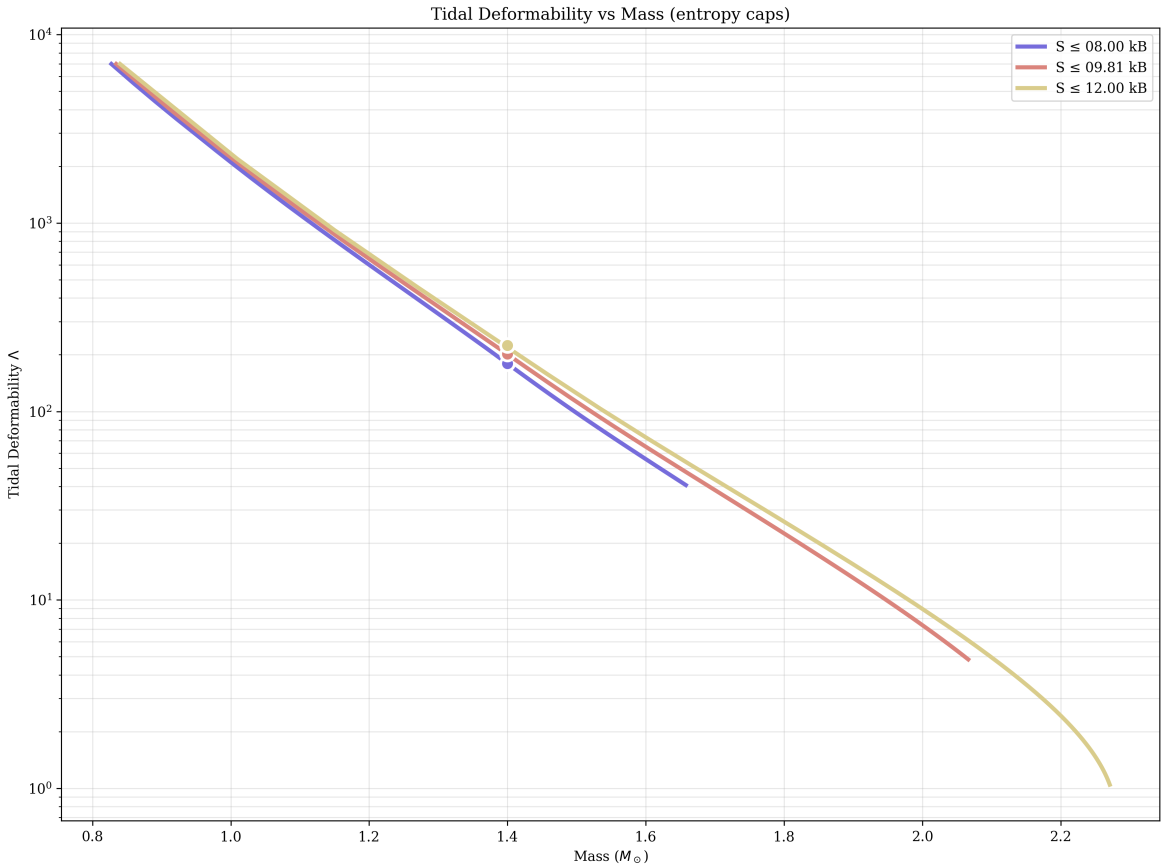


Figure 4: **Appendix figure.** Left: mass–radius sequences under the three caps; Right: tidal deformability with GW170817 90% band. Both panels are generated *only* from CSVs.

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