SpaceAndTime

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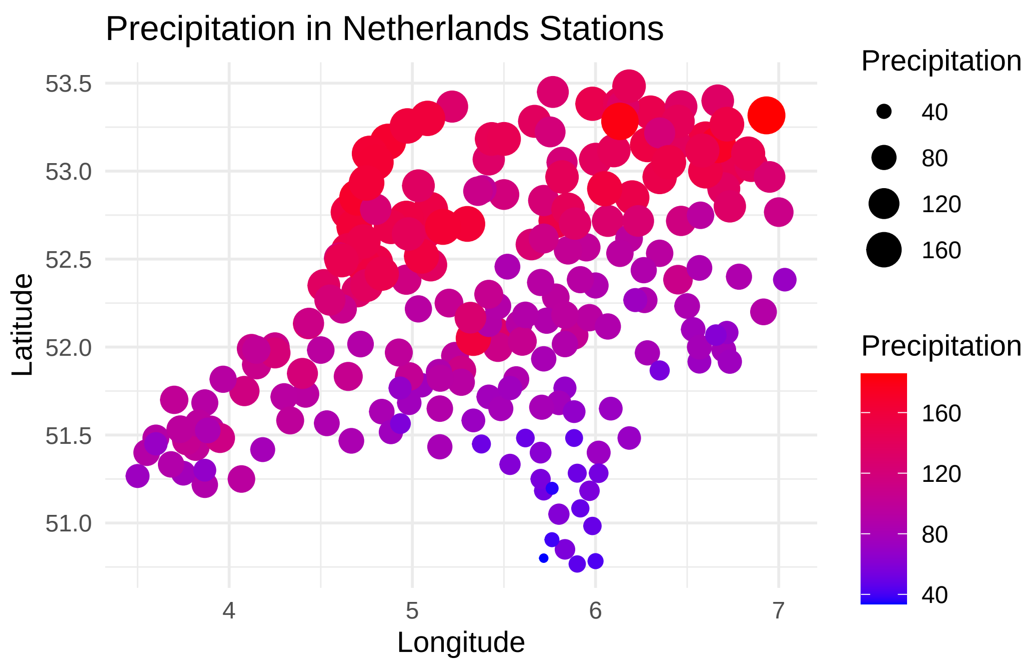
TODO check if I checked all the residuals for all models

Question 1 Spatial modelling Kingdom of the Netherlands

1 a)

```
ggplot(data = netherlandsDF) + geom_point(aes(x = longitude, y = latitude,
    size = precip, color = precip)) + scale_color_continuous(low = "blue",
    high = "red") + labs(title = "Precipitation in Netherlands Stations",
    x = "Longitude", y = "Latitude", size = "Precipitation", color = "Precipitation") +
    theme_minimal()
```



From what we can see from the data it does seem to be spatially correlated as we can that the Dutch provinces of north Holland, Friesland and Groningen has higher precipitation and as we go south the precipitation does decrease as we can see from the Dutch provinces of Zeeland, north Brabant and Limburg where precipitation is significantly lower than their northern counterparts.

From this data, latitude seems to be the biggest factor in the variation of the precipitation as the longitude only suggests some slight variations in the data.

```
# Create geodata object
precipitationNetherland_geoR = as.geodata(netherlandsDF, coords.col = c("longitude",
    "latitude"), data.col = "precip")

summary(precipitationNetherland_geoR)
```

```
Number of data points: 220

Coordinates summary
    longitude latitude
min     3.500   50.767
max     7.033   53.483
```

```
Distance summary
     min      max
0.001000 3.998498

Data summary
    Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
 33.9000  80.8500 100.1500 106.5705 137.3500 185.6000
```

As we can see from the numerical summary of the data the median is different from the mean, which indicates it is not a symmetric distribution of data points and is instead positively skewed since the mean is bigger than the median. As such there are more values on the left side of the distribution.

1 b)

```r
# set seed for reproducibility
set.seed(26041999)

# Select 3 random rows from the data frame
randomRowsPrecipitation = netherlandsDF %>%
    sample_n(3)

# Add a new column with labels
randomRowsPrecipitation$label = c("A", "B", "C")

# Print the randomly selected rows
randomRowsPrecipitation
```

```
# A tibble: 3 x 5
  station_name   longitude latitude precip label
  <chr>              <dbl>    <dbl>  <dbl> <chr>
1 NIJKERK             5.47     52.2   89.1 A
2 WOLPHAARTSDIJK      3.73     51.5   95.9 B
3 EEXT                6.73     53    147.  C
```

```r
# Remove the selected rows from the original dataset
netherlandsDF_filtered = netherlandsDF %>%
    anti_join(randomRowsPrecipitation)
```

```
Joining, by = c("station_name", "longitude", "latitude", "precip")
```

```
# Print the resulting dataframe
netherlandsDF_filtered
```

```
# A tibble: 217 x 4
   station_name      longitude latitude precip
   <chr>                 <dbl>    <dbl>  <dbl>
 1 WEST TERSCHELLING      5.22     53.4  130.
 2 GRONINGEN-1            6.6      53.2  157.
 3 HOORN                  5.07     52.6  146.
 4 HOOFDDORP              4.7      52.3  130.
 5 WINTERSWIJK            6.7      52.0   77.7
 6 KERKWERVE              3.87     51.7   91.8
 7 WESTDORPE-1            3.87     51.2   87.7
 8 OUDENBOSCH             4.53     51.6   84.2
 9 ROERMOND               5.97     51.2   56.4
10 PETTEN                 4.65     52.8  158.
# ... with 207 more rows
```

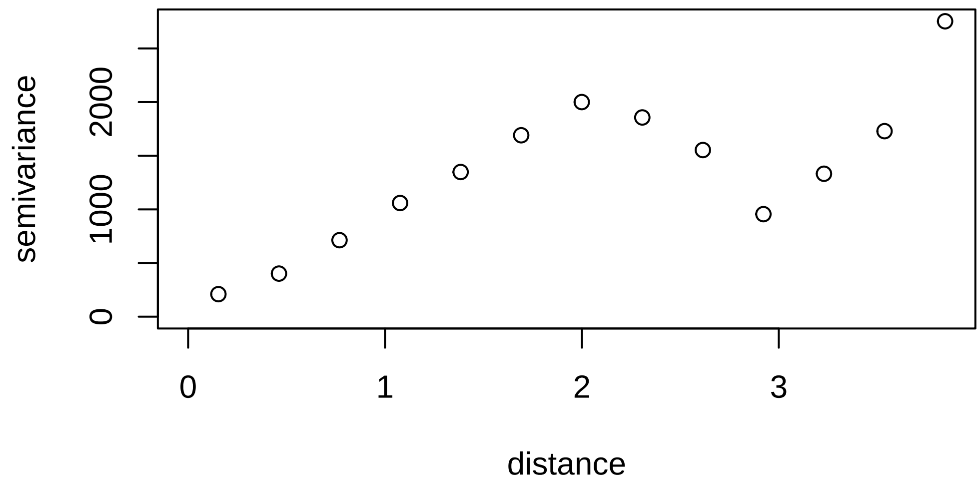
```
## recreate the geoData object with the new filtered dataframe
precipitationNetherland_geoR = as.geodata(netherlandsDF_filtered, coords.col = c("longitud
    "latitude"), data.col = "precip")
```

1 c)

```
# # Calculate empirical variogram
variogramPrecipitationNetherlands = variog(precipitationNetherland_geoR)
```

```
variog: computing omnidirectional variogram
```

```
# Plot empirical variogram
plot(variogramPrecipitationNetherlands)
```




```
variogramPrecipitationNetherlands$n
```

```
 [1] 1069 2482 3384 3834 3647 3221 2534 1541  787  468  318  133   17
```

From the plotted variogram we can see there a very clear need for a nugget as there is a non-zero value around zero distance, this values seems to be around 75 to 100 at the zero distance from how much is it decreasing.

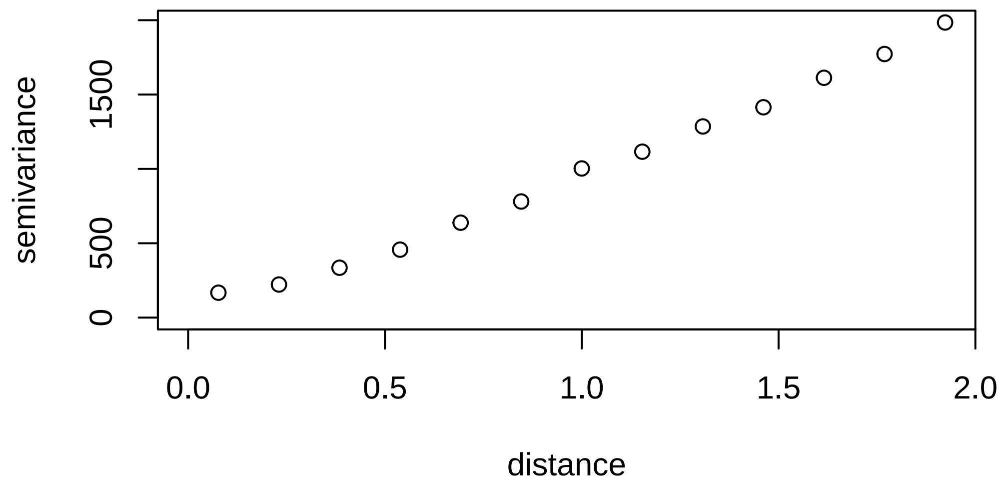
The semi variance continuous to increase with distance till around the distance of 2 degrees distance wise, after this there is a decrease in variance that is not representative of the data as we are more and more uncertain the further we are from our known points, as such we will choose the distance of two as the cut off for the maximum distance.

we know change the maximum distance change and recut our previous variogram.

```
variogramPrecipitationNetherlands = variog(precipitationNetherland_geoR,
    option = "bin", max.dist = 2)
```

```
variog: computing omnidirectional variogram
```

```r
plot(variogramPrecipitationNetherlands)
```



As we can see from the newly updated variogram the increase is almost linear with a curve near 0 where we can see the need for the nugget.

1 d)

Now that we have the variogram we will start by fitting a model to estimate the covariance via weighted least squares. Fitting this variogram we get the estimated values of σ^2, ϕ and τ^2 also known as the nugget

We will first start with the default Matrén = 0,5 which is equivalent to an exponential as the form observed in the previous variogram seems to not fully linear and therefore require the curvature from a function like the exponential function to account for the behaviour at the near 0 distance.

From this we will try different models to search for the model with the best fit.

```r
# ?variofit thau = nugget variability sigmasq = if the model can
# capture more or less of the total variability phi = if the
# correlation extends over a bigger or smaller distance loss value =
```

```r
# goodness of fit (smaller means better fit)

krigingVariogramFittedDefault = variofit(variogramPrecipitationNetherlands,
    nugget = 85)
```

```
variofit: covariance model used is matern
variofit: weights used: npairs
variofit: minimisation function used: optim
```

```
Warning in variofit(variogramPrecipitationNetherlands, nugget = 85): initial
values not provided - running the default search
```

```
variofit: searching for best initial value ... selected values:
              sigmasq   phi    tausq kappa
initial.value "1984.67" "1.54" "85"  "0.5"
status        "est"     "est"  "est" "fix"
loss value: 818327620.928191
```

```r
krigingVariogramFittedDefault
```

```
variofit: model parameters estimated by WLS (weighted least squares):
covariance model is: matern with fixed kappa = 0.5 (exponential)
parameter estimates:
     tausq    sigmasq        phi
      0.00 2561207.09    2590.45
Practical Range with cor=0.05 for asymptotic range: 7760.294

variofit: minimised weighted sum of squares = 35360382
```

Now we will increase the kappa of the Matrén to see if the increased flexibility and smoothness leads to a better fit

```r
krigingVariogramFittedMatrén1.5 = variofit(variogramPrecipitationNetherlands,
    kappa = 1.5, nugget = 85)
```

```
variofit: covariance model used is matern
variofit: weights used: npairs
variofit: minimisation function used: optim
```

```
Warning in variofit(variogramPrecipitationNetherlands, kappa = 1.5, nugget =
85): initial values not provided - running the default search
```

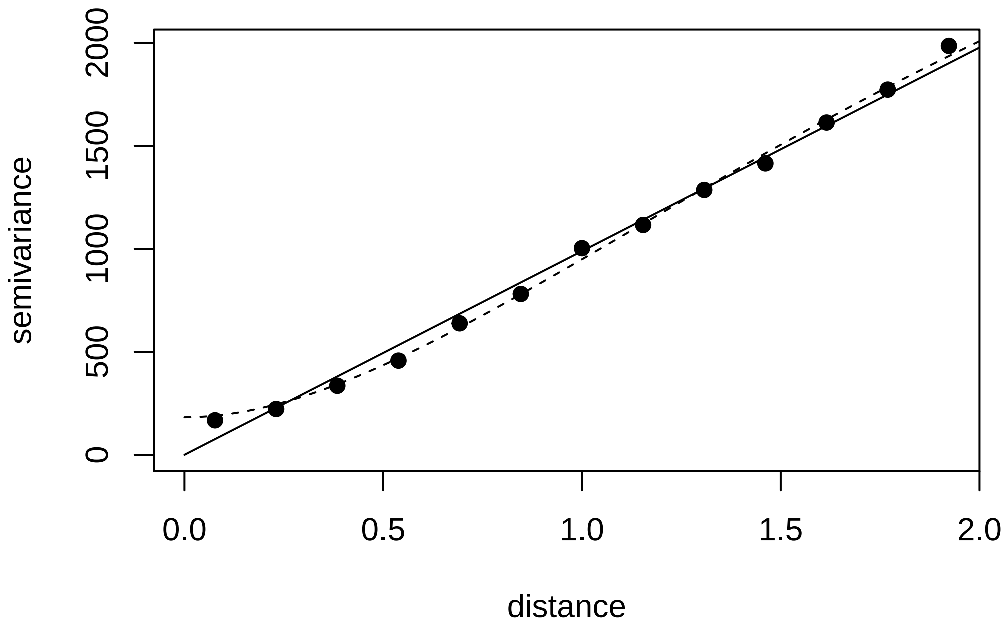
```
variofit: searching for best initial value ... selected values:
              sigmasq   phi    tausq kappa
initial.value "1984.67" "0.62" "85"  "1.5"
status        "est"     "est"  "est" "fix"
loss value: 190482114.063677
```

```
krigingVariogramFittedMatrén1.5
```

```
variofit: model parameters estimated by WLS (weighted least squares):
covariance model is: matern with fixed kappa = 1.5
parameter estimates:
    tausq   sigmasq       phi
 182.0638 3461.1492    1.1316
Practical Range with cor=0.05 for asymptotic range: 5.368367

variofit: minimised weighted sum of squares = 15435206
```

We will first visually compare these 2 models to see which one has a better

```
par(mar = c(4, 4, 2, 2))
plot(variogramPrecipitationNetherlands, pch = 19)
lines(krigingVariogramFittedDefault)
lines(krigingVariogramFittedMatrén1.5, lty = 2)
```




```
# lines(krigingVariogramFittedMatrén2.5, lty = 3)
```

Immediately we can see that the extra flexibility of the Matrén 1,5 not only better follows the actual data, it actually accounts correctly for the initial variance from the nugget which the Matrén 0,5 does not as it simply decreases to 0.

We now will test if any additional flexibility changes can improve the model fit

We will first try to again increase the kappa to see if the model again benefits from the extra flexibility

```
krigingVariogramFittedMatrén2.0 = variofit(variogramPrecipitationNetherlands,
    kappa = 2, nugget = 85)
```

```
variofit: covariance model used is matern
variofit: weights used: npairs
variofit: minimisation function used: optim
```

```
Warning in variofit(variogramPrecipitationNetherlands, kappa = 2, nugget = 85):
initial values not provided - running the default search
```

```
variofit: searching for best initial value ... selected values:
              sigmasq   phi    tausq    kappa
initial.value "1984.67" "0.62" "198.47" "2"
status        "est"     "est"  "est"    "fix"
loss value: 227233080.389154
```

```r
krigingVariogramFittedMatrén2.0
```

```
variofit: model parameters estimated by WLS (weighted least squares):
covariance model is: matern with fixed kappa = 2
parameter estimates:
    tausq   sigmasq       phi
 197.6127 3037.3813    0.8386
Practical Range with cor=0.05 for asymptotic range: 4.502076

variofit: minimised weighted sum of squares = 18197964
```

Here we will instead see if the model will benefit instead form a cut of flexibility to make it less smooth

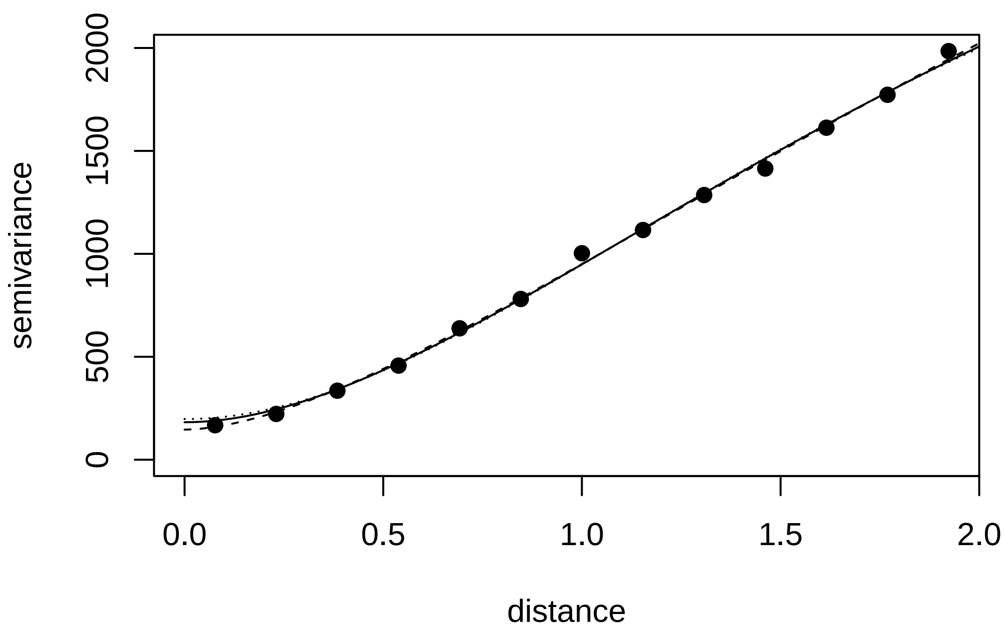
```r
krigingVariogramFittedMatrén1.0 = variofit(variogramPrecipitationNetherlands,
    kappa = 1, nugget = 85)
```

```
variofit: covariance model used is matern
variofit: weights used: npairs
variofit: minimisation function used: optim

Warning in variofit(variogramPrecipitationNetherlands, kappa = 1, nugget = 85):
initial values not provided - running the default search

variofit: searching for best initial value ... selected values:
              sigmasq   phi    tausq    kappa
initial.value "1984.67" "0.92" "198.47" "1"
status        "est"     "est"  "est"    "fix"
loss value: 352001017.756763
```

```r
krigingVariogramFittedMatrén1.0
```

```
variofit: model parameters estimated by WLS (weighted least squares):
covariance model is: matern with fixed kappa = 1
parameter estimates:
    tausq   sigmasq       phi
 146.1354 4832.1862    2.0470
Practical Range with cor=0.05 for asymptotic range: 8.185098

variofit: minimised weighted sum of squares = 12019899
```

```r
par(mar = c(4, 4, 2, 2))
plot(variogramPrecipitationNetherlands, pch = 19)
lines(krigingVariogramFittedMatrén1.5)
lines(krigingVariogramFittedMatrén1.0, lty = 2)
lines(krigingVariogramFittedMatrén2.0, lty = 3)
```



As we can see from the new graph it does seem that actually a lower flexibility Matrén has a better fit since the extra flexibility near the start and end of the data points made the models deviate too much from the points.

1 e)

To fit a model using the maximum likelihood we will have to try multiple initial values to make sure this is indeed the maximum likelihood and not just a local maximum.

```
# ?likest

maximumLikelihoodNetherlandsInitial10.1 = likfit(precipitationNetherland_geoR,
    ini.cov.pars = c(10, 1))
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```
maximumLikelihoodNetherlandsInitial1.10 = likfit(precipitationNetherland_geoR,
    ini.cov.pars = c(1, 10))
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```
maximumLikelihoodNetherlandsInitial100.10 = likfit(precipitationNetherland_geoR,
    ini.cov.pars = c(100, 10))
```

```
---------------------------------------------------------------
```

```
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```r
maximumLikelihoodNetherlandsInitial10.100 = likfit(precipitationNetherland_geoR,
    ini.cov.pars = c(10, 100))
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.

WARNING: estimated range is more than 10 times bigger than the biggest distance between two p
 1) excluding spatial dependence if estimated sill is too low and/or
 2) taking trends (covariates) into account
```

```r
maximumLikelihoodNetherlandsInitial1.1 = likfit(precipitationNetherland_geoR,
    ini.cov.pars = c(1, 1))
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```r
maximumLikelihoodNetherlandsInitial1000.1000 = likfit(precipitationNetherland_geoR,
    ini.cov.pars = c(1000, 1000))
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.

WARNING: estimated range is more than 10 times bigger than the biggest distance between two p
 1) excluding spatial dependence if estimated sill is too low and/or
 2) taking trends (covariates) into account
```

```r
maximumLikelihoodNetherlandsInitial500.500 = likfit(precipitationNetherland_geoR,
    ini.cov.pars = c(500, 500))
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.

WARNING: estimated range is more than 10 times bigger than the biggest distance between two p
 1) excluding spatial dependence if estimated sill is too low and/or
 2) taking trends (covariates) into account
```

```r
maximumLikelihoodNetherlandsInitial10.1
```

```
likfit: estimated model parameters:
```

```
      beta      tausq    sigmasq        phi 
" 102.376" " 114.249" "3036.627" "   7.132" 
Practical Range with cor=0.05 for asymptotic range: 21.36583

likfit: maximised log-likelihood = -896.9
```

```r
maximumLikelihoodNetherlandsInitial1.10
```

```
likfit: estimated model parameters:
      beta      tausq    sigmasq        phi 
" 102.449" " 114.921" "4184.402" "   9.991" 
Practical Range with cor=0.05 for asymptotic range: 29.9294

likfit: maximised log-likelihood = -896.9
```

```r
maximumLikelihoodNetherlandsInitial100.10
```

```
likfit: estimated model parameters:
      beta      tausq    sigmasq        phi 
" 102.449" " 114.921" "4184.402" "   9.991" 
Practical Range with cor=0.05 for asymptotic range: 29.9294

likfit: maximised log-likelihood = -896.9
```

```r
maximumLikelihoodNetherlandsInitial10.100
```

```
likfit: estimated model parameters:
     beta     tausq   sigmasq       phi 
"  102.7" "  117.7" "39625.5" "  100.0" 
Practical Range with cor=0.05 for asymptotic range: 299.5729

likfit: maximised log-likelihood = -897.8
```

```r
maximumLikelihoodNetherlandsInitial1.1
```

```
likfit: estimated model parameters:
      beta      tausq    sigmasq        phi 
```

```
" 102.376" " 114.249" "3036.627" "   7.132"
Practical Range with cor=0.05 for asymptotic range: 21.36583

likfit: maximised log-likelihood = -896.9
```

```
maximumLikelihoodNetherlandsInitial1000.1000
```

```
likfit: estimated model parameters:
      beta      tausq    sigmasq        phi
"   103.2" "   148.3" "246755.1" "  1000.0"
Practical Range with cor=0.05 for asymptotic range: 2995.732

likfit: maximised log-likelihood = -900.9
```

```
maximumLikelihoodNetherlandsInitial500.500
```

```
likfit: estimated model parameters:
      beta      tausq    sigmasq        phi
"   103.1" "   145.0" "130046.9" "   500.0"
Practical Range with cor=0.05 for asymptotic range: 1497.866

likfit: maximised log-likelihood = -900.1
```

As we can see from the maximised log-likelihoods it does seem that have indeed reached the maximum log-likelihood as none of the values are too fastly different and the likelihood worsedned as we started to increase much more our starting values.

Next we will try the REML that takes into account the fact that some of the parameters of the model are related to the variance of the residuals and not the mean.

```
REMLmaximumLikelihoodNetherlandsInitial10.1 = likfit(precipitationNetherland_geoR,
    ini.cov.pars = c(10, 1), lik.method = "REML")
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
```

```
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.

WARNING: estimated range is more than 10 times bigger than the biggest distance between two p
 1) excluding spatial dependence if estimated sill is too low and/or
 2) taking trends (covariates) into account
```

```r
REMLmaximumLikelihoodNetherlandsInitial1.10 = likfit(precipitationNetherland_geoR,
    ini.cov.pars = c(1, 10), lik.method = "REML")
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.

WARNING: estimated range is more than 10 times bigger than the biggest distance between two p
 1) excluding spatial dependence if estimated sill is too low and/or
 2) taking trends (covariates) into account
```

```r
REMLmaximumLikelihoodNetherlandsInitial100.1 = likfit(precipitationNetherland_geoR,
    ini.cov.pars = c(100, 1), lik.method = "REML")
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```
WARNING: estimated range is more than 10 times bigger than the biggest distance between two p
 1) excluding spatial dependence if estimated sill is too low and/or
 2) taking trends (covariates) into account
```

```r
REMLmaximumLikelihoodNetherlandsInitial1.100 = likfit(precipitationNetherland_geoR,
    ini.cov.pars = c(1, 100), lik.method = "REML")
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.

WARNING: estimated range is more than 10 times bigger than the biggest distance between two p
 1) excluding spatial dependence if estimated sill is too low and/or
 2) taking trends (covariates) into account
```

```r
REMLmaximumLikelihoodNetherlandsInitial10.1
```

```
likfit: estimated model parameters:
      beta      tausq    sigmasq        phi
"  102.61" "  114.60" "18253.12" "   43.35"
Practical Range with cor=0.05 for asymptotic range: 129.8698

likfit: maximised log-likelihood = -888.9
```

```r
REMLmaximumLikelihoodNetherlandsInitial1.10
```

```
likfit: estimated model parameters:
      beta      tausq    sigmasq        phi
"  102.62" "  114.83" "18124.93" "   43.19"
Practical Range with cor=0.05 for asymptotic range: 129.3933

likfit: maximised log-likelihood = -888.9
```

```r
REMLmaximumLikelihoodNetherlandsInitial100.1
```

```
likfit: estimated model parameters:
      beta      tausq    sigmasq        phi 
"  102.61" "  114.60" "18253.12" "   43.35" 
Practical Range with cor=0.05 for asymptotic range: 129.8698

likfit: maximised log-likelihood = -888.9
```

```r
REMLmaximumLikelihoodNetherlandsInitial1.100
```

```
likfit: estimated model parameters:
     beta     tausq   sigmasq       phi 
"  102.7" "  116.5" "40885.8" "  100.0" 
Practical Range with cor=0.05 for asymptotic range: 299.5729

likfit: maximised log-likelihood = -888.9
```

As we can see from these new models, the new likelihood method actually did improve our model as we have a lower log-likelihood.

Since the data did not seem to be perfectly stationary as seen in the previous questions, we will now check if adding a linear trend improves our model

```r
linearREMLmaximumLikelihoodNetherlandsInitial10.1 = likfit(precipitationNetherland_geoR,
    trend = "1st", ini.cov.pars = c(10, 1), lik.method = "REML")
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```r
linearREMLmaximumLikelihoodNetherlandsInitial1.10 = likfit(precipitationNetherland_geoR,
    trend = "1st", ini.cov.pars = c(1, 10), lik.method = "REML")
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```r
linearREMLmaximumLikelihoodNetherlandsInitial100.10 = likfit(precipitationNetherland_geoR,
    trend = "1st", ini.cov.pars = c(100, 10), lik.method = "REML")
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```r
linearREMLmaximumLikelihoodNetherlandsInitial10.100 = likfit(precipitationNetherland_geoR,
    trend = "1st", ini.cov.pars = c(10, 100), lik.method = "REML")
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
```

```
------------------------------------------------------------------
likfit: end of numerical maximisation.

WARNING: estimated range is more than 10 times bigger than the biggest distance between two p
 1) excluding spatial dependence if estimated sill is too low and/or
 2) taking trends (covariates) into account
```

```r
linearREMLmaximumLikelihoodNetherlandsInitial10.1
```

```
likfit: estimated model parameters:
       beta0        beta1        beta2        tausq      sigmasq          phi
"-2194.6747" "  -11.4587" "   45.2352" "  112.7395" "  191.5497" "    0.4033"
Practical Range with cor=0.05 for asymptotic range: 1.208203

likfit: maximised log-likelihood = -872
```

```r
linearREMLmaximumLikelihoodNetherlandsInitial1.10
```

```
likfit: estimated model parameters:
      beta0       beta1       beta2       tausq     sigmasq         phi
"-2084.141" "   -6.973" "   42.674" "  125.053" " 3238.867" "    9.905"
Practical Range with cor=0.05 for asymptotic range: 29.67433

likfit: maximised log-likelihood = -873
```

```r
linearREMLmaximumLikelihoodNetherlandsInitial100.10
```

```
likfit: estimated model parameters:
      beta0       beta1       beta2       tausq     sigmasq         phi
"-2084.141" "   -6.973" "   42.674" "  125.053" " 3238.867" "    9.905"
Practical Range with cor=0.05 for asymptotic range: 29.67433

likfit: maximised log-likelihood = -873
```

```r
linearREMLmaximumLikelihoodNetherlandsInitial10.100
```

```
likfit: estimated model parameters:
     beta0      beta1      beta2      tausq    sigmasq        phi 
"-2084.09" "   -6.91" "   42.67" "  125.89" "32191.57" "  100.00" 
Practical Range with cor=0.05 for asymptotic range: 299.5701

likfit: maximised log-likelihood = -873
```

From these models we can see that the linear trend does indeed improve our model, now we will check if there are any other covariance functions that can improve the model further.

```
Matren0.5linearREMLmaximumLikelihoodNetherlandsInitial10.1 = likfit(precipitationNetherlan
    trend = "1st", ini.cov.pars = c(10, 1), lik.method = "REML", cov.model = "matern",
    kappa = 0.5)
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```
Matren1.0linearREMLmaximumLikelihoodNetherlandsInitial10.1 = likfit(precipitationNetherlan
    trend = "1st", ini.cov.pars = c(10, 1), lik.method = "REML", cov.model = "matern",
    kappa = 1)
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```r
Matren1.5linearREMLmaximumLikelihoodNetherlandsInitial10.1 = likfit(precipitationNetherlan
    trend = "1st", ini.cov.pars = c(10, 1), lik.method = "REML", cov.model = "matern",
    kappa = 1.5)
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```r
Matren2.0linearREMLmaximumLikelihoodNetherlandsInitial10.1 = likfit(precipitationNetherlan
    trend = "1st", ini.cov.pars = c(10, 1), lik.method = "REML", cov.model = "matern",
    kappa = 2)
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```r
Matren2.5linearREMLmaximumLikelihoodNetherlandsInitial10.1 = likfit(precipitationNetherlan
    trend = "1st", ini.cov.pars = c(10, 1), lik.method = "REML", cov.model = "matern",
    kappa = 2.5)
```

```
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
```

```
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```
Matren0.5linearREMLmaximumLikelihoodNetherlandsInitial10.1
```

```
likfit: estimated model parameters:
       beta0         beta1         beta2         tausq       sigmasq           phi
"-2194.6747" "  -11.4587" "   45.2352" "  112.7395" "  191.5497" "    0.4033"
Practical Range with cor=0.05 for asymptotic range: 1.208203

likfit: maximised log-likelihood = -872
```

```
Matren1.0linearREMLmaximumLikelihoodNetherlandsInitial10.1
```

```
likfit: estimated model parameters:
       beta0         beta1         beta2         tausq       sigmasq           phi
"-2221.5117" "  -12.1627" "   45.8184" "  125.4406" "  160.1807" "    0.2088"
Practical Range with cor=0.05 for asymptotic range: 0.8347704

likfit: maximised log-likelihood = -871.3
```

```
Matren1.5linearREMLmaximumLikelihoodNetherlandsInitial10.1
```

```
likfit: estimated model parameters:
       beta0         beta1         beta2         tausq       sigmasq           phi
"-2233.7541" "  -12.4219" "   46.0784" "  129.6185" "  149.9124" "    0.1529"
Practical Range with cor=0.05 for asymptotic range: 0.7254113

likfit: maximised log-likelihood = -871.1
```

```
Matren2.0linearREMLmaximumLikelihoodNetherlandsInitial10.1
```

```
likfit: estimated model parameters:
       beta0         beta1         beta2         tausq       sigmasq           phi
"-2240.7056" "  -12.5595" "   46.2252" "  131.4685" "  144.9394" "    0.1248"
```

```
Practical Range with cor=0.05 for asymptotic range: 0.669996

likfit: maximised log-likelihood = -871.1
```

```
Matren2.5linearREMLmaximumLikelihoodNetherlandsInitial10.1
```

```
likfit: estimated model parameters:
      beta0        beta1        beta2        tausq      sigmasq          phi
"-2245.1703" "  -12.6453" "   46.3192" "  132.4280" "  142.0693" "    0.1074"
Practical Range with cor=0.05 for asymptotic range: 0.6358818

likfit: maximised log-likelihood = -871
```

It does seem that the matrén covariance function did indeed slightly improved the model so we will compare it to a model using a spherical covariance function. The spherical covariance function is appropriate for this scenario has the spatial correlation between data points decreases rapidly as the distance between the points increases and we are limited with the range of correlation has after 2 degrees of distance we loose sensible correlation, hence the cut in the variogram.

```
SphericallinearREMLmaximumLikelihoodNetherlandsInitial10.1 = likfit(precipitationNetherlan
    trend = "1st", ini.cov.pars = c(10, 1), lik.method = "REML", cov.model = "spherical")
```

```
kappa not used for the spherical correlation function
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```
SphericallinearREMLmaximumLikelihoodNetherlandsInitial1.10 = likfit(precipitationNetherlan
    trend = "1st", ini.cov.pars = c(1, 10), lik.method = "REML", cov.model = "spherical")
```

```
kappa not used for the spherical correlation function
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```
SphericallinearREMLmaximumLikelihoodNetherlandsInitial100.10 = likfit(precipitationNetherl
    trend = "1st", ini.cov.pars = c(100, 10), lik.method = "REML", cov.model = "spherical"
```

```
kappa not used for the spherical correlation function
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
likfit: end of numerical maximisation.
```

```
SphericallinearREMLmaximumLikelihoodNetherlandsInitial10.100 = likfit(precipitationNetherl
    trend = "1st", ini.cov.pars = c(10, 100), lik.method = "REML", cov.model = "spherical"
```

```
kappa not used for the spherical correlation function
---------------------------------------------------------------
likfit: likelihood maximisation using the function optim.
likfit: Use control() to pass additional
         arguments for the maximisation function.
        For further details see documentation for optim.
likfit: It is highly advisable to run this function several
        times with different initial values for the parameters.
likfit: WARNING: This step can be time demanding!
---------------------------------------------------------------
```

```
likfit: end of numerical maximisation.

WARNING: estimated range is more than 10 times bigger than the biggest distance between two p
 1) excluding spatial dependence if estimated sill is too low and/or
 2) taking trends (covariates) into account
```

```
SphericallinearREMLmaximumLikelihoodNetherlandsInitial10.1
```

```
likfit: estimated model parameters:
      beta0       beta1       beta2       tausq     sigmasq         phi
"-2182.422" "  -11.539" "   45.002" "  127.066" "  200.406" "    1.003"
Practical Range with cor=0.05 for asymptotic range: 1.00273

likfit: maximised log-likelihood = -872.2
```

```
SphericallinearREMLmaximumLikelihoodNetherlandsInitial1.10
```

```
likfit: estimated model parameters:
      beta0       beta1       beta2       tausq     sigmasq         phi
"-2082.850" "   -6.686" "   42.622" "  125.174" " 2151.559" "    9.905"
Practical Range with cor=0.05 for asymptotic range: 9.905214

likfit: maximised log-likelihood = -873
```

```
SphericallinearREMLmaximumLikelihoodNetherlandsInitial100.10
```

```
likfit: estimated model parameters:
      beta0       beta1       beta2       tausq     sigmasq         phi
"-2082.850" "   -6.686" "   42.622" "  125.174" " 2151.559" "    9.905"
Practical Range with cor=0.05 for asymptotic range: 9.905214

likfit: maximised log-likelihood = -873
```

```
SphericallinearREMLmaximumLikelihoodNetherlandsInitial10.100
```

```
likfit: estimated model parameters:
      beta0       beta1       beta2       tausq     sigmasq         phi
"-2083.890" "   -6.902" "   42.667" "  125.695" "21538.008" "   99.999"
Practical Range with cor=0.05 for asymptotic range: 99.99893

likfit: maximised log-likelihood = -873
```

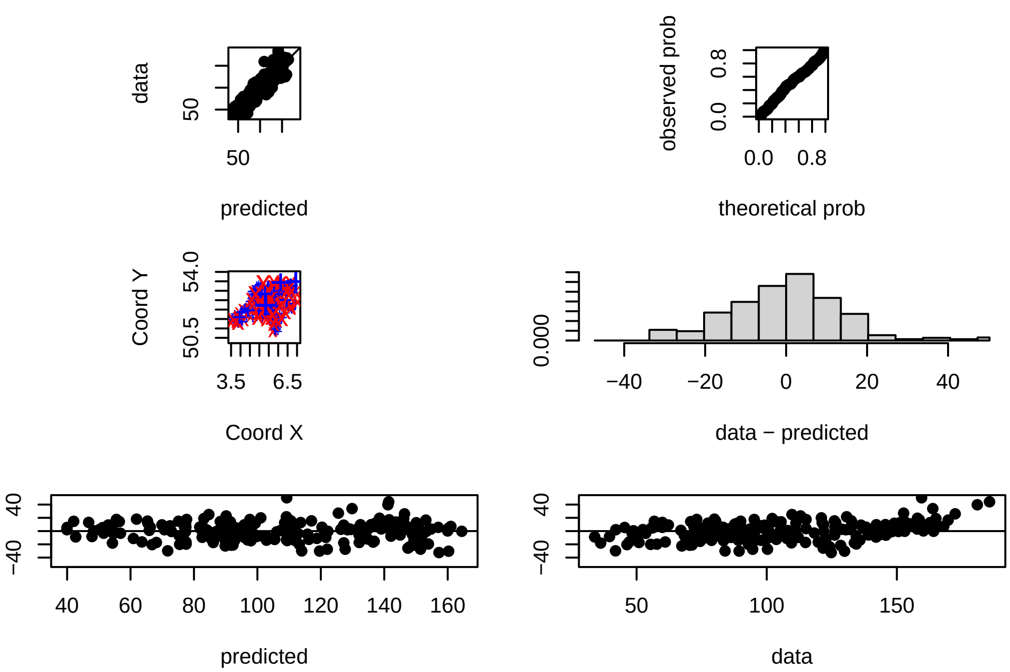
As we can see the spherical covariance function does not provide as good of a fit as the Matrén.

We will now validate the model by doing cross-validation on the model.

```
xv.ml = xvalid(precipitationNetherland_geoR, model = Matren2.5linearREMLmaximumLikelihoodN
```

```
xvalid: number of data locations       = 217
xvalid: number of validation locations = 217
xvalid: performing cross-validation at location ... 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13
xvalid: end of cross-validation
```

```
par(mfrow = c(3, 2), mar = c(4, 2, 2, 2))
plot(xv.ml, error = TRUE, std.error = FALSE, pch = 19)
```



From these plots we can see that the residuals seem mostly normal without any quickly identifiable patterns or bias.

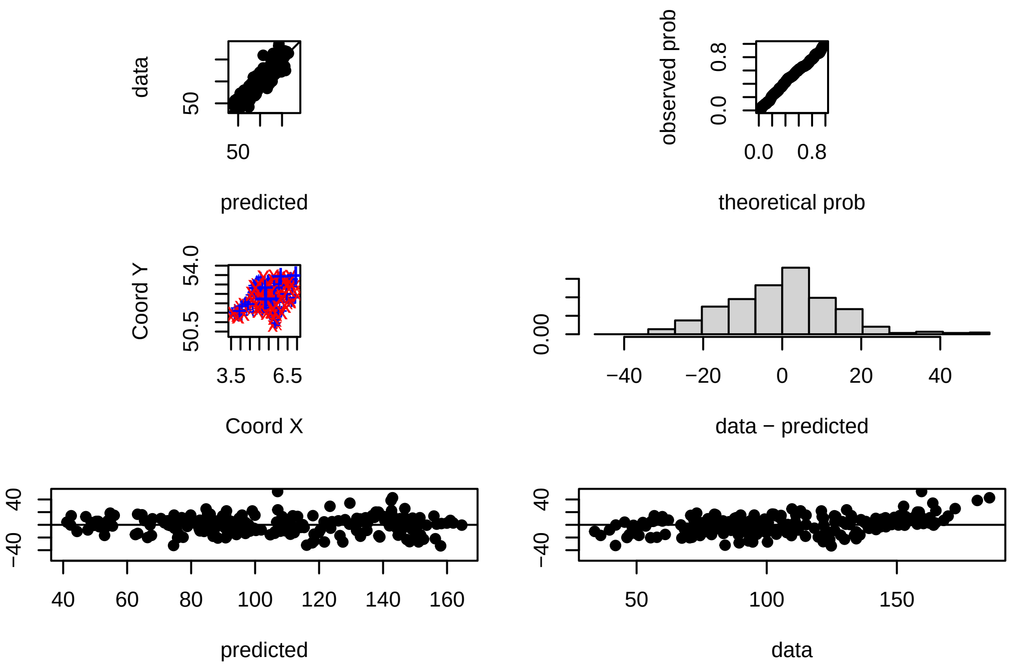
From the first top left graph we can see however that we seem to sightly underestimate more data points.

To account for bias we will also perform cross-validation to the next best performing model using the spherical function instead

```
xv.ml = xvalid(precipitationNetherland_geoR, model = SphericallinearREMLmaximumLikelihoodN
```

```
xvalid: number of data locations       = 217
xvalid: number of validation locations = 217
xvalid: performing cross-validation at location ... 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13
xvalid: end of cross-validation
```

```
par(mfrow = c(3, 2), mar = c(4, 2, 2, 2))
plot(xv.ml, error = TRUE, std.error = FALSE, pch = 19)
```



As we can see the data at the start seems to be systematically underestimated and at the end it seems to overestimated. Furthermore the theoretical data plot seems to be less linear.

This confirms that the spherical covariance function is indeed a worse model than the Matrén model.

1 f)

First we will start by making the predictions using the variogram

```
spatialPointsABC = randomRowsPrecipitation[, c("longitude", "latitude")]


# Set the krige.control parameters
krigeControl = krige.control(type.krige = "OK", cov.model = krigingVariogramFittedMatrén1.
    cov.pars = krigingVariogramFittedMatrén1.0$cov.pars)


# Kriging with the fitted variogram model
krigeResults = krige.conv(precipitationNetherland_geoR, locations = spatialPointsABC,
    krige = krigeControl)
```

```
krige.conv: model with constant mean
krige.conv: Kriging performed using global neighbourhood
```

```
# Extract predictions from the kriging results
predictions = krigeResults$predict

# Compare the predicted values with the actual precipitation values

actualPrecipitationValues = randomRowsPrecipitation[, 4]

comparisonvariogram = data.frame(actualPrecipitationValues, predictions)
```

Now for the maximum likelihood function

```
# done above spatialPointsABC = randomRowsPrecipitation[,
# c('longitude', 'latitude')]


# Set the krige.control parameters
krigeControl = krige.control(type.krige = "OK", cov.model = Matren2.5linearREMLmaximumLike
    cov.pars = Matren2.5linearREMLmaximumLikelihoodNetherlandsInitial10.1$cov.pars)
```

```r
# Kriging with the fitted variogram model
krigeResults = krige.conv(precipitationNetherland_geoR, locations = spatialPointsABC,
    krige = krigeControl)
```

```
krige.conv: model with constant mean
krige.conv: Kriging performed using global neighbourhood
```

```r
# Extract predictions from the kriging results
predictions = krigeResults$predict

# Compare the predicted values with the actual precipitation values

# done above actualPrecipitationValues = randomRowsPrecipitation[,4]
comparisonMaximumLikelihood = data.frame(actualPrecipitationValues, predictions)
```

Now that we have made the predictions for our 2 models we will check the predicted values compared to the real values for each of the models.

```r
comparisonvariogram
```

```
  precip predictions
1   89.1    95.41904
2   95.9    98.90558
3  147.2   147.82038
```

```r
comparisonMaximumLikelihood
```

```
  precip predictions
1   89.1    99.00780
2   95.9    99.36194
3  147.2   140.07734
```

```r
# Calculate Mean Absolute Error (MAE)
MAEVariogram = mean(abs(comparisonvariogram$precip - comparisonvariogram$predictions))
MAEMaximumLikelihood = mean(abs(comparisonMaximumLikelihood$precip - comparisonMaximumLike
```

```
# Calculate Mean Squared Error (MSE)
mseVariogram = mean((comparisonvariogram$precip - comparisonvariogram$predictions)^2)
mseMaximumLikelihood = mean((comparisonMaximumLikelihood$precip - comparisonMaximumLikelih


# Calculate Root Mean Squared Error (RMSE)
rmseVariogram = sqrt(mseVariogram)
rmseMaximumLikelihood = sqrt(mseMaximumLikelihood)


# Display the calculated metrics
cat("Mean Absolute Error (MAE) of the Variogram:", MAEVariogram, "\n")
```

```
Mean Absolute Error (MAE) of the Variogram: 3.315002
```

```
cat("Mean Squared Error (MSE) of the Variogram:", mseVariogram, "\n")
```

```
Mean Squared Error (MSE) of the Variogram: 16.44957
```

```
cat("Root Mean Squared Error (RMSE) of the Variogram:", rmseVariogram, "\n")
```

```
Root Mean Squared Error (RMSE) of the Variogram: 4.055807
```

```
cat("\n\n")
```

```
cat("Mean Absolute Error (MAE) of the maximum likelihood:", MAEMaximumLikelihood,
    "\n")
```

```
Mean Absolute Error (MAE) of the maximum likelihood: 6.830801
```

```
cat("Mean Squared Error (MSE) of the maximum likelihood:", mseMaximumLikelihood,
    "\n")
```

```
Mean Squared Error (MSE) of the maximum likelihood: 53.62729
```

```
cat("Root Mean Squared Error (RMSE) of the maximum likelihood:", rmseMaximumLikelihood,
    "\n")
```

```
Root Mean Squared Error (RMSE) of the maximum likelihood: 7.323066
```

As we can see from both the real values and the MAE, MSE and RMSE the variogram has as much better performance predicting those 3 points than our maximum likelihood model

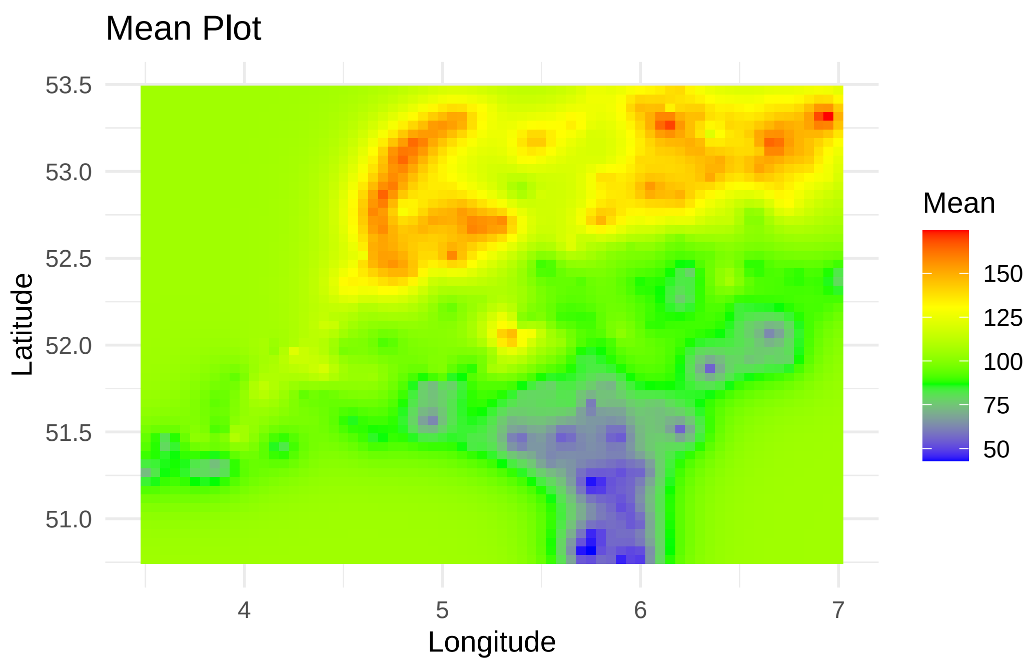
1 g)

```
# Determine the range of the coordinates
xRange = range(precipitationNetherland_geoR$coords[, 1])
yRange = range(precipitationNetherland_geoR$coords[, 2])

# Create a grid with 0.05-degree spacing
gridPoints = expand.grid(x = seq(xRange[1], xRange[2], by = 0.05), y = seq(yRange[1],
    yRange[2], by = 0.05))


# Kriging with the fitted variogram model
krigeResults = krige.conv(precipitationNetherland_geoR, locations = gridPoints,
    krige = krigeControl)
```

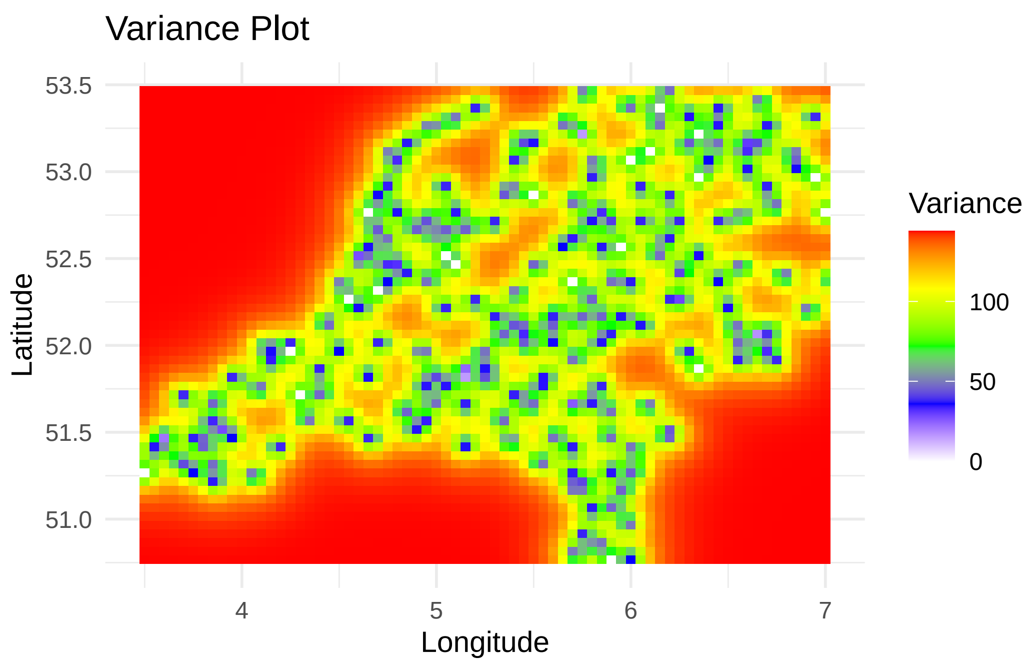
```
krige.conv: model with constant mean
krige.conv: Kriging performed using global neighbourhood
```

```
# Create a data frame for the grid points with the predicted mean and
# variance
gridData = data.frame(gridPoints, mean = krigeResults$predict, variance = krigeResults$kri

# Mean plot
meanPlot = ggplot(gridData, aes(x = x, y = y, fill = mean)) + geom_tile() +
    scale_fill_gradientn(colors = c("blue", "green", "yellow", "red")) +
    theme_minimal() + ggtitle("Mean Plot") + labs(x = "Longitude", y = "Latitude",
    fill = "Mean")

# Variance plot
variancePlot = ggplot(gridData, aes(x = x, y = y, fill = variance)) + geom_tile() +
```

```r
    scale_fill_gradientn(colors = c("white", "blue", "green", "yellow", "red")) +
    theme_minimal() + ggtitle("Variance Plot") + labs(x = "Longitude", y = "Latitude",
    fill = "Variance")

# Display the plots
print(meanPlot)
```




```r
print(variancePlot)
```



1 h)

For the priors I will be using the estimated values from our latest maximum likelihood model.

```
# Extract the estimated parameters

priorPhiVariogram = krigingVariogramFittedMatrén1.0$cov.pars[1]
priorTauSQVariogram = krigingVariogramFittedMatrén1.0$cov.pars[2]


priorPhiMaximumLikelihood = Matren1.0linearREMLmaximumLikelihoodNetherlandsInitial10.1$cov
priorTauSQMaximumLikelihood = Matren1.0linearREMLmaximumLikelihoodNetherlandsInitial10.1$c
```

The function does not support continuous priors directly so we will fit them as discrete priors.

```
# Creating discrete priors for phi and tau^2_rel AKA this is for the
# model with a nugget
phiDiscrete <- seq(min(priorPhiVariogram, priorPhiMaximumLikelihood) * 0.5,
    max(priorPhiVariogram, priorPhiMaximumLikelihood) * 1.5, length.out = 50)

tauSqDiscrete <- seq(min(priorTauSQVariogram, priorTauSQMaximumLikelihood) *
```

```r
    0.5, max(priorTauSQVariogram, priorTauSQMaximumLikelihood) * 1.5, length.out = 50)

# Informative priors based on the parameter estimates
phiProbability <- dnorm(phiDiscrete, mean = (priorPhiVariogram + priorPhiMaximumLikelihood
    sd = abs(priorPhiVariogram - priorPhiMaximumLikelihood)/2)
tauSqProbability <- dnorm(tauSqDiscrete, mean = (priorTauSQVariogram + priorTauSQMaximumLi
    sd = abs(priorTauSQVariogram - priorTauSQMaximumLikelihood)/2)

# Normalizing the probabilities
phiProbability <- phiProbability/sum(phiProbability)
tauSqProbability <- tauSqProbability/sum(tauSqProbability)


ex.grid <- as.matrix(expand.grid(seq(50.5, 53.5, l = 15), seq(3.5, 7, l = 15)))


# Fitting the krige.bayes model with the informative priors
krigeBayesModelWithNugget <- krige.bayes(geodata = precipitationNetherland_geoR,
    loc = ex.grid, prior = prior.control(phi.prior = phiProbability, phi.discrete = phiDis
        tausq.rel.prior = tauSqProbability, tausq.rel.discrete = tauSqDiscrete))
```

```
krige.bayes: model with constant mean
krige.bayes: computing the discrete posterior of phi/tausq.rel
krige.bayes: computing the posterior probabilities.
             Number of parameter sets:  2500
1, 101, 201, 301, 401, 501, 601, 701, 801, 901, 1001, 1101, 1201, 1301, 1401, 1501, 1601, 170

krige.bayes: sampling from posterior distribution
krige.bayes: sample from the (joint) posterior of phi and tausq.rel
                  [,1]
phi         80.0903555
tausq.rel    0.1043848
frequency 1000.0000000

krige.bayes: starting prediction at the provided locations
krige.bayes: phi/tausq.rel samples for the predictive are same as for the posterior
krige.bayes: computing moments of the predictive distribution
krige.bayes: sampling from the predictive
             Number of parameter sets:  1
1,
krige.bayes: preparing summaries of the predictive distribution
```

```r
krigeBayesModelWithoutNugget <- krige.bayes(geodata = precipitationNetherland_geoR,
    loc = ex.grid, prior = prior.control(phi.prior = phiProbability, phi.discrete = phiDis
```

```
krige.bayes: model with constant mean
krige.bayes: computing the discrete posterior of phi/tausq.rel
krige.bayes: computing the posterior probabilities.
             Number of parameter sets:  50
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 2

krige.bayes: sampling from posterior distribution
krige.bayes: sample from the (joint) posterior of phi and tausq.rel
              [,1]     [,2]     [,3]     [,4]     [,5]     [,6]     [,7]
phi       80.09036 226.3799 372.6695 518.9591 665.2486 811.5382 957.8278
tausq.rel  0.00000   0.0000   0.0000   0.0000   0.0000   0.0000   0.0000
frequency 16.00000  20.0000  14.0000  31.0000  23.0000  30.0000  23.0000
              [,8]     [,9]    [,10]    [,11]    [,12]    [,13]    [,14]
phi       1104.117 1250.407 1396.696 1542.986 1689.276 1835.565 1981.855
tausq.rel    0.000    0.000    0.000    0.000    0.000    0.000    0.000
frequency   24.000   21.000   28.000   21.000   27.000   28.000   31.000
             [,15]    [,16]    [,17]    [,18]    [,19]    [,20]    [,21]
phi       2128.144 2274.434 2420.723 2567.013 2713.303 2859.592 3005.882
tausq.rel    0.000    0.000    0.000    0.000    0.000    0.000    0.000
frequency   46.000   32.000   37.000   28.000   26.000   26.000   28.000
             [,22]    [,23]   [,24]   [,25]   [,26]    [,27]    [,28]    [,29]
phi       3152.171 3298.461 3444.75 3591.04 3737.33 3883.619 4029.909 4176.198
tausq.rel    0.000    0.000    0.00    0.00    0.00    0.000    0.000    0.000
frequency   30.000   37.000   20.00   18.00   23.00   14.000   24.000   20.000
             [,30]    [,31]    [,32]    [,33]    [,34]    [,35]    [,36]
phi       4322.488 4468.777 4615.067 4761.357 4907.646 5053.936 5200.225
tausq.rel    0.000    0.000    0.000    0.000    0.000    0.000    0.000
frequency   21.000   19.000   16.000   19.000   19.000   20.000   20.000
             [,37]    [,38]    [,39]    [,40]    [,41]    [,42]    [,43]
phi       5346.515 5492.804 5639.094 5785.384 5931.673 6077.963 6224.252
tausq.rel    0.000    0.000    0.000    0.000    0.000    0.000    0.000
frequency   17.000   14.000    8.000   17.000    5.000   12.000    9.000
             [,44]    [,45]    [,46]    [,47]  [,48]   [,49]    [,50]
phi       6370.542 6516.831 6663.121 6809.411 6955.7 7101.99 7248.279
tausq.rel    0.000    0.000    0.000    0.000    0.0    0.00    0.000
frequency    9.000    8.000    7.000    5.000    2.0    3.00    4.000

krige.bayes: starting prediction at the provided locations
krige.bayes: phi/tausq.rel samples for the predictive are same as for the posterior
```

```
krige.bayes: computing moments of the predictive distribution
krige.bayes: sampling from the predictive
             Number of parameter sets:  50
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26
krige.bayes: preparing summaries of the predictive distribution
```

```r
summary(krigeBayesModelWithoutNugget$posterior$sample)
```

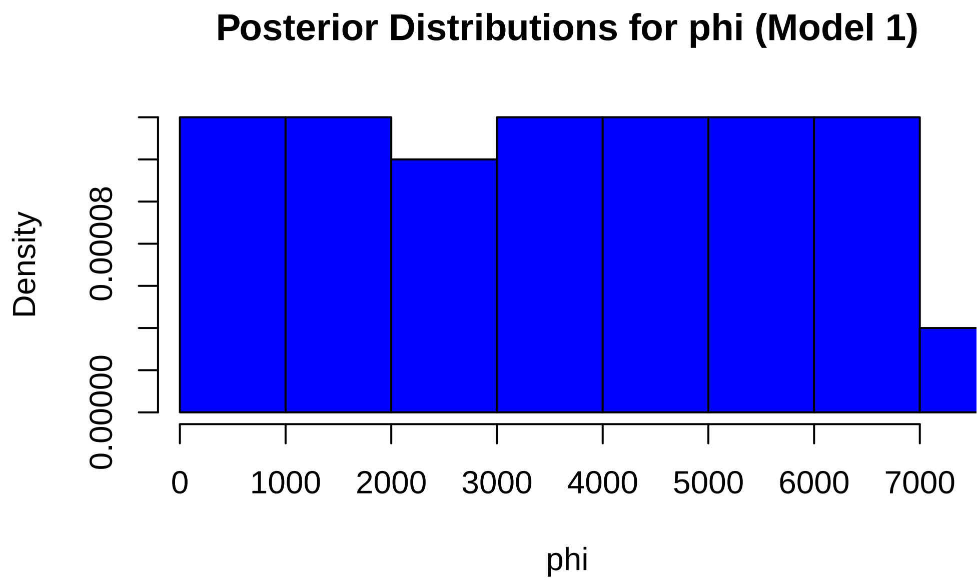
```
      beta              sigmasq               phi             tausq.rel
 Min.   :-14806.5   Min.   :  375543   Min.   :  80.09   Min.   :0
 1st Qu.: -2113.0   1st Qu.: 8331936   1st Qu.:1542.99   1st Qu.:0
 Median :   290.8   Median :14519487   Median :2713.30   Median :0
 Mean   :   299.1   Mean   :15700517   Mean   :2965.95   Mean   :0
 3rd Qu.:  2547.6   3rd Qu.:22223235   3rd Qu.:4322.49   3rd Qu.:0
 Max.   : 16575.4   Max.   :44053886   Max.   :7248.28   Max.   :0
```

```r
summary(krigeBayesModelWithNugget$posterior$sample)
```

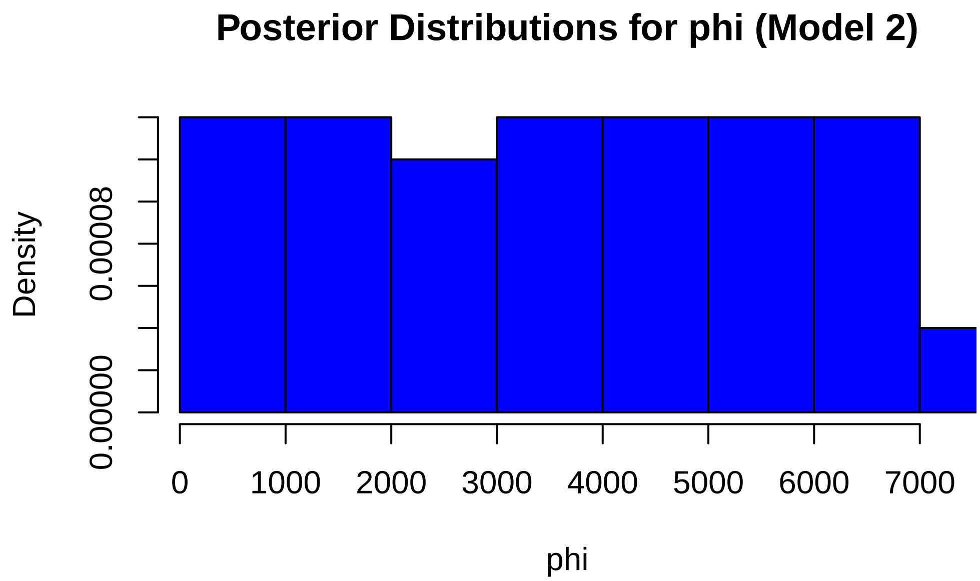
```
      beta            sigmasq          phi            tausq.rel
 Min.   :-49.52   Min.   :2216   Min.   :80.09   Min.   :0.1044
 1st Qu.: 66.57   1st Qu.:3000   1st Qu.:80.09   1st Qu.:0.1044
 Median :106.16   Median :3207   Median :80.09   Median :0.1044
 Mean   :105.66   Mean   :3238   Mean   :80.09   Mean   :0.1044
 3rd Qu.:145.57   3rd Qu.:3433   3rd Qu.:80.09   3rd Qu.:0.1044
 Max.   :282.96   Max.   :4455   Max.   :80.09   Max.   :0.1044
```

Now we will compare the posterior of both of the models to see the impact of the nugget

```r
# Extract posterior samples for phi and tau^2_rel
posterior_samples_model1 <- krigeBayesModelWithNugget$posterior$phi$phi.marginal$phi
posterior_samples_model2 <- krigeBayesModelWithoutNugget$posterior$phi$phi.marginal$phi

# Plot the posterior distributions
hist(posterior_samples_model1, freq = FALSE, main = "Posterior Distributions for phi (Mode
    xlab = "phi", col = "blue", xlim = range(c(posterior_samples_model1,
        posterior_samples_model2)))
```




```
hist(posterior_samples_model2, freq = FALSE, main = "Posterior Distributions for phi (Mode
    xlab = "phi", col = "blue", xlim = range(c(posterior_samples_model1,
        posterior_samples_model2)))
```




```r
# Compare summary statistics
summary_model1 <- summary(posterior_samples_model1)
summary_model2 <- summary(posterior_samples_model2)

cat("model 1 :\n")
```

```
model 1 :
```

```r
summary_model1
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  80.09 1872.14 3664.18 3664.18 5456.23 7248.28 
```

```r
cat("\n\n model 2 :\n")
```

```
 model 2 :
```

```
summary_model2
```

```
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
 80.09 1872.14 3664.18 3664.18 5456.23 7248.28 
```

```
# Reset the plot layout
par(mfrow = c(1, 1))
```

As we can seem with a low number of binds we can't see any significant difference in the summaries or histogram between the models with and without a nugget

Question 2

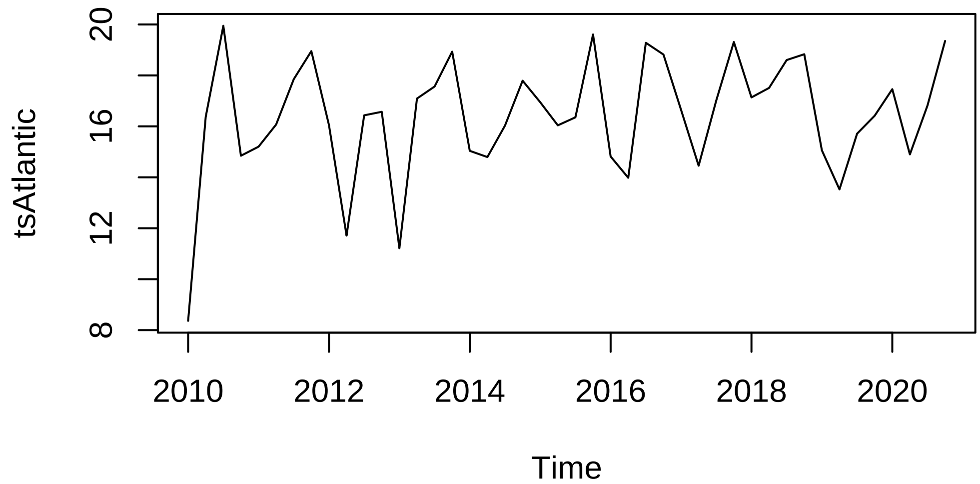
2 a)

We fist start by making the appropriate changes in the data to average the data to quarterly means

```
AMOCDF$Date = as.Date(AMOCDF$Date, format = "%d/%m/%Y")

## I will now make a column with the quarter and year that I will use
## to create the averages per quarter
AMOCDF$YearQuarter = paste(AMOCDF$Year, AMOCDF$Quarter, sep = "-")

YearQuarterAverage = AMOCDF %>%
    group_by(YearQuarter) %>%
    summarise(AverageStrength = mean(Strength))
```

Now we will convert the average data to a time series object to be able to plot it

```
tsAtlantic = ts(YearQuarterAverage, start = c(2010, 1), frequency = 4)

tsAtlantic = tsAtlantic[, "AverageStrength"]

plot.ts(tsAtlantic)
```



Trend analysis

From this graph we can see a yearly oscillation of Sverdrups. We can also identify that the peaks in Sverdrups are usually in the last quarter before the start of a new year and the valleys are on the second quarter of the year.

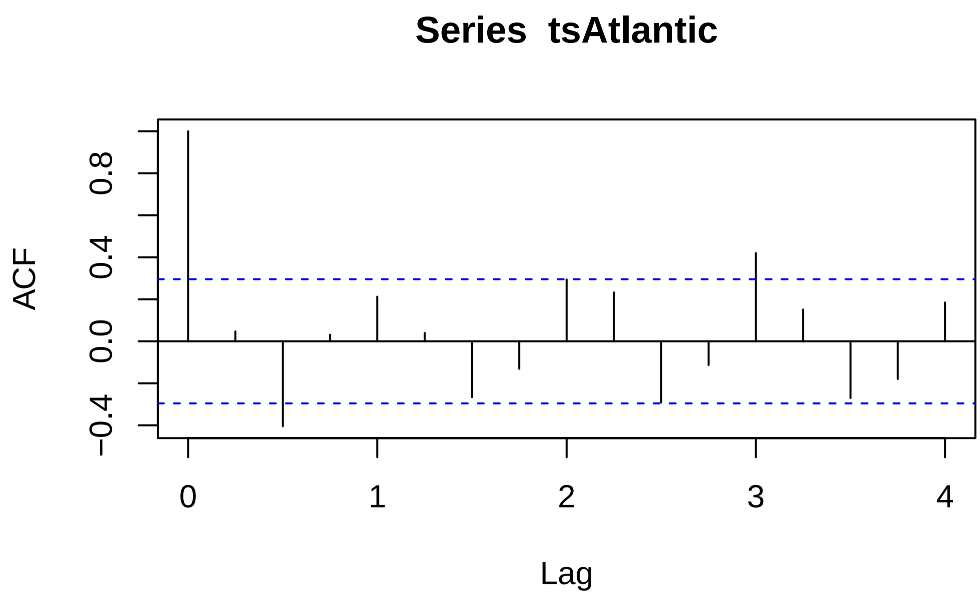
The data does seem stationary enough that if we were to differentiate we would start losing some of the structure.

2 b)

ACF

First we will start by checking the ACF(Autocorrelation Function) and PACF(Partial Autocorrelation Function) to check for if we have stationary data or not to help us decide between an ARMA or an ARIMA model.

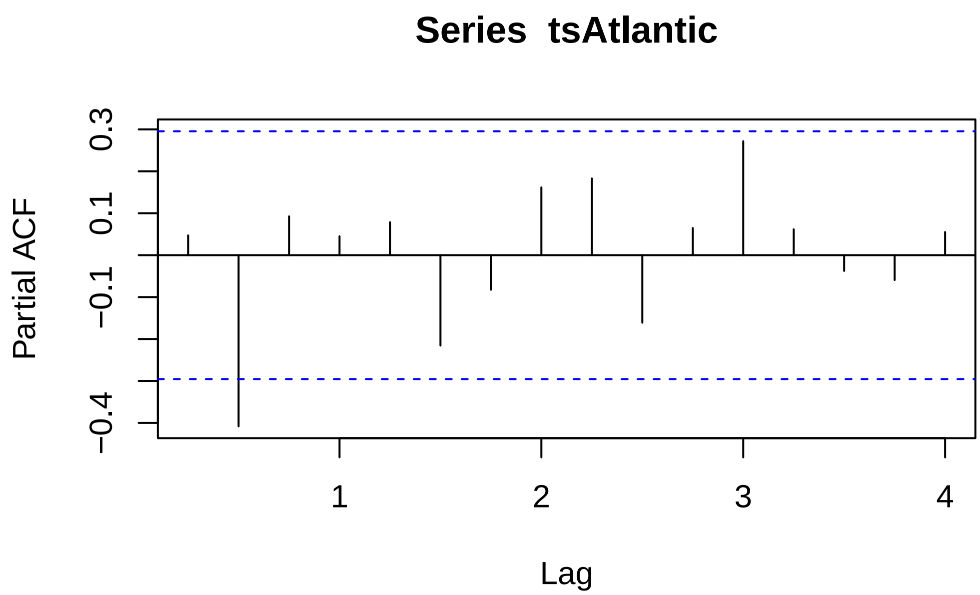
```
acf(tsAtlantic)
```



We can see that for ACF OF Average strength slowly decreases as lag increases to infinity with lag = 3 still being a significant values, meaning it is not a simple MA model as AR is clearly not quickly cut-off.

PACF

```
pacf(tsAtlantic)
```



The PACF seems to be cut-off at lag 0,5 indicating an AR model might be a best fit for our data to be a but with some almost significant values after the cut it might be also appropriate to some non-zero q values to confirm our initial assumption

As such we will now proceed to fit multiple model firstly with the initial assumption that, then I will both use models with non-zero q and the model given by the auto.arima function to double check that the assumptions made by the previous analyses is correct.

```
# it is always a good practice to try multiple values of p,d and q to
# see if we can do better we then obviously compare via the AIC of the
# models and their log likelihoods it is never enough to check those we
# also need to check the residuals

## order is p, d ,q

## initial models under our assumptions

model100 = Arima(tsAtlantic, order = c(1, 0, 0))
model200 = Arima(tsAtlantic, order = c(2, 0, 0))
model300 = Arima(tsAtlantic, order = c(3, 0, 0))

## now I will add postive q values
```

```r
model101 = Arima(tsAtlantic, order = c(1, 0, 1))
model102 = Arima(tsAtlantic, order = c(1, 0, 2))
model103 = Arima(tsAtlantic, order = c(1, 0, 3))

model201 = Arima(tsAtlantic, order = c(2, 0, 1))
model202 = Arima(tsAtlantic, order = c(2, 0, 2))
model203 = Arima(tsAtlantic, order = c(2, 0, 3))

model301 = Arima(tsAtlantic, order = c(3, 0, 1))
model302 = Arima(tsAtlantic, order = c(3, 0, 2))
model303 = Arima(tsAtlantic, order = c(3, 0, 3))



## lastly we will use auto.arima without seasonality to confirm our
## inital assumptions
modelAuto = auto.arima(tsAtlantic, max.d = 0, max.p = 5, max.q = 5, seasonal = FALSE)
```

best model selection

```r
model100
```

```
Series: tsAtlantic
ARIMA(1,0,0) with non-zero mean

Coefficients:
         ar1     mean
      0.0665  16.3878
s.e.  0.1788   0.3726

sigma^2 = 5.572:  log likelihood = -99.2
AIC=204.41   AICc=205.01   BIC=209.76
```

```r
model200
```

```
Series: tsAtlantic
ARIMA(2,0,0) with non-zero mean

Coefficients:
```

```
         ar1      ar2     mean
      0.0990  -0.5565  16.4298
s.e.  0.1576   0.1488   0.2113

sigma^2 = 4.321:  log likelihood = -93.45
AIC=194.9   AICc=195.92   BIC=202.04
```

```
model300
```

```
Series: tsAtlantic
ARIMA(3,0,0) with non-zero mean

Coefficients:
         ar1      ar2     ar3     mean
      0.1626  -0.5690  0.1464  16.4227
s.e.  0.1729   0.1479  0.1708   0.2409

sigma^2 = 4.35:  log likelihood = -93.09
AIC=196.17   AICc=197.75   BIC=205.1
```

As we can see from these inital models ARIMA(2,0,0) is the model that has the best fit has we can see from its lower AIC score of 194,9.

Now we will check against the other models to check the validity of our assumptions.

```
model101
```

```
Series: tsAtlantic
ARIMA(1,0,1) with non-zero mean

Coefficients:
          ar1     ma1     mean
      -0.4204  0.7718  16.3721
s.e.   0.2390  0.1466   0.4067

sigma^2 = 5.045:  log likelihood = -96.64
AIC=201.29   AICc=202.31   BIC=208.43
```

```
model102
```

```
Series: tsAtlantic
ARIMA(1,0,2) with non-zero mean

Coefficients:
         ar1     ma1      ma2     mean
      0.0230  0.1275  -0.4485  16.4289
s.e.  0.3051  0.2420   0.1348   0.2224

sigma^2 = 4.651:  log likelihood = -94.41
AIC=198.81   AICc=200.39   BIC=207.73
```

```
model103
```

```
Series: tsAtlantic
ARIMA(1,0,3) with non-zero mean

Coefficients:
          ar1     ma1      ma2      ma3     mean
      -0.5284  0.7214  -0.3646  -0.3072  16.4299
s.e.   0.9228  0.8649   0.2077   0.3545   0.2195

sigma^2 = 4.733:  log likelihood = -94.25
AIC=200.5   AICc=202.77   BIC=211.21
```

```
model201
```

```
Series: tsAtlantic
ARIMA(2,0,1) with non-zero mean

Coefficients:
          ar1      ar2     ma1     mean
      -0.0669  -0.5475  0.2187  16.4255
s.e.   0.2740   0.1555  0.2883   0.2300

sigma^2 = 4.366:  log likelihood = -93.15
AIC=196.31   AICc=197.88   BIC=205.23
```

```
model202
```

```
Series: tsAtlantic
ARIMA(2,0,2) with non-zero mean

Coefficients:
         ar1      ar2      ma1     ma2     mean
      0.0787  -0.9982  -0.0255  0.9999  16.4015
s.e.  0.0285   0.0066   0.0899  0.1158   0.2684

sigma^2 = 3.378:  log likelihood = -89.46
AIC=190.91   AICc=193.18   BIC=201.62
```

```
model203
```

```
Series: tsAtlantic
ARIMA(2,0,3) with non-zero mean

Coefficients:
         ar1      ar2     ma1     ma2     ma3     mean
      0.0325  -0.9621  0.0499  0.8487  0.4147  16.4028
s.e.  0.0645   0.0442  0.1987  0.2041  0.2511   0.3044

sigma^2 = 3.315:  log likelihood = -89.07
AIC=192.13   AICc=195.25   BIC=204.62
```

```
model301
```

```
Series: tsAtlantic
ARIMA(3,0,1) with non-zero mean

Coefficients:
         ar1      ar2     ar3      ma1     mean
      0.4092  -0.5931  0.2864  -0.2467  16.4191
s.e.  0.6330   0.1651  0.3580   0.6291   0.2537

sigma^2 = 4.449:  log likelihood = -93.03
AIC=198.06   AICc=200.34   BIC=208.77
```

```
model302
```

```
Series: tsAtlantic
ARIMA(3,0,2) with non-zero mean

Coefficients:
         ar1      ar2     ar3      ma1     ma2     mean
      0.2684  -0.9851  0.2222  -0.3030  1.0000  16.4144
s.e.  0.1999   0.0305  0.1995   0.1453  0.1921   0.2922

sigma^2 = 3.392:  log likelihood = -89.53
AIC=193.06   AICc=196.17   BIC=205.54
```

```
model303
```

```
Series: tsAtlantic
ARIMA(3,0,3) with non-zero mean

Coefficients:
          ar1      ar2      ar3     ma1     ma2     ma3     mean
      -0.3983  -0.9518  -0.4263  0.4381  0.7816  0.7352  16.4197
s.e.   0.3385   0.0412   0.3442  0.2946  0.1690  0.2422   0.2691

sigma^2 = 3.352:  log likelihood = -88.54
AIC=193.08   AICc=197.19   BIC=207.35
```

In this initial analysis we have found models that do have a lower AIC lower log likelihood than our previous best model, however these model ma's standard error are to close the the ma values indicating that while we are getting a better fit we might be overfitting to our data.

As such this does confirm our initial assumption for the choice of a zero q value.

Now lastly we will check if the auto.arima function does comfirm our initial assumptions.

```
modelAuto
```

```
Series: tsAtlantic
ARIMA(2,0,0) with non-zero mean

Coefficients:
         ar1      ar2     mean
      0.0990  -0.5565  16.4298
s.e.  0.1576   0.1488   0.2113
```

```
sigma^2 = 4.321:  log likelihood = -93.45
AIC=194.9   AICc=195.92   BIC=202.04
```

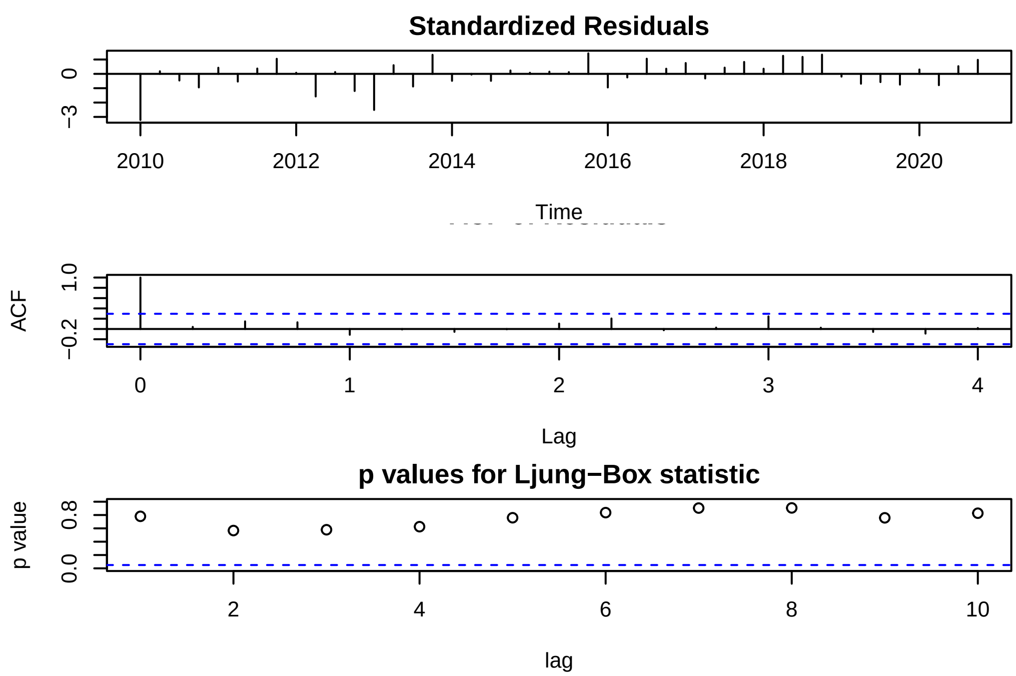
The function does confirm our assumption that ARIMA(2,0,0) is indeed the best model.

We will now check the residuals to verify if any of ou previously selected model validates well or if it is simply the best of bad models.

talk about the model being more easily explainability becaues MA = 0

Best model residual validation

```r
# Set smaller margins
par(mar = c(4, 4, 2, 2))

tsdiag(model200)
```




```r
# Reset margins
par(mar = c(5, 4, 4, 2) + 0.1)
```

Initially from the standardised residuals plot we can identify some sort of sinusoidal pattern, this implies that there is a seasonal trend that is not being accounted for in our model and as such this trends needs to be accounted in future models to better explain and increase the prediction power of a new model.

Forecasting

Now using the forecast function we will forecast the next 4 quarters of 2021

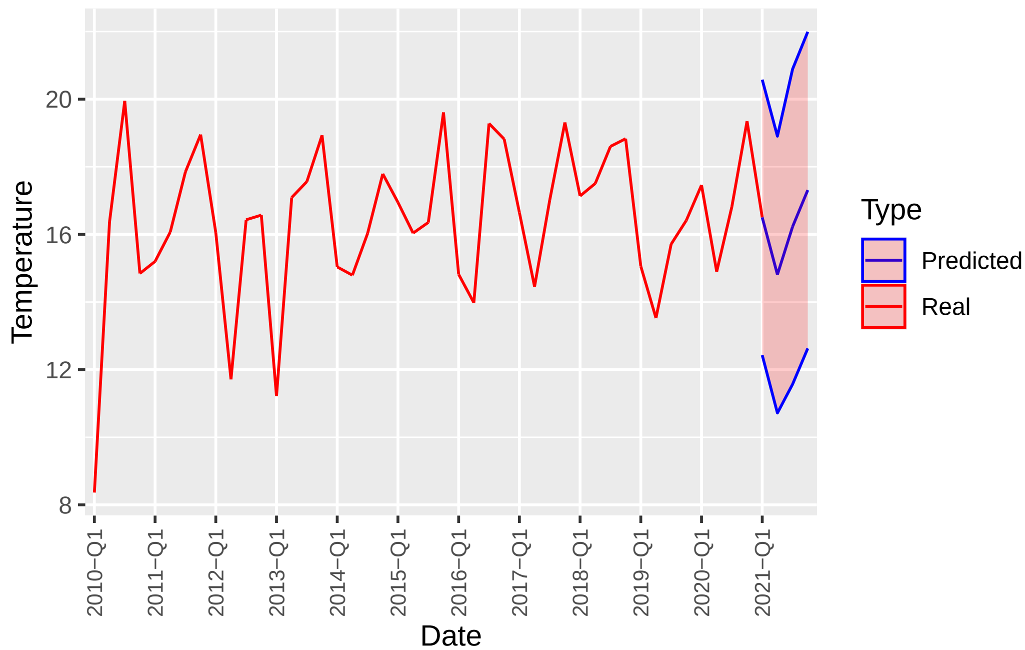
```r
forecast(model200, 4)
```

```
        Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
2021 Q1       16.50240 13.83841 19.16639 12.42818 20.57662
2021 Q2       14.81104 12.13403 17.48804 10.71691 18.90517
2021 Q3       16.22919 13.18168 19.27669 11.56843 20.88994
2021 Q4       17.31076 14.24941 20.37212 12.62882 21.99271
```

But this data is better visualized in a graph to better understand if the predictions are sensible compared to our real data.

```r
predictedArimaDF = data.frame(forecast(model200, 4))

predictedArimaDF$YearQuarter = c("2021-Q1", "2021-Q2", "2021-Q3", "2021-Q4")

# Combine real_data and pred_data into a single data frame
combinedDataframeAMOC = rbind(data.frame(Date = YearQuarterAverage$YearQuarter,
    Temperature = YearQuarterAverage$AverageStrength, Type = "Real"), data.frame(Date = pr
    Temperature = predictedArimaDF$Point.Forecast, Type = "Predicted"))

predictedArimaDF$Temperature = predictedArimaDF$Point.Forecast

predictedArimaDF$Type = "Predicted"


# Create the ggplot
plotARIMA = ggplot(combinedDataframeAMOC, aes(x = Date, y = Temperature,
    color = Type, group = 1)) + geom_line() + scale_color_manual(values = c("blue",
    "red"))

# Add the 95% confidence interval
plotARIMA = plotARIMA + geom_ribbon(data = predictedArimaDF, aes(x = YearQuarter,
    ymin = Lo.95, ymax = Hi.95), fill = "red", alpha = 0.2)
```

```r
# Adjust the x-axis labels
plotARIMA = plotARIMA + scale_x_discrete(breaks = combinedDataframeAMOC$Date[c(TRUE,
    rep(FALSE, 3))], labels = combinedDataframeAMOC$Date[c(TRUE, rep(FALSE,
    3))])

plotARIMA = plotARIMA + theme(axis.text.x = element_text(angle = 90, vjust = 0.5,
    size = 8))

# Display the plot
print(plotARIMA)
```



As we can see from the graph the ARIMA (2,0,0) seems to give us a sensible forecast for the 2021 quarter values, however as we can see the interval of the prediction accuracy our model is not too certain on the values most likely due to our model not accounting for the seasonal cycle of our data.

2 c)

Initial assumptions

From the previous exploratory analysis of the data we have established that the data did not need to be differentiated since it was constant, this translates to polynomial DLM component of order 2 that will use linear model to account for this type of changes in the data.

Furthermore, from the residual analysis we have inferred that there is an underlying seasonal trend present on the data, this seasonal trend will be represented by a seasonal component of frequency 4 to represent the 4 quarters per year.

model fitting

```
## linear model, order = 2, quadratic order = 3 , etc

## what we want is a linear model with a seasonal component so we add
## the 2 components together in a model

## things to try, another term like quadratic, or a arma component
## stacked on top of this

## Initial model with a linear polynomial and a seasonal component

buildFun = function(x) {
    dlmModPoly(order = 2, dV = exp(x[1]), dW = c(0, exp(x[2]))) + dlmModSeas(frequency = 4
        dV = 0, dW = c(exp(x[3]), rep(0, 2)))
}

linearDLM = dlmMLE(tsAtlantic, parm = c(0, 0, 0), build = buildFun)

linearDLM$par
```

```
[1]   1.151339 -18.078101  -2.189479
```

```
fittedLinearDLM = buildFun(linearDLM$par)

V(fittedLinearDLM)
```

```
         [,1]
[1,] 3.162425
```

```
W(fittedLinearDLM)
```

```
     [,1]         [,2]     [,3] [,4] [,5]
[1,]    0 0.000000e+00 0.000000    0    0
[2,]    0 1.408576e-08 0.000000    0    0
[3,]    0 0.000000e+00 0.111975    0    0
[4,]    0 0.000000e+00 0.000000    0    0
[5,]    0 0.000000e+00 0.000000    0    0
```

```
## second model with a quadratic polynomial and a seasonal component

buildFunQuad = function(x) {
    dlmModPoly(order = 3, dV = exp(x[1]), dW = c(0, exp(x[2]), exp(x[3]))) +
        dlmModSeas(frequency = 4, dV = 0, dW = c(exp(x[4]), rep(0, 2)))
}

quadraticDLM = dlmMLE(tsAtlantic, parm = c(0, 0, 0, 0), build = buildFunQuad)

quadraticDLM$par
```

```
[1]   1.161355 -17.807081 -28.603103  -2.352292
```

```
fittedQuadraticDLM = buildFunQuad(quadraticDLM$par)

V(fittedQuadraticDLM)
```

```
         [,1]
[1,] 3.194257
```

```
W(fittedQuadraticDLM)
```

```
     [,1]         [,2]         [,3]       [,4] [,5] [,6]
[1,]    0 0.000000e+00 0.000000e+00 0.00000000    0    0
[2,]    0 1.847069e-08 0.000000e+00 0.00000000    0    0
[3,]    0 0.000000e+00 3.782948e-13 0.00000000    0    0
[4,]    0 0.000000e+00 0.000000e+00 0.09515082    0    0
[5,]    0 0.000000e+00 0.000000e+00 0.00000000    0    0
[6,]    0 0.000000e+00 0.000000e+00 0.00000000    0    0
```

Now we will compare both models through their log likelihood using the dlmLL function and see if the extra flexibility from the extra polynomial function is providing a better fit

```r
dlmLL(tsAtlantic, fittedLinearDLM)
```

```
[1] 94.98804
```

```r
dlmLL(tsAtlantic, fittedQuadraticDLM)
```

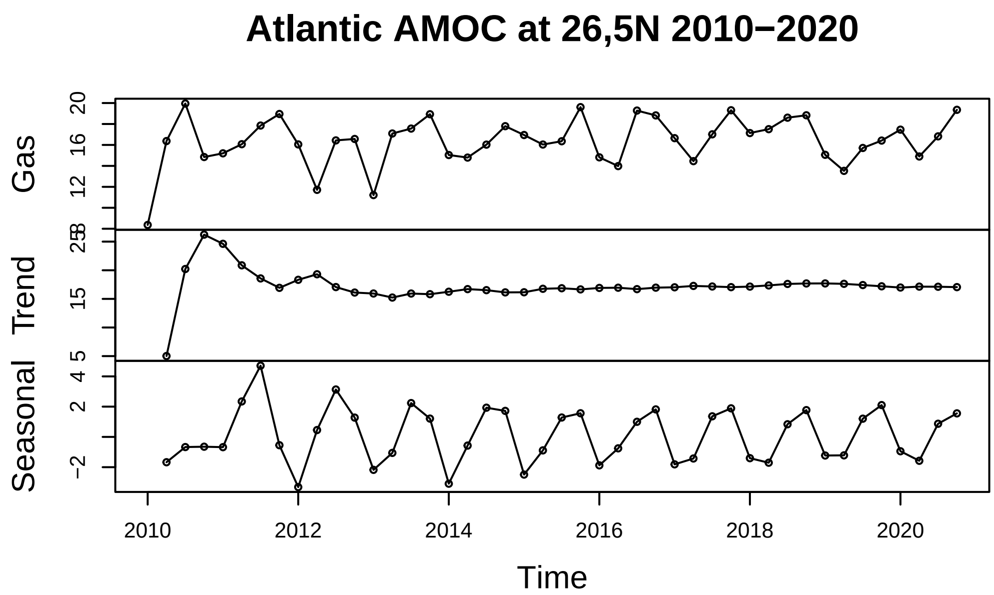
```
[1] 108.043
```

As we can see the dlm model using only a linear polynomial has a lower log likelihood than the model with an extra quadratic term, meaning this extra flexibility does not contribute to a better model fit and as such we will use the linear fitted model to do our forecasting.

```r
amocPredict = dlmFilter(tsAtlantic, mod = fittedLinearDLM)
summary(amocPredict)
```

```
    Length Class  Mode
y    44    ts     numeric
mod  10    dlm    list
m   225    mts    numeric
U.C  45    -none- list
D.C 225    -none- numeric
a   220    mts    numeric
U.R  44    -none- list
D.R 220    -none- numeric
f    44    ts     numeric
```

```r
x = cbind(tsAtlantic, dropFirst(amocPredict$a[, c(1, 3)]))
x = window(x, start = c(2010, 1))
colnames(x) = c("Gas", "Trend", "Seasonal")
plot(x, type = "o", main = "Atlantic AMOC at 26,5N 2010-2020")
```



Forecast

```
amocForecast = dlmForecast(amocPredict, nAhead = 4)
summary(amocForecast)
```

```
  Length Class  Mode
a 20     mts    numeric
R  4     -none- list
f  4     ts     numeric
Q  4     -none- list
```

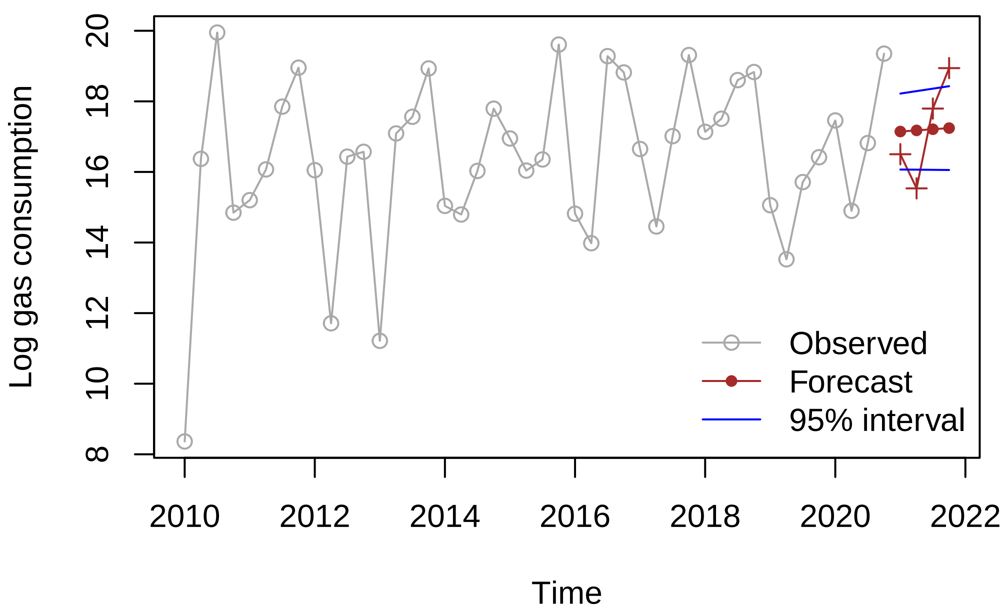
```
dim(amocForecast$a)
```

```
[1] 4 5
```

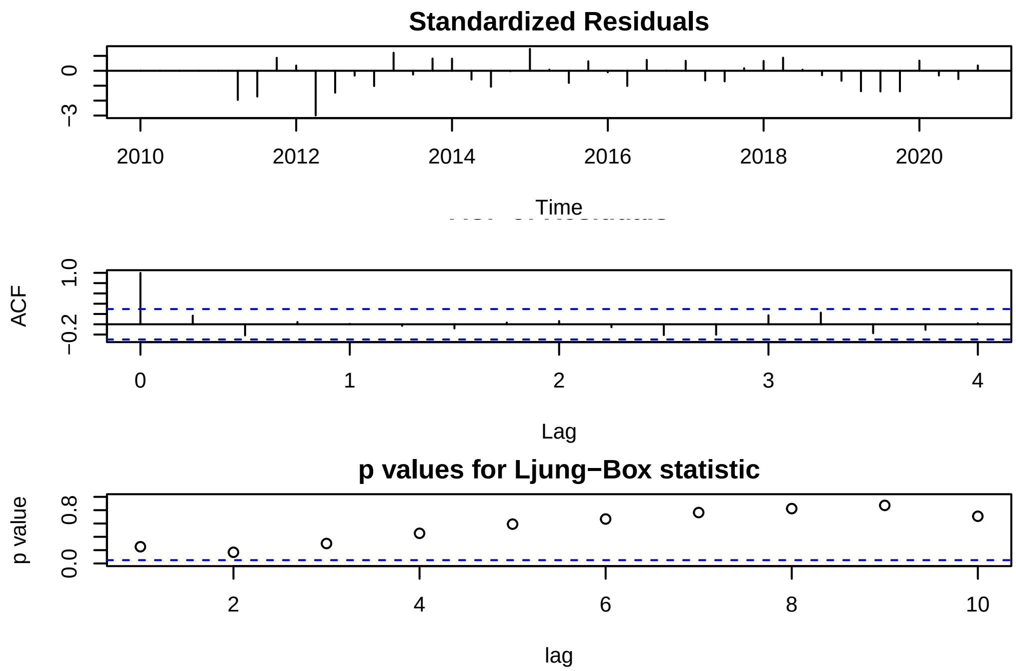
```
dim(amocForecast$f)
```

```
[1] 4 1
```

```r
sqrtR = sapply(amocForecast$R, function(x) sqrt(x[1, 1]))
pl = amocForecast$a[, 1] + qnorm(0.025, sd = sqrtR)
pu = amocForecast$a[, 1] + qnorm(0.975, sd = sqrtR)
x = ts.union(window(tsAtlantic, start = c(2010, 1)), amocForecast$a[, 1],
    amocForecast$f, pl, pu)
par(mar = c(4, 4, 2, 2))
plot(x, plot.type = "single", type = "o", pch = c(1, 20, 3, NA, NA), col = c("darkgrey",
    "brown", "brown", "blue", "blue"), ylab = "Log gas consumption")

legend("bottomright", legend = c("Observed", "Forecast", "95% interval"),
    bty = "n", pch = c(1, 20, NA), lty = 1, col = c("darkgrey", "brown",
        "blue"))
```




```r
# Set smaller margins
par(mar = c(4, 4, 2, 2))

tsdiag(amocPredict)
```

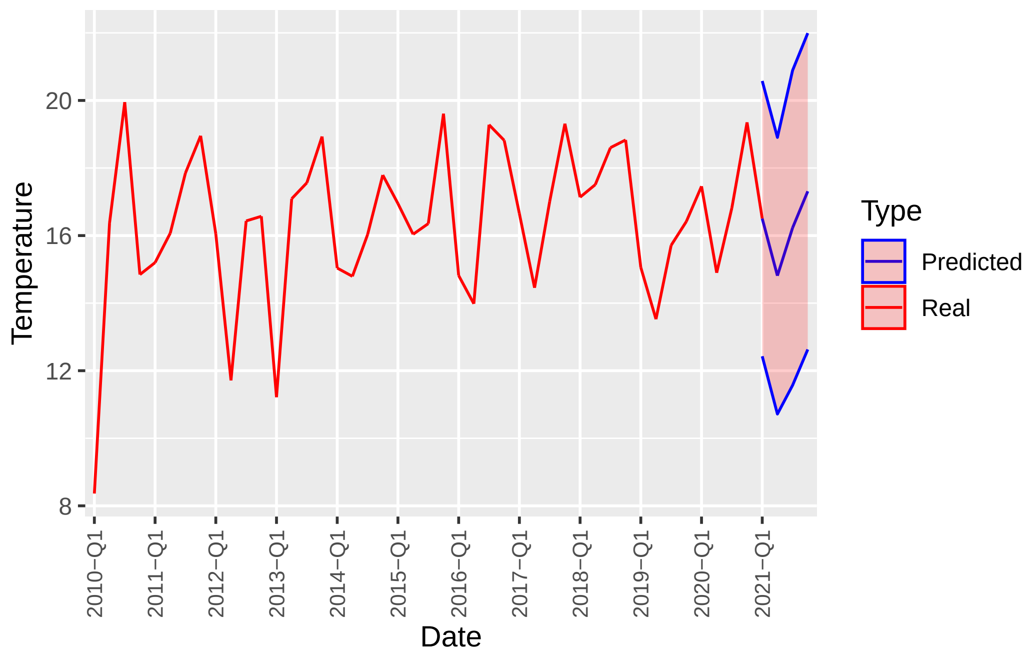



```
# Reset margins
par(mar = c(5, 4, 4, 2) + 0.1)
```

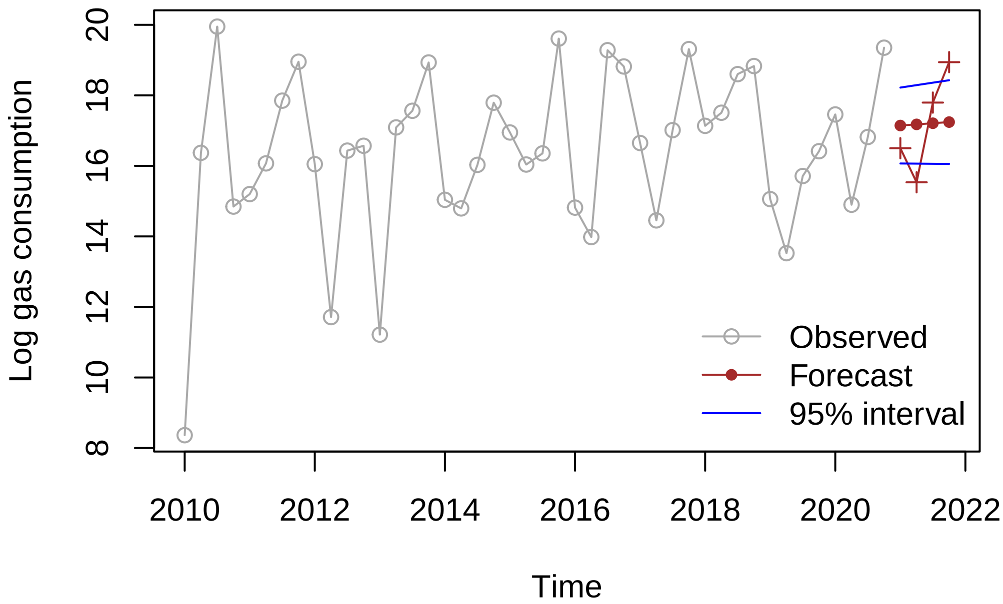
2 d)

Again comparing the forecast values and their respective prediction intervals as we can see from the graphs bellow the dlm model has smaller prediction intervals, most likely due to being able to explain the underlying seasonal trend reducing therefore the uncertainty in comparison the ARIMA model.

```
print(plotARIMA)
```




```r
sqaretRoot = sapply(amocForecast$R, function(x) sqrt(x[1, 1]))
predictionLow = amocForecast$a[, 1] + qnorm(0.025, sd = sqaretRoot)  ## Low
predictionUpper = amocForecast$a[, 1] + qnorm(0.975, sd = sqaretRoot)  ## Upper
x = ts.union(window(tsAtlantic, start = c(2010, 1)), amocForecast$a[, 1],
    amocForecast$f, predictionLow, predictionUpper)
par(mar = c(4, 4, 2, 2))
plot(x, plot.type = "single", type = "o", pch = c(1, 20, 3, NA, NA), col = c("darkgrey",
    "brown", "brown", "blue", "blue"), ylab = "Log gas consumption")

legend("bottomright", legend = c("Observed", "Forecast", "95% interval"),
    bty = "n", pch = c(1, 20, NA), lty = 1, col = c("darkgrey", "brown",
        "blue"))
```

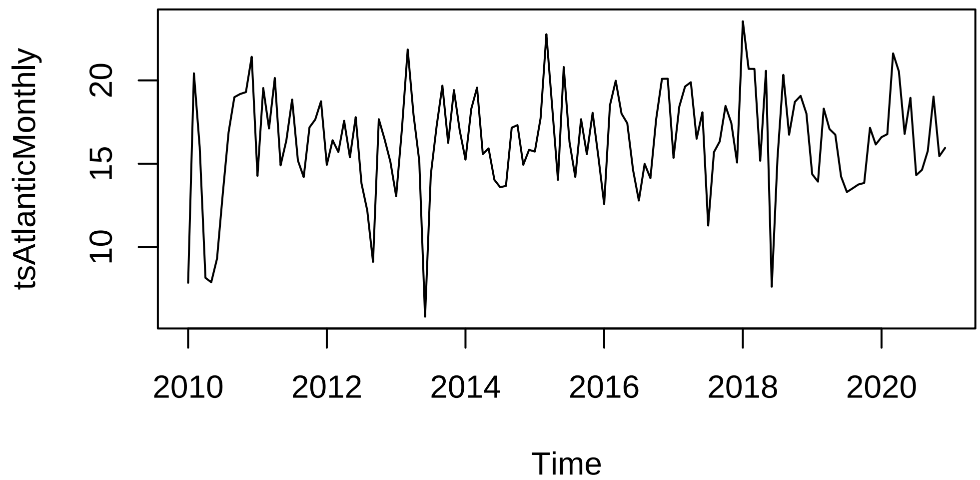


2 e)

```
# AMOCDFMonthly =AMOCDF %>% mutate(YearMonth = paste0(year(Date), '-',
# month(Date, label = TRUE, abbr = FALSE)))


## I will now make a column with the month and year that I will use to
## create the monthly averages
AMOCDF$YearMonth = paste(AMOCDF$Year, AMOCDF$Month, sep = "-")

YearMonthlyAverage = AMOCDF %>%
    group_by(YearMonth) %>%
    summarise(AverageStrength = mean(Strength))
```

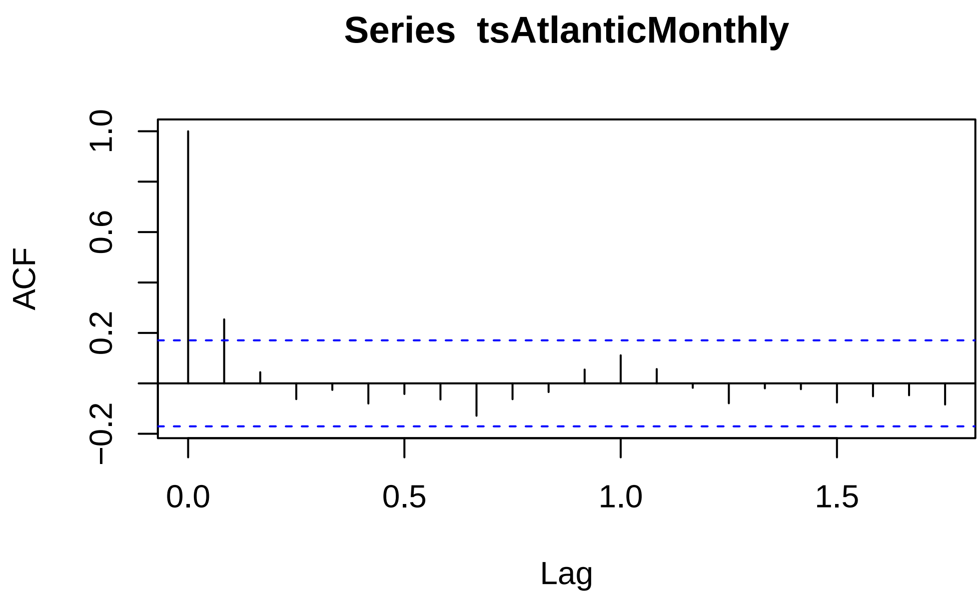
Now we will create a new montly time series object and make it univariate

```
tsAtlanticMonthly = ts(YearMonthlyAverage, start = c(2010, 1), frequency = 12)

tsAtlanticMonthly = tsAtlanticMonthly[, "AverageStrength"]

plot.ts(tsAtlanticMonthly)
```

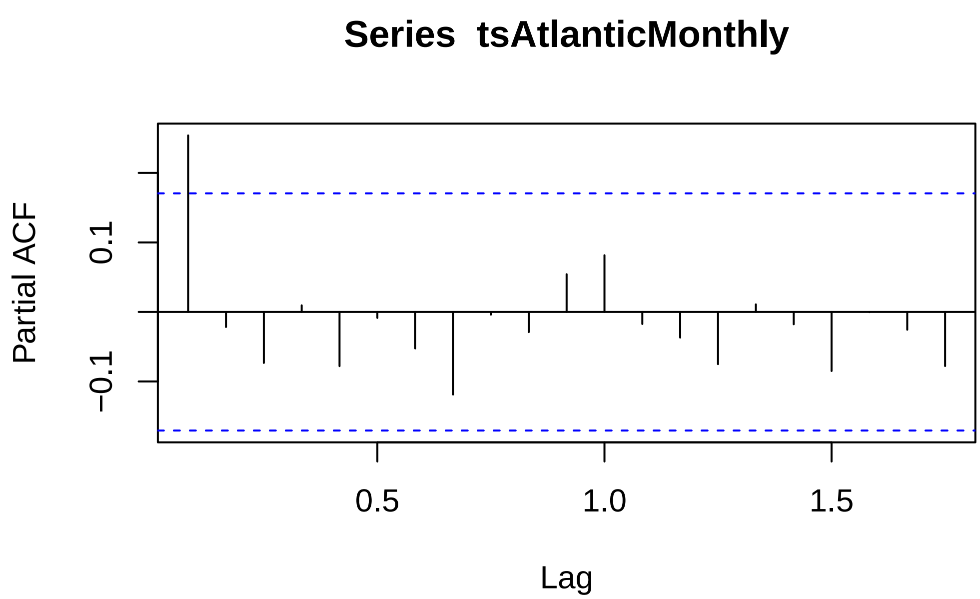


Seeing this graph we can observe that the data continues being stationary for the ARIMA model but a seasonal trend not only is more apparently but it also appear to need to be differentiated as it seems to have a decreasing linear trend

```
acf(tsAtlanticMonthly)
```




```
pacf(tsAtlanticMonthly)
```



The acf has a very clear cut-off as only 3 the values are significant which is very similar to what we had observed previously.

The main difference is in the pacf, where we can now say for sure that there is a very clear cut-off after the first value.

Model testing

These pattern suggests that an ARMA/ARIMA model might be the most appropriate so first we will check them out with the seasonal component of order 1, the so quick cut-off of both the ACF and PACF also might suggest that p and q will be smaller values.

Seasonal check

```
## initial assumption


modelMonthlySeasonal100.110 = Arima(tsAtlanticMonthly, order = c(1, 0, 0),
    seasonal = list(order = c(1, 1, 0), period = 12))

modelMonthlySeasonal100.011 = Arima(tsAtlanticMonthly, order = c(1, 0, 0),
    seasonal = list(order = c(0, 1, 1), period = 12))

modelMonthlySeasonal200.210 = Arima(tsAtlanticMonthly, order = c(2, 0, 0),
    seasonal = list(order = c(2, 1, 0), period = 12))
modelMonthlySeasonal200.012 = Arima(tsAtlanticMonthly, order = c(2, 0, 0),
    seasonal = list(order = c(0, 1, 2), period = 12))

modelMonthlySeasonal001.110 = Arima(tsAtlanticMonthly, order = c(0, 0, 1),
    seasonal = list(order = c(1, 1, 0), period = 12))

modelMonthlySeasonal001.011 = Arima(tsAtlanticMonthly, order = c(0, 0, 1),
    seasonal = list(order = c(0, 1, 1), period = 12))

modelMonthlySeasonal002.210 = Arima(tsAtlanticMonthly, order = c(0, 0, 2),
    seasonal = list(order = c(2, 1, 0), period = 12))
modelMonthlySeasonal002.012 = Arima(tsAtlanticMonthly, order = c(0, 0, 2),
    seasonal = list(order = c(0, 1, 2), period = 12))
```

```
modelMonthlySeasonal100.110
```

```
Series: tsAtlanticMonthly
ARIMA(1,0,0)(1,1,0)[12]

Coefficients:
         ar1     sar1
      0.1779  -0.4618
s.e.  0.0909   0.0839

sigma^2 = 12.06:  log likelihood = -320.1
AIC=646.2   AICc=646.4   BIC=654.56
```

```
modelMonthlySeasonal100.011
```

```
Series: tsAtlanticMonthly
ARIMA(1,0,0)(0,1,1)[12]

Coefficients:
         ar1     sma1
      0.1844  -0.8500
s.e.  0.0918   0.1123

sigma^2 = 8.74:  log likelihood = -306.88
AIC=619.76   AICc=619.97   BIC=628.12
```

```
modelMonthlySeasonal200.210
```

```
Series: tsAtlanticMonthly
ARIMA(2,0,0)(2,1,0)[12]

Coefficients:
        ar1     ar2     sar1     sar2
      0.115  0.0374  -0.7271  -0.4814
s.e.  0.094  0.0930   0.0932   0.0933

sigma^2 = 9.741:  log likelihood = -309.65
AIC=629.29   AICc=629.82   BIC=643.23
```

```
modelMonthlySeasonal200.012
```

```
Series: tsAtlanticMonthly
ARIMA(2,0,0)(0,1,2)[12]

Coefficients:
         ar1     ar2     sma1    sma2
      0.1566  0.0546  -0.9817  0.1895
s.e.  0.0947  0.0950   0.1384  0.1503

sigma^2 = 8.778:  log likelihood = -305.88
AIC=621.76   AICc=622.29   BIC=635.7
```

```
modelMonthlySeasonal001.110
```

```
Series: tsAtlanticMonthly
ARIMA(0,0,1)(1,1,0)[12]

Coefficients:
         ma1     sar1
      0.1680  -0.4634
s.e.  0.0861   0.0840

sigma^2 = 12.07:  log likelihood = -320.19
AIC=646.39   AICc=646.59   BIC=654.75
```

```
modelMonthlySeasonal001.011
```

```
Series: tsAtlanticMonthly
ARIMA(0,0,1)(0,1,1)[12]

Coefficients:
         ma1     sma1
      0.1606  -0.8446
s.e.  0.0847   0.1093

sigma^2 = 8.804:  log likelihood = -307.14
AIC=620.28   AICc=620.49   BIC=628.65
```

```
modelMonthlySeasonal002.210
```

```
Series: tsAtlanticMonthly
ARIMA(0,0,2)(2,1,0)[12]

Coefficients:
         ma1     ma2     sar1     sar2
      0.1145  0.0452  -0.7275  -0.4806
s.e.  0.0943  0.0882   0.0933   0.0936

sigma^2 = 9.745:  log likelihood = -309.66
AIC=629.33   AICc=629.85   BIC=643.26
```

```
modelMonthlySeasonal002.012
```

```
Series: tsAtlanticMonthly
ARIMA(0,0,2)(0,1,2)[12]

Coefficients:
         ma1     ma2     sma1    sma2
      0.1583  0.0745  -0.9786  0.1868
s.e.  0.0953  0.0893   0.1377  0.1495

sigma^2 = 8.784:  log likelihood = -305.89
AIC=621.78   AICc=622.3   BIC=635.72
```

So as suspected from both the time series plot and the last exercise analysis the added seasonality does increase our model goodness of fit while also penalising the increased in complexity with so far.

Now lets compare them to bigger p and q values to see if our initial assumptions do hold up

```
modelMonthlySeasonal301 = Arima(tsAtlanticMonthly, order = c(3, 0, 1), seasonal = list(ord
    1, 1), period = 12))
modelMonthlySeasonal302 = Arima(tsAtlanticMonthly, order = c(3, 0, 2), seasonal = list(ord
    1, 0), period = 12))
modelMonthlySeasonal303 = Arima(tsAtlanticMonthly, order = c(3, 1, 3), seasonal = list(ord
    1, 0), period = 12))

modelMonthlySeasonal103 = Arima(tsAtlanticMonthly, order = c(1, 0, 3), seasonal = list(ord
    1, 1), period = 12))
modelMonthlySeasonal203 = Arima(tsAtlanticMonthly, order = c(2, 1, 3), seasonal = list(ord
    1, 0), period = 12))
```

```r
modelMonthlySeasonal301
```

```
Series: tsAtlanticMonthly
ARIMA(3,0,1)(1,1,1)[12]

Coefficients:
          ar1     ar2      ar3     ma1     sar1     sma1
      -0.5728  0.1878  -0.0048  0.7442  -0.1158  -0.8073
s.e.   0.7595  0.1720   0.1369  0.7564   0.1224   0.1140

sigma^2 = 8.939:  log likelihood = -306.02
AIC=626.04   AICc=627.04   BIC=645.55
```

```r
modelMonthlySeasonal302
```

```
Series: tsAtlanticMonthly
ARIMA(3,0,2)(1,1,0)[12]

Coefficients:
          ar1      ar2     ar3     ma1     ma2     sar1
      -1.4367  -0.5902  0.1291  1.6983  1.0000  -0.4382
s.e.   0.0944   0.1532  0.0945  0.0436  0.0489   0.0871

sigma^2 = 11.34:  log likelihood = -316.59
AIC=647.19   AICc=648.19   BIC=666.7
```

```r
modelMonthlySeasonal303
```

```
Series: tsAtlanticMonthly
ARIMA(3,1,3)(1,1,0)[12]

Coefficients:
          ar1      ar2     ar3     ma1      ma2      ma3     sar1
      -1.4279  -0.5751  0.1373  0.6984  -0.6984  -1.0000  -0.4313
s.e.   0.0952   0.1547  0.0954  0.0533   0.0552   0.0576   0.0877

sigma^2 = 11.54:  log likelihood = -316.86
AIC=649.73   AICc=651.04   BIC=671.96
```

```r
modelMonthlySeasonal103
```

```
Series: tsAtlanticMonthly
ARIMA(1,0,3)(1,1,1)[12]

Coefficients:
          ar1     ma1     ma2      ma3     sar1     sma1
      -0.6951  0.8708  0.1931  -0.0013  -0.1134  -0.8029
s.e.   0.4854  0.4812  0.1503   0.1321   0.1194   0.1136

sigma^2 = 8.954:  log likelihood = -306.02
AIC=626.03   AICc=627.03   BIC=645.55
```

```r
modelMonthlySeasonal203
```

```
Series: tsAtlanticMonthly
ARIMA(2,1,3)(1,1,0)[12]

Coefficients:
          ar1      ar2     ma1      ma2      ma3     sar1
      -1.3087  -0.6017  0.5001  -0.6702  -0.8299  -0.4545
s.e.   0.2576   0.1557  0.2045   0.1332   0.1180   0.0979

sigma^2 = 12.02:  log likelihood = -318.1
AIC=650.19   AICc=651.2   BIC=669.65
```

As we can see here the initial assumption that a smaller p and q value would better fit the model.

Now we will use auto.arima to verify if our assumptions were indeed correct

auto arima check

```r
`?`(auto.arima)
```

```
starting httpd help server ... done
```

```r
monthlyModelAuto = auto.arima(tsAtlanticMonthly, max.d = 0, max.p = 5, max.q = 5,
    D = 1)
monthlyModelAuto
```

```
Series: tsAtlanticMonthly
ARIMA(1,0,0)(0,1,1)[12]

Coefficients:
         ar1     sma1
      0.1844  -0.8500
s.e.  0.0918   0.1123

sigma^2 = 8.74:  log likelihood = -306.88
AIC=619.76   AICc=619.97   BIC=628.12
```

From what we can see the auto arima has indeed confirmed our initial assumption by picking a model that we already had seen as the best performer ARIMA(1,0,0)(0,1,1)[12]

Forecasting

Now using the forecast function we will forecast the next 4 quarters of 2021

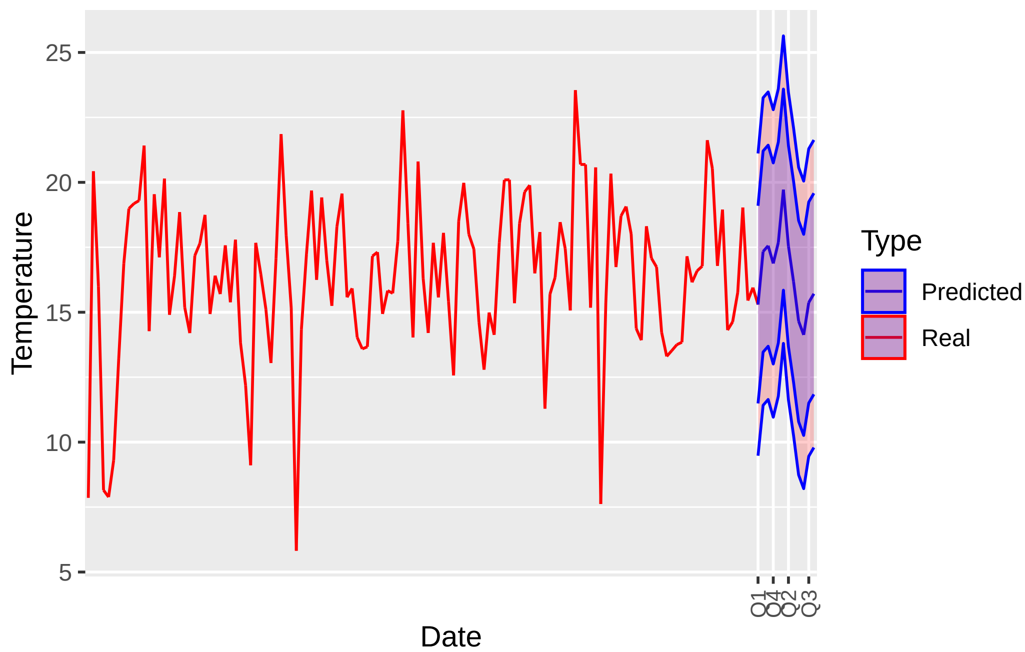
```r
forecast(modelMonthlySeasonal100.011, 12)
```

```
         Point Forecast    Lo 80    Hi 80     Lo 95    Hi 95
Jan 2021       15.29474 11.49064 19.09884  9.476872 21.11261
Feb 2021       17.68442 13.81636 21.55247 11.768741 23.60009
Mar 2021       19.72032 15.85011 23.59053 13.801347 25.63929
Apr 2021       17.53922 13.66894 21.40951 11.620140 23.45831
May 2021       16.17383 12.30355 20.04411 10.254741 22.09292
Jun 2021       14.65775 10.78746 18.52803  8.738657 20.57683
Jul 2021       14.12800 10.25772 17.99829  8.208914 20.04709
Aug 2021       15.37476 11.50447 19.24504  9.455670 21.29385
Sep 2021       15.71081 11.84052 19.58109  9.791718 21.62989
Oct 2021       17.33660 13.46632 21.20689 11.417514 23.25569
Nov 2021       17.56114 13.69086 21.43141 11.642055 23.48022
Dec 2021       16.87677 13.00663 20.74691 10.957906 22.79563
```

But this data is better visualized in a graph to better understand if the predictions are sensible compared to our real data.

```
predictedArimaSeasonalDF = data.frame(forecast(modelMonthlySeasonal100.011,
    12))

predictedArimaSeasonalDF$YearMonth = c("2021-1", "2021-2", "2021-3", "2021-4",
    "2021-5", "2021-6", "2021-7", "2021-8", "2021-9", "2021-10", "2021-11",
    "2021-12")

predictedArimaSeasonalDF$Type = "Predicted"

predictedArimaSeasonalDF$Temperature = predictedArimaSeasonalDF$Point.Forecast

YearMonthlyAverage$Type = "Real"
YearMonthlyAverage$Temperature = YearMonthlyAverage$AverageStrength


# Combine real_data and pred_data into a single data frame
combinedDataframeAMOC = rbind(data.frame(Date = YearMonthlyAverage$YearMonth,
    Temperature = YearMonthlyAverage$Temperature, Type = "Real"), data.frame(Date = predic
    Temperature = predictedArimaSeasonalDF$Temperature, Type = "Predicted"))
```

```
# Create the ggplot
plotARIMA2 = ggplot(combinedDataframeAMOC, aes(x = Date, y = Temperature,
    color = Type, group = 1)) + geom_line() + scale_color_manual(values = c("blue",
    "red"))

# Add the 80% and 95% confidence intervals
plotARIMA2 = plotARIMA2 + geom_ribbon(data = predictedArimaSeasonalDF, aes(x = YearMonth,
    ymin = Lo.95, ymax = Hi.95), fill = "red", alpha = 0.2) + geom_ribbon(data = predicted
    aes(x = YearMonth, ymin = Lo.80, ymax = Hi.80), fill = "blue", alpha = 0.2)

# Adjust the x-axis labels
plotARIMA2 = plotARIMA2 + scale_x_discrete(breaks = c("2021-1", "2021-4",
    "2021-8", "2021-12"), labels = c("Q1", "Q2", "Q3", "Q4"))

plotARIMA2 = plotARIMA2 + theme(axis.text.x = element_text(angle = 90, vjust = 0.5,
    size = 8))

# Display the plot
print(plotARIMA2)
```



DLM

model fitting

```
## linear model, order = 2, quadratic order = 3 , etc

## what we want is a linear model with a seasonal component so we add
## the 2 components together in a model

## things to try, another term like quadratic, or a arma component
## stacked on top of this

## Initial model with a linear polynomial and a seasonal component

buildFun = function(x) {
    dlmModPoly(order = 2, dV = exp(x[1]), dW = c(0, exp(x[2]))) + dlmModSeas(frequency = 1
        dV = 0, dW = c(exp(x[3]), rep(0, 10)))
}

linearDLM = dlmMLE(tsAtlanticMonthly, parm = c(0, 0, 0), build = buildFun)
```

```r
linearDLM$par
```

```
[1]   2.044026 -11.586366  -4.421181
```

```r
fittedLinearDLM = buildFun(linearDLM$par)
V(fittedLinearDLM)
```

```
         [,1]
[1,] 7.721632
```

```r
W(fittedLinearDLM)
```

```
      [,1]         [,2]       [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11]
 [1,]    0 0.000000e+00 0.00000000    0    0    0    0    0    0     0     0
 [2,]    0 9.291914e-06 0.00000000    0    0    0    0    0    0     0     0
 [3,]    0 0.000000e+00 0.01202003    0    0    0    0    0    0     0     0
 [4,]    0 0.000000e+00 0.00000000    0    0    0    0    0    0     0     0
 [5,]    0 0.000000e+00 0.00000000    0    0    0    0    0    0     0     0
 [6,]    0 0.000000e+00 0.00000000    0    0    0    0    0    0     0     0
 [7,]    0 0.000000e+00 0.00000000    0    0    0    0    0    0     0     0
 [8,]    0 0.000000e+00 0.00000000    0    0    0    0    0    0     0     0
 [9,]    0 0.000000e+00 0.00000000    0    0    0    0    0    0     0     0
[10,]    0 0.000000e+00 0.00000000    0    0    0    0    0    0     0     0
[11,]    0 0.000000e+00 0.00000000    0    0    0    0    0    0     0     0
[12,]    0 0.000000e+00 0.00000000    0    0    0    0    0    0     0     0
[13,]    0 0.000000e+00 0.00000000    0    0    0    0    0    0     0     0
      [,12] [,13]
 [1,]     0     0
 [2,]     0     0
 [3,]     0     0
 [4,]     0     0
 [5,]     0     0
 [6,]     0     0
 [7,]     0     0
 [8,]     0     0
 [9,]     0     0
[10,]     0     0
[11,]     0     0
```

```
[12,]     0     0
[13,]     0     0
```

```r
## second model with a quadratic polynomial and a seasonal component

buildFunQuad = function(x) {
    dlmModPoly(order = 3, dV = exp(x[1]), dW = c(0, exp(x[2]), exp(x[3]))) +
        dlmModSeas(frequency = 12, dV = 0, dW = c(exp(x[4]), rep(0, 10)))
}

quadraticDLM = dlmMLE(tsAtlanticMonthly, parm = c(0, 0, 0, 0), build = buildFunQuad)

quadraticDLM$par
```

```
[1]   2.047135 -21.060474 -56.104784 -12.300531
```

```r
fittedQuadraticDLM = buildFunQuad(quadraticDLM$par)

V(fittedQuadraticDLM)
```

```
         [,1]
[1,] 7.745678
```

```r
W(fittedQuadraticDLM)
```

```
      [,1]         [,2]         [,3]         [,4] [,5] [,6] [,7] [,8] [,9]
 [1,]    0 0.000000e+00 0.000000e+00 0.000000e+00    0    0    0    0    0
 [2,]    0 7.137603e-10 0.000000e+00 0.000000e+00    0    0    0    0    0
 [3,]    0 0.000000e+00 4.305286e-25 0.000000e+00    0    0    0    0    0
 [4,]    0 0.000000e+00 0.000000e+00 4.549326e-06    0    0    0    0    0
 [5,]    0 0.000000e+00 0.000000e+00 0.000000e+00    0    0    0    0    0
 [6,]    0 0.000000e+00 0.000000e+00 0.000000e+00    0    0    0    0    0
 [7,]    0 0.000000e+00 0.000000e+00 0.000000e+00    0    0    0    0    0
 [8,]    0 0.000000e+00 0.000000e+00 0.000000e+00    0    0    0    0    0
 [9,]    0 0.000000e+00 0.000000e+00 0.000000e+00    0    0    0    0    0
[10,]    0 0.000000e+00 0.000000e+00 0.000000e+00    0    0    0    0    0
[11,]    0 0.000000e+00 0.000000e+00 0.000000e+00    0    0    0    0    0
[12,]    0 0.000000e+00 0.000000e+00 0.000000e+00    0    0    0    0    0
```

```
[13,]    0 0.000000e+00 0.000000e+00 0.000000e+00    0    0    0    0    0
[14,]    0 0.000000e+00 0.000000e+00 0.000000e+00    0    0    0    0    0
      [,10] [,11] [,12] [,13] [,14]
 [1,]     0     0     0     0     0
 [2,]     0     0     0     0     0
 [3,]     0     0     0     0     0
 [4,]     0     0     0     0     0
 [5,]     0     0     0     0     0
 [6,]     0     0     0     0     0
 [7,]     0     0     0     0     0
 [8,]     0     0     0     0     0
 [9,]     0     0     0     0     0
[10,]     0     0     0     0     0
[11,]     0     0     0     0     0
[12,]     0     0     0     0     0
[13,]     0     0     0     0     0
[14,]     0     0     0     0     0
```

Now we will compare both models through their log likelihood using the dlmLL function and see if the extra flexibility from the extra polynomial function is providing a better fit

```r
dlmLL(tsAtlanticMonthly, fittedLinearDLM)
```

```
[1] 309.5446
```

```r
dlmLL(tsAtlanticMonthly, fittedQuadraticDLM)
```

```
[1] 324.4748
```

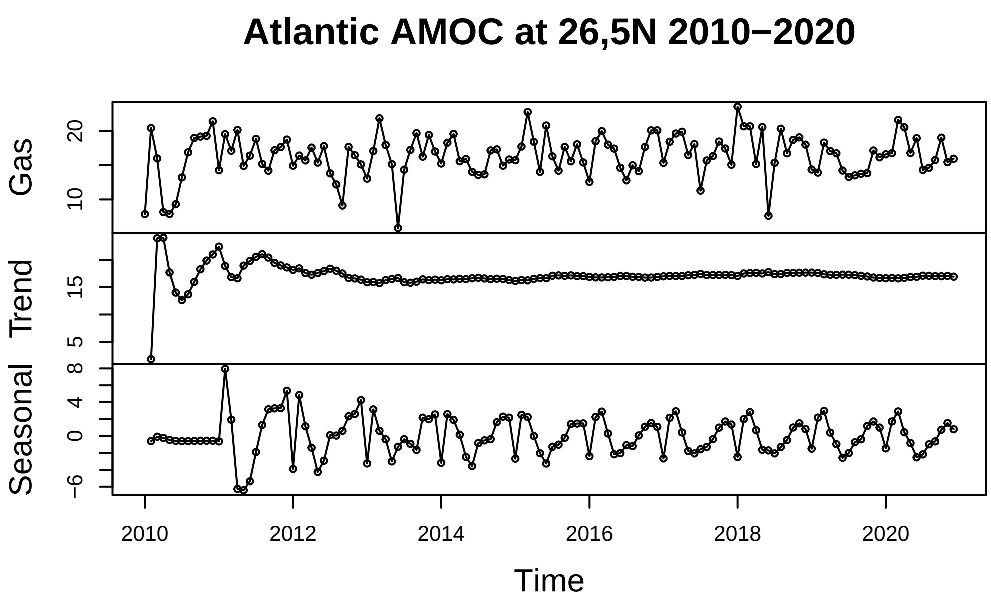
As we can see the dlm model using only a linear polynomial has a lower log likelihood than the model with an extra quadratic term, meaning this extra flexibility does not contribute to a better model fit and as such we will use the linear fitted model to do our forecasting.

```r
amocPredict = dlmFilter(tsAtlanticMonthly, mod = fittedLinearDLM)
summary(amocPredict)
```

```
    Length Class  Mode
y   132    ts     numeric
mod  10    dlm    list
```

```
m   1729   mts    numeric
U.C  133   -none- list
D.C 1729   -none- numeric
a   1716   mts    numeric
U.R  132   -none- list
D.R 1716   -none- numeric
f    132   ts     numeric
```

```r
x = cbind(tsAtlanticMonthly, dropFirst(amocPredict$a[, c(1, 3)]))
x = window(x, start = c(2010, 1))
colnames(x) = c("Gas", "Trend", "Seasonal")
plot(x, type = "o", main = "Atlantic AMOC at 26,5N 2010-2020")
```



Forecast

```r
amocForecastMonthly = dlmForecast(amocPredict, nAhead = 12)
summary(amocForecastMonthly)
```

```
  Length Class  Mode
a 156    mts    numeric
```

```
R  12    -none- list
f  12    ts     numeric
Q  12    -none- list
```

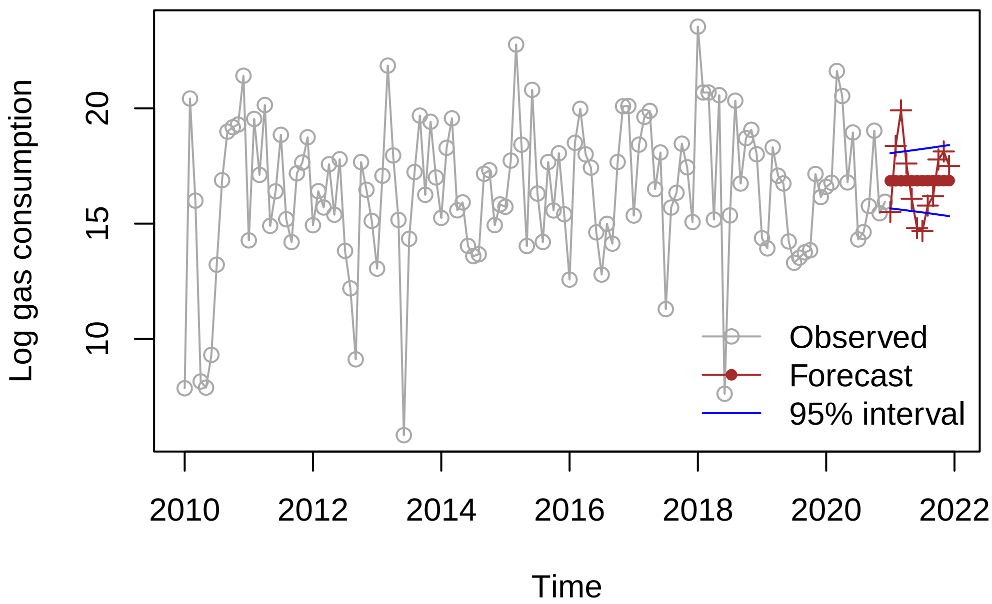
```r
dim(amocForecastMonthly$a)
```

```
[1] 12 13
```

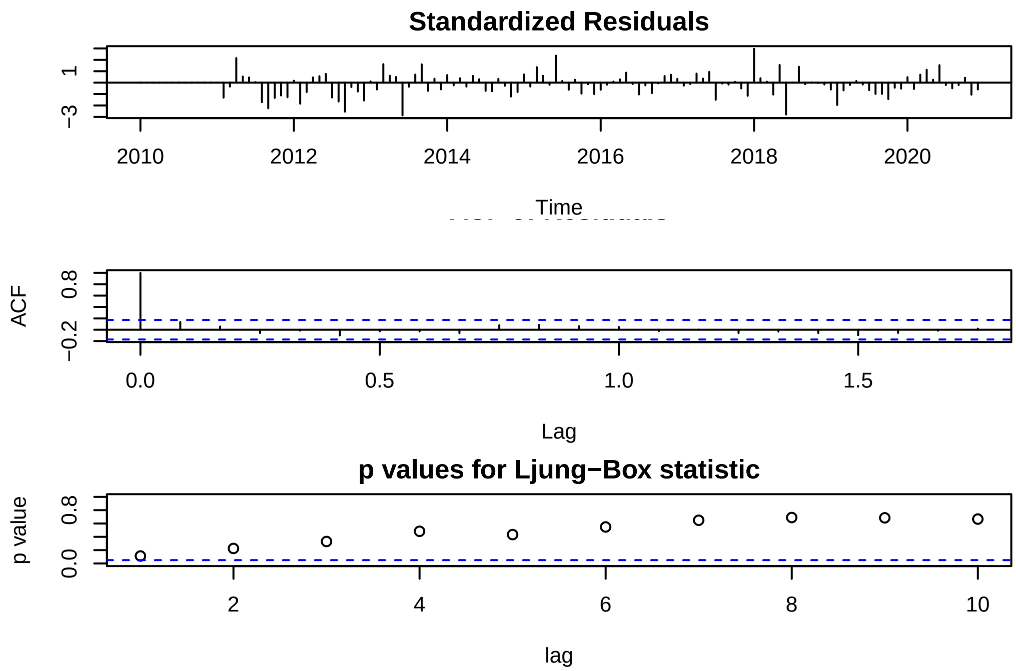
```r
dim(amocForecastMonthly$f)
```

```
[1] 12  1
```

```r
sqrtR = sapply(amocForecastMonthly$R, function(x) sqrt(x[1, 1]))
pl = amocForecastMonthly$a[, 1] + qnorm(0.025, sd = sqrtR)
pu = amocForecastMonthly$a[, 1] + qnorm(0.975, sd = sqrtR)

x = ts.union(window(tsAtlanticMonthly, start = c(2010, 1)), amocForecastMonthly$a[,
    1], amocForecastMonthly$f, pl, pu)
par(mar = c(4, 4, 2, 2))
plot(x, plot.type = "single", type = "o", pch = c(1, 20, 3, NA, NA), col = c("darkgrey",
    "brown", "brown", "blue", "blue"), ylab = "Log gas consumption")

legend("bottomright", legend = c("Observed", "Forecast", "95% interval"),
    bty = "n", pch = c(1, 20, NA), lty = 1, col = c("darkgrey", "brown",
        "blue"))
```




```
# Set smaller margins
par(mar = c(4, 4, 2, 2))

tsdiag(amocPredict)
```




```r
# Reset margins
par(mar = c(5, 4, 4, 2) + 0.1)
```

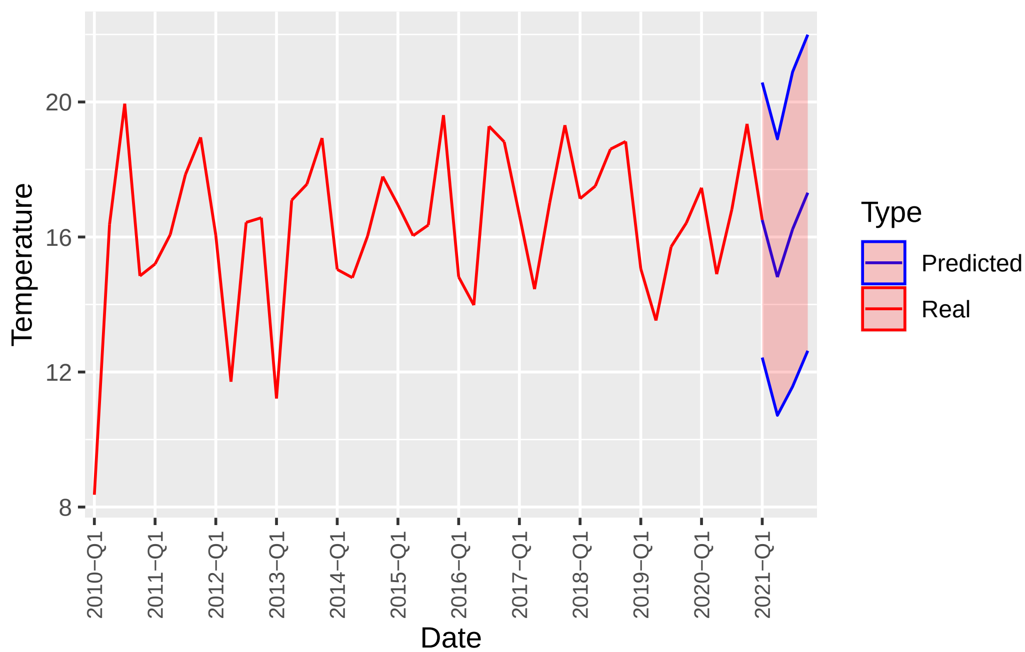
Lastly checking the residuals, they seem to be mostly normally distributed with a good mixture of over and under estimations, especially in the middle with some slight seasonality on both ends being present

2 f)

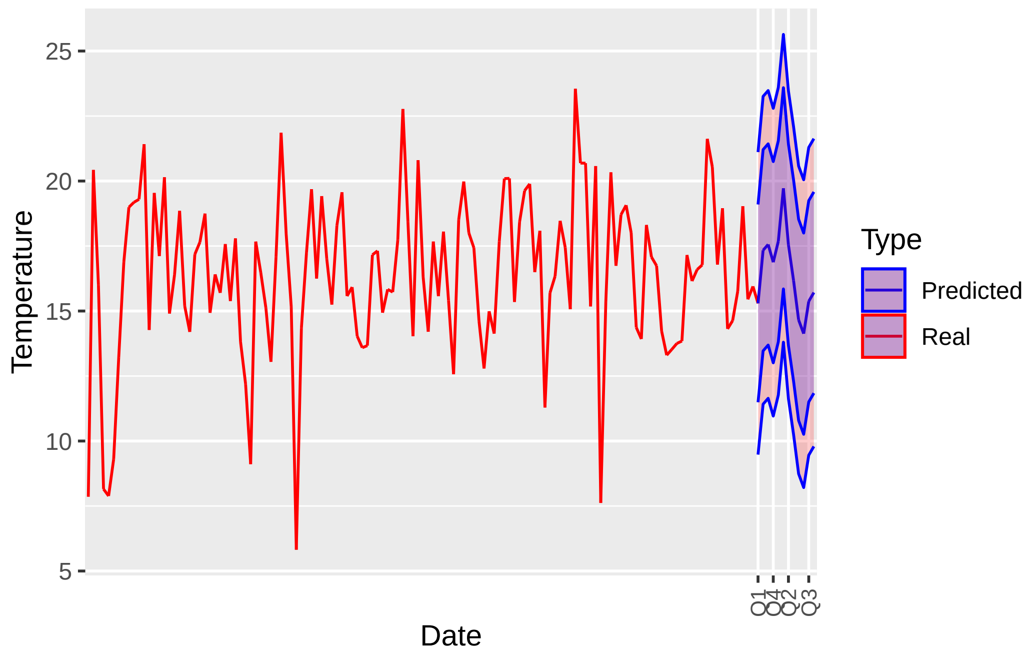
Now starting with the ARIMA models

```r
predictedArimaDF = data.frame(forecast(model200, 4))

predictedArimaDF$YearQuarter = c("2021-Q1", "2021-Q2", "2021-Q3", "2021-Q4")

# Combine real_data and pred_data into a single data frame
combinedDataframeAMOC = rbind(data.frame(Date = YearQuarterAverage$YearQuarter,
    Temperature = YearQuarterAverage$AverageStrength, Type = "Real"), data.frame(Date = pr
    Temperature = predictedArimaDF$Point.Forecast, Type = "Predicted"))

predictedArimaDF$Temperature = predictedArimaDF$Point.Forecast
```

```
predictedArimaDF$Type = "Predicted"


# Create the ggplot
plotARIMA = ggplot(combinedDataframeAMOC, aes(x = Date, y = Temperature,
    color = Type, group = 1)) + geom_line() + scale_color_manual(values = c("blue",
    "red"))

# Add the 95% confidence interval
plotARIMA = plotARIMA + geom_ribbon(data = predictedArimaDF, aes(x = YearQuarter,
    ymin = Lo.95, ymax = Hi.95), fill = "red", alpha = 0.2)

# Adjust the x-axis labels
plotARIMA = plotARIMA + scale_x_discrete(breaks = combinedDataframeAMOC$Date[c(TRUE,
    rep(FALSE, 3))], labels = combinedDataframeAMOC$Date[c(TRUE, rep(FALSE,
    3))])

plotARIMA = plotARIMA + theme(axis.text.x = element_text(angle = 90, vjust = 0.5,
    size = 8))

print(plotARIMA)
```



```r
print(plotARIMA2)
```



The most obvious difference is the level of detail on the seasonality, with the monthly averages capturing an almost opposite effect than the quarterly averages in both the predicted and also in some of the real data, the forecast also has the opposite trend, with an actual expected decrease in Sverdrups around between the 2nd quarter and the mid 3rd quarter which again seems to follow the opposite trend on the quartely data.

```r
sqrtR = sapply(amocForecastMonthly$R, function(x) sqrt(x[1, 1]))
pl = amocForecastMonthly$a[, 1] + qnorm(0.025, sd = sqrtR)
pu = amocForecastMonthly$a[, 1] + qnorm(0.975, sd = sqrtR)

x = ts.union(window(tsAtlanticMonthly, start = c(2010, 1)), amocForecastMonthly$a[,
    1], amocForecastMonthly$f, pl, pu)
par(mar = c(4, 4, 2, 2))
plot(x, plot.type = "single", type = "o", pch = c(1, 20, 3, NA, NA), col = c("darkgrey",
    "brown", "brown", "blue", "blue"), ylab = "Log gas consumption")

legend("bottomright", legend = c("Observed", "Forecast", "95% interval"),
    bty = "n", pch = c(1, 20, NA), lty = 1, col = c("darkgrey", "brown",
        "blue"))
```

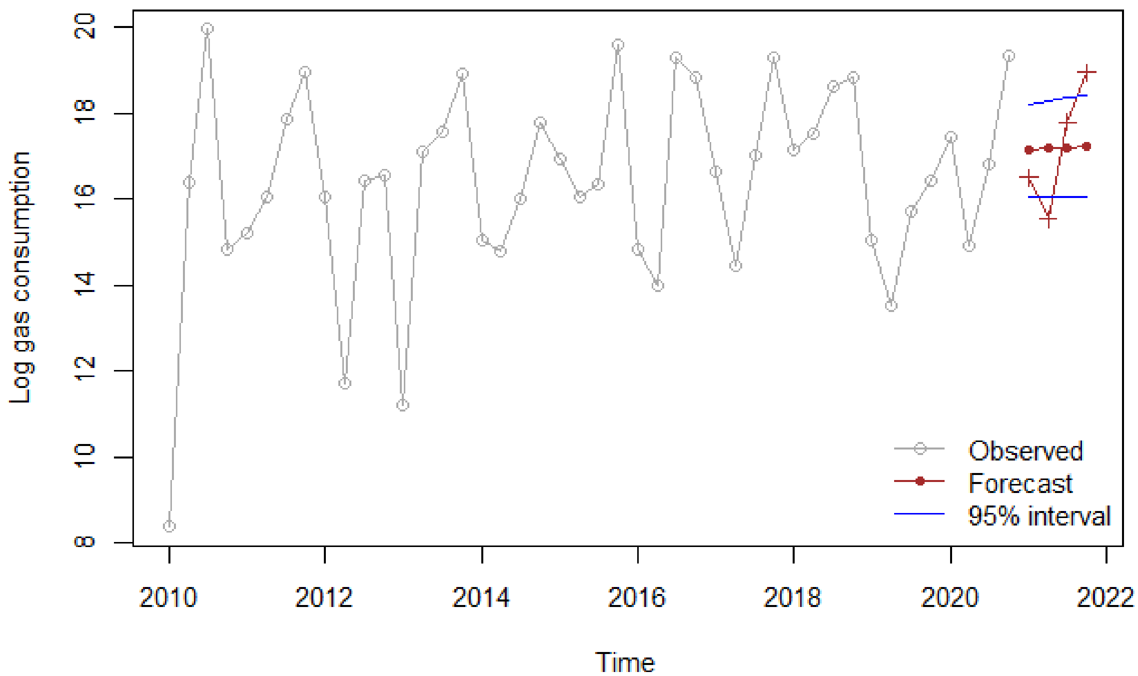
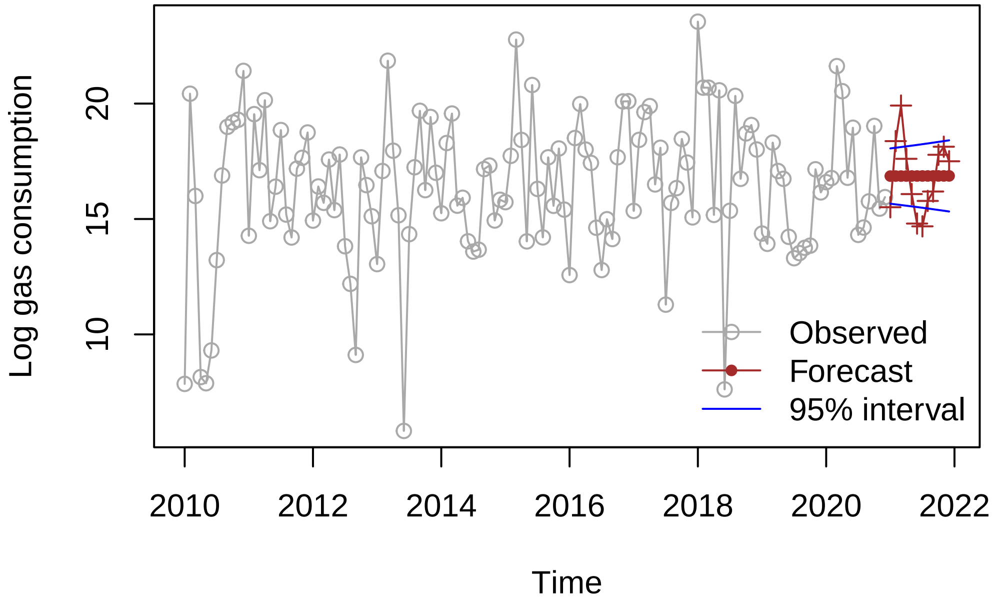


Figure 1: 1st dlm



Observing the 2 graphs of the dlm models we can see that although the quarterly predictions do not completly predict the spike during the first half of the year compared to the monthly data the second half of the year seems to be relative similarly forecasted with the exception of December where the monthly data show again a decrease but the quarterly data is not capable of capturing.

Despise these changes the predicted overall trend is quite similar with the quaterly trend very slighty increasing the monthly data seeming to remaind constant.

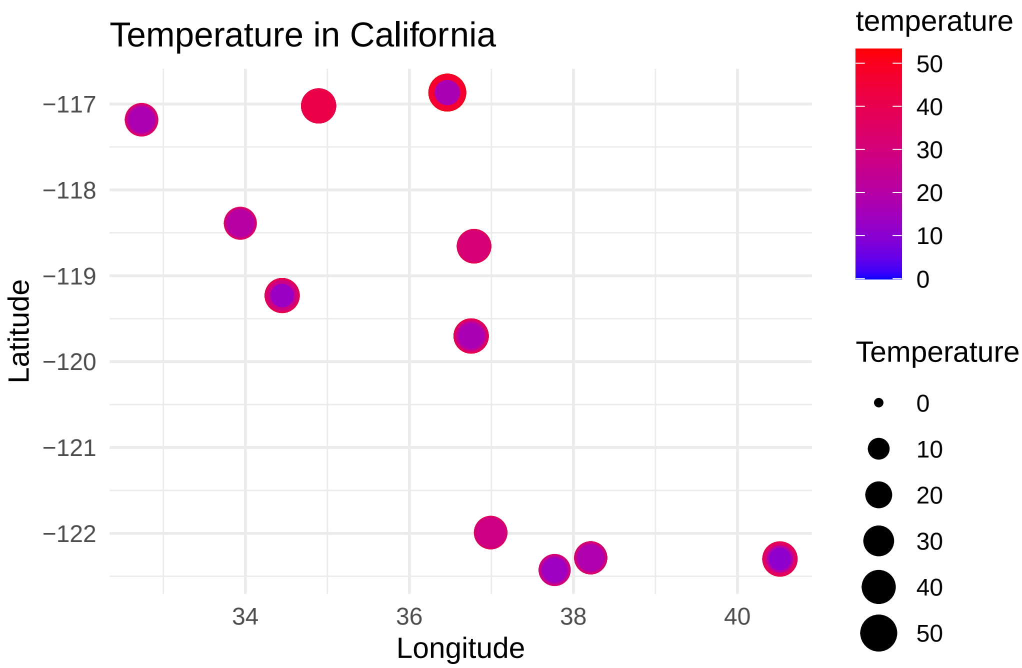
Question 3

Question 3 a)

my approach is to see the max temp in the entire state with 8 cities

```
californiaLongTempDF = pivot_longer(californiaTempDF, cols = -Date, names_to = "Location",
    values_to = "Temperature")

spatialTemperatureCaliforniaDF = merge(californiaLongTempDF, californiaSpatialDataDF)


ggplot(data = spatialTemperatureCaliforniaDF) + geom_point(aes(x = Lat, y = Long,
```

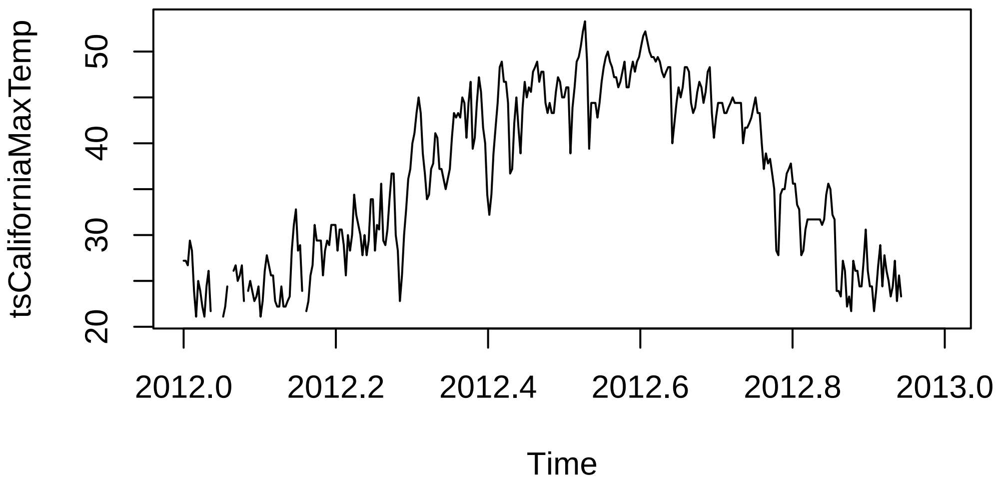
```
    color = Temperature, size = Temperature)) + scale_color_continuous(low = "blue",
    high = "red") + labs(title = "Temperature in California", x = "Longitude",
    y = "Latitude", color = "temperature") + theme_minimal()
```




```
californiaTempDF$Date = as.Date(as.character(californiaTempDF$Date), format = "%Y%m%d",
    origin = "1970-01-01")

californiaTempDF$max <- apply(californiaTempDF, 1, max, na.rm = TRUE)


tsCaliforniaMaxTemp = ts(californiaTempDF$max, start = c(2012, 1), frequency = 366)
```

```
plot.ts(tsCaliforniaMaxTemp)
```

```
Warning in xy.coords(x, NULL, log = log, setLab = FALSE): NAs introduced by
coercion

Warning in xy.coords(x, y): NAs introduced by coercion
```



As we can see from the time series trend, august to September seems to be the hottest months while january to feburary seems to be the coldest months in the californian state.

3 b)

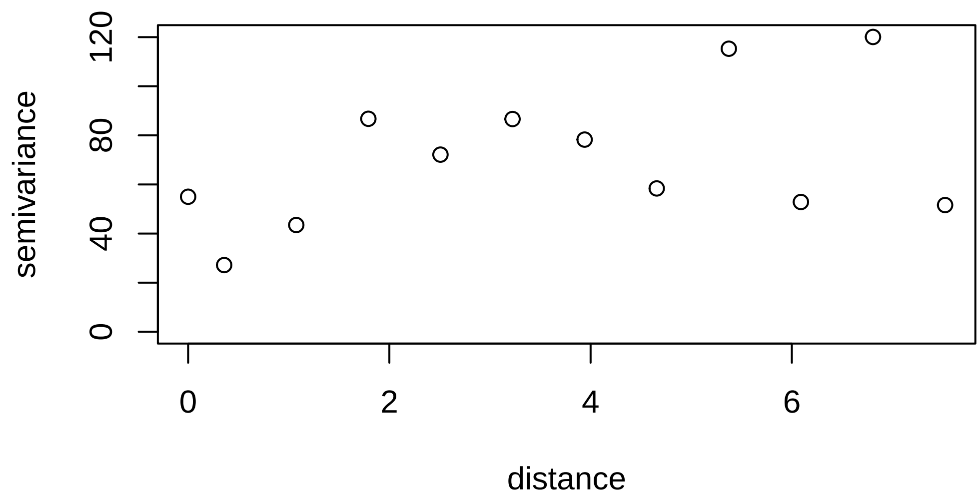
```
geoDataCalifornia = as.geodata(spatialTemperatureCaliforniaDF, coords.col = 4:5,
    data.col = "Temperature", covar.col = "Elev")
```

```
as.geodata: 4004 replicated data locations found.
 Consider using jitterDupCoords() for jittering replicated locations.
WARNING: there are data at coincident or very closed locations, some of the geoR's functions
 Use function dup.coords() to locate duplicated coordinates.
 Consider using jitterDupCoords() for jittering replicated locations
```

```
variogramCalifornia = variog(geoDataCalifornia)
```

```
variog: computing omnidirectional variogram
variog: co-locatted data found, adding one bin at the origin
```

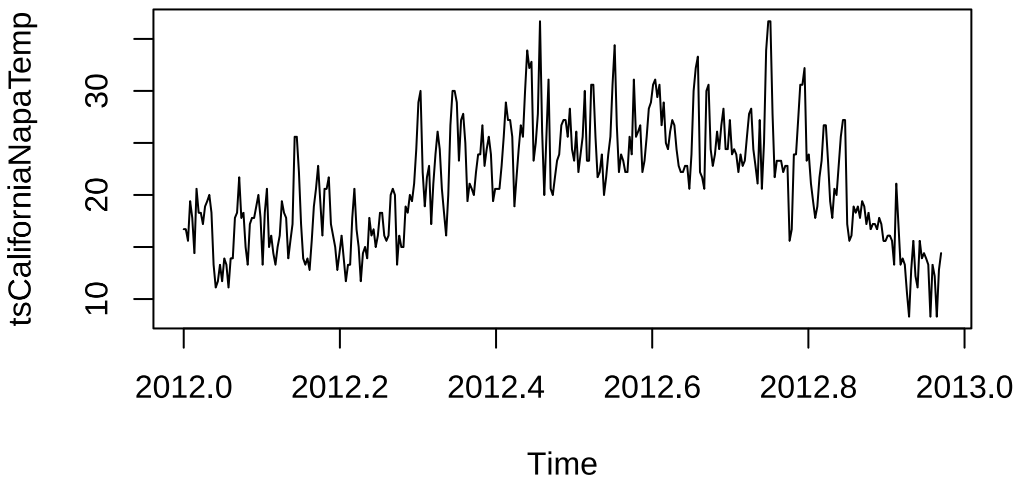
```r
plot(variogramCalifornia)
```



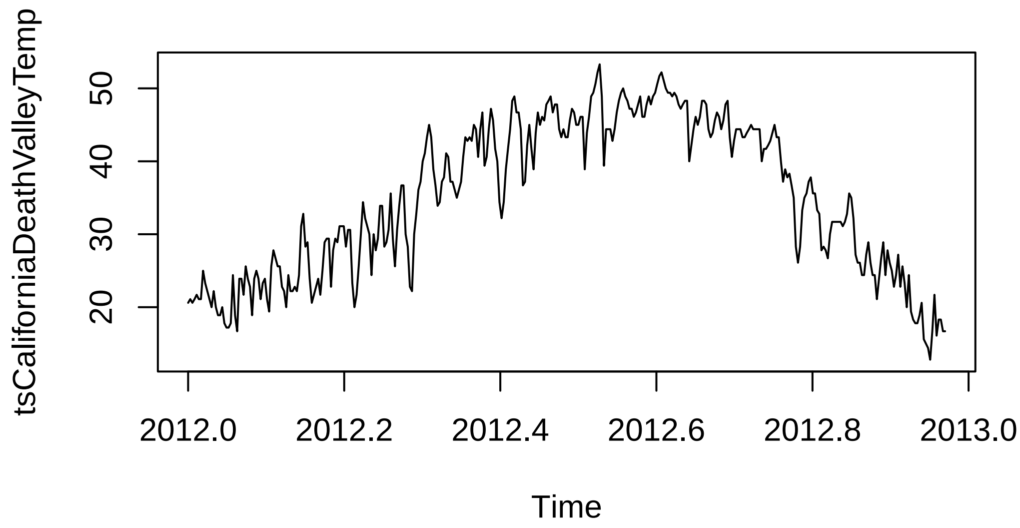
3 c)

Here we create a ts object with frequency 366 because 2012 does indeed have the 29th of February, removing the dates from the 9th to the 17th of november

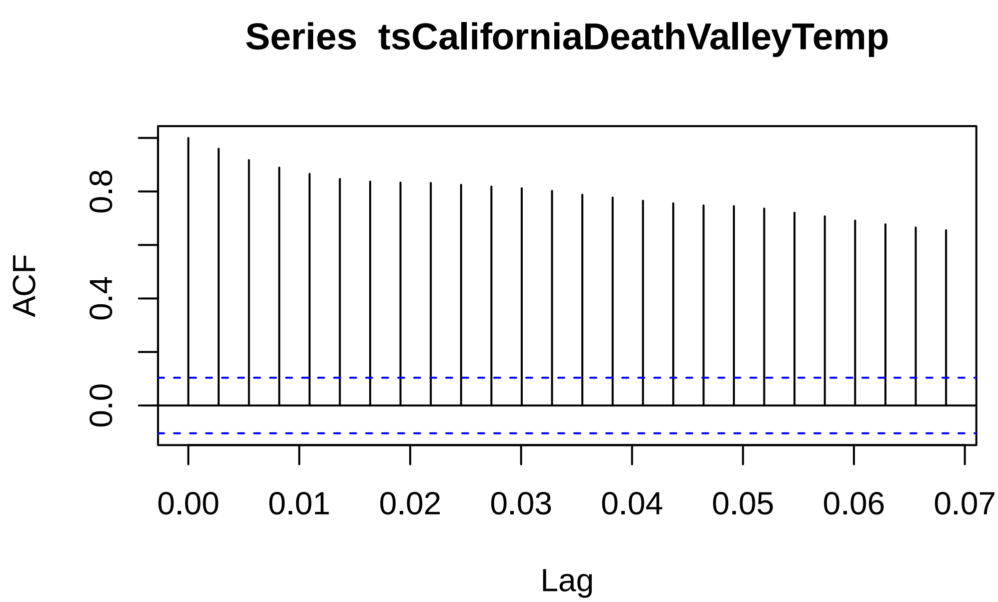
```r
toPredictValues = californiaTempDF %>%
    filter((Date >= as.Date("2012-11-09") & Date <= as.Date("2012-11-17")))

californiaTempDF = californiaTempDF %>%
    filter(!(Date >= as.Date("2012-11-09") & Date <= as.Date("2012-11-17")))


tsCaliforniaNapaTemp = ts(californiaTempDF$Napa, start = c(2012, 1), frequency = 366)

plot.ts(tsCaliforniaNapaTemp)
```



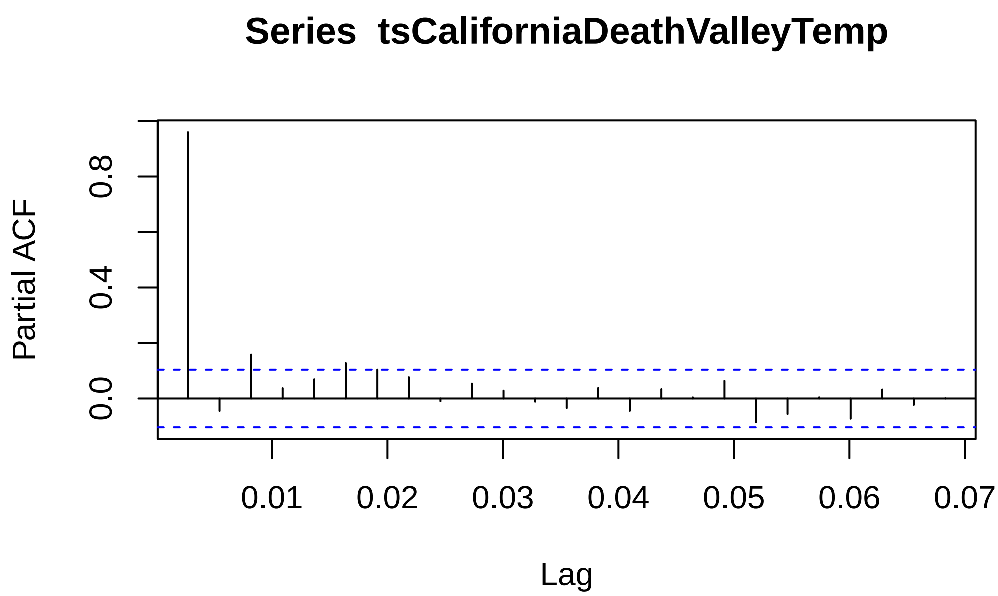

```
tsCaliforniaDeathValleyTemp = ts(californiaTempDF$`Death Valley`, start = c(2012,
    1), frequency = 366)

plot.ts(tsCaliforniaDeathValleyTemp)
```




```
acf(tsCaliforniaDeathValleyTemp)
```



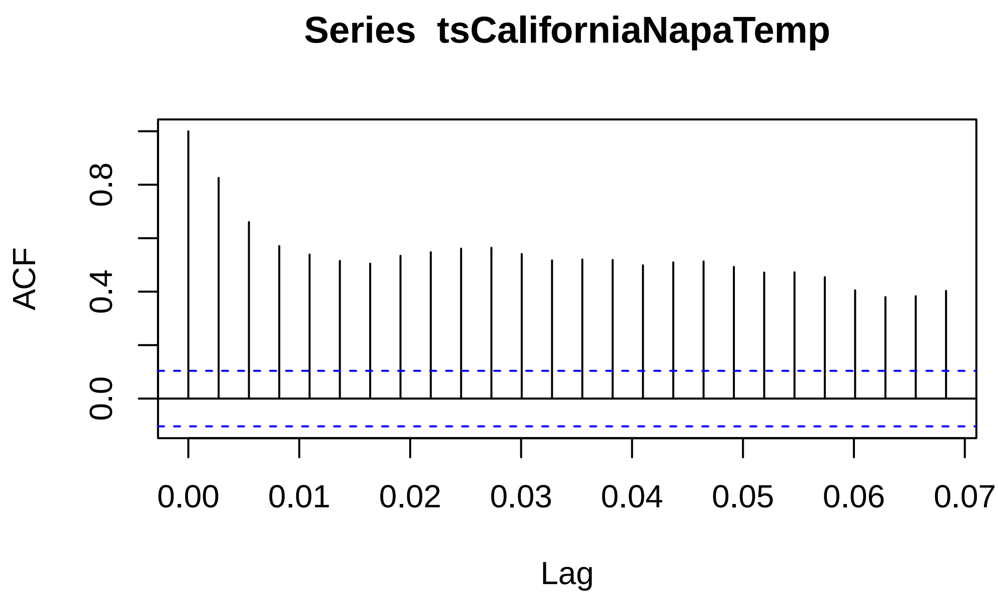
```r
pacf(tsCaliforniaDeathValleyTemp)
```



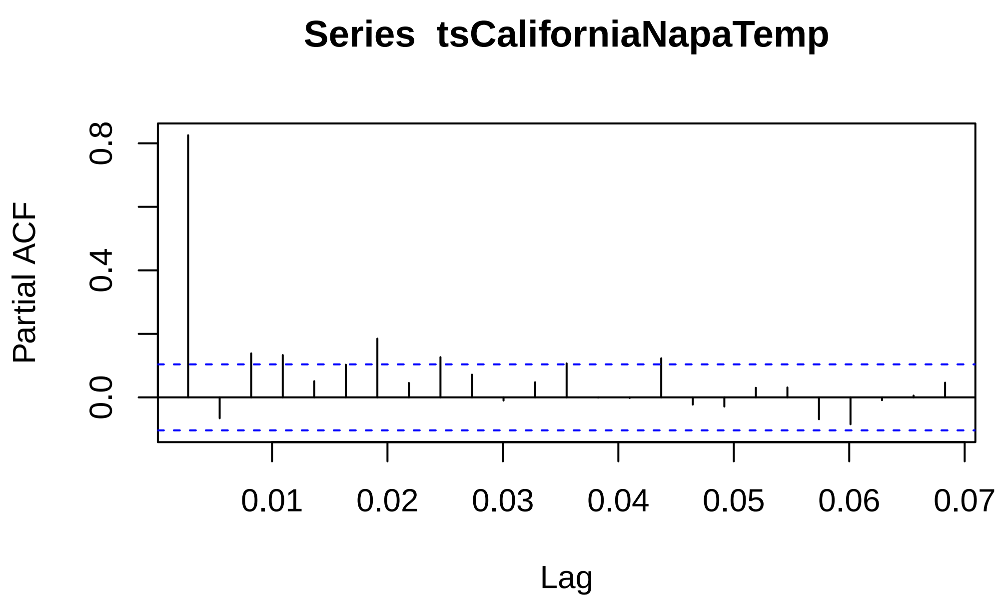
In death valley we can clearly see that ACF is very slowly descending without any cut-off and PAC seems to have a very quick cut off, this means that the best model will most likely will be AR with a possibly larger p value

Now checking for Napa

```r
acf(tsCaliforniaNapaTemp)
```




```
pacf(tsCaliforniaNapaTemp)
```



We can once again see the same pattern of a very slowly decreasing ACF but this time PACF does seem to at least not has such a clear cut off meanig there might a smaller q non-zero q value requiring an ARIMA for this city

The model for both cities does seem stationary with a linear trend on seasonality however it is not clear enough since we only have 1 year worth of data

model checking Nappa

```
## initial assumption


modelMonthlySeasonal100.110Napa = Arima(tsCaliforniaNapaTemp, order = c(1,
    0, 0), seasonal = list(order = c(1, 1, 0), period = 12))

modelMonthlySeasonal100.011Napa = Arima(tsCaliforniaNapaTemp, order = c(1,
    0, 0), seasonal = list(order = c(0, 1, 1), period = 12))

modelMonthlySeasonal200.210Napa = Arima(tsCaliforniaNapaTemp, order = c(2,
    0, 0), seasonal = list(order = c(2, 1, 0), period = 12))
modelMonthlySeasonal200.012Napa = Arima(tsCaliforniaNapaTemp, order = c(2,
    0, 0), seasonal = list(order = c(0, 1, 2), period = 12))

modelMonthlySeasonal001.110Napa = Arima(tsCaliforniaNapaTemp, order = c(0,
    0, 1), seasonal = list(order = c(1, 1, 0), period = 12))

modelMonthlySeasonal001.011Napa = Arima(tsCaliforniaNapaTemp, order = c(0,
    0, 1), seasonal = list(order = c(0, 1, 1), period = 12))

modelMonthlySeasonal002.210Napa = Arima(tsCaliforniaNapaTemp, order = c(0,
    0, 2), seasonal = list(order = c(2, 1, 0), period = 12))
modelMonthlySeasonal002.012Napa = Arima(tsCaliforniaNapaTemp, order = c(0,
    0, 2), seasonal = list(order = c(0, 1, 2), period = 12))
```

```
modelMonthlySeasonal100.110Napa
```

```
Series: tsCaliforniaNapaTemp
ARIMA(1,0,0)(1,1,0)[12]

Coefficients:
         ar1     sar1
```

```
      0.6323  -0.4915
s.e.  0.0418   0.0474

sigma^2 = 14.03:  log likelihood = -943.36
AIC=1892.73   AICc=1892.8   BIC=1904.25
```

```
modelMonthlySeasonal100.011Napa
```

```
Series: tsCaliforniaNapaTemp
ARIMA(1,0,0)(0,1,1)[12]

Coefficients:
         ar1     sma1
      0.8307  -0.9637
s.e.  0.0329   0.0780

sigma^2 = 10.12:  log likelihood = -900.8
AIC=1807.61   AICc=1807.68   BIC=1819.13
```

```
modelMonthlySeasonal200.210Napa
```

```
Series: tsCaliforniaNapaTemp
ARIMA(2,0,0)(2,1,0)[12]

Coefficients:
         ar1      ar2     sar1     sar2
      0.7592  -0.1464  -0.6312  -0.2890
s.e.  0.0535   0.0540   0.0539   0.0534

sigma^2 = 12.47:  log likelihood = -923.09
AIC=1856.19   AICc=1856.36   BIC=1875.39
```

```
modelMonthlySeasonal200.012Napa
```

```
Series: tsCaliforniaNapaTemp
ARIMA(2,0,0)(0,1,2)[12]

Coefficients:
         ar1      ar2     sma1    sma2
```

```
      0.8807  -0.0576  -0.9749  0.0349
s.e.  0.0546   0.0550   0.0730  0.0634

sigma^2 = 10.26:  log likelihood = -900.05
AIC=1810.11   AICc=1810.29   BIC=1829.31
```

```
modelMonthlySeasonal001.110Napa
```

```
Series: tsCaliforniaNapaTemp
ARIMA(0,0,1)(1,1,0)[12]

Coefficients:
         ma1     sar1
      0.5787  -0.4244
s.e.  0.0365   0.0493

sigma^2 = 15.15:  log likelihood = -955.99
AIC=1917.98   AICc=1918.05   BIC=1929.5
```

```
modelMonthlySeasonal001.011Napa
```

```
Series: tsCaliforniaNapaTemp
ARIMA(0,0,1)(0,1,1)[12]

Coefficients:
         ma1     sma1
      0.6150  -0.5909
s.e.  0.0358   0.0459

sigma^2 = 13.72:  log likelihood = -940.43
AIC=1886.85   AICc=1886.92   BIC=1898.37
```

```
modelMonthlySeasonal002.210Napa
```

```
Series: tsCaliforniaNapaTemp
ARIMA(0,0,2)(2,1,0)[12]

Coefficients:
         ma1     ma2     sar1     sar2
```

```
      0.7380  0.3614  -0.6225  -0.2583
s.e.  0.0488  0.0552   0.0561   0.0531

sigma^2 = 12.66:  log likelihood = -925.48
AIC=1860.95   AICc=1861.13   BIC=1880.16
```

```r
modelMonthlySeasonal002.012Napa
```

```
Series: tsCaliforniaNapaTemp
ARIMA(0,0,2)(0,1,2)[12]

Coefficients:
         ma1     ma2     sma1    sma2
      0.7903  0.3957  -0.7285  0.0584
s.e.  0.0494  0.0537   0.0609  0.0608

sigma^2 = 11.87:  log likelihood = -915.85
AIC=1841.71   AICc=1841.88   BIC=1860.91
```

So far it seems the second model is the most adequate

```r
NapaModelAuto = auto.arima(tsCaliforniaNapaTemp, max.d = 0, max.p = 5, max.q = 5)

NapaModelAuto
```

```
Series: tsCaliforniaNapaTemp
ARIMA(1,0,0) with non-zero mean

Coefficients:
         ar1     mean
      0.8280  21.0196
s.e.  0.0296   0.9505

sigma^2 = 9.813:  log likelihood = -911.23
AIC=1828.45   AICc=1828.52   BIC=1840.08
```

it seems there isn't enough data for auto.arima to fully caught the seasonality needed

Forecasting

Now using the forecast function we will produce an out of time cross-validation to forecast the values in the 2 weeks of November

```
forecast(modelMonthlySeasonal100.011Napa, 366)
```

```
          Point Forecast    Lo 80    Hi 80     Lo 95    Hi 95
2012.9727       14.92598 10.83022 19.02173  8.662061 21.18990
2012.9754       16.26633 10.94209 21.59057  8.123605 24.40905
2012.9781       18.45885 12.43152 24.48617  9.240851 27.67684
2012.9809       18.64136 12.17375 25.10897  8.750004 28.53272
2012.9836       18.76622 12.01212 25.52031  8.436723 29.09571
2012.9863       18.98751 12.04219 25.93284  8.365557 29.60947
2012.9891       19.08760 12.01335 26.16185  8.268467 29.90673
2012.9918       19.32427 12.16243 26.48611  8.371186 30.27735
2012.9945       18.47618 11.25455 25.69780  7.431651 29.52070
2012.9973       19.05158 11.78903 26.31414  7.944460 30.15870
2013.0000       20.09832 12.80771 27.38893  8.948296 31.24835
2013.0027       20.43042 13.12059 27.74024  9.251007 31.60983
2013.0055       19.93524 12.59879 27.27170  8.715105 31.15538
2013.0082       20.42736 13.07277 27.78195  9.179482 31.67524
2013.0109       21.91528 14.54846 29.28209 10.648704 33.18185
2013.0137       21.51250 14.13764 28.88735 10.233632 32.79137
2013.0164       21.15118 13.77133 28.53102  9.864681 32.43767
2013.0191       20.96862 13.58467 28.35257  9.675842 32.26140
2013.0219       20.73324 13.34646 28.12001  9.436144 32.03033
2013.0246       20.69125 13.30255 28.07995  9.391206 31.99129
2013.0273       19.61168 12.22167 27.00168  8.309643 30.91371
2013.0301       19.99481 12.60394 27.38567  8.691452 31.29816
2013.0328       20.88183 13.49042 28.27323  9.577646 32.18601
2013.0355       21.08125 13.68955 28.47295  9.776619 32.38588
2013.0383       20.47587 13.08008 27.87166  9.164985 31.78675
2013.0410       20.87644 13.47801 28.27486  9.561519 32.19135
2013.0437       22.28831 14.88833 29.68829 10.971011 33.60561
2013.0464       21.82237 14.42169 29.22304 10.504010 33.14072
2013.0492       21.40857 14.00797 28.80917 10.090331 32.72681
2013.0519       21.18243 13.78119 28.58367  9.863208 32.50165
2013.0546       20.91084 13.50917 28.31251  9.590959 32.23072
2013.0574       20.83878 13.43683 28.24073  9.518469 32.15909
2013.0601       19.73423 12.33211 27.13635  8.413663 31.05479
2013.0628       20.09660 12.69441 27.49880  8.775920 31.41729
```

2013.0656	20.96639	13.56419	28.36858	9.645705	32.28707
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2013.7568	21.15999	13.63400	28.68598	9.649976	32.67000
2013.7596	20.54127	13.01265	28.06990	9.027239	32.05531
2013.7623	20.93077	13.40051	28.46103	9.414230	32.44730
2013.7650	22.33344	14.80231	29.86457	10.815573	33.85131
2013.7678	21.85986	14.32850	29.39121	10.341644	33.37807

2013.7705	21.43971	13.90875	28.97068	9.922096	32.95733
2013.7732	21.20830	13.67692	28.73968	9.690043	32.72655
2013.7760	20.93233	13.40067	28.46399	9.413651	32.45101
2013.7787	20.85663	13.32479	28.38846	9.337684	32.37557
2013.7814	19.74905	12.21713	27.28098	8.229968	31.26814
2013.7842	20.10892	12.57697	27.64087	8.589791	31.62805
2013.7869	20.97662	13.44470	28.50853	9.457541	32.49569
2013.7896	21.15999	13.62817	28.69181	9.641067	32.67891
2013.7923	20.54127	13.00683	28.07572	9.018334	32.06421
2013.7951	20.93077	13.39469	28.46685	9.405327	32.45621
2013.7978	22.33344	14.79649	29.87039	10.806671	33.86021
2013.8005	21.85986	14.32268	29.39703	10.332742	33.38697
2013.8033	21.43971	13.90293	28.97650	9.913193	32.96623
2013.8060	21.20830	13.67110	28.74550	9.681141	32.73546
2013.8087	20.93233	13.39485	28.46981	9.404749	32.45991
2013.8115	20.85663	13.31897	28.39428	9.328783	32.38447
2013.8142	19.74905	12.21131	27.28680	8.221067	31.27704
2013.8169	20.10892	12.57115	27.64669	8.580889	31.63695
2013.8197	20.97662	13.43888	28.51435	9.448639	32.50459
2013.8224	21.15999	13.62235	28.69763	9.632165	32.68781
2013.8251	20.54127	13.00101	28.08154	9.009435	32.07311
2013.8279	20.93077	13.38887	28.47266	9.396430	32.46510
2013.8306	22.33344	14.79067	29.87621	10.797775	33.86910
2013.8333	21.85986	14.31686	29.40285	10.323847	33.39586
2013.8361	21.43971	13.89711	28.98232	9.904297	32.97513
2013.8388	21.20830	13.66528	28.75132	9.672245	32.74435
2013.8415	20.93233	13.38903	28.47562	9.395854	32.46880
2013.8443	20.85663	13.31316	28.40009	9.319888	32.39336
2013.8470	19.74905	12.20549	27.29261	8.212172	31.28593
2013.8497	20.10892	12.56533	27.65251	8.571995	31.64584
2013.8525	20.97662	13.43306	28.52017	9.439745	32.51349
2013.8552	21.15999	13.61653	28.70344	9.623271	32.69670
2013.8579	20.54127	12.99520	28.08735	9.000544	32.08200
2013.8607	20.93077	13.38306	28.47848	9.387541	32.47399
2013.8634	22.33344	14.78486	29.88202	10.788887	33.87799
2013.8661	21.85986	14.31105	29.40866	10.314958	33.40475
2013.8689	21.43971	13.89130	28.98813	9.895409	32.98402
2013.8716	21.20830	13.65947	28.75713	9.663357	32.75324
2013.8743	20.93233	13.38322	28.48143	9.386966	32.47769
2013.8770	20.85663	13.30735	28.40590	9.311000	32.40225
2013.8798	19.74905	12.19968	27.29843	8.203284	31.29482
2013.8825	20.10892	12.55952	27.65832	8.563107	31.65473
2013.8852	20.97662	13.42725	28.52598	9.430857	32.52238

```
2013.8880       21.15999 13.61072 28.70925  9.614383 32.70559
2013.8907       20.54127 12.98939 28.09316  8.991659 32.09089
2013.8934       20.93077 13.37725 28.48429  9.378658 32.48288
2013.8962       22.33344 14.77905 29.88783 10.780005 33.88688
2013.8989       21.85986 14.30525 29.41446 10.306077 33.41363
2013.9016       21.43971 13.88549 28.99394  9.886526 32.99290
2013.9044       21.20830 13.65366 28.76294  9.654476 32.76212
2013.9071       20.93233 13.37741 28.48724  9.378085 32.48657
2013.9098       20.85663 13.30154 28.41171  9.302119 32.41113
2013.9126       19.74905 12.19387 27.30423  8.194404 31.30370
2013.9153       20.10892 12.55371 27.66413  8.554226 31.66361
2013.9180       20.97662 13.42144 28.53179  9.421976 32.53126
2013.9208       21.15999 13.60492 28.71506  9.605502 32.71447
2013.9235       20.54127 12.98358 28.09897  8.982781 32.09977
2013.9262       20.93077 13.37145 28.49009  9.369782 32.49175
2013.9290       22.33344 14.77325 29.89363 10.771130 33.89575
2013.9317       21.85986 14.29944 29.42027 10.297202 33.42251
2013.9344       21.43971 13.87969 28.99974  9.877651 33.00177
2013.9372       21.20830 13.64786 28.76874  9.645601 32.77100
2013.9399       20.93233 13.37161 28.49304  9.369210 32.49545
2013.9426       20.85663 13.29574 28.41751  9.293245 32.42001
2013.9454       19.74905 12.18807 27.31004  8.185529 31.31258
2013.9481       20.10892 12.54791 27.66993  8.545352 31.67248
2013.9508       20.97662 13.41564 28.53759  9.413102 32.54013
2013.9536       21.15999 13.59911 28.72086  9.596628 32.72335
2013.9563       20.54127 12.97778 28.10477  8.973910 32.10864
2013.9590       20.93077 13.36565 28.49589  9.360913 32.50062
2013.9617       22.33344 14.76745 29.89943 10.762262 33.90462
2013.9645       21.85986 14.29364 29.42607 10.288334 33.43138
2013.9672       21.43971 13.87389 29.00554  9.868783 33.01064
2013.9699       21.20830 13.64206 28.77454  9.636733 32.77986
```

```r
toPredictValues$Napa
```

```
[1] 14.4 15.0 14.4 16.7 20.6 22.2 20.0 17.2 17.8
```

Death valley

```r
## initial assumption
```

```r
modelMonthlySeasonal100.110DeathValley = Arima(tsCaliforniaDeathValleyTemp,
    order = c(1, 0, 0), seasonal = list(order = c(1, 1, 0), period = 12))

modelMonthlySeasonal100.011DeathValley = Arima(tsCaliforniaDeathValleyTemp,
    order = c(1, 0, 0), seasonal = list(order = c(0, 1, 1), period = 12))

modelMonthlySeasonal200.210DeathValley = Arima(tsCaliforniaDeathValleyTemp,
    order = c(2, 0, 0), seasonal = list(order = c(2, 1, 0), period = 12))
modelMonthlySeasonal200.012DeathValley = Arima(tsCaliforniaDeathValleyTemp,
    order = c(2, 0, 0), seasonal = list(order = c(0, 1, 2), period = 12))

modelMonthlySeasonal001.110DeathValley = Arima(tsCaliforniaDeathValleyTemp,
    order = c(0, 0, 1), seasonal = list(order = c(1, 1, 0), period = 12))

modelMonthlySeasonal001.011DeathValley = Arima(tsCaliforniaDeathValleyTemp,
    order = c(0, 0, 1), seasonal = list(order = c(0, 1, 1), period = 12))

modelMonthlySeasonal002.210DeathValley = Arima(tsCaliforniaDeathValleyTemp,
    order = c(0, 0, 2), seasonal = list(order = c(2, 1, 0), period = 12))
modelMonthlySeasonal002.012DeathValley = Arima(tsCaliforniaDeathValleyTemp,
    order = c(0, 0, 2), seasonal = list(order = c(0, 1, 2), period = 12))
```

```r
modelMonthlySeasonal100.110DeathValley
```

```
Series: tsCaliforniaDeathValleyTemp
ARIMA(1,0,0)(1,1,0)[12]

Coefficients:
         ar1     sar1
      0.8082  -0.4482
s.e.  0.0321   0.0491

sigma^2 = 9.802:  log likelihood = -881.56
AIC=1769.13   AICc=1769.2   BIC=1780.65
```

```r
modelMonthlySeasonal100.011DeathValley
```

```
Series: tsCaliforniaDeathValleyTemp
ARIMA(1,0,0)(0,1,1)[12]
```

```
Coefficients:
         ar1     sma1
      0.9679  -0.8985
s.e.  0.0164   0.0443

sigma^2 = 7.777:  log likelihood = -850.22
AIC=1706.45   AICc=1706.52   BIC=1717.97
```

```
modelMonthlySeasonal200.210DeathValley
```

```
Series: tsCaliforniaDeathValleyTemp
ARIMA(2,0,0)(2,1,0)[12]

Coefficients:
         ar1      ar2     sar1     sar2
      0.9640  -0.1497  -0.5483  -0.1975
s.e.  0.0538   0.0546   0.0555   0.0578

sigma^2 = 9.163:  log likelihood = -869.61
AIC=1749.21   AICc=1749.39   BIC=1768.42
```

```
modelMonthlySeasonal200.012DeathValley
```

```
Series: tsCaliforniaDeathValleyTemp
ARIMA(2,0,0)(0,1,2)[12]

Coefficients:
         ar1      ar2     sma1     sma2
      1.0154  -0.0521  -0.8544  -0.0521
s.e.  0.0538   0.0544   0.0608   0.0560

sigma^2 = 7.769:  log likelihood = -849.32
AIC=1708.63   AICc=1708.81   BIC=1727.84
```

```
modelMonthlySeasonal001.110DeathValley
```

```
Series: tsCaliforniaDeathValleyTemp
ARIMA(0,0,1)(1,1,0)[12]
```

```
Coefficients:
         ma1     sar1
      0.7398  -0.2828
s.e.  0.0308   0.0525

sigma^2 = 13.16:  log likelihood = -931.29
AIC=1868.57   AICc=1868.64   BIC=1880.09
```

```
modelMonthlySeasonal001.011DeathValley
```

```
Series: tsCaliforniaDeathValleyTemp
ARIMA(0,0,1)(0,1,1)[12]

Coefficients:
         ma1     sma1
      0.7460  -0.2453
s.e.  0.0315   0.0468

sigma^2 = 13.31:  log likelihood = -933.06
AIC=1872.12   AICc=1872.19   BIC=1883.64
```

```
modelMonthlySeasonal002.210DeathValley
```

```
Series: tsCaliforniaDeathValleyTemp
ARIMA(0,0,2)(2,1,0)[12]

Coefficients:
         ma1     ma2     sar1    sar2
      0.9398  0.3858  -0.3944  0.0105
s.e.  0.0493  0.0432   0.0571  0.0564

sigma^2 = 10.94:  log likelihood = -899.09
AIC=1808.17   AICc=1808.35   BIC=1827.38
```

```
modelMonthlySeasonal002.012DeathValley
```

```
Series: tsCaliforniaDeathValleyTemp
ARIMA(0,0,2)(0,1,2)[12]
```

```
Coefficients:
         ma1     ma2     sma1    sma2
      0.9326  0.3751  -0.3968  0.1196
s.e.  0.0497  0.0429   0.0581  0.0464

sigma^2 = 10.99:  log likelihood = -899.77
AIC=1809.53   AICc=1809.71   BIC=1828.73
```

Forecast

```
forecast(modelMonthlySeasonal100.011DeathValley, 366)
```

```
          Point Forecast     Lo 80    Hi 80       Lo 95    Hi 95
2012.9727       17.55612 13.981621 21.13061 12.08939732 23.02284
2012.9754       17.24408 12.269606 22.21855  9.63627874 24.85188
2012.9781       17.65015 11.653802 23.64650  8.47952529 26.82078
2012.9809       18.51313 11.697162 25.32909  8.08900843 28.93725
2012.9836       19.08324 11.580327 26.58615  7.60852551 30.55795
2012.9863       19.63382 11.540130 27.72751  7.25559039 32.01205
2012.9891       20.56363 11.953235 29.17402  7.39516750 33.73209
2012.9918       19.34573 10.277995 28.41346  5.47782621 33.21363
2012.9945       20.52485 11.048715 30.00098  6.03235322 35.01734
2012.9973       20.43781 10.594486 30.28113  5.38374523 35.49188
2013.0000       20.60068 10.425416 30.77594  5.03895769 36.16240
2013.0027       20.53368 10.057025 31.01033  4.51102113 36.55633
2013.0055       21.26659 10.427741 32.10544  4.68999978 37.84318
2013.0082       20.83532  9.667849 32.00278  3.75614945 37.91448
2013.0109       21.12598  9.659248 32.59271  3.58912675 38.66283
2013.0137       21.87725 10.137131 33.61738  3.92228531 39.83222
2013.0164       22.33925 10.348714 34.32979  4.00130618 40.67720
2013.0191       22.78520 10.564706 35.00569  4.09556783 41.47483
2013.0219       23.61374 11.181703 36.04577  4.60058211 42.62689
2013.0246       22.29782  9.670850 34.92478  2.98653762 41.60910
2013.0273       23.38207 10.575197 36.18893  3.79565092 42.96848
2013.0301       23.20321 10.230095 36.17632  3.36254261 43.04388
2013.0328       23.27721 10.150285 36.40413  3.20131186 43.35310
2013.0355       23.12419  9.854822 36.39356  2.83044225 43.41794
2013.0383       23.77386 10.316638 37.23108  3.19281554 44.35490
2013.0410       23.26201  9.631187 36.89283  2.41546625 44.10855
2013.0437       23.47469  9.683250 37.26612  2.38250479 44.56687
2013.0464       24.15048 10.210292 38.09067  2.83080066 45.47016
```

2013.0492	24.53943	10.461345	38.61752	3.00885729	46.07000
2013.0519	24.91467	10.708583	39.12075	3.18833571	46.64100
2013.0546	25.67477	11.349829	39.99972	3.76666261	47.58288
2013.0574	24.29262	9.857236	38.72801	2.21560500	46.36964
2013.0601	25.31276	10.774696	39.85083	3.07870717	47.54682
2013.0628	25.07186	10.438271	39.70545	2.69171583	47.45201
2013.0656	25.08581	10.363311	39.80831	2.56969224	47.60192
2013.0683	24.87467	10.069390	39.67995	2.23194734	47.51739
2013.0710	25.46808	10.544278	40.39189	2.64409263	48.29208
2013.0738	24.90179	9.867821	39.93575	1.90931983	47.89426
2013.0765	25.06177	9.925359	40.19818	1.91262643	48.21091
2013.0792	25.68656	10.454834	40.91829	2.39164307	48.98148
2013.0820	26.02615	10.705690	41.34660	2.59553020	49.45676
2013.0847	26.35361	10.950449	41.75677	2.79650950	49.91071
2013.0874	27.06747	11.587250	42.54769	3.39251649	50.74242
2013.0902	25.64056	10.088512	41.19261	1.85575337	49.42537
2013.0929	26.61739	10.998360	42.23641	2.73014701	50.50463
2013.0956	26.33456	10.653064	42.01606	2.35178164	50.31734
2013.0984	26.30793	10.568152	42.04770	2.23601723	50.37984
2013.1011	26.05752	10.263354	41.85168	1.90242980	50.21260
2013.1038	26.61292	10.734721	42.49111	2.32931187	50.89652
2013.1066	26.00983	10.053336	41.96633	1.60647739	50.41318
2013.1093	26.13420	10.104731	42.16368	1.61923948	50.64917
2013.1120	26.72453	10.627018	42.82205	2.10550731	51.34356
2013.1148	27.03076	10.869790	43.19173	2.31468784	51.74683
2013.1175	27.32594	11.105694	43.54618	2.51921605	52.13266
2013.1202	28.00855	11.732991	44.28411	3.11722908	52.89988
2013.1230	26.55140	10.224201	42.87860	1.58110273	51.52170
2013.1257	27.49896	11.123540	43.87437	2.45491901	52.54299
2013.1284	27.18780	10.767358	43.60824	2.07490106	52.30070
2013.1311	27.13375	10.671252	43.59624	1.95653341	52.31096
2013.1339	26.85680	10.355018	43.35857	1.61950392	52.09409
2013.1366	27.38651	10.819799	43.95322	2.04991052	52.72311
2013.1393	26.75856	10.131272	43.38586	1.32931539	52.18781
2013.1421	26.85888	10.175056	43.54270	1.34317528	52.37458
2013.1448	27.42592	10.689338	44.16250	1.82952766	53.02231
2013.1475	27.70961	10.923776	44.49544	2.03789435	53.38132
2013.1503	27.98297	11.151075	44.81486	2.24081022	53.72512
2013.1530	28.64447	11.769550	45.51938	2.83650878	54.45243
2013.1557	27.16688	10.251768	44.08199	1.29744885	53.03631
2013.1585	28.09466	11.141988	45.04732	2.16778717	54.02152
2013.1612	27.76435	10.776592	44.75212	1.78381418	53.74489
2013.1639	27.69177	10.671215	44.71233	1.66107508	53.72247

2013.1667	27.39689	10.345679	44.44810	1.31931247	53.47447
2013.1694	27.90925	10.804432	45.01407	1.74968904	54.06881
2013.1721	27.26450	10.109641	44.41937	1.02840561	53.50060
2013.1749	27.34856	10.146966	44.55015	1.04099414	53.65612
2013.1776	27.89986	10.654633	45.14509	1.52555967	54.27416
2013.1803	28.16832	10.882333	45.45430	1.73168481	54.60495
2013.1831	28.42694	11.102802	45.75108	1.93195789	54.92192
2013.1858	29.07417	11.714381	46.43396	2.52466377	55.62368
2013.1885	27.58278	10.189663	44.97589	0.98230578	54.18324
2013.1913	28.49719	11.072926	45.92144	1.84908084	55.14529
2013.1940	28.15395	10.700574	45.60732	1.46131568	54.84658
2013.1967	28.06885	10.588255	45.54944	1.33458828	54.80311
2013.1995	27.76185	10.255808	45.26789	0.98867086	54.53502
2013.2022	28.26248	10.709905	45.81505	1.41813391	55.10682
2013.2049	27.60638	10.010346	45.20241	0.69556760	54.51719
2013.2077	27.67945	10.042816	45.31608	0.70654716	54.65235
2013.2104	28.22012	10.545562	45.89467	1.18921668	55.25102
2013.2131	28.47828	10.768295	46.18827	1.39319402	55.56337
2013.2158	28.72694	10.983768	46.47012	1.59109906	55.86278
2013.2186	29.36453	11.590337	47.13873	2.18124613	56.54782
2013.2213	27.86381	10.060608	45.66700	0.63616455	55.09145
2013.2240	28.76919	10.938872	46.59950	1.50007470	56.03830
2013.2268	28.41721	10.561543	46.27287	1.10932453	55.72509
2013.2295	28.32365	10.444278	46.20302	0.97951120	55.66778
2013.2322	28.00846	10.106924	45.90999	0.63042426	55.38649
2013.2350	28.50116	10.557696	46.44463	1.05899888	55.94333
2013.2377	27.83739	9.854753	45.82004	0.33531815	55.33947
2013.2404	27.90304	9.883795	45.92228	0.34498415	55.46109
2013.2432	28.43652	10.383081	46.48997	0.82616521	56.04688
2013.2459	28.68773	10.602334	46.77313	1.02850109	56.34697
2013.2486	28.92966	10.814317	47.04501	1.22463197	56.63469
2013.2514	29.56074	11.417396	47.70408	1.81289005	57.30859
2013.2541	28.05371	9.884185	46.22323	0.26582087	55.84159
2013.2568	28.95298	10.758982	47.14698	1.12765950	56.77831
2013.2596	28.59510	10.378207	46.81199	0.73476718	56.45543
2013.2623	28.49582	10.257524	46.73412	0.60275308	56.38889
2013.2650	28.17510	9.916785	46.43341	0.25141766	56.09878
2013.2678	28.66245	10.365308	46.95959	0.67938725	56.64551
2013.2705	27.99350	9.660078	46.32692	-0.04504888	56.03204
2013.2732	28.05412	9.686802	46.42145	-0.03627101	56.14452
2013.2760	28.58275	10.183750	46.98176	0.44390521	56.72160
2013.2787	28.82927	10.400653	47.25788	0.64513402	57.01340
2013.2814	29.06665	10.610280	47.52301	0.84006963	57.29322

2013.2842	29.69332	11.211003	48.17564	1.42705557	57.95959
2013.2869	28.18203	9.675442	46.68861	-0.12135186	56.48541
2013.2896	29.07718	10.547900	47.60646	0.73909220	57.41527
2013.2923	28.71530	10.164801	47.26581	0.34475782	57.08585
2013.2951	28.61216	10.041812	47.18252	0.21126221	57.01307
2013.2978	28.28770	9.698789	46.87662	-0.14158694	56.71699
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2013.9481        28.96569  5.255137 52.67625 -7.29647211 65.22786
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2013.9536        28.52226  4.785176 52.25934 -7.78047511 64.82499
2013.9563        28.99845  5.235001 52.76190 -7.34460935 65.34151
2013.9590        28.31870  4.530590 52.10681 -8.06207518 64.69948
2013.9617        28.36888  4.557701 52.18005 -8.04717399 64.78493
2013.9645        28.88739  5.054645 52.72014 -7.56164919 65.33643
2013.9672        29.12411  5.271193 52.97703 -7.35578034 65.60401
2013.9699        29.35202  5.480168 53.22387 -7.15682645 65.86086
```

```r
toPredictValues$`Death Valley`
```

```
[1] 31.7 23.9 20.0 19.4 20.6 21.7 22.2 21.1 21.7
```