

Group 34 course work

Table of contents

Introduction:	2
Exploratory Data Analysis:	2
TODO INSERT Reference to PAIR PLOT -1 AND SUMMARY-1 TABLE HERE.	3
TODO reference INSERT BRAZIL MAP HERE.	3
TODO insert the ggpairs	3
Model Selection:	4
Model Fitting:	5
TODO QQ-plot reference	5
Model Evaluation:	5
TODO dispersion parameter	5
TODO add three residual sum here	5
Results and Interpretation:	6
Conclusion:	6
Images	6
Code Appendix	7
Exploratory analyses	9
Poisson definition	11
Negative binomial	18
Again tidy up the plots of the Negative Binomial	27

Introduction:

Tuberculosis (TB) is a bacterial disease caused by the bacterium *Mycobacterium tuberculosis*. It primarily affects the lungs, but can also affect other parts of the body, such as the kidneys, spine, and brain.

TB is a major public health problem worldwide, affecting millions of people each year. According to the World Health Organization (WHO), TB is one of the top 10 causes of death worldwide, and in 2020 alone, there were an estimated of approximately 10 million cases of TB reported globally. Brazil is one of the countries with a high burden of TB with an estimate of 96000, and it is considered a priority country for TB control by the WHO. *Global Tuberculosis Report 2020* (2020).

The purpose of this report is to determine, with statistical certainty, whether any of the aforementioned covariates, including space and time, affect the rate of tuberculosis per unit population by using data on a series of socio-economic variables that were captured across a wide range of geographical regions in Brazil between the years of 2012 and 2014. The data will be taken from across a wide range of regions in Brazil between the years of 2012 and 2014. The socioeconomic variables that were recorded for each micro-region in our data set are as follows: the level of illiteracy, urbanisation, poverty, unemployment, and sanitation; the proportion of indigenous population; the dwelling density; and finally, a proxy indicator of the amount of resources in the form of the average amount of time between diagnosing a TB case and reporting it to the health system. In addition, the latitude and longitude of the respective 537 micro-regions, as well as the year in which the data was obtained, are supplied for each of these values. Because of this, we will be better able to explain the geographical, temporal, and spatio-temporal structure of any systematic risk that is not described by the covariates.

Exploratory Data Analysis:

The TBdata dataframe contains information on various socio-demographic and geographic factors in Brazil that may be associated with TB incidence in each microregion. These factors include indigenous population, illiteracy levels, urbanization rate, dwelling density, poverty levels, sanitation levels, unemployment rates, and timeliness of TB case reporting. The dataset also includes information on the number of TB cases and population size for each microregion, as well as unique ID numbers to distinguish between the different regions.

Simple exploration of our covariates and their potential relationships with the rate of tuberculosis in each microregion of Brazil was carried out before attempting any type of formal statistical analysis or regression on the data. This was done before attempting to draw any conclusions from the data. The correlations that existed between each of our co-variables and the total number of TB cases were analysed with the help of pairplots. Several of the findings were unexpected, such as the observation that a lower degree of sanitation did not appear to be associated with a higher rate of tuberculosis cases. The same was true in regard to the

levels of poverty. Nevertheless, the issue with attempting to infer statistical associations in such a straightforward manner is that we are unable to take into consideration the possibility of changes occurring in other variables for each of the data points. This is the primary reason why we need to use a formal model to investigate the impact of our covariates on the incidence of tuberculosis in relation to the total population.

TODO INSERT Reference to PAIR PLOT -1 AND SUMMARY-1 TABLE HERE.

While developing statistical models, we are always forced to choose between two competing priorities: interpretability and flexibility. In most cases, we make an effort to fit the data to a linear model whenever that is at all possible. Linear models are simple, and it is straightforward to draw conclusions and interpretations from them. However, the correlations of interest are frequently far too complicated (and non-linear) to be correctly represented by this method, as is the case with the data that we have regarding tuberculosis. On the opposite end of the spectrum is the use of machine learning models such as neural networks or boosted trees. These techniques provide very accurate predictions of modelled relationships; nevertheless, they call for a substantial quantity of data and, more crucially, they are notoriously challenging to interpret. Generalized additive models, often known as GAMs, provide a reasonable middle ground when compared to these other choices, which is why these models were chosen to serve as this analysis' preferred framework.

TODO reference INSERT BRAZIL MAP HERE.

An exploratory analysis of this dataset can reveal important insights of the potential risk factors for TB in Brazil and help guide public health interventions.

We will start by analysing the distributions of each of the variables to identify any patterns

TODO insert the ggpairs

Firstly, the dwelling density seems to follow a normal distribution that is skewed to the right and a mean of approximately 0,6.

Secondly illiteracy is very heavily skewed to the right but it still displays a normal bell curve around the 5% illiteracy level.

Poor sanitation is extremely skewed to the right.

Unemployment seems to follow a normal distribution with little to no skewness and a mean of approximately 6%.

As we can see from the matrix, the variable that is the most correlated from TB is the population, with illiteracy, poor sanitation and poverty having a negative correlation with TB.

Model Selection:

At first, the idea was to combine a log link and a Poisson generalised additive model. This was because we were working with count data in the form of TB cases broken down by each microregion. Nonetheless, it was essential to standardise this count because the populations in each region were distinct from one another. After doing some study on the topic, I discovered that using the log of the population in the model is the most effective way to carry out a population standardised regression. While we were trying to fit our model in R, it was necessary for us to include an offset for the log of the population. This provides us with the tuberculosis rate per capita:

Let the model be :

$$\text{Log}(\lambda_i) = \text{offset}(\text{log}(\text{Population}_i)) + f_1(\text{Indigenous}_i) + f_2(\text{Illiteracy}_i) + f_3(\text{Urbanisation}_i) + f_4(\text{Density}_i) + f_5(\text{Poverty}_i)$$

Confirming that our model was accurate was the next step in the process of developing our model. Unfortunately, a QQ-plot revealed that the quantiles in our data did not closely resemble what they would look like if they followed a theoretical poisson distribution. This was the conclusion that could be drawn from the plot.

After calculating a Pearson estimate of our dispersion parameter to determine whether or not this lack of fit was due to over-dispersion, the results were unequivocal: the parameter was almost 7 (when it should have been 1), which indicated that our data was indeed over dispersed for a Poisson model.

An alternative model to Poisson is a Negative Binomial model. The Negative Binomial (NB) model is likewise suitable for use with count data; however, it differs from the Poisson regression in that it contains an additional parameter that can alter the variance independently from the mean.

mean. Because of this, it is more flexible than the Poisson model, and as a result, it can fit data with a greater degree of fluctuation. A NB model was fitted to the data making use of the exact same specification as was used before. We can count ourselves fortunate that the QQ-plot⁵ for the revised model showed a good fit (the majority of the dots fell on the y=x line), and the estimate for the dispersion parameter was 1.133. The AIC was also reduced for our newly developed model. We arrived at the following model after making adjustments to the choice of rank for each of our smooth terms (covariates), until we were certain that they were not too low (edf not too near to k').

INSERT BOTH QQ PLOT HERE, FIGURE -2 at the last of the code.

Model Fitting:

TODO QQ-plot reference

Our QQ-plot suggest that the quantiles in our data are not similar to the line as it deviates from the line in nearly all the values, showing a very flawed fit. As this suggests that our current model doesn't fit the data correctly and required an extension to our model as the Poisson GAM is not accounted for enough deviance as seen in the residuals.

Since the model is not accounting for enough of the variance we will check if there is a significant difference between the variance and the mean. In this analyses we will use the Pearson estimate for the dispersion parameter, this method allow us to estimate the amount of extra variability, or over-dispersion in count data and therefore analyse if the Poisson distribution assumption of equal mean and variance holds.

Model Evaluation:

TODO dispersion parameter

As we can see from the dispersion parameter should be 1 for the assumption of equal mean and variance to hold true, so it seems that there is substantial over-dispersion in the Poisson GAM. This violates one of the Poisson assumptions that the mean and variance are equal therefore we will have to extend the model from e GAM Poisson to a Negative Binomial GAM

As we can see from the residual versus predictor plot, the values seem to be randomly scattered with no clear trend but with some distance from the zero line. As such we can determine that this scatter is due to random errors and not a unaccounted pattern in the model.

##negative binomial QQ-plot reference

The QQ-plot looks much better for the Negative Binomial model. The majority of points lie either on top of very near the $y=x$ line, except for a few towards the extremes. This indicates our assumption about the true distribution of the data is a lot more safe than it was before.

TODO add three residual sum here

```
sum(residuals(nb_model, type = "pearson")^2) / df.residual(nb_model)
```

The dispersion parameter is very close to 1, unlike for the Poisson model, meaning that the model that can account for most of the over-dispersion in the data. As such a dispersion parameter value close to 1 can be interpreted as the model is a good fit for the data due to the model adequately capture the variability of the response variable.

Results and Interpretation:

Conclusion:

Images

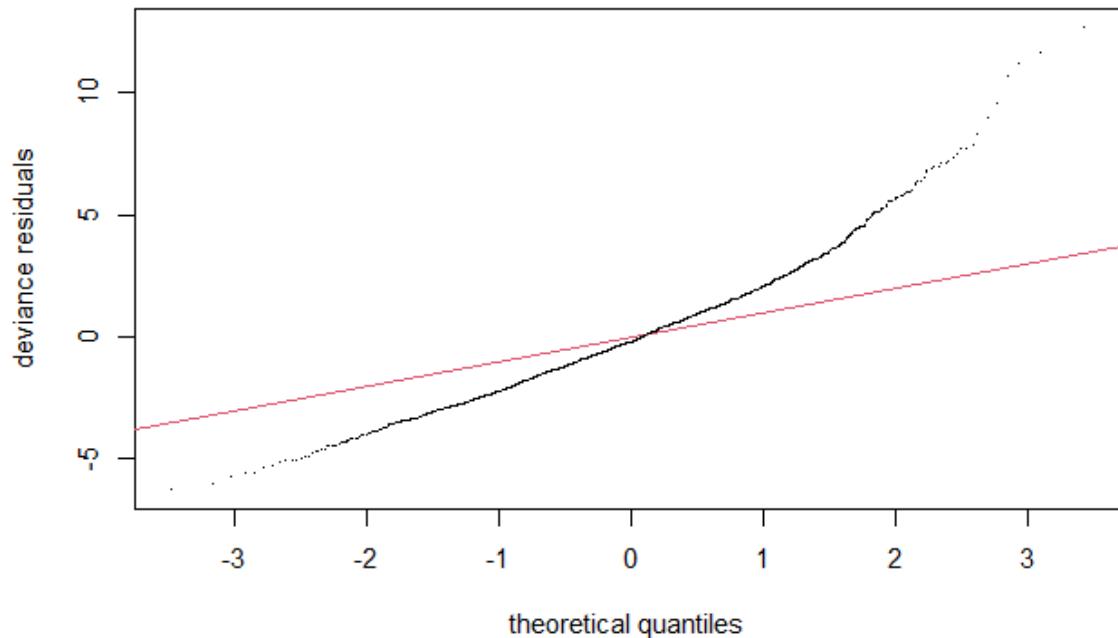
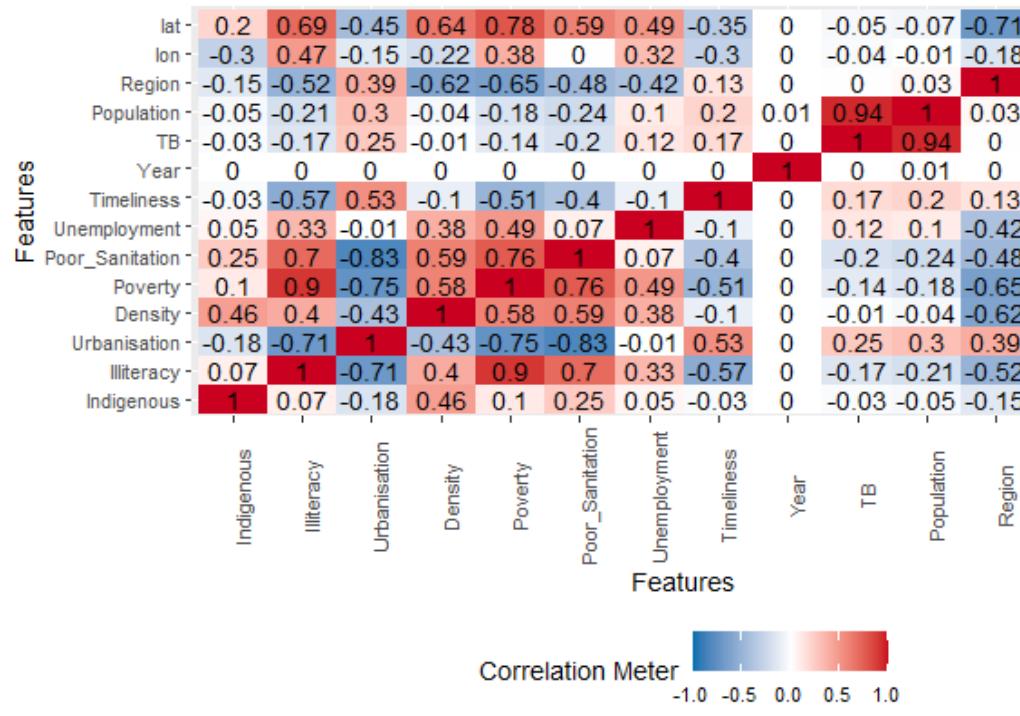


Figure 1: QQ Plot



{figure label="fig:heatMap"}

{caption: This is the heatmap that we will remove and get the better ggpairs} {/figure}

Code Appendix

```
## Plotting map of cases
par(mfrow = c(1,3))
plot.map(TBdata$TB[TBdata$Year==2012],n.levels=7,main="TB counts for 2012")
plot.map(TBdata$TB[TBdata$Year==2013],n.levels=7,main="TB counts for 2013")
plot.map(TBdata$TB[TBdata$Year==2014],n.levels=7,main="TB counts for 2014")
```

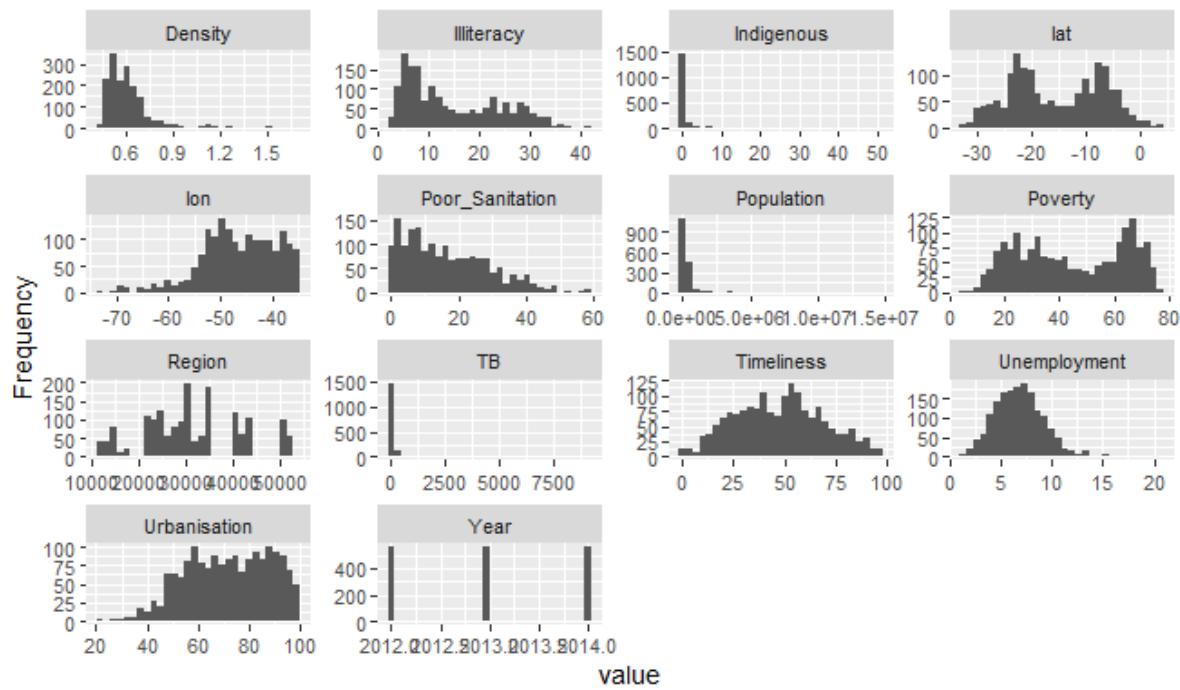
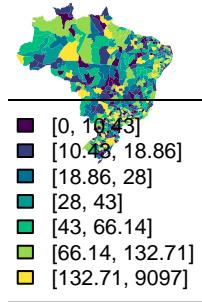
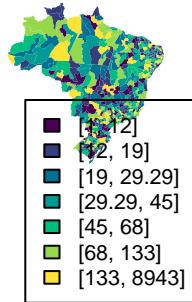
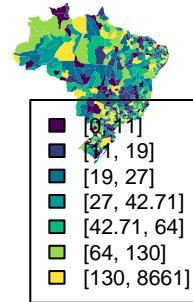


Figure 2: Initial variable analysis

TB counts for 2012**TB counts for 2013****TB counts for 2014**

```
par(mfrow = c(1,1))
```

Exploratory analyses

```
# INSERT THIS AS SUMMARY TABLE AS TABLE-1
summary_table <- summary(TBdata)
# Convert the summary table to a LaTeX table using the xtable() function
latex_table <- xtable(summary_table)

# Print the LaTeX table to the console
print(latex_table)
```

```
% latex table generated in R 4.2.2 by xtable 1.8-4 package
% Wed Mar 22 20:19:09 2023
\begin{table}[ht]
\centering
\begin{tabular}{rllllllllllll}
\hline
& Indigenous & Illiteracy & Urbanisation & Density & Poverty & Poor\_Sanitation \\
\hline

```

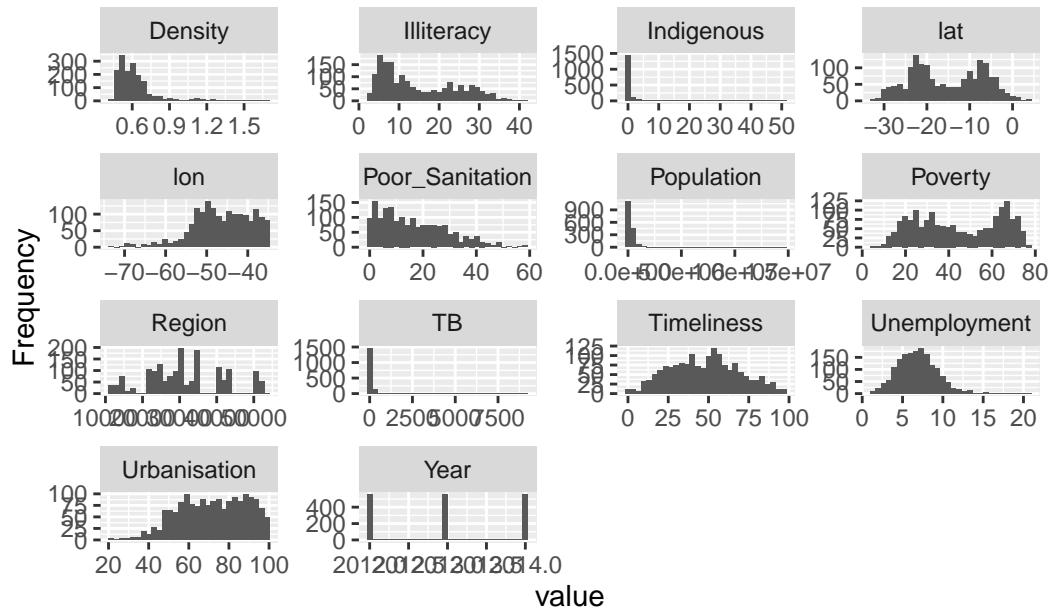
```

X & Min. : 0.01034 & Min. : 2.336 & Min. :22.34 & Min. :0.4223 & Min. : 5.9
X.1 & 1st Qu.: 0.06366 & 1st Qu.: 6.683 & 1st Qu.:58.45 & 1st Qu.:0.5166 & 1st Qu.:
X.2 & Median : 0.10577 & Median :11.516 & Median :72.66 & Median :0.5840 & Median :
X.3 & Mean : 0.84307 & Mean :14.802 & Mean :71.96 & Mean :0.6212 & Mean :
X.4 & 3rd Qu.: 0.23973 & 3rd Qu.:22.844 & 3rd Qu.:86.16 & 3rd Qu.:0.6585 & 3rd Qu.:
X.5 & Max. :50.64623 & Max. :41.137 & Max. :99.93 & Max. :1.6751 & Max. :

\hline
\end{tabular}
\end{table}

```

```
# AIM TO SKIP IT AS WE HAVE ALREADY USED CORRplot
plot_histogram(TBdata)
```



//TODO talk about the histograms and relevant distributions we can observe

Now investigating the correlation matrix of the numerical variables

```
# NOT REQUIRED AS WE HAVE ALREADY USED CORRplot
# plot_correlation(TBdata)
```

As we can see from the matrix, the variable that is the most correlated from TB is the population, with illiteracy, poor sanitation and poverty having a negative correlation with

TB.

Poisson definition

As the data is count data we will first fit a Poisson module since this distribution is a good fit for the nature of the data

```
poisson_model <- gam(TB ~ offset(log(Population)) + s(Indigenous, k = 20) + s(Illiteracy ,  
summary(poisson_model)
```

Family: poisson

Link function: log

Formula:

```
TB ~ offset(log(Population)) + s(Indigenous, k = 20) + s(Illiteracy ,  
k = 20) + s(Urbanisation, k = 20) + s(Density, k = 20) +  
s(Poverty, k = 20) + s(Poor_Sanitation, k = 20) + s(Unemployment ,  
k = 20) + s(Year, k = 3) + s(Timeliness, k = 20) + te(lon,  
Year, k = 3) + te(lat, Year, k = 3)
```

Parametric coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-8.467263	0.004435	-1909	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:

	edf	Ref.df	Chi.sq	p-value
s(Indigenous)	18.393	18.92	1051.3	<2e-16 ***
s(Illiteracy)	18.157	18.89	341.5	<2e-16 ***
s(Urbanisation)	18.879	18.99	1502.8	<2e-16 ***
s(Density)	17.396	18.05	1763.1	<2e-16 ***
s(Poverty)	18.636	18.95	2027.8	<2e-16 ***
s(Poor_Sanitation)	18.327	18.91	1251.0	<2e-16 ***
s(Unemployment)	18.648	18.98	2622.8	<2e-16 ***
s(Year)	1.999	2.00	1797.0	<2e-16 ***
s(Timeliness)	18.328	18.91	927.0	<2e-16 ***
te(lon,Year)	5.980	6.00	1440.1	<2e-16 ***
te(lat,Year)	5.986	6.00	1301.2	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

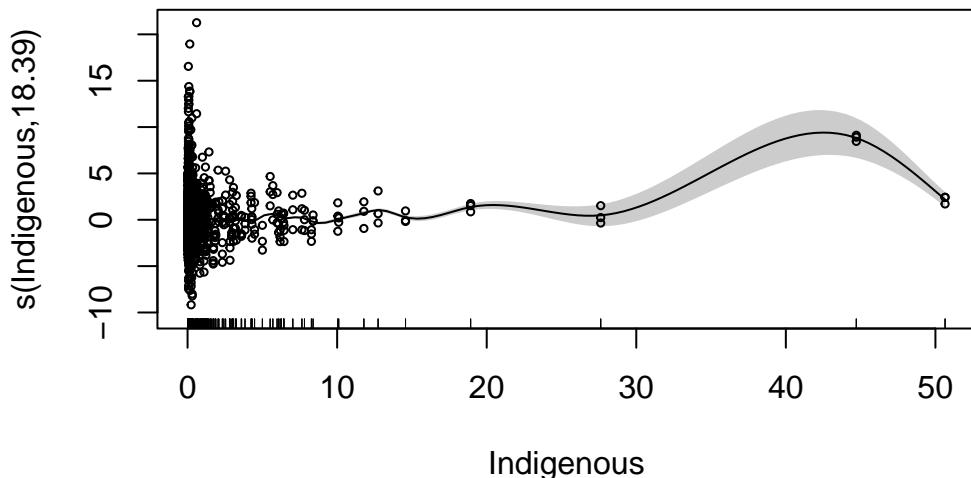
```
R-sq.(adj) = 0.995 Deviance explained = 82.9%
-REML = 11564 Scale est. = 1 n = 1671
```

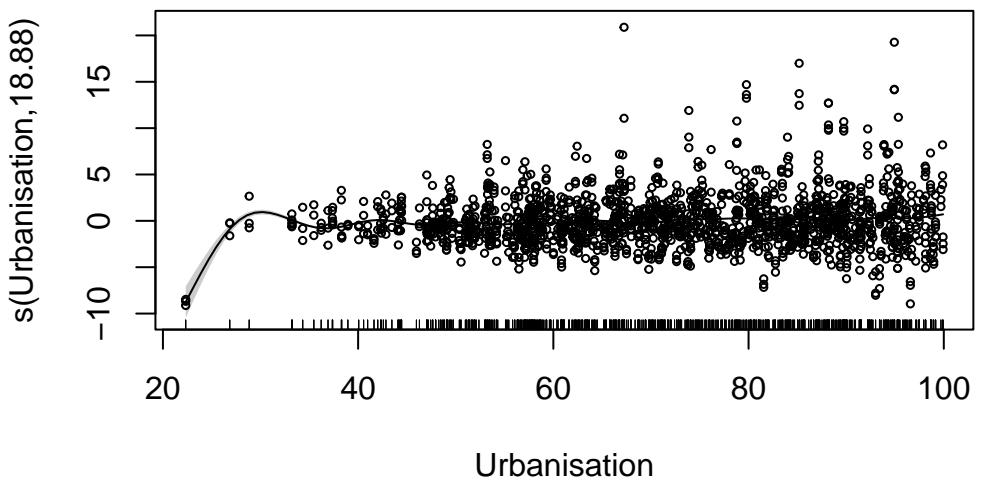
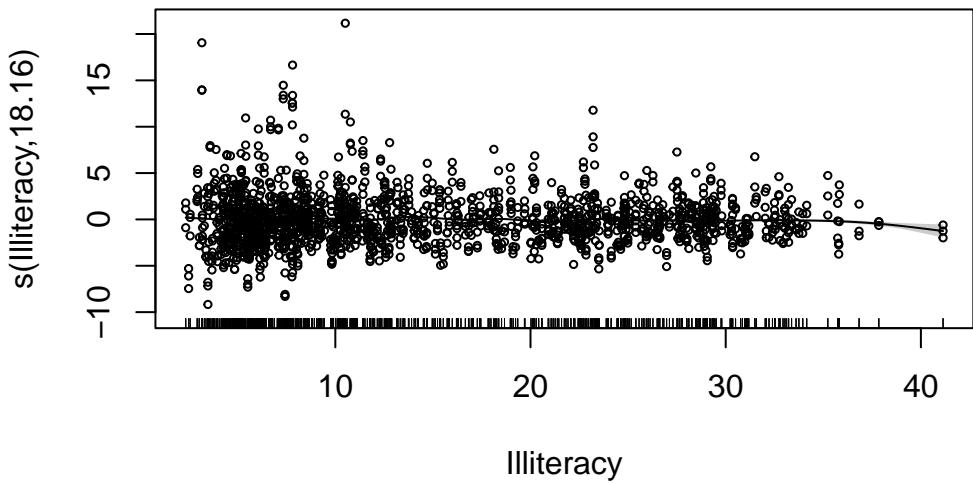
```
gam.check(poisson_model)
```

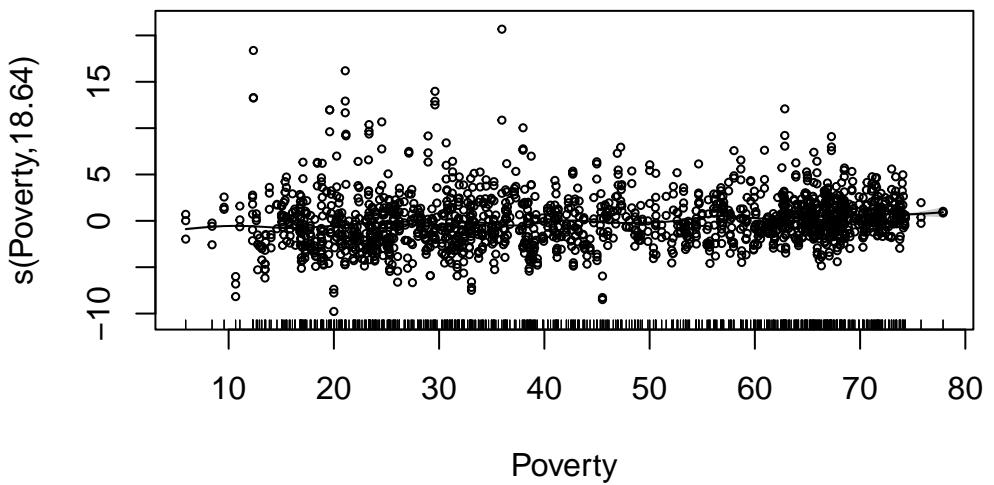
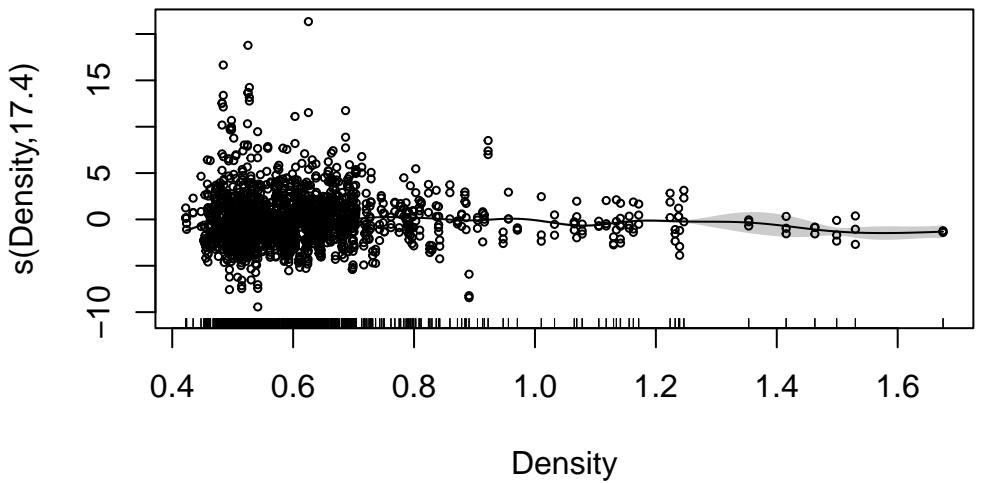
```
#Akaike Information Criterion:
poisson_model$aic
```

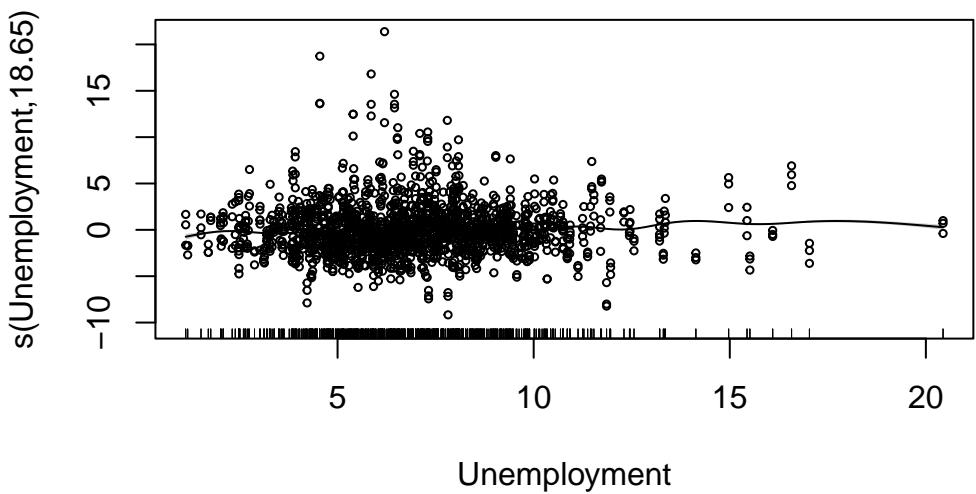
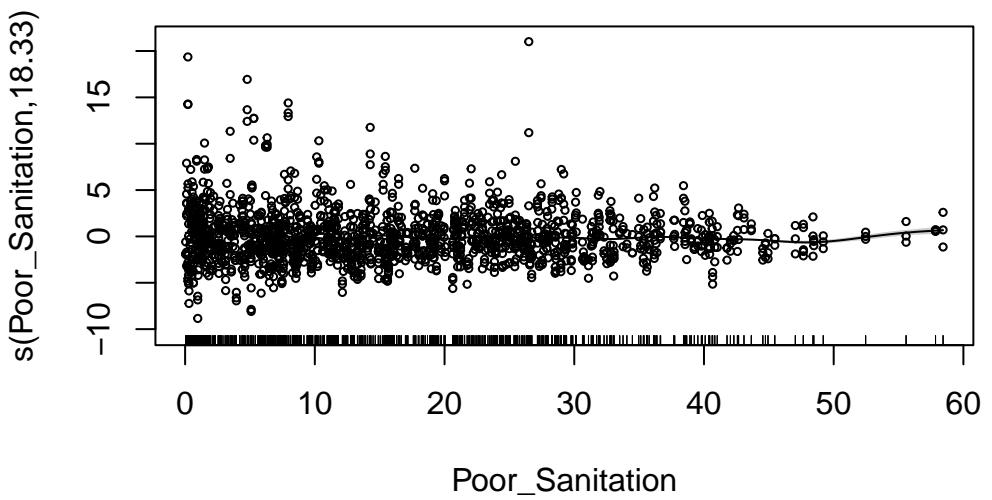
```
[1] 22271.66
```

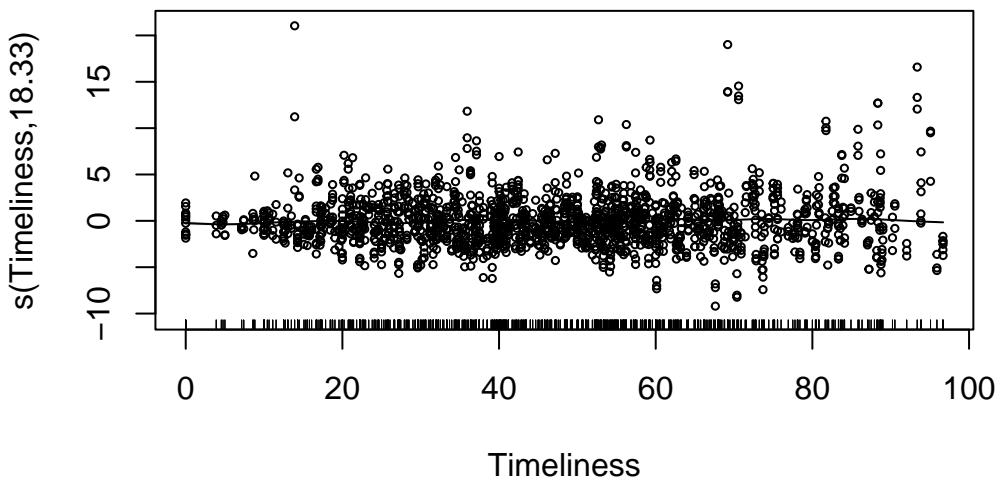
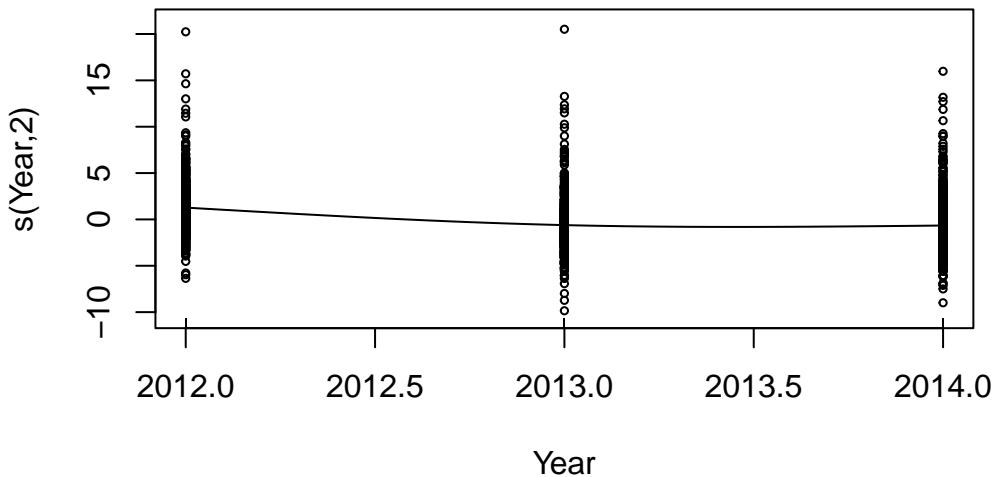
```
plot(poisson_model, shade=T, rug = TRUE, residuals = TRUE,
pch = 1, cex = 0.5)
```

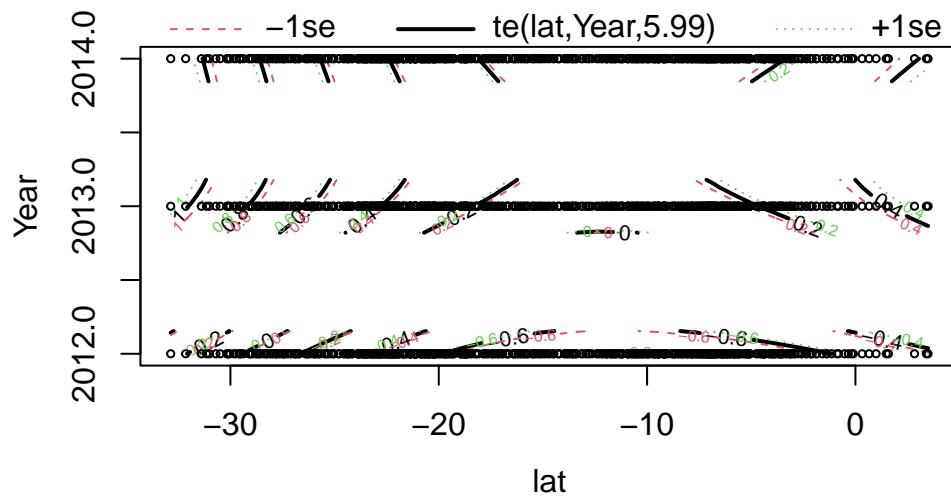
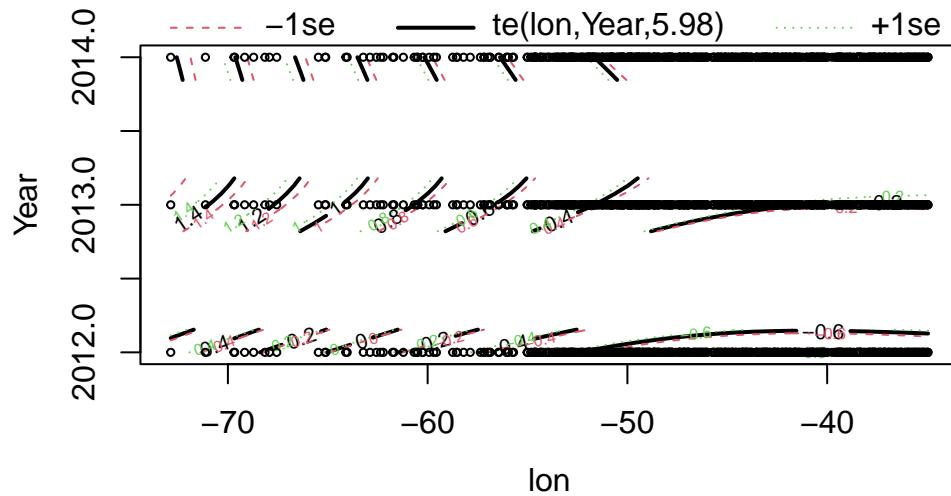












Our QQ-plot suggest that the poisson model fit deviates from the theoretical quantiles in nearly all the values, showing a very flawed fit. As this suggests that our current model doesn't fit the

data correctly and required an extension to our model as the Poisson GAM is not accounted for enough deviance as seen in the residuals.

Since the model is not accounting for enough of the variance we will check if there is a significant difference between the variance and the mean. In this analyses we will use the Pearson estimate for the dispersion parameter, this method allow us to estimate the amount of extra variability, or over-dispersion in count data and therefore analyse if the Poisson distribution assumption of equal mean and variance holds.

```
#Calculating Pearson estimate for dispersion parameter using Pearson residuals:  
sum(residuals(poisson_model, type = "pearson")^2) / df.residual(poisson_model)
```

```
[1] 9.353406
```

```
#The dispersion parameter should be 1, so it seems that there is substantial over-dispersi
```

As we can see from the dispersion parameter should be 1 for the assumption of equal mean and variance to hold true, so it seems that there is substantial over-dispersion in the Poisson GAM. This violates one of the Poisson assumptions that the mean and variance are equal therefore we will have to extend the model from e GAM Poisson to a Negative Binomial GAM

Negative binomial

```
#fitting a negative-binomial model to our TB data:  
nb_model <- gam(TB ~ offset(log(Population)) + s(Indigenous, k = 20) + s(Illiteracy , k =  
summary(nb_model)
```

```
Family: Negative Binomial(9)  
Link function: log
```

Formula:

```
TB ~ offset(log(Population)) + s(Indigenous, k = 20) + s(Illiteracy,  
k = 20) + s(Urbanisation, k = 20) + s(Density, k = 20) +  
s(Poverty, k = 20) + s(Poor_Sanitation, k = 20) + s(Unemployment,  
k = 20) + s(Year, k = 3) + s(Timeliness, k = 20) + te(lon,  
Year, k = 3) + te(lat, Year, k = 3)
```

Parametric coefficients:

Estimate	Std. Error	z value	Pr(> z)
----------	------------	---------	----------

```

(Intercept) -8.442775  0.009432 -895.1   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:
          edf Ref.df Chi.sq p-value
s(Indigenous) 1.001 1.002 22.58 1.90e-06 ***
s(Illiteracy) 10.893 13.279 37.81 0.000278 ***
s(Urbanisation) 11.070 13.410 58.68 < 2e-16 ***
s(Density)    11.231 13.495 155.64 < 2e-16 ***
s(Poverty)     7.109  8.897 36.63 2.36e-05 ***
s(Poor_Sanitation) 14.726 16.913 134.41 < 2e-16 ***
s(Unemployment) 7.203  8.927 99.22 < 2e-16 ***
s(Year)        1.986  1.998 102.59 < 2e-16 ***
s(Timeliness)  5.216  6.533 71.00 < 2e-16 ***
te(lon,Year)   5.700  5.961 101.69 < 2e-16 ***
te(lat,Year)   5.664  5.953 83.69 < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-sq.(adj) =  0.895  Deviance explained = 53.4%
-REML = 7203.2  Scale est. = 1           n = 1671

```

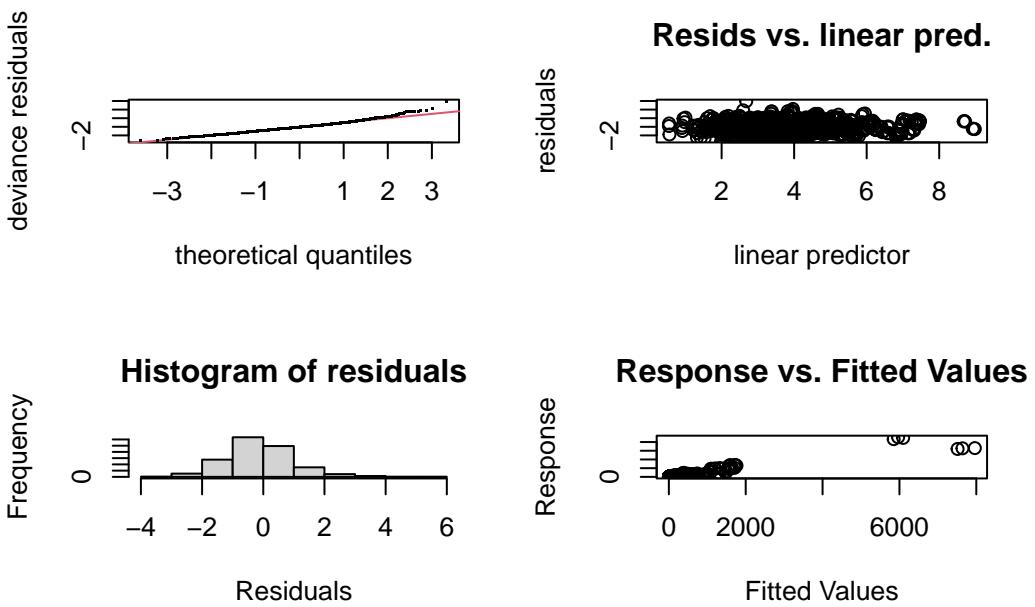
```
#Akaike Information Criterion
nb_model$aic
```

```
[1] 14199.83
```

As we can see from this Akaike Information Criterion(AIC) the Negative Binomial has a significantly lower value than the previous 18585,52 from the GAM Poisson, meaning this is already a better fitting model than the previous one.

Now we will check the residuals to check for any anomalies on our model prediction

```
gam.check(nb_model)
```



Method: REML Optimizer: outer newton
 full convergence after 12 iterations.
 Gradient range [-0.0002588633,5.095501e-05]
 (score 7203.166 & scale 1).
 Hessian positive definite, eigenvalue range [0.0002588044,2.739525].
 Model rank = 167 / 167

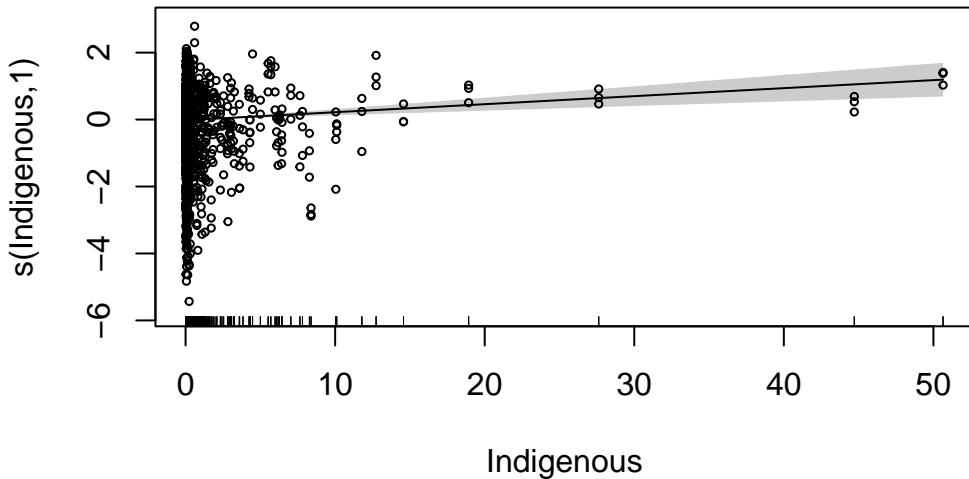
Basis dimension (k) checking results. Low p-value (k-index<1) may indicate that k is too low, especially if edf is close to k'.

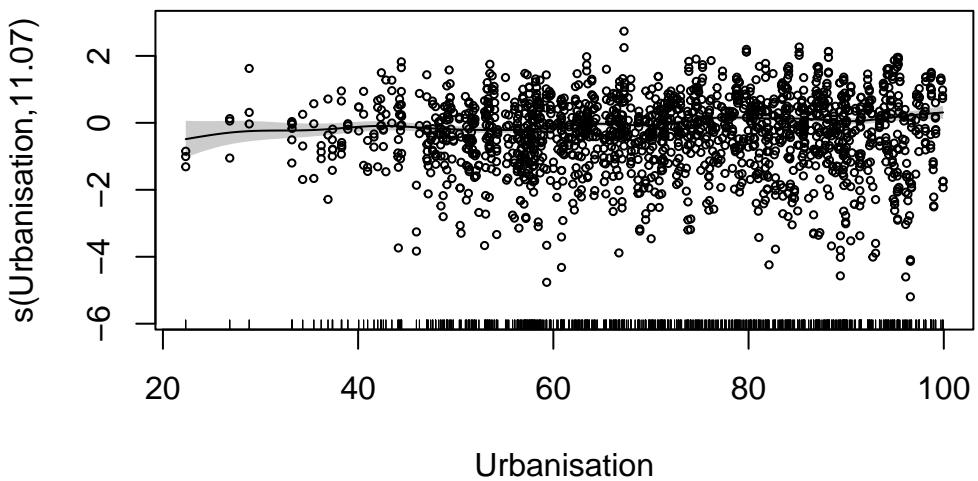
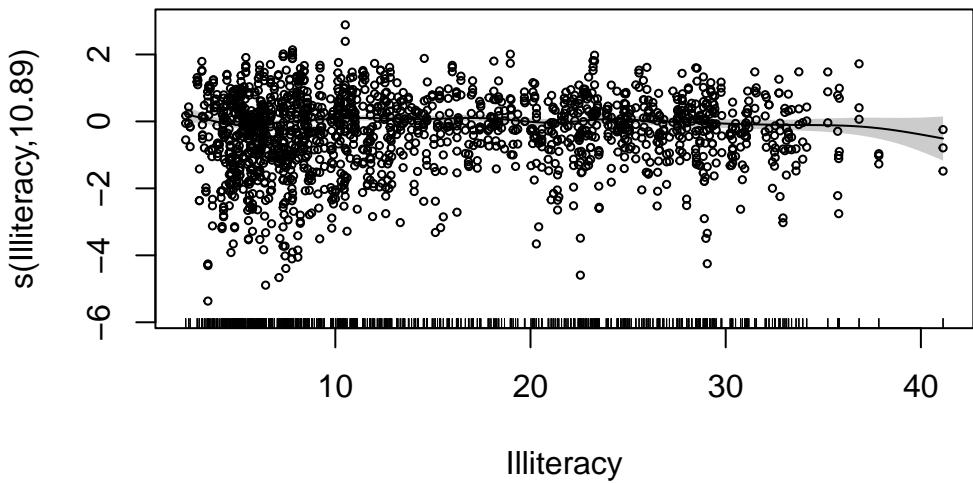
	k'	edf	k-index	p-value
s(Indigenous)	19.00	1.00	0.52	<2e-16 ***
s(Illiteracy)	19.00	10.89	0.51	<2e-16 ***
s(Urbanisation)	19.00	11.07	0.52	<2e-16 ***
s(Density)	19.00	11.23	0.52	<2e-16 ***
s(Poverty)	19.00	7.11	0.52	<2e-16 ***
s(Poor_Sanitation)	19.00	14.73	0.52	<2e-16 ***
s(Unemployment)	19.00	7.20	0.52	<2e-16 ***
s(Year)	2.00	1.99	0.73	<2e-16 ***
s(Timeliness)	19.00	5.22	0.57	<2e-16 ***
te(lon,Year)	6.00	5.70	0.97	0.22
te(lat,Year)	6.00	5.66	0.96	0.12

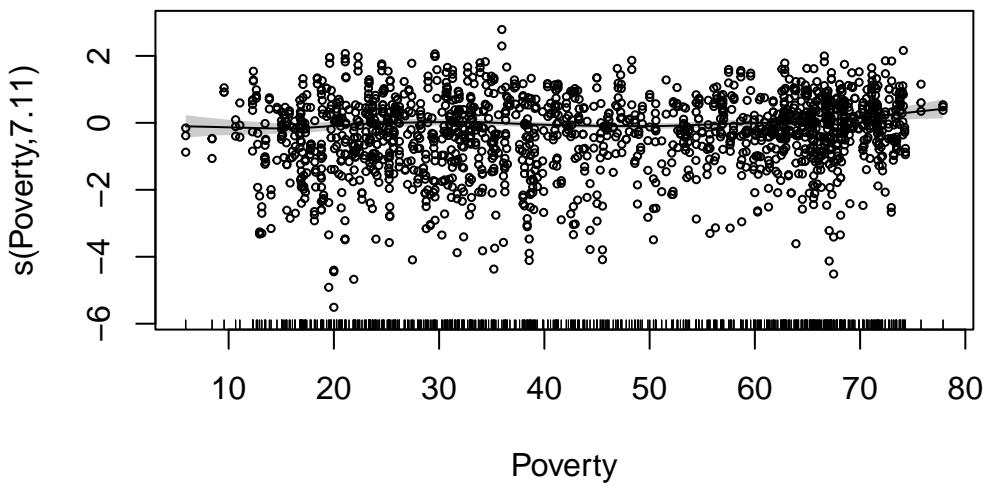
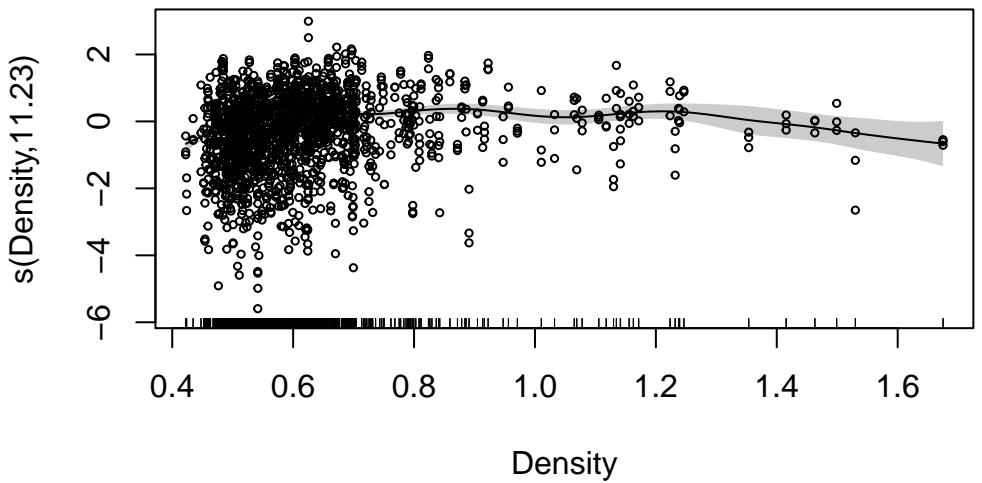
```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

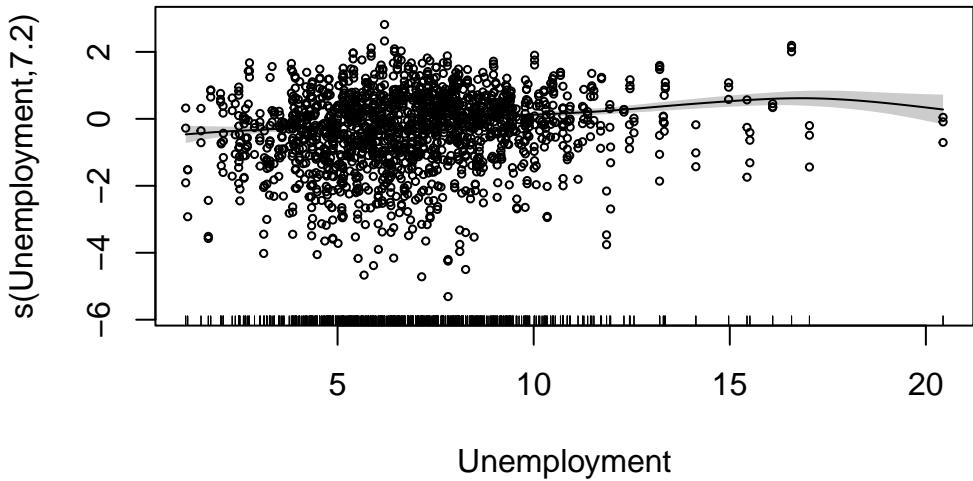
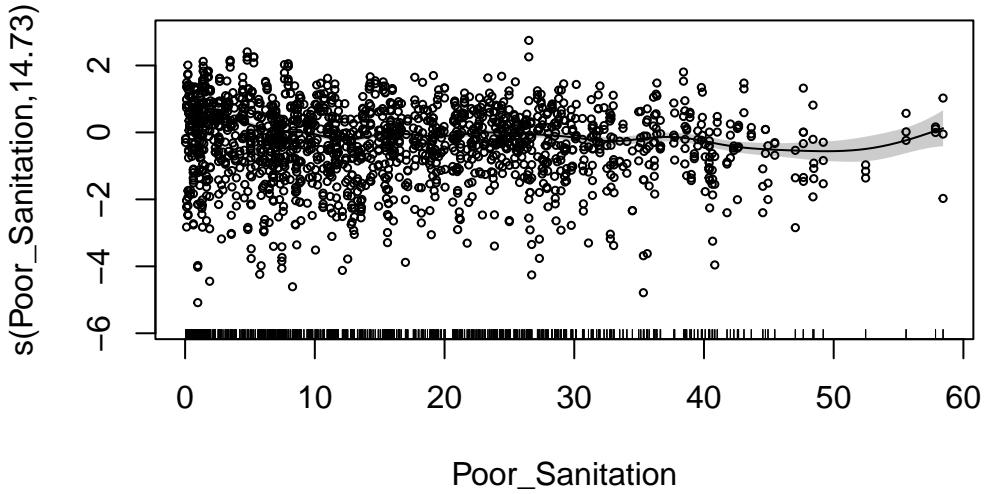
As we can see from the residual versus predictor plot, the values seem to be randomly scattered with no clear trend but with some distance from the zero line. As such we can determine that this scatter is due to random errors and not an uncounted pattern in the model.

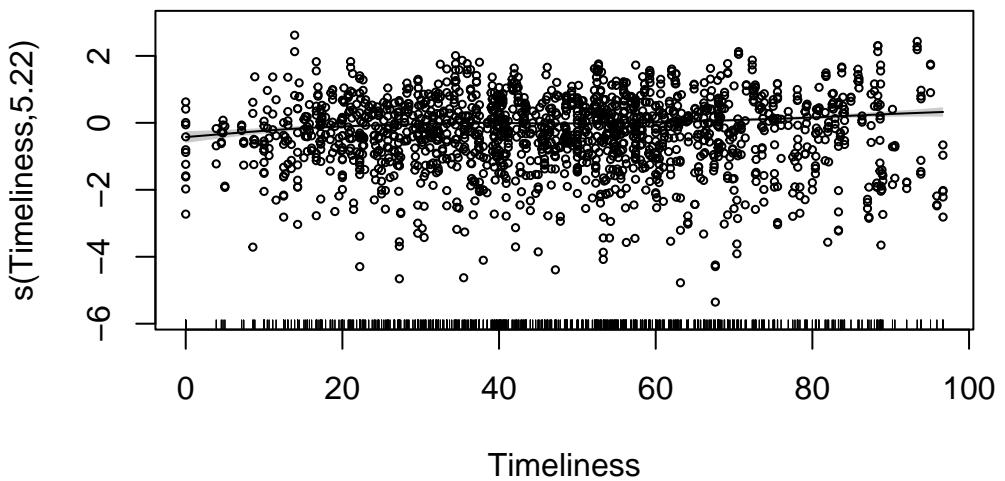
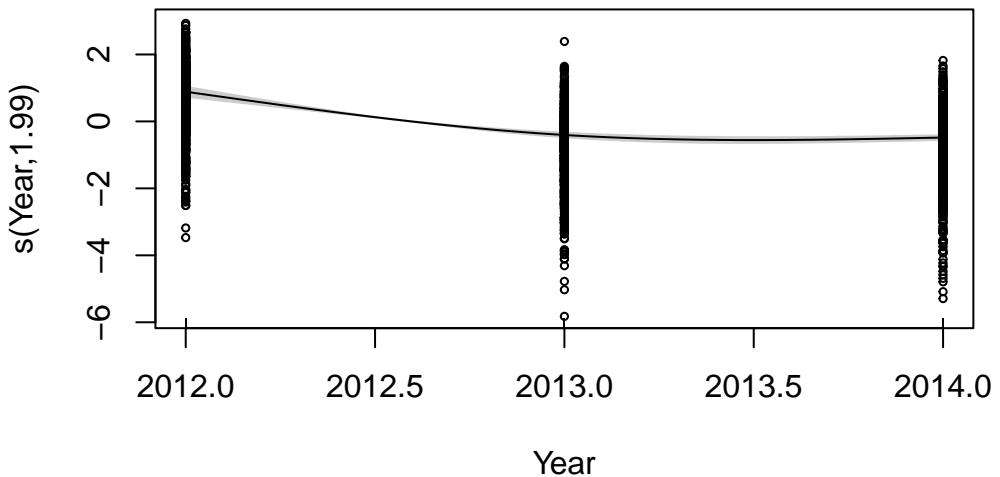
```
plot(nb_model, shade=T, rug = TRUE, residuals = TRUE,  
     pch = 1, scheme =1, cex = 0.5)
```

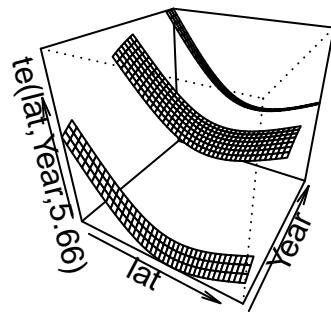
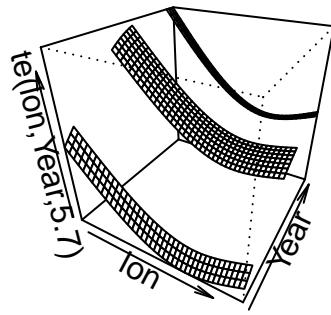












The QQ-plot looks much better for the Negative Binomial model. The majority of points lie either on top of or very near the $y=x$ line, except for a few towards the extremes. This indicates

our assumption about the true distribution of the data is a lot more safe than it was before.

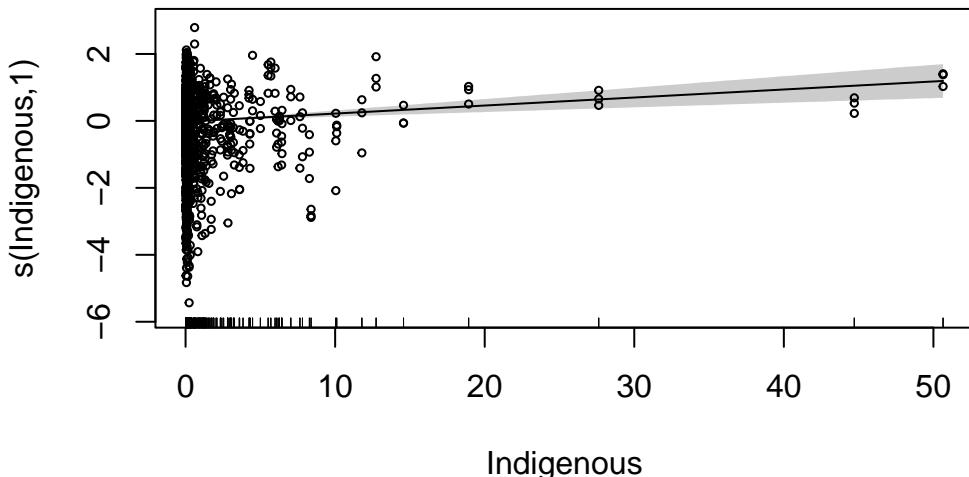
```
#Calculating Pearson estimate for dispersion parameter using Pearson residuals:  
sum(residuals(nb_model, type = "pearson")^2) / df.residual(nb_model)
```

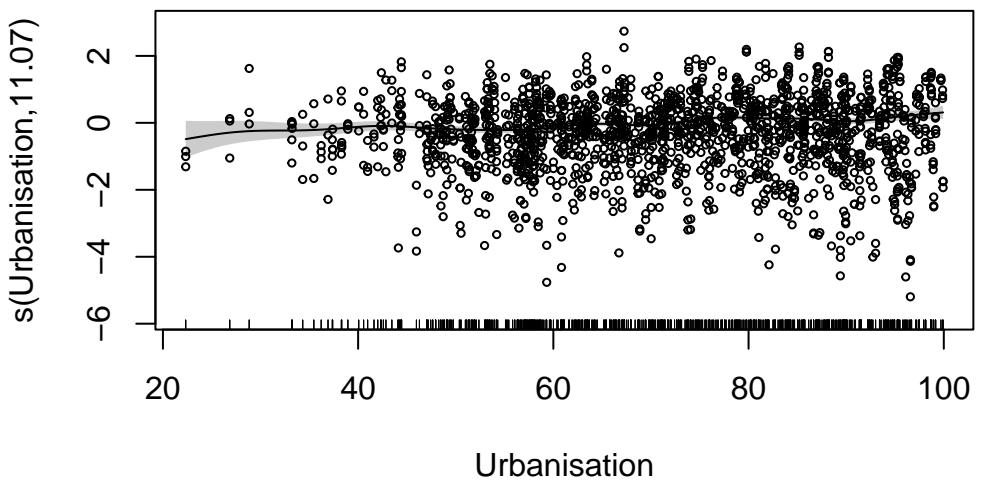
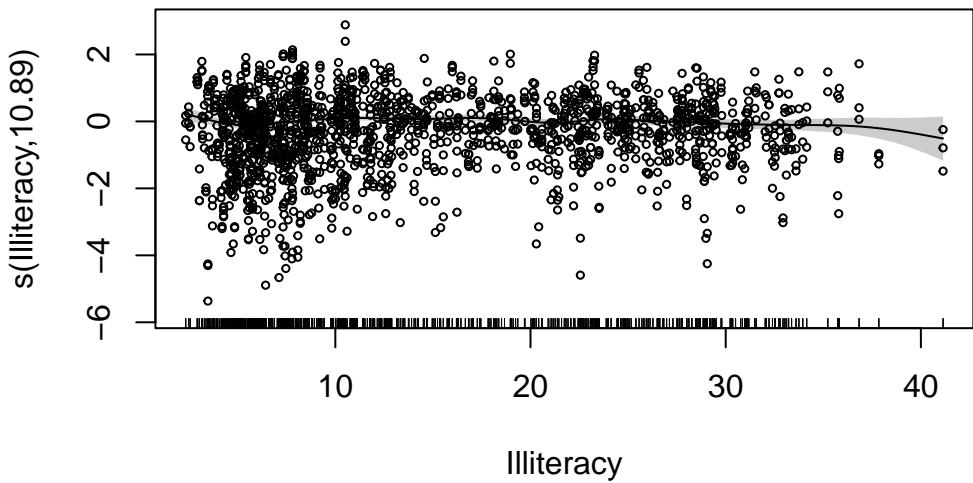
```
[1] 1.338156
```

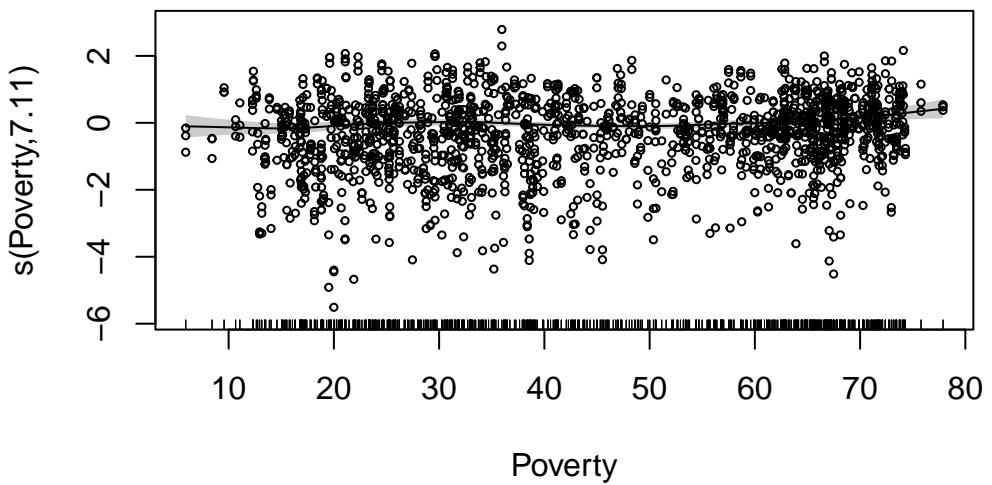
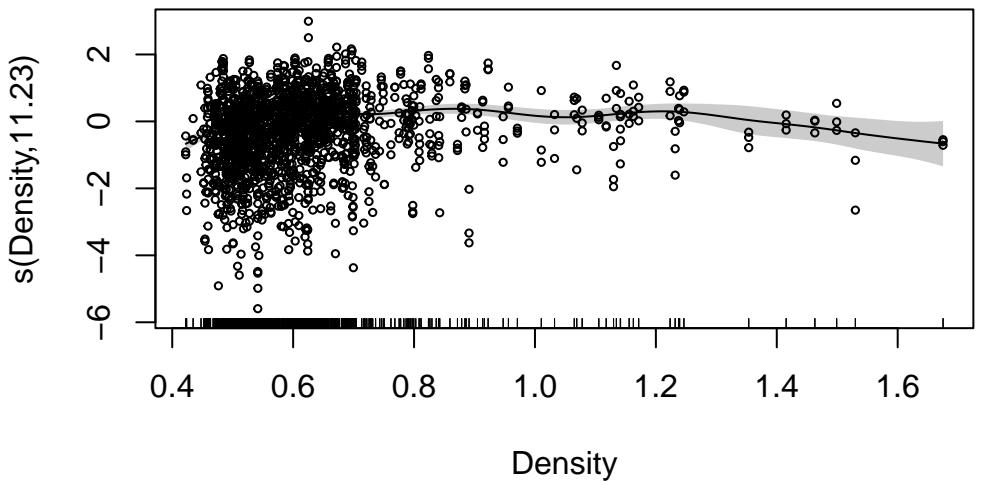
The dispersion parameter is very close to 1, unlike for the Poisson model, meaning that the model that can account for most of the over-dispersion in the data. As such a dispersion parameter value close to 1 can be interpreted as the model is a good fit for the data due to the model adequately capture the variability of the the response variable.

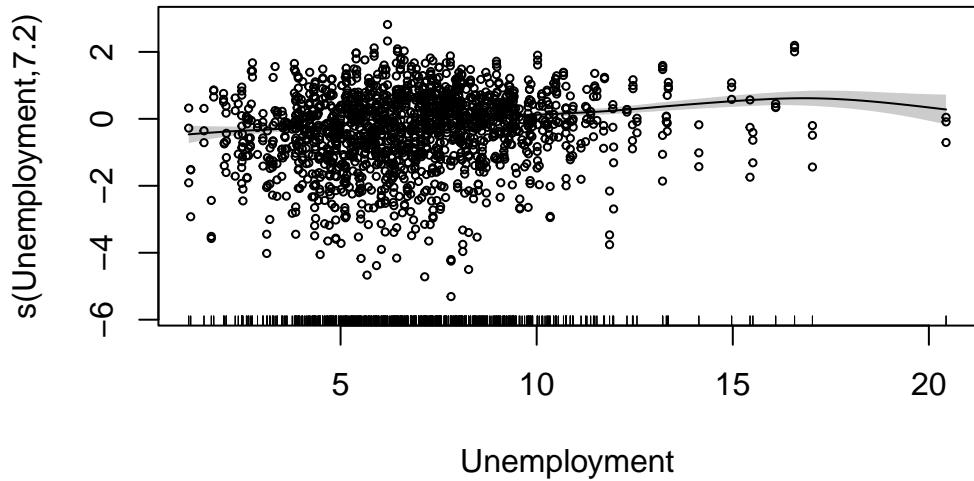
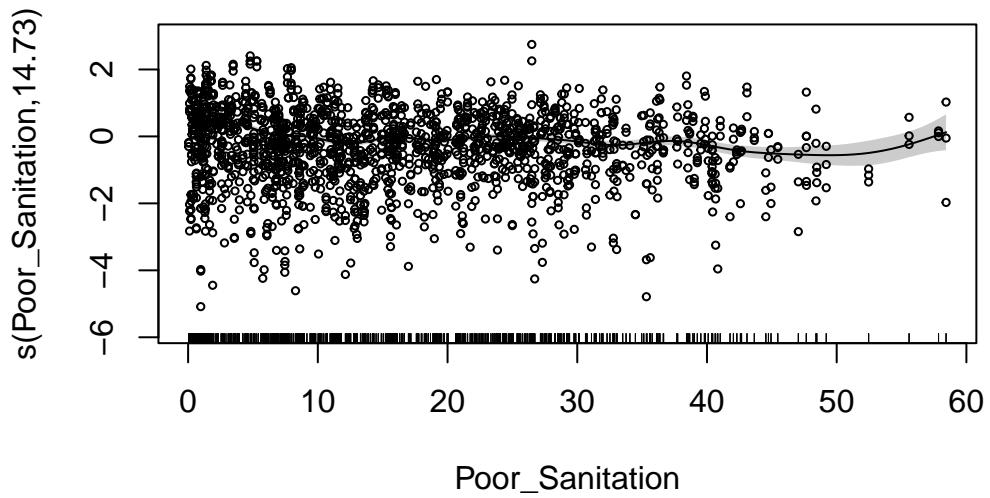
Again tidy up the plots of the Negative Binomial

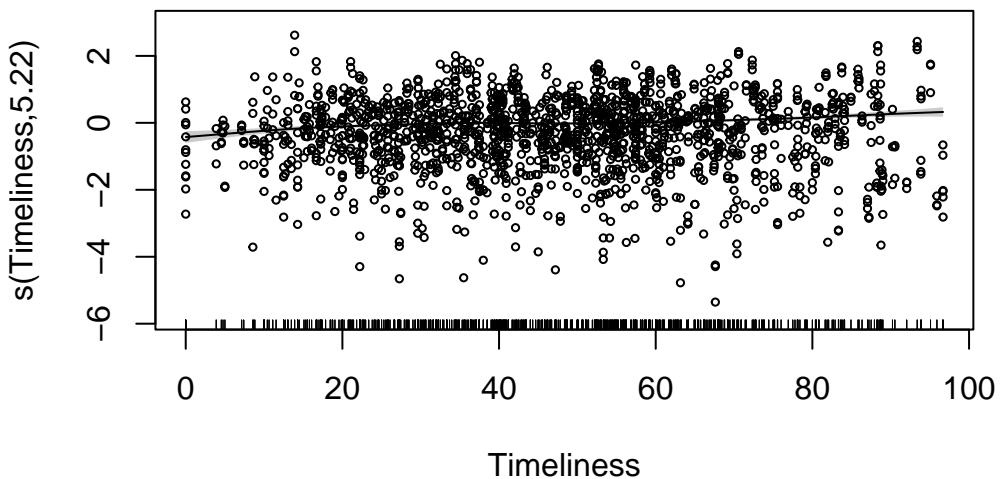
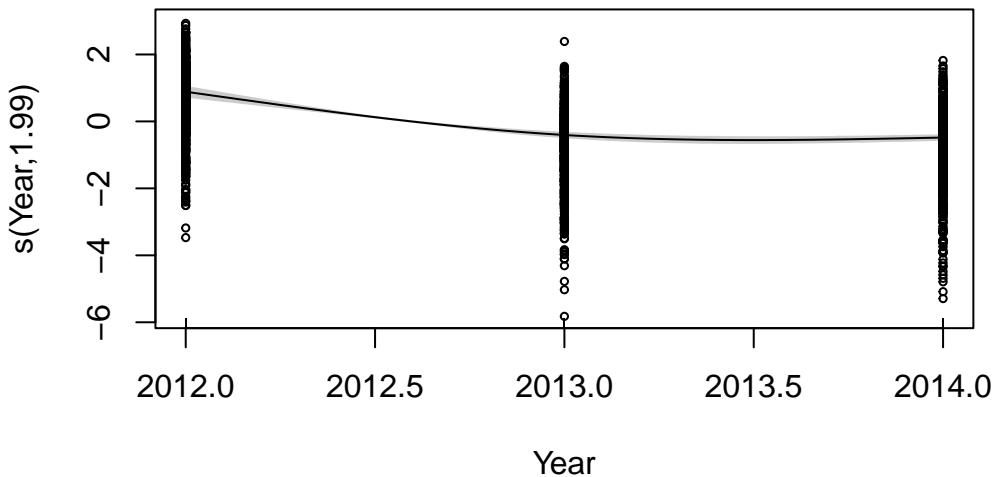
```
plot(nb_model, shade=T, rug = TRUE, residuals = TRUE, scheme=1,  
pch = 1, cex = 0.5)
```

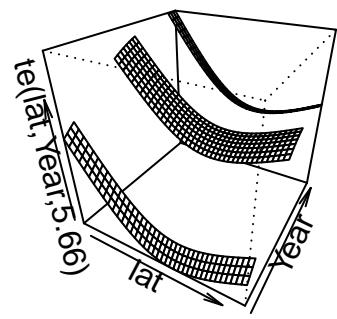
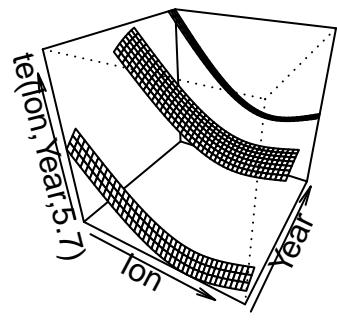




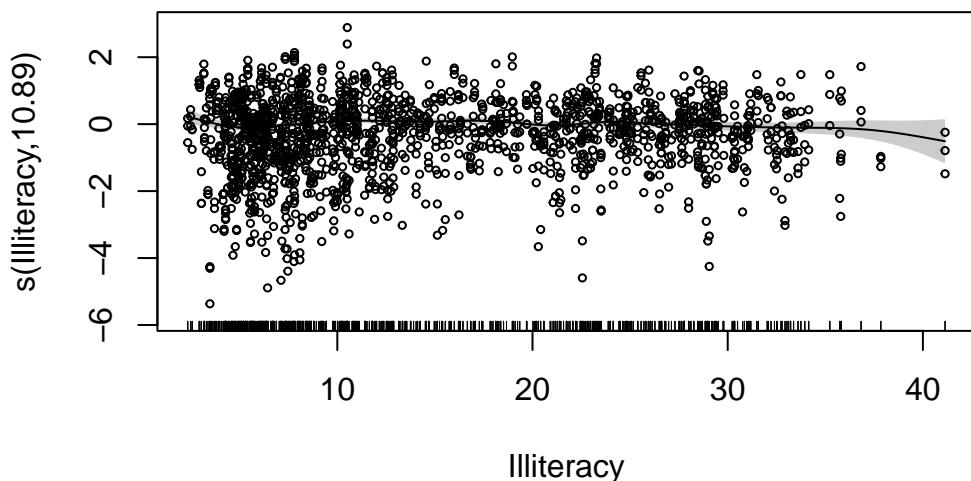
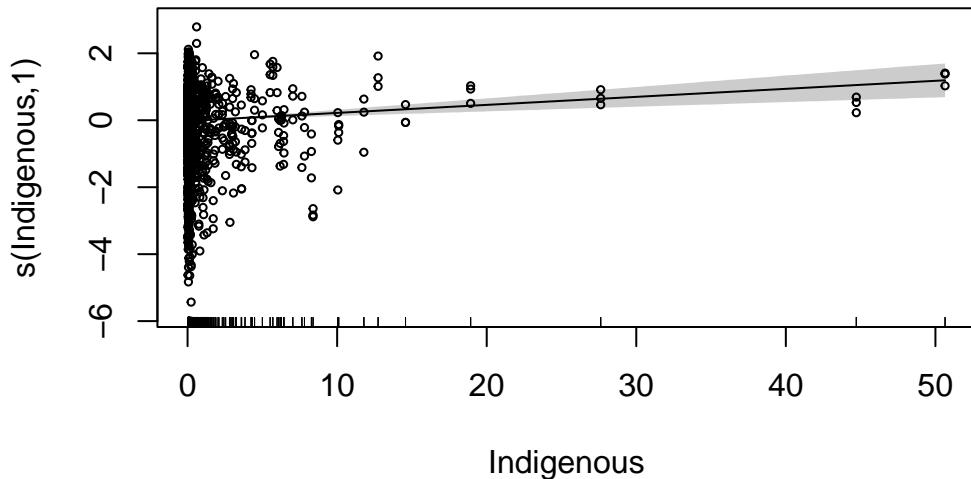


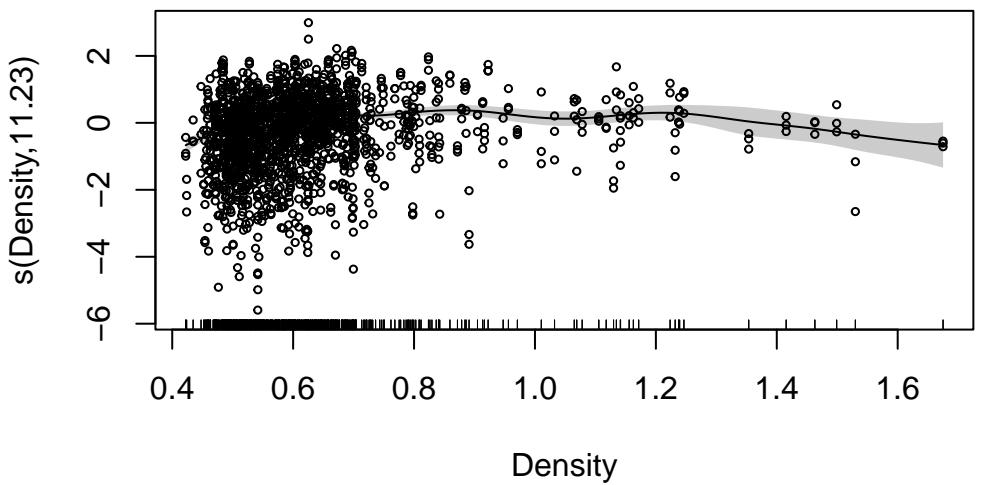
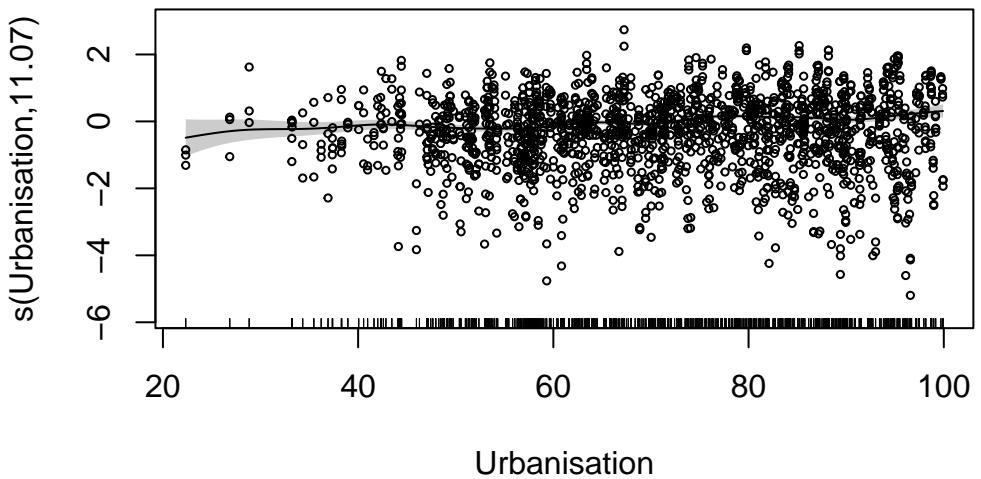


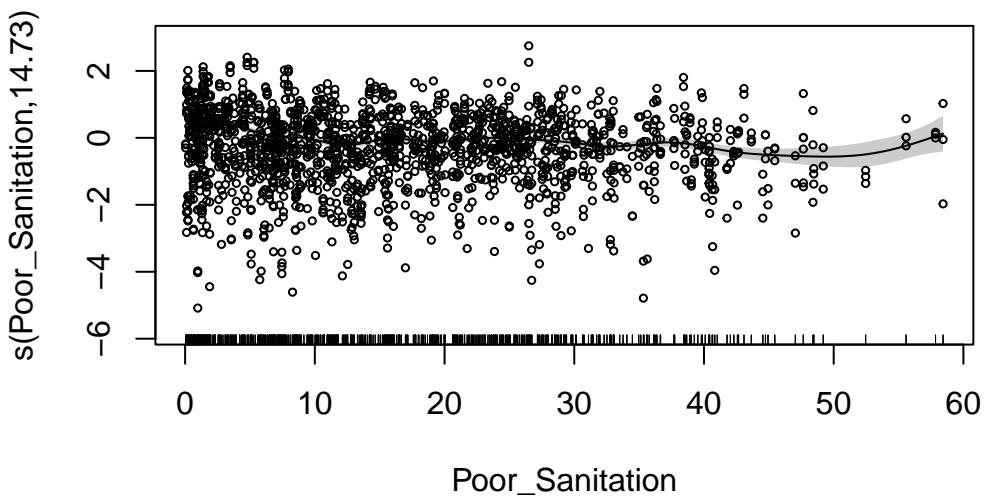
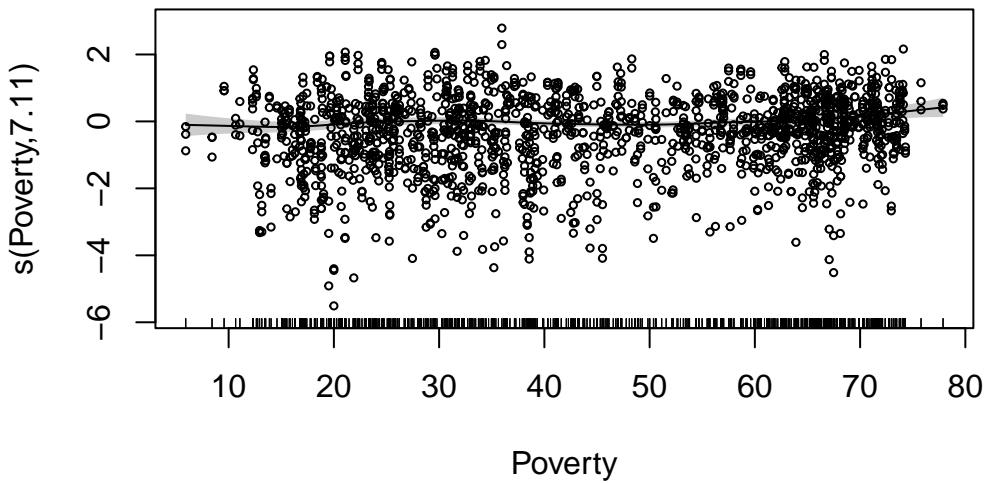


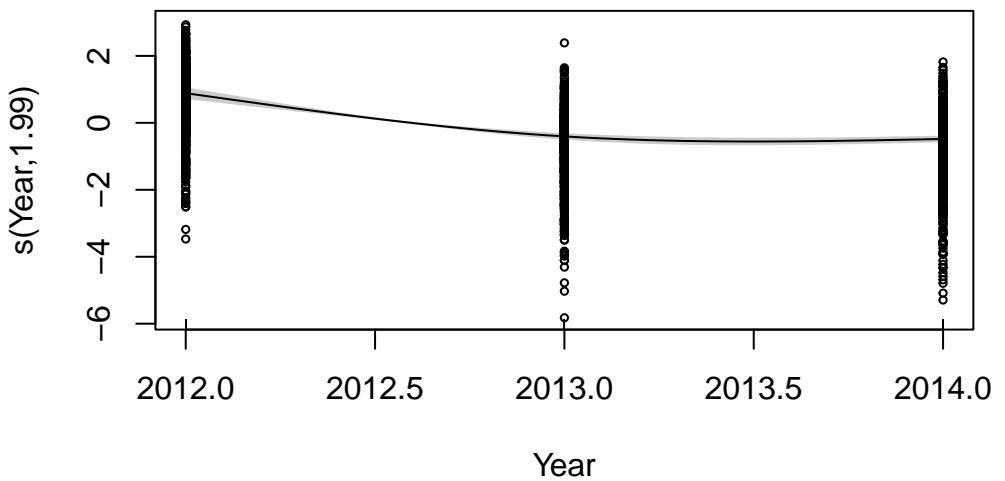
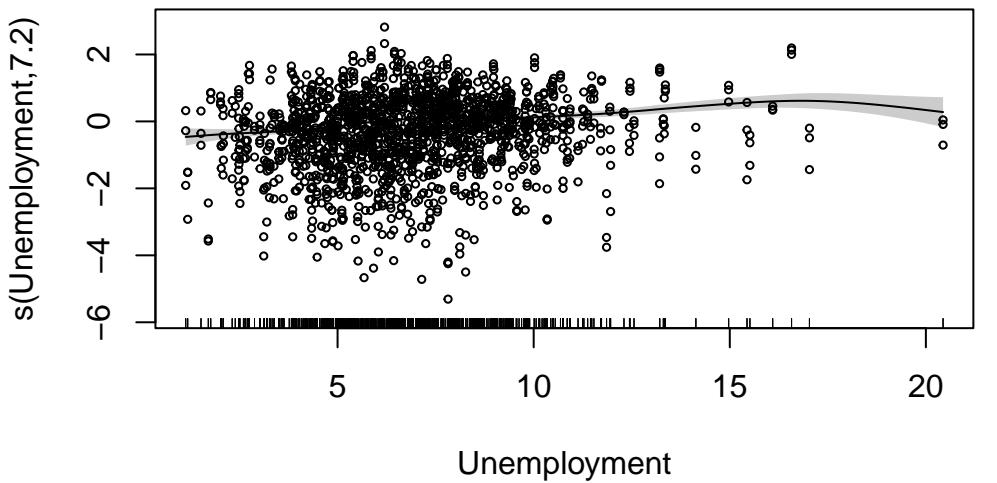


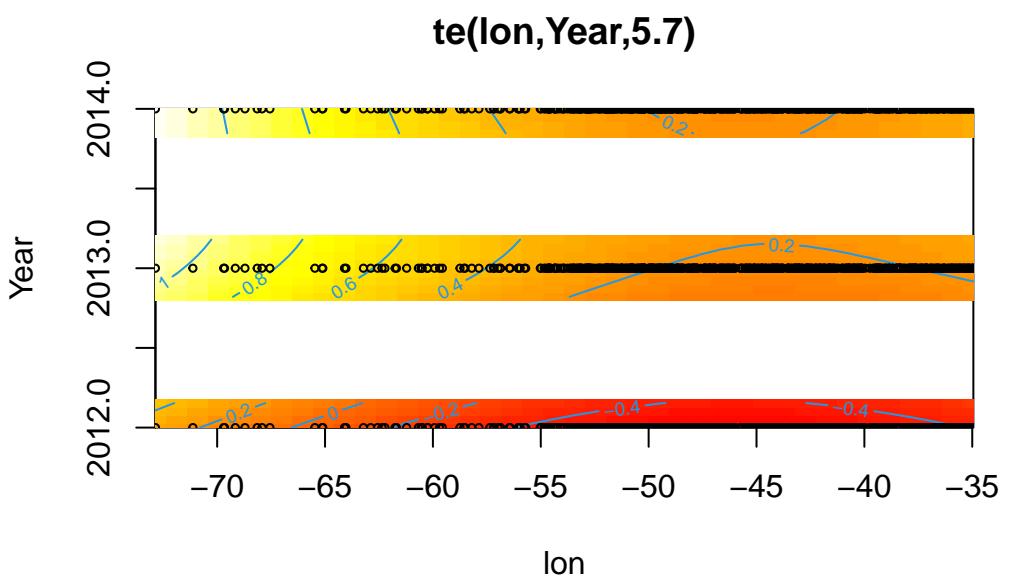
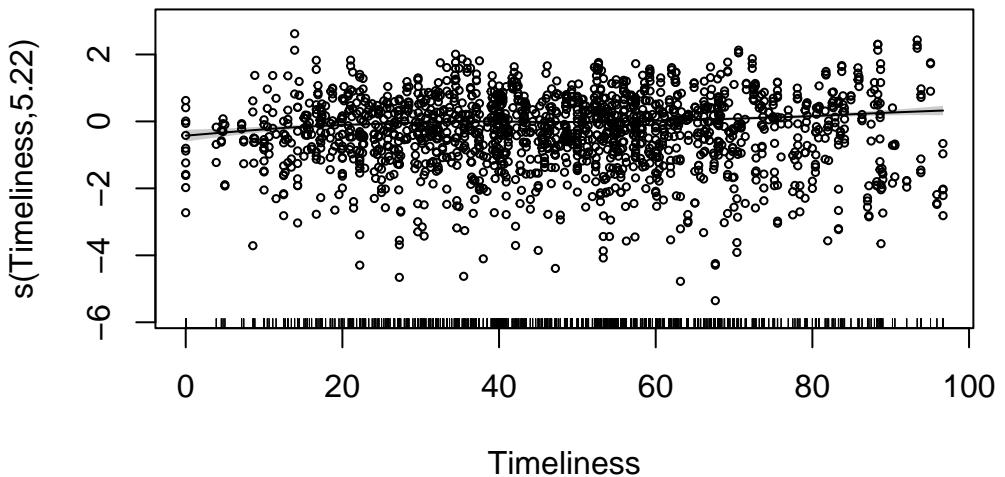
```
plot(nb_model, shade = T, rug = TRUE, residuals = TRUE, scheme = 2, pch = 1,  
cex = 0.5)
```

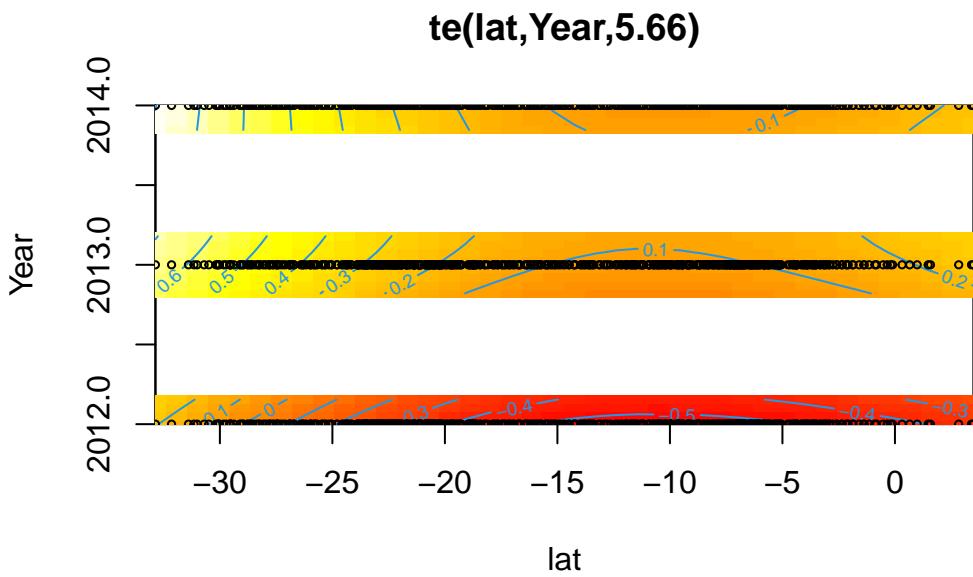








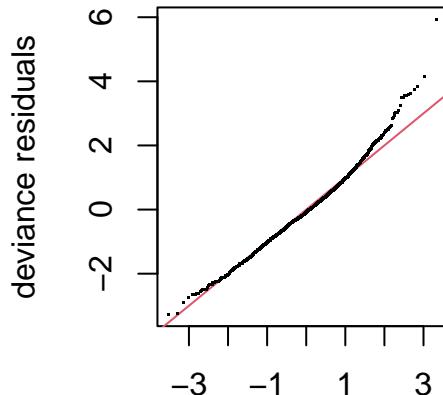
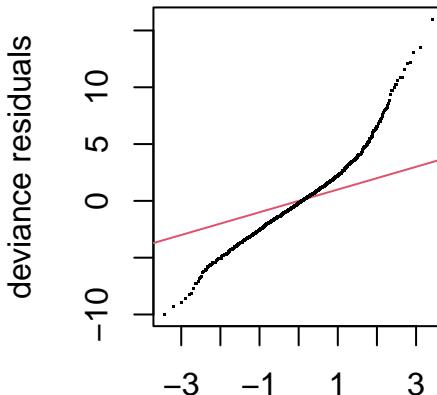




```
par(mfrow = c(1, 2))

qq.gam(poisson_model, main = "Q-Q Plot for Poisson Model")
qq.gam(nb_model, main = "Q-Q Plot for Negative Binomial Model")
```

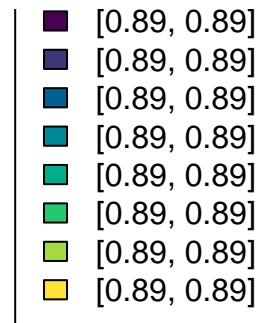
Q–Q Plot for Poisson Mode–Q Plot for Negative Binomial



```
preds = predict(nb_model, type = "terms", se.fit = FALSE)
year2012 <- preds[1:557, ]
year2013 <- preds[558:1114, ]
year2014 <- preds[1115:1671, ]
cord.pred_2012 <- as.numeric(year2012[, 8])
cord.pred_2013 <- as.numeric(year2013[, 9])
cord.pred_2014 <- as.numeric(year2014[, 10])
List = list(cord.pred_2012, cord.pred_2013, cord.pred_2014)
cord.preds.mean <- rowMeans(simplify2array(List))
cord.change_1 <- cord.pred_2013 - cord.pred_2012
cord.change_2 <- cord.pred_2014 - cord.pred_2013

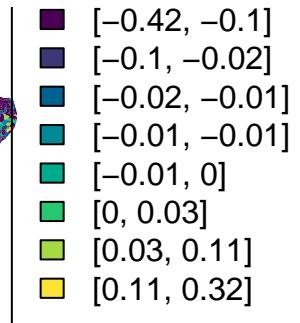
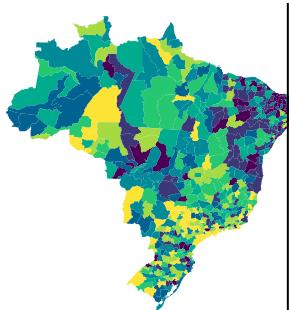
plot.map(cord.pred_2012, n.levels = 8, main = "Predicted TB risk in 2012")
```

Predicted TB risk in 2012



```
plot.map(cord.pred_2013, n.levels = 8, main = "Predicted TB risk in 2013")
```

Predicted TB risk in 2013



Global Tuberculosis Report 2020. 2020. Genève, Switzerland: World Health Organization.