Technical University
of Denmark

Technical University of Denmark

Department of Applied Mathematics and Computer Science

02458 Cognitive Modeling

Assignment 1 Report

Homework 1 Psychophysics

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II. Nomenclature

Acronyms

CDF	Cumulative distribution function
TPR	True Positive Rate
FPR	False Positive Rate
ANN	Artificial Neural Network
PPF	Percent Point Function
SVD	Singular-Value Decomposition

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1 Part 1: Psychophysics Fetchers and Stevens Law

In this part it is going to be shown, that the earlier Law of Gustav Theodor Fechner, which has a logarithmic behavior is invalid and has been corrected by the Stevens law. Many scientists have used the Fetchers Law for a long time. In the following is pointed out, the difference between those two Laws.

Given is the interval of brightness intensity stimullation (*luminance*).

The Steven's law is defined as the following:

$$(\psi = k * \phi s)$$

It allows to determine the perceived brightness at the eye of an observant.
The given variables therefore are

$$(\phi = luminance \text{ and } s = 0.33):$$

Hence, plotting the stimulus intensity against the measured response, which brings us to the observation of a logarithmic relationship for brightness stimuli shown in Figure 1.

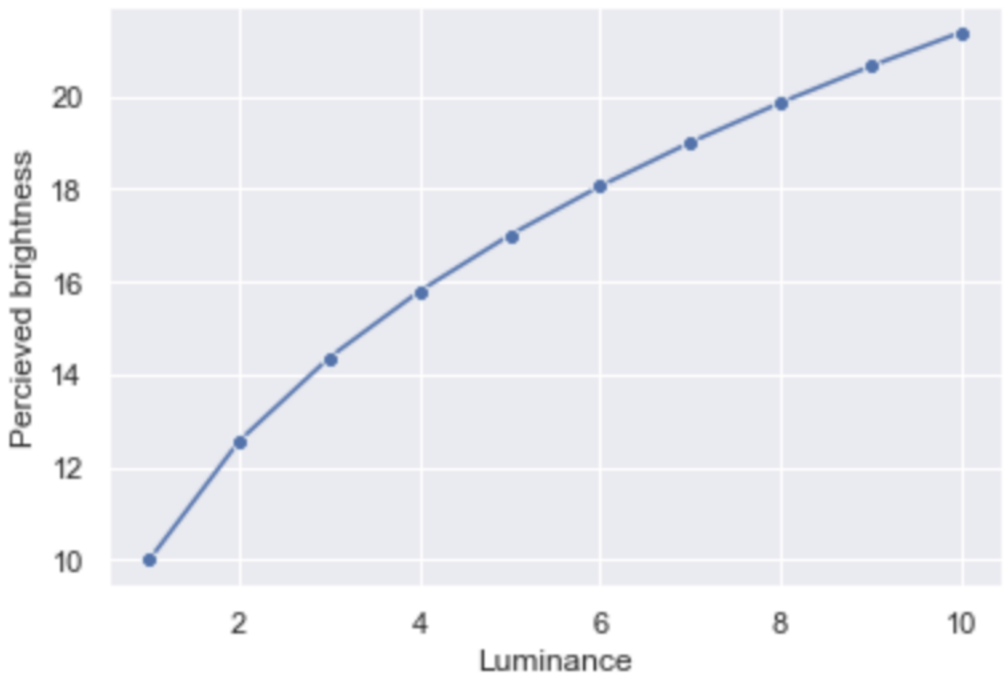

Figure 1 Stimulus intensity against the measured response

Moreover, it is wanted to compare this result against Fechner's earlier theory, let us construct our matrices X and Y needed for determination of the two unknown variables in Fechner's law: k and threshold. Fechner's law dictates that:

$$\psi = k * log(\frac{\phi}{\text{threshold}})$$

The matrix of β is easily findable with the formula: $Y = X * \beta$.
Therefore, the pseudoinverse of X has to be multiplied with the matrix Y. Computing the threshold, is is just needed to place k into the equation for b, which is able to devariate from β!

The matrix of results of the measured percieved stimulus, taken from Steven's law calculation is enabled through Y. Having all values of the model now available, let us compute the perceived brightness of observers.

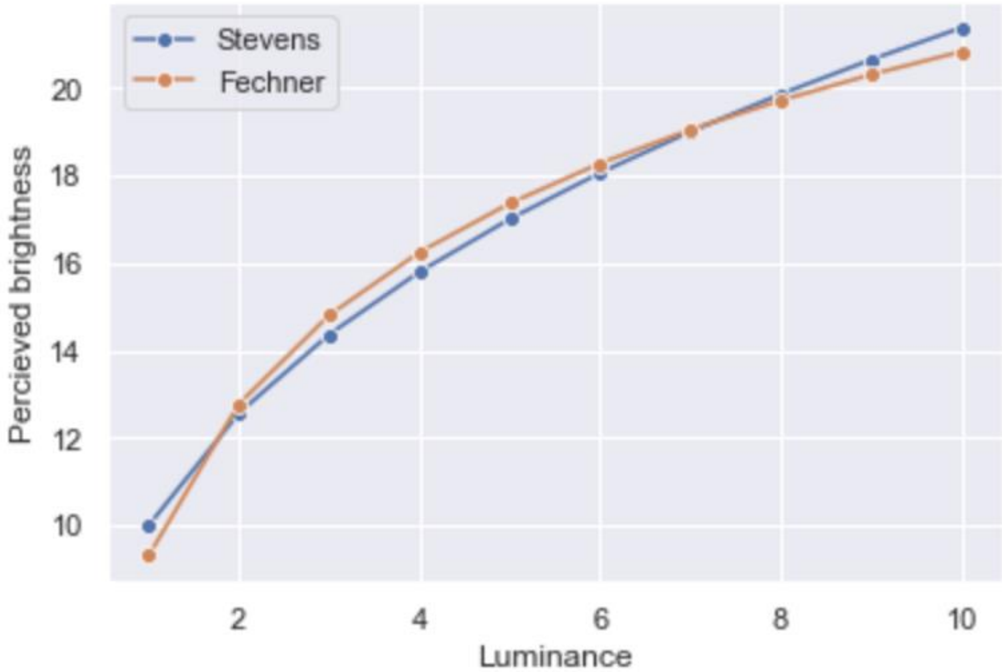

Figure 2 Similar measured response for Stevens and fetcher

Hence it is to acknowledge, that Fechner's law is a good approximation of the perceived brighness response.

However for the electric shock stimulation it will appear differently and it will be able to recognize the difference of those two laws (Figure 3, Figure 4).

The computations are identical to the previous experiment.

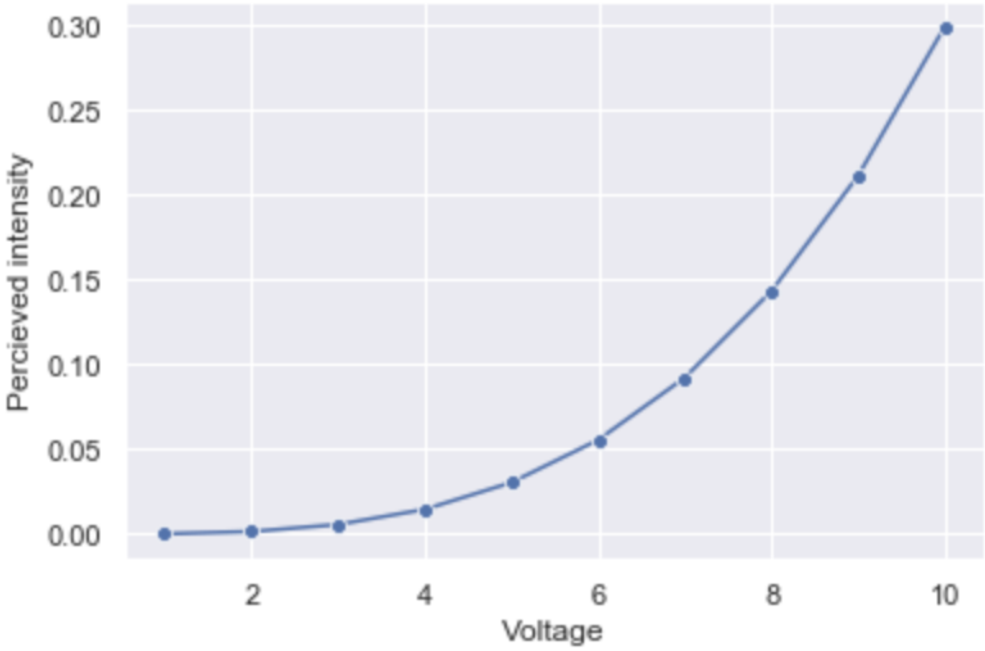

Figure 3 Voltage measured response for Stevens law

As mentioned before, it is able to observe that the logarithmic approach of Fechner's law no longer estimates correctly in terms of intensity and observation. Therefore many scientists have assumed the Fechner's law as correct but the problem is that it is only a case approximation where stimulation detection behaves logarithmically.

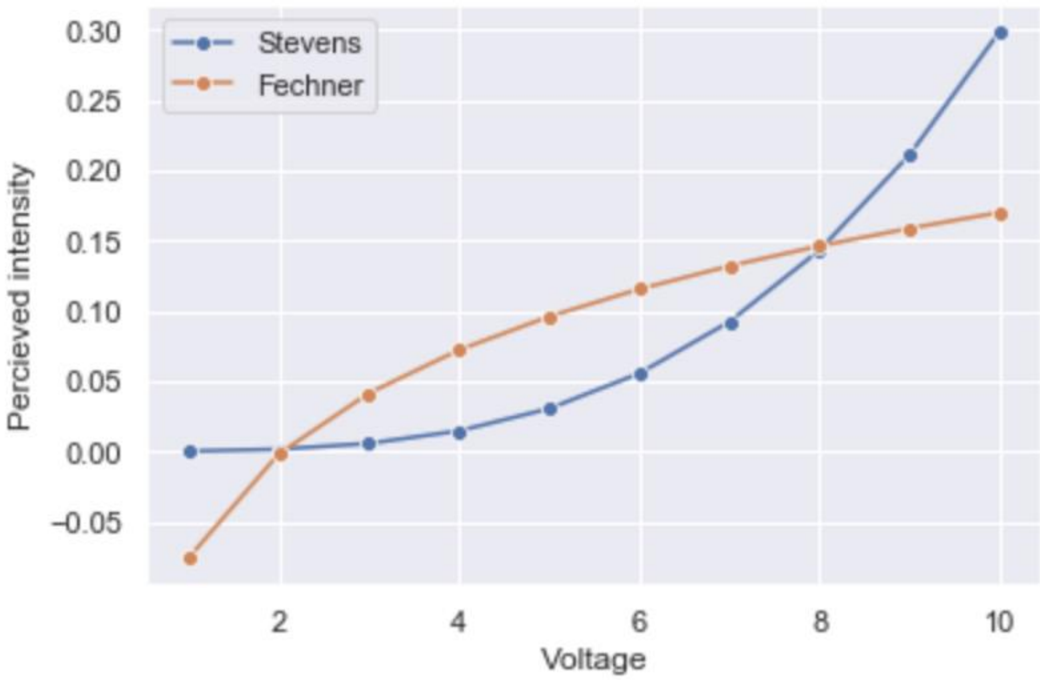

Figure 4 Voltage measured different responses for Stevens and Fechers lawsr

2 Part 2: Signal Detection Theory

2.1 Part 1:

It exists the three selected criterions λ:

1. λ is conservative and ~= **1.5**
2. λ is lax and ~= **-0.5**
3. λ is moderate and ~= **0.5**,

Additionally, 100 trials from an observer with similar variance properties and d'=1.
→300 trails
There are 100 trials and two normal distributions to show: In detail the noise and the signal distribution. Therefore, it will exist 50 trials for each of the distributions, for each experiment.

To expect is that a **lax** observant will have much higher *TPR* and *FPR* than the conservative one. Additionally, the moderate observant should be in general give median values.

If the stimulus intensity is below the criterion value, it will be recorded the response as a 'No'. If the stimulus intensity is higher than the observer's criterion, then they will respond with 'Yes' and it will recorded as soThat is why higher criterion levels (e.g. for conservative observers) will make the response to any stimulus value be less likely to be a 'Yes', as the threshold for responding 'Yes' has been set to a high value. The exact opposite happens for lower criterion levels (e.g. for lax observers).

Based on $P(Hit)$ and $P(FA)$, it is possible to compute d' for each of the three experiments. Taking into account that :

$$\phi(\lambda) = \mathrm{CDF}(\lambda)$$

$$\phi^{-1}(P)=\mathrm{PPF}(P)$$

Then it is possible to observe if rather the d is correct or incorrect. It should be d'= 1, because noise distibutions (μ=0) and the signal distribution (μ=1) are set at the beginning of the exercise.However the result is:

d' for conservative observer is: 1.5860501115173147
d' for lax observer is: 0.8222300530384167
d' for moderate observer is: 1.1110442045074251

All of our 3 experiments did not result correctly the value of d'. The moderate criterion observer is the closest to the actual value.

2.2 Part 2:

It is needed the **Gaussian** curve for the three experiments. Plotting it in the Cartesian coordinate system will result in a straight line similar to the function f(x)=x.

The line is determined by the inverse of the CDF function upon the probabilities $P(FA)$ and $P(Hit)$. Therefore, if have **P(FA)** and **P(Hit)** for each of our three experiments, it is possible to plot a regression line through them.

Observable from the calculations upon the Gaussian model below in Figure 5, the estimated value of d'=0.99 are reasonably close to the expected value of d'=1 for an equal variance observer.

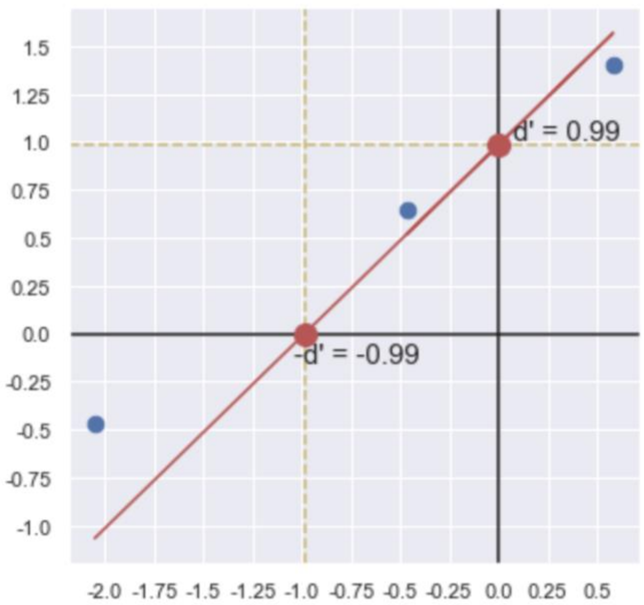

Figure 5 Gauss curve

2.3 **Part 3**

Now it is needed to simulate the same number of stimulus-vs-noise experiments as before, but this time with changing the distribution properties of the signal, so that the mean is now higher (μ_s=2, μ_n=0) and the variance is now different than that of the noise distribution (σ_s=1.5, σ_n=1). This creates strong implications.Hence, the measure of d' is not anymore equal to the difference between $\mu s-\mu n$. The formula right now needs to take into consideration the μs and σs within the new signal distribution. Originally, this values to are set to 2 and 1.5, respectively. The generated random sample for the signal will not have exactly this mean and std. Thus μ and σ are unknowns, and compute them from the formula:

$$\phi^{-1}(P(Hit))=1 / \sigma_s * \phi^{-1}(P(FA)) + \mu_s / \sigma_s$$

This is a linear equation of the form: $y=ax+b$

$$\rightarrow \; y=\phi^{-1}(P(Hit)) \text{ and } x=\phi^{-1}(P(FA)).$$

The unknowns σ_s and μ_s, have to be found based on the fact that $a=1/\sigma_s$ and $b=\mu_s\sigma_s$.

The calculations for an unequal variance observer are done exactly as in the previous part. Hence the following numbers for d result:

d' for conservative observer is: 2.022472393711362
d' for lax observer is: 1.3922272780009755
d' for moderate observer is: 1.2208458759422112

d' values received are much more further off the assumed difference of 1, because the difference between μ_s=2 and μ_n=0 is 2. These values are far away from the expectations and not very high quality to work with.
The actual unequal variance-related d' parameter needs to be calculated. Therefore, the three values of $\phi^{-1}(P(Hit))$ generate the Y matrix and $\phi^{-1}(P(FA))$ creating the X matrix. A second column of ones for the intercept. It is needed to compute the matrix β.

$$\beta= \begin{bmatrix} \text{slope(a)} \\ \text{intercept(b)} \end{bmatrix}$$

To perform the computation of finding β from the formula $Y{=}X{*}\beta$ it is needed the pseudoinverse.

$$X^{-1}{*}Y(=X^{-1}{*}X{*}\beta)=\beta$$

Finding a means finding σ_S.
To compute μ_S, σ_S is needed. It is placed into the the equation for b.
This results in:

$$\mu s=2.15$$
$$\sigma s=1.58$$

Thus, the sensitivity of the observer has been recorded.

Observable from the and calculations above, the intersection of a Gaussian curve for an unequal variance observer, it is not giving a 45°–45°–90° triangle anymore.

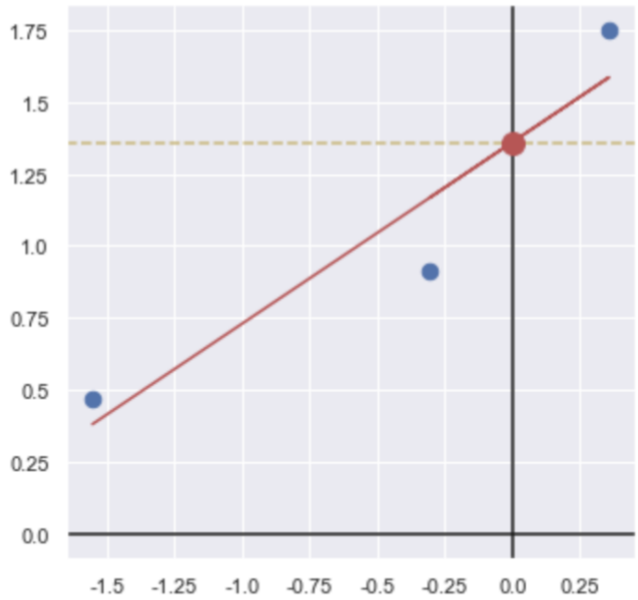

Figure 6 Gauss curve in Cartesian coordinates for unequal variance

3 Part 3: Problem Psychometric Function

Question:

An observer responds according to a psychometric function shaped like a cumulative Gaussian probability function in a signal detection task. The table lists the number of yesresponses out of 50 trials for five stimulus levels given in arbitrary units

Stimulus level	0.4	0.9	1.2	1.7	2.3
Number of yes Responses	1	6	13	32	49

Table 1 Number of Yes responses and Stimulus level

Fit the psychometric function to the data using the maximum likelihood principle. You can use any numerical optimization routine that you like.

• What are the estimates of the parameters of the psychometric function?

• In a follow-up experiment it is useed only intensity levels 1 and 2. The task of the observer is to say whether the intensity level is high or low. What value do we expect for the sensitivity (*d'*)?

Solution:

It is to estimate the parameters of the psychometric function. The function is represented as a CDF. Knowing this, it is possible to write the psychometric function. Hence it is needed ti inverse the CDF function in order to retrieve the probability density function of our experiment. This can be extracted from the book.

$$\phi[\mathrm{P(yes|x)}]^{-1} = \frac{x-\mu}{\sigma}$$

The two unknown parameters are μ and σ,can be extracted from the linear form:

$$a*x+b$$

$$\phi[\mathrm{P(yes|x)}]^{-1} = \frac{1}{\sigma}\, x + \left(-\frac{\mu}{\sigma}\right)$$

This is important because now:

$$a = \frac{1}{\sigma} \quad \text{and} \quad b = -\frac{\mu}{\sigma}.$$

The data can be plottet like this and created the following graph:

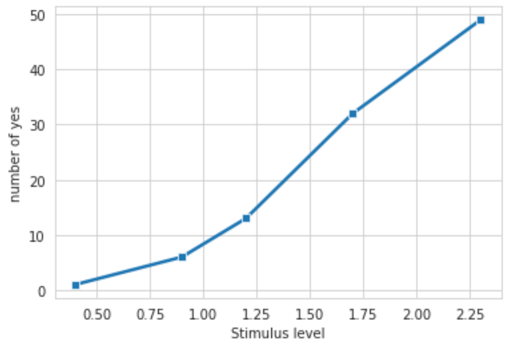

Figure 7 Number of Yes and stimulus plotted as lineplot

Interesting is $P(Yes|stimulus)$:

[0.02, 0.12, 0.26, 0.64, 0.98]

Hence, the list of the cummulative probabilities.

[-2.05374891 -1.17498679 -0.64334541 0.35845879 2.05374891]

The cumulative normal distribution is plotted below in Figure 8.

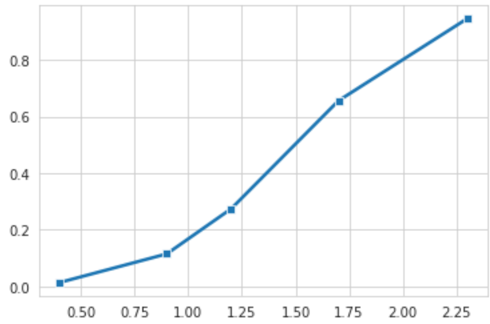

Figure 8 Cumulative normal distribution Number of Yes and stimulus

Now, it is needed to compute the parameters μ and σ, using K.
K has the inverse CDF of the probability of yes responses by observers.
Additionally the matrix X with two columns representing in nessecary.

1. Column of values for the original stimulus

2. Column of ones for the computation of the intercept (b)

Numpy has a useful tool linalg.pinv. It computes the (Moore-Penrose) pseudo-inverse of a matrix. Calculate the generalized inverse of a matrix using its singular-value decomposition (SVD) and including all large singular values.

Additionally computing the matrix β:

$$\beta = \begin{bmatrix} \text{slope(a)} \\ \text{intercept(b)} \end{bmatrix}$$

a, b = (2.1401101424610594, -3.074117866040939)

That is great, because now it is easily possible to find σ and μ.

σ= 1/a = 0.467265670191176

μ = 1.4364297449222871

The second part inspects the Psychometric function - sensitivity (d') computation:
The sensitivity of this observer refers to distance between the gaussian PDF of the stimulus of high intensity (centered around the mean μ_s=2) and the gaussian PDF of the stimulus of low intensity (centered around the mean μ_s=1).
The formula for the sensitivity d' of the equal variance observer is:

$$d' = \phi^{-1}(P(Hit)) - \phi^{-1}(P(FA)).$$

This can be extacted from the script or book.

It is needed to use the values of **P(Hit)** and **P(FA)** from the psychometric function.

$$\phi^{-1}(P(High|x=2)) = \phi^{-1}(P(Hit))$$
$$\phi^{-1}(P(High|x=1)) = \phi^{-1}(P(FA))$$

Placing x = 1 and x = 2, into the linear function.

$$\phi^{-1}(P(High|x=2)) = 1.2061024188811795$$
$$\phi^{-1}(P(High|x=1)) = -0.9340077235798798$$

The final value for the distance is **2.1401101424610594.**