9.1.5

EE24BTECH11030 - Kedarananda

Question

Solve the differential equation:

$$\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \ln(x)$$

Solution: Theoretical Approach

Given:

$$\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \ln(x) \tag{1}$$

Since the equation is nonlinear, direct analytical solutions are challenging. Computational approaches such as **Euler's Method** or higher-order methods are used for approximations.

Solution: Computational Approach

The given equation is reformulated as:

$$y_1 = y, \quad y_2 = \frac{dy}{dx} \tag{2}$$

$$\frac{dy_1}{dx} = y_2 \tag{3}$$

$$\frac{dy_2}{dx} = \ln(x) - 5x(y_2)^2 + 6y_1 \tag{4}$$

The system of equations is solved iteratively using numerical techniques.
Initial Conditions:

- $y(0.01) = 0.01$
- $y'(0.01) = 1$
- Step size $h = 0.01$

Using Euler's method:

$$y_{1,n+1} \tag{5}$$

$$y_{2,n+1} = y_{1,n} \tag{6}$$

$$y_{2,n} + h \cdot y_{2,n} \tag{7}$$

$$\ln(x_n) - 5x_n(y_{2,n})^2 + 6y_{1,n} \tag{8}$$

This update is applied iteratively for all x_n values to compute $y(x)$ and $y'(x)$.

Comparison of Results

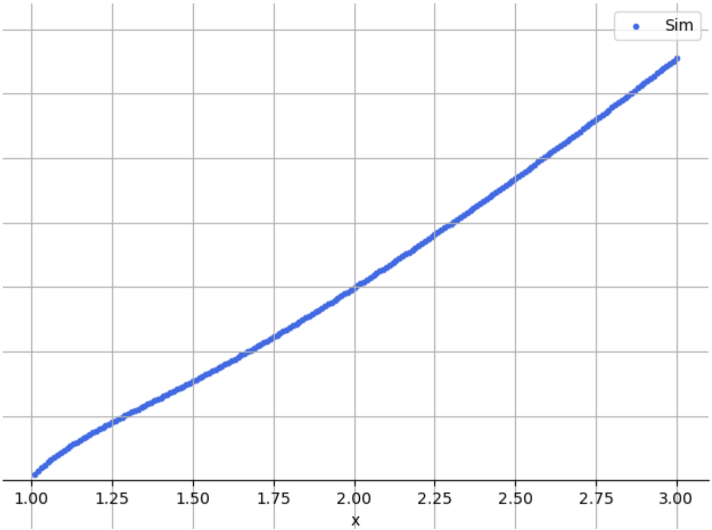

fig2.png

Code for Plot

fig3.png