12.8.1.12

EE24BTECH11030 - KEDARANANDA

Question: Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines.x = 0 and x = 2 is

Solution:

Theoritical Solution:

$$A = \int_0^2 \sqrt{4 - x^2}\, dx. \tag{1}$$

Substitute x = 2sin θ

$$A = \int_0^{\pi/2} 2\cos\theta \cdot 2\cos\theta\, d\theta \tag{2}$$

$$= \int_0^{\pi/2} 4\cos^2\theta\, d\theta. \tag{3}$$

Using the identity $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$A = \int_0^{\pi/2} 4 \cdot \frac{1 + \cos 2\theta}{2}\, d\theta \tag{4}$$

$$= \int_0^{\pi/2} 2\, d\theta + \int_0^{\pi/2} 2\cos 2\theta\, d\theta. \tag{5}$$

$$\int_0^{\pi/2} 2\, d\theta = 2\,[\theta]_0^{\pi/2} = 2 \cdot \frac{\pi}{2} - 0 = \pi, \tag{6}$$

$$\int_0^{\pi/2} 2\cos 2\theta\, d\theta = [\sin 2\theta]_0^{\pi/2} = \sin\pi - \sin 0 = 0. \tag{7}$$

$$\tag{8}$$

Thus, the area is: $A = \pi + 0 = \pi$.

Computational Solution:

We need to compute A = $\int_0^2 \sqrt{4 - x^2}\, dx$ using the trapezoidal rule with n = 500.The interval is [a, b] = [0, 2] and the number of subintervals is n = 500.

$$\text{The width of each subinterval is } h = \frac{b - a}{n} = \frac{2 - 0}{500} = 0.004. \tag{9}$$

$$\text{The function to integrate is f(x) = } \sqrt{4 - x^2}. \tag{10}$$

$$\text{The trapezoidal rule formula is: } A \approx h\left[\frac{1}{2}f(a) + f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + \frac{1}{2}f(b)\right]. \tag{11}$$

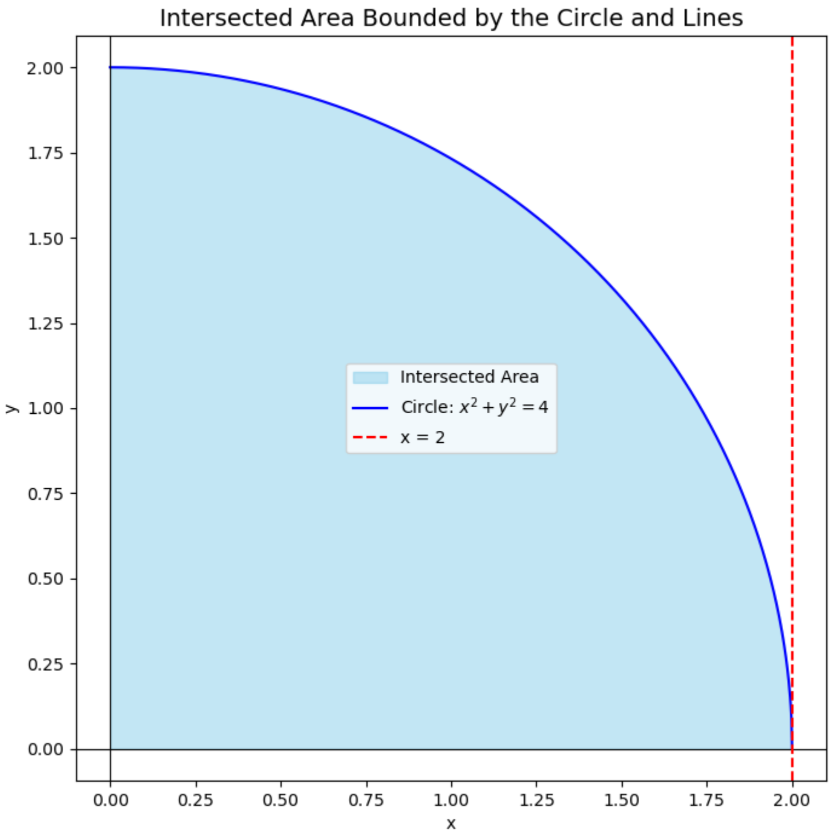
The points x_i are given by

$$x_i = a + ih = 0 + i \cdot 0.004, \text{ for } i = 0, 1, 2, \ldots, 500. \tag{12}$$

$$\text{Substitute the values: } A \approx 0.004\left[\frac{1}{2}f(0) + f(x_1) + f(x_2) + \cdots + f(x_{499}) + \frac{1}{2}f(2)\right]. \tag{13}$$

$$\text{Compute } f(0) = \sqrt{4 - 0^2} = 2, \text{ and } f(2) = \sqrt{4 - 2^2} = 0. \tag{14}$$

$$\text{Summing the terms and applying the formula, we find } A \approx 3.1415. \tag{15}$$



Intersected Area Bounded by the Circle and Lines
Intersected Area
Circle: $x^2 + y^2 = 4$
x = 2
y
x
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00