9.4.1

EE24BTECH11030 - J.KEDARANANDA

Question:

For each of the differential equations in Exercises 1 to 10, find the general solution: $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

Solution: Given differential equation:

$$\frac{dy}{dx} = \frac{2\sin^2\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)} \tag{0.1}$$

$$\tag{0.2}$$

$$\frac{dy}{dx} = \tan^2\left(\frac{x}{2}\right) \tag{0.3}$$

$$\int dy = \int \tan^2\left(\frac{x}{2}\right) dx \tag{0.4}$$

$$y = \int \sec^2\left(\frac{x}{2}\right) dx - \int 1\, dx \tag{0.5}$$

$$y = 2\tan\left(\frac{x}{2}\right) - x + C \tag{0.6}$$

logic behind the iteration used in code:
Method of finite differences : The finite difference method is rooted in the fundamental concept of approximating derivatives using finite differences.

The derivative of $y(x)$ can be approximated as

$$\frac{dy}{dx} = \frac{y(x+h) - y(x)}{h} \tag{0.7}$$

$$y(x+h) = y(x) + h\left(\frac{dy}{dx}\right) \tag{0.8}$$

Where h is a small value very close to zero.

$$y(x+h) = y(x) + h\left(\frac{1-\cos x}{1+\cos x}\right) \tag{0.9}$$

Let (x_0, y_0) be a point on the curve.
Let some $x_1 = x_0 + h$.Then,

$$y_1 = y_0 + h\left(\frac{1-\cos x}{1+\cos x}\right) \tag{0.10}$$

On Generalizing the above equation, we have

$$x_{n+1} = x_n + h \tag{0.11}$$

$$y_{n+1} = y_n + h\left(\frac{1-\cos x}{1+\cos x}\right) \tag{0.12}$$