

# jee-main-maths-02-09-2020-shift-2<sup>1</sup>

EE24BTECH11030 - J.KEDARANANDA

- 1) A line parallel to the straight line  $2x - y = 0$  is tangent to the hyperbola  $\frac{x^2}{4} - \frac{y^2}{2} = 1$  at the point  $(x_1, y_1)$ . Then  $x_1^2 + 5y_1^2$  is equal to:
- a) 6                                      b) 10                                      c) 8                                      d) 5
- 2) The domain of the function  $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$  is  $(-\infty, -a] \cup [a, \infty)$ . Then  $a$  is equal to:
- a)  $\frac{\sqrt{17}-1}{2}$                                       b)  $\frac{\sqrt{17}}{2}$                                       c)  $\frac{1+\sqrt{17}}{2}$                                       d)  $\frac{\sqrt{17}}{2} + 1$
- 3) If a function  $f(x)$  defined by  $f(x) = \begin{cases} ae^x + be^{-x} & , -1 \leq x < 1 \\ cx^2 & , 1 \leq x < 3 \\ ax^2 + 2cx & , 3 \leq x \leq 4 \end{cases}$  be continuous for some  $a, b, c \in \mathbb{R}$  and  $f'(0) + f'(2) = e$ , then the value of  $a$  is:
- a)  $\frac{1}{e^2-3e+13}$                                       b)  $\frac{e}{e^2-3e-13}$                                       c)  $\frac{e}{e^2+3e+13}$                                       d)  $\frac{e}{e^2-3e+13}$
- 4) The sum of the first three terms of a G.P. is  $S$  and their product is 27. Then all such  $S$  lie in:
- a)  $(-\infty, -9] \cup [3, \infty)$                                       c)  $(-\infty, 9]$   
b)  $[-3, \infty)$                                       d)  $(-\infty, -3] \cup [9, \infty)$
- 5) If  $R = \{(x, y) : x, y \in \mathbb{Z}, x^2 + 3y^2 \leq 8\}$  is a relation on the set of integers  $\mathbb{Z}$ , then the domain of  $R^{-1}$  is:
- a)  $\{-1, 0, 1\}$                                       b)  $\{-2, -1, 1, 2\}$                                       c)  $\{0, 1\}$                                       d)  $\{-2, -1, 0, 1, 2\}$
- 6) The value of  $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$  is:
- a)  $-\frac{1}{2}(1-i\sqrt{3})$                                       b)  $\frac{1}{2}(1-i\sqrt{3})$                                       c)  $-\frac{1}{2}(\sqrt{3}-i)$                                       d)  $\frac{1}{2}(\sqrt{3}-i)$
- 7) Let  $\mathbf{P}(h, k)$  be a point on the curve  $y = x^2 + 7x + 2$ , nearest to the line,  $y = 3x - 3$ . Then the equation of the normal to the curve at  $\mathbf{P}$  is:

a)  $x + 3y - 62 = 0$    b)  $x - 3y - 11 = 0$    c)  $x - 3y + 22 = 0$    d)  $x + 3y + 26 = 0$

- 8) Let  $A$  be a  $2 \times 2$  real matrix with entries from  $\{0, 1\}$  and  $|A| \neq 0$ . Consider the following two statements:

(P) If  $A \neq I_2$ , then  $|A| = -1$

(Q) If  $|A| = 1$ , then  $\text{tr}(A) = 2$ ,

where  $I_2$  denotes  $2 \times 2$  identity matrix and  $\text{tr}(A)$  denotes the sum of the diagonal entries of  $A$ . Then:

a) Both (P) and (Q) are false                      c) Both (P) and (Q) are true

b) (P) is true and (Q) is false                      d) (P) is false and (Q) is true

- 9) Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is:

a)  $\frac{4}{17}$                       b)  $\frac{8}{17}$                       c)  $\frac{2}{5}$                       d)  $\frac{2}{3}$

- 10) If  $p(x)$  be a polynomial of degree three that has a local maximum value 8 at  $x=1$  and a local minimum value 4 at  $x=2$ ; then  $p(0)$  is equal to:

a) 12                      b) -12                      c) -24                      d) 6

- 11) The contrapositive of the statement "If I reach the station in time, then I will catch the train" is:

a) If I will catch the train, then I reach the station in time.  
 b) If I do not reach the station in time, then I will catch the train.  
 c) If I do not reach the station in time, then I will not catch the train.  
 d) If I will not catch the train, then I do not reach the station in time.

- 12) Let  $\alpha$  and  $\beta$  be the roots of the equation,  $5x^2 + 6x - 2 = 0$ . If  $S_n = \alpha^n + \beta^n$ ,  $n=1,2,3,\dots$ , then:

a)  $5S_6 + 6S_5 + 2S_4 = 0$                       c)  $6S_6 + 5S_5 + 2S_4 = 0$   
 b)  $6S_6 + 5S_5 = 2S_4$                       d)  $5S_6 + 6S_5 = 2S_4$

- 13) If the tangent to the curve  $y = x + \sin y$  at a point  $(a,b)$  is parallel to the line joining  $(0, \frac{3}{2})$  and  $(\frac{1}{2}, 2)$ , then:

a)  $b = \frac{\pi}{2} + a$

b)  $|a + b| = 1$

c)  $|b - a| = 1$

d)  $b = a$

14) Area (in sq. units) of the region outside  $\frac{|x|}{2} + \frac{|y|}{3} = 1$  and inside the ellipse  $\frac{x^2}{4} + \frac{y^2}{9} = 1$  is:

a)  $3(\pi - 2)$

b)  $6(\pi - 2)$

c)  $6(4 - \pi)$

d)  $3(4 - \pi)$

15) If  $|x| < 1$ ,  $|y| < 1$  and  $x \neq y$ , then the sum to infinity of the following series  $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \dots$  is:

a)  $\frac{x+y+xy}{(1-x)(1-y)}$

b)  $\frac{x+y-xy}{(1-x)(1-y)}$

c)  $\frac{x+y+xy}{(1+x)(1+y)}$

d)  $\frac{x+y-xy}{(1+x)(1+y)}$