GATE 2013 XE(4-5)

EE24BTECH11030 - J.KEDARANANDA

1) Consider the function $f(z) = z^2\bar{z}$, $z \in \mathbb{C}$. At $z = 0$, the function f
   a) does not satisfy the Cauchy-Riemann equations
   b) satisfies the Cauchy-Riemann equations but is not differentiable
   c) is differentiable but not analytic
   d) is analytic

2) The integral $\oint_C \frac{(z+4)}{(z+1)(z-2)^3}\,dz$ along the contour $C : |z - (1 + i)| = 2$, oriented anti-clockwise, is equal to

   a) 0 b) $\frac{2\pi i}{9}$ c) $-\frac{2\pi i}{9}$ d) $\frac{4\pi i}{9}$

3) The integral $\int_0^1 \int_x^{x^2} \left(\frac{x}{y}\right) e^{-x^2/y}\,dy\,dx$ equals

   a) $\frac{e-2}{e}$ b) $\frac{e-1}{2e}$ c) $\frac{e-1}{2}$ d) $\frac{e-2}{2e}$

4) If the mean and variance of a binomial distribution are 6 and 2 respectively, then the probability of two failures is

   a) $4\left(\frac{2}{3}\right)^7$ b) $4\left(\frac{22}{37}\right)$ c) $17\left(\frac{2}{3}\right)^7$ d) $17\left(\frac{22}{37}\right)$

5) For the matrix $M = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & 1 & -2 \end{pmatrix}$, consider the following statements:
   (P) The characteristic equation of M is $\lambda^3 - \lambda = 0$.
   (Q) M^{-1} does not exist.
   (R) The matrix M is diagonalizable.
   Which of the above statements are true?

   a) P, Q and R
   b) P and R but not Q
   c) P and Q but not R
   d) Q and R but not P

6) The work done by the force $\mathbf{F} = (x + x^2)\,\hat{i} + (x^2 + y^3)\,\hat{j}$ in moving a particle once along the triangle with vertices $(0, 0), (1, 0)$ and $(0, 1)$ in the anti-clockwise direction is

a) 0 b) $\frac{1}{6}$ c) $\frac{1}{3}$ d) $\frac{5}{3}$

7) The general solution of the differential equation

$$x^3\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0, \quad x > 0$$

is

a) $C_1e^x + e^{x/2}\left\{C_2\cos\left(\frac{\sqrt{3}}{2}x\right) + C_3\sin\left(\frac{\sqrt{3}}{2}x\right)\right\}$

b) $C_1x + x^{-1/2}\left\{C_2\cos\left(\frac{\sqrt{3}}{2}\log_e x\right) + C_3\sin\left(\frac{\sqrt{3}}{2}\log_e x\right)\right\}$

c) $C_1e^x + e^{-x/2}\left\{C_2\cos\left(\frac{\sqrt{3}}{2}x\right) + C_3\sin\left(\frac{\sqrt{3}}{2}x\right)\right\}$

d) $C_1x + x^{1/2}\left\{C_2\cos\left(\frac{\sqrt{3}}{2}\log_e x\right) + C_3\sin\left(\frac{\sqrt{3}}{2}\log_e x\right)\right\}$

8) Using Euler's method to solve the differential equation

$$\frac{dy}{dx} = 2\cos\left(\frac{4\pi x}{3}\right) - y, \quad y(0) = 1$$

with step-size $h = 0.25$, the value of $y(0.5)$ is

a) 1.3125 b) 1.1875 c) 1.125 d) 1.0625

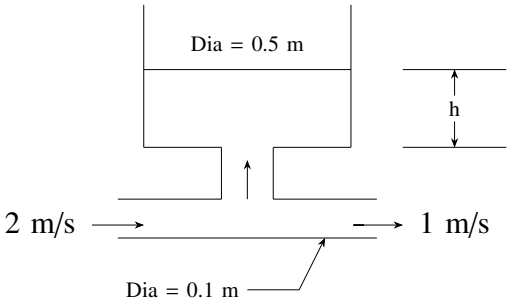
9) The gauge pressure inside a soap bubble of radius R, with σ denoting the surface tension between the soap solution and air, is:

a) $\frac{\sigma}{2\pi R}$ b) $\frac{4\sigma}{R}$ c) $\frac{2\sigma}{R}$ d) $\frac{\sigma}{4\pi R}$

10) Let M, B, and G represent respectively the metacentre, centre of buoyancy, and the centre of mass of a floating buoy. Which of the following statements is correct?

a) M is above G; Buoy unstable
b) B is above G; Buoy stable
c) M is above G; Buoy stable
d) B is above G; Buoy unstable

11) A reservoir connected to a pipeline is being filled with water, as shown in the Figure. At any time t, the free surface level in the reservoir is h. Find the time in seconds for the reservoir to get filled up to a height of 1 m, if the initial level is 0.2 m.



12) Bernoulli's equation is valid for the following type of flow:

a) Compressible, steady, inviscid
b) Incompressible, steady, viscous
c) Compressible, unsteady, viscous
d) Incompressible, steady, inviscid

13) If A is the area of a circle of radius r enclosing a plane forced vortex flow, with origin at the centre of the vortex and if ω is the angular velocity, ζ is the vorticity, $\mathbf{V}$ is the velocity vector, then the circulation around the contour of the circle is given by

a) $2\omega A$
b) $2\zeta A$
c) $2\mathbf{V}A$
d) 0