Question-9-9.3-13

EE24BTECH11030 - J.KEDARANANDA

Question

Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$.

Inputs

Variable	Description
$\vec{x_1}, \vec{x_2}, \vec{x_3}, \vec{x_4}$	Intersection points
$\vec{h}$	Point on the given line
$\vec{m}$	Direction vector of given line
A	Area of the region

Formulas

Conic	Expression
ellipse	$\vec{x}^\top \vec{V}\vec{x} + 2\vec{u}^\top \vec{x} + f = 0$
line	$\vec{x} = \vec{h} + \kappa\vec{m}$

Solution

$$\vec{V} = \begin{pmatrix} \frac{1}{16} & 0 \\ 0 & \frac{1}{4} \end{pmatrix} \tag{1}$$

$$\vec{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2}$$

$$f = -1 \tag{3}$$

For the given line $x = 2$, the values of $\vec{h}$ and $\vec{m}$ are:

$$\vec{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{4}$$

$$\vec{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{5}$$

Solution

The intersection points are:

$$\vec{x_1} = \begin{pmatrix} 2 \\ \sqrt{3} \end{pmatrix} \tag{6}$$

$$\vec{x_2} = \begin{pmatrix} 2 \\ -\sqrt{3} \end{pmatrix} \tag{7}$$

$$\vec{x_3} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{8}$$

$$\vec{x_4} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{9}$$

$$y = \pm\sqrt{4 - \frac{1}{4}x^2} \tag{10}$$

Solution

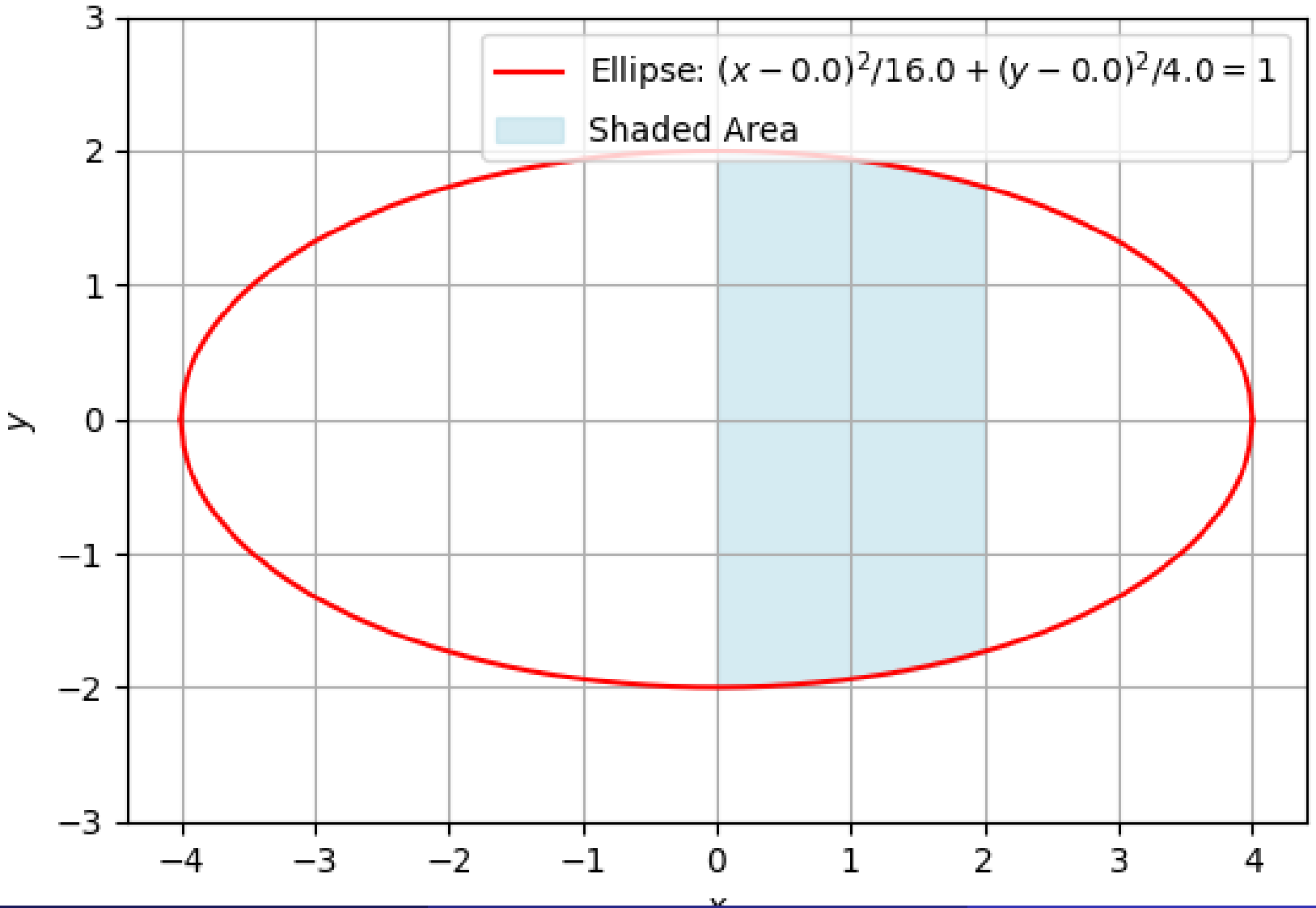
The area A between the curves from $x = 0$ to $x = 2$ is given by:

$$A = 4 \int_0^2 \sqrt{1 - \frac{x^2}{16}}\, dx \tag{11}$$

Simplifying, we get:

$$A = 2\sqrt{3} + \frac{4\pi}{3} \tag{12}$$

Diagram



C-Code

```c
#include <math.h>
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <unistd.h>
#include <sys/socket.h>
#include <netinet/in.h>

#include "libs/matfun.h"
#include "libs/geofun.h"

int main() {
    FILE *file;
    file = fopen("data.txt", "w");
```

C-Code

```
        if (file == NULL) {
            printf("Error opening file!\n");
            return 1; // Return error code if file couldn't be opened
        }
        // Ellipse parameters
        double a = 4.0; // semi-major axis
        double b = 2.0; // semi-minor axis
        // Store the parameters
        fprintf(file, "%lf\n%lf\n%lf\n%lf\n", a, b, 0.0, 0.0); // a, b, h, k
        fclose(file); // Close the file
        return 0; // Return success
}
```

Python-Code

```
# Code by GVV Sharma
# Modified for Problem Solution
# Released under GNU GPL
# Calculating area enclosed between curves
import sys  # For path to external scripts
sys.path.insert(0, '/home/kedar/assignments/matgeo/
    codes/CoordGeo')  # Path to my scripts

import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import quad
```

Python-Code

```
# Read the values from the C-generated text file using
    numpy.loadtxt
try:
    data = np.loadtxt('data.txt')
except OSError:
    print("Error: Unable to read 'data.txt'. Please
        ensure the file exists.")
    sys.exit(1)

# Extracting ellipse parameters
a = data[0]   # semi-major axis
b = data[1]   # semi-minor axis
h = data[2]   # center x-coordinate (h)
k = data[3]   # center y-coordinate (k)
```

Python-Code

```
# Ellipse equations
def ellipse_upper(x, a, b, h, k):
    """Returns the upper part of the ellipse."""
    return k + b * np.sqrt(1 - ((x - h) * 2) / (a * 2)
        )

def ellipse_lower(x, a, b, h, k):
    """Returns the lower part of the ellipse."""
    return k - b * np.sqrt(1 - ((x - h) * 2) / (a * 2)
        )
```

Python-Code

```
# Define the limits of integration
x_min = 0
x_max = 2

# Compute the area between the curves using
    integration
def area_between_curves(x):
    """Calculates the difference between the upper and
        lower ellipse."""
    return ellipse_upper(x, a, b, h, k) -
        ellipse_lower(x, a, b, h, k)

# Perform the integration from x_min to x_max
area, _ = quad(area_between_curves, x_min, x_max)

print(f"Area enclosed between the ellipse and the
    ordinates: {area:.4f}")
```

Python-Code

```
# Visualization
# Generating points for the ellipse to show the
    complete ellipse
x_vals = np.linspace(h - a, h + a, 400)
y_upper = ellipse_upper(x_vals, a, b, h, k)
y_lower = ellipse_lower(x_vals, a, b, h, k)

# Plot the ellipse's upper and lower parts
plt.plot(x_vals, y_upper, label=f'Ellipse: $(x - {h})
    ^2/{a*2} + (y - {k})^2/{b*2} = 1$', color='r')
plt.plot(x_vals, y_lower, color='r')
```

Python-Code

```
# Shade the area between the ellipse and the x-axis
    between x=0 and x=2
y_fill_upper = ellipse_upper(np.linspace(x_min, x_max,
    100), a, b, h, k)
y_fill_lower = ellipse_lower(np.linspace(x_min, x_max,
    100), a, b, h, k)

# Fill the area
plt.fill_between(np.linspace(x_min, x_max, 100),
    y_fill_upper, y_fill_lower, color='lightblue',
    alpha=0.5, label='Shaded Area')

# Labels and plot settings
plt.xlabel('$x$')
plt.ylabel('$y$')
plt.grid(True)
plt.legend(loc='upper right')
```

Python-Code

```
# Set the limits for y-axis to ensure complete
    visibility of the ellipse
plt.ylim(k - b - 1, k + b + 1)  # Slightly adjust as
    needed for aesthetics

# Set equal aspect ratio to avoid distortion
plt.gca().set_aspect('equal', adjustable='box')

# Show the plot
plt.show()
```