GATE 2013 MA(14-26)

EE24BTECH11030 - J.KEDARANANDA

1) Let $c \in \mathbb{Z}_3$ be such that $\frac{\mathbb{Z}_3[X]}{(X^3+cX+X+1)}$ is a field. Then c is equal to ______.

2) Let $V = C^1[0,1]$, $X = C([0,1], \|\ \|_\infty)$ and $Y = C([0,1], \|\ \|_2)$. Then V is

   a) dense in X but NOT in Y
   b) dense in Y but NOT in X
   c) dense in both X and Y
   d) neither dense in X nor dense in Y

3) Let $T : (C([0,1], \|\ \|_\infty) \to \mathbb{R}$ be defined by $T(f) = \int_0^1 2xf(x)\,dx$ for all $f \in C([0,1])$. Then $\|T\|$ is equal to ______.

4) Let τ_1 be the usual topology on $\mathbb{R}$. Let τ_2 be the topology on $\mathbb{R}$ generated by $\mathcal{B} = \{(a,b) \subset \mathbb{R} : -\infty < a < b < \infty\}$. Then the set $\{x \in \mathbb{R} : 4\sin^2 x \leq 1\} \cup \left\{\frac{\pi}{2}\right\}$ is
   a) closed in $(\mathbb{R}, \tau_1)$ but NOT in $(\mathbb{R}, \tau_2)$
   b) closed in $(\mathbb{R}, \tau_2)$ but NOT in $(\mathbb{R}, \tau_1)$
   c) closed in both $(\mathbb{R}, \tau_1)$ and $(\mathbb{R}, \tau_2)$
   d) neither closed in $(\mathbb{R}, \tau_1)$ nor closed in $(\mathbb{R}, \tau_2)$

5) Let X be a connected topological space such that there exists a non-constant continuous function $f : X \to \mathbb{R}$, where $\mathbb{R}$ is equipped with the usual topology. Let $f(X) = \{f(x) : x \in X\}$. Then
   a) X is countable but $f(X)$ is uncountable
   b) $f(X)$ is countable but X is uncountable
   c) both $f(X)$ and X are countable
   d) both $f(X)$ and X are uncountable

6) Let d_1 and d_2 denote the usual metric and the discrete metric on $\mathbb{R}$, respectively. Let $f : (\mathbb{R}, d_1) \to (\mathbb{R}, d_2)$ be defined by $f(x) = x, x \in \mathbb{R}$. Then
   a) f is continuous but f^{-1} is NOT continuous
   b) f^{-1} is continuous but f is NOT continuous
   c) both f and f^{-1} are continuous
   d) neither f nor f^{-1} is continuous

7) If the trapezoidal rule with single interval $[0,1]$ is exact for approximating the integral $\int_0^1 (x^3 - cx^2)dx$, then the value of c is equal to ______.

8) Suppose that the Newton-Raphson method is applied to the equation $2x^2+1-e^{x^2}=0$ with an initial approximation x_0 sufficiently close to zero. Then, for the root $x = 0$, the order of convergence of the method is equal to ______.

9) The minimum possible order of a homogeneous linear ordinary differential equation with real constant coefficients having $x^2 \sin(x)$ as a solution is equal to ______.

10) The Lagrangian of a system in terms of polar coordinates (r, θ) is given by

$$L = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - mgr\,(1 - \cos(\theta))\,,$$

where m is the mass, g is the acceleration due to gravity, and $\dot{s}$ denotes the derivative of s with respect to time. Then the equations of motion are

a) $2\ddot{r} = r\dot{\theta}^2 - g\,(1 - \cos(\theta))\,,\quad \frac{d}{dt}\left(r^2\dot{\theta}\right) = -gr\sin(\theta)$

b) $2\ddot{r} = r\dot{\theta}^2 + g\,(1 - \cos(\theta))\,,\quad \frac{d}{dt}\left(r^2\dot{\theta}\right) = -gr\sin(\theta)$

c) $2\ddot{r} = r\dot{\theta}^2 - g\,(1 - \cos(\theta))\,,\quad \frac{d}{dt}\left(r^2\dot{\theta}\right) = gr\sin(\theta)$

d) $2\ddot{r} = r\dot{\theta}^2 + g\,(1 - \cos(\theta))\,,\quad \frac{d}{dt}\left(r^2\dot{\theta}\right) = gr\sin(\theta)$

11) If $y(x)$ satisfies the initial value problem

$$(x^2 + y)dx = x\,dy, \quad y(1) = 2,$$

then $y(2)$ is equal to ______.

12) It is known that Bessel functions $J_n(x)$, for $n \geq 0$, satisfy the identity

$$e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = J_0(x) + \sum_{n=1}^{\infty} J_n(x)\left(t^n + \frac{(-1)^n}{t^n}\right)$$

for all $t > 0$ and $x \in \mathbb{R}$. The value of $J_0\left(\frac{\pi}{3}\right) + 2\sum_{n=1}^{\infty} J_{2n}\left(\frac{\pi}{3}\right)$ is equal to ______.

13) Let X and Y be two random variables having the joint probability density function

$$f(x,y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Then the conditional probability $P(X \leq \frac{2}{3} | Y = \frac{3}{4})$

a) $\frac{5}{9}$ b) $\frac{2}{3}$ c) $\frac{7}{9}$ d) $\frac{8}{9}$