jee-main-maths-25-06-2022-shift-2

EE24BTECH11030 - J.KEDARANANDA

1) Let $A = \begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}$
Then the number of elements in the set (n, m) : n, m $\in$ 1, 2, $\cdots$, 10 and $nA^n + mB^m = I$ is _____.

2) Let $f(x) = [2x^2+1]$ and $g(x) = \begin{cases} 2x-3 & , x < 0 \\ 2x+3 & , x \geq 0 \end{cases}$, where [t] is the greatest integer $\leq t$.
Then, in the open interval $(-1, 1)$, the number of points where $f(g(x))$ is discontinuous is equal to _____.

3) The value of b > 3 for which $12\int_3^b \frac{1}{(x^2-1)(x^2-4)}\,dx = \ln(\frac{49}{40})$, is equal to _____.

4) If the sum of the co-efficients of all the positive even powers of x in the binomial expansion of $\left(2x^3 + \frac{3}{x}\right)^{10}$ is $5^{10} - \beta 3^9$ then β equal to _____

5) If the mean deviation about the mean of the numbers 1, 2, 3, $\cdots$, n, where n is odd, is $\frac{5(n+1)}{n}$, then n is equal to _____.

6) $\vec{b} = \hat{i} + \hat{j} + \lambda\hat{k}$, $\lambda \in R$. If $\vec{b}$ is a vector such that $\vec{a} \times \vec{b} = 13\hat{i} - 1\hat{j} - 4\lambda\hat{k}$ and $\vec{a} \cdot \vec{b} + 21 = 0$, then $(\vec{b} - \vec{a}) \cdot (\hat{k} - \hat{j}) + (\vec{b} + \vec{a}) \cdot (\hat{i} - \hat{k})$ _____.

7) The total number of three-digit numbers, with one digit repeated exactly two times, is _____.

8) Let f(x) = $|(x-1)(x^2 - 2x - 3)| + x - 3$, x $\in$ R. If m and M are, respectively the number of points of local minimum and local maximum of f in the interval (0, 4), then m + M is equal to _____.

9) Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $\frac{5}{4}$. If the equation of the normal at the point $(\frac{8}{\sqrt{5}}, \frac{12}{5})$ on the hyperbola is $8\sqrt{5}x + \beta y = \lambda$, then $\lambda - \beta$ is equal to _____.

10) Let l_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively and l_2 be the line in zx-plane with x and z intercepts $\frac{-1}{8}$ and $\frac{-1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line l_1 and l_2, then d^{-2} is equal to _____.