GATE 2019 MA(14-26)

EE24BTECH11030 - J.KEDARANANDA

1) Let
$$u_n = \frac{n!}{1.3.5...(2n-1)}, \quad n \in \mathbb{N} \text{ (the set of all natural numbers).}$$
Then $\lim_{n\to\infty} u_n$ is equal to ___________.

2) If the differential equation
$$\frac{dy}{dx} = \sqrt{x^2+y^2}, \quad y(1) = 2$$
is solved using Euler's method with step-size $h = 0.1$, then $y(1.2)$ is equal to ___________ (round off to 2 places of decimal).

3) Let f be any polynomial function of degree at most 2 over $\mathbb{R}$ (the set of all real numbers).
If the constants a and b are such that
$$\frac{df}{dx} = a\,f(x) + 2\,f(x+1) + b\,f(x+2), \quad \text{for all } x \in \mathbb{R},$$
then $4a + 3b$ is equal to ___________ (round off to 2 places of decimal).

4) Let L denote the value of the line integral
$$\oint_C \left(3x - 4x^2y\right) dx + \left(4xy^2 + 2y\right) dy,$$
where C, a circle of radius 2 with center at origin of the xy-plane, is traversed once in the anti-clockwise direction. Then $\frac{L}{\pi}$ is equal to ___________.

5) The temperature $T : \mathbb{R}^3\backslash\{(0,0,0)\} \to \mathbb{R}$ at any point $P(x,y,z)$ is inversely proportional to the square of the distance of P from the origin. If the value of the temperature T at the point $R(0,0,1)$ is $\sqrt{3}$, then the rate of change of T at the point $Q(1,1,2)$ in the direction of $\overrightarrow{QR}$ is equal to ___________ (round off to 2 places of decimal).

6) Let f be a continuous function defined on $[0,2]$ such that $f(x) \geq 0$ for all $x \in [0,2]$. If the area bounded by $y = f(x), x = 0, y = 0$ and $x = b$ is $\sqrt{3+b^2} - \sqrt{3}$, where $b \in (0,2]$, then $f(1)$ is equal to ______ (round off to 1 place of decimal).

7) If the characteristic polynomial and minimal polynomial of a square matrix A are $(\lambda-1)(\lambda+1)^4(\lambda-2)^5$ and $(\lambda-1)(\lambda+1)(\lambda-2)$, respectively, then the rank of the matrix $A+I$ is ______, where I is the identity matrix of appropriate order.

8) Let ω be a primitive complex cube root of unity and $i = \sqrt{-1}$. Then the degree of the field extension $\mathbb{Q}\left(i, \sqrt{3}, \omega\right)$ over $\mathbb{Q}$ (the field of rational numbers) is ______.

9) Let

$$\alpha = \int_C \frac{e^{i\pi z}}{2z^2 - 5z + 2}\, dz, \quad C : \cos t + i \sin t,\ 0 \le t \le 2\pi,\ i = \sqrt{-1}.$$

Then the greatest integer less than or equal to $|\alpha|$ is ______.

10) Consider the system:

$$\begin{cases} 3x_1 + x_2 + 2x_3 - x_4 = a, \\ x_1 + x_2 + x_3 - 2x_4 = 3, \\ x_1, x_2, x_3, x_4 \ge 0. \end{cases}$$

If $x_1 = 1, x_2 = b, x_3 = 0, x_4 = c$ is a basic feasible solution of the above system (where a, b, and c are real constants), then $a + b + c$ is equal to ______.

11) Let $f : \mathbb{C} \to \mathbb{C}$ be a function defined by $f(z) = z^6 - 5z^4 + 10$. Then the number of zeros of f in $\{z \in \mathbb{C} : |z| < 2\}$ is ______.
($\mathbb{C}$ is the set of all complex numbers)

12) Let

$$\ell^2 = \{x = (x_1, x_2, \dots) : x_i \in \mathbb{C}, \sum_{i=1}^{\infty} |x_i|^2 < \infty\}$$

be a normed linear space with the norm

$$\|x\|_2 = \left(\sum_{i=1}^{\infty} |x_i|^2\right)^{\frac{1}{2}}.$$

Let $g : \ell^2 \to \mathbb{C}$ be the bounded linear functional defined by

$$g(x) = \sum_{n=1}^{\infty} \frac{x_n}{3^n} \quad \text{for all } x = (x_1, x_2, \dots) \in \ell^2.$$

Then

$$(\sup\{|g(x)| : \|x\|_2 \le 1\})^2$$

is equal to ______ (round off to 3 places of decimal).
($\mathbb{C}$ is the set of all complex numbers)

13) For the linear programming problem (LPP):

$$\text{Maximize } Z = 2x_1 + 4x_2$$

subject to

$$\begin{cases} -x_1 + 2x_2 \le 4, \\ 3x_1 + \beta x_2 \le 6, \\ x_1, x_2 \ge 0, \quad \beta \in \mathbb{R}, \end{cases}$$

($\mathbb{R}$ is the set of all real numbers)
consider the following statements:

I. The LPP always has a finite optimal value for any $\beta \geq 0$.
II. The dual of the LPP may be infeasible for some $\beta \geq 0$.
III. If for some β, the point $(1, 2)$ is feasible to the dual of the LPP, then $Z \leq 16$, for any feasible solution (x_1, x_2) of the LPP.
IV. If for some β, x_1 and x_2 are the basic variables in the optimal table of the LPP with $x_1 = \frac{1}{2}$, then the optimal value of dual of the LPP is 10.

Then which of the above statements are TRUE?

a) (A) I and III only
b) (B) I, III and IV only
c) (C) III and IV only
d) (D) II and IV only