jee-main-maths-27-08-2021-shift-1

EE24BTECH11030 - J.KEDARANANDA

1) Let $\vec{a} = \hat{i} + 5\hat{j} + \alpha\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \beta\hat{k}$ and $\vec{c} = -1\hat{i} + 2\hat{j} - 3\hat{k}$ be three vectors such that $|\vec{b} \times \vec{c}| = 5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest among the values of $|\vec{a}|^2$ is _____.

2) The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is _____.

3) Let the equation $x^2 + y^2 + px + (1-p)y + 5 = 0$ represent circles of varying radius r $\in$ (0, 5]. Then the number of elements in the set S = {q : q = p^2 and q is an integer} is _____.

4) If $A = \{x \in R : |x-2| > 1\}$, $B = \{x \in R : \sqrt{x^2-3} > 1\}$, $C = \{x \in R : |x-4| \geq 2\}$, and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap Z$ is _____.

5) If $\int \frac{dx}{(x^2+x+1)^2} = a\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + b\left(\frac{2x+1}{x^2+x+1}\right) + C$, x > 0 where C is the constant of integration, then the value of $9(\sqrt{3}a + b)$ is equal to _____.

6) If the system of linear equations
$2x + y - z = 3$
$x - y - z = \alpha$
$3x + 3y + \beta z = 3$
has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is equal to _____.

7) Let n be an odd natural number such that the variance of 1, 2, 3, 4, $\cdots$, n is 14. Then n is equal to _____.

8) If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$ and the co-ordinate axis is kab, then k is equal to _____.

9) A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is _____.

10) If $y^{\frac{1}{4}} + y^{\frac{-1}{4}} = 2x$, and $(x^2 - 1)\frac{d^2y}{dx^2} + \alpha x\frac{dy}{dx} + \beta y = 0$, then $|\alpha - \beta|$ is equal to _____.