

1.1.9.17

EE24BTECH11030 - J.KEDARANANDA

Question:

Write the coordinates of the point \mathbf{P} on the x-axis which is equidistant from the points $\mathbf{A}(-2, 0)$ $\mathbf{B}(6, 0)$. (10, 2019)

Solution:

Variable	Description	Formula
A	A Point to be plotted	$A = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$
B	A Point to be plotted	$B = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$
P	Midpoint of A and B	$(A - B)^\top P = \frac{\ A\ ^2 - \ B\ ^2}{2}$

TABLE 0

If \mathbf{P} is equidistant from the points \mathbf{A} and \mathbf{B}

$$\|\mathbf{P} - \mathbf{A}\| = \|\mathbf{P} - \mathbf{B}\| \quad (0.1)$$

$$\|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 \quad (0.2)$$

$$\|\mathbf{P}\|^2 - 2\mathbf{P}^\top \mathbf{A} + \|\mathbf{A}\|^2 = \|\mathbf{P}\|^2 - 2\mathbf{P}^\top \mathbf{B} + \|\mathbf{B}\|^2 \quad (0.3)$$

By simplifying further,

$$(\mathbf{A} - \mathbf{B})^\top \mathbf{P} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \quad (0.4)$$

$$\begin{pmatrix} -8 \\ 0 \end{pmatrix}^\top \mathbf{P} = \frac{\left\| \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right\|^2 - \left\| \begin{pmatrix} 6 \\ 0 \end{pmatrix} \right\|^2}{2} = -16 \quad (0.5)$$

Comparing with $n^\top x = c$

$$\mathbf{n} = \begin{pmatrix} -8 \\ 0 \end{pmatrix} \quad (0.6)$$

$$\mathbf{c} = -16 \quad (0.7)$$

$$-8x + 0y = -16 \quad (0.8)$$

$$x = 2, y = 0 \quad (0.9)$$

$$\mathbf{P} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (0.10)$$

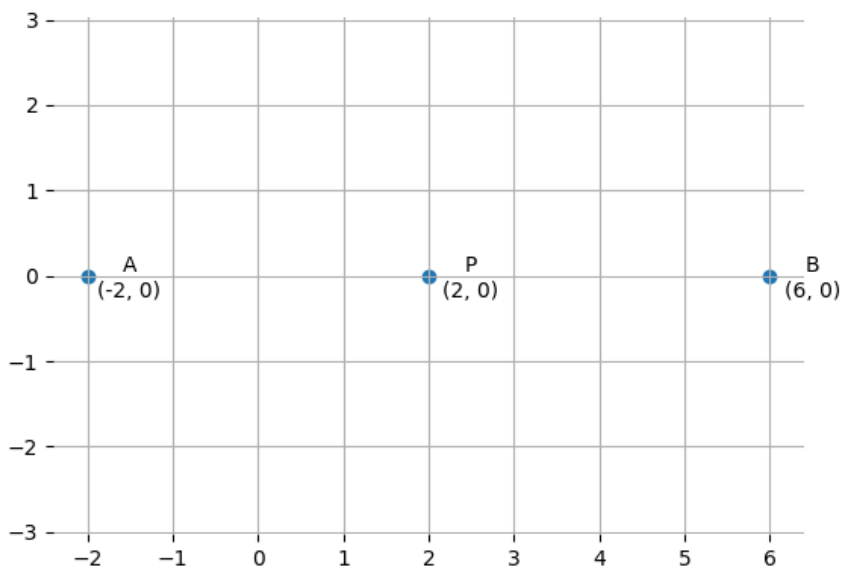


Fig. 0.1