1.11.12

EE24BTECH11030 - J.KEDARANANDA

Question:

If the sum of 2 vectors is a unit vector , prove that the magnitude of their difference is $\sqrt{3}$.
(12, 2018)

Solution:

Variable	Norm	Value
A	$\|\mathbf{A}\|$	1
B	$\|\mathbf{B}\|$	1
A + B	$(\|\mathbf{A} + \mathbf{B}\|)$	1
A − B	$(\|\mathbf{A} − \mathbf{B}\|)$	3

$$\|\mathbf{A}\| = \|\mathbf{B}\| = 1 \tag{0.1}$$

$$\|\mathbf{A} + \mathbf{B}\| = 1 \tag{0.2}$$

$$\|\mathbf{A} + \mathbf{B}\|^2 = \|\mathbf{A}\|^2 + \|\mathbf{B}\|^2 + 2\,(\mathbf{A} \cdot \mathbf{B}) \tag{0.3}$$

$$1 = 2 + 2\,(\mathbf{A} \cdot \mathbf{B}) \tag{0.4}$$

$$(\mathbf{A} \cdot \mathbf{B}) = \frac{-1}{2} \tag{0.5}$$

$$\|\mathbf{A} − \mathbf{B}\|^2 = \|\mathbf{A}\|^2 + \|\mathbf{B}\|^2 − 2\,(\mathbf{A} \cdot \mathbf{B}) \tag{0.6}$$

$$\|\mathbf{A} − \mathbf{B}\|^2 = 1 + 1 + 1 \tag{0.7}$$

$$\|\mathbf{A} − \mathbf{B}\|^2 = 3 \tag{0.8}$$

$$\|\mathbf{A} − \mathbf{B}\| = \sqrt{3} \tag{0.9}$$
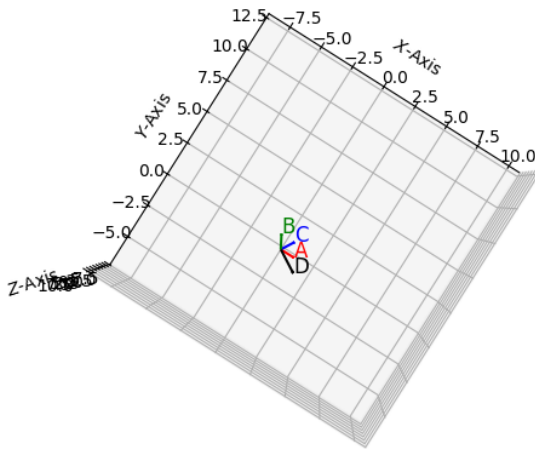
$$||D|| = ||A-B|| = \sqrt{3}$$ $$||C|| = ||A+B|| = 1$$

Fig. 0.1