GATE 2010 MA(1-13)

EE24BTECH11030 - J.KEDARANANDA

1) Let E and F be any two events with $P(E \cup F') = 0.8, P(E) = 0.4$ and $P(E|F') = 0.3$. Then $P(F')$ is

   a) $\frac{3}{7}$ b) $\frac{4}{7}$ c) $\frac{3}{5}$ d) $\frac{2}{5}$

2) Let X have a binomial distribution with parameters n and p, where n is an integer greater than 1 and $0 < p < 1$. If $P(X = 0) = P(X = 1)$, then the value of p is

   a) $\frac{1}{n-1}$ b) $\frac{n}{n+1}$ c) $\frac{1}{n+1}$ d) $\frac{1}{1+n^{n-1}}$

3) Let $u(x, y) = 2x(1 - y)$ for all real x and y. Then a function $\varphi(x, y)$, so that $f(z) = u(x, y) + i\varphi(x, y)$ is analytic, is

   a) $(x-1)^2 - y^2$ b) $(x-1)^2 + y^2$ c) $(x-1)^2 + y^2$ d) $(x+1)^2$

4) Let $f(z)$ be analytic on $D = \{z \in \mathbb{C} : |z - 1| < 1\}$ such that $f(0) = 1$. If $f(z) = f(z')$ for all $z \in D$, then which one of the following statements is NOT correct?

   a) $f(z) = [f(z)]^2$ for all $z \in D$ b) $f\left(\frac{z}{2}\right) = \frac{1}{2}f(z)$ for all $z \in D$ c) $f(z) = [f(z)]^2$ for all $z \in D$ d) $f'(1) = 0$

5) The maximum number of linearly independent solutions of the differential equation $\frac{d^n y}{dx^n} = 0$ with the condition $y(0) = 1$ is

   a) 4 b) 6 c) 2 d) 1

6) Which one of the following sets of functions is NOT orthogonal (with respect to the L^2-inner product) over the given interval?

   a) $\{\sin n\pi x : n \in \mathbb{N}\}, -\pi < x < \pi$ b) $\{\cos n\pi x : n \in \mathbb{N}\}, -\pi < x < \pi$ c) $\{\cos n\pi x : n \in \mathbb{N}\}, -1 < x < 1$ d) $\{1 : n \in \mathbb{N}\}, -1 < x < 1$

7) If $f : [1, 2] \to \mathbb{R}$ is a non-negative Riemann-integrable function such that

$$\int_1^2 \frac{f(x)}{x}dx = \int_1^1 f(x)dx = 0,$$

   then a belongs to the interval

a) $\left[0, \frac{1}{3}\right)$ b) $\left[\frac{1}{2}, \frac{2}{3}\right)$ c) $\left[\frac{2}{3}, 1\right)$ d) $\left[1, \frac{4}{3}\right)$

8) The set $X = \mathbb{R}$ with the metric $d(x, y) = \frac{|x-y|}{1+|x-y|}$ is

a) bounded but not compact
b) bounded but not complete
c) complete but not bounded
d) compact but not complete

9) Let $f(x, y) = \begin{cases} \frac{xy}{(x^2+y^2)^{3/2}} \left[1 - \cos(x^2 + y^2)\right], & (x, y) \neq (0, 0) \\ k, & (x, y) = (0, 0) \end{cases}$
Then the value of k for which $f(x, y)$ is continuous at $(0, 0)$ is

a) 0 b) $\frac{1}{2}$ c) 1 d) $\frac{3}{2}$

10) Let A and B be disjoint subsets of $\mathbb{R}$ and let m^* denote the Lebesgue outer measure on $\mathbb{R}$.
Consider the statements:
P: $m^*(A \cup B) = m^*(A) + m^*(B)$
Q: Both A and B are Lebesgue measurable
R: One of A and B is Lebesgue measurable
Which one of the following is correct?

a) If P is true, then Q is true
b) If P is NOT true, then R is true
c) If R is true, then P is NOT true
d) If R is true, then P is true

11) Let $f : \mathbb{R} \to [0, \infty)$ be a Lebesgue measurable function and E be a Lebesgue measurable subset of $\mathbb{R}$ such that $\int_E f\, dm = 0$, where m is the Lebesgue measure on $\mathbb{R}$. Then

a) $m(E) = 0$
b) $\{x \in \mathbb{R} : f(x) = 0\} = E$
c) $m(\{x \in E : f(x) \neq 0\}) = 0$
d) $m(\{x \in E : f(x) = 0\}) = 0$

12) If the nullity of the matrix $\begin{bmatrix} k & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{bmatrix}$ is 1, then the value of k is

a) -1 b) 0 c) 1 d) 2

13) If a 3×3 real skew-symmetric matrix has an eigenvalue $2i$, then one of the remaining eigenvalues is

a) $\frac{1}{2i}$ b) $-\frac{1}{2i}$ c) 0 d) 1