

# jee-main-maths-28-06-2022-shift-1<sup>1</sup>

EE24BTECH11030 - J.KEDARANANDA

1) If

$$\sum_{k=1}^{31} \binom{31}{k} \binom{31}{k-1} - \sum_{k=1}^{30} \binom{30}{k} \binom{30}{k-1} = \frac{\alpha \cdot (60!)}{(30!) \cdot (31!)}$$

where  $\alpha \in \mathbb{R}$ , then the value of  $16\alpha$  is equal to

- a) 1411                      b) 1320                      c) 1615                      d) 1855

2) Let a function  $f : \mathbb{N} \rightarrow \mathbb{N}$  be defined by

$$f(x) = \begin{cases} 2n & , n = 2, 4, 6, 8, \dots \\ n-1 & , n = 3, 7, 11, 15, \dots \\ \frac{n+1}{2} & , n = 1, 5, 9, 13, \dots \end{cases}$$

then,  $f$  is

- a) One-one but not onto  
b) Onto but not one-one  
c) Neither one-one nor onto  
d) One-one and onto

3) If the system of linear equations

$$2x + 3y - z = -2$$

$$x + y + z = 4$$

$$x - y + |\lambda|z = 4\lambda - 4$$

where  $\lambda \in \mathbb{R}$ , has no solution, then

- a)  $\lambda = 7$                       b)  $\lambda = -7$                       c)  $\lambda = 8$                       d)  $\lambda^2 = 1$

4) Let  $A$  be a matrix of order  $3 \times 3$  and  $\det(A) = 2$ . Then  $\det(\det(A) \operatorname{adj}(5 \operatorname{adj}(A^3)))$  is equal to

- a)  $512 \times 10^6$                       c)  $1024 \times 10^6$   
b)  $256 \times 10^6$                       d)  $256 \times 10^{11}$

5) The total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is

- a) 36                      b) 48                      c) 60                      d) 72

6) Let  $A_1, A_2, A_3, \dots$  be an increasing geometric progression of positive real numbers. If  $A_1 A_3 A_5 A_7 = \frac{1}{1296}$  and  $A_2 + A_4 = \frac{7}{36}$  then, the value of  $A_6 + A_8 + A_{10}$  is equal to

- a) 33                      b) 37                      c) 43                      d) 47

7) Let  $[t]$  denote the greatest integer less than or equal to  $t$ . Then, the value of the integral  $\int_0^1 [-8x^2 + 6x - 1] dx$  is equal to

- a) -1                      b)  $-\frac{5}{4}$                       c)  $\frac{\sqrt{17}-13}{8}$                       d)  $\frac{\sqrt{17}-16}{8}$

8) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined as

$$f(x) = \begin{cases} 0 & , x < 0 \\ ae^x - 1 & , 0 \leq x < 1 \\ b & , x = 1 \\ b - 1 & , 1 < x < 2 \\ -c & , x \geq 2 \end{cases}$$

Where  $a, b, c \in \mathbb{R}$  and  $[t]$  denotes greatest integer less than or equal to  $t$ . Then, which of the following statements is true?

- a) There exists  $a, b, c \in \mathbb{R}$  such that  $f$  is continuous on  $\mathbb{R}$  .  
 b) If  $f$  is discontinuous at exactly one point, then  $a + b + c = 1$   
 c) If  $f$  is discontinuous at exactly one point, then  $a + b + c \neq 1$   
 d)  $f$  is discontinuous at atleast two points, for any values of  $a, b$  and  $c$

9) The area of the region

$$\{(x, y) : y^2 \leq 8x, y \geq \sqrt{2}x, x \geq 1\}$$
 is

- a)  $\frac{13\sqrt{2}}{6}$                       b)  $\frac{11\sqrt{2}}{6}$                       c)  $\frac{5\sqrt{2}}{6}$                       d)  $\frac{19\sqrt{2}}{6}$

10) Let the solution curve  $y = y(x)$  of the differential equation  $\left[ \frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}} \right] x \frac{dy}{dx} =$

$x + \left[ \frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}} \right] y$  pass through the points  $(1, 0)$  and  $(2\alpha, \alpha)$ ,  $\alpha > 0$ . Then  $\alpha$  is equal to

- a)  $\frac{1}{2} \exp\left(\frac{\pi}{6} + \sqrt{e} - 1\right)$                       c)  $\exp\left(\frac{\pi}{6} + \sqrt{e} + 1\right)$   
 b)  $\frac{1}{2} \exp\left(\frac{\pi}{3} + \sqrt{e} - 1\right)$                       d)  $2 \exp\left(\frac{\pi}{3} + \sqrt{e} - 1\right)$

11) Let  $y = y(x)$  be the solution of the differential equation  $x(1 - x^2) \frac{dy}{dx} + 3x^2y - y - 4x^3 = 0$ ,  $x > 1$  with  $y(2) = -2$ . Then  $y(3)$  is equal to

