4-4.2-6

EE24BTECH11030 - J.KEDARANANDA

Question:
Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$.

Solution:

Variable	Description
$\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}, \mathbf{x_4}$	intersection points
$\mathbf{h}$	Point on the given line
$\mathbf{m}$	Direction vector of given line
A	Area of the region

TABLE 0: Variables Used

The equation of an ellipse in matrix form is

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.1}$$

The equation of a line in vector form is

$$\mathbf{x} = \mathbf{h} + \kappa \mathbf{m} \tag{0.2}$$

$$\mathbf{V} = \begin{pmatrix} \frac{1}{16} & 0 \\ 0 & \frac{1}{4} \end{pmatrix} \tag{0.3}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.4}$$

$$f = -1 \tag{0.5}$$

For the given line $x = 2$, the values of $\mathbf{h}$ and $\mathbf{m}$ are

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.6}$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.7}$$

The intersection points are

$$\mathbf{x_1} = \begin{pmatrix} 2 \\ \sqrt{3} \end{pmatrix} \tag{0.8}$$

$$\mathbf{x_2} = \begin{pmatrix} 2 \\ -\sqrt{3} \end{pmatrix} \tag{0.9}$$

$$\mathbf{x_3} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{0.10}$$

$$\mathbf{x_4} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{0.11}$$

The equation for y can be expressed as:

$$y = \pm \sqrt{4 - \frac{1}{4}x^2} \tag{0.12}$$

The area A between the curves from $x = 0$ to $x = 2$ is given by:

$$A = 2 \int_0^2 \sqrt{4 - \frac{1}{4}x^2}\, dx \tag{0.13}$$

This simplifies to:

$$A = 4 \int_0^2 \sqrt{1 - \frac{x^2}{16}}\, dx \tag{0.14}$$

Thus, the area becomes:

$$A = 2\sqrt{3} + \frac{4\pi}{3} \tag{0.15}$$

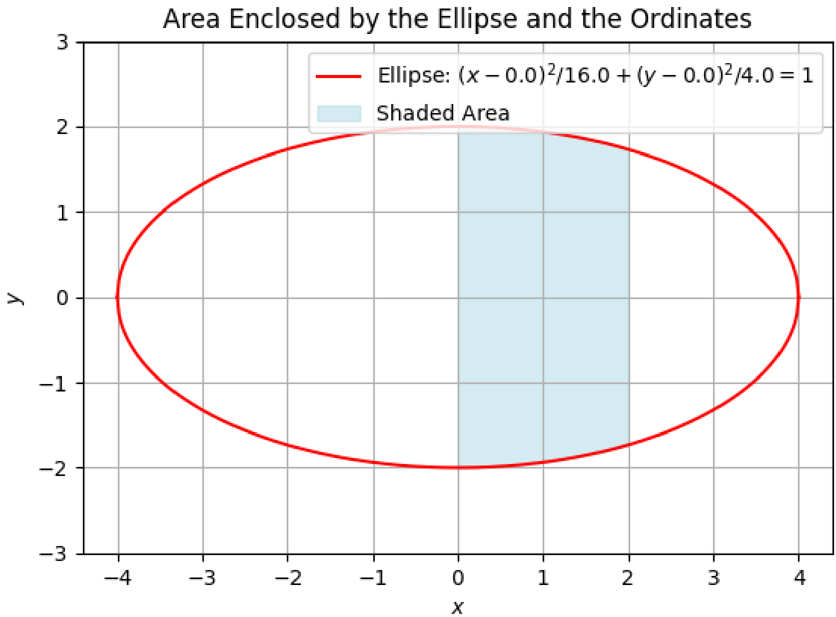

Fig. 0.1: Area Bounded by the Ellipse and the Ordinates