## jee-main-maths-28-06-2022-shift-1

## EE24BTECH11030 - J.KEDARANANDA

1) If

$$\sum_{k=1}^{31} {31 \choose k} {31 \choose k-1} - \sum_{k=1}^{30} {30 \choose k} {30 \choose k-1} = \frac{\alpha \cdot (60!)}{(30!) \cdot (31!)}$$

where  $\alpha \in \mathbb{R}$ , then the value of  $16\alpha$  is equal to

- a) 1411
- b) 1320
- c) 1615
- d) 1855

2) Let a function  $f: N \to N$  be defined by

Let a function 
$$f: N \to N$$
 be defined
$$f(x) = \begin{bmatrix} 2n & , n = 2,4,6,8,\cdots \\ n-1 & , n = 3,7,11,15,\cdots \\ \frac{n+1}{2} & , n = 1,5,9,13,\cdots \end{bmatrix}$$
then  $f$  is

- a) One-one but not onto
- b) Onto but not one-one
- c) Neither one-one nor onto
- d) One-one and onto
- 3) If the system of linear equations

$$2x + 3y - z = -2$$

$$x + y + z = 4$$

$$x - y + |\lambda|z = 4\lambda - 4$$

where  $\lambda \in \mathbb{R}$ , has no solution, then

a) 
$$\lambda = 7$$

b) 
$$\lambda = -7$$

c) 
$$\lambda = 8$$

b) 
$$\lambda = -7$$
 c)  $\lambda = 8$  d)  $\lambda^2 = 1$ 

4) Let A be a matrix of order  $3 \times 3$  and det (A) = 2. Then det (det (A) adj (5 adj (A3))) is equal to

a) 
$$512 \times 10^6$$

c) 
$$1024 \times 10^6$$

b) 
$$256 \times 10^6$$

d) 
$$256 \times 10^{11}$$

5) he total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is

2)	36

6) Let  $A_1, A_2, A_3, \cdots$  be an increasing geometric progression of positive real numbers. If  $A_1A_3A_5A_7 = \frac{1}{1296}$  and  $A_2 + A_4 = \frac{7}{36}$  then, the value of  $A_6 + A_8 + A_{10}$  is equal to

7) Let [t] denote the greatest integer less than or equal to t. Then, the value of the integral  $\int_0^1 [-8x^2 + 6x - 1] dx$  is equal to

a) 
$$-1$$

b) 
$$\frac{-5}{4}$$

c) 
$$\frac{\sqrt{17}-13}{8}$$
 d)  $\frac{\sqrt{17}-16}{8}$ 

d) 
$$\frac{\sqrt{17}-16}{8}$$

8) Let f:  $R \to R$  be defined as

$$f(x) = \begin{bmatrix} 0 & ,x < 0 \\ ae^x - 1 & ,0 \le x < 1 \\ b & ,x = 1 \\ b - 1 & ,1 < x < 2 \\ -c & ,x \ge 2 \end{bmatrix}$$

Where a, b,  $c \in R$  and [t] denotes greatest integer less than or equal to t. Then, which of the following statements is true?

- a) There exists a, b,  $c \in R$  such that f is continuous on  $\in R$ .
- b) If f is discontinuous at exactly one point, then a + b + c = 1
- c) If f is discontinuous at exactly one point, then  $a + b + c \neq 1$
- d) f is discontinuous at atleast two points, for any values of a, b and c
- 9) The area of the region

$$\{(x, y) : y^2 \le 8x, y \ge \sqrt{2}x, x \ge 1\}$$
 is

a) 
$$\frac{13\sqrt{2}}{6}$$

b) 
$$\frac{11\sqrt{2}}{6}$$
 c)  $\frac{5\sqrt{2}}{6}$ 

c) 
$$\frac{5\sqrt{2}}{6}$$

d) 
$$\frac{19\sqrt{2}}{6}$$

10) Let the solution curve y = y(x) of the differential equation  $\left| \frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}} \right| x \frac{dy}{dx} =$  $x + \left| \frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}} \right|$  y pass through the points (1, 0) and (2\alpha, \alpha), \alpha > 0. Then \alpha is equal

a) 
$$\frac{1}{2} exp(\frac{\pi}{6} + \sqrt{e} - 1)$$

c) 
$$exp(\frac{\pi}{6} + \sqrt{e} + 1)$$

b) 
$$\frac{1}{2} exp(\frac{\pi}{3} + \sqrt{e} - 1)$$

d) 
$$2exp(\frac{\pi}{3} + \sqrt{e} - 1)$$

11) Let y = y(x) be the solution of the differential equation  $x(1-x^2)\frac{dy}{dx} + 3x^2y - y - 4x^3 = 0$ , x>1 with y(2) = -2. Then y(3) is equal to

a)	-1	8
$a_{j}$	- 1	O

$$d) -3$$

12) The number of real solutions of  $x^7 + 5x^3 + 3x + 1 = 0$  is equal to

13) Let the eccentricity of the hyperbola

$$H: \frac{x^2}{a_E^2} + \frac{y^2}{b^2} = 1$$

be  $\frac{\sqrt[4]{5}}{2}$  and length of its latus rectum be  $6\sqrt{2}$ , If y = 2x + c is a tangent to the hyperbola H. then the value of  $c^2$  is equal to

14) If the tangents drawn at the points O(0,0) and  $P(1 + \sqrt{5},2)$  on the circle  $x^2 + y^2 -$ 2x - 4y = 0 intersect at the point Q, then the area of the triangle OPQ is equal to

a) 
$$\frac{3+\sqrt{5}}{2}$$

b) 
$$\frac{4+2\sqrt{5}}{2}$$
 c)  $\frac{5+3\sqrt{5}}{2}$ 

c) 
$$\frac{5+3\sqrt{3}}{2}$$

d) 
$$\frac{7+3\sqrt{5}}{2}$$

15) If two distinct points Q, R lie on the line of intersection of the planes -x+2y-z=0and 3x - 5y + 2z = 0 and PQ = PR =  $\sqrt{18}$  where the point P is (1, -2, 3), then the area of the triangle PQR is equal to

a) 
$$\frac{2}{3}\sqrt{38}$$

b) 
$$\frac{4}{3}\sqrt{38}$$

a) 
$$\frac{2}{3}\sqrt{38}$$
 b)  $\frac{4}{3}\sqrt{38}$  c)  $\frac{8}{3}\sqrt{38}$ 

d) 
$$\sqrt{\frac{152}{3}}$$