

9-9.3-13

EE24BTECH11030 - J.KEDARANANDA

Question:

Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$.

Solution:

Variable	Description
$\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4$	intersection points
\mathbf{h}	Point on the given line
\mathbf{m}	Direction vector of given line
A	Area of the region

TABLE 0

Conic	expression
ellipse	$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0$
line	$\mathbf{x} = \mathbf{h} + \kappa \mathbf{m}$

TABLE 0

$$\mathbf{V} = \begin{pmatrix} \frac{1}{16} & 0 \\ 0 & \frac{1}{4} \end{pmatrix} \quad (0.1)$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.2)$$

$$f = -1 \quad (0.3)$$

For the given line $x = 2$, The values of \mathbf{h} , \mathbf{m} are

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.4)$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (0.5)$$

0 The intersection points are

$$\mathbf{x}_1 = \begin{pmatrix} 2 \\ \sqrt{3} \end{pmatrix} \quad (0.6)$$

$$\mathbf{x}_2 = \begin{pmatrix} 2 \\ -\sqrt{3} \end{pmatrix} \quad (0.7)$$

$$\mathbf{x}_3 = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (0.8)$$

$$\mathbf{x}_4 = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \quad (0.9)$$

$$(0.10)$$

$$y = \pm \sqrt{4 - \frac{1}{4}x^2} \quad (0.11)$$

The area A between the curves from $x = 0$ to $x = 2$ is given by:

$$A = 4 \int_0^2 \sqrt{1 - \frac{x^2}{16}} dx \quad (0.12)$$

So by simplyfing :

$$A = 2\sqrt{3} + \frac{4\pi}{3} \quad (0.13)$$

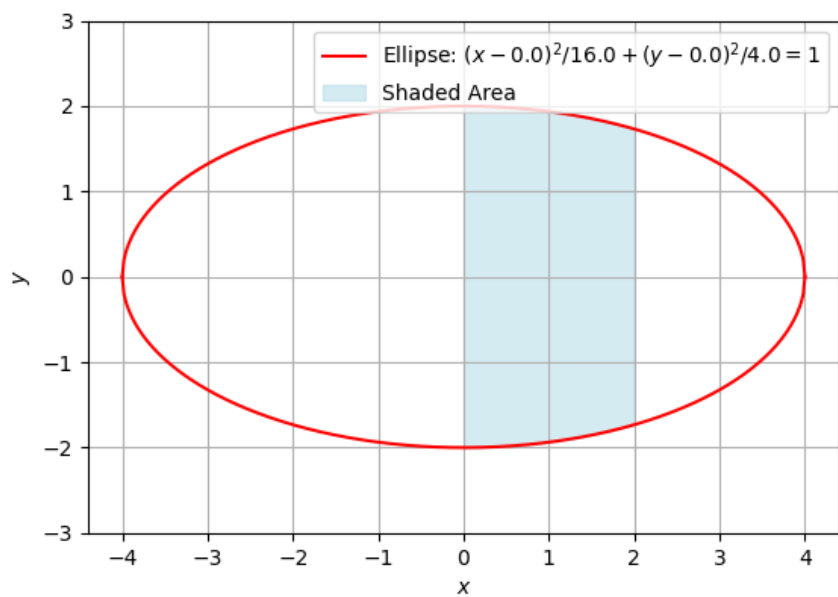


Fig. 0.1