

GATE 2019 MA(14-26)

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EE24BTECH11030 - J.KEDARANANDA

1) Let $u_n = \frac{n!}{1.3.5...(2n-1)}$, $n \in \mathbb{N}$ (the set of all natural numbers). Then $\lim_{n \rightarrow \infty} u_n$ is equal to _____.

2) If the differential equation

$$\frac{dy}{dx} = \sqrt{x^2 + y^2}, \quad y(1) = 2$$

is solved using Euler's method with step-size $h = 0.1$, then $y(1.2)$ is equal to _____ (round off to 2 places of decimal).

3) Let f be any polynomial function of degree at most 2 over \mathbb{R} (the set of all real numbers).

If the constants a and b are such that

$$\frac{df}{dx} = a f(x) + 2 f(x+1) + b f(x+2), \quad \text{for all } x \in \mathbb{R},$$

then $4a + 3b$ is equal to _____ (round off to 2 places of decimal).

4) Let L denote the value of the line integral

$$\oint_C (3x - 4x^2y) dx + (4xy^2 + 2y) dy,$$

where C , a circle of radius 2 with center at origin of the xy -plane, is traversed once in the anti-clockwise direction. Then $\frac{L}{\pi}$ is equal to _____.

5) The temperature $T : \mathbb{R}^3 \setminus \{(0, 0, 0)\} \rightarrow \mathbb{R}$ at any point $P(x, y, z)$ is inversely proportional to the square of the distance of P from the origin. If the value of the temperature T at the point $R(0, 0, 1)$ is $\sqrt{3}$, then the rate of change of T at the point $Q(1, 1, 2)$ in the direction of \vec{QR} is equal to _____ (round off to 2 places of decimal).

6) Let f be a continuous function defined on $[0, 2]$ such that $f(x) \geq 0$ for all $x \in [0, 2]$. If the area bounded by $y = f(x)$, $x = 0$, $y = 0$ and $x = b$ is $\sqrt{3 + b^2} - \sqrt{3}$, where $b \in (0, 2]$, then $f(1)$ is equal to _____ (round off to 1 place of decimal).

7) If the characteristic polynomial and minimal polynomial of a square matrix A are $(\lambda - 1)(\lambda + 1)^4(\lambda - 2)^5$ and $(\lambda - 1)(\lambda + 1)(\lambda - 2)$, respectively, then the rank of the matrix $A + I$ is _____, where I is the identity matrix of appropriate order.

8) Let ω be a primitive complex cube root of unity and $i = \sqrt{-1}$. Then the degree of the field extension $\mathbb{Q}(i, \sqrt{3}, \omega)$ over \mathbb{Q} (the field of rational numbers) is _____.

9) Let

$$\alpha = \int_C \frac{e^{i\pi z}}{2z^2 - 5z + 2} dz, \quad C : \cos t + i \sin t, \quad 0 \leq t \leq 2\pi, \quad i = \sqrt{-1}.$$

Then the greatest integer less than or equal to $|\alpha|$ is _____.

- 10) Consider the system:
$$\begin{cases} 3x_1 + x_2 + 2x_3 - x_4 = a, \\ x_1 + x_2 + x_3 - 2x_4 = 3, \\ x_1, x_2, x_3, x_4 \geq 0. \end{cases}$$
 If $x_1 = 1, x_2 = b, x_3 = 0, x_4 = c$ is a basic feasible solution of the above system (where a, b , and c are real constants), then $a + b + c$ is equal to _____.

- 11) Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be a function defined by $f(z) = z^6 - 5z^4 + 10$. Then the number of zeros of f in $\{z \in \mathbb{C} : |z| < 2\}$ is _____.
(\mathbb{C} is the set of all complex numbers)

- 12) Let $\ell^2 = \{x = (x_1, x_2, \dots) : x_i \in \mathbb{C}, \sum_{i=1}^{\infty} |x_i|^2 < \infty\}$ be a normed linear space with the norm $\|x\|_2 = \left(\sum_{i=1}^{\infty} |x_i|^2\right)^{\frac{1}{2}}$. Let $g : \ell^2 \rightarrow \mathbb{C}$ be the bounded linear functional defined by

$$g(x) = \sum_{n=1}^{\infty} \frac{x_n}{3^n} \quad \text{for all } x = (x_1, x_2, \dots) \in \ell^2.$$

Then

$$(\sup\{|g(x)| : \|x\|_2 \leq 1\})^2$$

is equal to _____ (round off to 3 places of decimal).
(\mathbb{C} is the set of all complex numbers)

- 13) For the linear programming problem (LPP):

$$\text{Maximize } Z = 2x_1 + 4x_2$$

$$\text{subject to } \begin{cases} -x_1 + 2x_2 \leq 4, \\ 3x_1 + \beta x_2 \leq 6, \\ x_1, x_2 \geq 0, \quad \beta \in \mathbb{R}, \end{cases} \quad (\mathbb{R} \text{ is the set of all real numbers})$$

consider the following statements:

- I. The LPP always has a finite optimal value for any $\beta \geq 0$.
- II. The dual of the LPP may be infeasible for some $\beta \geq 0$.
- III. If for some β , the point $(1, 2)$ is feasible to the dual of the LPP, then $Z \leq 16$, for any feasible solution (x_1, x_2) of the LPP.
- IV. If for some β , x_1 and x_2 are the basic variables in the optimal table of the LPP with $x_1 = \frac{1}{2}$, then the optimal value of dual of the LPP is 10.

Then which of the above statements are TRUE?

- a) (A) I and III only
- b) (B) I, III and IV only
- c) (C) III and IV only

d) (D) II and IV only