jee-main-maths-28-06-2022-shift-1

EE24BTECH11030 - J.KEDARANANDA

1) If
$\sum_{k=1}^{31} \binom{31}{k}\binom{31}{k-1} - \sum_{k=1}^{30} \binom{30}{k}\binom{30}{k-1} = \frac{\alpha\cdot(60!)}{(30!)\cdot(31!)}$
where $\alpha \in$ R, then the value of 16α is equal to

a) 1411 b) 1320 c) 1615 d) 1855

2) Let a function f : N $\rightarrow$ N be defined by
$$f(x) = \begin{bmatrix} 2n & ,n = 2,4,6,8,\cdots \\ n-1 & ,n = 3,7,11,15,\cdots \\ \frac{n+1}{2} & ,n = 1,5,9,13,\cdots \end{bmatrix}$$
then, f is

a) One-one but not onto
b) Onto but not one-one
c) Neither one-one nor onto
d) One-one and onto

3) If the system of linear equations
2x + 3y - z = -2
x + y + z = 4
x - y + $|\lambda|$z = 4λ - 4
where $\lambda\in$ R, has no solution, then

a) $\lambda = 7$ b) $\lambda = -7$ c) $\lambda = 8$ d) $\lambda^2 = 1$

4) Let A be a matrix of order 3 $\times$ 3 and det (A) = 2. Then det (det (A) adj (5 adj (A3))) is equal to

a) 512×10^6
b) 256×10^6
c) 1024×10^6
d) 256×10^{11}

5) he total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is

a) 36　　b) 48　　c) 60　　d) 72

6) Let $A_1, A_2, A_3, \cdots$ be an increasing geometric progression of positive real numbers. If $A_1A_3A_5A_7 = 1/1296$ and $A_2 + A_4 = \frac{7}{36}$ then, the value of $A_6 + A_8 + A_10$ is equal to

a) 33　　b) 37　　c) 43　　d) 47

7) Let [t] denote the greatest integer less than or equal to t. Then, the value of the integral $\int_0^1 [-8x^2 + 6x - 1] dx$is equal to

a) -1　　b) $\frac{-5}{4}$　　c) $\frac{\sqrt{17}-13}{8}$　　d) $\frac{\sqrt{17}-16}{8}$

8) Let f: $R \to R$ be defined as

$$f(x) = \begin{cases} 0 & , x < 0 \\ ae^x - 1 & , 0 \leq x < 1 \\ b & , x = 1 \\ b - 1 & , 1 < x < 2 \\ -c & , x \geq 2 \end{cases}$$

Where a, b, c ∈ R and [t] denotes greatest integer less than or equal to t. Then, which of the following statements is true?

a) There exists a, b, c ∈ R such that f is continuous on ∈ R .
b) If f is discontinuous at exactly one point, then a + b + c = 1
c) If f is discontinuous at exactly one point, then a + b + c ≠ 1
d) f is discontinuous at atleast two points, for any values of a, b and c

9) The area of the region $\left\{(x, y) : y^2 \leq 8x, y \geq \sqrt{2}x, x \geq 1\right\}$ is

a) $\frac{13\sqrt{2}}{6}$　　b) $\frac{11\sqrt{2}}{6}$　　c) $\frac{5\sqrt{2}}{6}$　　d) $\frac{19\sqrt{2}}{6}$

10) Let the solution curve y = y(x) of the differential equation $\left[\frac{x}{\sqrt{x^2-y^2}} + e^{\frac{y}{x}}\right] x\frac{dy}{dx} = x + \left[\frac{x}{\sqrt{x^2-y^2}} + e^{\frac{y}{x}}\right] y$ pass through the points (1, 0) and $(2\alpha, \alpha)$, $\alpha > 0$. Then α is equal to

a) $\frac{1}{2}exp(\frac{\pi}{6} + \sqrt{e} - 1)$　　c) $exp(\frac{\pi}{6} + \sqrt{e} + 1)$

b) $\frac{1}{2}exp(\frac{\pi}{3} + \sqrt{e} - 1)$　　d) $2exp(\frac{\pi}{3} + \sqrt{e} - 1)$

11) Let y = y(x) be the solution of the differential equation $x(1-x^2)\frac{dy}{dx} + 3x^2y - y - 4x^3 = 0$, x>1 with y(2) = -2. Then y(3) is equal to

a) -18 b) -12 c) -6 d) -3

12) The number of real solutions of $x^7 + 5x^3 + 3x + 1 = 0$ is equal to

a) 0 b) 1 c) 3 d) 5

13) Let the eccentricity of the hyperbola
H : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
be $\frac{\sqrt{5}}{2}$ and length of its latus rectum be $6\sqrt{2}$, If $y = 2x + c$ is a tangent to the hyperbola H. then the value of c^2 is equal to

a) 18 b) 20 c) 24 d) 32

14) If the tangents drawn at the points $\mathbf{O}\left(0, 0\right)$ and $\mathbf{P}\left(1 + \sqrt{5}, 2\right)$ on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at the point Q, then the area of the triangle OPQ is equal to

a) $\frac{3+\sqrt{5}}{2}$ b) $\frac{4+2\sqrt{5}}{2}$ c) $\frac{5+3\sqrt{5}}{2}$ d) $\frac{7+3\sqrt{5}}{2}$

15) If two distinct points Q, R lie on the line of intersection of the planes $-x+2y-z=0$ and $3x - 5y + 2z = 0$ and PQ = PR = $\sqrt{18}$ where the point P is (1, -2, 3), then the area of the triangle PQR is equal to

a) $\frac{2}{3}\sqrt{38}$ b) $\frac{4}{3}\sqrt{38}$ c) $\frac{8}{3}\sqrt{38}$ d) $\sqrt{\frac{152}{3}}$