Tutorial - 1

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Ans 1

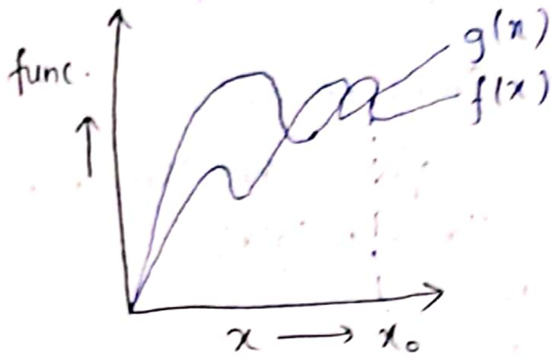
(i) Big $O(n)$

$f(n) \Rightarrow O(g(n))$

if $f(x) \leq g(x) \times c \ \forall \ x \geq x_0$

for some constant, $c > 0$

$g(x)$ is 'tight' upper bound of $f(x)$



(ii) Big Omega(Ω)

when $f(x) = \Omega(g(x))$

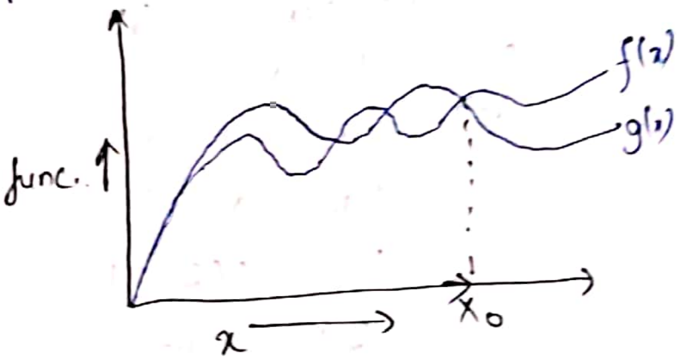
means $g(x)$ is "tight" lower bound of $f(x)$ i.e $f(x)$ can go beyond $g(x)$

i.e $f(x) = \Omega g(x)$

if and only if

$f(x) \geq c.g(x)$

$\forall \ x_2 > x_0$ and c = constant > 0



(iii) Big Theta (Θ)

when $f(x) = \Theta(g(x))$ gives the tight upperbound and lowerbound both.

i.e $f(x) = \Theta(g(x))$

if and only if

$c_1 * g(x_1) \leq f(x) \leq c_2 * g(x_2)$

for all $x \geq \max(x1, x2)$ some constant $c_1 > 0$ and $c_2 > 0$

i.e $f(x)$ can never go beyond $c_2 \, g(x)$ and will never come dow of $c_1 \, g(x)$

(iv) Small o(o)

when $f(x) = O\,g(x)$ gives the upper bound

i.e $f(x) = O\,g(x)$

if and only if

$f(x) < c\,g(x)$

$\forall\ n > n_0$ and $n > 0$

(v) Small omega (ω)

It gives the 'lower bound' i.e

$f(x) = \omega(g(x))$

where $g(x)$ is lower bound of $f(x)$

if and only if $f(x) > c * g(x)$

$\forall\ n > n_0$ of some constant, $c > 0$

Ans 2. for $i \Rightarrow 1, 2, 4, 6, 8 \ldots n$ times

i.e series $\rightarrow$ GP

so $a = 1$, $r = 2/1$

k^{th} value of GP

$t_k = a r^{k-1}$

$t_k = 1(2)^{k-1}$

$2n = 2^k$

$\log_2(2n) = k \log_2 2$

$\log_2 2 + \log_2 n = k$

$\log_2 n + 1 = k$

Time complexity, $\boxed{T(n) = O(\log n).}$

Ans 3. $T(n) = 3T(n-1)$ — ①

$T(n) = 1$

Put $n = n-1$ in — ①

$T(n-1) = 3T(n-2)$ — ②

put ② in ①

$T(n) = 3 \times 3T(n-2)$

$T(n) = 9T(n-2)$ — ③

put $n = n-2$ in ①

$T(n-2) = 3T(n-3)$.

put in ③

$T(n) = 27T(n-3)$ — ④

$\Rightarrow$ $T(k) = 3^k\, T(n-k)$ — ⑤

for kth terms, let $n-k = 1$

$k = n-1$

put in ⑤

$T(n) = 3^{n-1}\, T(1)$

$T(n) = 3^{n-1}$

$\boxed{T(n) = O(3^n)}$

Ans 4. $T(n) = 2T(n-1) - 1$ — ①

put $n = n-1$

$T(n-1) = 2T(n-2) - 1$ — ②.

put in equ ①

$T(n) = 2 \times (2T(n-2) - 1) - 1$

$= 4T(n-2) - 2 - 1$ — ③.

put $n = n-2$ in ①

$T(n-2) = 2T(n-3) - 1$

Put in ①

$\cancel{T(n-2) = 2T(n-3) - 1}$

$T(n) = 8T(n-3) - 4 - 2 - 1 \quad - (4)$

<u>Generalising series</u>

$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} \ldots 2^0$

k^{th} term

let $n-k = 1$

$k = n-1$

$T(n) = 2^{n-1} T(1) - 2^k \left(1/2 + 1/2^2 + \ldots + \frac{1}{2^k} \right)$

$= 2^{n-1} - 2^{n-1} \left(1/2 + 1/2^2 + \ldots + 1/2^{n-1} \right)$

$a = 1/2$ and $r = 1/2$

So, $T(n) = 2^{n-1} \left(1 - \left(1/2 \left(\frac{1 - (1/2)^{n-1}}{1 - 1/2} \right) \right) \right)$

$= 2^{n-1} \left(1 - 1 + (1/2)^{n-1} \right)$

$= \frac{2^{n-1}}{2^{n-1}}$

$\boxed{T(n) = O(1)}$

<u>Ans 5.</u> $i = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \ldots$

$S = 1 + 3 + 6 + 10 + 15 + \ldots n \quad - (1)$

also $S = 1 + 3 + 6 + 10 + \ldots T_{n-1} + T_n \quad - (2)$

$0 = 1 + 2 + 3 + 4 + \ldots n - T_n$

$T_k = 1 + 2 + 3 + 4 + \ldots + k$

$T_k = \frac{1}{2} k(k+1)$

for k iterations

$1 + 2 + 3 + \dots + k <= n$

$\frac{k(k+1)}{2} <= n$

$\frac{k^2+k}{2} <= n$

$O(k^2) <= n$

$k = O\sqrt{n}$

$\boxed{T(n) = O(\sqrt{n})}$

Ans 6. As $i^2 = n$

$i = \sqrt{n}$

$i = 1, 2, 3, 4 \dots \sqrt{n}$

$\sum_{i=1} 1 + 2 + 3 + 4 + \dots + \sqrt{n}$

$T(n) = \frac{\sqrt{n} * (\sqrt{n}+1)}{2}$

$T(n) = \frac{n * \sqrt{n}}{2}$

$\boxed{T(n) = O(n)}$

Ans 7. $\because$ ~~for~~ $a = 1, r = 2$

$\frac{a(r^n - 1)}{r-1}$

$= \frac{1(2^k - 1)}{1}$

$n = 2^k - 1$

$n + 1 = 2^k$

$\log_2(n) = k$

i	j	k
1	$\log n$	$\log(n) * \log(n)$
2	$\log n$	$\log(n) * \log n$
⋮	⋮	⋮
n	$\log n$	$\log(n) * \log(n)$

$T.C = O(n * \log n * \log n)$

$\boxed{T(n) = O(n \log^2(n))}$

Ans 8. $T(n) = n^2 + T(n-3)$

$T(n-3) = (n^2 \cdot 3)^2 + T(n-6)$

$T(n-6) = (n^3 6)^2 + T(n-9)$

and $T(1) = 1$

$\Rightarrow T(n) = n^2 + (n-3)^2 + (n-6)^2 + \dots + 1$

let $k^n - 3k = 1$

$k = (n-1)/3$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \dots + 1$

$T(n) \simeq (k-1)/3 - n^2$

so

$\boxed{T(n) = O(n^3)}$

Ins 10

As given n^k and c^n

relationship b/w x^k and c^n is

$$n^k = O(c^n)$$

$$n^k \leq a(c^n)$$

$\forall\ n \geq n_0$ and constant, $a > 0$

for $n_0 = 1$, $c = 2$

$\Rightarrow 1^k < a2$

$\Rightarrow \boxed{n_0 = 1 \ \& \ c = 2}$