Tutorial - 4

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Section → G

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Ans 1 $T(n) = 3T(n/2) + n^2$

$a = 3, b = 2, f(n) = n^2$

$c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$f(n) > n^c$

$\boxed{T(n) = \Theta(n^2)}$

Ans 2 $T(n) = 4T(n/2) + n^2$

$a = 4, b = 2, f(n) = n^2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

Ans 3 $T(n) = T(n/2) + 2^n$

$a = 1, b = 2, f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$\boxed{T(n) = \Theta(2^n)}$

Ans 4 $T(n) = 2^n T(n/2) + n^n$

$a = 2^n, b = 2, f(n) = n^n$

$c = \log_b a = \log_2 2^n = n$

$n^c = n^n$

$f(n) = n^c$

$\boxed{T(n) = \Theta(n^2 \log_2 n)}$

Ans 5 $T(n) = 16T(n/4) + n$

$a = 16, b = 4, f(n) = n$

$c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

$n^c > f(n)$

$\therefore \boxed{T(n) = \Theta(n^2)}$

Ans 6. $T(n) = 2T(n/2) + n\log n$

$a = 2, b = 2, f(n) = n\log n$

$c = \log_b a = \log_2 2 = 1$

$n^c = n$

$\because n\log n > n$

$\therefore \boxed{T(n) = \Theta(n\log n)}$

Ans7. $T(n) = 2T(n/2) + n/\log n$

$a=2,\ b=2,\ f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n$

$\because \frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore \boxed{T(n) = \Theta(n)}$

Ans 8 $T(n) = 2T(n/4) + n^{0.51}$

$a=2,\ b=4,\ f(n) = 0.51$

$c = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$n^{0.5} < n^{0.51}$

$\boxed{T(n) = \Theta(n^{0.51})}$

Ans9. $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5,\ b=2$

$a \geq 1$ but here $a < 1$

so cannot apply Masters Theorem.

Ans10 $T(n) = 16T(n/4) + n!$

$a=16,\ b=4,\ f(n) = n!$

$c = \log_4 16 = 2$

$n^c = n^2$

$n! > n^2$

$\boxed{T(n) = \Theta(n!)}$

Ans11 $T(n) = 4T(n/2) + \log n$

$a=4,\ b=2,\ f(n) = \log n$

$c = \log_2 4 = 2$

$f(n) = \log n < n^2$

$\boxed{T(n) = \Theta(n^2)}$

Ans12 $T(n) = \sqrt{n}\, T(n/2) + \log n$

$a = \sqrt{n},\ b=2,\ f(n) = \log n$

$c = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\frac{1}{2}\log_2 n \leq \log(n)$

$\boxed{T(n) = \Theta(\log n)}$

Ans13. $T(n) = 3T(n/2) + n$

$a=3;\ b=2,\ f(n) = n$

$c = \log_2 3 = 1.5849$

$n^c = n^{1.5849}$

$n < n^{1.5849}$

$\boxed{T(n) = \Theta(n^{1.5849})}$

Ans14. $T(n) = 3T(n/3) + \sqrt{n}$

$a=3,\ b=3,\ f(n) = \sqrt{n}$

$c = \log_3 3 = 1$

$n^c = n$

$\because \sqrt{n} < n$

$\boxed{T(n) = \Theta(n)}$

Ans 15. $T(n) = 4T(n/2) + n$

$a = 4, b = 2, f(n) = n$

$c = \log_2 4 = 2$

$n^c = n^2$

$\because n < n^2$

$\boxed{f(n) = \Theta(n^2)}$

Ans 16. $T(n) = 3T(n/4) + n\log n$

$a = 3, b = 4, f(n) = n\log n$

$c = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^c < n\log n$

$\boxed{T(n) = \Theta(n\log n)}$

Ans 17. $T(n) = 3T(n/3) + n/2$

$a = 3, b = 3, f(n) = n/2$

$c = \log_3 3 = 1$

$n^c = n$

$\because n > n/2$

$\boxed{T(n) = \Theta(n)}$

Ans 18. $T(n) = 6T(n/3) + n^2\log n$

$a = 6, b = 3, f(n) = n^2\log n$

$c = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

$n^{1.6309} < n^2\log n$

$\boxed{T(n) = \Theta(n^2\log n)}$

Ans 19. $T(n) = 4T(n/2) + n/\log n$

$a = 4, b = 2, f(n) = n/\log n$

$c = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$\boxed{T(n) = \Theta(n^2)}$

Ans 20. $T(n) = 64T(n/8) - n^2\log n$

$a = 64, b = 8, f(n) = n^2\log n$

$c = \log_8 64 = 2$

$n^c = n^2$

$n^2\log n > n^2$

$\boxed{T(n) = \Theta(n^2\log n)}$

Ans 21. $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3, f(n) = n^2$

$c = \log_3 7 = 1.7712$

$n^c = n^{1.7712} < n^2$

$\therefore \boxed{T(n) = \Theta(n^2)}$

Ans 22. $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, b = 2$

$c = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos n) > n^c$

$\boxed{T(n) = \Theta(n(2 - \cos n))}$