Project Proposal: Bayesian Nonparametric Adaptive Control Using Gaussian Processes

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The purpose of the project will be to implement a model reference adaptive control (MRAC) scheme in which Gaussian Processes (GPs) are employed to estimate the adaptive elements that MRAC features. This project will focus in implementing the algorithms and examples featured in [1].

The MRAC algorithm relies on a reference model with an adaptive term to mitigate errors caused by model uncertainty. Radial basis function neural networks (RBFNs) are widely used to approximate the adaptive term in MRAC ν_{ad}, such that $\nu_{ad} = W^T \Phi(z)$, with W as weights that will be updated and $\Phi(z)$ as the vector of radial functions that depend on vector z, which includes the states and inputs of the system, which would help in learning a representation of modelling uncertainty $\Delta(z)$.

For this project, it is assumed that uncertainty $\Delta(z)$ can be described by a time-varying prior mean and covariance and that GPs can be used to learn a continuous function that best describes it, such that:

$$\Delta(\cdot) \sim \mathcal{GP}(m(\cdot), k(\cdot, \cdot)),$$

in which $m(\cdot)$ is the mean function and $k(\cdot, \cdot)$ is a covariance kernel function (the paper in [1] uses a Gaussian kernel, but other kernels might be tested). Therefore the GP-MRAC scheme aims to model uncertainty using the GP-based adaptive element ν_{ad} such that:

$$\nu_{ad} \sim \mathcal{GP}(\hat{m}(z), k(z, z')),$$

where $\hat{m}(z)$ would be the mean of model uncertainty Δ. In the implementation of GP-MRAC, the adaptive signal ν_{ad} would be set to the estimate of $\hat{m}(z)$. Furthermore, since most implementations of GP regression assume that all data is available, which is unfeasible in an online control scheme, part of the algorithm will also deal with determining when and how to replace the samples to perform regression, which is also covered in [1].

The GP-MRAC algorithm will be tested by tracking a trajectory in the presence of the wing rock dynamics featured in [1]. Let θ denote the roll attitude of an aircraft, p denote its roll rate and δ_a the aileron control input, the dynamic model used will be:

$$\dot{\theta} = p, \ \dot{p} = L_{\delta_a} \delta_a + \Delta(x),$$

where L_{δ_a} is a constant term and $\Delta(x)$ represents the modelling error which whose function mean and covariance be approximated by a GP. While RBFN-MRAC is also implemented in the paper this project is based on, a greater focus will be given to the implementation of GP*MRAC, meaning that a comparison will be established if enough time is available to do so.

The aim of this project is to use Gaussian Processes in an online adaptive control scheme to successfully track a trajectory and diminish the effects of modelling uncertainty by approximating its mean function and covariance. Thus, the Gaussian Process Regression algorithm we learned in class will be implemented in an online setting, which we haven't attempted in any of the projects.

References

[1] G. Chowdhary, H. A. Kingravi, J. P. How, and P. A. Vela, "Bayesian nonparametric adaptive control using gaussian processes," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 26, pp. 537–550, March 2015.