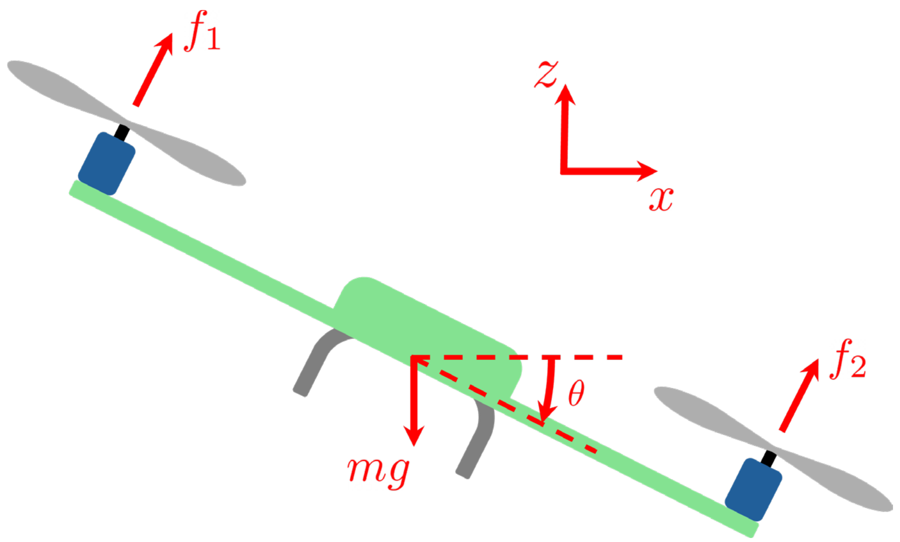
Time-optimal trajectory planning for multicopter lateral flight

Juan Augusto Paredes Salazar

Multicopter lateral flight dynamics without rotational dynamics



States

- x: Horizontal position of multicopter.
- z: Vertical position of multicopter.
- θ: Pitch angle of multicopter.

Inputs

- f_1, f_2: Thrust exerted by each of the multicopter rotors.
- $f_{\mathrm{T}} = f_1 + f_2$: Total thrust exerted by multicopter rotors.
- $\omega = \dot{\theta}$: Pitch rate of multicopter, can be directly modulated (assumption)

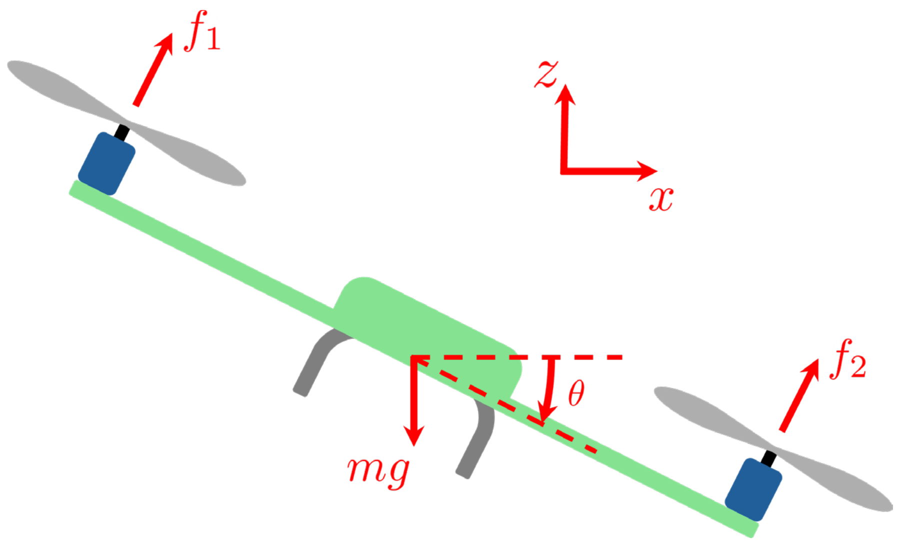
Parameters

- m: Multicopter mass.
- g: Gravitational acceleration constant.

Input Constraints

- $f_{\mathrm{T}} \in \left[\underline{f}_{\mathrm{T}},\ \overline{f}_{\mathrm{T}}\right]$.
- $\omega \in [-\overline{\omega},\ \overline{\omega}]$.

Multicopter lateral flight dynamics without rotational dynamics



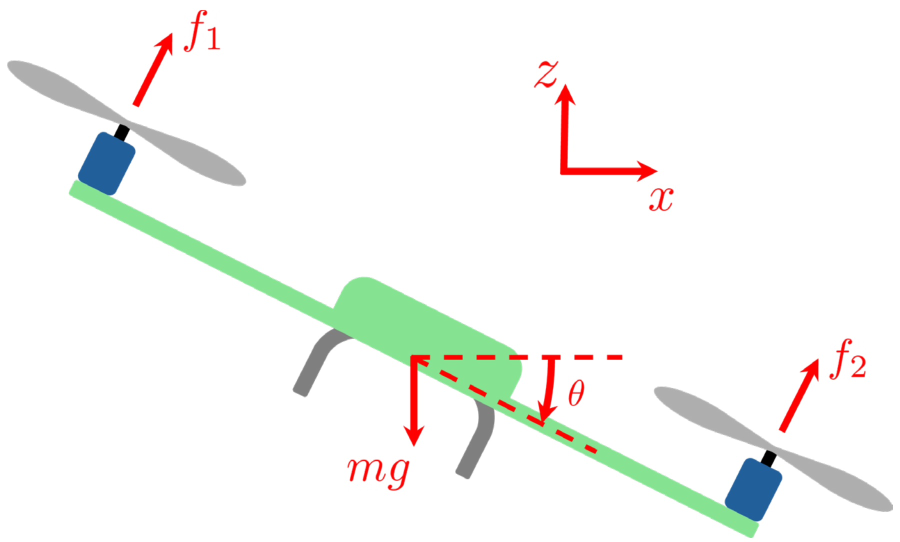
System Dynamics

$$\ddot{x} = \frac{f_{\mathrm{T}}}{m} \sin\theta$$

$$\ddot{z} = \frac{f_{\mathrm{T}}}{m} \cos\theta - g$$

$$\dot{\theta} = \omega$$

Multicopter lateral flight dynamics without rotational dynamics



State vector

- $\mathbf{x} \triangleq [x \quad \dot{x} \quad z \quad \dot{z} \quad \theta]^{\mathrm{T}}$

Input Vector

- $\mathbf{u} \triangleq [f_{\mathrm{T}}/m \quad \omega]^{\mathrm{T}} \triangleq [u_{\mathrm{T}} \quad u_{\mathrm{R}}]^{\mathrm{T}}$

Input Constraints

- $u_{\mathrm{T}} \in \left[\underline{f}_{\mathrm{T}}/m, \quad \overline{f}_{\mathrm{T}}/m\right].$
- $u_{\mathrm{R}} \in [-\overline{\omega}, \quad \overline{\omega}].$

System Dynamics

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} \dot{x} \\ u_T \sin\theta \\ \dot{z} \\ u_T \cos\theta - g \\ u_R \end{bmatrix}$$

Nonlinear programming for time-optimal trajectory planning

Discrete-time system setup

- Discrete-time nonlinear dynamics resulting from discretizing continuous-time dynamics $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$ using the Euler method are given by
$$\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta t\, f(\mathbf{x}_k, \mathbf{u}_k)$$
where $k \in \{0, 1, \ldots\}$ is the iteration step, $\Delta t > 0$ is the time between iterations, and $\mathbf{x}_k, \mathbf{u}_k$ are the discrete-time state and input vectors, respectively.

- Iteration step k corresponds to the time $t = k\, \Delta t$.

- Let $\mathbf{x}_{k,(i)}$ be the i-th component of $\mathbf{x}_k$, and let $\mathbf{u}_{k,(j)}$ be the j-th component of $\mathbf{u}_k$, and consider the state and input constraints
$$\mathbf{x}_{k,(i)} \in \left[\mathbf{x}_{(i),\min}, \mathbf{x}_{k,(i),\max}\right],$$
$$\mathbf{u}_{k,(j)} \in \left[\mathbf{u}_{(j),\min}, \mathbf{u}_{k,(j),\max}\right]$$

Nonlinear programming for time-optimal trajectory planning

Nonlinear programming

- For a given trajectory, let $T > 0$ be the final time, $\mathbf{x}_\mathrm{i}$ the initial state, $\mathbf{x}_\mathrm{f}$ the final state, and $\mathbf{u}_\mathrm{f}$ the final input.
- For a user-chosen maximum number of iteration steps N, the time-optimal trajectory optimization problem can be formulated using the discretized dynamics shown in the previous slide as

$$\min_{(\mathbf{x}_k)_{k=0}^{N},\, (\mathbf{u}_k)_{k=0}^{N-1},\, T} T + (\mathbf{x}_\mathrm{f} - \mathbf{x}_N)^\mathrm{T} Q_\mathrm{x} (\mathbf{x}_\mathrm{f} - \mathbf{x}_N) + (\mathbf{u}_\mathrm{f} - \mathbf{u}_{N-1})^\mathrm{T} Q_\mathrm{u} (\mathbf{u}_\mathrm{f} - \mathbf{u}_{N-1}),$$

where Q_x, Q_u are weighting matrices, subject to

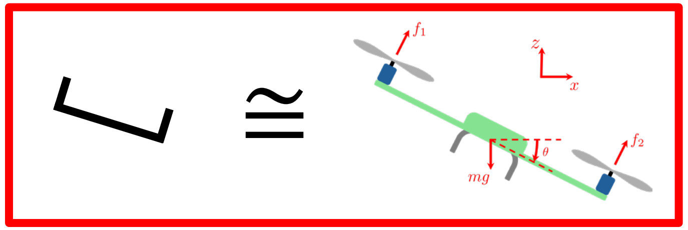
$$\mathbf{x}_{k+1} = \mathbf{x}_k + \frac{T}{N} f(\mathbf{x}_k, \mathbf{u}_k) \qquad \text{for all } k \in \{0, 1, \dots, N\}$$

$$\mathbf{x}_0 = \mathbf{x}_\mathrm{i}, \qquad \mathbf{x}_N = \mathbf{x}_\mathrm{f}, \qquad \mathbf{u}_{N-1} = \mathbf{u}_\mathrm{f}, \qquad T > 0$$

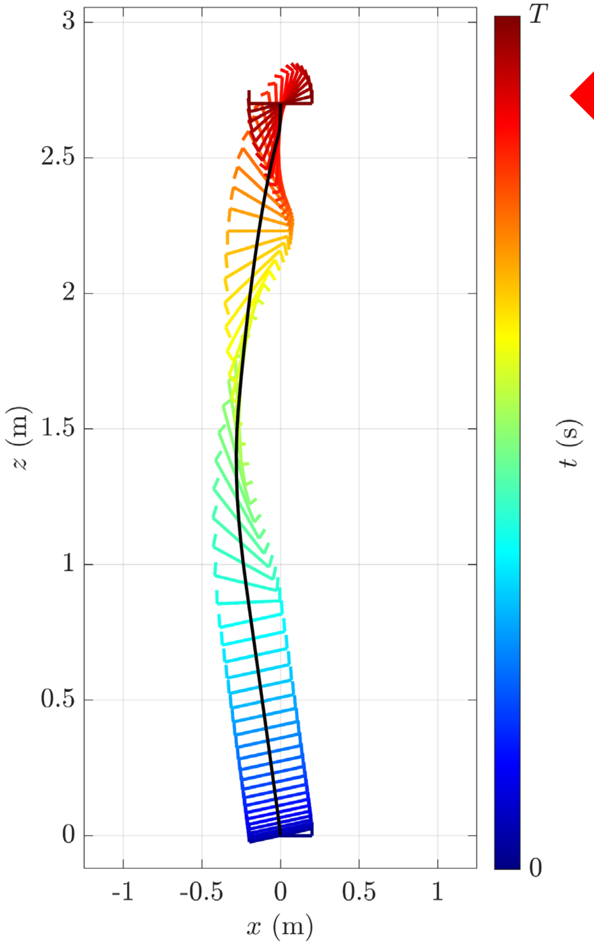
$$\mathbf{x}_{k,(i)} \in \left[\mathbf{x}_{(i),\min}, \mathbf{x}_{k,(i),\max}\right], \qquad \text{for all } k \in \{0, 1, \dots, N\}, \text{for all } i \text{ components of } \mathbf{x}_k$$

$$\mathbf{u}_{k,(j)} \in \left[\mathbf{u}_{(j),\min}, \mathbf{u}_{k,(j),\max}\right], \qquad \text{for all } k \in \{0, 1, \dots, N-1\}, \text{for all } j \text{ components of } \mathbf{u}_k$$

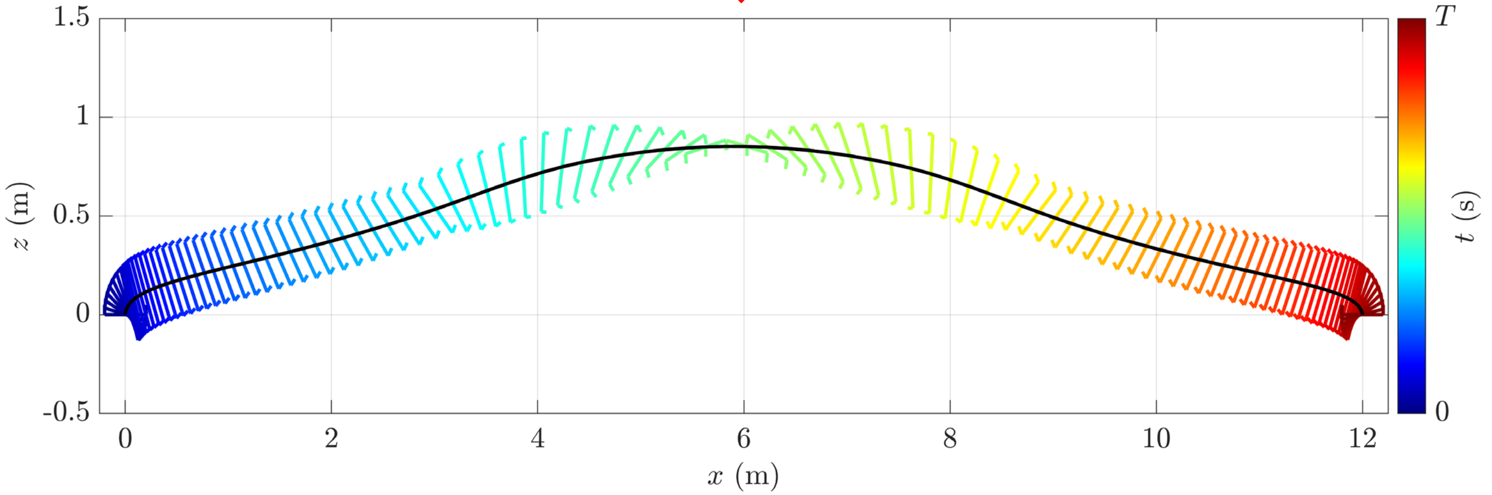
Test scenarios



without rotational dynamics

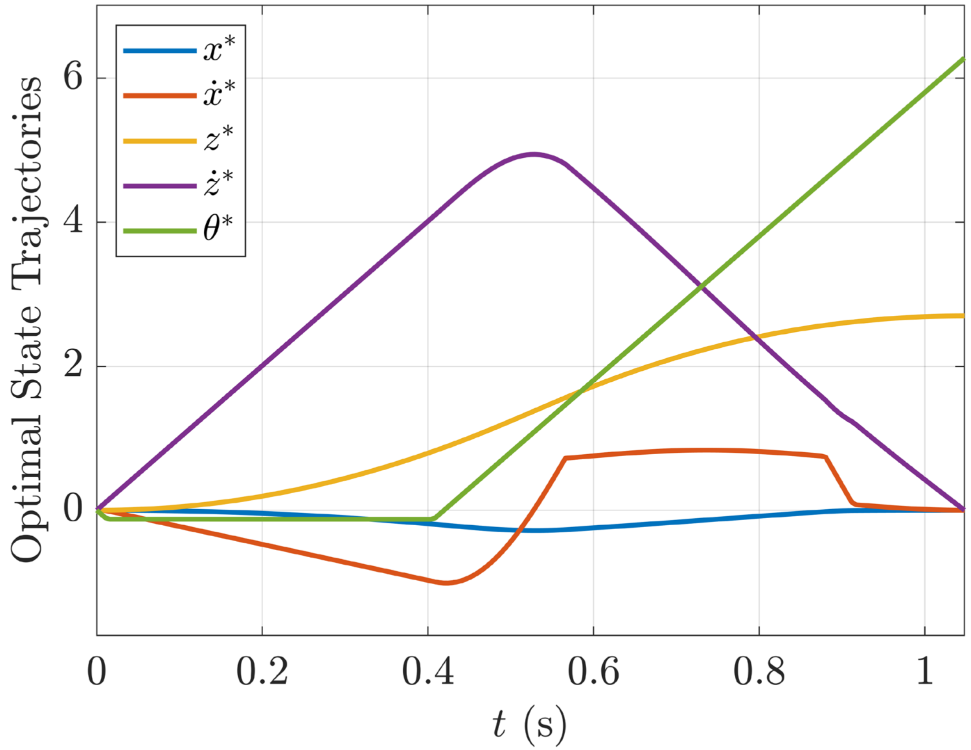


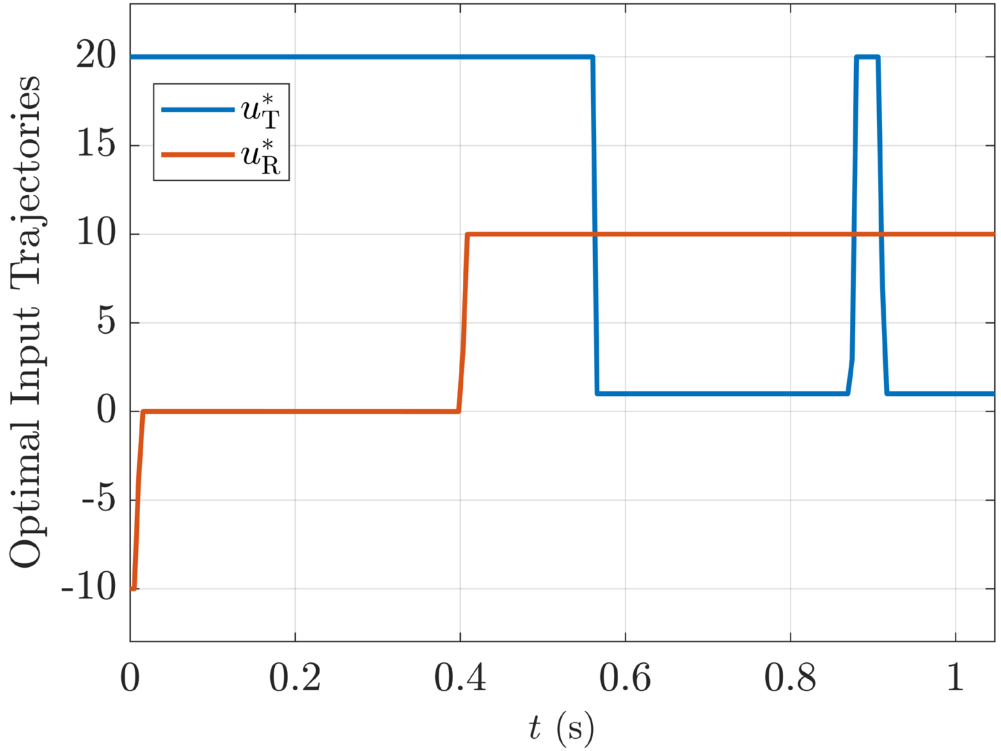
- Vertical displacement with flip maneuver ($x_T = 0$ m, $z_T = 2.7$ m, $\theta_T = 2\pi$ rad)
- Horizontal displacement with flip maneuver ($x_T = 12$ m, $z_T = 0$ m, $\theta_T = 2\pi$ rad)



Vertical Displacement with Flip

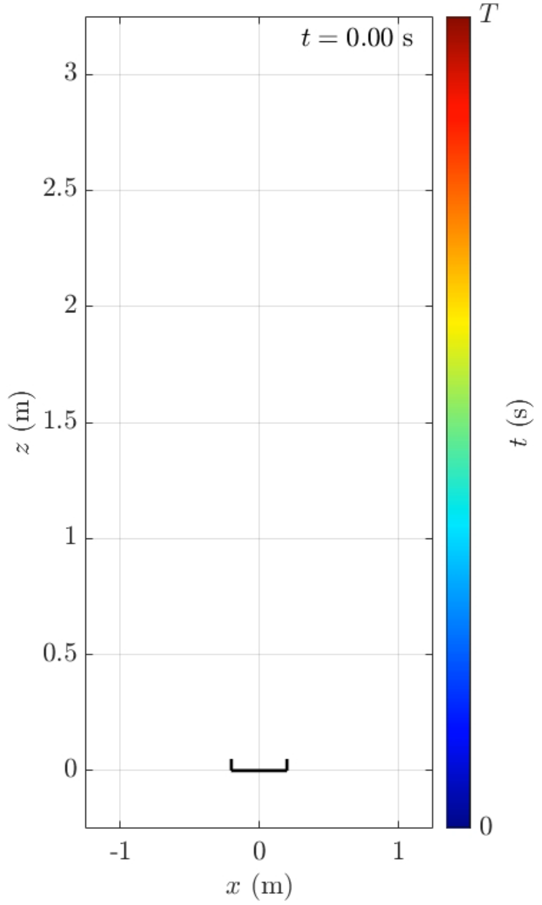
$x_T = 0$ m, $z_T = 2.7$ m, $\theta_T = 2\pi$ rad

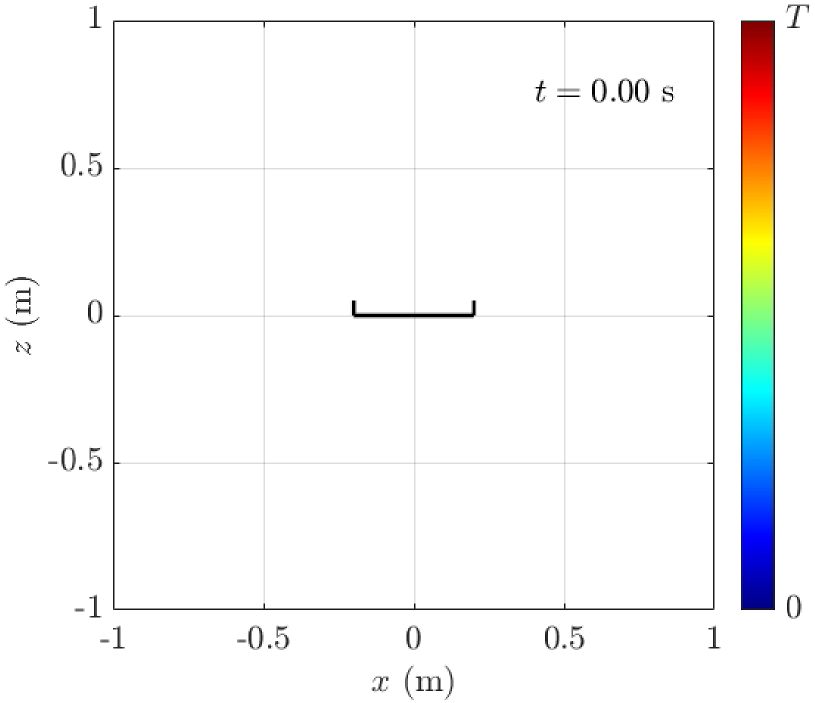




Vertical Displacement with Flip

$x_T = 0$ m, $z_T = 2.7$ m, $\theta_T = 2\pi$ rad

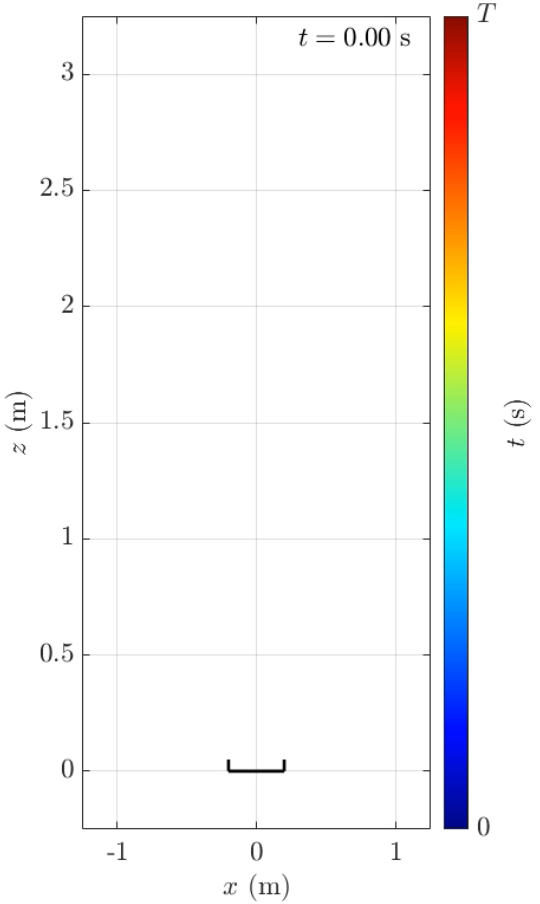


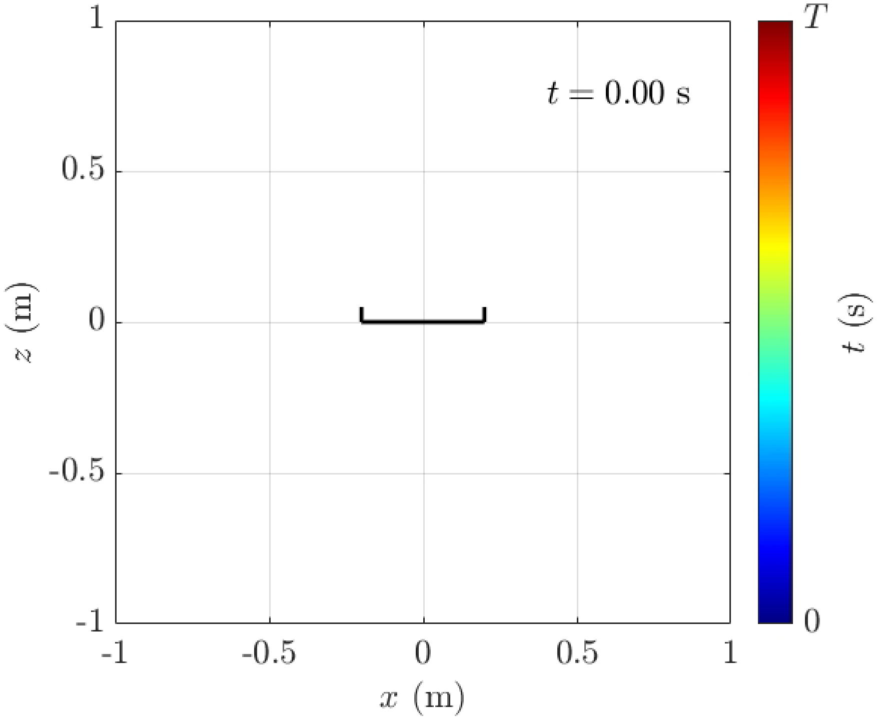


Playback speed: 1x

Vertical Displacement with Flip

$x_T = 0$ m, $z_T = 2.7$ m, $\theta_T = 2\pi$ rad

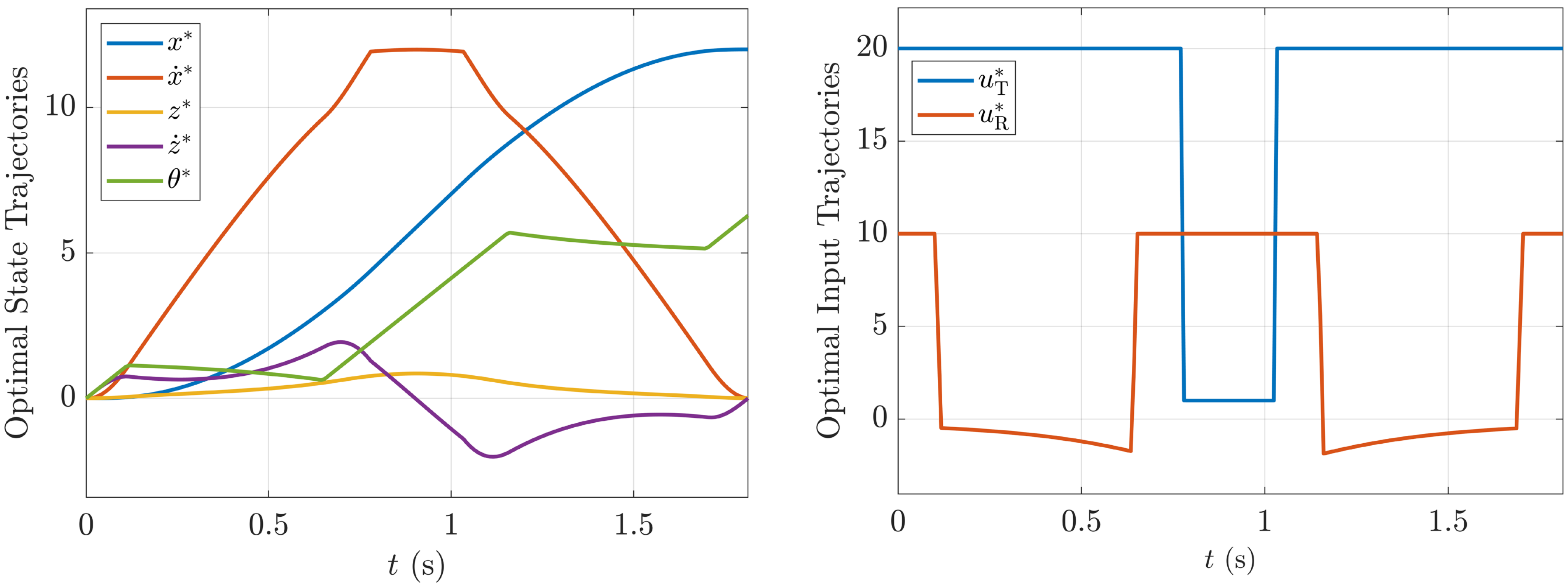




Playback speed: 0.2x

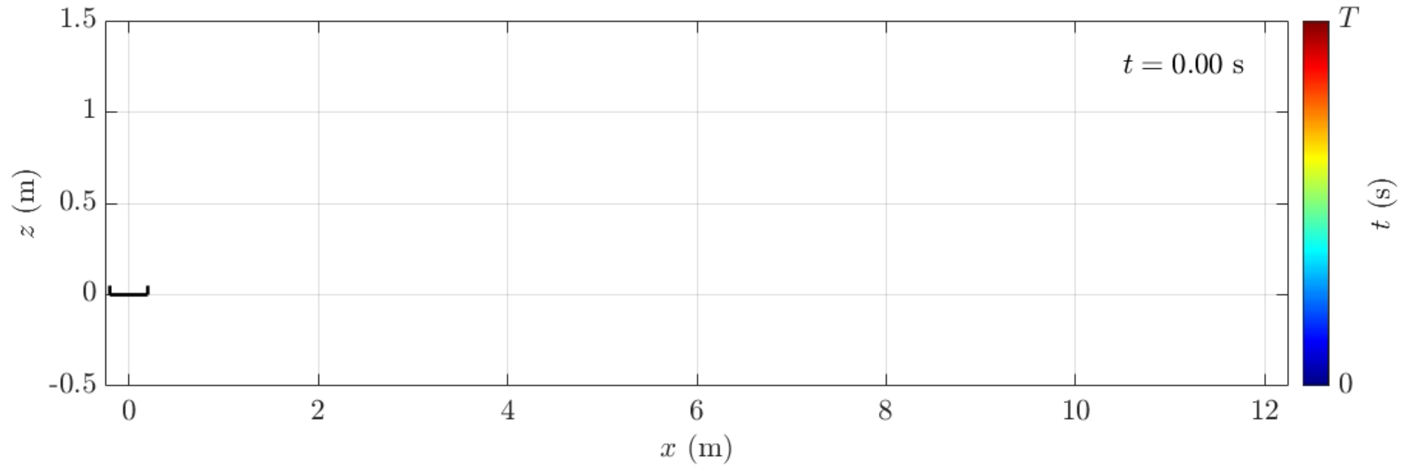
Horizontal Displacement with Flip

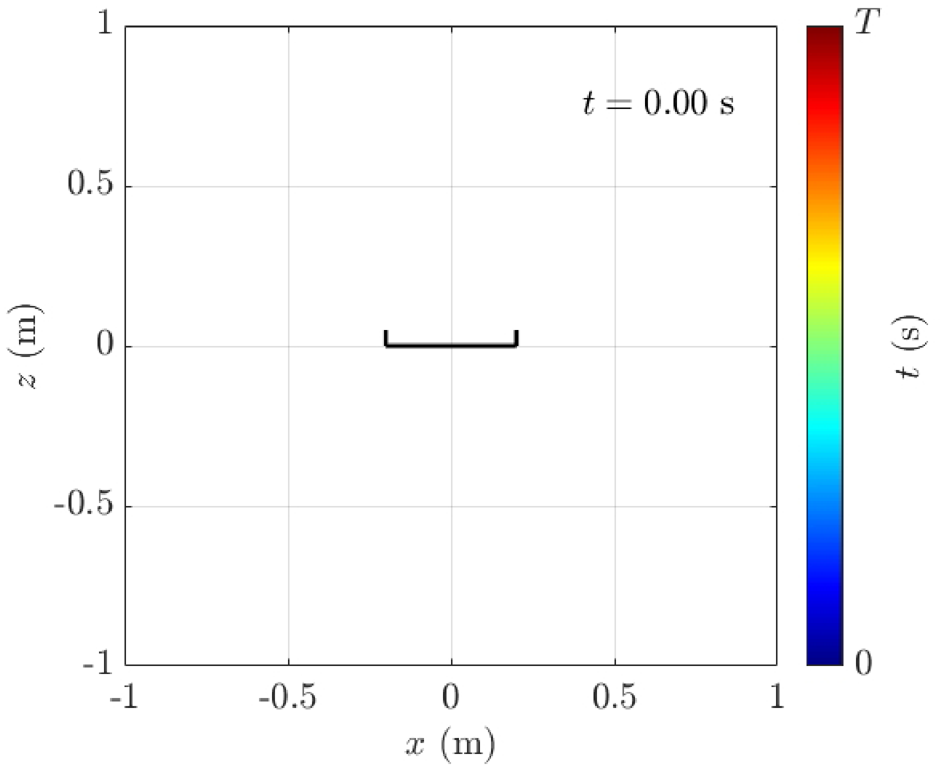
$x_T = 12$ m, $z_T = 0$ m, $\theta_T = 2\pi$ rad



Horizontal Displacement with Flip

$x_T = 12$ m, $z_T = 0$ m, $\theta_T = 2\pi$ rad

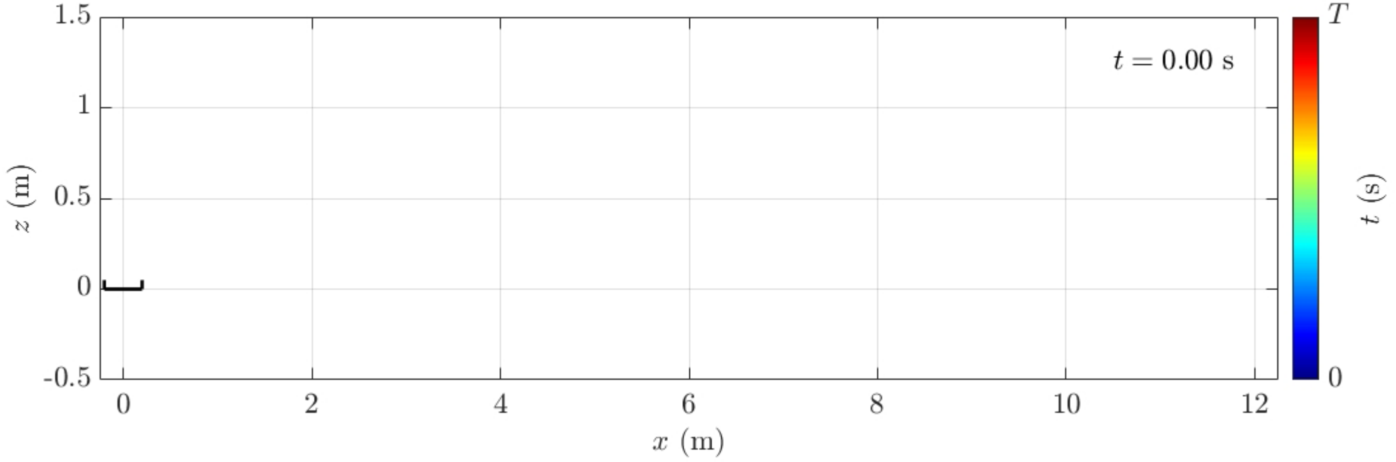


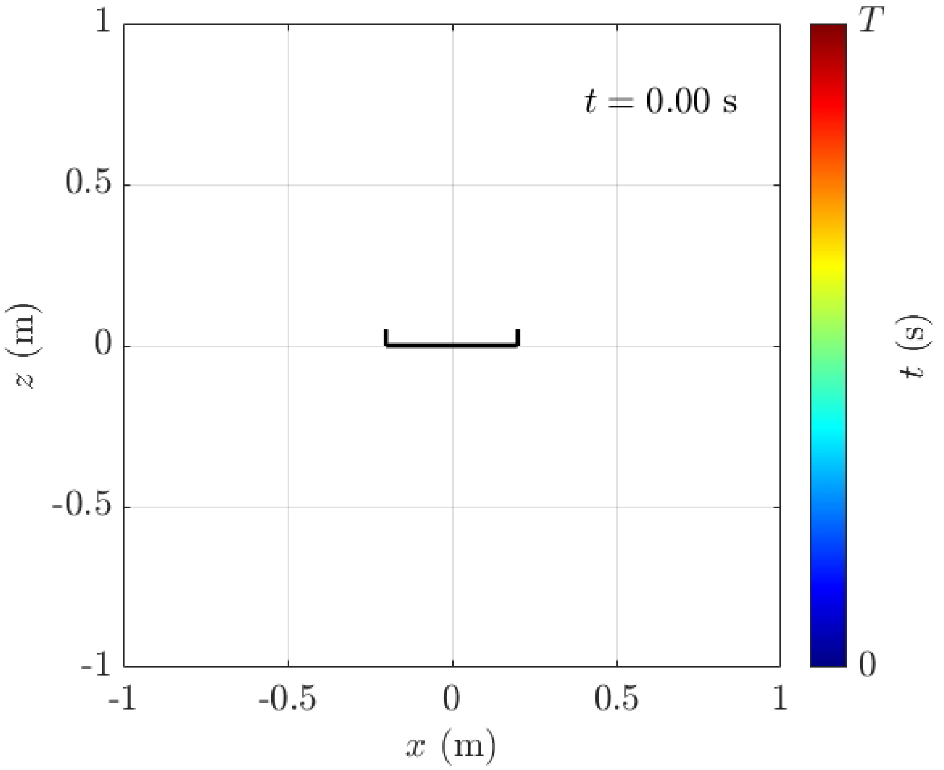


Playback speed: 1x

Horizontal Displacement with Flip

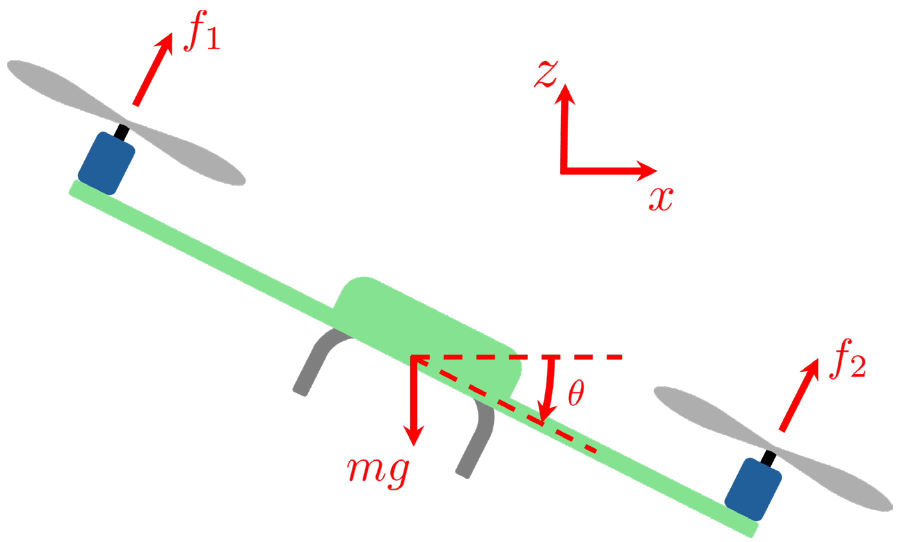
$x_T = 12$ m, $z_T = 0$ m, $\theta_T = 2\pi$ rad





Playback speed: 0.2x

Multicopter lateral flight dynamics with rotational dynamics



States

- x: Horizontal position of multicopter.
- z: Vertical position of multicopter.
- θ: Pitch angle of multicopter.

Inputs

- f_1, f_2: Thrust exerted by each of the multicopter rotors.
- $f_{\mathrm{T}} = f_1 + f_2$: Total thrust exerted by multicopter rotors.
- $\tau = (f_1 - f_2)/d$: Total moment exerted by the multicopter rotors separated by distance d.

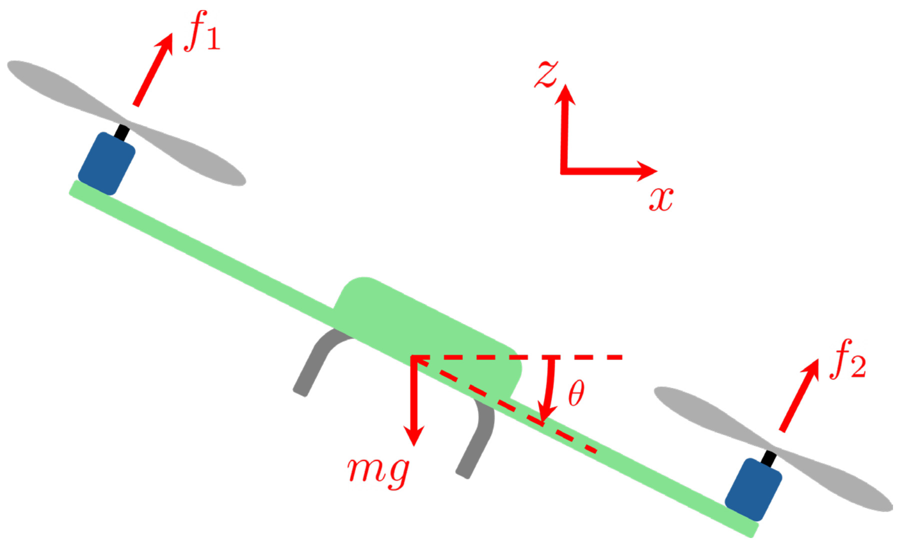
Parameters

- m: Multicopter mass.
- J_{I}: Multicopter moment of lateral inertia.
- g: Gravitational acceleration constant.

Input Constraints

- $f_{\mathrm{T}} \in \left[\underline{f}_{\mathrm{T}},\ \overline{f}_{\mathrm{T}}\right]$.
- $\tau \in [-\overline{\tau},\ \overline{\tau}]$.

Multicopter lateral flight dynamics with rotational dynamics



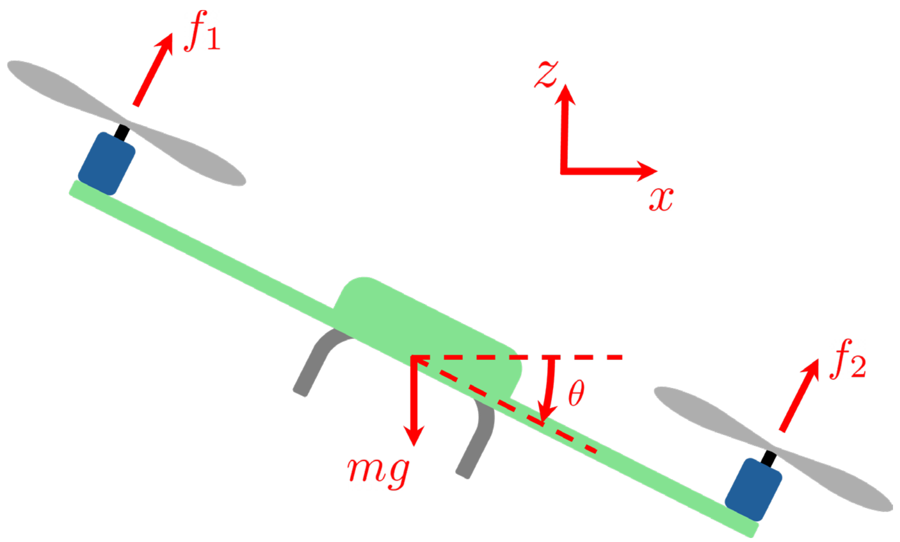
System Dynamics

$$\ddot{x} = \frac{f_{\mathrm{T}}}{m}\sin\theta$$

$$\ddot{z} = \frac{f_{\mathrm{T}}}{m}\cos\theta - g$$

$$\ddot{\theta} = \frac{\tau}{J_{\mathrm{I}}}$$

Multicopter lateral flight dynamics with rotational dynamics



State vector

- $\mathbf{x} \triangleq \begin{bmatrix} x & \dot{x} & z & \dot{z} & \theta & \dot{\theta} \end{bmatrix}^{\mathrm{T}}$

Input Vector

- $\mathbf{u} \triangleq [f_{\mathrm{T}}/m \quad \tau/J_{\mathrm{I}}]^{\mathrm{T}} \triangleq [u_{\mathrm{T}} \quad u_{\mathrm{R}}]^{\mathrm{T}}$

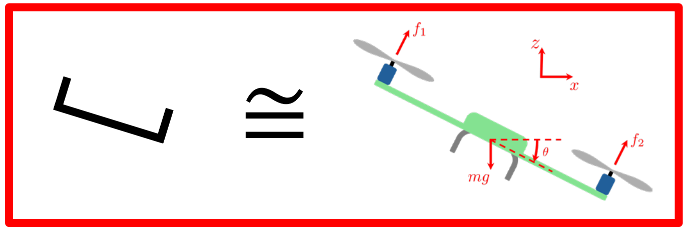
Input Constraints

- $u_{\mathrm{T}} \in \left[\underline{f}_{\mathrm{T}}/m, \quad \overline{f}_{\mathrm{T}}/m\right].$
- $u_{\mathrm{R}} \in [-\overline{\tau}/J_{\mathrm{I}}, \quad \overline{\tau}/J_{\mathrm{I}}].$

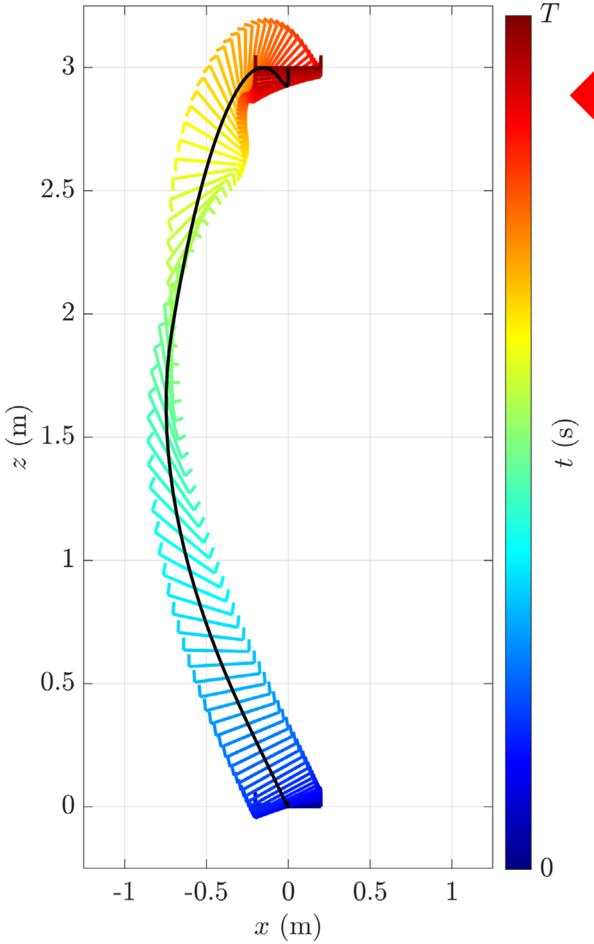
System Dynamics

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} \dot{x} \\ u_T \sin\theta \\ \dot{z} \\ u_T \cos\theta - g \\ \dot{\boldsymbol{\theta}} \\ u_R \end{bmatrix}$$

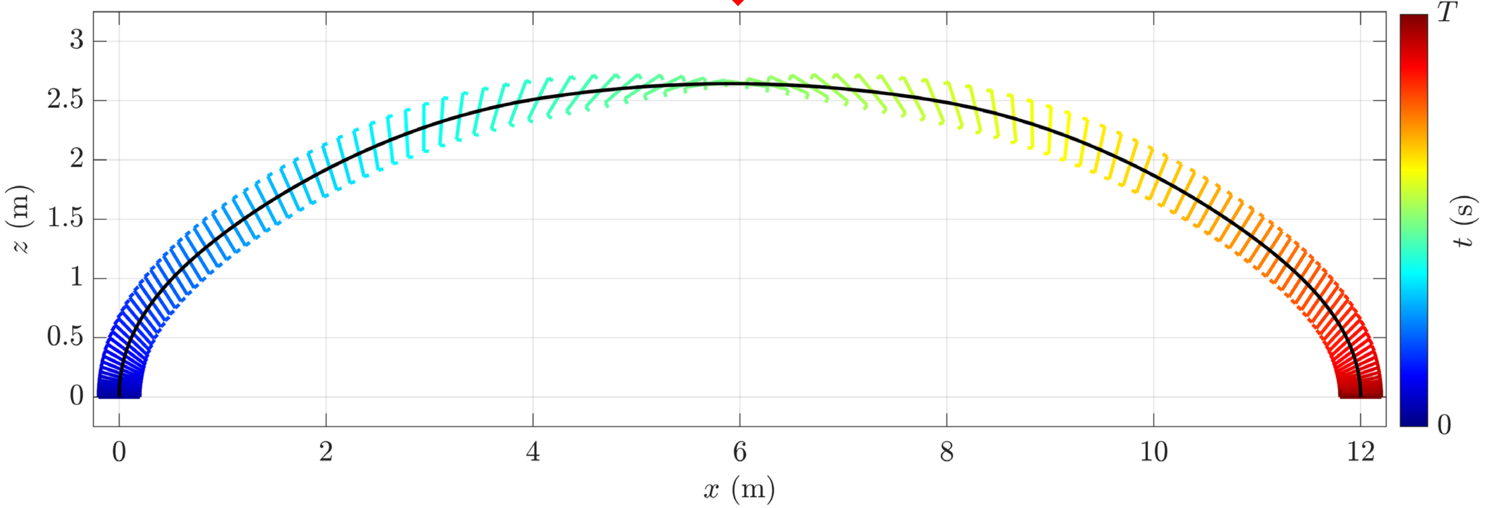
Test scenarios



with rotational dynamics

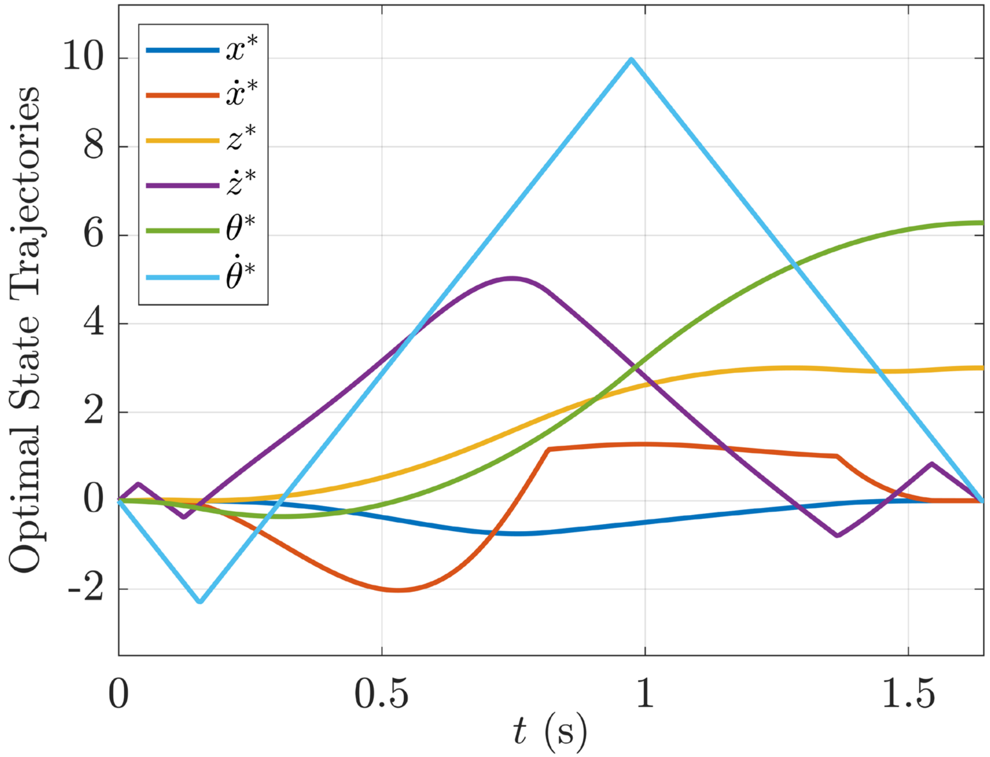


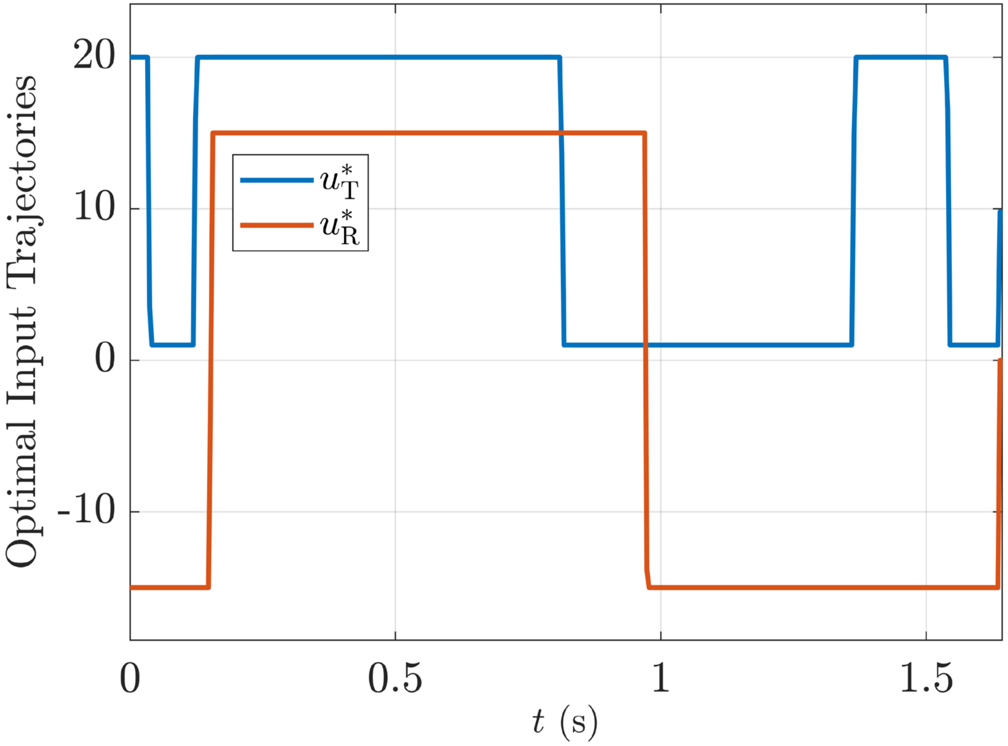
- Vertical displacement with flip maneuver ($x_T = 0$ m, $z_T = 3$ m, $\theta_T = 2\pi$ rad)
- Horizontal displacement with flip maneuver ($x_T = 12$ m, $z_T = 0$ m, $\theta_T = 2\pi$ rad)



Vertical Displacement with Flip

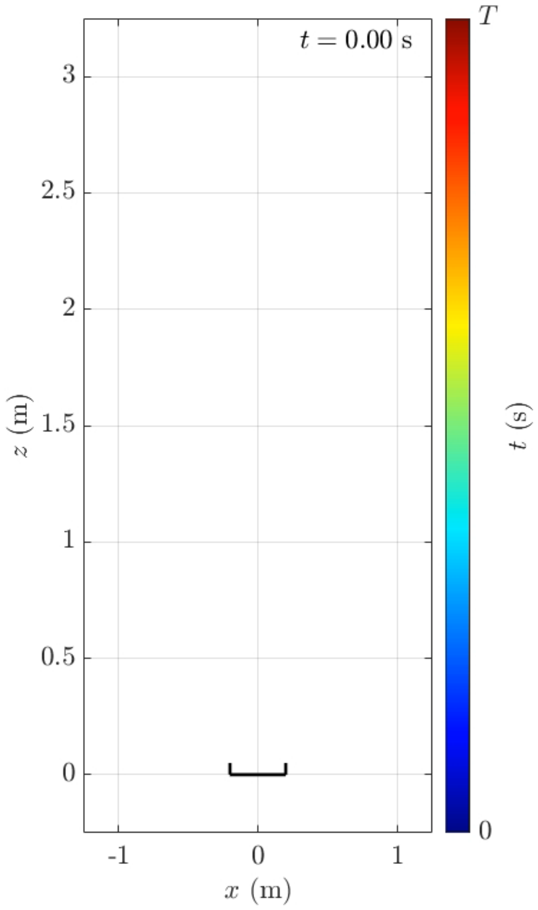
$x_T = 0$ m, $z_T = 3$ m, $\theta_T = 2\pi$ rad

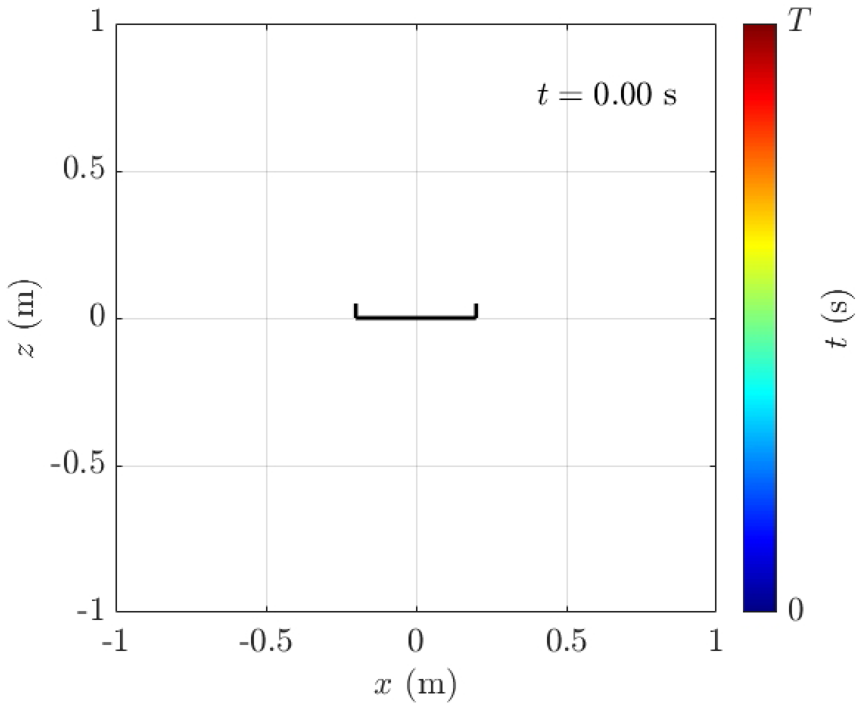




Vertical Displacement with Flip

$x_T = 0$ m, $z_T = 3$ m, $\theta_T = 2\pi$ rad

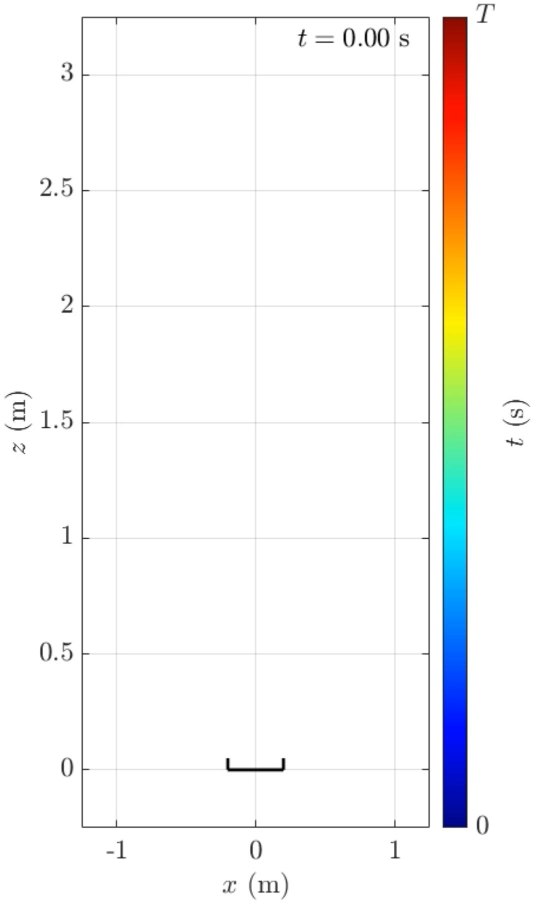


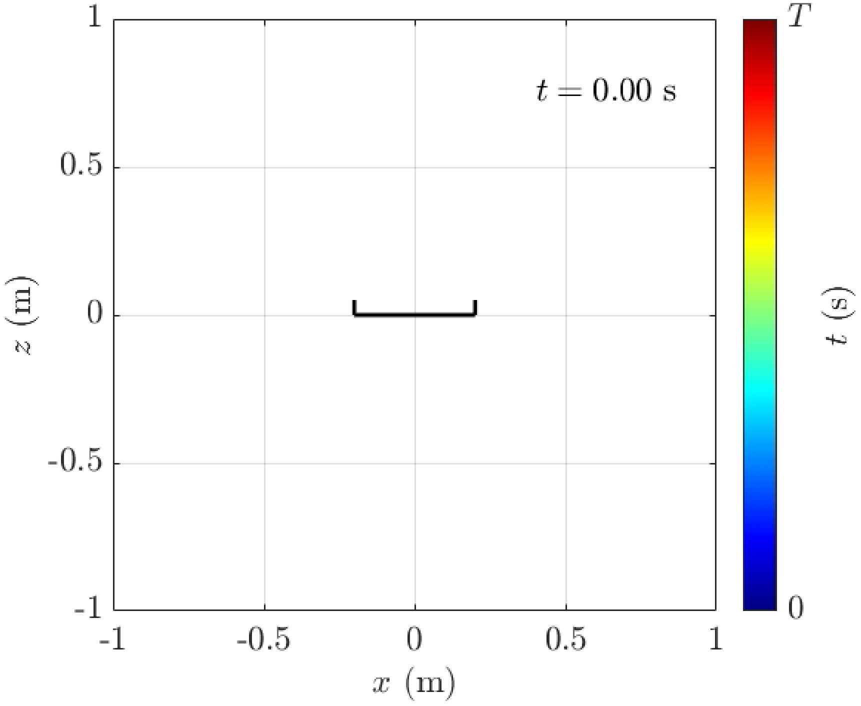


Playback speed: 1x

Vertical Displacement with Flip

$x_T = 0$ m, $z_T = 3$ m, $\theta_T = 2\pi$ rad

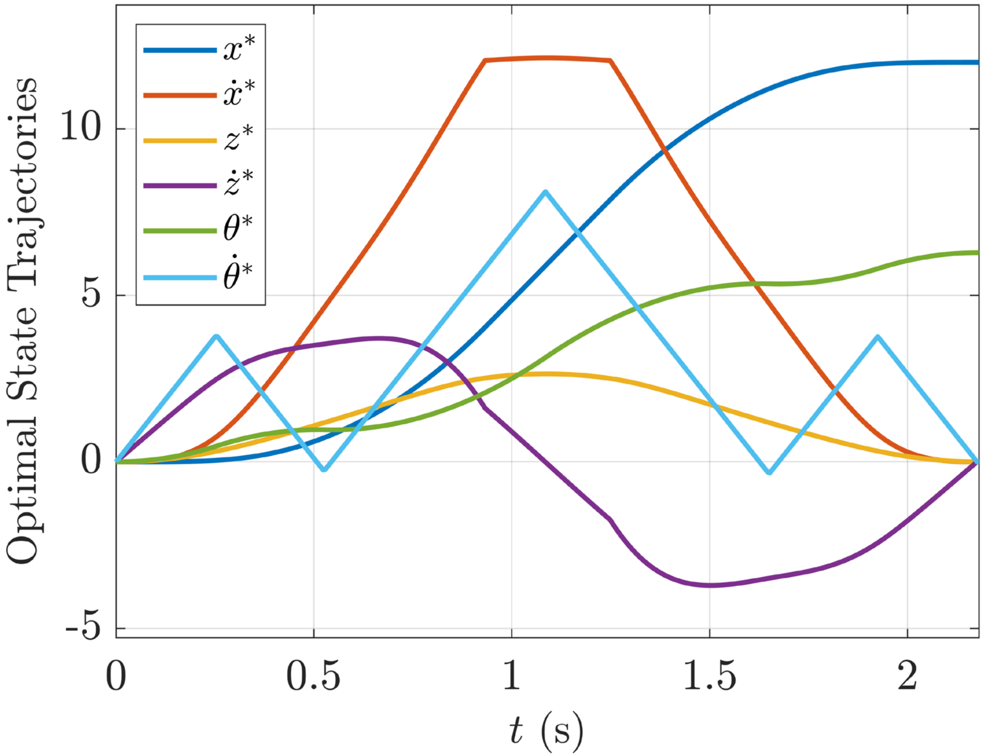


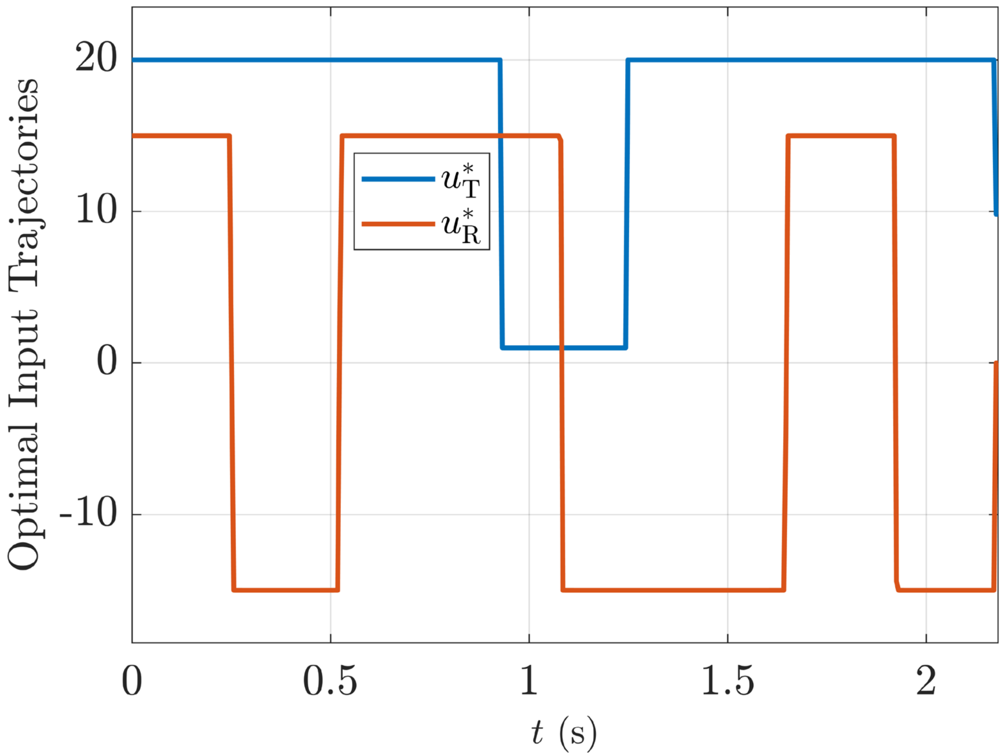


Playback speed: 0.2x

Horizontal Displacement with Flip

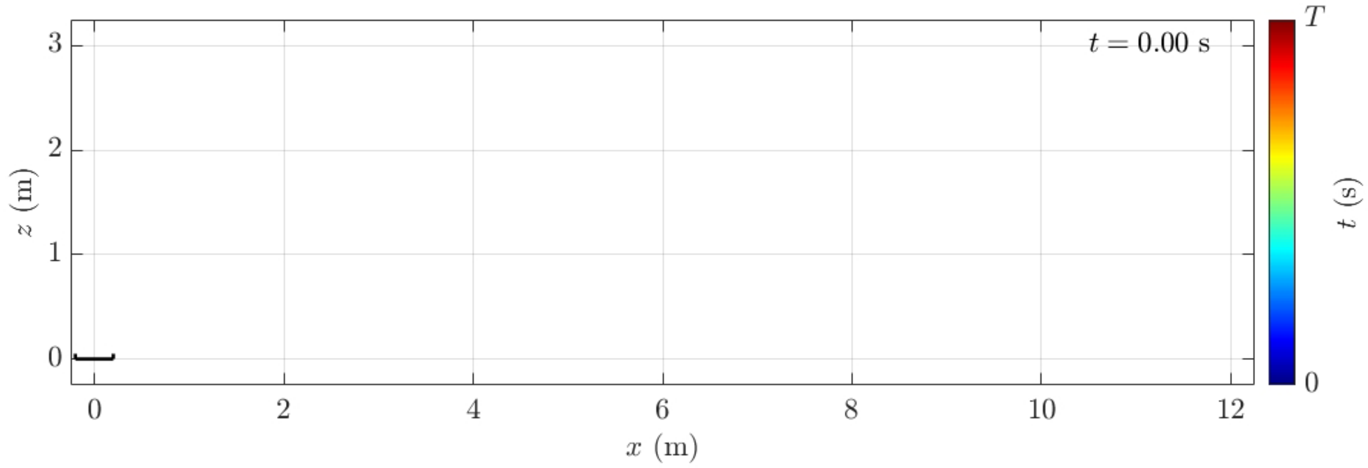
$x_T = 12$ m, $z_T = 0$ m, $\theta_T = 2\pi$ rad

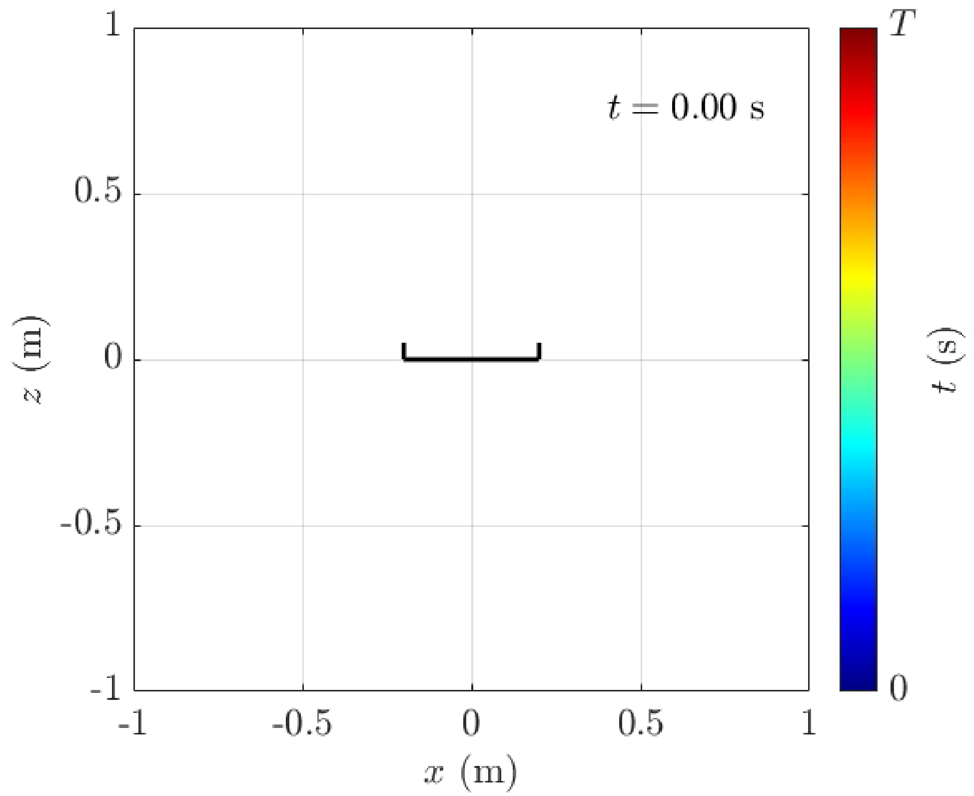




Horizontal Displacement with Flip

$x_T = 12$ m, $z_T = 0$ m, $\theta_T = 2\pi$ rad

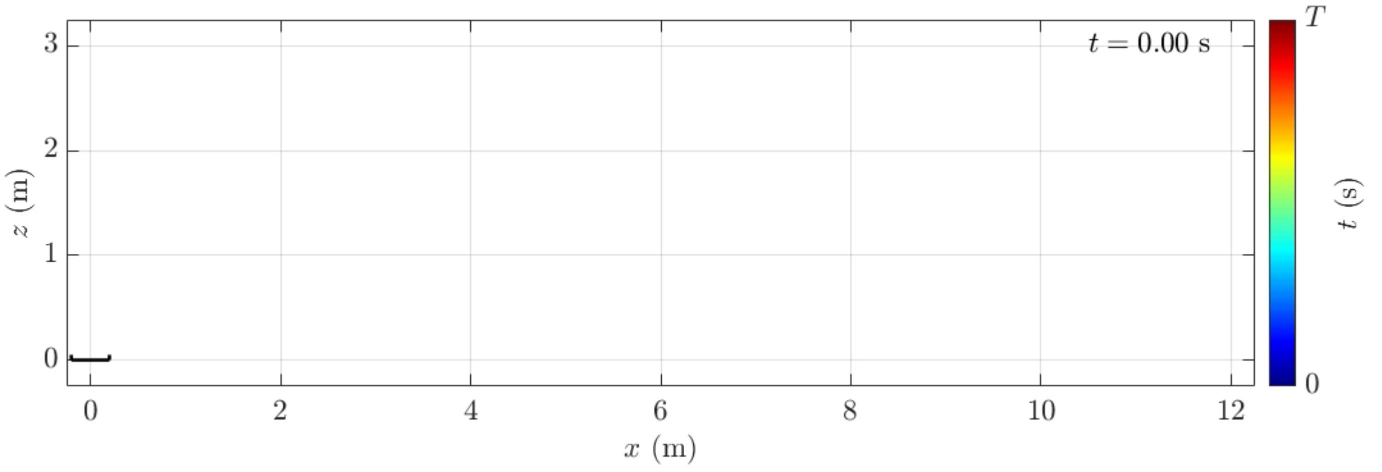


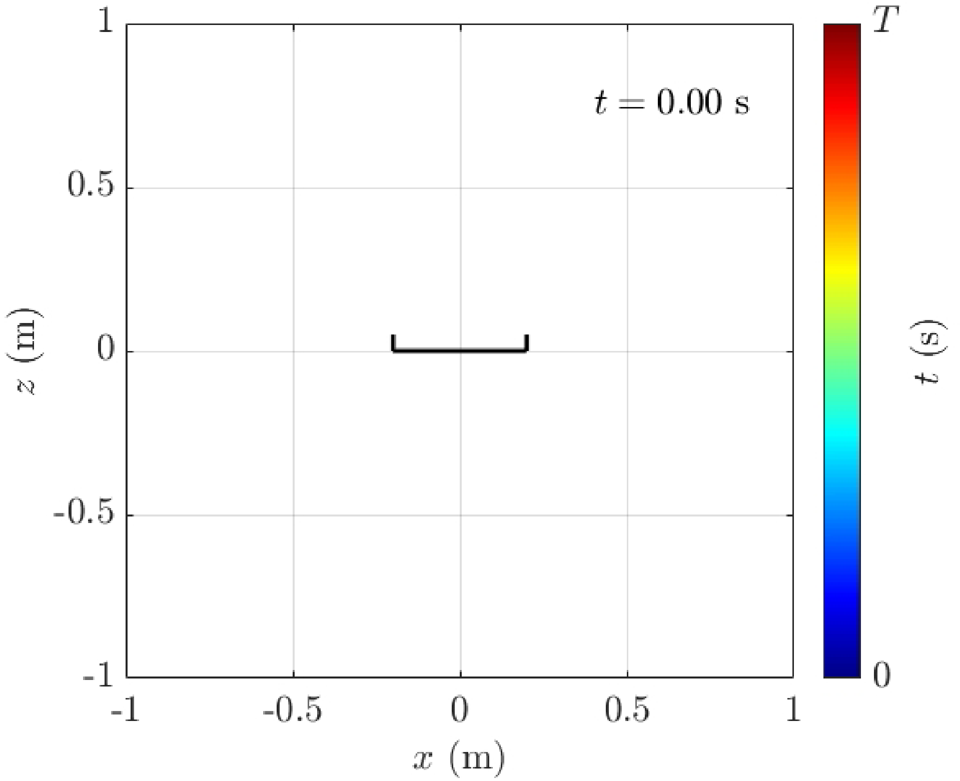


Playback speed: 1x

Horizontal Displacement with Flip

$x_T = 12$ m, $z_T = 0$ m, $\theta_T = 2\pi$ rad





Playback speed: 0.2x

For more information, go to

https://github.com/JAParedes/Trajectory_Optimization_for_Multicopter_UAV_Lateral_Flight_

Link in the description