Optimization and algorithms

Course overview

Team: João Xavier, João Sequeira, Hugo Pereira

Instituto Superior Técnico, Universidade de Lisboa

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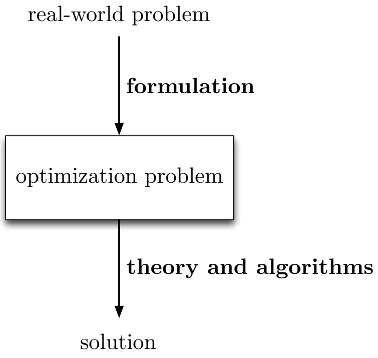
Outline

- What is the goal of optimization?
- What is the most important fact about optimization?
- Your first optimization problem: where to place a fire station?
- What is the plan for the lectures?
- How is your grade computed?

What is the goal of optimization?



Why study optimization?

Optimization is at the heart of numerous fields:

- communications
- control
- power systems
- computer vision
- machine learning
- finance
- networks
- data science
- ...

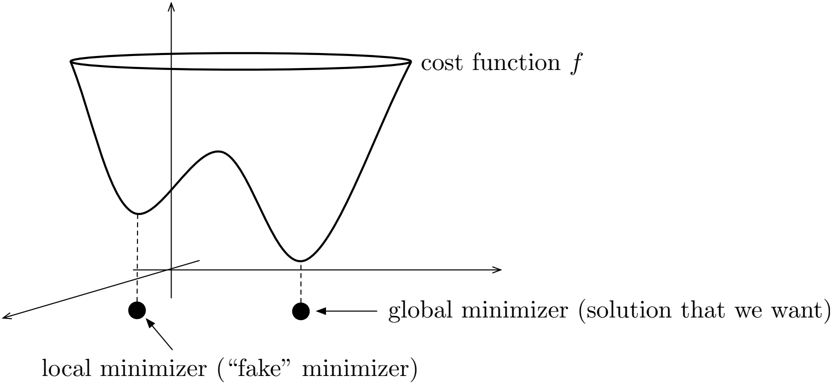
An optimization problem is a mathematical object of the following form:

$$
\begin{array}{lll}
\underset{x}{\text{minimize}} & f(x) & \\
\text{subject to} & h_1(x) & = 0 \\
 & & \vdots \\
 & h_p(x) & = 0 \\
 & g_1(x) & \leq 0 \\
 & & \vdots \\
 & g_m(x) & \leq 0
\end{array}
$$

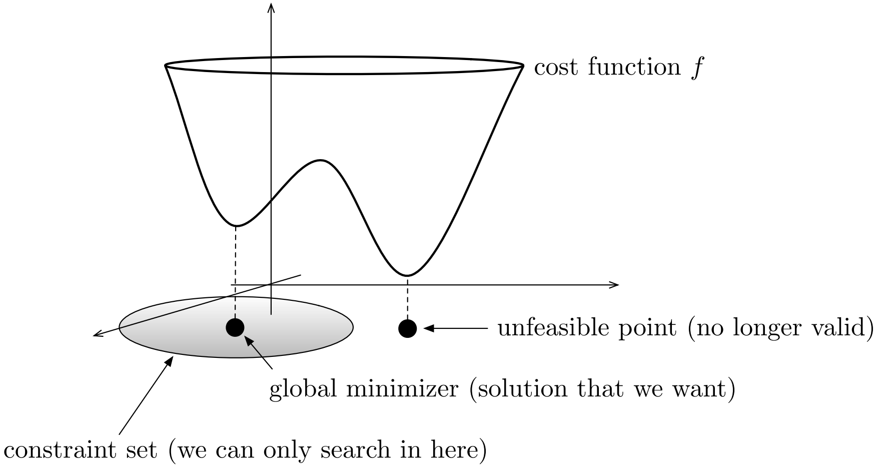
- $x \in \mathbf{R}^n$ is the optimization variable
- $f : \mathbf{R}^n \to \mathbf{R}$ is the objective or cost function that we want to minimize
- $h_1, \ldots, h_p, g_1, \ldots, g_m : \mathbf{R}^n \to \mathbf{R}$ are constraint functions

Solving an optimization problem means finding a global minimizer

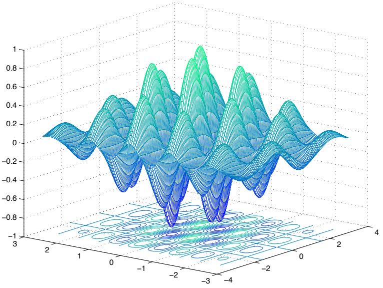
An unconstrained optimization problem:



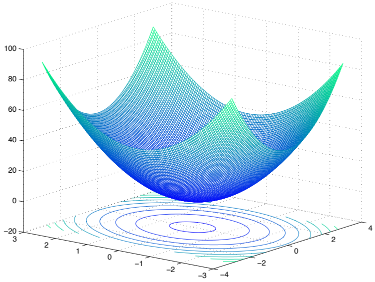
A constrained optimization problem:



What is the most important fact about optimization?



Nonconvex problem

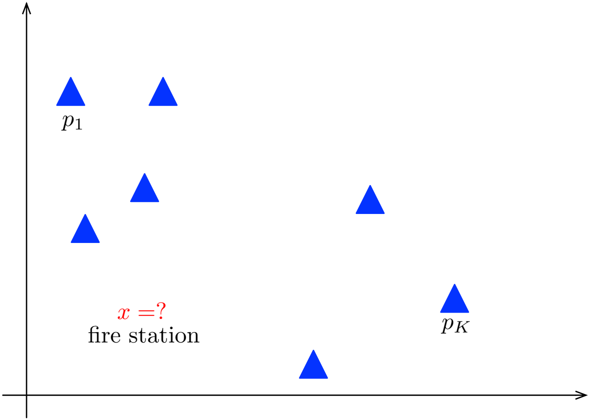


Convex problem

- Algorithms that solve typical nonconvex problems are very slow
- Algorithms that solve typical convex problems are **very fast**

Your first optimization problem: where to place a fire station?

- A fire station is going to serve K villages
- The K villages are located at given positions $p_1, p_2, \ldots, p_K \in \mathbf{R}^2$
- Where should you place the fire station?



- A possible problem formulation is as follows:

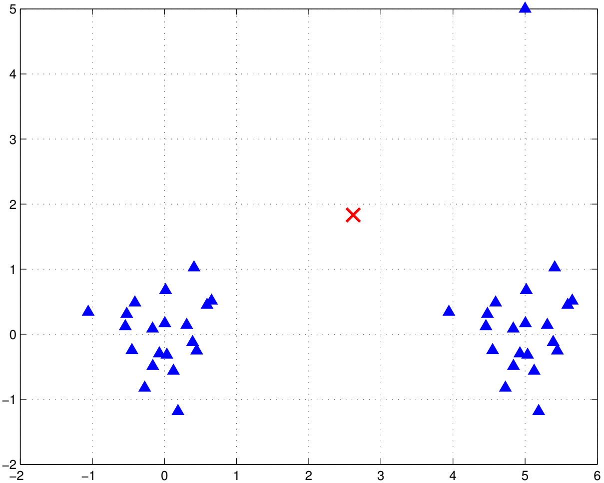
$$\underset{x}{\text{minimize}} \quad \underbrace{\max\left\{\|x - p_1\|, \|x - p_2\|, \ldots, \|x - p_K\|\right\}}_{f(x)}$$

with optimization variable $x \in \mathbf{R}^2$ (x is location of fire station)

- Is this a convex or a nonconvex problem?

```
% firestation.m; uses package CVX from http://cvxr.com/cvx
KK = 20; % choose K = number of villages
p1 = 0.5*randn(2,KK); % generate random positions
p2 = p1+[5 ; 0]*ones(1,KK);
p = [ p1 , p2 , [ 5 ; 5 ] ]; K = size(p,2);

% plot the villages
figure(1); clf;
plot(p(1,:),p(2,:),'^','MarkerSize',8,'MarkerFaceColor','b');
grid on;

% solve the optimization problem
cvx_begin quiet
    variable x(2,1);

    % build cost function
    f = norm(x - p(:,1));
    for i = 2:K
        f = max(f,norm(x-p(:,i)));
    end;

    minimize(f);
cvx_end;

%plot solution
hold on; plot(x(1),x(2),'rx','MarkerSize',15,'LineWidth',2);
```

- Here is a typical output:



The course consists of four modules:

- module 1: formulating optimization problems
- module 2: unconstrained optimization
  - basic theory
  - numerical algorithms
- module 3: convex functions
  - how to notice and build convex functions
- module 4: constrained optimization
  - basic theory
  - numerical algorithms

Plan for the theoretical lectures

number of lec	module
1	formulating optimization problems
2	formulating optimization problems
3	formulating optimization problems
4	unconstrained optimization
5	unconstrained optimization
6	unconstrained optimization
7	convex functions
8	convex functions
9	convex functions
10	constrained optimization
11	quiz (MAP45)
12	constrained optimization
13	constrained optimization
14	constrained optimization

Plan for the practice lectures

week	1st lec	2nd lec
1	no class	no class
2	exercises 1, 2, 3	exercises 5, 6, 7
3	project support	project support
4	exercises 12, 14, 19, 20	project support
5	exercises 31, 36, 38	exercises 32, 35, 37
6	exercises 39, 40, 42	exercises 48, 50, 51
7	project support	exercises 52, 55, 56

Grading

- Your grade is computed as follows:

$$\mathsf{G} = 25\%\mathsf{P} + \max\{50\%\mathsf{EX1} + 25\%\mathsf{MAP45}, 75\%\mathsf{EX1}, 75\%\mathsf{EX2}\}$$

  - G = final grade
  - P = project done (in MATLAB, Python, ...) in a group during the quarter. Minimum passing grade: 9.5
  - EX1 = closed-book exam in the normal period of evaluation (Nov 2024). Duration: 1h45m. Minimum passing grade: 8
  - MAP45 = closed-book quiz done in class the theoretical class on Monday, Oct 14 (week 6). Duration: 45min. Minimum passing grade: 0
  - EX2 = closed-book exam in the appeal period (Feb 2025). Duration: 1h30m. Minimum passing grade: 8

- For the special season of exams (Jul 2025), the grade is 100% from a closed-book exam with a duration of 2h

About the project

- The project is done in groups of up to 4 students
- The group must be formed until Sep 16, 2024 (start of week 2)
- The project consists in 10 tasks, some theoretical and some numerical
- Each groups delivers a report, which is a pdf file with the answers to the 10 tasks (for numerical tasks, the code MATLAB or Python should also be included)
- The deadline for submitting the report is Oct 25, 2024 (end of week 7)