Optimization and Algorithms
November 10, 2021

Write your name:	______________________________
Write your student number:	______________________________

Quiz

1. *A basic question.* (3 points) Task 1 of the project is about solving the problem

$$\begin{array}{ll} \underset{x,u}{\text{minimize}} & \sum_{t=1}^{T} \|Ex(t) - q(t)\| + \lambda \sum_{t=1}^{T-1} \|u(t)\|^2 \\ \text{subject to} & x(1) = x_{\text{initial}} \\ & x(t+1) = Ax(t) + Bu(t), \qquad \text{for } 1 \le t \le T-1. \end{array} \tag{1}$$

What does $Ex(t)$ represent?

(A) The velocity of the target at time t
(B) The position of our vehicle at time t
(C) The control signal at time t
(D) The velocity of our vehicle at time t
(E) The position of the target at time t
(F) The state of our vehicle at time t

Write your answer (A, B, C, D, E, or F) here: ____ B ____

2. *Levenberg-Marquardt.* (4 points) Suppose we use the Levenberg-Marquardt (LM) method to address the optimization problem

$$\underset{x \in \mathbf{R}^n}{\text{minimize}} \quad f(x), \tag{2}$$

where $f\colon \mathbf{R}^n \to \mathbf{R}$ is given by $f(x) = \sum_{p=1}^{P} (f_p(x))^2$. Assume that the function $f_p\colon \mathbf{R}^n \to \mathbf{R}$ is differentiable for $1 \le p \le P$.

Suppose the current iterate is x_k. To get the next iterate, the LM method starts by solving one of the following problems:

(A)

$$\underset{x \in \mathbf{R}^n}{\text{minimize}} \quad \sum_{p=1}^{P} f_p(x_k) + \nabla f_p(x_k)^T (x - x_k) + \lambda_k \|x - x_k\|^2$$

(B)

$$\underset{x \in \mathbf{R}^n}{\text{minimize}} \quad f(x_k) + \nabla f(x_k)^T (x - x_k) + \lambda_k \|x - x_k\|^2$$

(C)

$$\underset{x \in \mathbf{R}^n}{\text{minimize}} \quad \sum_{p=1}^{P} \left(f_p(x_k) + \nabla f_p(x_k)^T (x - x_k)\right)^2 + \lambda_k \|x - x_k\|^2$$

(D)

$$\underset{x \in \mathbf{R}^n}{\text{minimize}} \quad \sum_{p=1}^{P} \left(f_p(x_k) + \nabla f_p(x_k)^T (x - x_k) \right)^2 + \lambda_k \|x - x_k\|$$

(E)

$$\underset{x \in \mathbf{R}^n}{\text{minimize}} \quad \left(f(x_k) + \nabla f(x_k)^T (x - x_k) \right)^2 + \lambda_k \|x - x_k\|^2$$

(F)

$$\underset{x \in \mathbf{R}^n}{\text{minimize}} \quad \left(\sum_{p=1}^{P} f_p(x_k) + \nabla f_p(x_k)^T (x - x_k) \right)^2 + \lambda_k \|x - x_k\|^2$$

Which problem?

Write your answer (A, B, C, D, E, or F) here: _____ C _____

3. *Gradient.* (3 points) Consider the function $f\colon \mathbf{R}^3 \to \mathbf{R}$,

$$f(x_1, x_2, x_3) = \left(\left\| \begin{bmatrix} x_2 \\ x_3 \\ x_1 \end{bmatrix} \right\| + 1 \right)^2 .$$

The gradient $\nabla f\,(4, 0, 3)$ is one of the following vectors:

(A)

$$\begin{bmatrix} 0 \\ 36/5 \\ 48/5 \end{bmatrix}$$

(B)

$$\begin{bmatrix} 0 \\ 9 \\ 8 \end{bmatrix}$$

(C)

$$\begin{bmatrix} 48 \\ 0 \\ 36 \end{bmatrix}$$

(D)

$$\begin{bmatrix} 0 \\ 36 \\ 48 \end{bmatrix}$$

(E)

$$\begin{bmatrix} 9 \\ 0 \\ 8 \end{bmatrix}$$

(F)

$$\begin{bmatrix} 48/5 \\ 0 \\ 36/5 \end{bmatrix}$$

Which one?

Write your answer (A, B, C, D, E, or F) here: ______ F ______

4. *Random target.* (3 points) Task 5 of the project is about analyzing an optimization problem of the form

$$\begin{array}{ll} \underset{x_1,u_1,x_2,u_2}{\text{minimize}} & \underbrace{\sum_{k=1}^{K} p_k \left(\sum_{t=1}^{T} \|Ex_k(t) - q_k(t)\| + \lambda \sum_{t=1}^{T-1} \|u_k(t)\|^2 \right)}_{f(x_1,u_1,x_2,u_2)} \\ \text{subject to} & x_1(1) = x_{\text{initial}} \\ & x_1(t+1) = Ax_1(t) + Bu_1(t) \quad \text{for } 1 \le t \le T-1 \\ & x_2(1) = x_{\text{initial}} \\ & x_2(t+1) = Ax_2(t) + Bu_2(t) \quad \text{for } 1 \le t \le T-1. \end{array} \tag{3}$$

Recall that A, B, and x_{initial} are given constants. Also, $\lambda > 0$, $p_1 > 0$, $p_2 > 0$, and $p_1 + p_2 = 1$.

One of the following statements about problem (3) is true:

(A) The function f is strongly convex

(B) The function f is a quadratic

(C) The solution of the optimization problem (3) when $p_1 = 0.7$, $p_2 = 0.3$, and $\lambda = 1$ is the same as the solution of the optimization problem (3) when $p_1 = 0.7$, $p_2 = 0.3$, and $\lambda = 10$

(D) The solution of the optimization problem (3) when $p_1 = 0.7$, $p_2 = 0.3$, and $\lambda = 1$ is the same as the solution of the optimization problem (3) when $p_1 = 0.2$, $p_2 = 0.8$, and $\lambda = 1$

(E) We have $x_1^\star(t) = x_2^\star(t)$ and $u_1^\star(t) = u_2^\star(t)$ for $t = 1, 2, \ldots, 24$, where the symbol $(x_1^\star, u_1^\star, x_2^\star, u_2^\star)$ denotes the solution of optimization problem (3)

(F) The function f is not convex

Which one?

Write your answer (A, B, C, D, E, or F) here: ______ D ______

5. *Convexity.* (4 points) Consider the optimization problem

$$\underset{x \in \mathbf{R}^n}{\text{minimize}} \quad \|Ax - b\| + f(x), \tag{4}$$

where $A \in \mathbf{R}^{m\times n}$ and $b \in \mathbf{R}^n$. Consider the following choices for the form of function f:

(A) $f(x) = \|x - c\|$

(B) $f(x) = (c^T x)^2$

(C) $f(x) = e^{c^T x} + \|c\|^2$

(D) $f(x) = c^T x$

(E) $f(x) = (x - c)^T x$

(F) $f(x) = \|x\| + c^T x$.

For one of the six forms of f above, the optimization problem (4) is guaranteed to have a unique global minimizer, regardless of how the constants A, b, and c are chosen.

For which form?

Write your answer (A, B, C, D, E, or F) here: _____ E _____

6. *Trade-off.* (3 points) Consider the optimization problem

$$\underset{x \in \mathbf{R}^n}{\text{minimize}} \quad f(x) + \rho\, g(x), \tag{5}$$

where f and g are nonnegative functions $\mathbf{R}^n \to \mathbf{R}$ (that is, $f(x) \geq 0$ and $g(x) \geq 0$ for all $x \in \mathbf{R}^n$), and ρ is a positive number.

Suppose that x_1 is a global minimizer for (5) when $\rho = \rho_1$, and suppose that x_2 is a global minimizer for (5) when $\rho = \rho_2$. Consider that $\rho_2 > \rho_1 > 0$.

One of the following inequalities is guaranteed to be true:

(A) $f(x_1) + \rho_1 g(x_1) > f(x_2) + \rho_2 g(x_2)$

(B) $f(x_2) > f(x_1)$

(C) $g(x_2) \geq g(x_1)$

(D) $g(x_2) > g(x_1)$

(E) $f(x_1) + \rho_1 g(x_1) < f(x_2) + \rho_2 g(x_2)$

(F) $f(x_2) \geq f(x_1)$

Which inequality?

Write your answer (A, B, C, D, E, or F) here: _____ F _____