Instituto Superior Técnico - 1^o Semestre 2006/2007

Cálculo Diferencial e Integral I

LEA-pB, LEM-pB, LEN-pB, LEAN, MEAer e MEMec

5^a Ficha de exercícios para as aulas práticas: 23 Outubro - 3 Novembro de 2006

1. Estude quanto à natureza (convergência absoluta, convergência simples, divergência) cada uma das seguintes séries.

(1) $\sum_{n=1}^{+\infty} \frac{\sqrt{n-1}}{n^2+2}$ **(2)** $\sum_{n=1}^{+\infty} \frac{\sqrt{n+1}+\sqrt{n}}{n+1}$ **(3)** $\sum_{n=1}^{+\infty} \frac{n^2}{n^3+3}$ **(4)** $\sum_{n=1}^{+\infty} \frac{n}{n^3+3}$ **(5)** $\sum_{n=1}^{+\infty} \frac{2(-1)^{n+1}}{1+\log n}$

(6) $\sum_{n=1}^{+\infty} \frac{\sqrt[3]{3n+2}}{\sqrt{(n^2+1)(n+1)}}$ **(7)** $\sum_{n=1}^{+\infty} (-1)^n \frac{5+2(-1)^{n+1}}{\sqrt{n}}$ **(8)** $\sum_{n=1}^{+\infty} \frac{-2+(-1)^n}{n^3}$ **(9)** $\sum_{n=1}^{+\infty} \frac{1+(-1)^n}{2n}$

(10) $\sum_{n=1}^{+\infty} (-1)^n \left(1-\cos\frac{1}{n}\right)$ **(11)** $\sum_{n=1}^{+\infty} \frac{n+2^n}{n2^n}$ **(12)** $\sum_{n=1}^{+\infty} \operatorname{sen}\frac{n\pi}{2} \log\left(1+\frac{1}{n}\right)^5$ **(13)** $\sum_{n=1}^{+\infty} \frac{\sqrt[3]{n}}{\sqrt{n}+1}$

(14) $\sum_{n=1}^{+\infty} \frac{\operatorname{arctg}(-1)^n}{n!}$ **(15)** $\sum_{n=1}^{+\infty} \frac{\operatorname{arctg}((-1)^n)}{\sqrt{n(n+1)}}$ **(16)** $\sum_{n=3}^{+\infty} \frac{\operatorname{tg}(\pi/n)}{n}$ **(17)** $\sum_{n=1}^{+\infty} \operatorname{tg}\frac{1}{n+1}$

(18) $\sum_{n=1}^{+\infty} \operatorname{sen}\frac{1}{\sqrt{n^3}}$ **(19)** $\sum_{n=2}^{+\infty} \frac{1}{(-1)^n \log n}$ **(20)** $\sum_{n=1}^{+\infty} \log\frac{1}{n}$ **(21)** $\sum_{n=2}^{+\infty} \frac{1+\operatorname{sen}^2 n}{\log n}$ **(22)** $\sum_{n=2}^{+\infty} \frac{1}{\log^2 n}$

(23) $\sum_{n=1}^{+\infty} \frac{\log n}{n}$ **(24)** $\sum_{n=1}^{+\infty} \frac{\log n}{n^2}$ **(26)** $\sum_{n=1}^{+\infty} n^{-1-1/n}$ **(27)** $\sum_{n=1}^{+\infty} \log(1+e^{-n})$

(28) $\sum_{n=1}^{+\infty} \sqrt{\frac{n+1}{n^3+\sqrt{n}}}$ **(29)** $\sum_{n=1}^{+\infty} (-1)^n \frac{n\log n}{2+\sqrt{n^5}\log n}$ **(30)** $\sum_{n=1}^{+\infty} \frac{(-1)^{n+1}\sqrt[3]{n}+\cos n^3}{\sqrt{n^3}+2}$

(31) $\sum_{n=1}^{+\infty} \frac{\sqrt[4]{n}+\operatorname{sen} n!}{\sqrt{n^3}+n}$ **(32)** $\sum_{n=1}^{+\infty} \frac{\sqrt[3]{n^2+\log n}}{\sqrt{n^3}+1}$ **(33)** $\sum_{n=1}^{+\infty} \frac{\sqrt{2n^2-n}+\sqrt[3]{n^2(2+3n)}}{\sqrt{n^3}+\sqrt[3]{(n^2+1)n^3}}$

(34) $\sum_{n=0}^{+\infty} \frac{(-1)^n(2n^2-1)n}{\left(n^2\sqrt{n}+1\right)^2+4}$ **(35)** $\sum_{n=1}^{+\infty} \frac{\left(\sqrt{n}+1\right)^2}{(5n+1)\sqrt{n}+2}$ **(36)** $\sum_{n=0}^{+\infty} \left(\sqrt{n+1}-\sqrt{n}\right)^3$

(37) $\sum_{n=1}^{+\infty} \frac{(-1)^n n}{(n^3+1)\operatorname{arctg} n}$ **(38)** $\sum_{n=1}^{+\infty} \left(\operatorname{sen}\frac{1}{n^2}\right)(\sqrt{n}+1)$ **(39)** $\sum_{n=1}^{+\infty} \frac{\operatorname{sen}(n\pi/2)}{1+2\sqrt{n^3}}$

(40) $\sum_{n=1}^{+\infty} \frac{1}{[(-1)^n+10]^{2n}}$ **(41)** $\sum_{n=1}^{+\infty} \frac{n^3+1000}{\log 2^n+n^4}$ **(42)** $\sum_{n=2}^{+\infty} \frac{\sqrt{n^3+2n^2+3}}{\sqrt[3]{n-1}}$ **(43)** $\sum_{n=1}^{+\infty} (-1)^n \operatorname{sen}\frac{1}{n}$

(44) $\sum_{n=1}^{+\infty} \frac{\sqrt{n}-\sqrt{n-1}}{n}$ **(45)** $\sum_{n=1}^{+\infty} \frac{n^{n+1}}{(\sqrt{n}+1)n(-n)^n}$ **(46)** $\sum_{n=1}^{+\infty} \frac{2\cos[(n-1)\pi]}{1+2\sqrt{n}}$

(47) $\sum_{n=1}^{+\infty} \frac{\cos^2 n+\sqrt{n}e^n}{\sqrt{n^5}e^n+(n+1)^3}\operatorname{sen} n$ **(48)** $\sum_{n=0}^{+\infty} (-1)^n\left(\sqrt{1+n^2}-n\right)$ **(49)** $\sum_{n=1}^{+\infty} \frac{1}{n}\frac{3^{-n}}{1-3^{-n}}$

(50) $\sum_{n=1}^{+\infty} \frac{1}{n}\frac{2^n}{1-2^n}$ **(51)** $\sum_{n=1}^{+\infty} \frac{2+\operatorname{arctg}(n!)}{n^2+\log^2 n+2}$ **(52)** $\sum_{n=1}^{+\infty} (-1)^n \frac{n^n}{(n+1)^{n+1}}$ **(53)** $\sum_{n=1}^{+\infty} \frac{n^n}{(n+1)^{n+1}}$

(54) $\sum_{n=1}^{+\infty} \frac{(n+1)^n}{n^{n+1}}$ **(55)** $\sum_{n=2}^{+\infty} \frac{1}{\sqrt{n}\log n}$ **(56)** $\sum_{n=2}^{+\infty} \frac{1}{n^2 \log n}$ **(57)** $\sum_{n=1}^{+\infty} \frac{(-1)^n n}{n^2+1}$ **(58)** $\sum_{n=1}^{+\infty} e^{-\log n}$

(59) $\sum_{n=2}^{+\infty} (\log n)^{-p}$, $(p \in \mathbb{R})$ **(60)** $\sum_{n=1}^{+\infty} \frac{1}{n^2 \operatorname{sen} \frac{1}{n}}$ **(61)** $\sum_{n=1}^{+\infty} \frac{3^{n+3}+n!}{(n+2)!+n^{n+2}+4^{n+1}+\log n}$

(62) $\sum_{n=1}^{+\infty} \frac{\operatorname{sen}^3(n!n)+e^{-n}}{2n!n^2+(\sqrt[3]{n}+1)\sqrt{n}}$ **(63)** $\sum_{n=1}^{+\infty} \frac{2^n+(-n)^n}{n^{n+2}+n!n}$ **(64)** $\sum_{n=1}^{+\infty} \frac{n-(-3)^n}{3^n(n+1)}$

(65) $\sum_{n=1}^{+\infty} \frac{1+\sqrt{n}+(-1)^n n}{(n+1)\sqrt{n}}$

2. Estude quanto à natureza (convergência absoluta, convergência simples, divergência) cada uma das seguintes séries.

(1) $\sum_{n=1}^{+\infty} \frac{n^{1000}}{(1,001)^n}$ **(2)** $\sum_{n=1}^{+\infty} \frac{1000^n}{n!}$ **(3)** $\sum_{n=1}^{+\infty} \frac{e^n n^3}{n!2^n}$ **(4)** $\sum_{n=1}^{+\infty} \frac{n!(n+2)!}{(3.5.7.9.\cdots.(2n+1))^2}$

(5) $\sum_{n=1}^{+\infty} n^3(-e)^{-n}$ **(6)** $\sum_{n=1}^{+\infty} \frac{(-\pi)^{-n}}{n}$ **(7)** $\sum_{n=1}^{+\infty} n^2 2^n e^{-n}$ **(8)** $\sum_{n=1}^{+\infty} n! e^{-n}$ **(9)** $\sum_{n=1}^{+\infty} \frac{n!}{n^n}$

(10) $\sum_{n=1}^{+\infty} \frac{2^n n!}{n^n}$ **(11)** $\sum_{n=1}^{+\infty} \frac{3^n n!}{n^n}$ **(12)** $\sum_{n=0}^{+\infty} \frac{1.3.5.\cdots.(2n+1)}{3.6.9.\cdots.(3n+3)}$ **(13)** $\sum_{n=1}^{+\infty} \frac{n!}{2^{n^2}}$

(14) $\sum_{n=1}^{+\infty} \frac{(-1)^n (n!)^2}{2^n n^n}$ **(15)** $\sum_{n=1}^{+\infty} \frac{3^n+n!}{n!+n^n}$ **(16)** $\sum_{n=1}^{+\infty} (-1)^{n-1} \frac{1-2\sqrt{n}}{2^n+n^2}$ **(17)** $\sum_{n=1}^{+\infty} \frac{2^n n^n}{(7n+1)^n}$

(18) $\sum_{n=1}^{+\infty} \frac{(4+(-1)^n)^n n}{6^n}$ **(19)** $\sum_{n=1}^{+\infty} (-1)^n \frac{3^n+n^3}{n!+1}$ **(20)** $\sum_{n=3}^{+\infty} \frac{5.7.9...(2n+3)}{5^n}$ **(21)** $\sum_{n=0}^{+\infty} \frac{2^n}{1+3^{n+1}}$

(22) $\sum_{n=1}^{+\infty} \frac{(2n)!}{(2n)^n}$ **(23)** $\sum_{n=0}^{+\infty} \frac{(5n)!}{(3n)!(2n)!}$ **(24)** $\sum_{n=1}^{+\infty} \frac{2^n (2n)!}{3^n (2n+1)!}$ **(25)** $\sum_{n=1}^{+\infty} e^{-n} \log n$

(26) $\sum_{n=0}^{+\infty} \frac{(n+1)^n}{n!(-3)^n}$ **(27)** $\sum_{n=1}^{+\infty} \frac{3^n \cos n}{n!}$ **(28)** $\sum_{n=1}^{+\infty} \frac{(-1)^{n+1} n \operatorname{arctg}(n^3)}{(n+1)!+\sqrt{n}}$ **(29)** $\sum_{n=1}^{+\infty} \frac{2^n+n^3}{2^{n+1}(n+1)^3}$

(30) $\sum_{n=1}^{+\infty} \frac{e^n n!}{n^n}$ **(31)** $\sum_{n=1}^{+\infty} \frac{(-3)^n+n^5}{2^n+n!+\log^2(n!)}$ **(32)** $\sum_{n=1}^{+\infty} \operatorname{sen} \frac{n^3}{3^n}$ **(33)** $\sum_{n=1}^{+\infty} n^2 \operatorname{sen} \frac{\pi}{2^n}$

(34) $\sum_{n=1}^{+\infty} \frac{e^{\frac{n!}{n^n}}-1}{\frac{3^n}{(n!)^2}}$ **(35)** $\sum_{n=2}^{+\infty} \frac{2^n}{n!-\operatorname{sen} n}$ **(36)** $\sum_{n=1}^{+\infty} \frac{n^2}{n!+\sqrt[3]{n^2}}$ **(37)** $\sum_{n=1}^{+\infty} \frac{n^2}{n!+\log n^2}$

(38) $\sum_{n=1}^{+\infty} \frac{1}{1+a^n}$, $(a \in \mathbb{R}^+)$ **(39)** $\sum_{n=1}^{+\infty} \frac{n}{(-2)^n+n^2}$

3. Estude quanto à natureza (convergência absoluta, convergência simples, divergência) cada uma das seguintes séries.

(1) $\sum_{n=1}^{+\infty} \left(1-\frac{1}{n}\right)^{n^2}$ **(2)** $\sum_{n=1}^{+\infty} \left(1+\frac{1}{n^2}\right)^{n^3}$ **(3)** $\sum_{n=1}^{+\infty} \left(1+\frac{1}{n^3}\right)^{n^2}$ **(4)** $\sum_{n=1}^{+\infty} \left(\frac{n+3}{n+1}\right)^{n^2}$

(5) $\sum_{n=1}^{+\infty} e^{-n^2}$ **(6)** $\sum_{n=1}^{+\infty} \frac{1}{\sqrt{n^3}} \left(1+\frac{1}{n}\right)^n$ **(7)** $\sum_{n=2}^{+\infty} \frac{1}{n(\log n)^n}$ **(8)** $\sum_{n=1}^{+\infty} \left(n^{1/n}-1\right)^n$

(9) $\sum_{n=1}^{+\infty}\left(\operatorname{arctg}\frac{1}{n}\right)^n$ **(10)** $\sum_{n=1}^{+\infty}\left(\frac{2n-1}{3n+1}\right)^{2n}$ **(11)** $\sum_{n=0}^{+\infty}\left(\frac{n+5}{n^2+1}\right)^n$ **(12)** $\sum_{n=1}^{+\infty}\left(\frac{n}{n+1}\right)^{n^2}$

(13) $\sum_{n=1}^{+\infty}\left(\frac{1}{n}-e^{-n^2}\right)$ **(14)** $\sum_{n=1}^{+\infty}\frac{\sqrt[3]{n^n}}{\sqrt{n^n}}$ **(15)** $\sum_{n=1}^{+\infty}\left(\frac{n^2}{2+n^2}\right)^{2^n}$ **(16)** $\sum_{n=1}^{+\infty}\left(\frac{n^2}{2-n^2}\right)^{2^n}$

(17) $\sum_{n=1}^{+\infty}\left(\frac{n!+n^4}{3^n+n!}\right)^{n!}$ **(18)** $\sum_{n=1}^{+\infty}(\log n)^{-n}$ **(19)** $\sum_{n=1}^{+\infty}\left(1+\frac{1}{n}\right)^{-n^2}$

4. Determine o raio de convergência de cada uma das seguintes séries de potências de $x-\alpha$ (para um certo $\alpha\in\mathbb{R}$) e estude-as quanto à natureza (convergência absoluta, convergência simples, divergência).

(1) $\sum_{n=0}^{+\infty}x^n$ **(2)** $\sum_{n=1}^{+\infty}\left(\frac{1}{2^n}+\frac{1}{n+1}\right)x^n$ **(3)** $\sum_{n=0}^{+\infty}\frac{2n+1}{n^2+1}(x+1)^{2n}$ **(4)** $\sum_{n=0}^{+\infty}\left(\frac{3^n}{n!}+\frac{4^n}{(n+1)!}\right)(x-1)^n$

(5) $\sum_{n=1}^{+\infty}\frac{\log n}{n^2}x^n$ **(6)** $\sum_{n=0}^{+\infty}\cos(n\pi)\frac{(x-1)^{2n+1}}{2^{n+1}\sqrt{2n+1}}$ **(7)** $\sum_{n=1}^{+\infty}n^{-\sqrt{n}}x^{2n}$ **(8)** $\sum_{n=1}^{+\infty}\frac{(x+1)^n}{n\operatorname{arctg}n}$

(9) $\sum_{n=0}^{+\infty}\frac{e^n+1}{n!}x^n$ **(10)** $\sum_{n=0}^{+\infty}\frac{n!\,(x-1)^n}{n!+4^n}$ **(11)** $\sum_{n=1}^{+\infty}\frac{(x+2)^n}{n^n}$ **(12)** $\sum_{n=1}^{+\infty}n^n(x+2)^n$

5. Estude quanto à natureza (convergência absoluta, convergência simples, divergência) cada uma das seguintes séries, onde x designa um parâmetro real.

(1) $\sum_{n=2}^{+\infty}\frac{n-\sqrt{n}}{2^n+1}(2-x)^n$ **(2)** $\sum_{n=1}^{+\infty}(-1)^n\frac{(2x-1)^n}{(2n-1)(2n+1)}$ **(3)** $\sum_{n=2}^{+\infty}\frac{(2x+3)^n}{2^n(n^2-n)}$

(4) $\sum_{n=1}^{+\infty}\frac{(-2)^n}{1+n}(1-2x)^{2n-1}$ **(5)** $\sum_{n=1}^{+\infty}\left(\frac{1}{1+|x|}\right)^{n-1}$ **(6)** $\sum_{n=0}^{+\infty}\frac{2^n(n!)^2}{(2n)!}(x^2-x)^n$

(7) $\sum_{n=0}^{+\infty}(1-|x|)^n$ **(8)** $\sum_{n=1}^{+\infty}\left(\frac{1}{1+|x|^n}\right)^{n-1}$ **(9)** $\sum_{n=0}^{+\infty}x$ **(10)** $\sum_{n=0}^{+\infty}\frac{(nx)^n}{(n+1)^n}$ **(11)** $\sum_{n=0}^{+\infty}\left(\frac{nx}{x+1}\right)^n$