CÁLCULO DIFERENCIAL E INTEGRAL I LEIC-TAGUS, LERCI, LEGI E LEE – 1º SEM. 2006/07

1ª FICHA DE EXERCÍCIOS

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1. Mostre que:
  1.1. \{x \in \mathbb{R} : |x+2| = 3\} = \{-5, 1\}
  1.2. \{x \in \mathbb{R} : |x+2| \le 1\} = [-3, -1]
  1.3. \{x \in \mathbb{R} : |3 - x| > 2\} = ]-\infty, 1[\cup ]5, +\infty[
  1.4. \{x \in \mathbb{R} : 2 < |x| < 3\} = ]-3, -2[\cup]2, 3[
  1.5. \{x \in \mathbb{R} : 3 < 2|x-1| \le 5\} = \left[-\frac{3}{2}, -\frac{1}{2}\right] \cup \left[\frac{5}{2}, \frac{7}{2}\right]
  1.6. \{x \in \mathbb{R} : |x-3| > 2 \land x \ge 0\} = [0, \tilde{1}[\cup]5, +\infty[
  1.7. \{x \in \mathbb{R} : |x+2| \le 3 \land x+1 > 0\} = [-1,1]
  1.8. \{x \in \mathbb{R} : |3 - 4x| < 1\} = \lfloor \frac{1}{2}, 1 \rfloor
  1.9. \{x \in \mathbb{R} : |4x+3| > 1\} = ]-\infty, -1[\cup]-\frac{1}{2}, +\infty[
1.10. \{x \in \mathbb{R} : |5-3x| \leq 2\} = |1, \frac{7}{3}|
1.11. \{x \in \mathbb{R} : |5+3x| \ge 2\} = \left] -\infty, -\frac{7}{3} \right] \cup [-1, +\infty[
1.12. \{x \in \mathbb{R} : |4x+1| > 5\} = \left] -\infty, -\frac{3}{2} \right[ \cup ]1, +\infty[
1.13. \{x \in \mathbb{R} : |1 - 4x| < 5\} = [-1, \frac{3}{2}]
1.14. \{x \in \mathbb{R} : |5x + 2| \ge 3\} = ]-\infty, -1] \cup \left[\frac{1}{5}, +\infty\right[
1.15. \{x \in \mathbb{R} : |2 - 5x| \le 3\} = \left[-\frac{1}{5}, 1\right]
1.16. \{x \in \mathbb{R} : |3x - 4| \le 1\} = \lfloor 1, \frac{5}{3} \rfloor
1.17. \{x \in \mathbb{R} : |3x+4| \ge 1\} = \left]-\infty, -\frac{5}{3}\right] \cup [-1, +\infty[
1.18. \{x \in \mathbb{R} : |2x+3| > 5\} = ]-\infty, -4[\cup]1, +\infty[
1.19. \{x \in \mathbb{R} : |3 - 2x| < 5\} = ]-1, 4[
1.20. \{x \in \mathbb{R} : |2 - 3x| < 1\} = \left\lfloor \frac{1}{3}, 1 \right\rfloor
1.21. \{x \in \mathbb{R} : |2+3x| > 1\} = ]-\infty, -1[\cup]-\frac{1}{3}, +\infty[
1.22. \{x \in \mathbb{R} : |5x - 4| \le 1\} = \left[\frac{3}{5}, 1\right]
1.23. \{x \in \mathbb{R} : |5x+4| \ge 1\} = ]-\infty, -1] \cup \left[-\frac{3}{5}, +\infty\right[
1.24. \{x \in \mathbb{R} : |5 - 2x| < 1\} = [2, 3]
1.25. \{x \in \mathbb{R} : |2x+5| > 1\} = ]-\infty, -3[\cup]-2, +\infty[
1.26. \{x \in \mathbb{R} : |5 - 6x| \le 1\} = \left[\frac{2}{3}, 1\right]
1.27. \{x \in \mathbb{R} : |6x - 5| > 1\} = \left[-\infty, \frac{2}{3}\right] \cup \left[1, +\infty\right]
1.28. \{x \in \mathbb{R} : |9 - 2x| < 1\} = |4, 5|
1.29. \{x \in \mathbb{R} : |2x - 9| \ge 1\} = ]-\infty, 4] \cup [5, +\infty[
1.30. \{x \in \mathbb{R} : |4 - 3x| < 8\} = \left[-\frac{4}{3}, 4\right]
1.31. \{x \in \mathbb{R} : |3x - 4| \ge 8\} = \left] -\infty, -\frac{4}{3}\right] \cup [4, +\infty[
1.32. \{x \in \mathbb{R} : |3 - 4x| \le 7\} = \left[-1, \frac{5}{2}\right]
1.33. \{x \in \mathbb{R} : |4x - 3| > 7\} = ]-\infty, -1[\cup]^{\frac{5}{2}}, +\infty[
1.34. \{x \in \mathbb{R} : |7 - 2x| \le 1\} = [3, 4]
1.35. \{x \in \mathbb{R} : |2x - 7| > 1\} = ]-\infty, 3[\cup]4, +\infty[
1.36. \{x \in \mathbb{R} : |5 - 2x| < 9\} = ]-2, 7[
1.37. \{x \in \mathbb{R} : |2x - 5| \ge 9\} = ]-\infty, -2] \cup [7, +\infty[
1.38. \{x \in \mathbb{R} : |5 - 3x| < 1\} = \left[\frac{4}{3}, 2\right[
1.39. \{x \in \mathbb{R} : |3x - 5| \ge 1\} = \left] -\infty, \frac{4}{3} \right] \cup [2, +\infty[
1.40. \{x \in \mathbb{R} : 2 < 3|x+1| \le 5\} = \left[-\frac{8}{3}, -\frac{5}{3}\right] \cup \left[-\frac{1}{3}, \frac{2}{3}\right]
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2. Mostre que:
 2.1. \{x \in \mathbb{R} : |3-2x| \ge |x+2|\} = \left]-\infty, \frac{1}{3}\right] \cup [5, +\infty[
 2.2. \{x \in \mathbb{R} : |x| = |x - 2|\} = \{1\}
 2.3. \{x \in \mathbb{R} : |x| \le |x-2|\} = ]-\infty, 1]
 2.4. \{x \in \mathbb{R} : |2x - 5| \ge |1 - x|\} = ]-\infty, 2] \cup [4, +\infty[
 2.5. \{x \in \mathbb{R} : |6x - 5| < |1 - 8x|\} = ]-\infty, -2[\cup]^{\frac{3}{7}}, +\infty[
 2.6. \{x \in \mathbb{R} : |5 - 6x| \ge |8x - 1|\} = \left[-2, \frac{3}{7}\right]
 2.7. \{x \in \mathbb{R} : |2x - 9| < |1 - 8x|\} = \left[-\infty, -\frac{4}{3}\right] \cup \left[1, +\infty\right]
 2.8. \{x \in \mathbb{R} : |9 - 2x| \ge |8x - 1|\} = \left[-\frac{4}{3}, 1\right]
2.9. \{x \in \mathbb{R} : |3x - 4| \le |8 - 9x|\} = \left]-\infty, \frac{2}{3}\right] \cup [1, +\infty[
2.10. \{x \in \mathbb{R} : |4-3x| > |9x-8|\} = \frac{2}{3}, 1
2.11. \{x \in \mathbb{R} : |4x - 3| < |7 - 6x|\} = ]-\infty, 1[\cup ]2, +\infty[
2.12. \{x \in \mathbb{R} : |3 - 4x| \ge |6x - 7|\} = [1, 2]
2.13. \{x \in \mathbb{R} : |2x - 7| < |1 - 6x|\} = \left] -\infty, -\frac{3}{2} \left[ \cup \right] 1, +\infty \right[
2.14. \{x \in \mathbb{R} : |7 - 2x| \ge |6x - 1|\} = \left[-\frac{3}{2}, 1\right]
2.15. \{x \in \mathbb{R} : |2x - 5| \le |9 - 4x|\} = ]-\infty, 2] \cup \left[\frac{7}{3}, +\infty\right[
2.16. \{x \in \mathbb{R} : |5-2x| > |4x-9|\} = |2, \frac{7}{3}|
2.17. \{x \in \mathbb{R} : |3x - 5| \le |1 - 4x|\} = [-\infty, -4] \cup [\frac{6}{7}, +\infty[
2.18. \{x \in \mathbb{R} : |5 - 3x| > |4x - 1|\} = \left] -4, \frac{6}{7} \right[
2.19. \{x \in \mathbb{R} : 3|2-x| \le |x|\} = \left|\frac{3}{2}, 3\right|
2.20. \{x \in \mathbb{R} : 3|x-2| > |x|\} = \left[-\infty, \frac{3}{2}\right] \cup \left[3, +\infty\right]
2.21. \{x \in \mathbb{R} : |4x - 9| \ge |6 - x|\} = ]-\infty, 1] \cup [3, +\infty[
2.22. \{x \in \mathbb{R} : |9-4x| < |6-x|\} = ]1,3[
2.23. \{x \in \mathbb{R} : |3x+4| \le |x+8|\} = [-3,2]
2.24. \{x \in \mathbb{R} : |3x+4| > |x+8|\} = ]-\infty, -3[\cup]2, +\infty[
2.25. \{x \in \mathbb{R} : |5x - 2| \ge |x + 2|\} = ]-\infty, 0] \cup [1, +\infty[
2.26. \{x \in \mathbb{R} : |2 - 5x| < |x + 2|\} = [0, 1]
2.27. \{x \in \mathbb{R} : |7 - 4x| \le |2x + 1|\} = [1, 4]
2.28. \{x \in \mathbb{R} : |4x - 7| > |2x + 1|\} = ]-\infty, 1[\cup ]4, +\infty[
2.29. \{x \in \mathbb{R} : |5x - 4| \ge |x + 4|\} = ]-\infty, 0] \cup [2, +\infty[
2.30. \{x \in \mathbb{R} : |4 - 5x| < |x + 4|\} = ]0, 2[
2.31. \{x \in \mathbb{R} : |7 - 2x| \le |x + 1|\} = [2, 8]
2.32. \{x \in \mathbb{R} : |2x - 7| > |x + 1|\} = ]-\infty, 2[\cup ]8, +\infty[
2.33. \{x \in \mathbb{R} : |5 - 2x| < |x - 1|\} = ]2, 4[
2.34. \{x \in \mathbb{R} : |2 - x| \ge |3 + 2x|\} = \left[-5, -\frac{1}{3}\right]
2.35. \{x \in \mathbb{R} : |3 - 5x| < |7x - 6|\} = \left] -\infty, \frac{3}{4} \left[ \cup \right] \frac{3}{2}, +\infty \right[
2.36. \{x \in \mathbb{R} : |5x - 3| \ge |6 - 7x|\} = \left[\frac{3}{4}, \frac{3}{2}\right]
2.37. \{x \in \mathbb{R} : |3x - 2| > |4 - 9x|\} = \frac{1}{3}, \frac{1}{2}
2.38. \{x \in \mathbb{R} : |2 - 3x| \le |9x - 4|\} = \left[-\infty, \frac{1}{3}\right] \cup \left[\frac{1}{2}, +\infty\right[
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2.39. $\{x \in \mathbb{R} : |2x - 5| > |4 - x|\} =]-\infty, 1[\cup 3, +\infty[$

2.40. $\{x \in \mathbb{R} : |5-2x| \le |x-4|\} = [1,3]$

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3. Mostre que:
  3.1. \{x \in \mathbb{R} : 4 < x^2 < 9\} = ]-3, -2[\cup]2, 3[
  3.2. \{x \in \mathbb{R} : 9 \le (x-1)^2 < 25\} = [-4, -2] \cup [4, 6]
  3.3. \{x \in \mathbb{R} : x^2 - 1 > 0 \land x - 3 \le 0\} = ]-\infty, -1[\cup ]1, 3]
  3.4. \{x \in \mathbb{R} : x^2 - 4 \le 0 \land x + 1 > 0\} = [-1, 2]
  3.5. \{x \in \mathbb{R} : x^2 - 2x - 3 \ge 0\} = ]-\infty, -1] \cup [3, +\infty[
  3.6. \{x \in \mathbb{R} : 2 - x - x^2 > 0\} = ]-2,1[
  3.7. \{x \in \mathbb{R} : |x^2 - 2| \le 1\} = [-\sqrt{3}, -1] \cup [1, \sqrt{3}]
  3.8. \{x \in \mathbb{R} : |3 - 2x + x^2| = 5\} = \{1 - \sqrt{3}, 1 + \sqrt{3}\}
  3.9. \{x \in \mathbb{R} : |3 - 2x + x^2| < 5\} = \left[1 - \sqrt{3}, 1 + \sqrt{3}\right]
3.10. \{x \in \mathbb{R} : |15 + 2x - x^2| \ge 9\} = ]-\infty, -4] \cup [1 - \sqrt{7}, 1 + \sqrt{7}] \cup [6, +\infty[
3.11. \{x \in \mathbb{R} : |x^2 + 2x - 15| < 9\} = \left] -6, -1 - \sqrt{7} \right[ \cup \left] -1 + \sqrt{7}, 4 \right[
3.12. \{x \in \mathbb{R} : |4x - 3x^2| > 1\} = \left] -\infty, \frac{2-\sqrt{7}}{3} \right[ \cup \left] \frac{1}{3}, 1 \right[ \cup \left] \frac{2+\sqrt{7}}{3}, +\infty \right[
3.13. \{x \in \mathbb{R} : |3x^2 + 4x| \le 1\} = \left[\frac{-2 - \sqrt{7}}{3}, -1\right] \cup \left[-\frac{1}{3}, \frac{-2 + \sqrt{7}}{3}\right]
3.14. \{x \in \mathbb{R} : |3x^2 - 5x + 1| \ge 1\} = ]-\infty, 0] \cup \left[\frac{2}{3}, 1\right] \cup \left[\frac{5}{3}, +\infty\right[
3.15. \{x \in \mathbb{R} : |3x^2 + 5x + 1| < 1\} = \left] -\frac{5}{3}, -1 \right[ \cup \left] -\frac{2}{3}, 0 \right]
3.16. \{x \in \mathbb{R} : |x^2 + 4x - 3| > 2\} = ]-\infty, -5[\cup]-2-\sqrt{5}, -2+\sqrt{5}[\cup]1, +\infty[
3.17. \{x \in \mathbb{R} : |3+4x-x^2| \le 2\} = [-1, 2-\sqrt{5}] \cup [2+\sqrt{5}, 5]
3.18. \{x \in \mathbb{R} : |2x^2 - 5x| \ge 3\} = ]-\infty, -\frac{1}{2}] \cup [1, \frac{3}{2}] \cup [3, +\infty[
3.19. \{x \in \mathbb{R} : |2x^2 + 5x| < 3\} = ]-3, -\frac{3}{2}[\cup]-1, \frac{1}{2}[
3.20. \{x \in \mathbb{R} : |1 + 4x - 3x^2| > 1\} = \left] -\infty, \frac{2 - \sqrt{10}}{3} \left[ \cup \right] 0, \frac{4}{3} \left[ \cup \right] \frac{2 + \sqrt{10}}{3}, +\infty \right[
3.21. \{x \in \mathbb{R} : |3x^2 + 4x + 1| \le 1\} = \left[ \frac{-2 - \sqrt{10}}{3}, -\frac{4}{3} \right] \cup \left[ 0, \frac{-2 + \sqrt{10}}{3} \right]
3.22. \{x \in \mathbb{R} : |x^2 + 3x - 2| \ge 2\} = ]-\infty, -4] \cup [-3, 0] \cup [1, +\infty[
3.23. \{x \in \mathbb{R} : |2+3x-x^2| < 2\} = ]-1,0[\cup]3,4[
3.24. \{x \in \mathbb{R} : |x^2 - 5x + 2| \ge 2\} = ]-\infty, 0] \cup [1, 4] \cup [5, +\infty[
3.25. \{x \in \mathbb{R} : |x^2 + 5x + 2| < 2\} = [-5, -4] \cup [-1, 0]
3.26. \{x \in \mathbb{R} : |2x^2 - 3x - 1| > 1\} = ]-\infty, -\frac{1}{2}[\cup]0, \frac{3}{2}[\cup]2, +\infty[
3.27. \{x \in \mathbb{R} : |2x^2 + 3x - 1| \le 1\} = [-2, -\frac{3}{2}] \cup [0, \frac{1}{2}]
3.28. \{x \in \mathbb{R} : |2x^2 + 4x - 3| > 3\} = ]-\infty, -3[\cup]-2, 0[\cup]1, +\infty[
3.29. \{x \in \mathbb{R} : |3+4x-2x^2| \le 3\} = [-2,0] \cup [4,6]
3.30. \{x \in \mathbb{R} : |x^2 + 3x - 7| \ge 3\} = ]-\infty, -5] \cup [-4, 1] \cup [2, +\infty[
3.31. \{x \in \mathbb{R} : |x^2 - 3x - 7| < 3\} = ]-2, -1[\cup]4, 5[
3.32. \{x \in \mathbb{R} : |4-x-x^2| > 2\} = ]-\infty, -3] \cup [-2, 1] \cup [2, +\infty[
3.33. \{x \in \mathbb{R} : |x^2 - x - 4| < 2\} = ]-2, -1[\cup]2, 3[
3.34. \{x \in \mathbb{R} : |3x^2 + 2x - 3| > 2\} = \left] -\infty, -\frac{5}{3} \left[ \cup \right] -1, \frac{1}{3} \left[ \cup \right] 1, +\infty \right[
3.35. \left\{x \in \mathbb{R} : \left|3 + 2x - 3x^2\right| \le 2\right\} = \left[-1, -\frac{1}{3}\right] \cup \left[1, \frac{5}{3}\right]
3.36. \left\{x \in \mathbb{R} : |5x^2 + 4x - \frac{1}{2}| > \frac{1}{2}\right\} = \left]-\infty, -1[\cup] - \frac{4}{5}, 0[\cup] \frac{1}{5}, +\infty[3.37. \left\{x \in \mathbb{R} : |5x^2 - 4x - \frac{1}{2}| \le \frac{1}{2}\right\} = \left[-\frac{1}{5}, 0\right] \cup \left[\frac{4}{5}, 1\right]
3.38. \{x \in \mathbb{R} : |5x^2 + 4x - 5| \ge 4\} = ]-\infty, -\frac{9}{5}] \cup [-1, \frac{1}{5}] \cup [1, +\infty[
3.39. \{x \in \mathbb{R} : |5 + 4x - 5x^2| < 4\} = ]-1, -\frac{1}{5}[\cup]1, \frac{9}{5}[
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4. Mostre que:

$$\begin{array}{l} 4.1. \ \{x \in \mathbb{R} : \ |x(x-3)| = |1-3x|\} = \left\{-1, 3-2\sqrt{2}, 1, 3+2\sqrt{2}\right\} \\ 4.2. \ \{x \in \mathbb{R} : \ |x(x-3)| > |1-3x|\} =]-\infty, -1[\cup] \ 3-2\sqrt{2}, 1[\cup] \ 3+2\sqrt{2}, +\infty[$$

$$4.3. \ \{x \in \mathbb{R} : \ |x^2+x| \leq |x+\frac{3}{4}|\} = \left[\frac{-3}{2}, \frac{-\sqrt{3}}{2}\right] \cup \left[\frac{1}{2}, \frac{\sqrt{3}}{2}\right] \\ 4.4. \ \{x \in \mathbb{R} : \ |x-x^2| \leq |x-\frac{3}{4}|\} = \left[\frac{-\sqrt{3}}{2}, \frac{1}{2}\right] \cup \left[\frac{\sqrt{3}}{2}, \frac{3}{2}\right] \\ 4.5. \ \{x \in \mathbb{R} : \ |3x+4| > |x^2+3x|\} = \left]-3-\sqrt{5}, -2[\cup]-3+\sqrt{5}, 2[$$

$$4.6. \ \{x \in \mathbb{R} : \ |4-3x| > |3x-x^2|\} = \right]-2, 3-\sqrt{5}[\cup]2, 3+\sqrt{5}[$$

$$4.7. \ \{x \in \mathbb{R} : \ |2x^2+5x| \leq |5x-8|\} = \left[-2,1\right] \cup \left[2,4\right] \\ 4.8. \ \{x \in \mathbb{R} : \ |2x^2+5x| \leq |5x+8|\} = \left[-4,-2\right] \cup \left[-1,2\right] \\ 4.9. \ \{x \in \mathbb{R} : \ |2x-x^2| < |1-2x|\} = \right]-1, 2-\sqrt{3}[\cup]1, 2+\sqrt{3}[$$

$$4.10. \ \{x \in \mathbb{R} : \ |x^2+2x| < |2x+1|\} = \left]-2,-1[\cup]-\frac{1}{2},1[$$

$$4.11. \ \{x \in \mathbb{R} : \ |5x+4| > |4x^2+5x|\} = \left]-2,-1[\cup]-\frac{1}{2},1[$$

$$4.12. \ \{x \in \mathbb{R} : \ |5x-4| > |4x^2-5x|\} = \right]-1, \frac{1}{2}[\cup]1, 2[$$

$$4.13. \ \{x \in \mathbb{R} : \ |3-2x| \geq |2x-x^2|\} = \left[-\sqrt{3},1\right] \cup \left[\sqrt{3},3\right] \\ 4.14. \ \{x \in \mathbb{R} : \ |x^2+3x| \leq |3x+5|\} = \left[-5,-\sqrt{5}\right] \cup \left[-1,\sqrt{5}\right] \\ 4.16. \ \{x \in \mathbb{R} : \ |x^2-3x| \leq |3x-5|\} = \left[-\sqrt{5},1\right] \cup \left[\sqrt{5},5\right] \\ 4.17. \ \{x \in \mathbb{R} : \ |2x^2+3x| < |3x+4|\} = \left]-2,-\sqrt{2}\left[\cup\right]-1,\sqrt{2}\left[-\frac{4}{2}\right] \\ 4.19. \ \{x \in \mathbb{R} : \ |2x^2+3x| < |3x+4|\} = \left]-\infty,-1\left[\cup\right]+\infty[$$

$$4.20. \ \{x \in \mathbb{R} : \ |x-2x^2| \geq |1-2x|\} = \left]-\infty,-1\left[\cup\right]+\infty[$$

$$4.21. \ \{x \in \mathbb{R} : \ |3x^2+4x| \geq |3x+1|\} = \left]-\infty,-1\left[\cup\right]+\infty[$$

$$4.22. \ \{x \in \mathbb{R} : \ |3x^2+4x| \geq |3x+2|\} = \left]-\infty,-1\left[\cup\right]-\frac{1}{2},\frac{1}{3}\left[\cup\right],+\infty[$$

$$4.23. \ \{x \in \mathbb{R} : \ |3x^2+4x| \geq |3x+2|\} = \left]-\infty,-2\left[\cup\left[-1,-\frac{1}{3}\right] \cup \left[\frac{2}{3},+\infty[$$

$$4.24. \ \{x \in \mathbb{R} : \ |3x^2+4x| \geq |3x+2|\} = \left]-\infty,-\frac{2}{3}\left[\cup\right],\frac{1}{3},1\left[\cup\right]+\infty[$$

$$4.25. \ \{x \in \mathbb{R} : \ |3x+4| \geq |2x^2+2x|\} = \left]-\infty,-\frac{2}{3}\left[\cup\right],\frac{1}{3},1\left[\cup\right]+\infty[$$

$$4.26. \ \{x \in \mathbb{R} : \ |3x+4| \geq |2x^2+2x|\} = \left]-\infty,-\frac{2}{3}\left[\cup\right],\frac{1}{3},1\left[\cup\right]+\infty[$$

$$4.28. \ \{x \in \mathbb{R} : \ |3x+4| \geq |2x^2+2x|\} = \left]-\infty,-\frac{2}{3}\left[\cup\right],\frac{1}{4},\frac{3}{4}\left[\cup\right],\frac{1}{2},+\infty[$$

$$4.28. \ \{x \in \mathbb{R} : \ |3x+4| \geq |2x^2+2x|\} = \left]-\infty,-\frac{2}{3}\left[\cup\right],\frac{1}{4},\frac{3}{4}\left[\cup\right],\frac{1}{2},+\infty[$$

$$4.29. \ \{x \in \mathbb{R} : \ |3x+4| \geq |2x+4|\} = \left$$

4.30. $\{x \in \mathbb{R} : 3|6-x| < |4x-x^2|\} = \left]-\infty, \frac{1-\sqrt{73}}{2}\right[\cup \left] \frac{1+\sqrt{73}}{2}, +\infty\right[$