1.

a) $N = n \times N_A$, $n = \frac{m}{M} \Rightarrow$

$\Rightarrow N(Cu) = \frac{m(Cu)}{M(Cu)} N_A = \frac{5}{63{,}5} \times 6{,}02 \times 10^{23}$

$N(Cu) = 4{,}74 \times 10^{22}$.

$Z(Cu) = 29 \Rightarrow \underbrace{29\,p^+ \text{ e } 29\,e^-}_{\text{em cada átomo de Cu}}$

$Q = 29\, q_e \times N(Cu) = \underline{-220211\,C.}$

b) $F_{1,2} = k_e \frac{|Q_1||q_2|}{R^2} = 1{,}9792 \times 10^{21}\,N.$ (?)

$|q_2| \to 1C$, $R \to 0{,}001\,m$

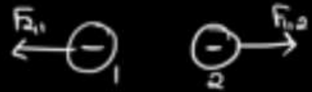
c) $N(Cu) = 4{,}74 \times 10^{22}$

99%: $4{,}6926 \times 10^{22}$ — $29e^-$

1%: $4{,}74 \times 10^{20}$ — $28e^-$

$Q = 29\,q_e \times 4{,}6926 \times 10^{22} + 28\,q_e \times 4{,}74 \times 10^{20}$

$\hookrightarrow Q = -220134\,C$



$F = \frac{|Q_1||Q_2|}{R^2} \times K = \underline{1{,}74 \times 10^{21}\,N.}$

Força necessária $= 2 \times F = \underline{3{,}5 \times 10^{21}\,N.}$ (?)

2.

$F_e = K \frac{|q_1||q_2|}{d^2} = 8{,}9876 \times 10^9 \times \frac{(1{,}602 \times 10^{-19})^2}{1^2}$

$F_e' = 8{,}9876 \times 10^9 \times \frac{(1{,}602 \times 10^{-19})^2}{(10^{-9})^2}$

$F_e = 2,3066 \times 10^{-28}\,N$; $F_e' = 2,3066 \times 10^{-10}\,N$

$$F_g = G\frac{m_{p^+}^2}{d^2} \rightarrow F_g = 1,8672 \times 10^{-64}\,N$$

$$\rightarrow F_g' = 1,8672 \times 10^{-46}\,N$$

$\frac{F_g}{F_e} = 8,095 \times 10^{-37}\,N$; $\frac{F_g'}{F_e'} = 8,095 \times 10^{-37}\,N$

3. $Q_1 = Q_2 = 1C$

$$F_{1,2} = K\frac{Q_1 Q_2}{R^2} = 8,9876 \times 10^{9}\,N$$

c) $F_{1y} = 0 \Leftrightarrow F_{2,1} = F_{g_2} \Leftrightarrow$

$$\Leftrightarrow m_2 = \frac{F_{2,1}}{g} = \frac{F_{1,2}}{g}$$

$$m_2 = \frac{8,9876 \times 10^{9}}{g} =$$

$= 9,171 \times 10^{8}$ kg.

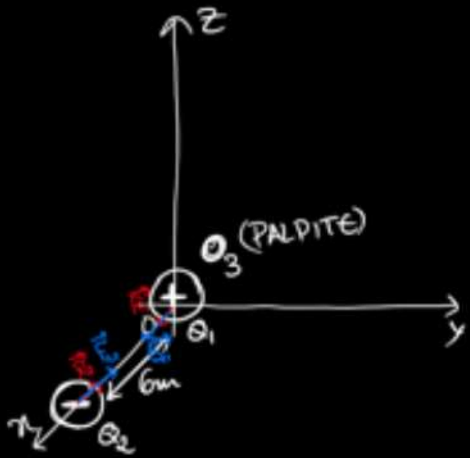
4. $Q_1 = 6\mu C$

$m_1 = 6g$

$Q_2 = -8\mu C$

$\vec{R_1} = 6\,\vec{e_n}$ m

$\vec{R_2} = -6\,\vec{e_n}$



$$F_{1,2} = K\frac{Q_1 Q_2}{R_1^2} = F_{2,1} = K\frac{Q_1 Q_2}{R_2^2} = 0,012\,N$$

$\vec{F_{1,2}} = 1,2 \times 10^{-2}\,\vec{e}\,(N)$; $\vec{F_{2,1}} = -1,2 \times 10^{-2}\,\vec{e}\,(N)$

$$F_R = ma \Rightarrow a = \frac{F_{1,2}}{m_1} = 2\,ms^{-2}.$$

g) $\vec{F}_{R_3} = 0 \Leftrightarrow \underset{>0}{\vec{F}_{1,3}} + \underset{<0}{\vec{F}_{2,3}} = 0 \Rightarrow$

$$\oplus_1\!\rightarrow \quad \leftarrow\!\ominus_3 \quad \leftarrow\!\ominus_2 \qquad \ominus_3\!\rightarrow$$

$$\Rightarrow F_{1,3} - F_{2,3} = 0 \Leftrightarrow \boxed{F_{1,3} = F_{2,3}}. \Leftrightarrow$$

$$\Leftrightarrow K\frac{|Q_1||Q_3|}{R_1^2} = K\frac{|Q_2||Q_3|}{R_2^2} \Leftrightarrow$$

$$\Leftrightarrow \frac{|Q_1||Q_3|}{R_1^2} = \frac{|Q_2||Q_3|}{R_2^2}, \quad R_2 = 6 + R_1 \Rightarrow$$

$$\Rightarrow \frac{|Q_1||Q_3|}{R_1^2} = \frac{|Q_2||Q_3|}{(R_1+6)^2} \Leftrightarrow$$

$$\Leftrightarrow R_1^2 + 12R_1 + 36 = \left|\frac{Q_2}{Q_1}\right| R_1^2 \Leftrightarrow$$

$$\Leftrightarrow R_1^2\left(1 - \frac{4}{3}\right) + 12R_1 + 36 = 0 \Leftrightarrow$$

$$\Leftrightarrow -\frac{1}{3}R_1^2 + 12R_1 + 36 = 0 \Leftrightarrow$$

$$\Leftrightarrow R_1^2 - 36R_1 - 108 = 0 \Leftrightarrow$$

$$\Leftrightarrow R_1 = \frac{36 \pm \sqrt{36^2 - 4\times1\times(-108)}}{2} \Leftrightarrow$$

$$\Leftrightarrow \underbrace{R_1 = -2{,}785}_{\times} \vee R_1 = 38{,}785 \quad \boxed{R_1 > 0}.$$

Colocar em: $(-38{,}8; 0; 0)$ m.

(5.) $R = 0{,}529 \times 10^{-10}$ m

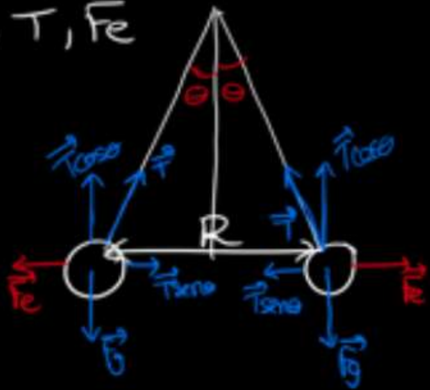
a) $F_e = K\frac{|q_p||q_e|}{R^2} \Leftrightarrow$

$\Leftrightarrow \underline{F_e = 8{,}2425 \times 10^{-8}\, N}$.

b) $F_R = F_e = m\dfrac{v^2}{R} \Leftrightarrow$

$\Leftrightarrow v = \sqrt{\dfrac{F_e R}{m_{\bar{e}}}} \Rightarrow \underline{v_{\bar{e}} = 2187822\ ms^{-1}}$

⑥ $\underline{\text{Forças}}$: F_g, T, F_e



$F_g = mg$

$T = \dfrac{mg}{\cos\theta}$

$F_e = K\dfrac{Q^2}{R^2}$

$\dfrac{R}{2} = l\,\text{sen}\,\theta \Rightarrow \underline{R = 2l\,\text{sen}\,\theta}$.

$F_e = K\dfrac{Q^2}{4l^2\,\text{sen}^2\theta}$

$F_e = T\,\text{sen}\,\theta \Leftrightarrow K\dfrac{Q^2}{R^2} = mg\tan\theta \Leftrightarrow$

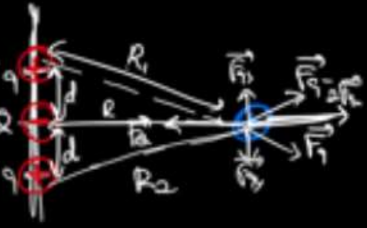
$\Leftrightarrow \underline{R = |Q|\sqrt{\dfrac{K}{mg\tan\theta}}}$

(?)

SOLUÇÃO

(?) $T = -(F_e + P)$

⑦ $\vec{F}_{e_T} = \vec{F}_{e_1} + \vec{F}_{e_2} + \vec{F}_e =$



$= Kq^2\left(\dfrac{\vec{R}_1}{|\vec{R}_1|^3} + \dfrac{\vec{R}_2}{|\vec{R}_2|^3} - 2\dfrac{\vec{R}}{|\vec{R}|^3}\right) =$

$= Kq^2\left(\dfrac{2R\,\vec{e}_n}{(R^2+d^2)^{3/2}} - \dfrac{2R\,\vec{e}_n}{R^3}\right)$

$= Kq^2\dfrac{2}{R^2}\left(\dfrac{R^3}{(R^2+d^2)^{3/2}} - 1\right)\vec{e}_n = \dfrac{q}{2\pi\varepsilon_0 R^2}\left(\dfrac{1}{\left(\frac{(R^2+d^2)^{1/2}}{R^2}\right)^3} - 1\right)$

$= \dfrac{q}{2\pi\varepsilon_0}\dfrac{1}{R^2}\left(\dfrac{1}{\left(\frac{R^2\left(1+\left(\frac{d}{R}\right)^2\right)^{1/2}}{R^2}\right)^3} - 1\right)\vec{e}_n =$

$$= \frac{q}{2\pi\varepsilon_0}\frac{1}{R^2}\left(\frac{1}{\left(1+\left(\frac{d}{R}\right)^2\right)^{3/2}}-1\right)\vec{e_n}.$$

b) $R \gg d \Rightarrow \frac{d}{R} \approx 0 \Rightarrow \frac{q}{2\pi\varepsilon_0}\frac{1}{R^2}\left(\frac{1}{\left(1+\left(\frac{d}{R}\right)^2\right)^{3/2}}-1\right)\vec{e_n} \approx$

$$\approx \frac{q}{2\pi\varepsilon_0}\frac{1}{R^2}\left(1-\frac{3}{2}\left(\frac{d}{R}\right)^2-1\right)\vec{e_n} = \frac{q}{2\pi\varepsilon_0}\frac{1}{R^2}\left(-\frac{3}{2}\frac{d^2}{R^2}\right)\vec{e_n}$$

$$= \frac{-3}{4\pi\varepsilon_0}\frac{d^2}{R^4}\vec{e_n}.$$

c) 0.

(8.) $R(\theta) = (R\,\mathrm{sen}\theta, R\cos\theta),\ \theta \in [0, \frac{\pi}{2}]$

$R'(\theta) = (R\cos\theta, -R\,\mathrm{sen}\theta)$

$$W = \int_0^{\frac{\pi}{2}} (F_0, 0)\cdot(R\cos\theta, -R\,\mathrm{sen}\theta)\,d\theta =$$

$$= \int_0^{\frac{\pi}{2}} F_0 R\cos\theta\,d\theta = F_0 R\,\mathrm{sen}\theta\Big|_0^{\frac{\pi}{2}} =$$

$= F_0 R$, sendo $R = x_B = y_A$.

(9.) $Q = -5\mu C$ $\quad \vec{E} = \frac{\vec{F}}{Q} = -4\times10^6\,\vec{u_n}\,(N)$

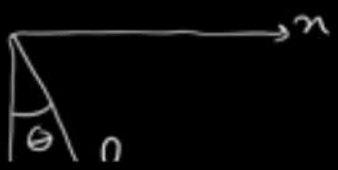
$\vec{F} = 20\,\vec{u_n}\,(N)$

(10.) a)

b) $\vec{E} = 2K\frac{e}{R^2}\vec{e_R} =$

$$= \frac{2{,}8796}{R^2}\vec{e_R}.$$

(11.) $\vec{E} = E\vec{u_n}$

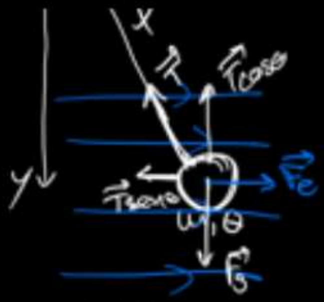


$\vec{E} = \frac{\vec{F_e}}{...}$

$\mathcal{E} = \frac{\quad}{Q} \Rightarrow$

$\Rightarrow \vec{F_e} = \vec{\mathcal{E}} Q \Rightarrow$

$\Rightarrow F_e = \mathcal{E} Q \vec{u_n}$

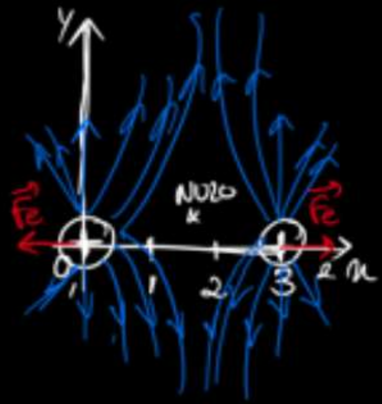


b) $T\cos\theta = F_g \Rightarrow T = \frac{mg}{\cos\theta}$

$T\operatorname{sen}\theta = F_e \Rightarrow mg\tan\theta = \mathcal{E}Q \Rightarrow$

$\Rightarrow \tan\theta = \frac{\mathcal{E}Q}{mg} \Rightarrow \underline{\theta = \operatorname{arctg}\left(\frac{\mathcal{E}Q}{mg}\right)}$

(12) $Q_1 = 6\mu C$
$Q_2 = 6\mu C$



b) $\vec{\mathcal{E}_1} = -K\frac{q_2}{R^2}\vec{e_n} = -\frac{6K}{R^2}\vec{e_n}$

$\vec{Fe_{2,1}} = -\frac{36K}{R^2}\vec{e_n}$

c) $\vec{\mathcal{E}_2} = -\vec{\mathcal{E}_1}$; $\vec{F_{1,2}} = -\vec{Fe_{2,1}}$

d) $\vec{\mathcal{E}} = \left(K\frac{q_1}{R_1^2} + K\frac{q_2}{R_2^2}\right)\vec{e_n} =$

$= Kq\left(\frac{1}{6^2} + \frac{1}{3^2}\right)\vec{e_n}$

e) Ponto: $(1,5;0,0)$.

$w_{int} = \int^b \vec{Fe} \cdot \vec{ds} =$

$$= \int_a^b q\vec{E}\cdot d\vec{s} = q\int_a^b \vec{E}\cdot d\vec{s}$$

$$W_{int} = -\Delta U \Rightarrow -\Delta U = q\int_a^b \vec{E}\cdot d\vec{s}$$

$$\Delta V = -\int_a^b \vec{E}\cdot d\vec{s} \leftarrow \Delta V q = \Delta U$$

(13.) $\Delta V = 20\,KV$; $\Delta U = \Delta V q \Rightarrow$

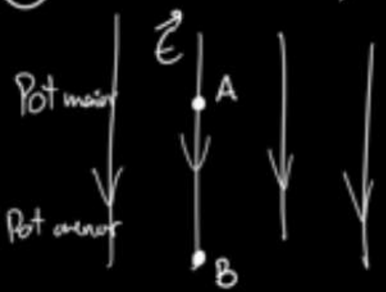
$\rightarrow KV = 20\times10^3 V$

$\Rightarrow \Delta U = 20Q = -W_{int} \Rightarrow$

$\Rightarrow W = -20\times10^3 \times 5 = -100\times10^3 J$

$W = -100\,KJ.$

(14.) $\vec{E} = -120\,\vec{u}_y\ Vm^{-1}$



$$\Delta V = -\int_A^B \vec{E}\cdot d\vec{s} =$$

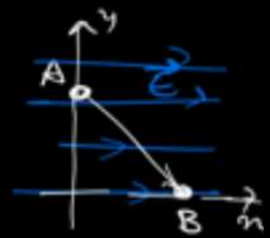
$$= -\int_A^B E\cdot \vec{u}_y\, dy =$$

$$= -\int_0^2 120\,\vec{u}_y \cdot \vec{u}_y\, dy = -\underline{\underline{240}}V$$

$\Delta V = -240V.$

(15.) $\vec{E} = E_0\,\vec{u}_x\ (Vm^{-1})$

$A = (0, y_A)$; $B(x_B, 0)$



a) $\Delta V_{AB} = \int_A^B \vec{E}\cdot d\vec{s} \overset{(E)}{\rightarrow} \vec{E}\cdot d\vec{y} = 0$

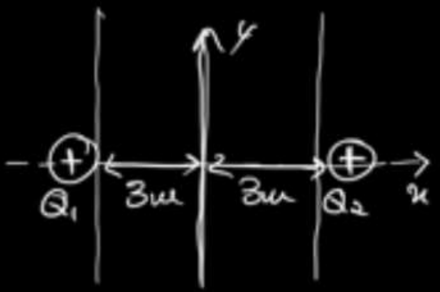
$\Rightarrow \Delta V_{AB} = \int_{x_A}^{x_B} E \cdot dx = \underline{E_0 x_B}$ (V)

b) $W_{EXT} = -\Delta U = -Q\Delta V = -\underline{QE_0 x_B}$ (J)

(16.) $Q_1 = 6\mu C$

$Q_2 = 6\mu C$

$Q = Q_1 = Q_2$



a) Definindo $V_1(\infty) = V_2(\infty) = 0$

$$V = V_1 + V_2 = KQ\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\left.\begin{array}{l} R_1 = 3+x \\ R_2 = 3-x \end{array}\right\} V = KQ\left(\frac{1}{3+x} + \frac{1}{3-x}\right) =$$

$$= KQ\left(\frac{3-x+3+x}{3^2-x^2}\right) = 6KQ\left(\frac{1}{9-x^2}\right) =$$

$$= \frac{32355}{9-x^2}\,V = \underline{\frac{3{,}24\times10^5}{9+x^2}}\ (V).$$

b) $\vec{E} = 0 \Rightarrow x_1 = x_2 \Rightarrow \underline{x=0}$.

$$V(0) = \frac{3{,}24\times10^5}{9} = \underline{3{,}60\times10^4\ (V)}.$$

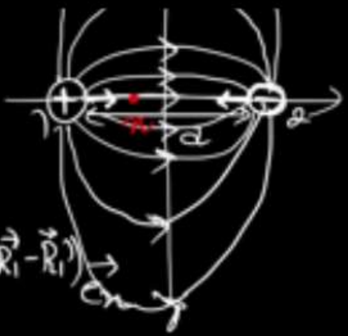
c) $\Delta V_{AB} = \int_A^B \vec{E}\cdot d\vec{\ell} = \int_{x_A}^{x_B} E_x\,dx =$

$$= \int_{-3}^{3} E_x\,dx = \underline{0}.$$

d) Movimento oscilatório.

(17.)

a)



b) $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$\vec{E}_1 = -Kq \frac{1}{|\vec{R}_1-\vec{R}_1'|^2}(\vec{R}_1-\vec{R}_1')\vec{e}_x$$

$$\vec{E}_2 = -Kq \frac{1}{|\vec{R}_2-\vec{R}_2'|^3}(\vec{R}_2-\vec{R}_2')\vec{e}_x$$

$$\vec{E}_1 = -Kq \frac{1}{|x-\frac{d}{2}|^3}\left(x-\frac{d}{2}\right)\vec{e}_x$$

$$\vec{E}_2 = -Kq \frac{1}{|x+\frac{d}{2}|^3}\left(x+\frac{d}{2}\right)\vec{e}_x$$

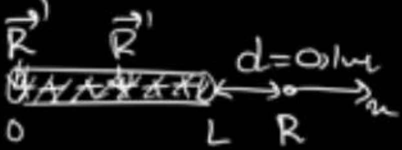
$$\vec{E} = -Kq\left(\frac{1}{|x-\frac{d}{2}|^3}\left(x-\frac{d}{2}\right)+\frac{1}{|x+\frac{d}{2}|^3}\left(x+\frac{d}{2}\right)\right)\vec{e}_x =$$

$$= -Kq\left(\frac{|x+\frac{d}{2}|^3\left(x-\frac{d}{2}\right)+|x-\frac{d}{2}|^3\left(x+\frac{d}{2}\right)}{|x-\frac{d}{2}|^3\,|x+\frac{d}{2}|^3}\right)\vec{e}_x =$$

$$= -Kq\left(\frac{\quad}{\left[x^2-\left(\frac{d}{2}\right)^2\right]^3}\right)\vec{e}_x$$

18. $L = 2{,}5\ m$

$Q = 5\mu C$



a) $\lambda = \frac{Q}{L} = 2\times10^{-6}\ C\,m^{-1}$

b)

b) $d\vec{E}_n = K\,dq \frac{1}{|\vec{R}-\vec{R'}|^3}(\vec{R}-\vec{R'})\vec{e_n}$

$$d\vec{E}_n(n) = K\,dq \frac{1}{|l+d-n|^3}(l+d-n)\vec{e_n}$$

$$\vec{E}_n(n) = \int_L K\,dq \frac{1}{|l+d-n|^3}(l+d-n)\vec{e_n} =$$

$$\Leftrightarrow \vec{E}_n = \int_0^l K\lambda \frac{(l+d-n)}{|l+d-n|^3}\,dl\,\vec{e_n} \Leftrightarrow$$

$$\Leftrightarrow \vec{E}_n = K\frac{Q}{l}\int_0^l \frac{1}{(l+d-n)^2}\,dl\,\vec{e_n} =$$

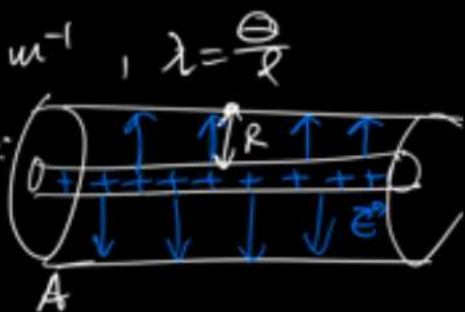
$$= K\frac{Q}{l}\left[-\frac{1}{l+d-n}\right]_0^l \vec{e_n} =$$

$$= K\frac{Q}{l}\left[-\frac{1}{d}+\frac{1}{l+d}\right]\vec{e_n} =$$

$$= K\frac{Q}{l}\left(\frac{-(l+d)+d}{d(l+d)}\right)\vec{e_n} =$$

$$= KQ\frac{-1}{d(l+d)}\vec{e_n} = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{d(l+d)}\vec{e_n}$$

(2) $\lambda = 0{,}3\ \mu C\ m^{-1}$, $\lambda = \frac{Q}{l}$



Pela Lei de Gauss:

$$\Phi = \frac{Q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

$$\Phi = \int \vec{E}\cdot d\vec{S} = \int E \cdot dA\ \vec{e_R} = *$$

$\forall A$ $\forall A$

$\vec{n} \parallel \vec{E}$, pois $\vec{n} = \vec{e}_R$

Como R é constante $\Rightarrow$ E constante

$* = E\int_A dA$, $dA = R\,d\theta\,dz$

$= E\int_0^{\ell}\int_0^{2\pi} R\,d\theta\,dz = E\times R\times 2\pi\times \ell$.

$\Phi = \int_A \vec{E}\cdot d\vec{S} \Leftrightarrow \frac{\lambda \cancel{\ell}}{\varepsilon_0} = E\times R\times 2\pi\times\cancel{\ell} \Leftrightarrow$

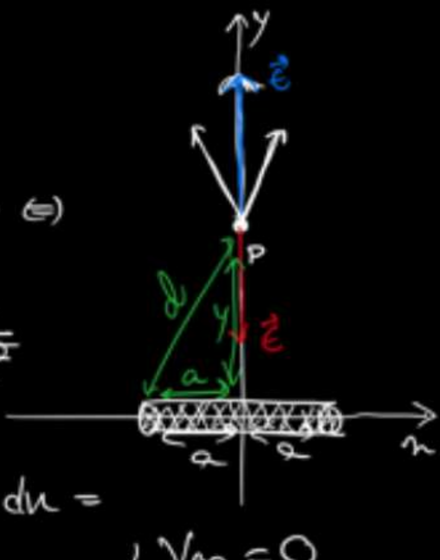
$\Leftrightarrow E = \frac{\lambda}{\varepsilon_0\times R\times 2\pi}$.

Como E só tem componente radial $\rightarrow$

$\Rightarrow \vec{E} = \frac{\lambda}{\varepsilon_0 R 2\pi}\vec{e}_R$.

20.

a) $dV = k\frac{dq}{d} \Leftrightarrow$



$\Leftrightarrow dV = k\frac{\lambda\, dx}{\sqrt{y^2+x^2}}$

$V = \int_{-a}^{a} k\lambda \frac{1}{\sqrt{y^2+x^2}}\,dx =$, $V_\infty = 0$

$= k\lambda \log\left(x+\sqrt{x^2+y^2}\right)\Big|_{x=-a}^{x=a} =$

$= k\lambda\left[\log\left(a+\sqrt{a^2+y^2}\right) - \log\left(-a+\sqrt{a^2+y^2}\right)\right]$

$\vec{E} = -\frac{dV}{dy}\vec{e}_y$, pois só existe $\vec{E}$ em y.

$$\vec{E} = -K\lambda \left[\frac{\frac{y}{\sqrt{a^2+y^2}}}{a+\sqrt{a^2+y^2}} + \frac{\frac{y}{\sqrt{a^2+y^2}}}{a-\sqrt{a^2+y^2}} \right] \vec{e}_y$$

$$\vec{E} = -K\lambda \frac{\frac{y}{\sqrt{a^2+y^2}}\left(a - \cancel{\sqrt{a^2+y^2}} + a + \cancel{\sqrt{a^2+y^2}}\right)}{\cancel{a^2} - (\cancel{a^2}+y^2)} \vec{e}_y$$

$$\vec{E} = \cancel{+}K\lambda \frac{2a\cancel{y}}{\cancel{+}y^{\cancel{2}}\sqrt{a^2+y^2}} \vec{e}_y = K\lambda \frac{2a}{y\sqrt{a^2+y^2}} \vec{e}_y$$

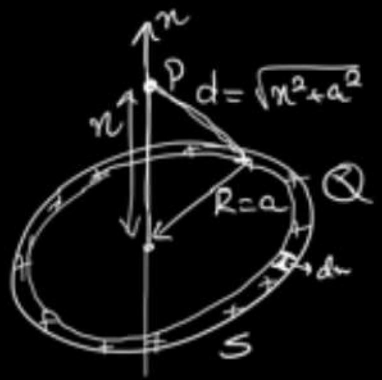
$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0} \frac{a}{y\sqrt{y^2+a^2}} \vec{e}_y.$$

$y^2+a^2 \approx y^2$

b) $$\frac{E_\infty}{E} = \frac{\frac{\lambda}{\varepsilon_0 2\pi \sqrt{y^2+a^2}}}{\frac{a\lambda}{2\pi\varepsilon_0 y\sqrt{y^2+a^2}}} \approx \frac{\frac{1}{\cancel{y}}}{\frac{a}{\cancel{y}\sqrt{y^2+a^2}}} = \frac{\sqrt{y^2+a^2}}{a}$$

i) 1,08.

21.



a) $$dV = K\frac{dq}{d} \Leftrightarrow$$

$$\Leftrightarrow dV = K\frac{dq}{\sqrt{x^2+a^2}}$$

$$\lambda = \frac{Q}{2\pi a} = \frac{dq}{dx} \Leftrightarrow dq = \lambda\, dx.$$

$$V = \int_S dV = \int_S K\frac{\lambda}{\sqrt{x^2+a^2}}\, dx =$$

$$= K\lambda \int_0^{2\pi} \frac{a}{\sqrt{x^2+a^2}}\, d\theta = \frac{K\lambda a\, 2\pi}{\sqrt{x^2+a^2}}$$

$$\boxed{V = \frac{KQ}{\sqrt{x^2+a^2}}}. \quad , V_\infty = 0$$

b) $\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\vec{e_x} - \frac{\partial V}{\partial y}\vec{e_y} - \frac{\partial V}{\partial z}\vec{e_z}$

$\vec{E} = -\frac{dV}{dx}\vec{e_x}$, pois V só depende de x

$$\vec{E} = \frac{KxQ}{(x^2+a^2)^{3/2}}\vec{e_x}$$

c) $x^2+a^2 \approx x^2 \Rightarrow \vec{E} = \frac{KxQ}{(\sqrt{x^2})^3}\vec{e_x} \Leftrightarrow$

$\Leftrightarrow \underline{\vec{E} = K\frac{Q}{x^2}\vec{e_x}}$ (igual a carga pontual)

22.

a) $\sigma = \frac{dq}{dA} \Leftrightarrow \underline{dq = \sigma dA}$

$$dV = K\frac{dq}{d} = K\frac{\sigma dA}{\sqrt{z^2+R^2}}$$

$$V = \int_S dV = \int_S K\frac{\sigma dA}{\sqrt{z^2+R^2}} = K\sigma\int_0^{2\pi}\int_0^a \frac{R}{\sqrt{z^2+R^2}}dR\,d\theta =$$

$$= K\sigma 2\pi\left(\sqrt{z^2+R^2}\right)_0^a =$$

$$= K\sigma 2\pi\left(\sqrt{z^2+a^2} - |z|\right),\ V_\infty = 0$$

$$\vec{E} = \vec{\nabla} V = -\frac{dV}{dz}\vec{e_z} =$$

$$= \underline{-K\sigma 2\pi\left(\frac{z}{\sqrt{z^2+a^2}} - \frac{z}{|z|}\right)\vec{e_z}}$$

b) $a \to \infty$

$$\vec{E} = -K\sigma 2\pi\left(1-\frac{z}{|z|}\right)\vec{e}_z$$

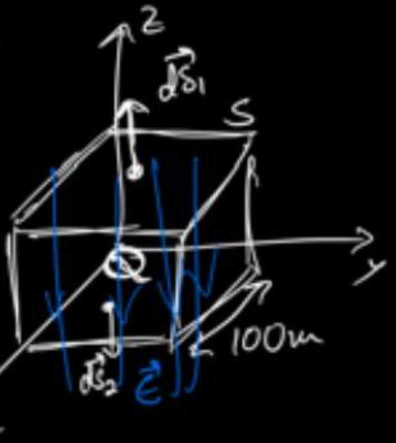
$$\vec{E} = K\sigma 2\pi\left(\frac{z}{|z|}-1\right)\vec{e}_z.$$

23. $\vec{E} = -(180-0{,}4y)\,\vec{u}_y$, $y \in]200,300[$

$\vec{E}_1(300m) = -100\,\vec{u}_y$

$\vec{E}_2(200m) = -60\,\vec{u}_y$

$$\Phi = \sum_{i=0}^{6}\int_{S_i}\vec{E}_i\cdot d\vec{S}_i =$$



$$= \int_{S_1}\vec{E}_1\cdot d\vec{S}_1 + \int_{S_2}\vec{E}_2\cdot d\vec{S}_2$$

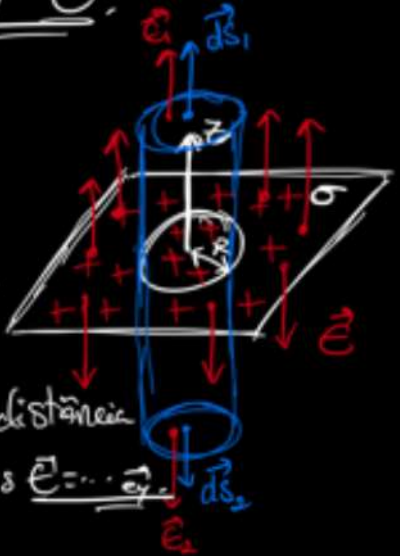
$$= -E_1\,dS_1 + E_2\,dS_2 = 10^4(-E_1+E_2) \Leftrightarrow$$

$$\Leftrightarrow \Phi = 10^4(100-60) = 4\times10^5$$

$$\Phi = \frac{Q}{\varepsilon_0} \Leftrightarrow Q = \Phi\,\varepsilon_0 \Leftrightarrow$$

$$\Leftrightarrow Q = 3{,}5\times10^{-6}\,C.$$

24.



a) $\vec{E}$ é o mesmo para qualquer ponto a uma distância R do plano, pois $\vec{E} = \dots\,\vec{e}_z$

$$\Phi = \frac{Q}{\varepsilon_0} = \frac{\sigma\,dA}{\varepsilon_0}$$

$\Phi = \Phi_1 + \Phi_2$, pois na lateral do cilindro considerada $\underline{\vec{dS} \cdot \vec{E} = 0}$

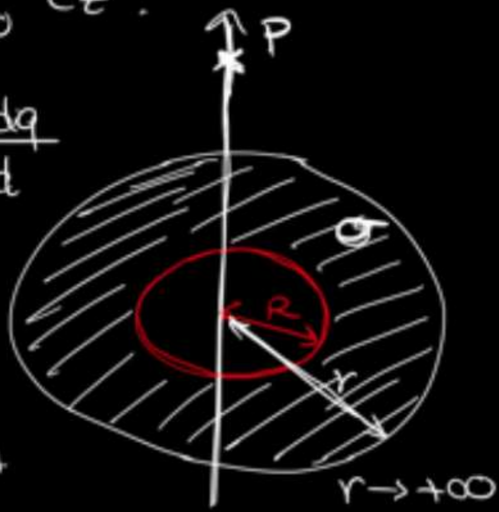
$$\Phi = \int_{S_1} \vec{E_1} \cdot \vec{dS_1} + \int_{S_2} \vec{E_2} \cdot \vec{dS_2} =$$

$$= 2\int_{S_1} E\, dS_1$$, sendo que $\underline{dS_1 = dS_2 = dA}$

visto que $\vec{E}$ só tem $\vec{u_y}$.

$$\Phi = 2E\cancel{dA} = \frac{\sigma \cancel{dA}}{\varepsilon_0} \Leftrightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}.$$

- $\vec{E} = \frac{z}{|z|} \frac{2\sigma}{\varepsilon_0} \vec{e_z}$.



b) $dV = K \frac{dq}{d}$

$\sigma = \frac{dq}{dA} \Leftrightarrow$

$\Leftrightarrow \underline{dq = \sigma dA}$.

$$dV = K \frac{\sigma}{d} dA$$

$$d = \sqrt{z^2 + r^2} \Rightarrow dV = \frac{\sigma}{4\pi\varepsilon_0} \frac{1}{\sqrt{z^2+r^2}} dA$$

$$V = \frac{\sigma}{4\pi\varepsilon_0} \int_0^{2\pi} d\theta \int_R^r \frac{1}{\sqrt{z^2+r^2}} r\, dr\, d\theta \Leftrightarrow$$

$$\Leftrightarrow V = \frac{\sigma}{2\varepsilon_0} \left(\sqrt{z^2+r^2} - \sqrt{z^2+R^2} \right)$$

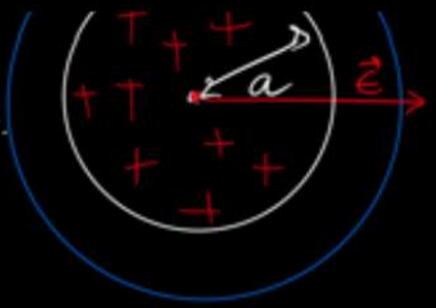
$$-\vec{\nabla} V = \vec{E} \Rightarrow \vec{E} = -\frac{dV}{dz} \vec{e_z} \Leftrightarrow$$

$$\Leftrightarrow \vec{E} = -\frac{\sigma}{2\varepsilon_0} \left(\frac{z}{\sqrt{z^2+r^2}} - \frac{z}{\sqrt{z^2+R^2}} \right) \vec{e_z}$$

$\underline{r \to +\infty} \Rightarrow \vec{E} = \frac{\sigma}{2\varepsilon_0} \frac{z}{\sqrt{z^2+R^2}} \vec{e_z}$.

25.

a) $\vec{E} = 0\,\vec{e}_R$,
pois está no interior.



b) $\vec{E} = K\frac{Q}{R^2}\vec{e}_R$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\,\frac{\sigma\times 4\pi a^2}{R^2}\,\vec{e}_R \qquad \vec{E} = \cdots\,\vec{e}_R$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\times\frac{a^2}{R^2}\,\vec{e}_R \rightarrow \underline{\vec{E} = 8{,}5\times10^3\ \text{V m}^{-1}\,\vec{e}_R}$$

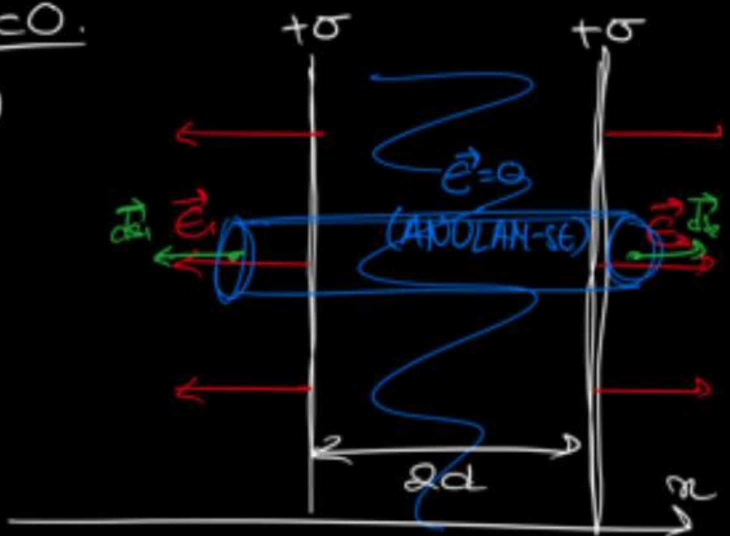
c) $dV = K\frac{dq}{R} = K\,\sigma\,dA$

$$V = \int_S \frac{K\sigma}{a}\,dA = \frac{K\sigma}{a}\int_0^{2\pi}\int_0^{\pi} a^2 \sin\phi\, d\phi\, d\theta$$

$$V = \frac{\sigma}{\cancel{4\pi}\varepsilon_0}\times\cancel{4\pi}a = \frac{\sigma}{\varepsilon_0}a = \underline{1{,}36\ \text{kV}}.$$

$\underline{V_\infty = 0.}$

(27)



No interior dos 2 planos $\vec{E}=0$.
No exterior, pela Lei de Gauss, temos $\Phi = \frac{Q}{\varepsilon_0} = \frac{2\sigma\, dA}{\varepsilon_0}$

e $\Phi = \int_{S_1}\vec{E_1}\cdot d\vec{S_1} + \int_{S_2}\vec{E_2}\cdot d\vec{S_2} =$

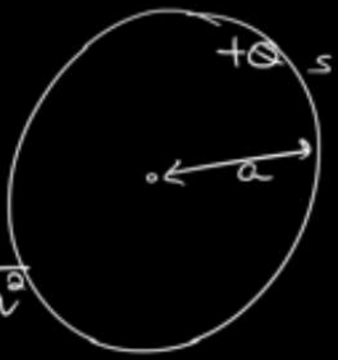
$= \underline{2E\,dA}$

$2\sigma\,dA \rightarrow \sigma$

$4 \not{\pi} E\, dA = \frac{\quad}{\varepsilon_0} \rightarrow E = \frac{\quad}{\varepsilon_0}$

$$\vec{E} = \frac{x}{|x|}\frac{\sigma}{\varepsilon_0}\vec{e_x}.$$

28.



a) $\rho = 0$; $\sigma = \frac{Q}{A}$

$E = 0 \Rightarrow$
$\Rightarrow Q = 0.$ $\quad \sigma = \frac{Q}{4\pi a^2}$

b) $R < a$: $\vec{E} = 0\,\vec{e_R}$ $\qquad dq = \sigma dA$

$R > a$: $\vec{E} = \int_S K\frac{dq}{R^2}\vec{e_R} = \int_S K\frac{\sigma}{R^2}dA =$

$$= \int_0^{2\pi}\int_0^{\pi} K\frac{\sigma}{R^2}a^2 \operatorname{sen}\phi\,\vec{e_R}\,d\phi\,d\theta =$$

$$= 4\pi K\frac{a^2}{R^2}\sigma\,\vec{e_R} = \frac{\sigma}{\varepsilon_0}\left(\frac{a}{R}\right)^2\vec{e_R}$$

c) $\Delta U = W = q_2\Delta V = q_2 V_0$, $V_\infty = 0$

Como $V_0 = 0$, temos que considerar até um ponto da superfície: $(R=a)$

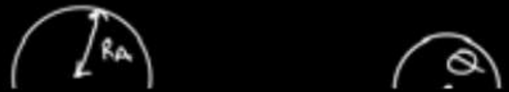
$$\vec{E} = \frac{\sigma}{\varepsilon_0}\left(\frac{a}{R}\right)^2\vec{e_R} = -\frac{dV}{dR}\vec{e_R}$$

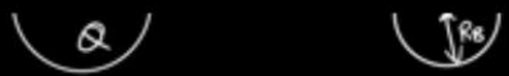
$$V = -\int_a^{+\infty}\underbrace{\vec{E}\cdot\vec{dR}}_{(-E\,dR)} = \int_a^{+\infty}\frac{\sigma}{\varepsilon_0}\frac{a^2}{R^2}dR =$$

$$= -\frac{\sigma}{\varepsilon_0}\frac{a^2}{R}\Big|_a^{+\infty} = \boxed{\frac{\sigma}{\varepsilon_0}a}.$$

$$U = qV \Rightarrow U = \frac{\sigma q a}{\varepsilon_0}.$$

29.





a) Está toda na superfície das esferas.

$$\rho = \frac{Q}{V} = 0 \ ; \ \sigma = \frac{Q}{A} = \frac{Q}{4\pi R^2}$$

$$\sigma_A = \frac{Q}{4\pi R_A^2} \ ; \ \sigma_B = \frac{Q}{4\pi R_B^2}$$

b) $\vec{E}_A = \frac{\sigma_A}{\varepsilon_0}\vec{e}_R = \frac{Q}{4\pi R_A^2 \varepsilon_0}\vec{e}_R$

$\vec{E}_B = \frac{\sigma_B}{\varepsilon_0}\vec{e}_R = \frac{Q}{4\pi R_B^2 \varepsilon_0}\vec{e}_R$

c)

Equilíbrio: $\phi_A(R_A) = \phi_B(R_B) = \phi$, $\phi(+\infty) = 0$.

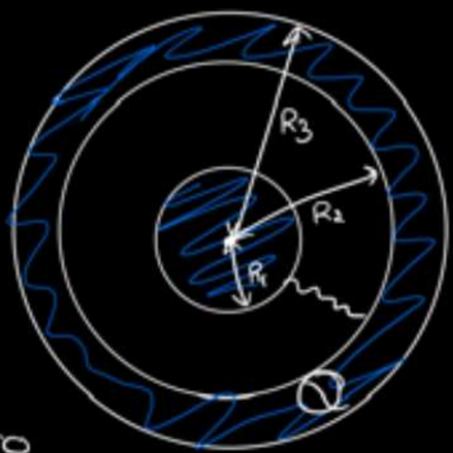
$$\phi = K\frac{Q_A}{R_A} = K\frac{Q_B}{R_B}$$

$\hookrightarrow Q_A = \frac{R_A}{R_B}Q_B$ e $2Q = Q_A + Q_B$

$$Q_A = \frac{R_A}{R_B}(2Q - Q_A) \Leftrightarrow Q_A\left(1 + \frac{R_A}{R_B}\right) = 2Q\frac{R_A}{R_B}$$

$$\Leftrightarrow Q_A = \frac{2Q\frac{R_A}{R_B}}{\frac{R_B+R_A}{R_B}} \Leftrightarrow \boxed{Q_A = \frac{2R_A Q}{R_A + R_B}}.$$

30



a) $\Delta\phi_{AB} = 0 =$
$= \phi_A(R) - \phi_B(R)$
$\Downarrow$
$\phi_A(R) = \phi_B(R)$

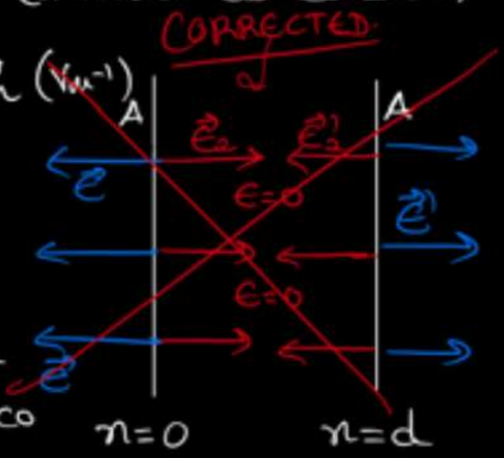
b) Retirando o fio as cargas movem-se todas para a superfície exterior (R_3).

c) Não.

31.

a) $\mathcal{E}=0$. (interior do condutor)

b) $\vec{\mathcal{E}} = \mathcal{E}_0 \, \vec{u}_n$ (V m^{-1})



$\mathcal{E} = \frac{\sigma}{\varepsilon_0}$

$\sigma = \mathcal{E}_0 \, \varepsilon_0.$

Para contrariar o campo elétrico criado por Q aparece $-Q$ no interior, de forma a que $\mathcal{E}=0$.

Logo cria-se um campo simétrico no interior.

Exterior: $\vec{\mathcal{E}} = \mathcal{E}_0 \vec{e}_n \rightarrow$ não se cria nada.

Interior: $\vec{\mathcal{E}} = -\mathcal{E}_0 \vec{e}_n$

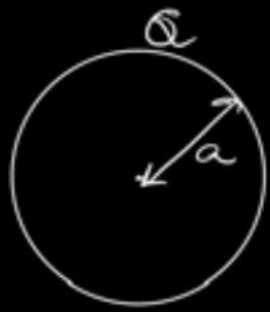
c) Interior: $\vec{\mathcal{E}} = 0$ (SEMPRE)

Temos no exterior: $\vec{\mathcal{E}} = \mathcal{E}_0 \vec{e}_n$ +

+ Carga Q: $\vec{\mathcal{E}} = \frac{Q}{4\pi R^2 \varepsilon_0}$

$$\vec{\mathcal{E}} = \mathcal{E}_0 + \frac{Q}{4\pi R^2 \varepsilon_0} \frac{x}{|x|} \vec{e}_n$$

32.



a) $d\vec{\mathcal{E}} = K \frac{dq}{R^2} \vec{e}_R$

$\sigma = \frac{dq}{dA} \Rightarrow dq = \sigma \, dA$

$\vec{\mathcal{E}} = \int d\vec{\mathcal{E}} = \frac{\sigma}{\ldots} \int \frac{1}{\ldots} dA = \frac{\sigma}{\ldots} 4\pi R^2$

$$E = \int dE = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma}{R^2}\,dA = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 4\pi R^2}{R^2}\vec{e}_R$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{e}_R = \frac{Q}{4\pi R^2 \varepsilon_0}\vec{e}_R$$

↳ $R<a$: $Q=0 \Rightarrow \vec{E}=0$

↳ $R>a$: $Q=Q \Rightarrow \vec{E} = \dfrac{Q}{4\pi R^2 \varepsilon_0}\vec{e}_R$

b) $E_{MAX} \Leftrightarrow R_{min} \Rightarrow$

$$\Rightarrow 3\times10^6 = \frac{0{,}3\times10^{-6}}{4\pi \times R_{min}^2\,\varepsilon_0} \Rightarrow$$

$$\Rightarrow R_{min} = \sqrt{\frac{0{,}1\times10^{-12}}{4\pi}\frac{1}{\varepsilon_0}}$$

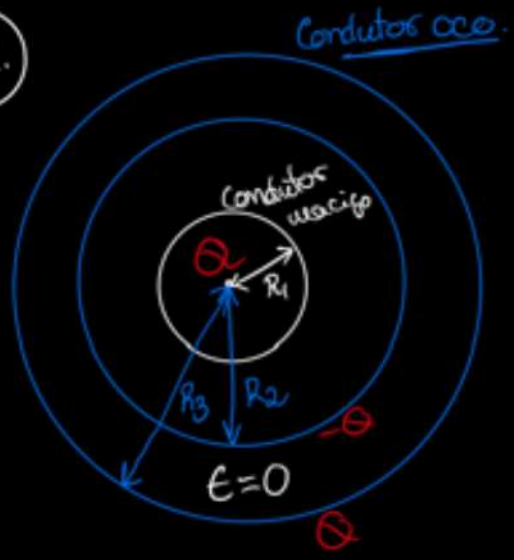
$R_{min} = 0{,}029979 \Rightarrow \underline{R_{min} = 0{,}03\,m}$

33. $\underline{V_{TERRA} = 0} \Rightarrow V_R = 60\,kV$

$$V_R = k\frac{Q}{R} = ER \Rightarrow R_{min} = \frac{V_R}{E_{MAX}}$$

$\underline{R_{min}} = 0{,}02\,m.$

34.



a)

Maciço: $\sigma_1 = \dfrac{Q}{4\pi R_1^2}$

Oco: $\sigma_2 = -\dfrac{Q}{4\pi R_2^2}$ $\quad\sigma_3 = \dfrac{Q}{4\pi R_3^2}$

b) $\vec{E}(R_1 < R < R_2) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\vec{e_R}$

c) $\phi(R_2) - \phi(R_1) = \int_{R_1}^{R_2} \vec{E}\, d\vec{R} =$

$= \int_{R_1}^{R_2} E\, dR = \int_{R_1}^{R_2} \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\, dR = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}\Big|_{R_1}^{R_2} =$

$= -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) = -\underline{\frac{Q}{4\pi\varepsilon_0}\frac{R_1 - R_2}{R_1 R_2}}.$

d) Na alínea a) $Q = 0$ (R_3).

35.

$\vec{E}(R < a) = 0$, pois é o interior de um condutor.

$\vec{E}(R > b) = 0$, pois está ligado a Terra.

Considerando uma superfície de Gauss: (cilindro $a < R < b$)

$\Phi = \frac{Q}{\varepsilon_0} = \int \vec{E} \cdot d\vec{S} = E \int \vec{e_R} \cdot \vec{e_R}\, dS =$

$= E \int_0^{l}\int_0^{2\pi} R\, d\theta\, dz = \underline{2\pi R l E}.$

$\frac{Q}{\varepsilon_0} = 2\pi R l E \Rightarrow \underline{\vec{E} = \frac{\lambda}{\varepsilon_0 2\pi R}\vec{e_R}}.$

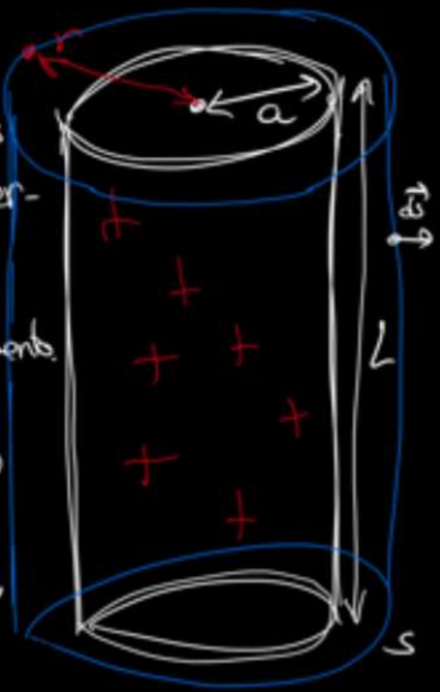
$\underline{(a < R < b)}$

b) $\phi_{AB} = \phi(A) - \phi(B) = \int_{R_A}^{R_B} \vec{E}\cdot d\vec{R} =$

$= \frac{\lambda}{2\pi\varepsilon_0}\int \frac{1}{R}\,\vec{e_R}\cdot\vec{e_R}\, dR =$

$= \frac{\lambda}{2\pi\varepsilon_0}\left(\log(R_B)-\log(R_A)\right) = \underline{\frac{\lambda}{2\pi\varepsilon_0}\log\left(\frac{R_B}{R_A}\right)}$

(26) Desprezamos o efeito das superfícies planas do cilindro devido ao seu comprimento.



a) $\frac{Q}{\varepsilon_0} = \int \vec{E}\cdot d\vec{S} \Leftrightarrow$

$\Leftrightarrow \frac{Q}{\varepsilon_0} = \int E\,\vec{e_r}\cdot\vec{e_r}\, dS \Leftrightarrow$

$\Leftrightarrow \frac{Q}{\varepsilon_0} = E\int dS \Leftrightarrow$

$\Leftrightarrow E = \frac{Q}{\varepsilon_0\int dS} = \frac{Q}{\varepsilon_0\int_0^{2\pi}\int_0^{L} r\,dz\,d\theta} \Leftrightarrow$

$\Leftrightarrow E = \frac{Q}{\varepsilon_0 2\pi r L}$

$\rho = \frac{Q}{V} = \frac{Q}{\pi a^2 L}$

$\vec{E} = \frac{\rho \cancel{\pi} a^2 \cancel{L}}{\varepsilon_0 2\cancel{\pi} r \cancel{L}}\,\vec{e_r}$

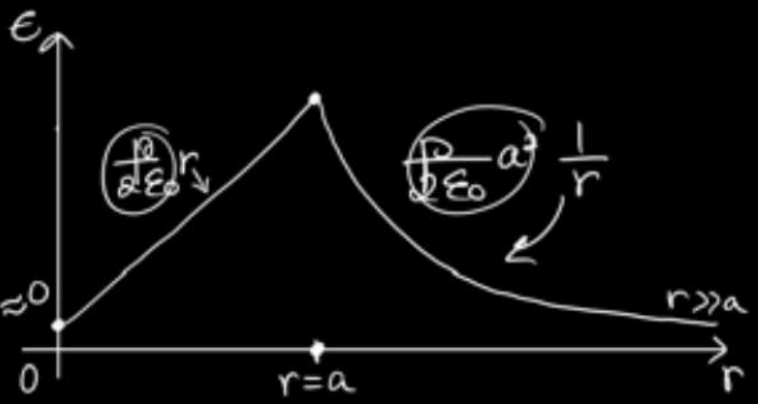
$\vec{E}(r>a) = \frac{\rho}{2\varepsilon_0 r}a^2\,\vec{e_r}.$

b) $\vec{E}(r\leq a) = \frac{\rho\cancel{\pi} r^{\cancel{2}} \cancel{L}}{\varepsilon_0 2\cancel{\pi}\cancel{r}\cancel{L}}\,\vec{e_r}$

$\vec{E}(r<a) = \frac{\rho r}{\ }\,\vec{e_r}.$

$2\varepsilon_0 \quad a$

e)



d) $\underline{r \leq a}$: $\vec{E}(r \leq a) = \frac{\rho}{2\varepsilon_0} r \, \vec{e}_r$

$$-\vec{\nabla}\phi = \vec{E} \Leftrightarrow -\int_0^R \vec{\nabla}\phi \, dr = \int_0^R E \, dr \Leftrightarrow$$

$$\Leftrightarrow -\left(\phi(R) - \phi(0)\right) = \frac{\rho}{2\varepsilon_0}\int_0^R r \, dr \Leftrightarrow$$

$$\Leftrightarrow -\phi(R) + \phi(0) = \frac{\rho}{2\varepsilon_0}\frac{R^2}{2} \Leftrightarrow$$

$$\Leftrightarrow \boxed{\phi(R) = \phi(0) - \frac{\rho}{4\varepsilon_0} R^2}, \; R \leq a$$

- Considerando $\underline{\phi(0) = 0}$,

$$\phi(R) = -\frac{\rho}{4\varepsilon_0} R^2, \; R \leq a$$

→ $\underline{R > a}$:

$$-\vec{\nabla}\phi = \vec{E} \Leftrightarrow -\int_a^R \vec{\nabla}\phi \, dr = \int_a^R E \, dr \Leftrightarrow$$

$$\Leftrightarrow \phi(a) - \phi(R) = \frac{\rho}{2\varepsilon_0} a^2 \int_a^R \frac{1}{r} \, dr \Leftrightarrow$$

$$\Leftrightarrow \phi(R) = \phi(a) - \frac{\rho}{2\varepsilon_0}\left(\log(R) - \log(a)\right) \Leftrightarrow$$

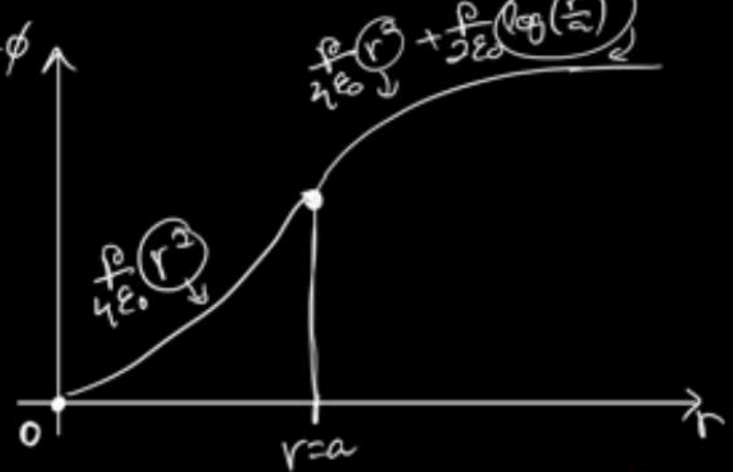
$$\Leftrightarrow \phi(R) = \phi(a) - \frac{\rho}{\ldots} \log\left(\frac{R}{\ldots}\right), \; R > a$$

$\phi(r) = \phi(a) - \frac{\rho}{2\varepsilon_0}\log\left(\frac{r}{a}\right)$

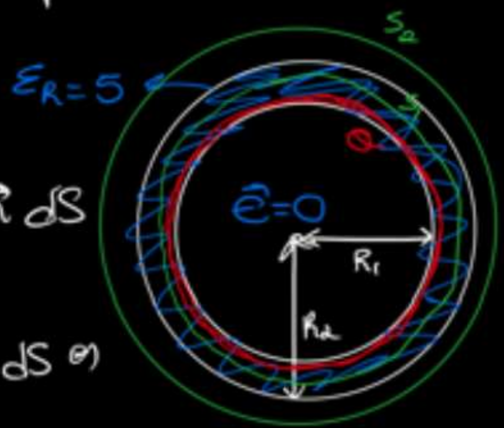
$$\phi(a) = -\frac{\rho}{4\varepsilon_0}a^2 \Rightarrow$$

$$\Rightarrow \boxed{\phi(R) = -\frac{\rho}{4\varepsilon_0}a^2 - \frac{\rho}{2\varepsilon_0}\log\left(\frac{R}{a}\right)}, R>a$$

e)



(37)



a) $Q = \int_S \vec{D} \cdot \vec{n}\, dS$

$Q = \int_S D_r\, \vec{e_r} \cdot \vec{e_r}\, dS \Leftrightarrow$

$\Leftrightarrow Q = D_r \int_S dS \Leftrightarrow \underline{D_r = \frac{Q}{4\pi r^2}}$

$\underline{\vec{D} = D_r \vec{e_r} = \frac{1}{4\pi}\frac{Q}{r^2}\vec{e_r}}.$

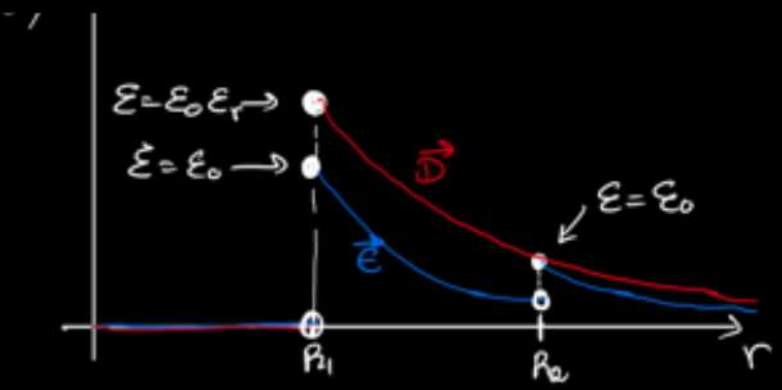
b) $\vec{E}(r<R_1) = 0.$

$\vec{D} = \varepsilon\vec{E} \Rightarrow \vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{Q}{4\pi\varepsilon}\frac{1}{r^2}\vec{e_r}$

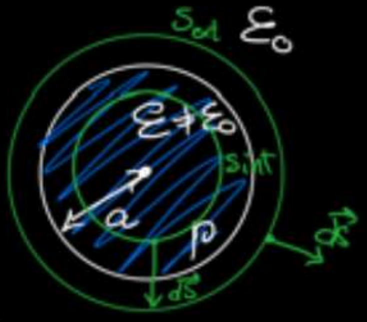
$\vec{E}(R_1<r<R_2) = \frac{Q}{4\pi\varepsilon}\frac{1}{r^2}\vec{e_r}$, $\underline{\varepsilon = \varepsilon_0\varepsilon_r}$

$\vec{E}(r>R_2) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}\vec{e_r}.$

c) ↑



(38)



a) $\vec{E} = E_r \vec{e_r}$

$$\Phi = \frac{Q}{\varepsilon} = \int_S \vec{E} \cdot \vec{dS} \Leftrightarrow$$

$$\Leftrightarrow E_r = \frac{Q}{\varepsilon \int_S \vec{e_r} \cdot \vec{e_r}\, dS} \Rightarrow$$

$$\Rightarrow \vec{E} = E_r \vec{e_r} = \frac{Q}{\varepsilon 4\pi r^2} \vec{e_r}.$$

$$\vec{E} = \begin{cases} \dfrac{Q}{\varepsilon_0 4\pi} \dfrac{1}{r^2} \vec{e_r}, & r > a \\ \dfrac{Q}{\varepsilon 4\pi} \dfrac{1}{r^2} \vec{e_r}, & r < a \end{cases}$$

$$Q(R > a) = \rho \frac{4}{3} \pi a^3$$

$$Q(R < a) = \rho \frac{4}{3} \pi r^3$$

$$\vec{E} = \begin{cases} \dfrac{\rho}{3\varepsilon_0} \dfrac{a^3}{r^2} \vec{e_r}, & r > a \\ \dfrac{\rho}{3\varepsilon} r \vec{e_r}, & r < a \end{cases}$$

b) $-\vec{\nabla}\phi = \vec{E}$

$\underline{R > a:}$ $-\int_r^{+\infty} \vec{\nabla}\phi \cdot \vec{dr} = \int_r^{+\infty} \vec{E} \cdot \vec{dr} \Leftrightarrow$

$$\Leftrightarrow -\phi(+\infty) + \phi(r) = \int E_r\, dr \Leftrightarrow$$

Vr

$$\Leftrightarrow \phi(r)=\int_r^{+\infty}\frac{\rho}{3\varepsilon_0}\frac{a^3}{r^2}\,dr+\phi(+\infty) \Leftrightarrow$$

$$\Leftrightarrow \phi(r)=\frac{\rho}{3\varepsilon_0}a^3\left[\frac{1}{+\infty}-\frac{1}{r}\right]+\phi(+\infty) \Leftrightarrow$$

$$\Leftrightarrow \underline{\phi(r)=-\frac{\rho}{3\varepsilon_0}\frac{a^3}{r}+\phi(+\infty)}.$$

$$(r=a)\Rightarrow \phi(a)=-\frac{\rho}{3\varepsilon_0}a^2+\phi(+\infty)$$

$$\phi(+\infty)=0 \rightarrow \underline{\phi(a)=-\frac{\rho}{3\varepsilon_0}a^2}.$$

$\underline{R<a}$: $$\int_r^a -\vec{\nabla}\phi\, \vec{dr}=\int_r^a \vec{e}\cdot\vec{dr} \Leftrightarrow$$

$$\Leftrightarrow \phi(r)-\phi(a)=\int_r^a e_r\,dr \Leftrightarrow$$

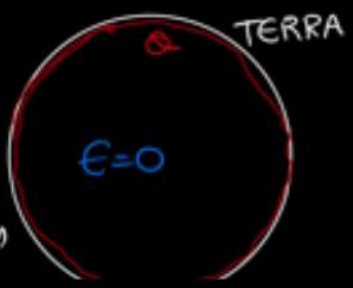
$$\Leftrightarrow \phi(r)-\phi(a)=\frac{\rho}{3\varepsilon}\left[\frac{a^2}{2}-\frac{r^2}{2}\right] \Leftrightarrow$$

$$\Leftrightarrow \phi(r)=\phi(a)+\frac{\rho}{3\varepsilon}\frac{a^2-r^2}{2}$$

$$\begin{cases}\underline{\phi(r)=\frac{\rho}{6\varepsilon}(a^2-r^2)-\frac{\rho}{3\varepsilon_0}a^2} & (R<a)\\ \underline{\phi(r)=-\frac{\rho}{3\varepsilon_0}a^2} & (R>a).\end{cases}$$

39.

a) $$\int_T \vec{D}\cdot\vec{dS}=Q \Leftrightarrow$$



$$\Leftrightarrow \int D_r\,\vec{e_r}\cdot\vec{e_r}\,dS=Q \Leftrightarrow$$

$\Leftrightarrow D_r \int_T dS = Q \Rightarrow \underline{E_r = \frac{Q}{4\pi r^2 \varepsilon}}$

$$-\vec{\nabla}\phi = \vec{E} \Rightarrow \int_r^{+\infty} -\vec{\nabla}\phi \cdot d\vec{r} = \int_r^{+\infty} \vec{E} \cdot d\vec{r} \Leftrightarrow$$

$$\Leftrightarrow -\left(\phi(+\infty) - \phi(r)\right) = \frac{Q}{4\pi\varepsilon} \int_r^{+\infty} \frac{1}{r^2} \vec{e}_r \cdot \vec{e}_r \, dr \Leftrightarrow$$

$$\Leftrightarrow \phi(r) = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{+\infty} + \frac{1}{r}\right]$$

$$\underline{\phi(r) = \frac{Q}{\varepsilon_0 4\pi r}}$$

$$C = \frac{Q}{V} = \frac{Q}{\phi(r) - \phi(+\infty)} = 4\pi r$$

$$C = \frac{Q}{\frac{Q}{4\pi\varepsilon_0 r}} = 4\pi\varepsilon_0 r = \underline{7{,}089 \times 10^{-7} F}$$

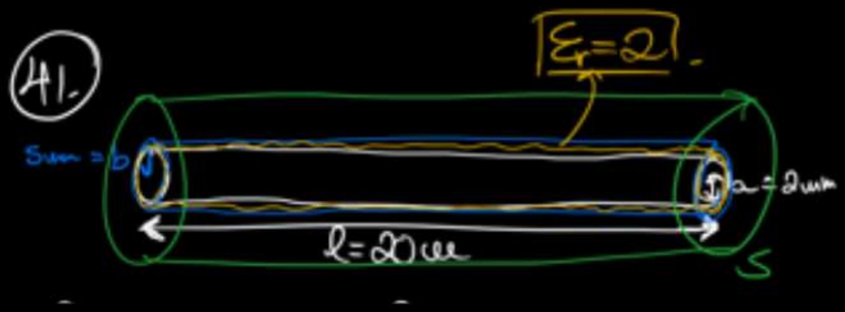
b) $C = 1 \Rightarrow r = \frac{1}{4\pi\varepsilon_0} = \underline{8{,}988 \times 10^{9} m}$

40.

$\left.\begin{array}{l} Q = 5\mu C \\ V = 10{,}0\ V \end{array}\right\}$ a) $C = \frac{Q}{V} = \underline{5 \times 10^{-7} F}$

b) $C = \frac{Q}{V}$, mas como C apenas depende da distância entre os terminais e ε, quando $V\uparrow \Rightarrow Q\uparrow$. Logo $\underline{C = \text{constante}}$.

41.



$$\int_S \vec{D}\cdot d\vec{S} = Q \Leftrightarrow \int_S D_r\, \vec{e_r}\cdot\vec{e_r}\, dS = Q \Leftrightarrow$$

$$\Leftrightarrow \varepsilon E_r \int_S dS = Q \Leftrightarrow \underline{E_r = \frac{Q}{\varepsilon 2\pi r l}}$$

(Desprezando o $\vec{E}$ produzido nas pontas do cilindro, pois $l >>> a$).

$$-\vec{\nabla}\phi = \vec{E} \Rightarrow \int_{r_1}^{r_2} -\vec{\nabla}\phi\cdot d\vec{r} = \int_{r_1}^{r_2} \vec{E}\cdot d\vec{r} \Leftrightarrow$$

$$\Leftrightarrow V = \int_{r_1}^{r_2} E_r\, \vec{e_r}\cdot\vec{e_r}\, dr \Leftrightarrow V = \int_{r_1}^{r_2} E_r\, dr \Leftrightarrow$$

$$\Leftrightarrow V = \int_{r_1}^{r_2} \frac{Q}{2\pi\varepsilon l}\,\frac{1}{r}\, dr \Rightarrow$$

$$\Rightarrow \underline{V = \frac{Q}{2\pi\varepsilon l}\log\left(\frac{R_2}{R_1}\right)}$$

$$V_{ab} \Rightarrow V = \frac{Q}{2\pi\varepsilon l}\log\left(\frac{b}{a}\right) \Rightarrow$$

(entre o condutor e a película metálica exterior)

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

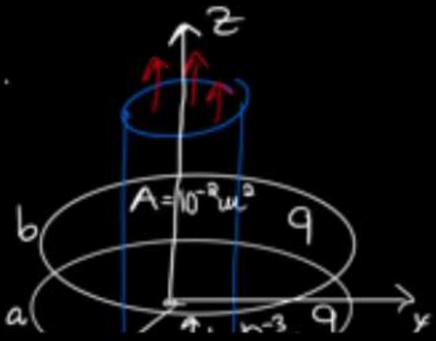
$$\Rightarrow V = \frac{Q}{4\pi l \varepsilon_0}\log\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{\frac{Q}{4\pi l\varepsilon_0}\log\left(\frac{b}{a}\right)} = \frac{4\pi l\,\varepsilon_0}{\log\left(\frac{b}{a}\right)}$$

$$\underline{C = 2{,}43\times10^{-9}\,F}$$

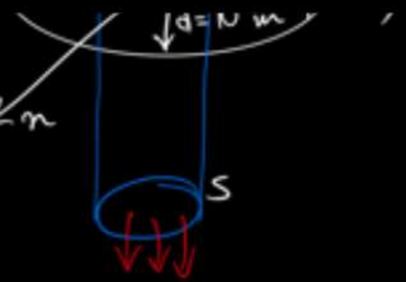
(42.)

a) $\int \vec{D}\cdot d\vec{S} = Q$



V_S

$$\int_S E_z \vec{e_z} \cdot \vec{e_z}\, dS = \frac{Q}{\varepsilon}$$



$$\boxed{E_z = \frac{Q}{\varepsilon A}}$$

↳ $\vec{E}$ independente de z.

$$-\vec{\nabla}\phi = \vec{E} \Rightarrow \int_a^b -\vec{\nabla}\phi \cdot \vec{dz} = \int_a^b \vec{E} \cdot \vec{dz} \Leftrightarrow$$

$$\Leftrightarrow V = E_z \times d \rightarrow V = \frac{dQ}{\varepsilon A}.$$

$$C = \frac{Q}{\frac{dQ}{\varepsilon A}} \rightarrow \boxed{C = \frac{\varepsilon_0 A}{d}}$$

$$C = 8{,}854 \times 10^{-11}\, F.$$

b) $C = 7{,}089 \times 10^{-7}\, F \Rightarrow$

$$\Rightarrow d = \frac{\varepsilon_0 \times A}{7{,}089 \times 10^{-7}} \Rightarrow d = 1{,}249 \times 10^{-7}\, m$$

c) $V = 1{,}5\, V$, $C = 8{,}854 \times 10^{-11}\, F$

$$Q = CV \Rightarrow Q = 1{,}328 \times 10^{-10}\, C.$$

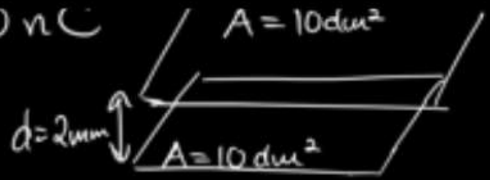
d) $dW = dq\, V$

$$dW = \frac{Q}{C}\, dq \Rightarrow W = \int_0^q \frac{Q}{C}\, dq \Leftrightarrow$$

$$\Leftrightarrow W = \frac{1}{2}\frac{Q^2}{C}\Big|_0^q \Leftrightarrow W = \frac{1}{2}\frac{q^2}{C}.$$

$$W = \frac{1}{2}\frac{(1{,}328 \times 10^{-10})^2}{8{,}854 \times 10^{-10}} = 9{,}96 \times 10^{-12}\, J.$$

(44.) $Q = 10\,nC$



a) $\underline{C} = \frac{\varepsilon_0 A}{d} = \underline{4{,}4 \times 10^{-10}\,F.}$

b) $U = \frac{1}{2}\frac{Q^2}{C} = \underline{1{,}136 \times 10^{-7}\,J.}$

c) $U(2mm) = 1{,}136 \times 10^{-7}\,J$

$U(4mm) = \frac{1}{2} C V_4^2$

$V = \frac{Q}{\varepsilon A} d \Rightarrow V_4 = 2V_2 \;(d_4 = 2d)$

$\hookrightarrow \underline{W = \Delta U = 1{,}136 \times 10^{-7}\,J.}$

$W = \int_2^4 \vec{F_e} \cdot \vec{dr} = F_e d \Rightarrow F_e = \frac{W}{d}$

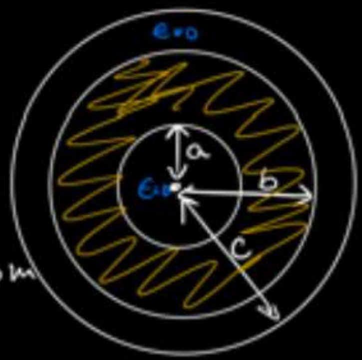
$\underline{F_e = 5{,}68 \times 10^{-8}\,N.}$

(45.) $Q_1 = 5\,nC$

$Q_2 = 2\,nC$

$a = 0{,}01\,m$

$b = 0{,}02\,m\,;\; c = 0{,}025\,m$



a) $\int_S \vec{D} \cdot \vec{dS} = Q \Rightarrow D_r \int_S dS = Q \Leftrightarrow$

$\Leftrightarrow \boxed{E_r = \frac{Q}{\varepsilon 4\pi r^2}}$

$-\vec{\nabla}\phi = \vec{E} \Rightarrow \int_a^b -\vec{\nabla}\phi \cdot \vec{dr} = \int_a^b \vec{E} \cdot \vec{dr} \Leftrightarrow$

$\Leftrightarrow V_{ab} = \int^b \frac{Q}{\varepsilon 4\pi r^2}\, dr \Rightarrow$

V_a ...

$$\Rightarrow V_{ab} = \frac{Q}{\varepsilon 4\pi}\left[-\frac{1}{b}+\frac{1}{a}\right]$$

$$V_{ab} = \frac{Q}{\varepsilon 4\pi}\left[-\frac{a-b}{ba}\right] \Rightarrow$$

$$\Rightarrow V_{ab} = -\frac{Q}{4\pi\varepsilon}\,\frac{a-b}{ab}$$

$$C = \frac{Q}{V_{ab}} = \frac{4\pi\varepsilon ab}{b-a}.$$

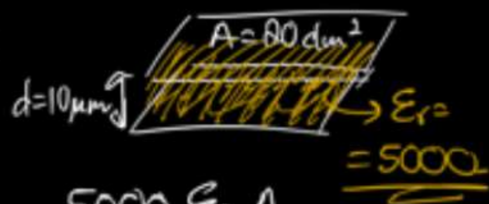
$\underline{\varepsilon = \varepsilon_0.}$ $\Rightarrow$ $\underline{C = 2{,}226\times10^{-12}\,F.}$

b) $U = \frac{1}{2}\frac{Q^2}{V} = \frac{1}{2}QC =$ SOLUÇÃO

$= \frac{1}{2}\times 7\times10^{-9}\times 2{,}226\times10^{-12} =$ (?)

c) Não. Porque haveria de ser?
Não há nada que o anule.
As cargas do interior produzem
campo em todo lado.

(46) $V = 12V.$



a) $C = \frac{\varepsilon A}{d} = \frac{5000\,\varepsilon_0 A}{d}$

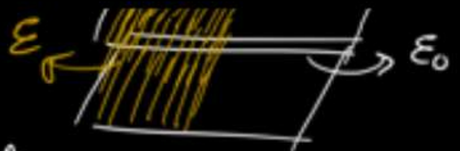
$$\underline{C = 8{,}8541\times10^{-4}\,F.}$$

b) $C = \frac{Q}{V} \Rightarrow Q = CV = \underline{0{,}01063\,C}.$

$U = \frac{1}{2}Q^2\frac{1}{V} = \frac{1}{2}CV^2 = \underline{0{,}06375\,J.}$

(47)

a) $C = C_1 + C_2$



$$C = \frac{\varepsilon \frac{A}{2}}{d} + \frac{\varepsilon_0 \frac{A}{2}}{d} = (\varepsilon + \varepsilon_0)\frac{A}{2d}$$

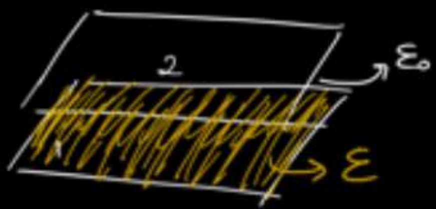
b) $$U = \frac{1}{2}\frac{Q^2}{C} \Rightarrow \begin{cases} U_f = \frac{1}{2} Q^2 \frac{d \times 2}{(\varepsilon + \varepsilon_0)A} \\ U_0 = \frac{1}{2} Q^2 \frac{d}{\varepsilon A} \end{cases}$$

$$\frac{U_f}{U_0} = \frac{\frac{2}{\varepsilon + \varepsilon_0}}{\frac{1}{\varepsilon}} = \frac{2\varepsilon}{\varepsilon + \varepsilon_0} < 1 \Rightarrow$$

↳ $\varepsilon >> \varepsilon_0 \Rightarrow \frac{U_f}{U_0} \approx 2$

Pelo que ligar o condensador em paralelo aumenta C e aumenta a energia acumulada no interior

48. $V_1 \neq V_2$
$(\varepsilon \neq \varepsilon_0)$



a) $\vec{E} = E_z \vec{e}_z$

$$E_z = \frac{Q}{\varepsilon A} \Rightarrow -\vec{\nabla}\phi = \vec{E} \Rightarrow$$

$$\Rightarrow \int -\vec{\nabla}\phi \cdot d\vec{z} = \int \vec{E} \cdot d\vec{z} \Rightarrow$$

$$\Rightarrow V = \int E_z \, dz = \frac{Q}{\varepsilon A}\frac{d}{2}$$

$$C = \frac{Q}{V} \hookrightarrow \begin{cases} C_1 = \frac{\varepsilon A}{d} \times 2 \\ C_2 = \frac{\varepsilon_0 A}{d} \times 2 \end{cases}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$, pois estão em série $(V_1 \neq V_2)$

↓

$$\frac{1}{C} = \frac{C_2 + C_1}{\ldots} \Rightarrow C = \frac{C_1 C_2}{\ldots}$$

$$\frac{1}{C} = \frac{\quad}{C_1 C_2} \Rightarrow C = \frac{\quad}{C_1 + C_2} \Rightarrow$$

$$\Rightarrow C = \frac{\frac{4\varepsilon\varepsilon_0 A^2}{d^2}}{(\varepsilon+\varepsilon_0)A \times 2} = \underline{\frac{2\varepsilon\varepsilon_0 A}{(\varepsilon+\varepsilon_0)d}}.$$

$$\varepsilon \gg \varepsilon_0 \Rightarrow C \approx \frac{2\varepsilon_0 A}{d} =$$

b) $$U_{\varepsilon_f} = \frac{1}{2}\frac{Q^2}{C_f} = \frac{1}{2}\frac{Q^2 d}{2\varepsilon_0 A} = \frac{1}{4}\frac{Q^2 d}{\varepsilon_0 A}$$

$$U_{\varepsilon_0} = \frac{1}{2}\frac{Q^2}{C_0} = \frac{1}{2}\frac{Q^2 d}{\varepsilon A}$$

$$\frac{U_{\varepsilon f}}{U_{\varepsilon_0}} = \frac{\frac{1}{4}\frac{\cancel{Q^2}\cancel{d}}{\varepsilon_0 \cancel{A}}}{\frac{1}{2}\frac{\cancel{Q^2}\cancel{d}}{\varepsilon \cancel{A}}} = \frac{1}{2}\frac{\varepsilon}{\varepsilon_0} \Rightarrow$$

$$\Rightarrow \underline{U_{\varepsilon f} \approx \frac{1}{2}\frac{\varepsilon}{\varepsilon_0} U_{\varepsilon_0}}.$$

(49)

a) $$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{C^2}{2C} = \frac{1}{2}C = \underline{1\mu F}.$$

b) $$C = C_1 + C_2 = 2C = \underline{4\mu F}.$$

(50)

a) Desprezando o $\vec{E}$ criado pelas pontas do cabo, por ser muito comprido:

$$\vec{E} = E_r \vec{e_r}$$

$$\int_S \vec{D} \cdot d\vec{S} = Q \Rightarrow E_r = \frac{Q}{\varepsilon \times A_{\square}} = \frac{Q}{2\pi r \ell \varepsilon}$$

$$\cap b \qquad \cap b$$

$$-\vec{\nabla}\phi = \vec{E} \Rightarrow \int_a^r -\vec{\nabla}\phi \cdot d\vec{r} = \int_a^r \vec{E} \cdot d\vec{r} \Rightarrow$$

$$\Rightarrow V_{ab} = \int_a^b \frac{Q}{2\pi\varepsilon l}\frac{1}{r}\,dr = \underline{\frac{Q}{2\pi\varepsilon l}\log\left(\frac{b}{a}\right)}$$

$$C = \frac{Q}{V_{ab}} = \frac{2\pi\varepsilon_0 l}{\log\left(\frac{b}{a}\right)}. \qquad \left(\underline{\varepsilon = \varepsilon_0.}\right)$$

$$C/m = C/l = \frac{2\pi\varepsilon_0}{\log\left(\frac{b}{a}\right)}.$$

b) $\frac{C}{l} = \frac{\frac{Q}{l}}{V} \Rightarrow Q/l = \frac{2\pi\varepsilon_0}{\log\left(\frac{b}{a}\right)} \times \overset{=9}{V}$

c) $U_E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{\left(\frac{2\pi\varepsilon_0 V}{\log(b/a)}l\right)^2}{\frac{2\pi\varepsilon_0}{\log(b/a)}l} =$

$$= \frac{1}{2}\frac{\frac{2^2\pi^2\varepsilon_0^2 V^2 l^2}{\log(b/a)^2}}{\frac{2\pi\varepsilon_0 l}{\log(b/a)}} = \frac{\pi\varepsilon_0 V^2 l}{\log(b/a)}$$

$$u_E = \frac{U_E}{V_B} = \frac{U_E}{\pi r^2 l} = \underline{\frac{\varepsilon_0 V^2}{r^2\log(b/a)}}.$$

(PORQUE NÃO?)

c) $u_E = \frac{1}{2}\varepsilon_0 E_r^2 = \frac{1}{2}\varepsilon_0\frac{Q^2}{r^2}\left(\frac{1}{2\pi l\varepsilon_0}\right)^2$

$$Q = \frac{2\pi\varepsilon_0 l V}{\log(b/a)} \Rightarrow$$

$$\rightarrow u_E = \frac{\varepsilon_0}{2r^2}\left(\frac{\cancel{2\pi\varepsilon_0 l}V}{\cancel{2\pi l\varepsilon_0}\log(b/a)}\right)^2 = \frac{\varepsilon_0 V^2}{2\log(b/a)^2}\frac{1}{r^2}.$$

$\hookrightarrow \underline{u_E = 7{,}46\times10^{-10}\ J/m^3}$

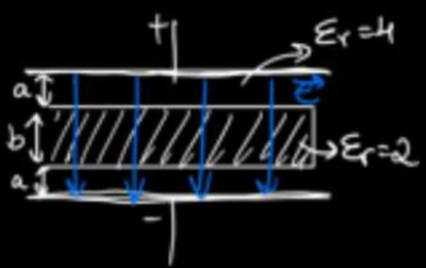
d) $\cancel{u_E(J/m) = u_E \times \pi r^2 = \frac{\pi \varepsilon_0 V^2}{2\log(b/a)}}$

$$U_E = \frac{1}{2} C V^2 = \frac{1}{2}\frac{2\pi \varepsilon_0 l}{\log(b/a)} V^2 = \frac{\pi \varepsilon_0 l}{\log(b/a)} V^2$$

↳ CORRECTED.

51.

$a = 0{,}025\,m$
$b = 0{,}05\,m$
$V = 45\,V$



a) $\int -\vec{\nabla}\phi \cdot \vec{dz} = V = \int \vec{E} \cdot \vec{dz} = \int E_z dz$

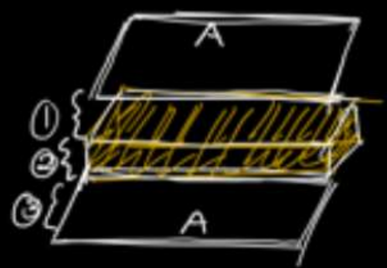
$$V = E_z \int dz = E_z (a+b) \Rightarrow$$

$$\Rightarrow E_z = \frac{V}{a+b} = \frac{45}{0{,}05 + 0{,}025} \Rightarrow \underline{E_z = 600\ V m^{-1}}$$

b) $A = 45\ cm^2 = \underline{4{,}5 \times 10^{-3}\, m^2}$

$$C = \frac{Q}{V} = \frac{\varepsilon A}{d}$$

$$C_1 = \frac{4\varepsilon_0 A}{a} = C_3$$



$$C_2 = \frac{2\varepsilon_0 A}{b}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_1 + C_2 + C_3}{C_1 C_2 C_3} \Rightarrow$$

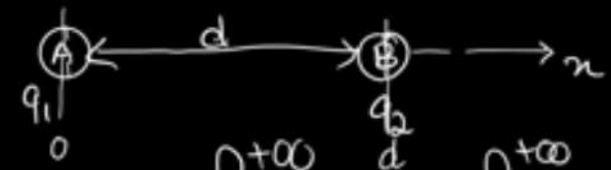
$$\Rightarrow C = \frac{C_1 C_2 C_3}{C_1 + C_2 + C_3} = \frac{32 \frac{(\varepsilon_0 A)^3}{a^2 b}}{8\frac{\varepsilon_0 A}{a} + \frac{2\varepsilon_0 A}{b}} =$$

$$\cancel{2\varepsilon_0 A}\left(16 \frac{(\varepsilon_0 A)^2}{\ldots}\right) \quad 16 \frac{(\varepsilon_0 A)^2}{\ldots}$$

$$= \frac{[illegible]}{2\varepsilon_0 A\left(\frac{4}{a}+\frac{1}{b}\right)} = \frac{[illegible]}{\frac{4b+a}{ab}} =$$

$$= \frac{16}{(4b+a)a} \times (\varepsilon_0 A)^2 = (?)$$

52.



- $-\vec{\nabla}\phi = \vec{E} \Rightarrow \int_0^{+\infty} -\vec{\nabla}\phi \cdot dx = \int_0^{+\infty} \vec{E} \cdot dx \Leftrightarrow$

$\Leftrightarrow \phi(a) - \phi(+\infty) = \int_0^{+\infty} \frac{1}{4\pi\varepsilon_0} \frac{q_1}{(d-x)^2}\, dx \Leftrightarrow$

$\Leftrightarrow \phi(a) = \frac{q_1}{4\pi\varepsilon_0}\left[-\frac{1}{+\infty} + \frac{1}{d}\right]$

$\phi(a) = \frac{q_1}{4\pi\varepsilon_0}\frac{1}{d}.$

- $\int_d^{+\infty} -\vec{\nabla}\phi \cdot \vec{dx} = \int_d^{+\infty} \vec{E} \cdot \vec{dx} \Leftrightarrow$

$\Leftrightarrow \phi(b) - \phi(+\infty) = \int_d^{+\infty} \frac{1}{4\pi\varepsilon_0}\frac{q_2}{x^2}\, dx \Leftrightarrow$

$\Leftrightarrow \phi(b) = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{d}.$

b) $U_E = \int_a^b \vec{F_e} \cdot \vec{dx} = F_e \times d =$

$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2}\, d = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d}.$

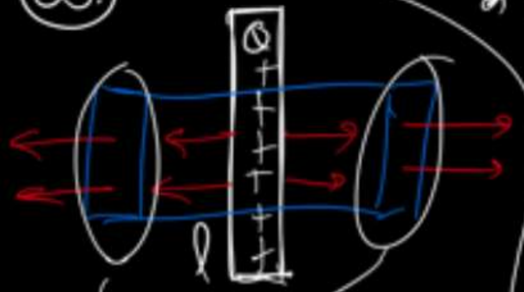
c) $U_e = F_e \times d \Rightarrow F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2}$

d) $\Delta U = 0 \Rightarrow 0 = q_3(V_1 + V_2) \Rightarrow \frac{q_3 q_1}{r_{1,3}} = -\frac{q_3 q_2}{r_{2,3}}$

$\Rightarrow \frac{-q_3 q_2}{r_{1,3}} = -\frac{q_3 q_2}{r_{2,3}} \Rightarrow$

$\Rightarrow \underline{r_{1,3} = r_{2,3}}$

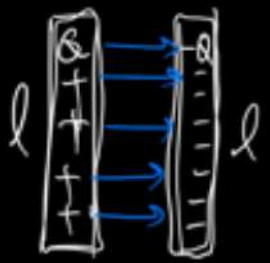
53.



a) $2\int_S \vec{D} \cdot \vec{dS} = Q \Rightarrow E = \frac{Q}{2\varepsilon_0 \int_S dS} = \frac{Q}{2\varepsilon_0 A}$

$$\underline{\vec{E} = \frac{Q}{2\varepsilon_0 l^2}\vec{e_n}}$$

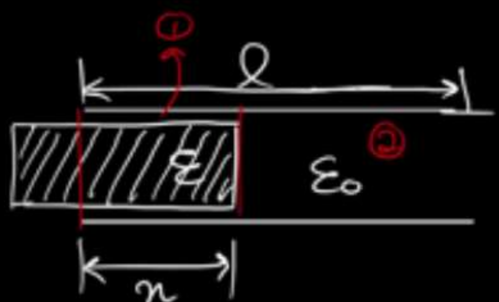
b)



$F_e = (-Q)E_Q \Rightarrow$

$$\Rightarrow \underline{\vec{F_e} = -\frac{Q^2}{2\varepsilon_0 l^2}\vec{e_n}}.$$

54.



a) $C = C_1 + C_2$, $A_1 \neq A_2$

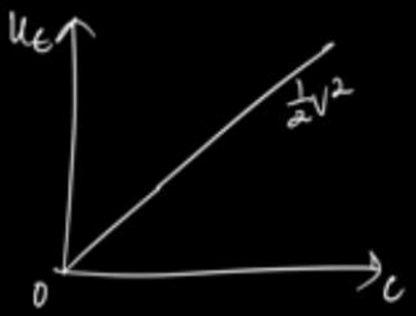
$A_1 = xl$; $A_2 = (l-x)l$

$C_1 = \frac{\varepsilon A_1}{d} = \frac{\varepsilon x l}{d}$

$C_2 = \frac{\varepsilon_0 A_2}{d} = \frac{\varepsilon_0 (l-x) l}{d}$

$$C = \frac{l}{d}\left(\varepsilon x + \varepsilon_0 (l-x)\right).$$

b) $U_E = \frac{1}{2} C V^2 \Rightarrow U_E = \underbrace{\left(\frac{1}{2}V^2\right)}_{\text{const}} C$

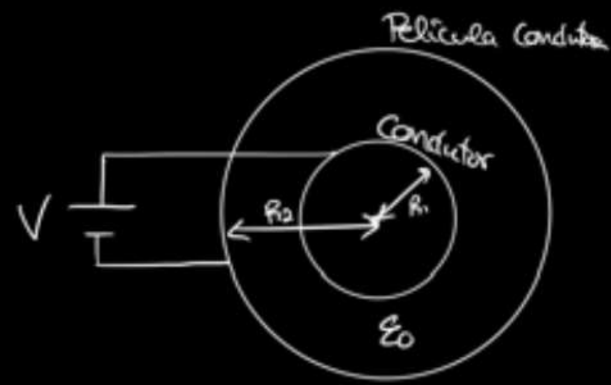


c) $\underline{V = \text{const}} \Rightarrow$ F interior:

$$\vec{F_e} = \frac{dU_E}{dx}\vec{e_x} = \frac{1}{2}V^2 \frac{d}{dx}\left(\frac{l}{d}\left(\varepsilon x + \varepsilon_0 (l-x)\right)\right)\vec{e_x}$$

$$= \frac{1}{2}V^2 \frac{l}{d}(\varepsilon - \varepsilon_0)\vec{e_x}$$

55.



a) $\underline{V = \text{const.}}$

$$U_E = \frac{1}{2} C V^2$$

$$C = \frac{Q}{V}, \quad V = \int_{R_1}^{R_2} \vec{E}\cdot d\vec{r} = \frac{Q}{4\pi\varepsilon_0}\int_{R_1}^{R_2} \frac{1}{r^2}\, dr \Leftrightarrow$$

$$\Leftrightarrow V = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{R_2} + \frac{1}{R_1}\right] =$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[-\frac{R_1 - R_2}{R_1 R_2}\right] = \frac{Q}{4\pi\varepsilon_0}\frac{R_2 - R_1}{R_1 R_2}$$

$4\pi\varepsilon_0 R_1 R_2$

$$C = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}$$

$$U_E = \frac{2\pi\varepsilon_0 R_1 R_2}{R_2 - R_1} V^2.$$

b) $P = \frac{F}{A}$, $A = \pi R_2^2$

$F_e = \frac{dU_E}{dr}$, $R_2 - R_1 = r \Rightarrow$

$\Rightarrow U_E = \frac{2\pi\varepsilon_0 (R_1(r+R_1))}{r} V^2$

$\Downarrow$

$F_e = -\frac{\pi\varepsilon_0 R_1^2}{r^2} V^2 \quad \Big| = V^2 \left(\frac{2\pi\varepsilon_0 R_1 r}{r} + \frac{2\pi\varepsilon_0 R_1^2}{r} \right)$

$$P = \frac{F}{A} = -\varepsilon_0 V^2 \frac{R_1^2}{R_2^2} \frac{1}{(R_2 - R_1)^2}.$$

50

a) $\frac{C}{l} = ?$ $\quad\Big|\quad \int_S \vec{D} \cdot d\vec{S} = Q -$

$C = \frac{Q}{V}$

$$E_r = \frac{Q}{\varepsilon} \frac{1}{\int_S dS} = \frac{Q}{\varepsilon 2\pi r l}.$$

$$-\vec{\nabla}\phi = \vec{E} \Rightarrow V = \int_a^b \frac{Q}{\varepsilon 2\pi l} \frac{1}{r} dr \Rightarrow$$

$$\Rightarrow V = \frac{Q}{2\pi l \varepsilon} \log\left(\frac{b}{a}\right).$$

$$C = \frac{2\pi l \varepsilon}{\log(b/a)} \longrightarrow \frac{C}{l} = \frac{2\pi\varepsilon}{\log(b/a)}.$$

b) $\frac{Q}{l} = ?$ $C = \frac{Q}{V} \Leftrightarrow \frac{C}{l} = \frac{\frac{Q}{l}}{V} \Leftrightarrow$

$\Leftrightarrow \frac{Q}{l} = \frac{2\pi\varepsilon}{\log(b/a)} \times V = \underline{\underline{\frac{18\pi\varepsilon}{\log(b/a)}}}$

c) $u_E = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}\varepsilon\left(\frac{Q}{\varepsilon 2\pi r l}\right)^2 =$

$= \frac{1}{2}\cancel{\varepsilon}\frac{Q^2}{\varepsilon^{\cancel{2}} 4\pi^2 r^2 l^2} = \underline{\underline{\frac{Q^2}{8\varepsilon\pi^2 r^2 l^2}}}$.

$Q = CV = \frac{2\pi l \varepsilon}{\log(b/a)} V \Rightarrow$

$\Rightarrow u_E = \frac{\cancel{4\pi^2 l^2}\varepsilon^{\cancel{2}} V^2}{8\cancel{\varepsilon}\cancel{\pi^2} r^2 \cancel{l^2}\log(b/a)^2} = \frac{1}{2}\frac{\varepsilon_0 V^2}{\log(b/a)^2 r^2}$

$u_E = \frac{0{,}746 \times 10^{-9}}{r^2}$ J/m².

d) $\int_V u_E\, dV = \int_0^{2\pi}\int_a^b\int_0^l u_E\, dl\, dr\, d\theta =$

$= 2\pi\int_a^b \frac{1}{2}\frac{\varepsilon_0 V^2}{\log(b/a)^2}\frac{1}{r^2} r\, dr =$

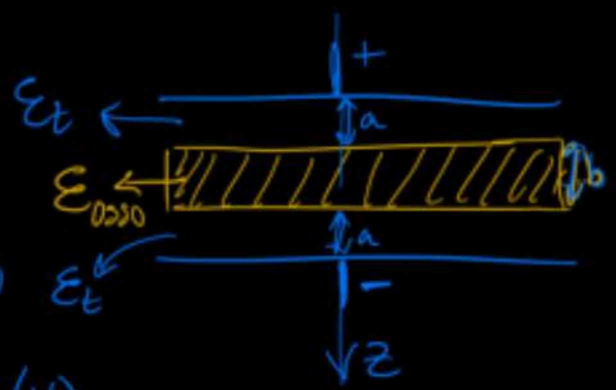
$= \frac{\pi\varepsilon_0 V^2}{\log(b/a)^2}\left[\log(r)\right]_a^b = \underline{\underline{\frac{\pi\varepsilon_0 V^2}{\log(b/a)}}}$.

$U_E = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{2\pi\varepsilon}{\log(b/a)}\right)V^2 =$

$= \underline{\underline{\frac{\pi\varepsilon V^2}{\log(b/a)}}} \Rightarrow \underline{\int_V u_E\, dV = U_E}$.

(51.)

$$\begin{cases} \varepsilon_t = 4\varepsilon_0 \\ \varepsilon_{osso} = 2\varepsilon_0 \end{cases}$$



$\underline{V = 45\ (V).}$

a) $$\int_S \vec{D}\cdot d\vec{S} = \int_S \varepsilon \mathcal{E}\, \vec{e}_z \cdot \vec{e}_z\, dS = Q \Rightarrow$$

$$\Rightarrow \mathcal{E} = \frac{Q}{\varepsilon}\frac{1}{\int_S dS} \Rightarrow \boxed{\mathcal{E} = \frac{\sigma}{\varepsilon}}.$$

$$\Rightarrow \begin{cases} \mathcal{E}_t = \dfrac{\sigma}{\varepsilon_t} \\ \mathcal{E}_{osso} = \dfrac{\sigma}{\varepsilon_{osso}} \end{cases}$$

$$-\vec{\nabla}\phi = \vec{\mathcal{E}} \Rightarrow V = \int \vec{\mathcal{E}}\cdot dz(\vec{e}_z)$$

$$\Rightarrow V = \mathcal{E}_t (2a) + \mathcal{E}_{osso}\, b \Rightarrow$$

$$\Rightarrow V = \frac{2a\sigma}{\varepsilon_t} + \frac{\sigma b}{\varepsilon_{osso}} \Rightarrow$$

$$\Rightarrow \sigma = \frac{V}{\frac{2a}{4\varepsilon_0} + \frac{b}{2\varepsilon_0}} = \frac{2V\varepsilon_0}{a+b}$$

$$\mathcal{E}_{osso} = \frac{\cancel{2}V\cancel{\varepsilon_0}}{a+b\,(\cancel{2}\cancel{\varepsilon_0})} = \underline{\frac{V}{a+b}}.$$

b) $$\frac{1}{C} = \frac{2}{C_t} + \frac{1}{C_{osso}} \Rightarrow$$

$$\Rightarrow C = \frac{1}{\frac{2}{C_t} + \frac{1}{C_{osso}}}$$

$$C_t = \frac{Q}{\ldots} = \frac{Q}{\ldots} = \frac{Q}{\ldots} = \frac{Q\,\varepsilon_t}{Q\ldots} = \frac{A\varepsilon_t}{a}$$

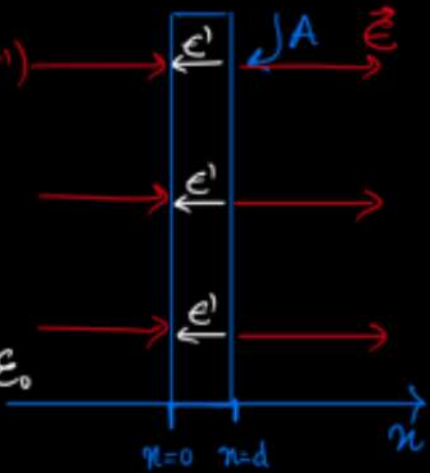
$C_t = \frac{Q}{V_t} = \frac{Q}{E a} = \frac{Q}{\frac{\sigma}{\varepsilon_t} a} = \frac{Q}{\frac{Q}{A} a} = $

$$C_{osso} = \frac{Q}{V_{osso}} = \frac{Q}{E b} = \frac{Q}{\frac{\sigma}{\varepsilon_{osso}} b} = \frac{Q\varepsilon_{osso}}{\frac{Q}{A} b} = \frac{A\varepsilon_{osso}}{b}$$

$$C = \frac{1}{\frac{2A\varepsilon_t}{a} + \frac{A\varepsilon_{osso}}{b}} = \frac{ab}{A\varepsilon_0(8b+2a)} =$$

$$= \frac{a(2a)}{A\varepsilon_0(16a+2a)} = \frac{a}{A\varepsilon_0 \times 9} =$$

31. $\vec{E} = E_0 \vec{e}_n \ (\mathrm{V\,m^{-1}})$



a) $\vec{E}_{int} = \vec{E} + \vec{E}'$

$\underline{\vec{E}_{int} = 0}$

b) $E = \frac{\sigma}{\varepsilon_0} \Rightarrow \sigma = E\varepsilon_0$

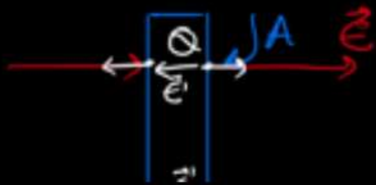
$\sigma(x=0) = -E_0\varepsilon_0$

$\sigma(x=d) = E_0\varepsilon_0$

$\vec{E}'_{int} = -E_0\vec{e}_n$

$\vec{E}_{ext} = 0$

c) $\vec{E}_{int} = 0$



$\vec{E}_{ext} = \vec{E} + \vec{E}_Q$