$$P\left(\frac{1}{(x^2+1)^2}\right)$$

1ª resolução – por substituição:

$$x = tg(t) = g(t) \rightarrow t = arctg(x)$$

$$g'(t) = \frac{1}{\cos^2 t}$$

$$= P\left(\frac{1}{(tg^2 t+1)^2}\frac{1}{\cos^2 t}\right) = P\left(\frac{1}{(\sec^2 t)^2}\sec^2 t\right) = P\left(\frac{1}{\sec^2 t}\right) = P(\cos^2 t) = P\left(\frac{1+\cos(2t)}{2}\right) =$$

$$= P\left(\frac{1}{2}\right) + P\left(\frac{1}{2}\cos(2t)\right) = \frac{1}{2}t + \frac{1}{4}sen(2t)$$

$$(\quad fórmulas \quad de \quad duplicação) : sen(2t) = \frac{2tgt}{1+tg^2 t} = \frac{2x}{1+x^2}$$

$$Então:$$

$$= \frac{1}{2}arctg(x) + \frac{1}{4}\frac{2x}{1+x^2} = \frac{1}{2}arctg(x) + \frac{1}{2}\frac{x}{(1+x^2)} + C$$

2ª resolução – por partes:

$$P\left(\frac{1}{(x^2+1)^2}\right) = P\left(\frac{1+x^2-x^2}{(x^2+1)^2}\right) = P\left(\frac{1+x^2}{(x^2+1)^2} - \frac{x^2}{(x^2+1)^2}\right) = P\left(\frac{1}{x^2+1} - \frac{x^2}{(x^2+1)^2}\right) =$$

$$P\left(\frac{1}{x^2+1}\right) - \frac{1}{2}P\left(x\frac{2x}{(x^2+1)^2}\right) =$$

$$u = x \rightarrow u' = 1$$

$$v' = \frac{2x}{(x^2+1)^2} \rightarrow v = \frac{(1+x^2)^{-1}}{-1} = -\frac{1}{1+x^2}$$

$$= arctg(x) - \frac{1}{2}\left[-\frac{x}{1+x^2} - P\left(-\frac{1}{1+x^2}\right)\right] = arctg(x) - \frac{1}{2}\left[-\frac{x}{1+x^2} + P\left(\frac{1}{1+x^2}\right)\right] =$$

$$= arctg(x) + \frac{1}{2}\frac{x}{(1+x^2)} - \frac{1}{2}arctg(x) = \frac{1}{2}arctg(x) + \frac{1}{2}\frac{x}{(1+x^2)} + C$$