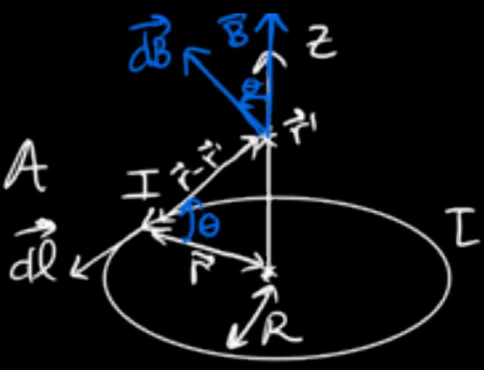
① $R = 0{,}02\,m$

$I = 5\times10^{-3}\,A$



a) $\begin{cases} \vec{r} = x\,\vec{e_x} + y\,\vec{e_y} \\ \vec{r}' = z\,\vec{e_z} \end{cases}$ $\qquad \cos\theta = \dfrac{B}{dB} = \dfrac{|\vec{r}|}{|\vec{r}-\vec{r}'|}$

$$\vec{r}-\vec{r}' = x\,\vec{e_x} + y\,\vec{e_y} - z\,\vec{e_z}$$

$$|\vec{r}-\vec{r}'| = (x^2+y^2+z^2)^{1/2} = (R^2+z^2)^{1/2}$$

$$\vec{dB} = \frac{\mu_0}{4\pi}\,\frac{I\,\vec{dl}\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\vec{B} = \int_I \vec{dB} = \int_I dB_z\,\vec{e_z}\ , \quad \int_I dB_x + dB_y = 0$$

$$dB = \frac{\mu_0}{4\pi}\,\frac{I\,dl}{|\vec{r}-\vec{r}'|^2} \Rightarrow dB_z = \frac{\mu_0}{4\pi}\,I\,\frac{dl\cos\theta}{|\vec{r}-\vec{r}'|^2}$$

$$dB_z = \frac{\mu_0}{4\pi}\,I\,dl\,\frac{R}{(R^2+z^2)^{3/2}}$$

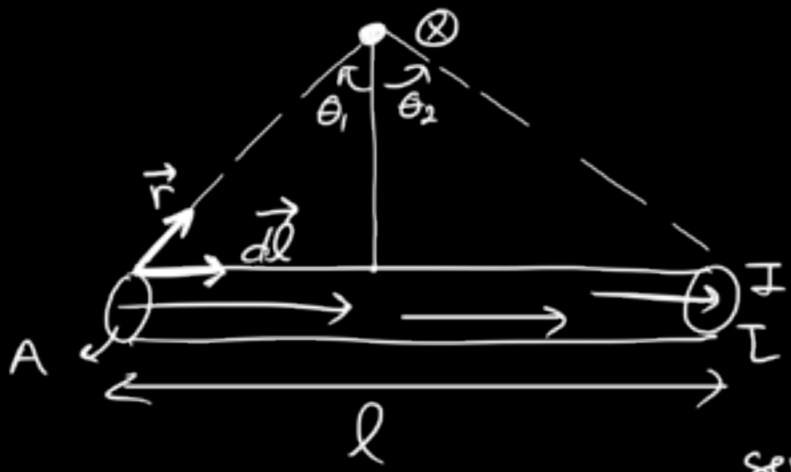
$$\vec{B} = \int_I dB_z\,\vec{e_z} = \frac{\mu_0}{4\pi}\,I\,\frac{R}{(R^2+z^2)^{3/2}}\int_0^{2\pi} R\,d\theta\,\vec{e_z}$$

$\Rightarrow \quad \mu_0 \quad I \quad R^2 \quad \to$

$$\vec{B} = \frac{\mu_0}{2} I \frac{R^2}{(R^2+z^2)^{3/2}} \vec{e}_z .$$

b) $$\underset{(z=0,2)}{\vec{B}(0)} = \frac{\mu_0}{2} \frac{(0{,}02)^2}{(0{,}02^2+0{,}2^2)^{3/2}} \vec{e}_z .$$

(2)



$$\vec{dB} = \frac{\mu_0}{4\pi} I \frac{\vec{dl} \times \vec{r}}{|\vec{r}|^3} = \frac{\mu_0}{4\pi} I \frac{dl\,|\vec{r}|\overset{\operatorname{sen}(\frac{\pi}{2}-\theta)}{\overset{\shortparallel}{\cos\theta}}}{|\vec{r}|^3} \vec{e}_\otimes$$

$$\vec{dB} = \frac{\mu_0}{4\pi} I \frac{dl \cos\theta}{|\vec{r}|^2}$$

$$\vec{B} = \int_L \vec{dB} = \frac{\mu_0}{4\pi} I \vec{e}_\otimes \int_L \frac{\cos\theta}{|\vec{r}|^2} dl =$$

$$= \frac{\mu_0}{4\pi} I \vec{e}_\otimes \int_{\theta_1}^{\theta_2} \frac{\cos\theta}{|\vec{r}|^2} |\vec{r}|\, d\theta =$$

$$= \frac{\mu_0}{4\pi} I \vec{e}_\otimes \frac{1}{|\vec{r}|} (\operatorname{sen}\theta_1 - \operatorname{sen}\theta_2)$$

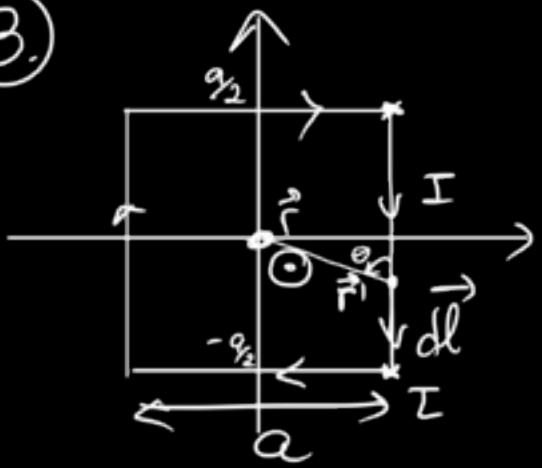
- $l \to +\infty \Rightarrow \theta_1 \to \frac{\pi}{2} \wedge \theta_2 \to -\frac{\pi}{2}$.

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \frac{1}{|\vec{r}|}\left(\operatorname{sen}\frac{\pi}{2} + \operatorname{sen}\left(\frac{\pi}{2}\right)\right)\vec{e}_\otimes$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{2\pi}\frac{I}{|\vec{r}|}\vec{e}_\otimes$$

$$\vec{B}(|\vec{r}|=0{,}05) = \frac{\mu_0}{2\pi}\frac{I}{0{,}05}\vec{e}_\otimes = \underline{64\,\mu T\,\vec{e}_\otimes}$$

3.



$$\begin{cases} \vec{r} = 0 \\ \vec{r}' = x'\vec{e}_x + y'\vec{e}_y \end{cases}$$

$$\vec{B} = \frac{\mu_0}{4\pi} I \int_{\Gamma} \frac{\vec{dl} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} =$$

$$= \frac{\mu_0}{4\pi} I \int_{\Gamma} \frac{\vec{dl} \times (x'\vec{e}_x + y'\vec{e}_y)}{(x'^2 + y'^2)^{3/2}} =$$

$$= -\frac{\mu_0}{4\pi} I \int_{\Gamma} \frac{dy'\, x'\, \vec{e}_\odot}{(x'^2 + y'^2)^{3/2}} =$$

$$= -\frac{\mu_0}{4\pi} I \int_{-\frac{a}{2}}^{a/2} \frac{x'\vec{e}_\odot}{(x'^2 + y'^2)^{3/2}}\, dy' =$$

$$= -\frac{\mu_0}{4\pi} I \frac{a}{2} \vec{e}_\Theta \left[\frac{1}{y'^2} \frac{y'}{\sqrt{y'^2 + \left(\frac{a}{2}\right)^2}} \right]_{y'=-\frac{a}{2}}^{y'=a} =$$

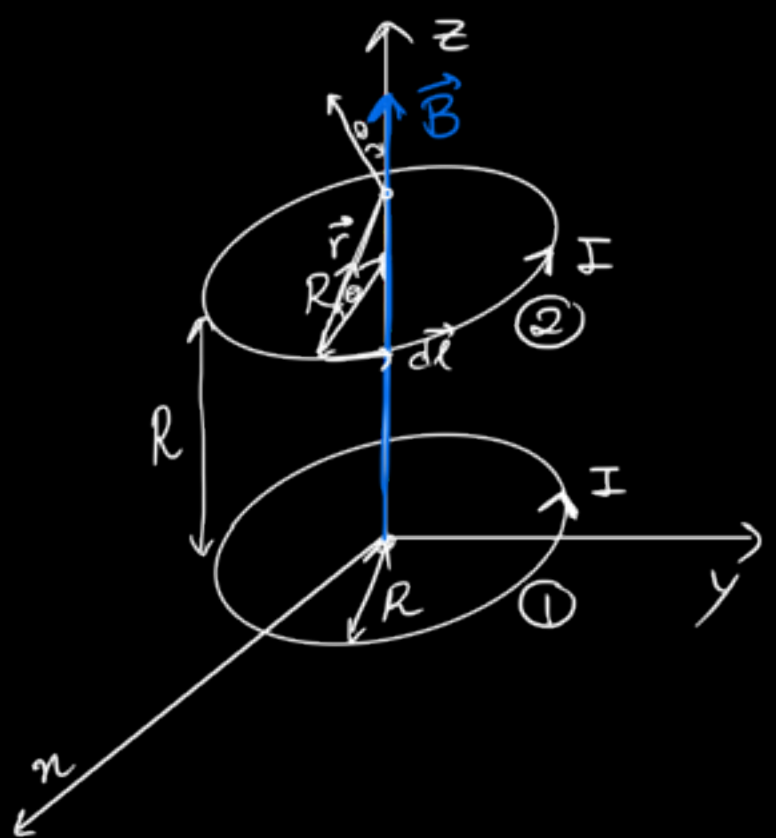
$$= -\frac{\mu_0}{4\pi} I \frac{a}{2} \vec{e}_\Theta \left[\frac{1}{\left(\frac{a}{2}\right)^2} \frac{\frac{a}{2}}{\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}} - \frac{1}{\left(\frac{a}{2}\right)^2} \frac{-\frac{a}{2}}{\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}} \right]$$

$$= -\frac{\mu_0}{4\pi} I \frac{a}{2} \vec{e}_\Theta \frac{a}{\frac{a^2}{4} a \sqrt{1/2}} =$$

$$= -\frac{\mu_0}{4\pi} I \frac{4}{a\sqrt{2}} \vec{e}_\Theta = -\frac{\mu_0 \sqrt{2}}{2\pi} I \frac{1}{a} \vec{e}_\Theta$$

$$\vec{B}_{4FACES} = 4\vec{B} = -\frac{2\sqrt{2}\,\mu_0}{a\pi} I \vec{e}_\Theta$$

4.



a) CAMPO MAGNÉTICO DE 1 ESPIRA:

$$d\vec{B} = dB_z \vec{e_z} \quad , \quad \vec{B} = \int_{esp.} dB_z \vec{e_z}$$

$$B = \frac{\mu_0}{4\pi} I \int_{espira} \frac{d\vec{l} \times \vec{r}}{|\vec{r}|^3} \cos\theta =$$

$$= \frac{\mu_0}{4\pi} I \int_{esp.} \frac{dl\, r}{r^3} \frac{R}{r} =$$

$$= \frac{\mu_0}{4\pi} I R \int_{esp.} \frac{1}{(z^2+R^2)^{3/2}} dl =$$

$$= \frac{\mu_0}{4\pi} I R \frac{1}{(z^2+R^2)^{3/2}} \times 2\pi R$$

$$\vec{B} = \frac{\mu_0}{2} I \frac{R^2}{(z^2+R^2)^{3/2}} \vec{e_z}.$$

$$\vec{B_1} = \frac{\mu_0}{2} I \frac{R^2}{(z^2+R^2)^{3/2}} \vec{e_z}$$

$$\vec{B_2} = \frac{\mu_0}{2} I \frac{R^2}{((R-z)^2+R^2)^{3/2}} \vec{e_z}$$

$$\vec{B}_{TOTAL} = \vec{B_1} + \vec{B_2} = \frac{\mu_0}{2} I R^2 \left((z^2+R^2)^{-\frac{3}{2}} + ((R-z)^2+R^2)^{-\frac{3}{2}} \right) \vec{e_z}.$$

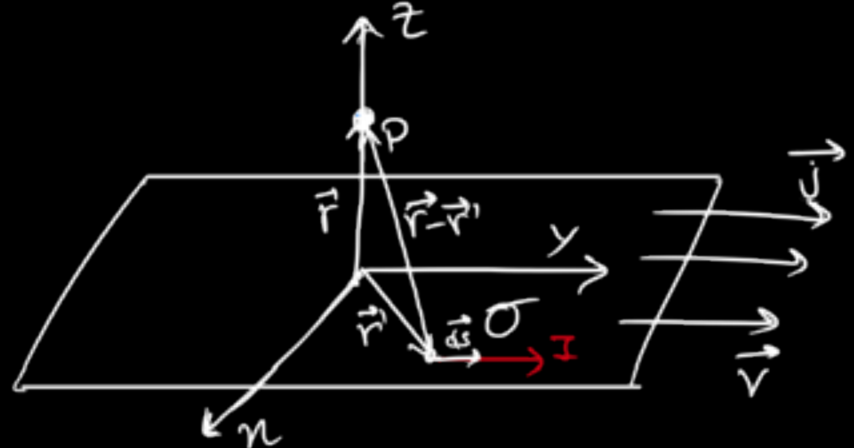
b) $\vec{B}\left(z=\frac{R}{2}\right)=\frac{\mu_0}{2}IR^2\left(\left(\frac{R^2}{4}+R^2\right)^{-\frac{3}{2}}+\left(\left(R-\frac{R}{2}\right)^2+R^2\right)^{-\frac{3}{2}}\right)\vec{e_z}$

$=\frac{\mu_0}{2}IR^2\left(\left(\frac{5R^2}{4}\right)^{-\frac{3}{2}}+\left(\frac{5R^2}{4}\right)^{-\frac{3}{2}}\right)\vec{e_z}=$

$=\underline{\mu_0 I R^2\left(\frac{5}{4}R^2\right)^{-\frac{3}{2}}}$

$\frac{dB}{dz}=0$ (não depende de z).

⑤



a) $\vec{j}=Nq\vec{v}=\sigma\vec{v}=\sigma v\,\vec{e_y}$

b) $\vec{r}'=x'\vec{e_x}+y'\vec{e_x}$

$\vec{r}=z\vec{e_z}$

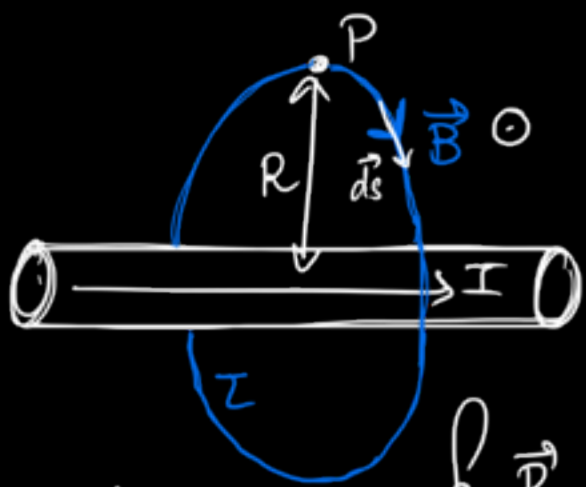
$$d\vec{B}=\frac{\mu_0}{4\pi}I\frac{d\vec{l}\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}=$$

$$=\frac{\mu_0}{4\pi}I\frac{\vec{e_y}\times(z\vec{e_z}-x'\vec{e_x}+y'\vec{e_y})}{\left(\sqrt{x'^2+y'^2+z^2}\right)^3}=$$

$$= \frac{\mu_0}{4\pi} I \, \frac{x'\vec{e_z} + z\vec{e_x}}{(x'^2+y'^2+z^2)^{3/2}} =$$

$$= \frac{\mu_0}{4\pi}$$

6.



Pela lei de Ampere: $\oint_L \vec{B}\cdot\vec{ds} = \mu_0 I \Leftrightarrow$

$$\Leftrightarrow B\int_L ds = \mu_0 I \Leftrightarrow B = \frac{\mu_0 I}{2\pi R}$$

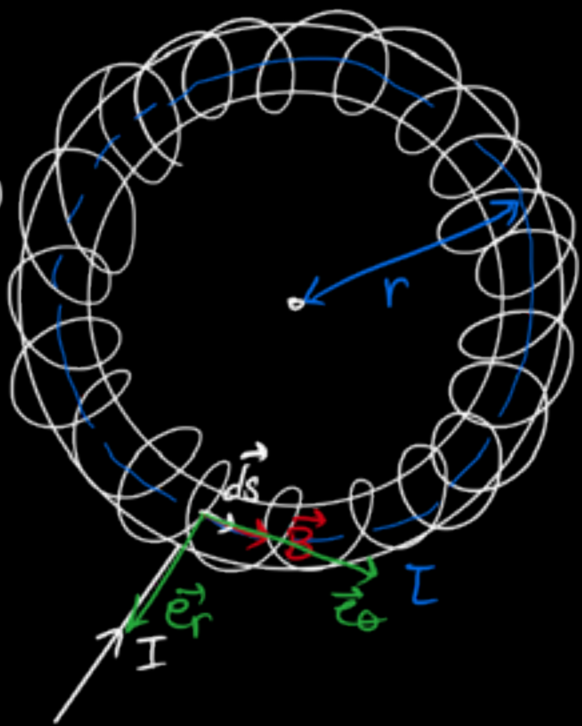
$$\vec{B} = \frac{\mu_0 I}{\ }$$

$2\pi R \quad \vec{e}_\theta$

• Sim é possível resolver o 2.2. recorrendo à lei de ampere.

• No entanto o 2.3 não, devido ao sentido da corrente alterar de face para face, levando a um campo magnético em vários sentidos, em certos locais.

8.



$$\oint_L \vec{B}\cdot\vec{ds} = \mu_0 I \Leftrightarrow$$

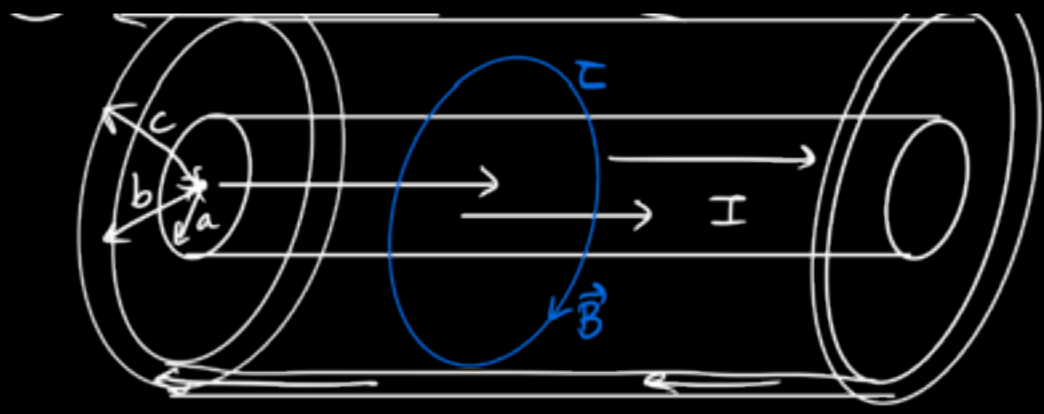
$$\Leftrightarrow B\oint_L ds = \mu_0 I \Leftrightarrow$$

$$\Leftrightarrow B = \frac{\mu_0 I}{2\pi r}$$

$$B_{TOTAL} = NB = \underline{\mu_0 I n}.$$

$$\vec{B}_T = \mu_0 I n \vec{e}_\theta$$

9. I



a) $\oint_C \vec{B}\cdot\vec{ds} = \mu_0 I' \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}\left(\dfrac{r}{a}\right)^2$

$\dfrac{I'}{I} = \dfrac{\pi r^2}{\pi a^2} \Rightarrow \underline{I' = \left(\dfrac{r}{a}\right)^2 I}$

$$\vec{B} = \frac{\mu_0 I}{2\pi}\frac{r}{a^2}\vec{e_\theta}.$$

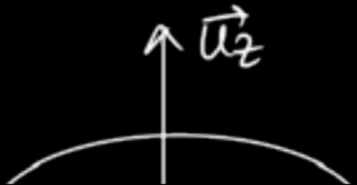
b) $\vec{B} = \dfrac{\mu_0 I}{2\pi r}\vec{e_\theta}.$

c) $I' = I\left(1 - \dfrac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2}\right) \Rightarrow$

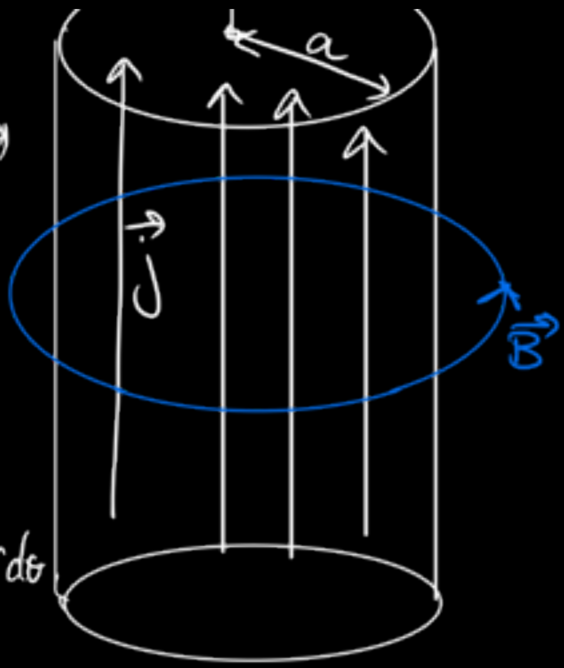
$\Rightarrow \vec{B} = \dfrac{\mu_0}{2\pi r} I \left(1 - \dfrac{r^2 - b^2}{c^2 - b^2}\right)\vec{e_\theta}.$

d) $I' = I - I = 0 \Rightarrow \underline{\vec{B} = 0}.$

(10)



10.



a) $I = \int_S \vec{j}\cdot\vec{n}\, dS \Rightarrow$

$\Rightarrow I = \int_S j_0 r\, dS \Rightarrow$

$\Rightarrow I = j_0 \int_0^{2\pi}\int_0^{a} r^2\, dr\, d\theta$

$$\underline{I = j_0 \frac{2\pi}{3} a^3}$$

b) $\underline{r<a}$: $I = j_0 \frac{2\pi}{3} r^3$

$\oint_L \vec{B}\cdot d\vec{s} = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$

$$\underline{\vec{B} = \frac{\mu_0}{3} j_0 r^2 \vec{e}_\theta}$$

$\underline{r>a}$: $I = j_0 \frac{2\pi}{3} a^3$

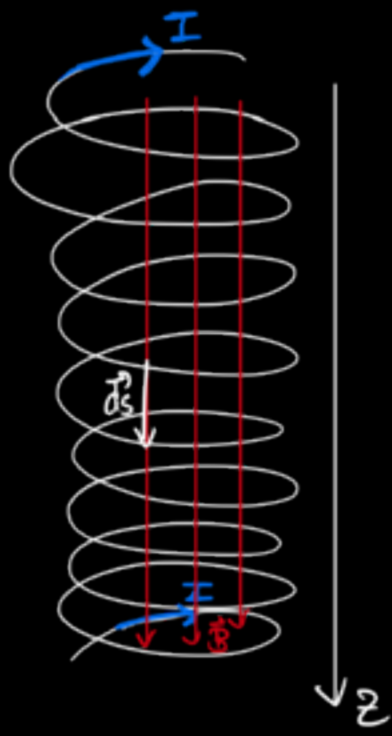
$\oint_L \vec{B}\cdot d\vec{s} = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$

$\vec{B} = \frac{\mu_0}{3} j_0 \frac{a^3}{r} \vec{e}_\theta$

c) $\vec{j} = j_0 r \, \vec{u_z} = \sigma_c \vec{E} = -\sigma_c \vec{\nabla}\phi \Leftrightarrow$

$\Leftrightarrow j_0 r = \sigma_c V' \Rightarrow \underline{\sigma_c = \frac{j_0 r}{V'}}$

7. Pela lei de Ampere, para 1 espira circular:



$$\int \vec{B}\cdot d\vec{s} = \mu_0 I \Leftrightarrow$$

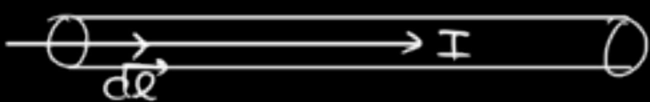
$$\Leftrightarrow B = \frac{\mu_0 I}{\int ds} = \frac{\mu_0 I}{\ell}$$

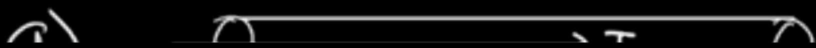
$$\vec{B} = \frac{\mu_0 I}{\ell}\vec{e_z}$$

$$n = \frac{N}{\ell} \Rightarrow \underline{N = n\ell}$$

$$\vec{B}_{TOTAL} = \sum_{i=0}^{N} \vec{B_i} = \frac{\mu_0 I}{\ell} N \vec{e_z} = \underline{n\mu_0 I \vec{e_z}}$$

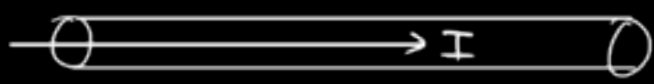
15.

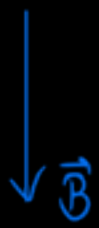




a) 

$$d\vec{F} = I\, d\vec{l} \times \vec{B} = 0 \,,\quad d\vec{l} \times \vec{B} = 0 \qquad \underline{\vec{F} = 0}.$$

b) 

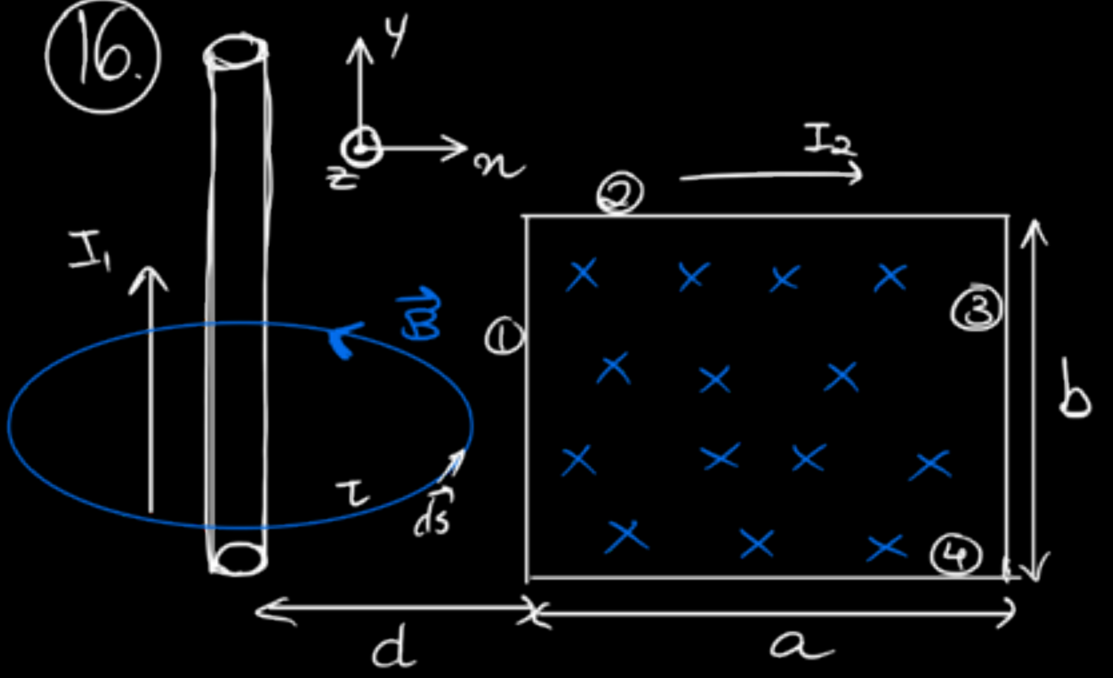


$$d\vec{l} \times \vec{B} = dl\,B$$

$$dF = I\, dl\, B$$

$$F_{(0,5)} = \int_0^{0,5} I\, dl\, B = IB \times 0,5 = \underline{4 \times 10^{-2}\ \text{N}}.$$

16.



a) $d\vec{F_2} = I_2\, d\vec{l_2} \times \vec{B_2} = I_2\, dl_2\, B_2\, \vec{e_y}$, $\vec{B_2} = \vec{B_4}$

$d\vec{F_4} = I_2\, d\vec{l_4} \times \vec{B_4} = -I_2\, dl_4\, B_4\, \vec{e_y}$

$\vec{F_2} = \int_{(2)} d\vec{F_2} = I_2\, a\, B\, \vec{e_y}$; $\vec{F_4} = -I_2\, a\, B\, \vec{e_y}$

$$\vec{F_2}+\vec{F_4}=0$$

$$d\vec{F_1}=I_2 d\vec{l_1}\times\vec{B_1}=-I_2 dl_1 B_1 \vec{e_n}$$

$$d\vec{F_3}=I_2 d\vec{l_3}\times\vec{B_3}=I_2 dl_3 B_3 \vec{e_n}$$

$$\vec{F_1}=\int_{①} d\vec{F_1}=-I_2 b B_1 \vec{e_n} \quad ; \quad \vec{F_3}=I_2 b B_3 \vec{e_n}$$

$B_1 \neq B_3$:

$$\oint_{L}\vec{B_n}\cdot d\vec{s}=\mu_0 I_1 \Rightarrow$$

$$\Rightarrow B_n=\frac{\mu_0 I_1}{\int_L ds} \Rightarrow \vec{B_n}=\frac{\mu_0 I_1}{2\pi r_n}\vec{e_\varphi}$$

$$\vec{B_1}=\frac{\mu_0 I_1}{2\pi d}\vec{e_z} \quad ; \quad \vec{B_3}=\frac{\mu_0 I_1}{2\pi(d+a)}\vec{e_z}$$

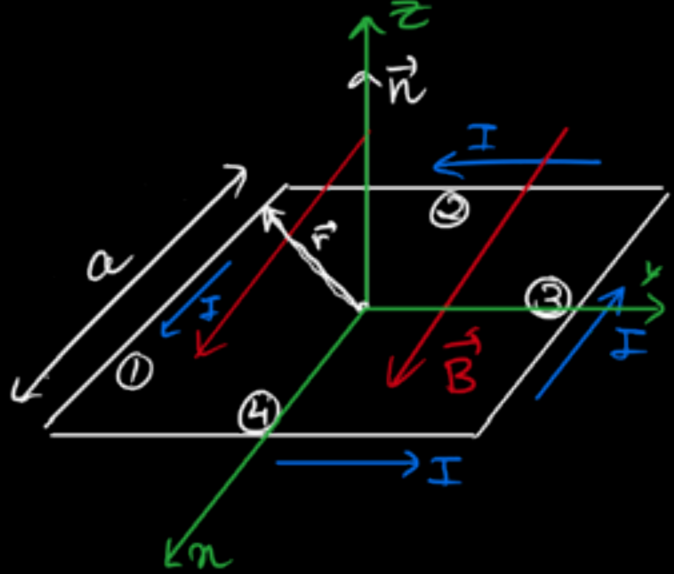
$$\vec{F}=\vec{F_1}+\vec{F_2}+\vec{F_3}+\vec{F_4}=\vec{F_1}+\vec{F_3}$$

$$=I_2 b(B_3-B_1)\vec{e_z}=\frac{I_1 I_2 \mu_0 b}{2\pi}\left(\frac{1}{d+a}-\frac{1}{d}\right)\vec{e_z}=$$

$$=\frac{I_1 I_2 \mu_0 b}{2\pi}\left(-\frac{a}{d(d+a)}\right)\vec{e_z}.$$

b) $$\vec{F}_{esp-fio}=-\vec{F}_{fio-esp}=\frac{I_1 I_2 \mu_0}{2\pi}\frac{ba}{d(d+a)}\vec{e_z}.$$

(17.)



a) $\vec F_1 = \vec F_3 = 0$

$\vec F_2 = I a B \vec e_z$

$\vec F_4 = -I a B \vec e_z$

b) $\vec F_R = \vec F_1 + \vec F_2 + \vec F_3 + \vec F_4 = 0$

c) $\vec N = \vec r \times \vec F$, $d\vec N = \vec r \times d\vec F$

$\vec F_1 = 0 \Rightarrow \vec N_1 = 0$ | $d\vec F = \pm I\, dy\, B \vec e_z$

$\vec F_3 = 0 \Rightarrow \vec N_3 = 0$ | $\vec r = x \vec e_x + y \vec e_y$

$$\vec N_2 = \int_{②} d\vec N_2 = \int_{-\frac{a}{2}}^{\frac{a}{2}} (x \vec e_x + y \vec e_y) \times (I B \vec e_z\, dy) =$$

$$= 2\int_0^{\frac{a}{2}} \underbrace{-\left(-\frac{a}{2}\right) I B \vec e_y\, dy} + \underbrace{\int_{-\frac{a}{2}}^{\frac{a}{2}} y I B \vec e_x\, dy}_{=0} =$$ (ímpar)

$$= 2\left(\frac{a}{2}\right)^2 I B \vec e_y = \underline{\frac{a^2}{2} I B \vec e_y}.$$

$$\vec N_4 = \int_{④} d\vec N_4 = \int_{-\frac{a}{2}}^{\frac{a}{2}} (x \vec e_x + y \vec e_y) \times (-I B \vec e_z\, dy) =$$

$$= 2\int_0 IB\left(\frac{a}{2}\right)\vec{e_y} + \int_{-\frac{a}{2}} IBy\,\vec{e_z}\,dy =$$

$$= 2IB\frac{a^2}{4}\vec{e_y} + 0 = \underline{IB\frac{a^2}{2}\vec{e_y}}.$$

$$\vec{N} = \vec{N_1} + \vec{N_2} + \vec{N_3} + \vec{N_4} = \underline{IBa^2\vec{e_y}}.$$

d) $\vec{m} = I\vec{A} = IA\vec{n}$

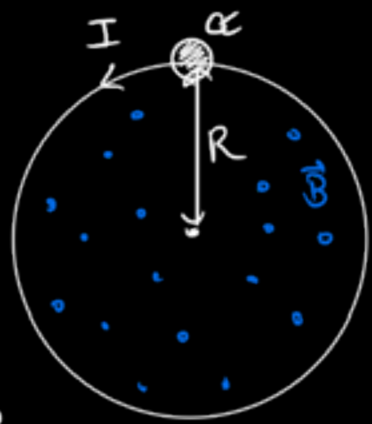
$$\vec{N} = IBa^2\vec{e_y} = IBA\vec{n} = Bm\vec{n}$$

Como $\vec{B} = B\vec{e_n}$ e $\vec{m} = m\vec{n} = m\vec{e_y}$

$$\vec{e_n} \perp \vec{e_y} \Rightarrow \vec{B}\times\vec{m} = Bm\vec{n} \Rightarrow$$

$$\Rightarrow \underline{\vec{N} = \vec{B}\times\vec{m}}.$$

(18)



a) $\underline{\theta = 90°}$

b) $F_R = ma_c = m\frac{v^2}{R}$

$$F_m = I\,\vec{dl}\times\vec{B} = q\,\vec{v}\times\vec{B}$$

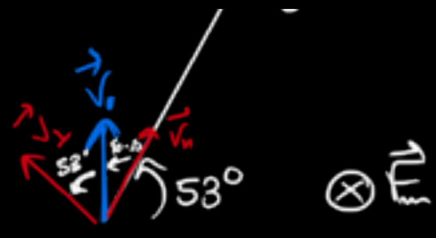
$$\vec{v}\perp\vec{B} \Rightarrow F_m = qvB = \underline{2evB}.$$

$$2evB = m\frac{v^{\cancel{2}}}{R} \Rightarrow v = \frac{2eBR}{m}.$$

(19) $B = 43{,}8\ \mu T$ $\nearrow\vec{B}$

$q = 1\,\mu C$

$\vec{V_0} = 5\,\vec{e_y}\ ms^{-1}$



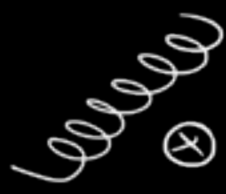
a) $V_{\parallel} = V_0 \cos(90^0 - 53^0) = 4\ ms^{-1}$

$V_{\perp} = V_0 \cos(53^0) = 3 ms^{-1}$

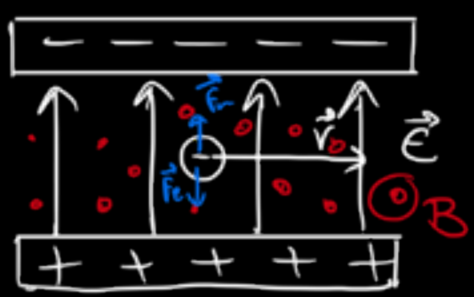
b) $F_m = q\,\vec{v_0} \times \vec{B} = q\,V_0\,B \sin(90^\circ - 53^\circ) =$

$= \underline{1{,}31798 \times 10^{-10}\ N}$

c) $\left.\begin{array}{l} V_{\perp} \text{ não se altera.} \\ F_m \text{ em } \otimes \end{array}\right\}$



(20.) $\vec{F_R} = \vec{F_e} + \vec{F_m}$



$\underline{F_R = 0} \Rightarrow$

$\Rightarrow F_m - F_e = 0 \Leftrightarrow$

$\Leftrightarrow qVB = qE \Rightarrow v = \frac{E}{B} \Rightarrow v = \frac{1000}{0{,}3} = \underline{3333{,}3\ ms^{-1}}$

(21.)

a) $\Delta U + \Delta K = 0 \Leftrightarrow -\Delta U = \frac{1}{2} m v^2 \Leftrightarrow$

$\Leftrightarrow \underline{v = \sqrt{\frac{2QV}{m}}} \hookrightarrow \underline{\frac{V_1}{V_2} = \sqrt{\frac{m_2}{m_1}}}$

b) $F_m = Q\cancel{v}B = m\frac{v^{\cancel{2}}}{R} \Leftrightarrow$

$\Leftrightarrow \underline{R = \frac{m}{QB}v}$.

$$\frac{R_1}{R_2} = \frac{m_1}{m_2}\frac{v_1}{v_2} = \frac{m_1}{m_2}\sqrt{\frac{m_2}{m_1}} =$$

$$= \sqrt{\frac{m_1^{\cancel{2}}}{m_2^{\cancel{2}}}\frac{\cancel{m_2}}{\cancel{m_1}}} = \underline{\sqrt{\frac{m_1}{m_2}}}.$$

23.

a) $\vec{F_m} = q\,\vec{v}\times\vec{B} = evB\,\vec{e_x}$

b) $\vec{F_e} = -\vec{F_m} = -evB\,\vec{e_x}$.

c) $V = \mathcal{E}L \Leftrightarrow V = \frac{F_e}{e}L = \underline{-vBL}$.

d) $R = \frac{V}{I} \Leftrightarrow I = \frac{V}{R}$, mas $\underline{V=0}$.

↳ $\underline{I=0}$.

24.

a) $F_B = q\,\vec{v}\times\vec{B} = evB\,\vec{e_z}$

b) Equilíbrio: $\vec{F_R} = 0 \Leftrightarrow$

$\Leftrightarrow \vec{F_e} + \vec{F_B} = 0 \Leftrightarrow \underline{F_e = F_B} \Longrightarrow$

$$\Rightarrow qE = q v_d B \Leftrightarrow \underline{E = v_d B}. \Rightarrow \vec{E} = \underline{v_d B \vec{e_z}}.$$

$$-\vec{\nabla}\phi = \vec{E} \Rightarrow \int_a^c -\vec{\nabla}\phi \cdot \vec{dz} = \int_a^c \vec{E}\cdot\vec{dz} \Leftrightarrow$$

$$\Leftrightarrow V = E d \Leftrightarrow \underline{V = v_d B d}.$$

c) $V = v_d B d$

$$j = N q v_d \Leftrightarrow \frac{I}{A} = N q v_d \Leftrightarrow$$

$$\Leftrightarrow v_d = \frac{I}{ANq}$$

$$V = \frac{BI}{ANq} d = \frac{BI}{\cancel{d} t N q}\cancel{d} = \underline{\frac{BI}{tNe}}.$$

d) Perpendicular ao $\vec{B}$. $(\vec{n} \parallel \vec{B})$

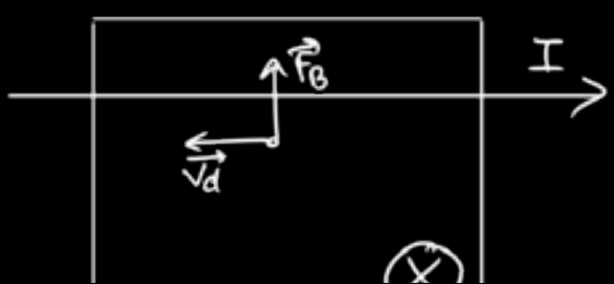
$\hookrightarrow \int_S \vec{B}\cdot\vec{n}\, dS = \text{MAX}$ se $\vec{B} \parallel \vec{n}$

e) $B = \frac{V}{I} t N e$ (substituir)

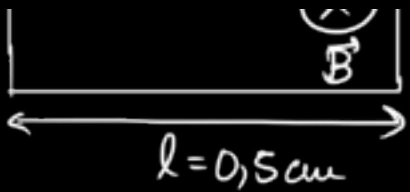
25.

$\vec{F_R} = 0 \Leftrightarrow$

$\Leftrightarrow F_e = F_B \Leftrightarrow$



$\Rightarrow \mathcal{E} = v_d B$



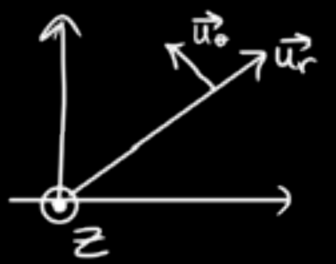
$$j = Nq\,v_d = \frac{I}{A} = \frac{I}{\ell^2}$$

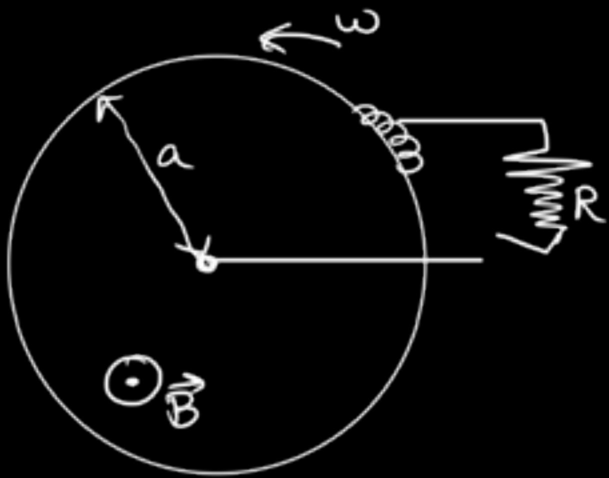
$$N = \frac{I}{q\ell^2 v_d} = \frac{I}{e\ell^2 \frac{\mathcal{E}}{B}} = \frac{BI}{\ell^2 \mathcal{E} e}$$

$$V = \mathcal{E} d \Rightarrow V = \mathcal{E}\ell \Rightarrow \mathcal{E} = \frac{V}{\ell}$$

$$N = \frac{BI}{V\ell e} = \underline{\underline{9{,}98 \times 10^{25}}}\ (e^-/m^3)$$

(26) $\vec{B} = B_0\, \vec{e}_z$





a) $\vec{F}_m = q\,\vec{v}_d \times \vec{B} = (-e)\,v_d B_0\, \vec{u}_r$

$v_d = \omega r \Rightarrow \underline{\underline{\vec{F}_m = -e\omega r B_0 \vec{u}_r}}\ (N).$

b) $\vec{F}_r = 0 \Rightarrow F_e = F_m \Rightarrow$

$\Rightarrow \cancel{+e}\,\mathcal{E} = \cancel{+e}\,\omega r B_0 \Rightarrow$

$\Rightarrow \mathcal{E} = \omega r B_0 \Rightarrow \underline{\underline{\vec{\mathcal{E}} = -\omega r B_0\, \vec{u}_r}}\ (V/m)$

c)

$$\int_S \vec{D}\cdot\vec{n}\, dS = \int_V \rho\, dV \Leftrightarrow \rho = \frac{-\varepsilon_0 E \int_S dS}{\int_V dV} \Leftrightarrow$$

$$\Leftrightarrow \rho = \frac{-\varepsilon_0 E (2\cancel{\pi} r \cancel{b})}{\cancel{\pi} a^2 \cancel{b}} = \underline{-2\varepsilon_0 E \frac{r}{a^2}}$$

$$E = \omega r B_0 \Rightarrow \rho = -2\varepsilon_0 \omega B_0 \left(\frac{r}{a}\right)^2$$

Como $r = a \Rightarrow \rho = \underline{-2\varepsilon_0 \omega B_0}.$

$$E = \frac{\sigma}{\varepsilon_0} \Rightarrow \sigma = \underline{-\omega r B_0 \varepsilon_0}.$$

$$\underline{r = a} \Rightarrow \sigma = \underline{-\omega B_0 \varepsilon_0 a}.$$

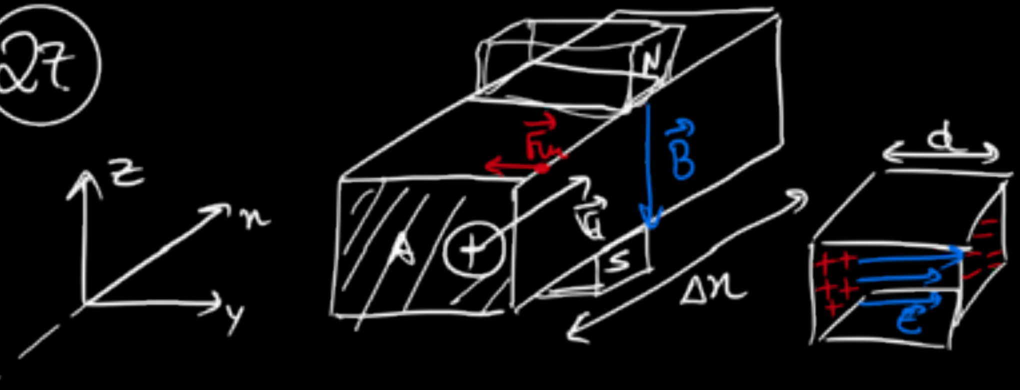
d) $-\vec{\nabla}\phi = \vec{E} \Leftrightarrow V = \int_0^a \vec{E}\cdot d\vec{r} \Leftrightarrow$

$$\Leftrightarrow V = \int_0^a -\omega r B_0\, dr \Leftrightarrow \underline{V = -\omega B_0 \frac{a^2}{2}}.$$

e) $$\left.\begin{array}{l} P = VI = RI^2 \\ R = \frac{V}{I} \Rightarrow I = \frac{V}{R^2} \end{array}\right\} \underline{P = \frac{V^2}{R}}$$

$$P = \frac{\omega^2 B_0^2 a^4}{4R}$$

(27)



a) $$\dot{V} = \frac{A\Delta x}{dt} = \frac{d^2 \Delta x}{dt} = d^2 v_d \Rightarrow$$

$$\Rightarrow v_d = \frac{\dot{V}}{d^2} \Rightarrow v_d = \frac{2\times10^{-3}}{0{,}02^2} \Rightarrow \underline{v_d = 5\,ms^{-1}}$$

$$j = \frac{I}{A} = Nqv_d \Rightarrow I = d^2 N q v_d = \underline{64\times10^{-12}\,A.}$$

b) $$\vec{F_m} = q\vec{v}\times\vec{B} = -q v_d B \vec{e_y} = \underline{-1{,}6\times10^{-21}\vec{e_y}}.$$

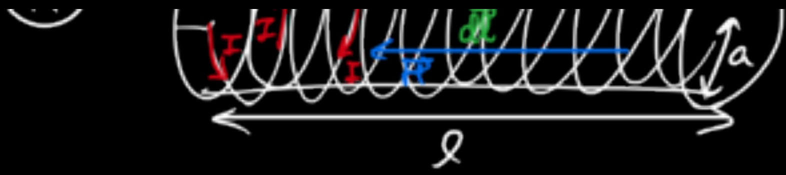
c) $$-\vec{\nabla}\phi = \vec{E} \Leftrightarrow V = Ed$$

Quando equilíbrio: $\vec{F_R} = 0 \Leftrightarrow$

$\Leftrightarrow F_e = F_m \Leftrightarrow E = v_d B \Longrightarrow$

$$\Longrightarrow \underline{V = v_d B d.} \qquad \underline{V = 2\times10^{-4}\,V}$$

(11)



$$\nabla \times \vec{H} = \vec{J} \Rightarrow$$

$$\Rightarrow \int_S \nabla \times \vec{H} \cdot \vec{n}\, dS = \int_S \vec{J} \cdot \vec{n}\, dS \Rightarrow$$

$$\Rightarrow \oint_L \vec{H} \cdot d\vec{\ell} = \int_S \vec{J} \cdot \vec{n}\, dS \Rightarrow$$

$$\Rightarrow \int_L \frac{B}{\mu}\, d\ell = I n d \Rightarrow$$

$$\Rightarrow B \cancel{d} = \mu I n \cancel{d} \Rightarrow \underline{B = \mu I n}.$$

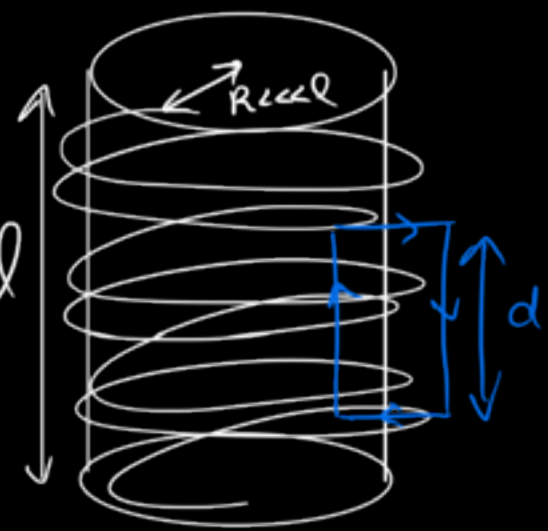
Substituir μ de acordo com o material em a), b), c).

(12) $\chi_m = 2$

$N = 150$ espiras

$\ell = 0{,}2\,m$

$I = 2A$



a) $\mu = \mu_0 (1 + \chi_m) \Rightarrow \mu = 3\mu_0$

b) $\oint_{\Gamma} \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot\vec{n}\, dS \Leftrightarrow$

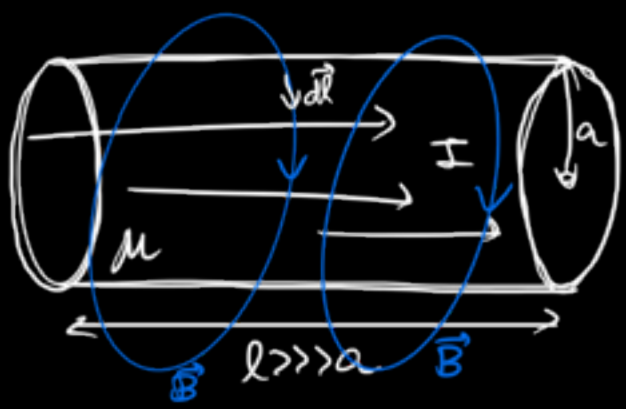
$\Leftrightarrow H \oint_{\Gamma} dl = n d\, I \Leftrightarrow \underline{H = \frac{N}{l} I}$

$B = \mu H \Rightarrow \underline{B = \mu \frac{N}{l} I}$

$\vec{B} = \mu_0 (\vec{M} + \vec{H}) \Leftrightarrow$

$\Leftrightarrow M = \frac{B}{\mu_0} - H \Rightarrow \underline{M = \frac{N}{l} I \left(\frac{\mu}{\mu_0} - 1\right)}$

13.



a) $\oint_{\Gamma} \vec{H}\cdot d\vec{l} = \int_S \vec{j}\cdot\vec{n}\, dS \Leftrightarrow$

$\Leftrightarrow \oint_{\Gamma} H\, dl = j \int_S dS \Leftrightarrow$

$\Leftrightarrow H\,(2\pi r) = j\, \pi a^2 \Rightarrow H = \frac{1}{2} j \frac{a^2}{}$

$H(2\pi r) = j\pi a^2 \quad , \quad H = \frac{j a^2}{2r}$

$$j = \frac{I}{A} = \frac{I}{\pi a^2} \Rightarrow H = \frac{1}{2}\frac{I}{\pi r}\frac{a^{\cancel{2}}}{a^2} \Rightarrow$$

$$\Rightarrow \vec{H} = \frac{I}{2\pi r}\vec{e}_\varphi$$

$$\vec{B} = \mu_0 \vec{H} \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi r}\vec{e}_\varphi$$

↳ (Exterior)

b) $$\oint_L \vec{H}\cdot d\vec{l} = \int_S \vec{j}\cdot\vec{n}\,dS \Rightarrow$$

$$\Rightarrow H(2\pi r) = \frac{I}{\pi a^2}(\pi r^2) \Rightarrow$$

$$\Rightarrow H = \frac{I}{2\pi}\frac{r}{a^2} \Rightarrow \underline{\vec{H} = \frac{I}{2\pi}\frac{r}{a^2}\vec{e}_\varphi}$$

$$\vec{B} = \mu\vec{H} \Rightarrow \vec{B} = \frac{\mu I}{2\pi}\frac{r}{a^2}\vec{e}_\varphi$$

c) $$\vec{B} = \mu_0(\vec{M} + \vec{H}) \Rightarrow \vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H} \Rightarrow$$

$$\Rightarrow \underline{\vec{M} = \frac{I}{2\pi}\frac{r}{a^2}\left(\frac{\mu}{\mu_0} - 1\right)\vec{e}_\varphi}$$

↙ ↘

$\underline{r < a}$:

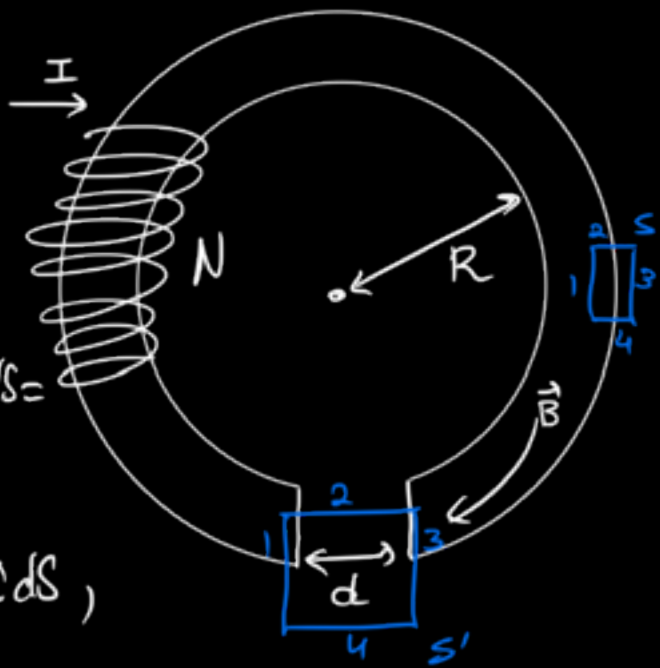
$\mu \neq \mu_0$:

$\underline{r > a}$:

$\mu = \mu_0$:

$$\vec{M} = \underline{\frac{I}{2\pi}\frac{r}{a^2}\left(\frac{\mu}{\mu_0}-1\right)\vec{e}_\varphi} \qquad \underline{\vec{M}=0}.$$

(14.)



a) $\int_S \vec{B}\cdot\vec{n}\, dS =$

$= \int_{S_1} + \int_{S_2} + \int_{S_3} + \int_{S_4} \vec{B}\cdot\vec{n}\, dS =$

$= \int_{S_1} \vec{B}\cdot\vec{n}\, dS + \int_{S_3} \vec{B}\cdot\vec{n}\, dS ,$

Considerando faces ② e ③ infinitamente pequenas.

Como $\int_S \vec{B}\cdot\vec{n}\, dS = 0 \Rightarrow \int_{S_1} \vec{B}\cdot\vec{n}_1\, dS = -\int_{S_3} \vec{B}\cdot\vec{n}_3\, dS \Leftrightarrow$

$\Leftrightarrow \int_{S_1} B\, dS = \int_{S_3} B\, dS$, $\vec{n}_1 = -\vec{n}_3 \Rightarrow \underline{B_{S_1} = B_{S_3}}.$

b) $\vec{\nabla}\times\vec{H} = \vec{J} \Leftrightarrow \oint_{\Gamma_2} \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot\vec{n}\, dS \Leftrightarrow$

$\Leftrightarrow H \int_{\Gamma_2} dl = I \Leftrightarrow \underline{B = \frac{I}{\mu d}}.$

$B_{ar} = \frac{I}{\mu_0 d \times 10^5}.$

(28.)

a) $\mathcal{E} = -\dfrac{d\Phi}{dt} = -\dfrac{d}{dt}\displaystyle\int_S \vec{B}\cdot\vec{n}\,dS =$

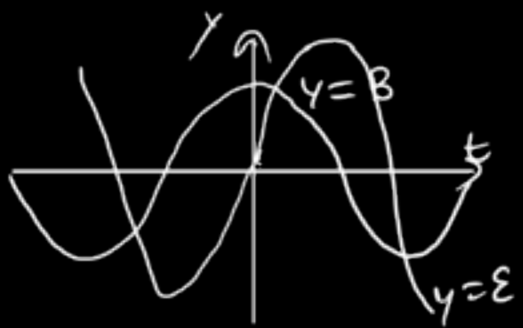
$= -\dfrac{d}{dt} B \displaystyle\int_S dS = -\dfrac{d}{dt} 10^{-2}\cos(50\pi t) \times A =$

$= 10^{-2} A \times 50\pi \operatorname{sen}(50\pi t).$

$\underline{\mathcal{E} = 10^{-3}\,\pi \operatorname{sen}(50\pi t).}\ (V)$

b) $R = \dfrac{V}{I} = \dfrac{\mathcal{E}}{I} \Rightarrow I = \dfrac{\mathcal{E}}{R} = \underline{10^{-2}\pi \operatorname{sen}(50\pi t)}.$

c)



B aumenta ⇒
⇒ ε diminui

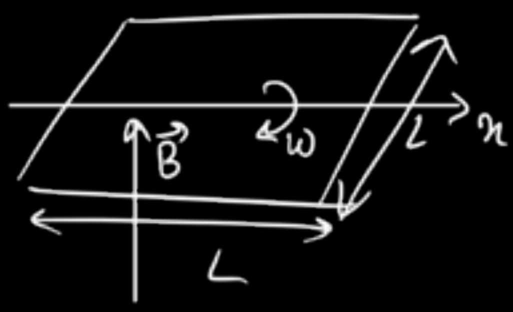
B diminui ⇒
⇒ ε aumenta

(29.)

$\omega = 100 \times 2\pi\ s^{-1}$

$R = 20\,\Omega$

$L = 0{,}2\,m$



a) $\phi = \displaystyle\int_S \vec{B}\cdot\vec{n}\,dS = \int_S B\cos(\omega t)\,dS =$

$= B\cos(\omega t)\,L^2 = \underline{0{,}04\cos(200\pi t)}$

b) $R = \dfrac{V}{I} = \dfrac{\mathcal{E}}{I} \Rightarrow I = \dfrac{\mathcal{E}}{R} = -\dfrac{1}{R}\dfrac{d\Phi}{dt} \Rightarrow$

$$\Rightarrow I = -\frac{0{,}04}{R}\frac{d}{dt}\cos(200\pi t) \Leftrightarrow$$

$$\Leftrightarrow I = -\frac{0{,}04}{20}(-200\pi)\,\text{sen}(200\pi t) \Rightarrow$$

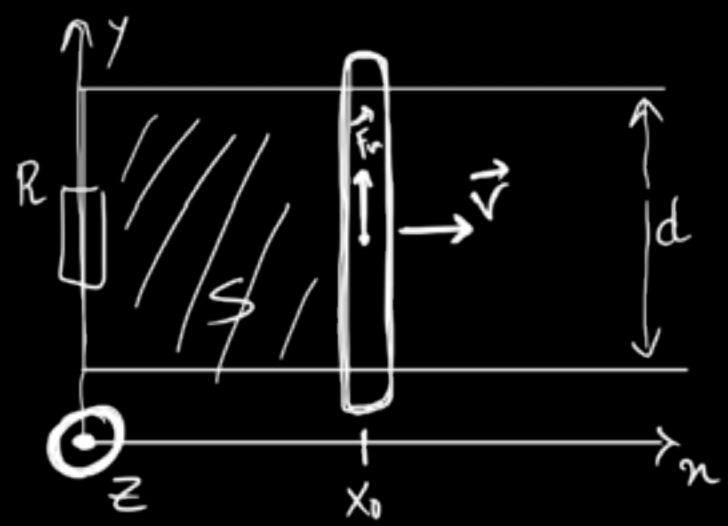
$$\Rightarrow \underline{I = 0{,}4\pi\ \text{sen}(200\pi t)}\ (A)$$

c) $P = \frac{dW}{dt} \Rightarrow dW = P\,dt \Leftrightarrow dW = I^2 R\,dt$

$$\underset{(2\text{mins})}{W} = \int_0^{120} I^2 R\ dt = R \times (0{,}4\pi)^2 \int_0^{120} \text{sen}^2(200\pi t)\ dt$$

$$= 20 \times (0{,}4\pi)^2 \times \left(\frac{t}{2} - \frac{\sin(400\pi t)}{800\pi}\right)$$

(30)



a) $\vec{F_m} = q\,\vec{v} \times \vec{B} = -e\,v_0 B\ \vec{u_x} \times \vec{u_z} \Leftrightarrow$

$\Leftrightarrow \underline{\vec{F_m} = e\,v_0\,B\ \vec{u_y}}.$

b) $\vec{F_R} = 0 \Leftrightarrow F_e = F_m \Leftrightarrow -e v_0 B = -e\mathcal{E} \Leftrightarrow$

$\Leftrightarrow \mathcal{E} = v_0 B$

$= \vec{E}d\ell = \vec{\mathcal{E}} \Leftrightarrow V = \int \vec{\mathcal{E}} \cdot \vec{d\ell} \Rightarrow V = \mathcal{E} d$

$\nabla\varphi = E \Rightarrow V = \int_{BARRA} E\, dy \Rightarrow \underline{V = Ed.}$

$$\underline{V = V_0 B d}$$

c) $R = \frac{V}{I} \Rightarrow I = \frac{V}{R} = \frac{V_0 B d}{R}$ I

d) $\underline{\mathcal{E} = V_0 B d.}$

e) $\vec{F}_B = q\,\vec{v} \times \vec{B} = I\,\vec{l} \times \vec{B} = I d B(-\vec{u_n})$

$$\vec{F}_B = -\frac{V_0 B^2 d^2}{R}\vec{u_n}$$

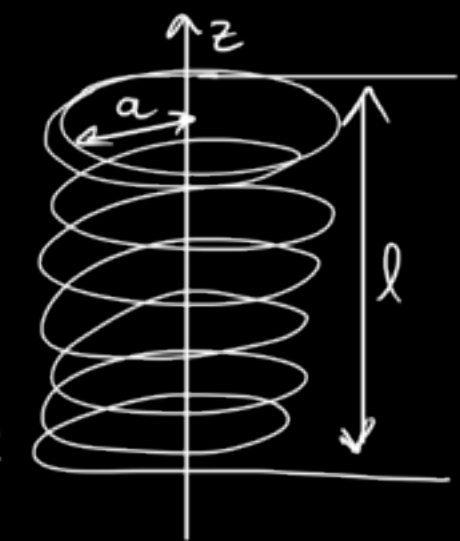
f) $\vec{P} = \vec{F}_{app} \cdot \vec{v} = (-\vec{F}_B) \cdot \vec{v} = \frac{V_0^2 B^2 d^2}{R} = \frac{V^2}{R}.$

31. $n = 1000$ espiras m^{-1}

$i_1 = 10A \quad i_2 = 1A$

$\Delta i = i_2 - i_1 = -9$

$\frac{\Delta i}{\Delta t} = \frac{-9}{0{,}1} = \underline{-90 A s^{-1}}.$



a) $I = I_0 - \frac{\Delta I}{\Delta t}\Delta t \Rightarrow$

$\Rightarrow I(t) = I_0 - 90t.$

$$\vec{\nabla} \times \vec{H} = \vec{J} \Rightarrow \int_S \vec{\nabla} \times \vec{H} \cdot \vec{n}\, ds = \int_S \vec{J} \cdot \vec{n}\, dS \Rightarrow$$

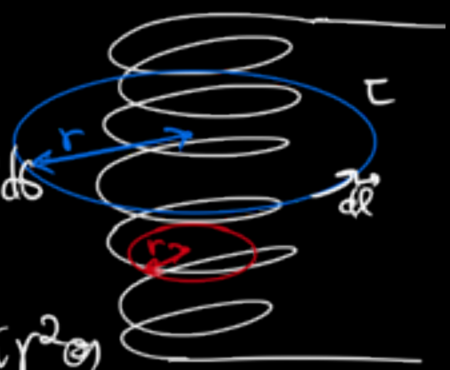
$$\Rightarrow \oint \vec{H} \cdot \vec{dl} = NI \Rightarrow \frac{B}{\mu}\oint dl = n l I \Rightarrow$$

VI μ VI

$$B = \mu_0 \frac{nIl}{l} \Rightarrow \vec{B} = \mu_0 n I(t) \vec{e_z}.$$

$$\vec{B} = \mu_0 n (I_0 - 90t)\vec{e_z}$$

b) $\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$



$$\int_S \vec{\nabla} \times \vec{E} \cdot \vec{n}\, dS = \int_S -\mu_0 n \frac{dI(t)}{dt} dS$$

$$\oint_I \vec{E} \cdot d\vec{l} = -\mu_0 n (-90) \pi r^2$$

$$E \cdot (2\pi r) = 90 \mu_0 n \pi r^2$$

$$E = \frac{90 \mu_0 n \cancel{\pi} r^{\cancel{2}}}{2 \cancel{\pi} \cancel{r}} \Rightarrow \underline{\vec{E} = 45 \mu_0 n r \vec{e_\varphi}}.$$

$(r<a)$

<u>r>a:</u> $\oint_I \vec{E} \cdot d\vec{l} = \int_S -\mu_0 n \frac{dI(t)}{dt} dS =$

$$= E \times 2\pi r = -\mu_0 n (-90) \times \pi a^2$$

$$E = \frac{\mu_0 n\, 90\, \cancel{\pi} a^2}{2 \cancel{\pi} r} \Rightarrow \underline{\vec{E} = 45 \mu_0 n \frac{a^2}{r} \vec{e_\varphi}}$$

<u>r=b</u> (b>a): $\vec{E} = 45 \mu_0 n \frac{a^2}{b} \vec{e_\varphi}$.

c) $\mathcal{E} = -\frac{d\Phi_{ANEL}}{dt}$

$$\Phi_{ANEL} = \int_S \vec{B}\cdot\vec{n}\, dS = B\int_S dS = \mu_0 n I(t)\pi a^2$$

$$\mathcal{E} = -\mu_0 n \pi a^2 \frac{dI(t)}{dt} = \underline{90\mu_0 n\pi a^2}$$

d) $R = \frac{V}{I} = \frac{\mathcal{E}}{I} \Rightarrow I = \frac{\mathcal{E}}{R}$

$$R = \frac{V}{jA} = \frac{V}{\sigma_c A E} = \frac{E\ell}{\sigma_c A \cancel{E}} = \frac{\ell}{\sigma_c A}.$$

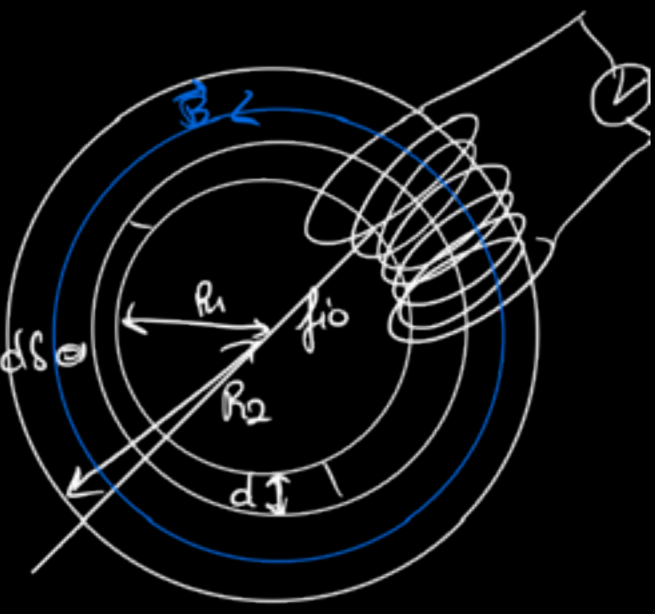
$$R = \frac{1}{\sigma_c}\frac{2\pi b}{S} \Rightarrow I = \frac{\mathcal{E}}{\frac{2\pi b}{\sigma_c S}} \Leftrightarrow$$

$$\Leftrightarrow I = \frac{\sigma_c S \mathcal{E}}{2\pi b}.$$

e) ~~É a mesma.~~

Não, apenas o anel de $r=a$ tem que estar todo dentro do de $r=b$.

32.



a) $\vec{\nabla}\times\vec{H} = \vec{J} \Leftrightarrow$

$\Leftrightarrow \int \vec{\nabla}\times\vec{H}\cdot\vec{n}\, dS = \int_S \vec{J}\cdot\vec{n}\, dS \Leftrightarrow$

$\Leftrightarrow \oint_\Gamma \vec{H}\cdot d\vec{\ell} = I \Leftrightarrow$

$\Leftrightarrow 2\pi r \frac{B}{\mu} = I \rightarrow \underline{B = \frac{\mu I}{2\pi r}}.$

b) $\phi = \int_S \vec{B}_e \cdot \vec{n}\, dS$

$$\oint_\Gamma \vec{H}_e \cdot \vec{dl} = \int_S \vec{J}\cdot\vec{n}\, dS \Leftrightarrow \frac{B}{\mu}\oint_\Gamma dl = NI \Leftrightarrow$$

$$\Leftrightarrow \frac{B}{\mu} 2\pi r = NI \Leftrightarrow \underline{B = \mu\frac{NI}{2\pi r}}$$

$$\phi = \int_S \vec{B_e}\cdot\vec{n}\, dS = \int_0^d dx \int_{R_1}^{R_2} \mu \frac{NI}{2\pi r}\, dr\, dx =$$

$$= \underline{\mu\frac{NI}{2\pi} d \log\left(\frac{R_2}{R_1}\right)}.$$

c) $i(t) = 16\cos(100\pi t)$

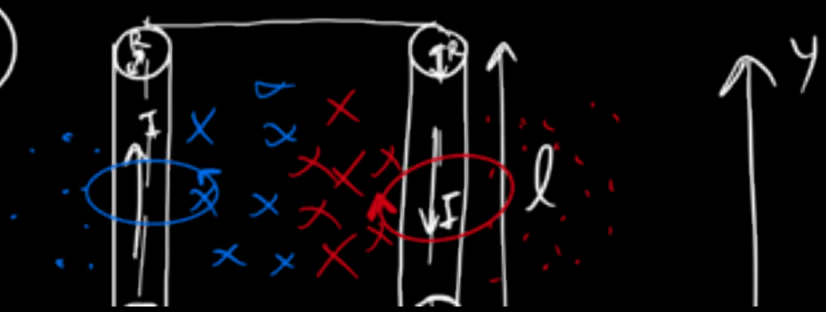
$\mu_r = 1000 \Rightarrow \frac{\mu}{\mu_0} = 1000 \Rightarrow \underline{\mu = 1000\mu_0}.$

$$\mathcal{E} = -\frac{d\phi}{dt} = -16\, \frac{d\cos(100\pi t)}{dt} =$$

$$= -16\times 100\pi\,(-\sin(100\pi t)) = \underline{1600\pi \sin(100\pi t)}$$

$V = \mathcal{E} = 1600\pi \sin(100\pi t)$.

33.



a) $$\phi_1 = \int_S \vec{B}_1 \cdot \vec{n}\, dS$$

$$\vec{\nabla} \times \vec{H} = \vec{J} \Leftrightarrow \oint_\Gamma \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot \vec{n}\, dS \Leftrightarrow$$

$$\Leftrightarrow \oint_\Gamma \frac{\vec{B}_1}{\mu_0} \cdot d\vec{l} = I \Leftrightarrow \vec{B}_1 = \frac{\mu_0 I}{2\pi r} \vec{e}_\varphi .$$

$$\phi_1 = \int_S \frac{\mu_0 I}{2\pi r}\, dS = \frac{\mu_0}{2\pi} I \int_S \frac{1}{r}\, dS =$$

$$= \frac{\mu_0}{2\pi} I \int_0^l dy \int_R^{d-R} \frac{1}{r}\, dr = \underline{\frac{\mu_0}{2\pi} I l \log\left(\frac{d-R}{R}\right)}$$

b) $$\phi_2 = \int_S \vec{B}_2 \cdot \vec{n}\, dS \;,\; \vec{B}_2 = \frac{\mu_0 I}{2\pi r} (-\vec{e}_\varphi)$$

$$\phi_2 = \int_S B_2 (-\vec{e}_\varphi) \cdot \vec{n}\, dS = -\frac{\mu_0 I}{2\pi} \int_S \frac{1}{r}\, dS =$$

$$= -\frac{\mu_0 I}{2\pi} \int_0^l dy \int_{d-R}^{R} \frac{1}{r}\, dr = \frac{\mu_0 I}{2\pi} \int_0^l dy \int_R^{d-R} \frac{1}{r}\, dr =$$

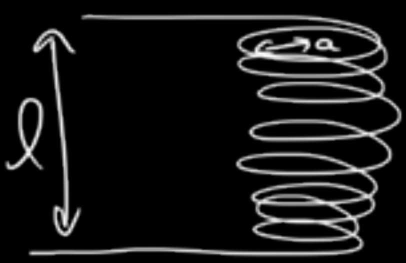
$$= \frac{\mu_0 I}{2\pi} \log\left(\frac{d-R}{R}\right) l$$

$$\phi_T = \phi_1 + \phi_2 = \frac{\mu_0 I}{\pi} l \log\left(\frac{d-R}{R}\right).$$

c) $\Phi_T = L I \Rightarrow \underline{L = \frac{\mu_0}{\pi} l \log\left(\frac{d-R}{R}\right)}$

(34) $\mu_r = 500$

$i = 0{,}5\cos(100\pi t)$



a) $\phi = L i \Rightarrow L = \frac{\phi}{i}$

$\phi = \int_S \vec{B}\cdot\vec{n}\, dS \;,\; \vec{\nabla}\times\vec{H} = \vec{J} \Rightarrow \int_L \vec{B}\cdot d\vec{s} = \mu I \Rightarrow$

$\Rightarrow Bl = \mu n l i \Rightarrow \underline{B = \mu n i}$

$\phi = \int_S \vec{B}\cdot\vec{n}\, dS = n i \int_S dS = \underline{\mu n i n l \pi a^2}$

$L = \frac{n^2 l \pi a^2 i \mu}{i} \Rightarrow \underline{L = \mu n^2 l \pi a^2}$ (H)

b) $\mathcal{E} = -\frac{d\phi}{dt} = -\mu n^2 l a^2 \frac{di}{dt} = -L\frac{di}{dt} =$

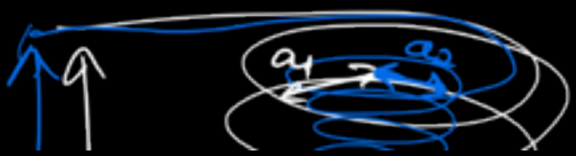
$= -L\frac{d}{dt}(0{,}5\cos(100\pi t)) =$

$= \underline{50\pi L\, \mathrm{sen}(100\pi t)}.$ (V)

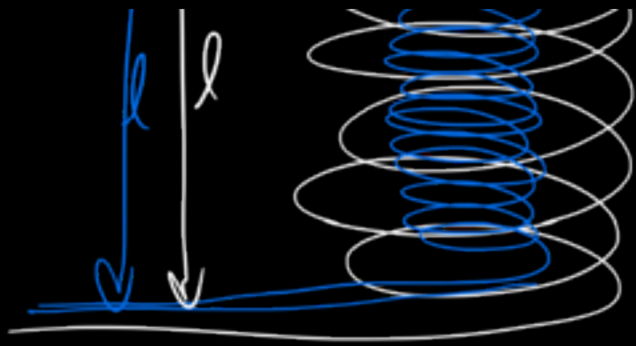
c) $U = \frac{1}{2}\phi i = \frac{1}{2}L i^2 = \frac{1}{2}\mu l n^2 a^2 i^2$ (J).

(35)

Bobina 1:



Raio a_1
N_1 espiras



Bobina 2:
Raio a_2
N_2 espiras

Ambas comprimento ℓ.

a) $M = M_{12} = M_{21}$

$$M_{12} = \frac{\phi_{12} N_2}{i_1}$$

$$\vec{\nabla} \times \vec{H}_1 = \vec{j} \Leftrightarrow \oint_I \vec{H}_1 \cdot d\vec{\ell} = I \Leftrightarrow \frac{B_1}{\mu} \ell = N_1 i$$

$$\cdot\ B_1 = \frac{\mu N_1 i}{\ell}$$

$$\phi_{12} = \int_{S_2} \vec{B}_1 \cdot \vec{n}\, dS = \frac{\mu N_1 i}{\ell} \int_{S_2} dS \Rightarrow$$

$$\Rightarrow \phi_{12} = \frac{\mu N_1 i}{\ell} \pi a_1^2 .$$

$a_1 < a_2$ e $1 < 2$.

$$M_{12} = \frac{\phi_{12} N_2}{i_1} = \mu \frac{N_1 N_2}{\ell} \pi a_1^2 .$$

b) $$\vec{\nabla} \times \vec{H} = \vec{j} \Leftrightarrow \int_S \vec{\nabla} \times \vec{H} \cdot \vec{n}\, dS = \int_S \vec{j} \cdot \vec{n}\, dS \Leftrightarrow$$

$$\Leftrightarrow B \oint_I ds = \mu I \Leftrightarrow B = \frac{\mu N i}{\ell}$$

$$B_2 = \mu \frac{N_2 i}{\ell}.$$

$$\phi_2 = L_2 i \Leftrightarrow L_2 = \frac{\int_S \vec{B_2}\cdot\vec{n}\,dS}{i} \Leftrightarrow$$

$$\Leftrightarrow L_2 = \frac{\mu \frac{N_2 i}{\ell} N_2 \int_S dS}{i} = \mu \frac{N_2^2}{\ell} \pi a_2^2$$

$$\oint_C \vec{E}\cdot d\vec{\ell} = \mathcal{E} = -\frac{d\phi}{dt} = -\frac{di}{dt}(L+M) \Leftrightarrow$$

$$\Leftrightarrow Ri + \frac{di}{dt}(L+M) = 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{R}{L+M} i + \frac{di}{dt} = 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{R}{L+M} = -\frac{di}{dt}/i \Leftrightarrow$$

$$\Leftrightarrow \int \frac{R}{L+M}dt = -\int \frac{\frac{di}{dt}}{i}dt \Leftrightarrow$$

$$\Leftrightarrow -\int \frac{R}{L+M}dt = \log(i(t)) \Leftrightarrow$$

$$\Leftrightarrow i(t) = e^{-\int \frac{R}{L+M}dt}$$

DUVIDA

37

a) $\vec{\nabla}\times\vec{H} = \vec{J} \Leftrightarrow \int_S \vec{\nabla}\times\vec{H}\cdot\vec{n}\,dS = \int_S \vec{J}\cdot\vec{n}\,dS$

$$\Leftrightarrow \oint_S \frac{\vec{B}}{\mu}\cdot\vec{n}\,dS = I \Leftrightarrow B = \frac{Ni}{2\pi r}\mu = \frac{Ni}{2\pi r}\mu$$

$$B_1 = \frac{N_1 i_1}{2\pi r}\mu \; ; \; B_2 = \frac{N_2 i_2}{2\pi r}\mu$$

$$B_T = B_1 + B_2 = \frac{\mu}{2\pi r}(N_1 i_1 + N_2 i_2)$$

$$\vec{B} = \frac{\mu}{2\pi r}(N_1 i_1 + N_2 i_2)\,\vec{e_\varphi}$$

b) $$M = \frac{\phi_{12} N_2}{i_1}$$

$$\phi_{12} = \int_{S_2} \vec{B_1}\cdot\vec{n}\, dS = \frac{N_1 i_1}{2\pi}\mu \int_{S_2} \frac{1}{r}\, dS =$$

$$= \frac{N_1 i_1}{2\pi}\mu \int_0^h dy \int_a^b \frac{1}{r}\, dr = \frac{N_1 i_1}{2\pi} h \log\left(\frac{b}{a}\right)\mu$$

$$M = \frac{N_1 N_2}{2\pi}\mu h \log\left(\frac{b}{a}\right).$$

c) $$\phi_1 = N_1 \phi \; ; \; \phi_2 = N_2 \phi$$

$$\varepsilon = -\frac{d\phi}{dt} \Rightarrow \begin{cases} \varepsilon_1 = -N_1 \dfrac{d\phi}{dt} \\ \varepsilon_2 = -N_2 \dfrac{d\phi}{dt} \end{cases}$$

$$\frac{\varepsilon_1}{\varepsilon_2} = -\frac{N_1}{N_2}.$$

38.

a) $\vec{\nabla}\times\vec{H}=\vec{J} \iff \oint_L \vec{B}\cdot d\vec{l} = \mu_0\int_S \vec{J}\cdot\vec{n}\,dS \iff$

$\iff B(2\pi r) = \mu_0 I_1 \Rightarrow \underline{B=\mu_0\frac{I_1}{2\pi r}}$

$I_1 = I\pi a^2 \Rightarrow B = \mu_0\frac{I\pi a^2}{2\pi r}$

$\vec{B} = \mu_0 \frac{I}{2}\frac{a^2}{r}\vec{e}_\theta$

$u_m = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\mu_0\frac{I^2}{4r^2}a^4$

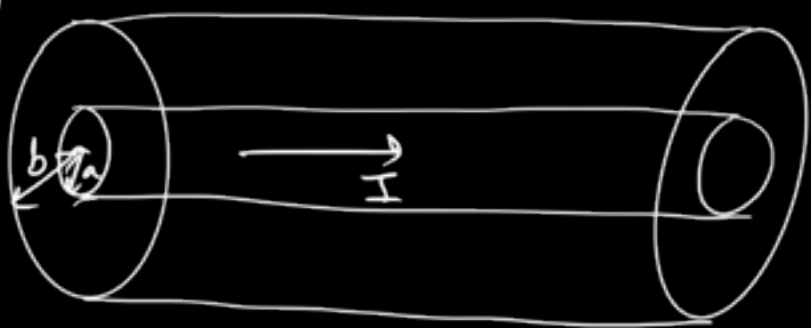
Solução

$u_m = \frac{1}{8}\mu_0\left(\frac{Ir}{\pi a^2}\right)^2 = \frac{1}{2}\frac{1}{\mu_0}\left(\mu_0\frac{Ir}{\pi a^2}\right)^2$

$B = \mu_0\frac{Ir}{\pi a^2} = \mu_0\frac{I}{\pi a^2}r = \underline{\mu_0 I_1 r}$ (?)

DUVIDA.

39.



a) $\vec{\nabla}\times\vec{H}=\vec{J} \iff \int_S \vec{\nabla}\times\vec{H}\cdot\vec{n}\,dS = \int_S \vec{J}\cdot\vec{n}\,dS \iff$

$\Rightarrow \oint \vec{H}\cdot d\vec{l} = I \iff H = \frac{I}{2\pi r} \Rightarrow \vec{B} = \mu\frac{I}{2\pi r}\vec{e}_\varphi$

U_I ... $2\pi r$ ... $\int 2\pi r \cdot$

$$u_m = \frac{1}{2}\frac{B^2}{\mu} = \frac{1}{8}\mu\left(\frac{I}{\pi r}\right)^2, \quad \mu = \mu_0.$$

b) $$U_m = \int_S u_m \, dS = \int_0^{2\pi} d\theta \int_a^b \frac{1}{8}\mu \frac{I^2}{\pi^2}\frac{1}{r^2} r \, dr =$$

$$= \frac{1}{8} \times 2\cancel{\pi} \times \mu \times \frac{I^2}{\pi^{\cancel{2}}} \times \log\left(\frac{b}{a}\right) =$$

$$\Rightarrow U_m = \frac{I^2}{4\pi}\mu_0 \log\left(\frac{b}{a}\right) \ (J/m).$$

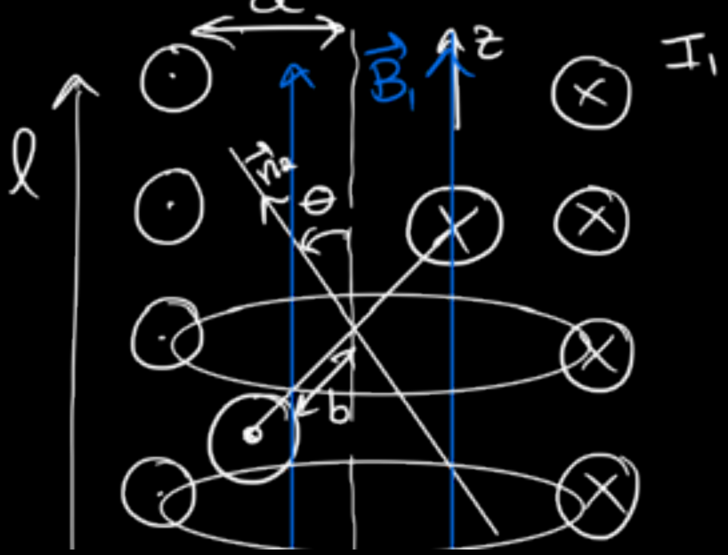
c) $$U_m = \frac{1}{2}\Phi I \Leftrightarrow \frac{I^{\cancel{2}}}{4\pi}\mu_0 \log\left(\frac{b}{a}\right) = \frac{1}{\cancel{2}}\Phi \cancel{I} \Leftrightarrow$$

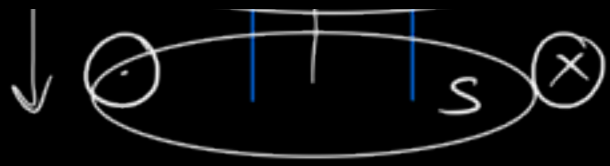
$$\Leftrightarrow \Phi = \frac{\mu_0}{2\pi}\log\left(\frac{b}{a}\right) I$$

$$\Phi = LI \Rightarrow L = \frac{\mu_0}{2\pi}\log\left(\frac{b}{a}\right).$$

d) Área de uma face lateral de um cilindro de raio entre a e b.

(40)





a) $\vec{\nabla}\times\vec{H}=\vec{J} \Leftrightarrow \oint_L \vec{B}_1\cdot d\vec{l} = \mu_0 \int_S \vec{J}\cdot\vec{n}\, dS \Leftrightarrow$

$\Leftrightarrow B_1 = \mu_0 \frac{N_1 I_1}{l}$, $\Phi_1 = \int_S \vec{B}\cdot\vec{n}\, dS = \mu_0 \frac{N_1^2 I_1}{l}\pi a^2$.

$\Phi_1 = L I_1 \Leftrightarrow L = \frac{\Phi_1}{I_1} \Leftrightarrow \underline{L = \mu_0 N_1^2 \frac{\pi a^2}{l}}$ (H)

b) $M = \frac{\Phi_{12} N_2}{I_1}$;

$\Phi_{12} = \int_{S_2} \vec{B}_1 \cdot \vec{n}_2\, dS = \int_{S_2} B_1 \cos\theta\, dS =$

$= \underline{B_1 \cos\theta\, \pi b^2 N_2}$

$M = \frac{B_1 N_2^2 \pi b^2 \cos\theta}{I_1} = \mu_0 \frac{N_2^2 N_1}{l} \pi b^2 \cos\theta$.

$\underline{N_2 = 1}$ (1 espira) $\Rightarrow \underline{M = \mu_0 \frac{N_1}{l} \pi b^2 \cos\theta}$.

c) $U_m = \frac{1}{2}\Phi_1 I_1 + \frac{1}{2}\Phi_2 I_2$

$\Phi_1 = L_1 I_1 + M I_2$; $\Phi_2 = L_2 I_2 + M I_1$

$\underline{L_2 = 0} \Rightarrow \Phi_2 = M I_1$

$U_m = \frac{1}{2}(L_1 I_1^2 + M I_1 I_2 + M I_1 I_2) =$

$= \frac{1}{2} L_1 I_1^2 + M I_1 I_2 =$

$$= \frac{1}{2}\mu_0 \frac{N_1^2}{\ell} \pi a^2 I_1^2 + \mu_0 \frac{N_1}{\ell} \pi b^2 \cos\theta \, I_1 I_2 .$$

d) $$\left.\begin{aligned} \vec{\tau} &= \vec{m} \times \vec{B} \\ \vec{m} &= \pi b^2 I_2 \, \vec{n_2} \\ \vec{B} &= \vec{B_1} = \mu_0 \frac{N_1 I_1}{\ell} \vec{e_z} \end{aligned}\right\} \Rightarrow$$

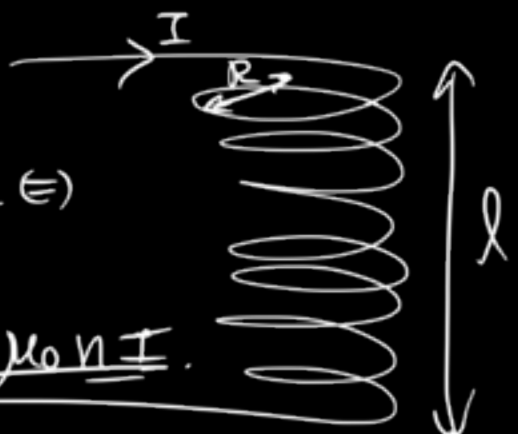
$$\Rightarrow \vec{\tau} = \pi b^2 I_2 \mu_0 \frac{N_1 I_1}{\ell} \, \vec{n} \times \vec{e_z} . \Leftrightarrow$$

$$\Leftrightarrow \vec{\tau} = \pi b^2 I_2 \mu_0 \frac{N_1 I_1}{\ell} (-\vec{e_n}) \operatorname{sen}\theta$$

$$\vec{\tau} = -\pi b^2 I_2 \mu_0 \frac{N_1 I_1}{\ell} \operatorname{sen}\theta .$$

$\vec{\tau} = 0$ quando $\underline{\underline{\theta = 0}} \rightarrow$ Equilíbrio

41. n espiras/m



a) $$\int_S \vec{B} \cdot \vec{n} \, dS = \mu_0 N I \Leftrightarrow$$

$$\Leftrightarrow B = \mu_0 \frac{n \ell I}{\ell} \Rightarrow \underline{B = \mu_0 n I} .$$

$$U_m = \frac{1}{2} \frac{B^2}{\mu_0} = \frac{1}{2} \frac{1}{\mu_0} (\mu_0 n I)^2 = \underline{\underline{\frac{1}{2} \mu_0 (n I)^2}}$$

b) $$U_m = \int_V u_m \, dv = \frac{1}{2} \mu_0 (n I)^2 \int_0^{2\pi} d\theta \int_0^{\ell} dz \int_0^{R} r \, dr$$

$$U_m = \frac{1}{\cancel{2}}\mu_0 (nI)^2 \times \cancel{2}\pi \times x \times \frac{1}{2} \Rightarrow$$

$$\Rightarrow U_m = \frac{1}{2}\mu_0 \pi \ell (nIR)^2 \text{ (J)}.$$

c) $U_m = \frac{1}{2}\Phi I = \frac{1}{2} L I^2 \Rightarrow L = \frac{2U_m}{I^2} \Rightarrow$

$$\Rightarrow L = \mu_0 \pi \ell (nR)^2. \text{ (H)}$$

d) $\vec{F_S} = \frac{dU_m}{dR} = \frac{1}{2}\mu_0 \pi \ell (nI)^2 \frac{dR^2}{dR} =$

$= \mu_0 \pi \ell (nI)^2 R . \vec{e_r}$: Explosão.

e) $P = \frac{F_S}{A} = \frac{F_S}{2\pi R \ell} = \frac{\mu_0}{2}(nI)^2.$

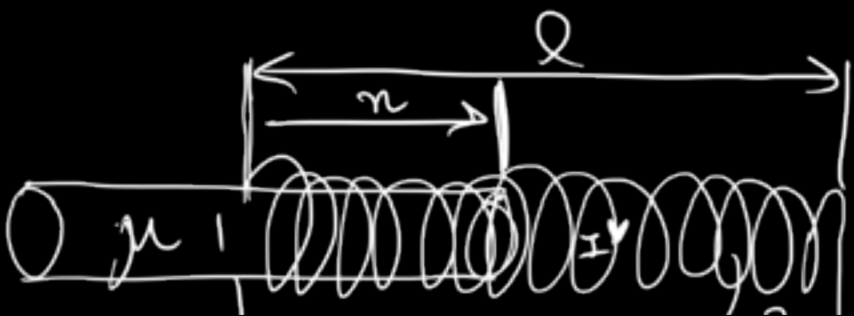
DUVIDA (Porque não se mantém ϕ constante)

(42.) a) $u_m = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2\mu_0}$

$$U_m = \int_V u_m \, dV = \frac{Sd}{2\mu_0} = 1{,}6 \times 10^4 \, d \text{ (J)}$$

b) $\vec{F_S} = -\frac{dU_m}{d\ell}\Big|_\phi = -\frac{S}{2\mu_0} \text{ (N)}$

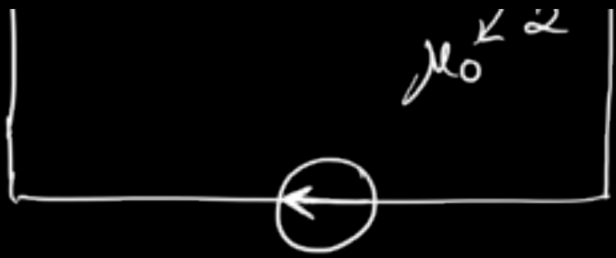
(43.)



Raio R

$l >> R$

n espiras/m



a) $\vec{\nabla} \times \vec{H} = \vec{J} \Leftrightarrow \int_S \vec{\nabla} \times \vec{H} \cdot \vec{n}\, dS = \int_S \vec{J} \cdot \vec{n}\, dS \Leftrightarrow$

$\Leftrightarrow \oint_\Gamma \vec{H} \cdot d\vec{l} = NI \Leftrightarrow \underline{B = nI\mu}.$

$$\Phi = \Phi_1 + \Phi_2 = \int_S B_1 \vec{n}\, dS + \int_S \vec{B_2} \cdot \vec{n}\, dS =$$

$$= nI \int_S dS (N_1 B_1 + N_2 B_2) =$$

$$= n^2 I \pi R^2 (x\mu + (l-x)\mu_0)$$

$$\Phi = LI \Rightarrow L = n^2 \pi R (\mu x + \mu_0 (l-x)).$$

b) $P = \frac{dU_m}{dt} = \frac{U_f - U_i}{dt} = \frac{U_m}{dt}, \underline{U_i = 0}$

$U_m = \frac{1}{2}\Phi I = \frac{1}{2} L I^2 \Rightarrow \underline{P = \frac{1}{2}\frac{LI^2}{\Delta t}}.$

c) $\vec{F} = \frac{dU_m}{d\ldots}\Big| = \frac{1}{2} n^2 \pi R \frac{d}{d\ldots} (x\mu + (l-x)\mu_0) \vec{e_x}$