Autonomous Agents and Multiagent Systems

MSc in Computer Science and Engineering

Second Exam (2021-2022)

Instructions

- You have 120 minutes to complete the exam.
- Make sure that your exam has a total of 13 pages. Also, check if there are no missing sheets, then write your full name and student number on this page (and your student number on all pages).
- The exam has 14 questions, with a maximum score of 20 points. The questions have different levels of difficulty. The point value of each question is provided next to the question number.
- *If you get stuck in a question, move on.* You should start with the more straightforward questions to secure those points before moving on to the more complex questions.
- *No interaction with the faculty is allowed during the exam.* If you are unclear about a question, clearly indicate the unclear part and answer the question to the best of your ability.
- Please provide your answer in the space below each question. If you make a mess, clearly indicate your answer.
- This exam is a closed-book assessment, whereby students are NOT allowed to bring books or other reference material into the examination room. You may bring only ONE A4 page of handwritten notes, in your OWN handwriting. Typed notes or a copy of someone else's notes are not allowed.
- You may use a calculator, but any other type of electronic or communication equipment is not allowed.
- **Good luck!**

1 Agent architectures

Question 1. (1.5 pts.)
What is social ability in a multiagent system? What are the key social behaviors in multiagent systems? Explain each social behavior.

Write your answer here:

Solution 1.
Social ability in a multiagent system is an agent's ability to interact with other agents.
The key social behaviors in multiagent systems are cooperation, coordination, and negotiation.
Cooperation is working together as a team to achieve a shared goal.
Coordination is managing the interdependencies between activities.
Negotiation is the ability to reach agreements on matter of common interest.

Question 2. (1.5 pts.)
What are deliberation and means-ends reasoning in practical reasoning? Explain.

Write your answer here:

Solution 2.
Deliberation is when an agent is deciding what state of affairs to achieve from (possibly conflicting) desires.
Means-ends reasoning is when an agent is deciding how he wants to achieve these states of affairs.

2 Normal-form games

Consider the following normal-form game:

	a	b	c
A	1, 3	0, 2	4, 4
B	1, 2	5, 5	1, 3
C	2, 1	6, 0	2, 0

Question 3. (**1 pts.**)
What are the pure strategy Nash equilibria?

Write your answer here:

Solution 3.
In order to find the pure strategy Nash equilibria, we first find the best responses for agent 1 and agent 2 and underline them in the payoff matrix:

	a	b	c
A	1, 3	0, 2	$\underline{4}$, $\underline{4}$
B	1, 2	5, $\underline{5}$	1, 3
C	$\underline{2}$, $\underline{1}$	$\underline{6}$, 0	2, 0

A joint action satisfies the definition of a NE if each agent's action is the best response to the other's. We thus have a Nash equilibrium if both payoffs are underlined in a cell of the payoff matrix above. In conclusion, the Nash equilibria is (A, c) and (C, a).

Question 4. (**1 pts.**)
What is the result if you use the Iterated Elimination of Strictly Dominated Actions (IESDA)? What can you say about IESDA and Nash equilibrium in this game?

Write your answer here:

Solution 4.
B is strictly dominated by C. Hence, we can eliminate B.

	a	b	c
A	1, 3	0, 2	4, 4
C	2, 1	6, 0	2, 0

After eliminating B, a strictly dominates b. Hence, we can eliminate b.

	a	c
A	1, 3	4, 4
C	2, 1	2, 0

No other actions can be eliminated.

The Nash equilibrium is a stronger solution concept than IESDA. Also, note that the pure strategy Nash equilibria (A, c) and (C, a) in the resulting subgame after using IESDA (below) are the same Nash equilibria of the original game.

	a	c
A	1, 3	$\underline{4}$, $\underline{4}$
C	$\underline{2}$, $\underline{1}$	2, 0

Question 5. (**1.5 pts.**)
What is the mixed strategy Nash equilibrium? (Hint: use the result from IESDA to find the mixed strategy Nash equilibrium)

Write your answer here:

Solution 5.
Considering rational agents, where they do not play strictly dominated actions. We can assume that the mixed strategy NE in the subgame below is the same as the mixed strategy NE in the original game.

	a	c
A	1, 3	4, 4
C	2, 1	2, 0

In order to find the mixed strategy NE, let us suppose that Agent 1 believes that Agent 2 will choose a with probability q and c with probability $1 - q$.

If Agent 1 best-responds with a mixed strategy, then Agent 2 must make him indifferent between A and C:

$$EU_1(\text{A}) = EU_1(\text{C})$$
$$1q + 4(1 - q) = 2q + 2(1 - q)$$
$$4 - 3q = 2$$
$$3q = 2$$
$$q = \tfrac{2}{3}$$

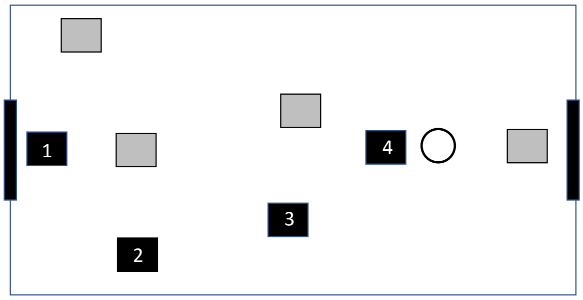
Suppose that Agent 2 believes that Agent 1 will choose A with probability r and C with probability $1 - r$.
If Agent 2 best-responds with a mixed strategy, then Agent 1 must make her indifferent between a and c:

$$EU_2(\text{a}) = EU_2(\text{c})$$
$$3r + 1(1 - r) = 4r + 0(1 - r)$$
$$2r + 1 = 4r$$
$$2r = 1$$
$$r = \tfrac{1}{2}$$

Hence, the mixed strategy Nash equilibrium is $(\frac{2}{3}, 0, \frac{1}{3}), (\frac{1}{2}, 0, \frac{1}{2})$

3 Coordination

Consider the state below of a robot soccer match.

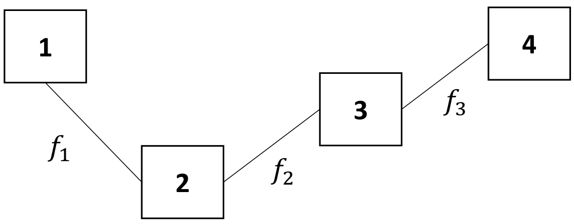


Use Coordination Graphs to solve the coordination problem above between the 4 robots (each robot is numbered from 1 to 4 in the figure above). In addition, the action sets of the robots are the following: $A_1, A_2, A_3, A_4 = \{pass, shoot, stay\}$.

Question 6. (1 pts.)
Draw the coordination graph and present the optimization problem that the Coordination Graph is solving. Factors should include at most two players. In addition, consider that the need for coordination among the different players increases as the distance between the players decreases.

Write your answer here:

Solution 6.



$$\max_a u(a) = \max_{a_1,a_2,a_3,a_4} f_1(a_1, a_2) + f_2(a_2, a_3) + f_3(a_3, a_4)$$

Question 7. (2 pts.)
Use the Variable Elimination algorithm to solve the coordination problem and present each step of the first pass that eliminates each variable.

Write your answer here:

Solution 7.
$F = \{f_1, f_2, f_3\}$

$\underline{i = 1}$

$B_1(a_2) = \underset{a_1}{\operatorname{argmax}}[f_1(a_1, a_2)]$
$f_4(a_2) = \max_{a_1}[f_1(a_1, a_2)]$
$F = \{f_2, f_3, f_4\}$

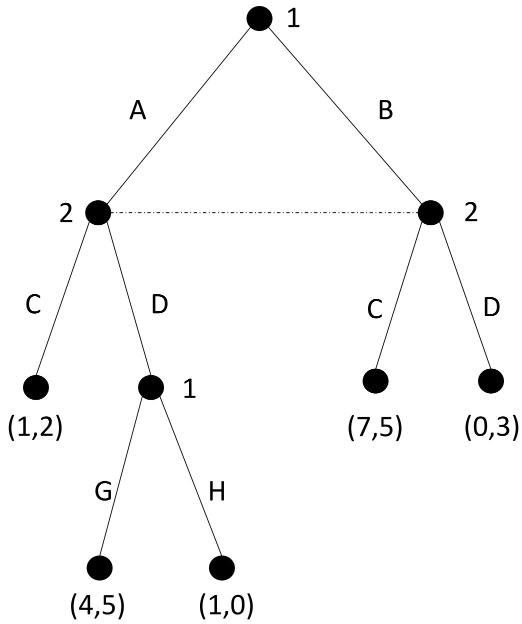
$\underline{i = 2}$
$B_2(a_3) = \underset{a_2}{\operatorname{argmax}}[f_2(a_2, a_3) + f_4(a_2)]$
$f_5(a_3) = \max_{a_2}[f_2(a_2, a_3) + f_4(a_2)]$
$F = \{f_3, f_5\}$

$\underline{i = 3}$
$B_3(a_4) = \underset{a_3}{\operatorname{argmax}}[f_3(a_3, a_4) + f_5(a_3)]$
$f_6(a_4) = \max_{a_3}[f_3(a_3, a_4) + f_5(a_3)]$
$F = \{f_6\}$

$\underline{i = 4}$
$B_4(a_5) = \underset{a_4}{\operatorname{argmax}}[f_6(a_4)]$

4 Extensive-form games

Consider the following imperfect-information extensive-form game.



Question 8. (**0.5 pts.**)
What are the pure strategies of the extensive-form game above?

Write your answer here:

Solution 8.
The pure strategies are:
$S_1 = \{(A, G), (A, H), (B, G), (B, H)\}$
$S_2 = \{C, D\}$

Question 9. (**1 pts.**)
Convert the extensive-form game to an equivalent normal-form game and find the Nash equilibria.

Write your answer here:

Solution 9.
This is the equivalent normal-form game:

	C	D
(A,G)	1,2	4,5
(A,H)	1,2	1,0
(B,G)	7,5	0,3
(B,H)	7,5	0,3

In order to find the pure strategy Nash equilibria, we first find the best responses for agent 1 and agent 2 and underline them in the payoff matrix:

	C	D
(A,G)	1,2	4,5
(A,H)	1,2	1,0
(B,G)	7,5	0,3
(B,H)	7,5	0,3

A joint action satisfies the definition of a NE if each agent's action is the best response to the other's. We thus have a Nash equilibrium if both payoffs are underlined in a cell of the payoff matrix above. In conclusion, the Nash equilibria is {(A,G),D}, {(B,G),C)}, and {(B,H),C}}.

5 Bayesian Games

Consider a two-agent game below, whereby the payoffs depend on β:

For $\beta = 0$:

	U	D
U	0, 4	4, 2
D	2, 4	6, 6

For $\beta = 1$:

	U	D
U	1, 1	2, 0
D	0, 0	0, 2

where $P(\beta = 0) = P(\beta = 1) = \frac{1}{2}$. Only agent 1 knows whether $\beta = 0$ or $\beta = 1$.

Question 10. (1 pts.)

Write this problem as a Bayesian game.

Write your answer here:

Solution 10.

A Bayesian game is tuple (N, A, θ, P, u) where:

- $N = \{Agent1, Agent2\}$ is a set of agents
- $A_1 = A_2 = \{U, D\}$ is a set of actions for each agent and $A = A_1 \times A_2$
- $\theta_1 = \{0, 1\}$ and $\theta_2 = \{t_2\}$ are the set of types for each agent and $\theta = \theta_1 \times \theta_2$
- $P(0, t_2) = P(1, t_2) = \frac{1}{2}$ is a common prior over types
- $u_1(a_1, a_2, \theta_1, \theta_2)$ and $u_2(a_1, a_2, \theta_1, \theta_2)$ are given by the payoff matrices above and $u = (u_1, u_2)$

Question 11. (2 pts.)

Find a Bayesian Nash equilibrium of this game.

Write your answer here:

Solution 11.

Considering Agent 1:

For $\theta_1 = 0$: D strictly dominates U. Hence, $a_1^*(\theta_1 = 0) = D$.

For $\theta_1 = 1$: U strictly dominates D. Hence, $a_1^*(\theta_1 = 1) = U$.

Considering Agent 2:

Calculating the expected utility (EU) for action U:

$EU_2(U|\theta_2) = P(\theta_1 = 0|\theta_2 = t_2) \times u_2(a_1^*(\theta_1 = 0), U, \theta_1 = 0, \theta_2 = t_2) + P(\theta_1 = 1|\theta_2 = t_2) \times u_2(a_1^*(\theta_1 = 1), U, \theta_1 = 1, \theta_2 = t_2)$

$EU_2(U|\theta_2) = P(\theta_1 = 0|\theta_2 = t_2) \times u_2(D, U, 0, t_2) + P(\theta_1 = 1|\theta_2 = t_2) \times u_2(U, U, 1, t_2)$
$EU_2(U|\theta_2) = \frac{1}{2} \times 4 + \frac{1}{2} \times 1 = 2.5$

Calculating the expected utility (EU) for action D:
$EU_2(D|\theta_2) = P(\theta_1 = 0|\theta_2 = t_2) \times u_2(a_1^*(\theta_1 = 0), D, \theta_1 = 0, \theta_2 = t_2) + P(\theta_1 = 1|\theta_2 = t_2) \times u_2(a_1^*(\theta_1 = 1), D, \theta_1 = 1, \theta_2 = t_2)$
$EU_2(D|\theta_2) = P(\theta_1 = 0|\theta_2 = t_2) \times u_2(D, D, 0, t_2) + P(\theta_1 = 1|\theta_2 = t_2) \times u_2(U, D, 1, t_2)$
$EU_2(D|\theta_2) = \frac{1}{2} \times 6 + \frac{1}{2} \times 0 = 3$

Since $EU_2(D|\theta_2) > EU_2(U|\theta_2)$ then $a_2^*(t_2) = D$

The Bayesian Nash equilibrium is $a^* = (a_1^*, a_2^*)$ where $a_1^*(\theta_1 = 0) = D$, $a_1^*(\theta_1 = 1) = U$, and $a_2^*(t_2) = D$.

6 Repeated games

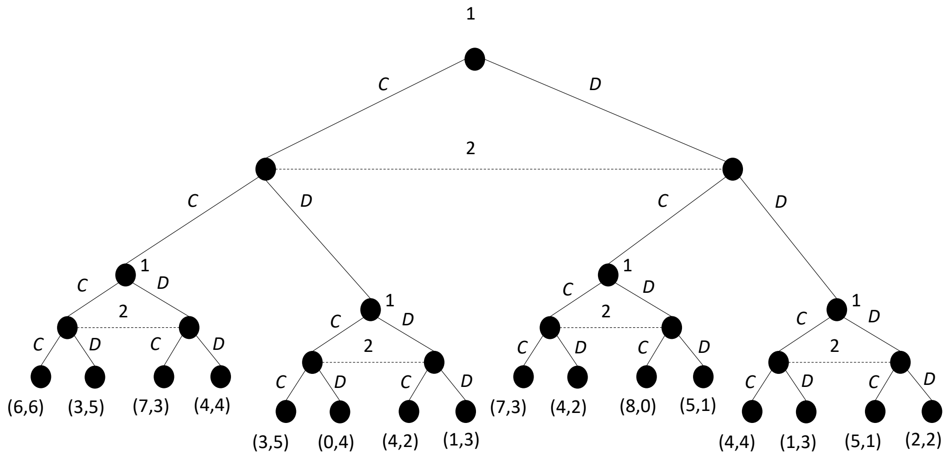
The following stage game is played repeatedly:

	C	D
C	3, 3	0, 2
D	4, 0	1, 1

Question 12. (**1 pts.**)
If the stage game above is played twice, use an extensive-form game to represent this repeated game.

Write your answer here:

Solution 12.



Question 13. (**2 pts.**)
Could we sustain cooperation if the stage game above is played an infinite number of steps? Use future discounted rewards in order to obtain the solution and give an interpretation of the final result.

Write your answer here:

Solution 13.
If agents 1 and 2 always cooperate:
$3 + \beta 3 + \beta^2 3 + \beta^3 3 + \ldots = \frac{3}{1-\beta}$

If agent 1 defects and agent 2 cooperates in the first time step, then agent 2 will defect in all the other time steps (trigger strategy):

$4 + \beta 1 + \beta^2 1 + \beta^3 1 + \ldots = 4 + \beta \frac{1}{1-\beta}$

The difference between these two strategies is:
$-1 + \beta 2 + \beta^2 2 + \beta^3 2 + \ldots = -1 + \beta \frac{2}{1-\beta}$

To sustain cooperation:
$-1 + \beta \frac{2}{1-\beta} \geq 0$
$\beta \frac{2}{1-\beta} \geq 1$
$2\beta \geq 1 - \beta$
$\beta \geq \frac{1}{3}$

Interpretation: if we want to sustain cooperation, the agent needs to care about tomorrow at least $\frac{1}{3}$ more than he cares about today!

7 Learning in Games

Question 14. (**3 pts.**)
Suppose two agents are playing the repeated game of the Stag Hunt:

	C	D
C	3, 3	0, 2
D	2, 0	1, 1

Using fictitious play, calculate the beliefs and actions for both agents in the first 5 rounds. Use the following initial beliefs $w_1 = (1.5, 1)$ and $w_2 = (1, 1.5)$.

Write your answer here:

Solution 14.

Round	1's action	2's action	1's beliefs	2's beliefs
0			(1.5,1)	(1,1.5)
1	C	D	(1.5,2)	(2,1.5)
2	D	C	(2.5,2)	(2,2.5)
3	C	D	(2.5,3)	(3,2.5)
4	D	C	(3.5,3)	(3,3.5)
5	C	D	(3.5,4)	(4,3.5)