Ficha 2
Resolução dos exercícios propostos

I.1 Calcule as seguintes primitivas, utilizando a regra de primitivação:

$$\boxed{P\,u'\cdot u^k = \frac{u^{k+1}}{k+1}+C,\ k\neq -1}$$

a) Px^2

Resolução

$$Px^2 \underset{\substack{\uparrow\\ u=x,\ k=2\\ u'=1}}{=} \frac{x^{2+1}}{2+1}+C=\frac{x^3}{3}+C$$

b) $P(x+5)^3$

Resolução

$$P(x+5)^3 \underset{\substack{\uparrow\\ u=x+5,\ k=3\\ u'=1}}{=} \frac{(x+5)^{3+1}}{3+1}+C=\frac{(x+5)^4}{4}+C$$

c) $P(2x+3)^2$

Resolução

$$P(2x+3)^2 \underset{\substack{\uparrow\\ u=2x+3,\ k=2\\ u'=2}}{=} \frac{1}{2}P2(2x+3)^2=\frac{1}{2}\frac{(2x+3)^{2+1}}{2+1}+C=\frac{(2x+3)^3}{6}+C$$

d) $Px^2(2x^3+2)$

Resolução

$$Px^2(2x^3+2) \underset{\substack{\uparrow\\ u=2x^3+2,\ k=1\\ u'=2\cdot 3x^2=6x^2}}{=} \frac{1}{6}P6x^2(2x^3+2)=\frac{1}{6}\frac{(2x^3+2)^2}{2}+C=\frac{(2x^3+2)^2}{12}+C$$

e) $P2x(5+6x^2)^{\frac{1}{2}}$

Resolução

$$P2x(5+6x^2)^{\frac{1}{2}} \underset{\substack{\uparrow\\ u=5+6x^2,\ k=\frac{1}{2}\\ u'=12x}}{=} \frac{1}{6}P6\cdot 2x(5+6x^2)^{\frac{1}{2}}=\frac{1}{6}\frac{(5+6x^2)^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=\frac{1}{6}\frac{(5+6x^2)^{\frac{3}{2}}}{\frac{3}{2}}+C$$

$$=\frac{1}{6}\frac{2}{3}\sqrt{(5+6x^2)^3}+C=\frac{1}{9}(5+6x^2)\sqrt{5+6x^2}+C$$

f) $P\frac{2}{(x+2)^2}$

Resolução

$$P\frac{2}{(x+2)^2}=P2(x+2)^{-2}=2P(x+2)^{-2} \underset{\substack{\uparrow\\ u=x+2,\ k=-2\\ u'=1}}{=} 2\frac{(x+2)^{-2+1}}{-2+1}+C=2\frac{(x+2)^{-1}}{-1}+C=\frac{-2}{x+2}+C$$

g) $P\frac{x}{(x^2+3)^3}$

Resolução

$$P\frac{x}{(x^2+3)^3} \underset{\substack{\uparrow\\ u=x^2+3,\ k=-3\\ u'=2x}}{=} \frac{1}{2}P2x(x^2+3)^{-3}=\frac{1}{2}\frac{(x^2+3)^{-3+1}}{-3+1}=\frac{1}{2}\frac{(x^2+3)^{-2}}{-2}+C=-\frac{1}{4(x^2+3)^2}+C$$

Elaborado por Maria Cristina Jorge e João Prata

h) $P\frac{x}{2\sqrt{x}}$

Resolução

$$P\frac{x}{2\sqrt{x}}=\frac{1}{2}P\frac{x}{x^{\frac{1}{2}}}=\frac{1}{2}Px\cdot x^{-\frac{1}{2}}=\frac{1}{2}Px^{1-\frac{1}{2}}=\frac{1}{2}Px^{\frac{1}{2}}\underset{\substack{\uparrow\\ u=x,\ k=\frac{1}{2}\\ u'=1}}{=}\frac{1}{2}\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=\frac{1}{2}\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+C=\frac{1}{2}\frac{2}{3}\sqrt{x^3}+C=\frac{1}{3}x\sqrt{x}+C$$

i) $P\frac{2}{\sqrt[3]{(x+4)^2}}$

Resolução

$$P\frac{2}{\sqrt[3]{(x+4)^2}}=P\frac{2}{(x+4)^{\frac{2}{3}}}=2P(x+4)^{-\frac{2}{3}}\underset{\substack{\uparrow\\ u=x+4,\ k=-\frac{2}{3}\\ u'=1}}{=}2\frac{(x+4)^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}+C=2\frac{(x+4)^{\frac{1}{3}}}{\frac{1}{3}}+C=6\sqrt[3]{x+4}+C$$

j) $P(x+1)\sqrt{2x+2+x^2}$

Resolução

$$P(x+1)\sqrt{2x+2+x^2}\underset{\substack{\uparrow\\ u=2x+2+x^2,\ k=\frac{1}{2}\\ u'=2+2x=2(x+1)}}{=}\frac{1}{2}P2(x+1)\left(2x+2+x^2\right)^{\frac{1}{2}}=\frac{1}{2}\frac{\left(2x+2+x^2\right)^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=\frac{1}{2}\frac{\left(2x+2+x^2\right)^{\frac{3}{2}}}{\frac{3}{2}}+C$$

$$=\frac{1}{2}\frac{2}{3}\sqrt{\left(2x+2+x^2\right)^3}+C=\frac{1}{3}\left(2x+2+x^2\right)\sqrt{2x+2+x^2}+C$$

l) $P\left(3x^2+5x+2\right)$

Resolução

$$P\left(3x^2+5x\right)=P3x^2+P5x=3Px^2+5Px=3\frac{x^3}{3}+5\frac{x^2}{2}+C=x^3+\frac{5}{2}x^2+C$$

I.2 Calcule as seguintes primitivas, utilizando a regra de primitivação:

$$\boxed{Pu'\cdot e^u=e^u+C}$$

a) Pe^{x+1}

Resolução

$$Pe^{x+1}\underset{\substack{\uparrow\\ u=x+1\\ u'=1}}{=}e^{x+1}+C$$

b) $P\left(x^2e^{2x^3}\right)$

Resolução

$$P\left(x^2e^{2x^3}\right)\underset{\substack{\uparrow\\ u=2x^3\\ u'=6x^2}}{=}\frac{1}{6}P6x^2e^{2x^3}=\frac{e^{2x^3}}{6}+C$$

I.3 Calcule as seguintes primitivas, utilizando a regra de primitivação:

$$\boxed{P\frac{u'}{u}=\ln|u|+C}$$

a) $P\frac{1}{x+2}$

Resolução

$$P\frac{1}{x+2}\underset{\substack{\uparrow\\ u=x+2\\ u'=1}}{=}\ln|x+2|+C$$

b) $P\dfrac{5x}{x^2+4}$

Resolução

$$P\frac{5x}{x^2+4} \underset{\substack{\uparrow \\ u=x^2+5 \\ u'=2x}}{=} \frac{5}{2}P\frac{2x}{x^2+4} = \frac{5}{2}\ln\left|x^2+4\right|+C$$

c) $P(5x+4)^{-1}$

Resolução

$$P(5x+4)^{-1} = P\frac{1}{5x+4} \underset{\substack{\uparrow \\ u=5x+4 \\ u'=5}}{=} \frac{1}{5}P\frac{5}{5x+4} = \frac{1}{5}\ln\left|5x+4\right|+C$$

I.4 Calcule as seguintes primitivas, utilizando a regra de primitivação:

$$\boxed{P\,u'\cdot a^u = \frac{a^u}{\ln a}+C}$$

a) $P4^x$

Resolução

$$P4^x \underset{\substack{\uparrow \\ u=x \\ u'=1}}{=} \frac{4^x}{\ln 4}+C = \frac{4^x}{\ln 2^2}+C = \frac{4^x}{2\ln 2}+C$$

b) $P\left(\operatorname{sen}x\cdot 2^{\cos x}\right)$

Resolução

$$P\left(\operatorname{sen}x\cdot 2^{\cos x}\right) \underset{\substack{\uparrow \\ u=\cos x \\ u'=-\operatorname{sen}x}}{=} -P\left(-\operatorname{sen}x\, 2^{\cos x}\right) = -\frac{2^{\cos x}}{\ln 2}+C$$

I.5 Calcule as seguintes primitivas, utilizando a regra de primitivação:

$$\boxed{P\,u'\cos u = \operatorname{sen}u + C}$$

a) $P\cos(2x)$

Resolução

$$P\cos(2x) \underset{\substack{\uparrow \\ u=2x \\ u'=2}}{=} \frac{1}{2}P2\cos(2x) = \frac{1}{2}\operatorname{sen}(2x)+C$$

b) $P\left(\dfrac{-4}{(x+1)^2}\cos\left(\dfrac{2x}{x+1}\right)\right)$

Resolução

$$P\left(\frac{-4}{(x+1)^2}\cos\left(\frac{2x}{x+1}\right)\right) \underset{\uparrow}{=} -2P\frac{2}{(x+1)^2}\cos\left(\frac{2x}{x+1}\right) = -2\operatorname{sen}\left(\frac{2x}{x+1}\right)+C$$

$$u=\frac{2x}{x+1}$$

$$u'=\left(\frac{2x}{x+1}\right)' = \frac{(2x)'(x+1)-(2x)(x+1)'}{(x+1)^2}$$

$$=\frac{(2x)'(x+1)-(2x)(x+1)'}{(x+1)^2} = \frac{2(x+1)-(2x)1}{(x+1)^2}$$

$$=\frac{2x+2-2x}{(x+1)^2} = \frac{2}{(x+1)^2}$$

I.6 Calcule as seguintes primitivas, utilizando a regra de primitivação:

$$\boxed{P\,u'\operatorname{sen} u = -\cos u + C}$$

a) $P\left(xe^{x^2+1}\operatorname{sen}\left(e^{x^2+1}\right)\right)$

Resolução

$$P\left(xe^{x^2+1}\operatorname{sen}\left(e^{x^2+1}\right)\right) \underset{\substack{\uparrow\\ u=e^{x^2+1}\\ u'=2xe^{x^2+1}}}{=} \frac{1}{2}P\left(2xe^{x^2+1}\operatorname{sen}\left(e^{x^2+1}\right)\right) = -\frac{1}{2}\cos\left(e^{x^2+1}\right)+C$$

b) $P\left(x\operatorname{sen}\left(2x^2-\frac{\pi}{3}\right)\right)$

Resolução

$$P\left(x\operatorname{sen}\left(2x^2-\frac{\pi}{3}\right)\right) \underset{\substack{\uparrow\\ u=2x^2-\frac{\pi}{3}\\ u'=4x}}{=} \frac{1}{4}P\,4x\cdot\operatorname{sen}\left(2x^2-\frac{\pi}{3}\right) = -\frac{1}{4}\cos\left(2x^2-\frac{\pi}{3}\right)+C$$

I.7 Calcule as seguintes primitivas, utilizando a regra de primitivação:

$$\boxed{P\frac{u'}{\sqrt{1-u^2}} = \operatorname{arc\,sen} u + C}$$

a) $P\dfrac{2x+1}{\sqrt{1-\left(3x^2+3x\right)^2}}$

Resolução

$$P\frac{2x+1}{\sqrt{1-\left(3x^2+3x\right)^2}} \underset{\substack{\uparrow\\ u=3x^2+3x\\ u'=6x+3}}{=} \frac{1}{3}P\frac{3(2x+1)}{\sqrt{1-\left(3x^2+3x\right)^2}} = \frac{1}{3}\operatorname{arc\,sen}\left(3x^2+3x\right)+C$$

b) $P\dfrac{x}{\sqrt{4-9x^4}}$

Resolução

$$P\frac{x}{\sqrt{4-9x^4}} = P\frac{x}{\sqrt{4\left(1-\frac{9}{4}x^4\right)}} = P\frac{x}{2\sqrt{\left(1-\left(\frac{3}{2}x^2\right)^2\right)}} = \frac{1}{2}P\frac{x}{\sqrt{1-\left(\frac{3}{2}x^2\right)^2}} \underset{\substack{\uparrow\\ u=\frac{3}{2}x^2\\ u'=\frac{3}{2}\cdot 2x=3x}}{=} \frac{1}{2}\cdot\frac{1}{3}P\frac{3x}{\sqrt{1-\left(\frac{3}{2}x^2\right)^2}}$$

$$=\frac{1}{6}\operatorname{arc\,sen}\left(\frac{3}{2}x^2\right)+C$$

c) $P\dfrac{1}{\sqrt{1-4x^2}}$

Resolução

$$P\frac{1}{\sqrt{1-4x^2}} = P\frac{1}{\sqrt{1-(2x)^2}} \underset{\substack{\uparrow\\ u=2x\\ u'=2}}{=} \frac{1}{2}P\frac{2}{\sqrt{1-(2x)^2}} = \frac{1}{2}\operatorname{arc\,sen}(2x)+C$$

I.8 Calcule as seguintes primitivas, utilizando a regra de primitivação:

$$\boxed{P\frac{u'}{1+u^2} = \operatorname{arc\,tg}(u) + C}$$

a) $P\left(\frac{x^3}{x^8+4}\right)$

Resolução:

$$P\left(\frac{x^3}{x^8+4}\right) = P\left(\frac{x^3}{4\left(\frac{1}{4}x^8+1\right)}\right) = \frac{1}{4}P\left(\frac{x^3}{\frac{1}{2^2}\left(x^4\right)^2+1}\right) = \frac{1}{4}\frac{1}{2}P\left(\frac{2x^3}{\left(\frac{x^4}{2}\right)^2+1}\right) \underset{P\frac{u'}{1+u^2}=\operatorname{arc\,tg} u+C}{\uparrow} = \frac{1}{8}\operatorname{arc\,tg}\left(\frac{x^4}{2}\right)+C$$

b) $P\left(\frac{x^5}{9x^{12}+16}\right)$

Resolução:

$$P\left(\frac{x^5}{9x^{12}+16}\right) = P\left(\frac{x^5}{16\left(\frac{9}{16}x^{12}+1\right)}\right) = \frac{1}{16}P\left(\frac{x^5}{\left(\frac{3^2}{4^2}\left(x^6\right)^2+1\right)}\right) = \frac{1}{16}\frac{2}{9}P\left(\frac{\frac{9}{2}x^5}{\left(\left(\frac{3}{4}x^6\right)^2+1\right)}\right) \underset{P\frac{u'}{1+u^2}=\operatorname{arc\,tg} u+C}{\uparrow} = \frac{1}{72}\operatorname{arc\,tg}\left(\frac{3}{4}x^6\right)+C$$

Elaborado por Maria Cristina Jorge e João Prata