Autonomous Agents and Multiagent Systems

MSc in Computer Science and Engineering

Second Exam (2022-2023)

Instructions

- You have 120 minutes to complete the exam.
- Make sure that your exam has a total of 10 pages. Also, check if there are no missing sheets, then write your full name and student number on this page (and your student number on all pages).
- The exam has 14 questions, with a maximum score of 20 points. The questions have different levels of difficulty. The point value of each question is provided next to the question number.
- *If you get stuck in a question, move on.* You should start with the more straightforward questions to secure those points before moving on to the more complex questions.
- *No interaction with the faculty is allowed during the exam.* If you are unclear about a question, clearly indicate the unclear part and answer the question to the best of your ability.
- Please provide your answer in the space below each question. If you make a mess, clearly indicate your answer.
- This exam is a closed-book assessment, whereby students are NOT allowed to bring books or other reference material into the examination room. You may bring only ONE A4 page of handwritten notes, in your OWN handwriting. Typed notes or a copy of someone else's notes are not allowed.
- You may use a calculator, but any other type of electronic or communication equipment is not allowed.
- **Good luck!**

1 Agent architectures

Question 1. (1 pts.)

What are the main components of a BDI model? Explain each one.

Write your answer here:

Solution 1.

The BDI model has three main components. Beliefs, that represent the information that the agent has about the environment, other agents, and itself.

Desires, that express the state of affairs to achieve (e.g., goals).

Intentions, that represent commitments to achieving particular goals.

Question 2. (1 pts.)

What is a rational agent? Explain.

Write your answer here:

Solution 2.

A rational agent is an agent that acts (i.e., makes decisions) in a way that maximizes some utility function.

2 Normal-form games

Question 3. (**1 pts.**)

Model the following problem as a normal-form game. Alex and Bella are dating. They are planning to go together to a restaurant. They have two options of restaurants: an Indian and a Thai. However, Alex forgot his cell phone at home so he cannot call Bella to decide on the restaurant. Alex prefers Indian food and Bella prefers Thai. Both prefer to go together to any of these restaurants than going alone.

Write your answer here:

Solution 3.

N = {Alex, Bella}

A1 = A2 = {Indian, Thai}

The payoff matrix of the game:

	Indian	Thai
Indian	2, 1	0, 0
Thai	0, 0	1, 2

Alex is represented as player 1 (row) and Bella as player 2 (column).

Question 4. (**2 pts.**)

Given the following payoff matrix for a 2-agent normal-form game. Find and eliminate strictly dominated actions.

	x	y	z
a	3, 1	10, 0	5, 1
b	1, 3	5, 4	7, 5
c	2, 2	2, 1	8, 3

Write your answer here:

Solution 4.

Action y is strictly dominated by action z, since $u_2(z, a) > u_2(y, a)$ and $u_2(z, b) > u_2(y, b)$ and $u_2(z, c) > u_2(y, c)$.

Eliminate y. The following game is obtained:

	x	z
a	3, 1	5, 1
b	1, 3	7, 5
c	2, 2	8, 3

Action b is strictly dominated by action c, since $u_1(c, x) > u_1(b, x)$ and $u_1(c, z) > u_1(b, z)$

Eliminate b. The following game is obtained:

	x	z
a	3, 1	5, 1
c	2, 2	8, 3

Question 5. (**2 pts.**)

Determine the pure Nash Equilibria of the previous game.

	x	y	z
a	3, 1	10, 0	5, 1
b	1, 3	5, 4	7, 5
c	2, 2	2, 1	8, 3

Write your answer here:

Solution 5.

We should start with the simplified version of the game-

	x	z
a	3, 1	5, 1
c	2, 2	8, 3

The best action for A1 if A2 plays x is a. The best action for A1 if A2 plays z is c.

The best action for A2 if A1 plays a is indifferent (either option works). The best action for A2 if A1 plays c is z.

	x	z
a	$\underline{3}$, $\underline{1}$	5, $\underline{1}$
c	2, 2	$\underline{8}$, $\underline{3}$

The Nash Equilibria are {(a,x),(c,z)}.

Question 6. (**1 pts.**)

Consider the game of question 3. Explain how it can be solved by using social conventions.

Write your answer here:

Solution 6.

Using Social Conventions as a coordination mechanism defines an ordering scheme.

First ordering the agents. For example: Alex $\succ$ Bella

Then ordering the actions. For example: Indian $\succ$ Thai

Hence, according to the ordering scheme, the selected joint action is (Indian, Indian).

3 Applications of a Nash equilibrium: Cournot Model

Question 7. (**2 pts.**)
Consider a market with two firms that produce homogeneous products. Firm A has a constant marginal cost of \$20 per unit, while firm B has a constant marginal cost of \$10 per unit. The market demand function is given by $P = 100 - 2Q$, where Q is the total quantity and P is the price. Calculate the Cournot equilibrium output for each firm and the price.

Write your answer here:

Solution 7.
To find the Cournot equilibrium output and price, we can use the following steps:

- Firm A's payoff is $\pi_A = Pq_A - c_A = (100 - 2(q_A + q_B))q_A - 20q_A = -2q_A^2 - 2q_Bq_A + 80q_A$
- Firm B's payoff is $\pi_B = Pq_B - c_B = (100 - 2(q_A + q_B))q_B - 10q_B = -2q_B^2 - 2q_Aq_B + 90q_B$

- Firm A's first order condition: $\frac{d}{dq_A}\pi_A = 0$

  $-4q_A - 2q_B + 80 = 0$

  $q_A = \frac{40-q_B}{2}$

- Firm B's first order condition: $\frac{d}{dq_B}\pi_B = 0$

  $-4q_B - 2q_A + 90 = 0$

  $q_B = \frac{45-q_A}{2}$

- We now need to solve the following pair of equations:

  $q_A = \frac{40-q_B}{2}$

  $q_B = \frac{45-q_A}{2}$

- Hence, the Nash equilibrium is:

  $q_A = \frac{35}{3}$

  $q_B = \frac{50}{3}$

- And the price is $P = \frac{130}{3}$

4 Extensive-form games

Question 8. (**1 pts.**)

Define what are pure strategies of an agent in an extensive-form game.

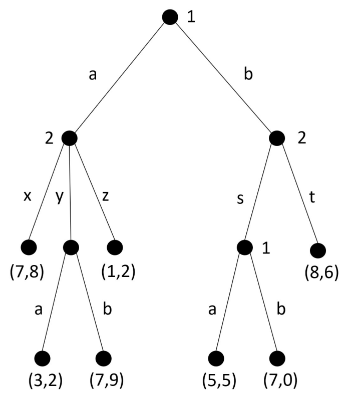
Write your answer here:

Solution 8.

A pure strategy for an agent in a perfect information game is the complete specification of which deterministic action to take at every node belonging to that agent. The pure strategies of an agent consist of the Cartesian product of all its actions in the node of the tree representing the game.

Question 9. (**1 pts.**)

Consider the following extensive form game:



Identify the pure strategies of the two agents.

Write your answer here:

Solution 9.

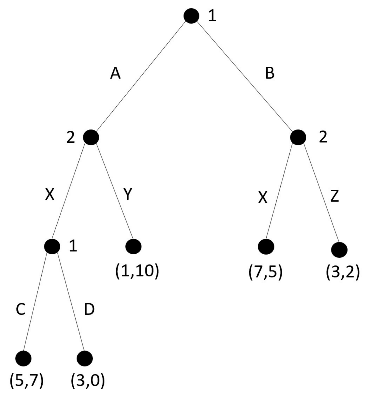
The pure strategies are:

S1 = {(a,a,a), (a,a,b), (a,b,a), (a,b,b), (b, a, a), (b,a,b), (b,b,a), (b,b,b)}

S2 = {(x,s), (x,t), (y,s), (y,t), (z,s), (z,t)}

Question 10. (**2 pts.**)

Consider the following extensive-form game:

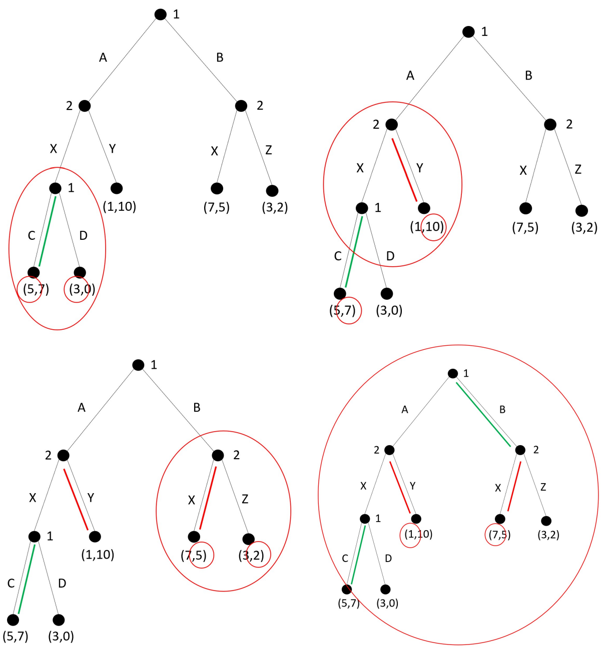


Solve the extensive-form game above using backward induction.

Write your answer here:

Solution 10.

Applying backward induction.



Hence, the subgame-perfect equilibrium is {(B,C),(X,Y)}

5 Bayesian games

Question 11. (1 pts.)
Define what is an agent type in a Bayesian game and explain its use.

Write your answer here:

Solution 11.
A Bayesian game is a strategic interaction among agents where each agent has incomplete information about the game being played. Agent types capture the private information agents can have about the game (e.g. the payoffs) and define different profiles that specify a concrete game. Types have a prior distribution that represents the probability of each player assuming each type.

Question 12. (3 pts.)
Consider the strategic game where Agent 1 is unsure about the kind of opponent it faces. For each opponent Agent 1 would need a different approach represented by the following payoff matrices:

for Opponent of kind A:

	X	Y
X	0, 4	3, 2
Y	3, 3	5, 2

for Opponent of kind B:

	X	Y
X	1, 3	2, 5
Y	3, 1	1, 3

where $P(Opponent = A) = p$ and $P(Opponent = B) = (1 - p)$, and $p \in [0, 1]$ is an unknown parameter.

First, formalize the game above as a Bayesian game. Second, find the interval of values of p such that Agent 1 plays action Y under the Bayesian Nash equilibrium.

Write your answer here:

Solution 12.

The game can be formalized as a Bayesian game as follows:

- $N = \{\text{Agent}_1, \text{Agent}_2\}$ is the set of agents;
- $A_1 = A_2 = \{X, Y\}$ is the set of actions for each agent with $A = A_1 \times A_2$;
- $\theta_1 = \{t_1\}$ and $\theta_2 = \{A, B\}$ are the set of types for each agent with $\theta = \theta_1 \times \theta_2$;
- $P(\theta_1 = t_1, \theta_2 = A) = p$ and $P(\theta_1 = t_1, \theta_2 = B) = (1-p)$ is a prior over types;
- $u_1(a_1, a_2, \theta_1, \theta_2)$ and $u_2(a_1, a_2, \theta_1, \theta_2)$ are given by the payoff matrices above and $u = (u_1, u_2)$.

Considering Agent 2:

For $\theta_2 = A$, X strictly dominates Y. Hence, $a_2^*(\theta_2 = A) = X$.

For $\theta_2 = B$, Y strictly dominates X. Hence, $a_2^*(\theta_2 = B) = Y$.

Considering Agent 1:

The expected utility for action X is given by:

$$\begin{aligned} EU_1(X|\theta_1) &= P(\theta_2 = A|\theta_1 = t_1) \cdot u_1(X, a_2^*(\theta_2 = A), t_1, A) + P(\theta_2 = B|\theta_1 = t_1) \cdot u_1(X, a_2^*(\theta_2 = B), t_1, B) \\ &= P(\theta_2 = A|\theta_1 = t_1) \cdot u_1(X, X, t_1, A) + P(\theta_2 = B|\theta_1 = t_1) \cdot u_1(X, Y, t_1, B) \\ &= p \cdot 0 + (1-p) \cdot 2 \end{aligned}$$

The expected utility for action Y is given by:

$$\begin{aligned} EU_1(Y|\theta_1) &= P(\theta_2 = A|\theta_1 = t_1) \cdot u_1(Y, a_2^*(\theta_2 = A), t_1, A) + P(\theta_2 = B|\theta_1 = t_1) \cdot u_1(Y, a_2^*(\theta_2 = B), t_1, B) \\ &= P(\theta_2 = A|\theta_1 = t_1) \cdot u_1(Y, X, t_1, A) + P(\theta_2 = B|\theta_1 = t_1) \cdot u_1(Y, Y, t_1, B) \\ &= p \cdot 3 + (1-p) \cdot 1 \end{aligned}$$

For Agent 1 to select action Y under the Bayesian Nash equilibrium we need to have that $EU_1(Y|\theta_1) > EU_1(X|\theta_1)$:

$$\begin{aligned} EU_1(Y|\theta_1) &> EU_1(X|\theta_1) \\ 3p + 1 \cdot (1-p) &> 2 \cdot (1-p) \\ p &> 1/4 \end{aligned}$$

Thus, Agent 1 selects $a_1^*(t_1) = Y$ in the Bayesian Nash equilibrium $a^* = (a_1^*, a_2^*)$ whenever $p > 1/4$.

6 Repeated games

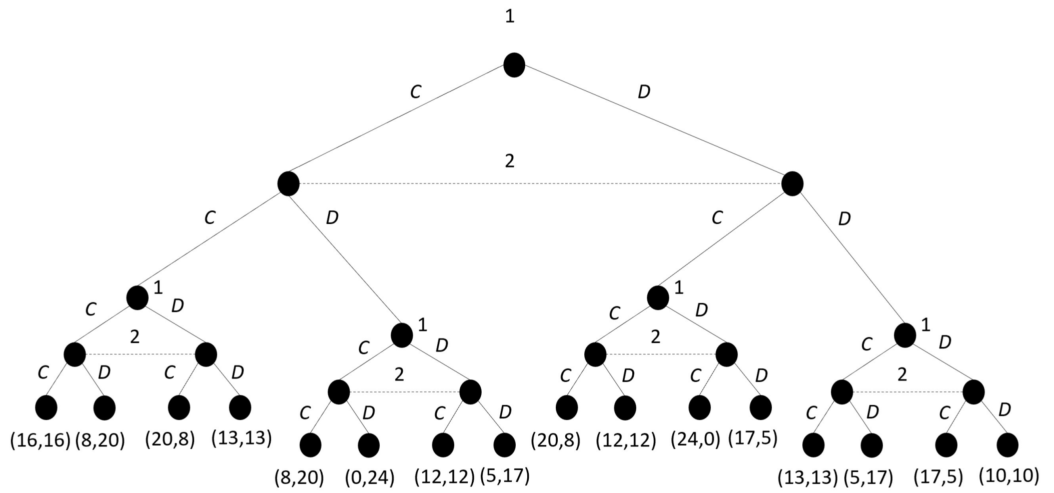
The following stage game of the Prisoner's Dilemma is played repeatedly:

	C	D
C	8, 8	0, 12
D	12, 0	5, 5

Question 13. (**1 pts.**)
If the stage game above is played twice, use an extensive-form game to represent this repeated game.

Write your answer here:

Solution 13.



Question 14. (**1 pts.**)
What is the Nash Equilibrium of this repeated game?

Write your answer here:

Solution 14.
If a stage game G has a unique Nash equilibrium then, for any finite repetitions the game G has a unique outcome: the Nash equilibrium of G is played in every subgame perfect stage. Hence the Nash Equilibrium of the repeated game is the one expected for a Prisoner's Dilemma, {(D,D),(D,D)}.