Ficha 2
Resolução dos exercícios de auto-avaliação

III 1 Calcule as seguintes primitivas:

a) $P\left(x\left(x^2+2\right)^3\right)$

Resolução

$$P\left(x\left(x^2+2\right)^3\right) \underset{\substack{\uparrow \\ u=x^2+2,\ k=3 \\ u'=2x}}{=} \frac{1}{2}P\left(2x\left(x^2+2\right)^3\right)=\frac{1}{2}\frac{\left(x^2+2\right)^{3+1}}{3+1}+C=\frac{1}{2}\frac{\left(x^2+2\right)^4}{4}+C=\frac{\left(x^2+2\right)^4}{8}+C$$

b) $P\left(x+\sqrt{x}\right)$

Resolução

$$P\left(x+\sqrt{x}\right)=P\left(x+x^{\frac{1}{2}}\right) \underset{\substack{\uparrow \\ u=x,\ k=1 \\ u'=1 \\ u_1=x,\ k=\frac{1}{2} \\ u_1'=1}}{=} \frac{x^{1+1}}{1+1}+\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=\frac{x^2}{2}+\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+C=\frac{x^2}{2}+\frac{2}{3}\sqrt{x^3}+C$$

c) $P\dfrac{3}{\sqrt[4]{6x}}$

Resolução

$$P\frac{3}{\sqrt[4]{6x}}=P\frac{3}{(6x)^{\frac{1}{4}}}=P\left(3(6x)^{-\frac{1}{4}}\right) \underset{\substack{\uparrow \\ u=6x,\ k=-\frac{1}{4} \\ u'=6}}{=} \frac{1}{2}P\left(2\cdot 3(6x)^{-\frac{1}{4}}\right)=\frac{1}{2}\frac{(6x)^{-\frac{1}{4}+1}}{-\frac{1}{4}+1}+C$$

$$=\frac{1}{2}\frac{(6x)^{\frac{3}{4}}}{\frac{3}{4}}+C=\frac{4\sqrt[4]{(6x)^3}}{3\cdot 2}=\frac{2\sqrt[4]{(6x)^3}}{3}+C$$

d) $P\left(x^2e^{2x^3}\right)$

e) $P\left(\dfrac{x}{1+x^4}e^{\operatorname{arc\,tg} x^2}\right)$

Resolução

$$P\left(\frac{x}{1+x^4}e^{\operatorname{arc\,tg} x^2}\right) \underset{\substack{\uparrow \\ u=\operatorname{arc\,tg} x^2 \\ u'=\frac{\left(x^2\right)'}{1+\left(x^2\right)^2}=\frac{2x}{1+x^4}}}{=} \frac{1}{2}P\frac{2x}{1+x^4}e^{\operatorname{arc\,tg} x^2}=\frac{1}{2}e^{\operatorname{arc\,tg} x^2}+C$$

f) $P\left((x+1)\cos\left(x^2+x\right)e^{\operatorname{sen}\left(x^2+x\right)}\right)$

Resolução

$$P\left((x+1)\cos\left(x^2+x\right)e^{\operatorname{sen}\left(x^2+x\right)}\right) \underset{\substack{\uparrow \\ u=\operatorname{sen}\left(x^2+x\right) \\ u'=\left(x^2+2x\right)'\cos\left(x^2+x\right) \\ =(2x+2)\cos\left(x^2+x\right)}}{=} \frac{1}{2}P2(x+1)\cos\left(x^2+x\right)e^{\operatorname{sen}\left(x^2+x\right)}$$

$$=\frac{1}{2}P(2x+2)\cos\left(x^2+x\right)e^{\operatorname{sen}\left(x^2+x\right)}=\frac{1}{2}e^{\operatorname{sen}\left(x^2+x\right)}+C$$

g) $P\left(\frac{5}{x+1}e^{\ln(4x+4)}\right)$

Resolução

$$P\left(\frac{5}{x+1}e^{\ln(4x+4)}\right)\underset{\substack{\uparrow \\ u=\ln(4x+4) \\ u'=\frac{(4x+4)'}{4x+4}=\frac{4}{4x+4}=\frac{1}{x+1}}}{=} 5P\frac{1}{x+1}e^{\ln(4x+4)}=5e^{\ln(4x+4)}+C$$

h) $P\left(\frac{1+xe^x}{x}3^{(\ln x+e^x)}\right)$

Resolução

$$P\left(\frac{1+xe^x}{x}3^{(\ln x+e^x)}\right)\underset{\substack{\uparrow \\ u=\ln x+e^x \\ u'=\frac{1}{x}+e^x=\frac{1+xe^x}{x}}}{=}\frac{3^{(\ln x+e^x)}}{\ln 3}+C$$

i) $P\left(\frac{1}{x}\operatorname{sen}(\ln 2x)\right)$

Resolução

$$P\left(\frac{1}{x}\operatorname{sen}(\ln 2x)\right)\underset{\substack{\uparrow \\ u=\ln 2x \\ u'=\frac{(2x)'}{2x}=\frac{2}{2x}=\frac{1}{x}}}{=}-\cos(\ln 2x)+C$$

III 2 Primitive as seguintes funções:

a) $\operatorname{sen} x(1+\cos x)^2$

Resolução:

$$P\operatorname{sen} x(1+\cos x)^2=-P(-\operatorname{sen} x)(1+\cos x)^2\underset{\substack{\uparrow \\ \text{Regra de primitivação: } P\,u'\cdot u^k=\frac{u^{k+1}}{k+1}+C,\ k\neq -1 \\ \text{em que } \begin{cases} u=1+\cos x, & k=2 \\ u'=-\operatorname{sen} x \end{cases}}}{=}-\frac{(1+\cos x)^{2+1}}{2+1}+C=-\frac{(1+\cos x)^3}{3}+C$$

b) $\frac{3\operatorname{sen} x}{(1+\cos x)^2}$

Resolução

$$P\frac{3\operatorname{sen} x}{(1+\cos x)^2}=-3P(-\operatorname{sen} x)(1+\cos x)^{-2}\underset{\substack{\uparrow \\ \text{Regra de primitivação: } P\,u'\cdot u^k=\frac{u^{k+1}}{k+1}+C,\ k\neq -1 \\ \text{em que } \begin{cases} u=1+\cos x, & k=-2 \\ u'=-\operatorname{sen} x \end{cases}}}{=}-3\frac{(1+\cos x)^{-2+1}}{-2+1}+C=-3\frac{(1+\cos x)^{-1}}{-1}+C=\frac{3}{1+\cos x}+C$$

c) $\dfrac{3\text{sen } x}{\sqrt{1+\cos x}}$

Resolução

$$P\frac{3\text{sen } x}{\sqrt{1+\cos x}} = P\frac{3\text{sen } x}{(1+\cos x)^{\frac{1}{2}}} = P3\text{sen } x\,(1+\cos x)^{-\frac{1}{2}} = -3P\,(-\text{sen } x)(1+\cos x)^{-\frac{1}{2}} \underset{\uparrow}{=} -3\frac{(1+\cos x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C$$

Regra de primitivação: $P\,u'\cdot u^k = \frac{u^{k+1}}{k+1}+C,\ k\neq -1$

em que $\begin{cases} u=1+\cos x, & k=-\frac{1}{2} \\ u'=-\text{sen } x \end{cases}$

$$= -3\frac{(1+\cos x)^{\frac{1}{2}}}{\frac{1}{2}}+C = -6\sqrt{1+\cos x}+C$$

d) $\dfrac{3\text{sen } x}{1+\cos x}$

Resolução

$$P\,\frac{3\text{sen } x}{1+\cos x} = -3P\frac{-\text{sen } x}{1+\cos x} \underset{\uparrow}{=} -3\ln|1+\cos x|+C$$

Usando a regra de primitivação: $P\frac{u'}{u}=\ln|u|+C$

em que $\begin{cases} u=1+\cos x \\ u'=-\text{sen} x \end{cases}$

e) $\dfrac{x^3}{\sqrt{1-x^4}}$

Resolução

$$P\,\frac{x^3}{\sqrt{1-x^4}} = P\,\frac{x^3}{(1-x^4)^{\frac{1}{2}}} = -\frac{1}{4}P\left(-4x^3(1-x^4)^{-\frac{1}{2}}\right) \underset{\uparrow}{=} -\frac{1}{4}\frac{(1-x^4)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C = -\frac{1}{4}\frac{(1-x^4)^{\frac{1}{2}}}{\frac{1}{2}}+C$$

Regra de primitivação: $P\,u'\cdot u^k = \frac{u^{k+1}}{k+1}+C,\ k\neq -1$

em que $\begin{cases} u=1-x^4, & k=-\frac{1}{2} \\ u'=-4x^3 \end{cases}$

$$= -\frac{1}{4}\frac{2}{1}\sqrt{1-x^4}+C = -\frac{1}{2}\sqrt{1-x^4}+C$$

f) $\dfrac{e^{6x}}{\sqrt{1-e^{6x}}}$

Resolução

$$P\,\frac{e^{6x}}{\sqrt{1-e^{6x}}} = P\,\frac{e^{6x}}{(1-e^{6x})^{\frac{1}{2}}} = -\frac{1}{6}P\left(-6e^{6x}(1-e^{6x})^{-\frac{1}{2}}\right) \underset{\uparrow}{=} -\frac{1}{6}\frac{(1-e^{6x})^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C = -\frac{1}{6}\frac{(1-e^{6x})^{\frac{1}{2}}}{\frac{1}{2}}+C$$

Regra de primitivação: $P\,u'\cdot u^k = \frac{u^{k+1}}{k+1}+C,\ k\neq -1$

em que $\begin{cases} u=1-e^{6x}, & k=-\frac{1}{2} \\ u'=-6e^{6x} \end{cases}$

$$= -\frac{1}{6}\frac{2}{1}\sqrt{1-e^{6x}}+C = -\frac{1}{3}\sqrt{1-e^{6x}}+C$$

g) $\dfrac{x}{1+x^2}$

Resolução

$$P\ \frac{x}{1+x^2}=\frac{1}{2}P\ \underset{\uparrow}{\frac{2x}{1+x^2}}=\frac{1}{2}\ln\left|1+x^2\right|+C=\frac{1}{2}\ln\left(1+x^2\right)+C$$

Usando a regra de primitivação: $P\frac{u'}{u}=\ln|u|+C$

em que $\begin{cases}u=1+x^2\\ u'=2x\end{cases}$

h) $\dfrac{x^5}{1+x^6}$

Resolução

$$P\frac{x^5}{1+x^6}=\frac{1}{6}P\underset{\uparrow}{\frac{6x^5}{1+x^6}}=\frac{1}{6}\ln\left|1+x^6\right|+C=\frac{1}{6}\ln\left(1+x^6\right)+C$$

Usando a regra de primitivação: $P\frac{u'}{u}=\ln|u|+C$

em que $\begin{cases}u=1+x^6\\ u'=6x^5\end{cases}$

i) $\dfrac{3\operatorname{sen} x}{\left(1+\cos x\right)^2}$

Resolução

$$P\frac{3\operatorname{sen} x}{\left(1+\cos x\right)^2}=-3P\ \underset{\uparrow}{\left(-\operatorname{sen} x\left(1+\cos x\right)^{-2}\right)}=-3\frac{\left(1+\cos x\right)^{-2+1}}{-2+1}+C=-3\frac{\left(1+\cos x\right)^{-1}}{-1}+C=\frac{3}{1+\cos x}+C$$

Regra de primitivação: $P\,u'\cdot u^k=\frac{u^{k+1}}{k+1}+C,\ k\neq-1$

em que $\begin{cases}u=1+\cos x, & k=2\\ u'=-\operatorname{sen} x\end{cases}$

j) $x\sqrt{1+x^2}$

Resolução

$$P\ x\sqrt{1+x^2}=P\ x\left(1+x^2\right)^{\frac{1}{2}}=\frac{1}{2}P\underset{\uparrow}{\left(2x\left(1+x^2\right)^{\frac{1}{2}}\right)}=\frac{1}{2}\frac{\left(1+x^2\right)^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=\frac{1}{2}\frac{\left(1+x^2\right)^{\frac{3}{2}}}{\frac{3}{2}}+C=\frac{1}{3}\left(1+x^2\right)\sqrt{1+x^2}+C$$

Regra de primitivação: $P\,u'\cdot u^k=\frac{u^{k+1}}{k+1}+C,\ k\neq-1$

em que $\begin{cases}u=1+x^2, & k=\frac{1}{2}\\ u'=2x\end{cases}$

l) $\dfrac{\operatorname{sen}x-\cos x}{\operatorname{sen}x+\cos x}$

Resolução

$$P\frac{\operatorname{sen} x-\cos x}{\operatorname{sen} x+\cos x}=-P\underset{\uparrow}{\frac{-\left(\operatorname{sen} x-\cos x\right)}{\operatorname{sen} x+\cos x}}=-\ln\left|\operatorname{sen} x+\cos x\right|+C$$

Usando a regra de primitivação: $P\frac{u'}{u}=\ln|u|+C$

em que $\begin{cases}u=\operatorname{sen} x+\cos x\\ u'=\cos x-\operatorname{sen} x=-\left(\operatorname{sen} x-\cos x\right)\end{cases}$

m) $\operatorname{tg}(2x)$

Resolução

$$P\operatorname{tg}\left(2x\right)=-\frac{1}{2}P\underset{\uparrow}{\frac{-2\operatorname{sen}\left(2x\right)}{\cos\left(2x\right)}}=-\frac{1}{2}\ln\left|\cos\left(2x\right)\right|+C$$

Usando a regra de primitivação: $P\frac{u'}{u}=\ln|u|+C$

em que $\begin{cases}u=\cos\left(2x\right)\\ u'=-2\operatorname{sen} x=-\left(\operatorname{sen} x-\cos x\right)\end{cases}$

n) $\dfrac{1}{x^2+2}$

Resolução

$$P\frac{1}{x^2+2}=P\frac{1}{2\left(\frac{1}{2}x^2+1\right)}=\frac{1}{2}P\frac{1}{\frac{1}{2}x^2+1}=\frac{1}{2}P\frac{1}{\left(\frac{1}{\sqrt{2}}x\right)^2+1}=\frac{\sqrt{2}}{2}P\frac{\frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}x\right)^2+1}=\text{arc tg}\left(\frac{1}{\sqrt{2}}x\right)+C$$

$\uparrow$ Regra de primitivação: $P\frac{u'}{1+u^2}=\text{arc tg}\,u+C$ em que $\begin{cases} u=\frac{1}{\sqrt{2}}x \\ u'=\frac{1}{\sqrt{2}} \end{cases}$

o) $\text{sen}^3x\cdot\cos^3x$

Resolução

$$P\,\text{sen}^3x\cdot\cos^3x=P\,\text{sen}^3x\left(1-\text{sen}^2x\right)\cos x=P\,\left(\text{sen}^3x\cos x-\text{sen}^5x\cos x\right)=P\,\cos x\,\text{sen}^3x-P\cos x\,\text{sen}^5x$$

$\uparrow$ $\cos^3x=\cos^2x\cos x=\left(1-\text{sen}^2x\right)\cos x$

$$=\frac{(\text{sen}\,x)^{3+1}}{3+1}+\frac{(\text{sen}\,x)^{5+1}}{5+1}+C=\frac{(\text{sen}\,x)^4}{4}+\frac{(\text{sen}\,x)^6}{6}+C=\frac{\text{sen}^4x}{4}+\frac{\text{sen}^6x}{6}+C$$

$\uparrow$ Regra de primitivação: $P\,u'\cdot u^k=\frac{u^{k+1}}{k+1}+C,\ k\neq-1$

p) $\dfrac{1}{\left(1+x^2\right)\text{arc tg}\,x}$

Resolução

$$P\frac{1}{\left(1+x^2\right)\text{arc tg}\,x}=P\frac{\frac{1}{1+x^2}}{\text{arc tg}\,x}=\ln\left|\text{arc tg}\,x\right|+C$$

$\uparrow$ Usando a regra de primitivação: $P\frac{u'}{u}=\ln|u|+C$ em que $\begin{cases} u=\text{arc tg}\,x \\ u'=\frac{1}{1+x^2} \end{cases}$

q) $\dfrac{1}{(1+x)\sqrt{x}}$

Resolução

$$P\frac{1}{(1+x)\sqrt{x}}=P\frac{\frac{1}{\sqrt{x}}}{1+x}=2P\frac{\frac{1}{2\sqrt{x}}}{1+\left(\sqrt{x}\right)^2}=2\text{arc tg}\left(\sqrt{x}\right)+C$$

$\uparrow$ Regra de primitivação: $P\frac{u'}{1+u^2}=\text{arc tg}\,u+C$ em que $\begin{cases} u=\sqrt{x} \\ u'=\frac{1}{2\sqrt{x}} \end{cases}$

r) $\dfrac{e^x}{4+e^{2x}}$

Resolução

$$P\frac{e^x}{4+e^{2x}}=P\frac{e^x}{4\left(\frac{1}{4}e^{2x}+1\right)}=\frac{1}{4}P\frac{e^x}{\frac{1}{4}\left(e^x\right)^2+1}=\frac{1}{4}P\frac{e^x}{\left(\frac{1}{2}e^x\right)^2+1}=\frac{1}{4}2P\frac{\frac{1}{2}e^x}{\left(\frac{1}{2}e^x\right)^2+1}=\frac{1}{2}\text{arc tg}\left(\frac{1}{2}e^x\right)+C$$

$\uparrow$ Regra de primitivação: $P\frac{u'}{1+u^2}=\text{arc tg}\,u+C$ em que $\begin{cases} u=\frac{1}{2}e^x \\ u'=\frac{1}{2}e^x \end{cases}$