TÉCNICO LISBOA

Autonomous Agents and Multiagent Systems

MSc in Computer Science and Engineering

First Exam (2021-2022)

Instructions

- You have 120 minutes to complete the exam.
- Make sure that your exam has a total of 16 pages. Also, check if there are no missing sheets, then write your full name and student number on this page (and your student number on all pages).
- The exam has 15 questions, with a maximum score of 20 points. The questions have different levels of difficulty. The point value of each question is provided next to the question number.
- *If you get stuck in a question, move on.* You should start with the more straightforward questions to secure those points before moving on to the more complex questions.
- *No interaction with the faculty is allowed during the exam.* If you are unclear about a question, clearly indicate the unclear part and answer the question to the best of your ability.
- Please provide your answer in the space below each question. If you make a mess, clearly indicate your answer.
- This exam is a closed-book assessment, whereby students are NOT allowed to bring books or other reference material into the examination room. You may bring only ONE A4 page of handwritten notes, in your OWN handwriting. Typed notes or a copy of someone else's notes are not allowed.
- You may use a calculator, but any other type of electronic or communication equipment is not allowed.
- **Good luck!**

1 Agent architectures

Question 1. (1.5 pts.)
What are the typical behaviors that intelligent agents exhibit? Explain each behavior.

Write your answer here:

Solution 1.
The behaviors are reactive, pro-active, and social.
An agent with a reactive behavior maintains an ongoing interaction with its environment and responds to changes that occur in it (in time for the response to be useful).
Pro-active behavior is the same as a goal-oriented behavior, whereby an agent is generating goals and attempting to achieve these goals.
Social behavior is an agent's ability to interact with other agents through cooperation, coordination, and negotiation.

Question 2. (1.5 pts.)
What are the key limitations of a reactive agent architecture? Explain.

Write your answer here:

Solution 2.
These are the key limitations of a reactive agent architecture:

- Decisions are only based on local information: agents have a short-term view
- Agents need sufficient local information to make decisions
- No learning. Hence, reactive rules cannot evolve
- Not trivial to engineer agents with complex behavior
- Not easy to predict complex behavior when agents have a high number of layers

2 Normal-form games

Consider the following scenario. Max and Ann are dating. They would like to go together to an Indian or Thai restaurant tonight. However, Max forgot his cell phone at home so he cannot call Ann to decide on the restaurant. Ann prefers Indian food and Max prefers Thai. Both prefer to go together to any of these restaurants than going alone.

Question 3. (1 pts.)
Model this problem as a normal-form game.

Write your answer here:

Solution 3.

N = {Max, Ann}

$A_1 = A_2$ = {Indian, Thai}

The payoff matrix of the game:

	Indian	Thai
Indian	1, 2	0, 0
Thai	0, 0	2, 1

Question 4. (**2 pts.**)

How many Nash equilibria are there in this game? If possible, find the pure strategy Nash equilibria and mixed strategy Nash equilibrium.

Write your answer here:

Solution 4.

In order to find the pure strategy Nash equilibria, we first find the best responses for Max (agent 1) and Ann (agent 2) and underline them in the payoff matrix:

	Indian	Thai
Indian	$\underline{1}$, $\underline{2}$	0, 0
Thai	0, 0	$\underline{2}$, $\underline{1}$

A joint action satisfies the definition of a NE if each agent's action is the best response to the other's. We thus have a Nash equilibrium if both payoffs are underlined in a cell of the payoff matrix above. In conclusion, the Nash equilibria is (Indian, Indian) and (Thai, Thai).

In order to find the mixed strategy NE, let us suppose that Max believes that Ann will choose Indian with probability q and Thai with probability $1 - q$.

If Max best-responds with a mixed strategy, then Ann must make him indifferent between Indian and Thai:

$$EU_{Max}(Indian) = EU_{Max}(Thai)$$
$$1q + 0(1 - q) = 0q + 2(1 - q)$$
$$1q = 2 - 2q$$
$$3q = 2$$
$$q = \frac{2}{3}$$

Suppose that Ann believes that Max will choose Indian with probability r and Thai with probability $1 - r$.
If Ann best-responds with a mixed strategy, then Max must make her indifferent between Indian and Thai:

$$EU_{Ann}(Indian) = EU_{Ann}(Thai)$$
$$2r + 0(1 - r) = 0r + 1(1 - r)$$
$$2r = 1 - r$$

$3r = 1$

$r = \frac{1}{3}$

Hence, the mixed strategy Nash equilibrium is $(\frac{2}{3}, \frac{1}{3}), (\frac{1}{3}, \frac{2}{3})$

Question 5. (0.5 pts.)

Are there any Pareto optimal Nash equilibria in this game? Explain.

Write your answer here:

Solution 5.

The Pareto optimal Nash Equilibria are (Indian, Indian) and (Thai, Thai) because these joint actions are a NE and are not Pareto dominated by any other joint action.

Question 6. (0.5 pts.)

Propose a mechanism to coordinate Max and Ann and solve the coordination problem.

Write your answer here:

Solution 6.

We could use Social Conventions as a coordination mechanism.

To use Social Convetions, we must propose an ordering scheme:

Order the agents: Ann $\succ$ Max

Order the actions: Indian $\succ$ Thai

Hence, according to the ordering scheme, the selected joint action is (Indian, Indian)

3 Applications of a Nash equilibrium: Cournot Model

Consider a duopoly in which two risk-neutral firms compete to sell a homogeneous product. The two firms choose quantities simultaneously, are economically rational, and act strategically. Let q_1 and q_2 denote the quantities (of the homogeneous product) produced by firms 1 and 2, respectively. In addition, assume the following production cost $c_1(q_1) = 2q_1$ for firm 1 and $c_2(q_2) = 3q_2$ for firm 2. Lastly, let $P(Q) = 100 - 2Q$ be the market-clearing price, where $Q = q_1 + q_2$ and $P(Q) \geq 0$.

Question 7. (3 pts.)

Derive the Cournot model and find the quantities of both firms that are a Nash equilibrium.

Write your answer here:

Solution 7.
Firm 1's payoff is $\pi_1 = P(q_1+q_2)q_1 - c_1(q_1) = (100-2(q_1+q_2))q_1 - 2q_1 = -2q_1^2 - 2q_2q_1 + 98q_1$
Firm 2's payoff is $\pi_2 = P(q_1+q_2)q_2 - c_2(q_2) = (100-2(q_1+q_2))q_2 - 3q_2 = -2q_2^2 - 2q_1q_2 + 97q_2$

Firm 1's first order condition:
$\frac{d}{dq_1}\pi_1 = 0$
$-4q_1 - 2q_2 + 98 = 0$
$q_1 = \frac{49-q_2}{2}$

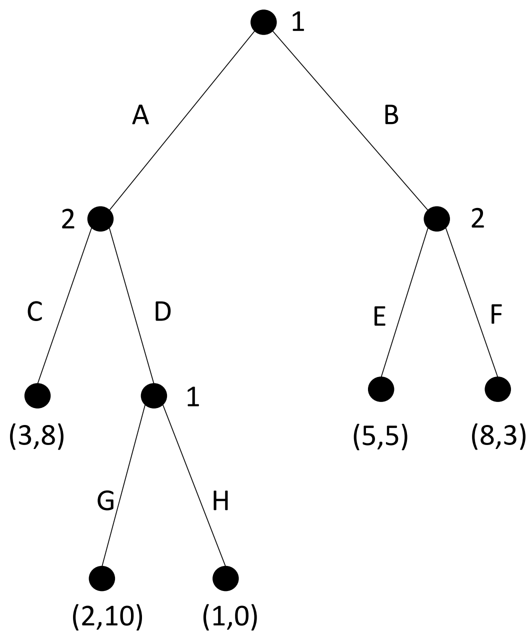
Firm 2's first order condition:
$\frac{d}{dq_2}\pi_2 = 0$
$-4q_2 - 2q_1 + 97 = 0$
$q_2 = \frac{97-2q_1}{4}$

We now need to solve the following pair of equations:
$q_1 = \frac{49-q_2}{2}$
$q_2 = \frac{97-2q_1}{4}$

Hence, the Nash equilibrium is:
$q_1 = \frac{33}{2}$
$q_2 = 16$

4 Extensive-form games

Consider the following perfect-information extensive-form game.



Question 8. (**0.5 pts.**)

What are the pure strategies of the extensive-form game above?

Write your answer here:

Solution 8.

The pure strategies are:

$S_1 = \{(A, G), (A, H), (B, G), (B, H)\}$

$S_2 = \{(C, E), (C, F), (D, E), (D, F)\}$

Question 9. (**1.5 pts.**)

Convert the extensive-form game to an equivalent normal-form game and find the Nash equilibria.

Write your answer here:

Solution 9.

This is the equivalent normal-form game:

	(C,E)	(C,F)	(D,E)	(D,F)
(A,G)	3,8	3,8	2,10	2,10
(A,H)	3,8	3,8	1,0	1,0
(B,G)	5,5	8,3	5,5	8,3
(B,H)	5,5	8,3	5,5	8,3

In order to find the pure strategy Nash equilibria, we first find the best responses for agent 1 and agent 2 and underline them in the payoff matrix:

	(C,E)	(C,F)	(D,E)	(D,F)
(A,G)	3,8	3,8	2,$\underline{10}$	2,$\underline{10}$
(A,H)	3,$\underline{8}$	3,$\underline{8}$	1,0	1,0
(B,G)	$\underline{5}$,$\underline{5}$	$\underline{8}$,3	$\underline{5}$,$\underline{5}$	$\underline{8}$,3
(B,H)	$\underline{5}$,$\underline{5}$	$\underline{8}$,3	$\underline{5}$,$\underline{5}$	$\underline{8}$,3

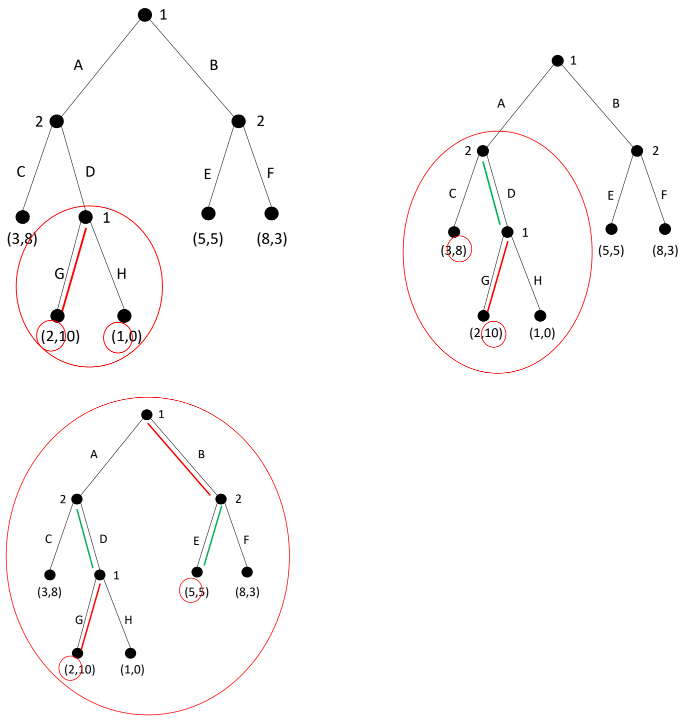
A joint action satisfies the definition of a NE if each agent's action is the best response to the other's. We thus have a Nash equilibrium if both payoffs are underlined in a cell of the payoff matrix above. In conclusion, the Nash equilibria is {(B,G),(C,E)}, {(B,G),(D,E)}, {(B,H),(C,E)}, and {(B,H),(D,E)}.

Question 10. (**1.5 pts.**)

What is the subgame-perfect equilibrium? Use backward induction to find the subgame-perfect equilibrium and explain all the steps of the algorithm.

Write your answer here:

Solution 10.



Hence, the subgame-perfect equilibrium is {(B,G),(D,E)}

Question 11. (**0.5 pts.**)

What is the relationship between subgame-perfect equilibrium and Nash equilibrium?

Write your answer here:

Solution 11.

A subgame-perfect equilibrium is a Nash equilibrium but not all Nash equilibria in the game are a subgame-perfect equilibrium.

For example, {(B,G),(C,E)}, {(B,H),(C,E)}, and {(B,H),(D,E)} are a Nash equilibria but are not a subgame-perfect equilibrium.

5 Bayesian games

Suppose that the payoffs of a game are given by the following table:

	C	D
A	-1,-1	α,0
B	α,2	2,β

where $\alpha \in \{0, 1\}$ is known by agent 1, $\beta \in \{1, 3\}$ is known by agent 2, and all pairs of (α, β) have probability of $\frac{1}{4}$ (i.e., $P(\alpha = 0, \beta = 1) = P(\alpha = 0, \beta = 3) = P(\alpha = 1, \beta = 1) = P(\alpha = 1, \beta = 3) =$

$\frac{1}{4}$).

Question 12. (**1 pts.**)

Write this problem as a Bayesian game.

Write your answer here:

Solution 12.

A Bayesian game is tuple (N, A, θ, P, u) where:

- $N = \{1, 2\}$ is a set of agents
- $A_1 = \{A, B\}$ and $A_2 = \{C, D\}$ are the sets of actions for each agent and $A = A_1 \times A_2$
- $\theta_1 = \{0, 1\}$ and $\theta_2 = \{1, 3\}$ are the set of types for each agent and $\theta = \theta_1 \times \theta_2$
- $P(\theta_1 = 0, \theta_2 = 1) = P(\theta_1 = 0, \theta_2 = 3) = P(\theta_1 = 1, \theta_2 = 1) = P(\theta_1 = 1, \theta_2 = 3) = \frac{1}{4}$ is a common prior over types
- $u_1(a_1, a_2, \theta_1, \theta_2)$ and $u_2(a_1, a_2, \theta_1, \theta_2)$ are given by the payoff matrices above and $u = (u_1, u_2)$

Question 13. (**2 pts.**)

Compute the Bayesian Nash equilibrium.

Write your answer here:

Solution 13.

Considering Agent 1:

For $\theta_1 = 0$ and $\theta_1 = 1$: B strictly dominates A. Hence, $a_1^*(\theta_1 = 0) = B$ and $a_1^*(\theta_1 = 1) = B$.

Considering Agent 2:

For $\theta_2 = 3$: D strictly dominates C. Hence, $a_2^*(\theta_2 = 3) = D$

For $\theta_2 = 1$, we can calculate the expected utility (EU) for action C as follows:

$EU_C = P(\theta_1 = 0|\theta_2 = 1) \times u_2(a_1^*(\theta_1 = 0), C, \theta_1 = 0, \theta_2 = 1) + P(\theta_1 = 1|\theta_2 = 1) \times u_2(a_1^*(\theta_1 = 1), C, \theta_1 = 1, \theta_2 = 1)$

$EU_C = P(\theta_1 = 0|\theta_2 = 1) \times u_2(B, C, \theta_1 = 0, \theta_2 = 1) + P(\theta_1 = 1|\theta_2 = 1) \times u_2(B, C, \theta_1 = 1, \theta_2 = 1)$

$EU_C = \frac{1}{2} \times 2 + \frac{1}{2} \times 2 = 2$

For $\theta_2 = 1$, we can calculate the expected utility (EU) for action D as follows: :

$EU_D = P(\theta_1 = 0|\theta_2 = 1) \times u_2(a_1^*(\theta_1 = 0), D, \theta_1 = 0, \theta_2 = 1) + P(\theta_1 = 1|\theta_2 = 1) \times u_2(a_1^*(\theta_1 = 1), D, \theta_1 = 1, \theta_2 = 1)$

$EU_D = P(\theta_1 = 0|\theta_2 = 1) \times u_2(B, D, \theta_1 = 0, \theta_2 = 1) + P(\theta_1 = 1|\theta_2 = 1) \times u_2(B, D, \theta_1 = 1, \theta_2 = 1)$

$EU_D = \frac{1}{2} \times 1 + \frac{1}{2} \times 1 = 1$

Since $EU_C > EU_D$ then $a_2^*(\theta_2 = 1) = C$

The Bayesian Nash equilibrium is $a^* = (a_1^*, a_2^*)$ where $a_1^*(\theta_1 = 0) = B$, $a_1^*(\theta_1 = 1) = B$, $a_2^*(\theta_2 = 1) = C$, and $a_2^*(\theta_2 = 3) = D$.

6 Repeated games

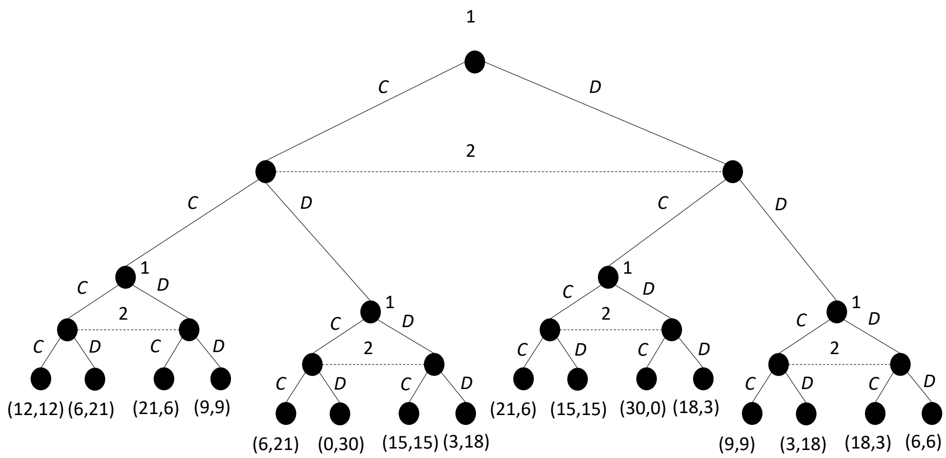
The following stage game of the Prisoner's Dilemma is played repeatedly:

	C	D
C	6, 6	0, 15
D	15, 0	3, 3

Question 14. (**1.5 pts.**)
If the stage game above is played twice, use an extensive-form game to represent this repeated game.

Write your answer here:

Solution 14.



Question 15. (**1.5 pts.**)
Could we sustain cooperation if the stage game above is played an infinite number of steps? Use future discounted rewards in order to obtain the solution and give an interpretation of the final result.

Write your answer here:

Solution 15.

If agents 1 and 2 always cooperate:

$6 + \beta 6 + \beta^2 6 + \beta^3 6 + ... = \frac{6}{1-\beta}$

If agent 1 defects and agent 2 cooperates in the first time step, then agent 2 will defect in all the other time steps (trigger strategy):

$15 + \beta 3 + \beta^2 3 + \beta^3 3 + ... = 15 + \beta \frac{3}{1-\beta}$

The difference between these two strategies is:

$-9 + \beta 3 + \beta^2 3 + \beta^3 3 + ... = -9 + \beta \frac{3}{1-\beta}$

To sustain cooperation:

$-9 + \beta \frac{3}{1-\beta} \geq 0$

$\beta \frac{3}{1-\beta} \geq 9$

$3\beta \geq 9 - 9\beta$

$\beta \geq \frac{3}{4}$

Interpretation: if we want to sustain cooperation, the agent needs to care about tomorrow at least $\frac{3}{4}$ more than he cares about today!