Ficha 8
Resolução dos exercícios de auto-avaliação

III.1 Determine os domínios das seguintes funções e represente-os graficamente:

a) $f(x,y)=\sqrt[4]{x\cdot \operatorname{sen}(y)}-e^{2-xy^2}+\sqrt[5]{2\text{-}x\text{-}y^2}$

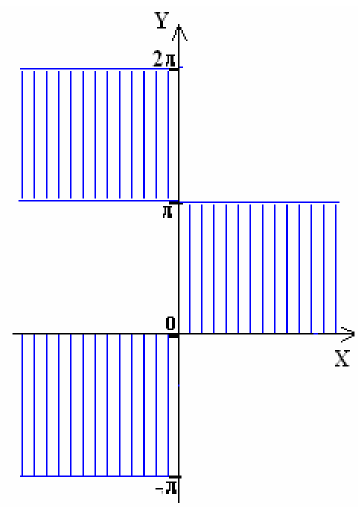
Resolução:
Domínio:

$$D_f=\left\{(x,y)\in\mathbb{R}^2: x\operatorname{sen}(y)\geq 0\right\}$$
$$\underset{(*)}{=}\left\{(x,y)\in\mathbb{R}^2:\left(x\geq 0\wedge 2\pi k\leq y\leq(2k+1)\pi,\ k\in\mathbb{Z}\right)\vee\left(x\leq 0\wedge(2k+1)\pi\leq y\leq(k+1)2\pi,\ k\in\mathbb{Z}\right)\right\}$$
$$=\left(\mathbb{R}_0^+\times\bigcup_{k\in\mathbb{Z}}\left[2\pi k,(2k+1)\pi\right]\right)\cup\left(\mathbb{R}_0^-\times\bigcup_{k\in\mathbb{Z}}\left[(2k+1)\pi,(k+1)2\pi\right]\right)$$

Cálculos auxiliares: $(*)$

$$x\sin y\geq 0\Leftrightarrow(x\geq 0\wedge\sin y\geq 0)\vee(x\leq 0\wedge\sin y\leq 0)$$
$$\Leftrightarrow(x\geq 0\wedge 0+2k\pi\leq y\leq\pi+2k\pi,\ k\in\mathbb{Z})\vee(x\leq 0\wedge\pi+2k\pi\leq y\leq 2\pi+2k\pi,\ k\in\mathbb{Z})$$
$$\Leftrightarrow\left(x\geq 0\wedge 2\pi k\leq y\leq(2k+1)\pi,\ k\in\mathbb{Z}\right)\vee\left(x\leq 0\wedge(2k+1)\pi\leq y\leq(k+1)2\pi,\ k\in\mathbb{Z}\right)$$

Representação gráfica do domínio:

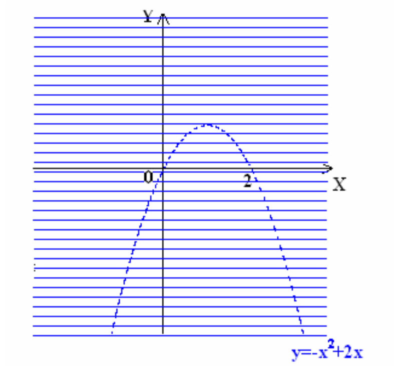


b) $f(x,y)=\operatorname{sen}(3x-y)+\sqrt[7]{2\text{-}x\text{-}y^2}-\dfrac{1}{x^2+y-2x}$

Resolução:
Domínio:

$$D_f=\left\{(x,y)\in\mathbb{R}^2: x^2+y-2x\neq 0\right\}=\left\{(x,y)\in\mathbb{R}^2: y\neq -x^2+2x\right\}$$

Representação gráfica do domínio:



c) $f(x,y)=\text{arc sen}\left(2y\left(1+x^2\right)-1\right)-e^{3-x^2y^2}+\sqrt[9]{10-x-y^2}$

Resolução:

Domínio:

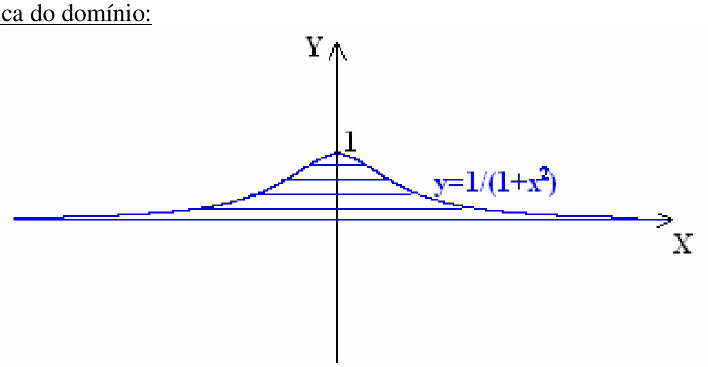
$$D_f=\left\{(x,y)\in\mathbb{R}^2:-1\le 2y\left(1+x^2\right)-1\le 1\right\}\underset{(*)}{=}\left\{(x,y)\in\mathbb{R}^2: y\le\frac{1}{\left(1+x^2\right)}\wedge y\ge 0\right\}$$

Cálculos auxiliares: $(*)$

$$-1+1\le 2y\left(1+x^2\right)\le 1+1\Leftrightarrow 0\le 2y\left(1+x^2\right)\le 2\Leftrightarrow 0\le y\left(1+x^2\right)\le 1$$

$$\Leftrightarrow y\left(1+x^2\right)\le 1\wedge y\left(1+x^2\right)\ge 0\underset{1+x^2>0,\,\forall x\in\mathbb{R}}{\Leftrightarrow} y\le\frac{1}{1+x^2}\wedge y\ge 0$$

Representação gráfica do domínio:



d) $f(x,y)=e^{3-y^2}+\sqrt[9]{2-x-y^2}+\text{sen}\left(2xy^2\right)-\sqrt{\left(e^y-e^{-y}\right)\cdot\cos(x)}$

Resolução:

Domínio:

$$D_f=\left\{(x,y)\in\mathbb{R}^2:\left(e^y-e^{-y}\right)\cos x\ge 0\right\}$$

$$\underset{(*)}{=}\left\{(x,y)\in\mathbb{R}^2:\left(y\ge 0\wedge -\frac{\pi}{2}+2k\pi\le x\le\frac{\pi}{2}+2k\pi,\ k\in\mathbb{Z}\right)\vee\left(y\le 0\wedge\frac{\pi}{2}+2k\pi\le x\le\frac{3\pi}{2}+2k\pi,\ k\in\mathbb{Z}\right)\right\}$$

$$=\left(\bigcup_{k\in\mathbb{Z}}\left[-\frac{\pi}{2}+2\pi k,\frac{\pi}{2}+2k\pi\right]\times\mathbb{R}_0^+\right)\cup\left(\bigcup_{k\in\mathbb{Z}}\left[\frac{\pi}{2}+2k\pi,\frac{3\pi}{2}+2k\pi\right]\times\mathbb{R}_0^-\right)$$

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Cálculos auxiliares: $(*)$

$$\left(e^{y}-e^{-y}\right)\cos x\geq 0$$

$$\Leftrightarrow\left(e^{y}-e^{-y}\geq 0\wedge\cos x\geq 0\right)\vee\left(e^{y}-e^{-y}\leq 0\wedge\cos x\leq 0\right)$$

$$\Leftrightarrow\left(e^{y}-\frac{1}{e^{y}}\geq 0\wedge\cos x\geq 0\right)\vee\left(e^{y}-\frac{1}{e^{y}}\leq 0\wedge\cos x\leq 0\right)\Leftrightarrow\left(\frac{e^{y}\cdot e^{y}-1}{e^{y}}\geq 0\wedge\cos x\geq 0\right)\vee\left(\frac{e^{y}\cdot e^{y}-1}{e^{y}}\leq 0\wedge\cos x\leq 0\right)$$

$$\Leftrightarrow\left(\frac{e^{2y}-1}{e^{y}}\geq 0\wedge\cos x\geq 0\right)\vee\left(\frac{e^{2y}-1}{e^{y}}\leq 0\wedge\cos x\leq 0\right)\underset{\left(e^{y}>0,\ \forall y\in\mathbb{R}\right)}{\Leftrightarrow}\left(e^{2y}-1\geq 0\wedge\cos x\geq 0\right)\vee\left(e^{2y}-1\leq 0\wedge\cos x\leq 0\right)$$

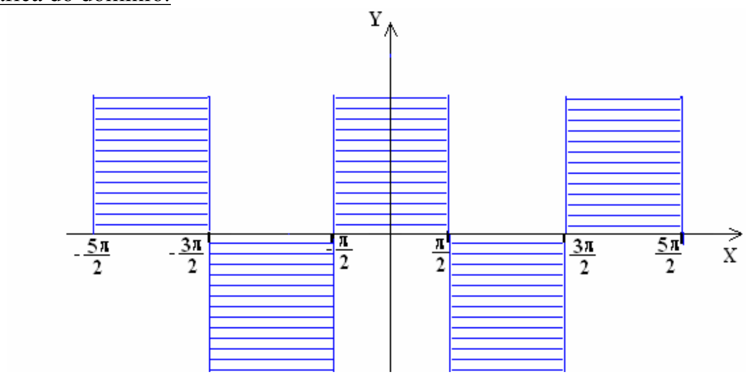
$$\Leftrightarrow\left(e^{2y}\geq 1\wedge\cos x\geq 0\right)\vee\left(e^{2y}\leq 1\wedge\cos x\leq 0\right)\Leftrightarrow\left(e^{2y}\geq e^{0}\wedge\cos x\geq 0\right)\vee\left(e^{2y}\leq e^{0}\wedge\cos x\leq 0\right)$$

$$\underset{\uparrow}{\Leftrightarrow}\left(2y\geq 0\wedge\cos x\geq 0\right)\vee\left(2y\leq 0\wedge\cos x\leq 0\right)\Leftrightarrow\left(y\geq 0\wedge\cos x\geq 0\right)\vee\left(y\leq 0\wedge\cos x\leq 0\right)$$

Como a=e>1,então a
função exponencial é crescente. Logo o
sinal da inequação não se altera.

$$\Leftrightarrow\left(y\geq 0\wedge-\frac{\pi}{2}+2k\pi\leq x\leq\frac{\pi}{2}+2k\pi,\ k\in\mathbb{Z}\right)\vee\left(y\leq 0\wedge\frac{\pi}{2}+2k\pi\leq x\leq\frac{3\pi}{2}+2k\pi,\ k\in\mathbb{Z}\right)$$

Representação gráfica do domínio:



e) $f(x,y)=\sqrt[3]{2-x-y^{2}}+\cos(xy)+\log_{3}(\operatorname{sen}(x))$

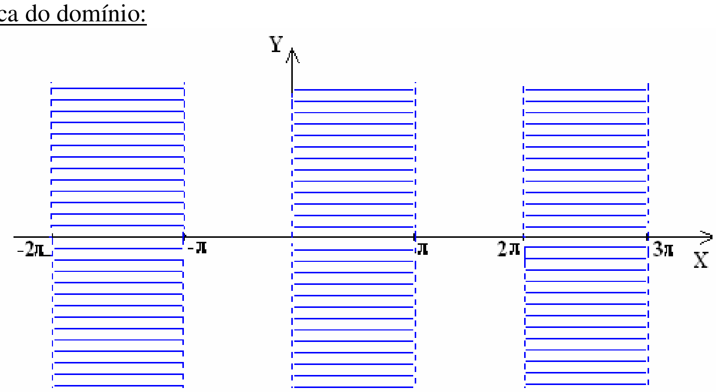
Resolução:

Domínio:

$$D_f=\left\{(x,y)\in\mathbb{R}^2:\operatorname{sen}(x)>0\right\}=\left\{(x,y)\in\mathbb{R}^2:0+2k\pi<x<\pi+2k\pi,\ k\in\mathbb{Z}\right\}$$

$$=\left\{(x,y)\in\mathbb{R}^2:2k\pi<x<(2k+1)\pi,\ k\in\mathbb{Z}\right\}=\left(\bigcup_{k\in\mathbb{Z}}\left]2k\pi,(2k+1)\pi\right[\right)\times\mathbb{R}$$

Representação gráfica do domínio:



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III 2. Sejam $f(x,y)=\dfrac{\log_2\left(4-x^2-y^2\right)}{\sqrt[6]{y-|x|}}$ e $g(x,y)=x^2+2x+y^2-4y+5$.

a) Determine e esboce o domínio de $f(x,y)$.

Resolução:

Domínio:

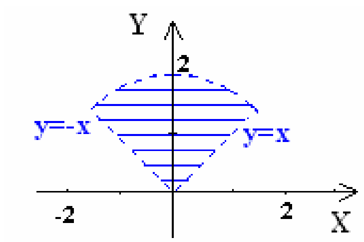
$$
\begin{aligned}
D_f &= \left\{(x,y)\in\mathbb{R}^2 : 4-x^2-y^2>0 \wedge \sqrt{y-|x|}\neq 0 \wedge y-|x|\geq 0\right\} \\
&= \left\{(x,y)\in\mathbb{R}^2 : -x^2-y^2>-4 \wedge y-|x|\neq 0 \wedge y-|x|\geq 0\right\} \\
&= \left\{(x,y)\in\mathbb{R}^2 : x^2+y^2<4 \wedge y-|x|>0\right\} \\
&\underset{(*)}{=} \left\{(x,y)\in\mathbb{R}^2 : x^2+y^2<2^2 \wedge (x<y \wedge x>-y) \wedge y>0\right\}
\end{aligned}
$$

Cálculos auxiliares: $(*)$

$$
y-|x|>0 \Leftrightarrow -|x|>-y \Leftrightarrow \left(|x|<y \wedge y\in\mathbb{R}\right) \Leftrightarrow \underbrace{\left(|x|<y \wedge y\leq 0\right)}_{\text{Condição impossível}} \vee \left(|x|<y \wedge y>0\right)
$$

$$
\Leftrightarrow |x|<y \wedge y>0 \Leftrightarrow (x<y \wedge x>-y) \wedge y>0
$$

Representação gráfica do domínio:



b) Sempre que possível represente as curvas de nível 0,1 e 2

Resolução:

As curvas de nível são dadas por:

$$
\begin{aligned}
L(c) &= \left\{(x,y)\in\mathbb{R}^2 : c=g(x,y)\right\} = \left\{(x,y)\in\mathbb{R}^2 : c=x^2+2x+y^2-4y+5\right\} \\
&= \left\{(x,y)\in\mathbb{R}^2 : c=x^2+2x+1-1+y^2-4y+4-4+5\right\} = \left\{(x,y)\in\mathbb{R}^2 : c=(x+1)^2+(y-2)^2\right\}
\end{aligned}
$$

Assim a curva de nível associado

- ao valor 0:

$$
\begin{aligned}
L(0) &= \left\{(x,y)\in\mathbb{R}^2 : 0=g(x,y)\right\} = \left\{(x,y)\in\mathbb{R}^2 : 0=(x+1)^2+(y-2)^2\right\} \\
&= \left\{(x,y)\in\mathbb{R}^2 : x=-1 \wedge y=2\right\} = \left\{(-1,2)\right\}
\end{aligned}
$$

- ao valor 1:

$$
L(1) = \left\{(x,y)\in\mathbb{R}^2 : 1=(x+1)^2+(y-2)^2\right\} = \left\{(x,y)\in\mathbb{R}^2 : (x-(-1))^2+(y-2)^2=1\right\}
$$

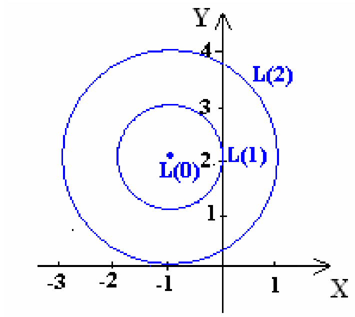
Representa uma circunferência de centro (-1, 2) e de raio 1

- ao valor 4:

$$
L(4) = \left\{(x,y)\in\mathbb{R}^2 : 4=(x+1)^2+(y-2)^2\right\} = \left\{(x,y)\in\mathbb{R}^2 : (x-(-1))^2+(y-2)^2=2^2\right\}
$$

Representa uma circunferência de centro (-1, 2) e de raio 2

Representação gráfica destas curvas de nível:



III 3. Sejam $f(x,y)=\dfrac{\log_3\left[4-(x+1)^2-y^2\right]}{\sqrt{4-(x-1)^2-y^2}}$ e $g(x,y)=|x-y|$.

a) Determine e esboce o domínio de $f(x,y)$.

Resolução:

Domínio:

$$D_f=\left\{(x,y)\in\mathbb{R}^2: 4-(x+1)^2-y^2>0\wedge\sqrt{4-(x-1)^2-y^2}\neq 0\wedge 4-(x-1)^2-y^2\geq 0\right\}$$

$$\underset{(*)}{=}\left\{(x,y)\in\mathbb{R}^2: 4-(x+1)^2-y^2>0\wedge 4-(x-1)^2-y^2\neq 0\wedge 4-(x-1)^2-y^2\geq 0\right\}$$

$$=\left\{(x,y)\in\mathbb{R}^2: 4-(x+1)^2-y^2>0\wedge 4-(x-1)^2-y^2>0\right\}$$

Cálculos auxiliares $(*)$ **:**

$$\sqrt{4-(x-1)^2-y^2}\neq 0=\sim\left(\sqrt{4-(x-1)^2-y^2}=0\right)=\sim\left(\left|4-(x-1)^2-y^2\right|=0\right)$$

$$=\sim\left(4-(x-1)^2-y^2=0\right)=4-(x-1)^2-y^2\neq 0$$

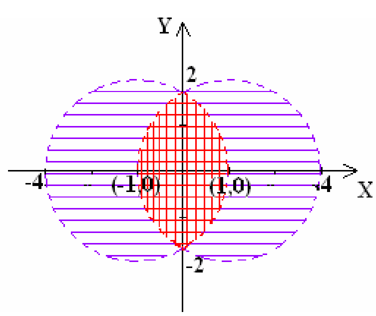
Representação gráfica do domínio:
(A representação gráfica do domínio encontra-se a quadriculado de cor vermelha)

- $4-(x+1)^2-y^2>0\Leftrightarrow-(x+1)^2-y^2>-4\Leftrightarrow(x+1)^2+y^2<4\Leftrightarrow(x-(-1))^2+y^2<2^2$

Representa o interior de uma circunferência de centro (-1,0) e de raio 2

- $4-(x-1)^2-y^2>0\Leftrightarrow-(x-1)^2-y^2>-4\Leftrightarrow(x-1)^2+y^2<4\Leftrightarrow(x-1)^2+y^2<2^2$

Representa o interior de uma circunferência de centro (1,0) e de raio 2



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b) Represente graficamente as curvas de nível 0,1 e 2 de $g(x,y)$.

Resolução:

As curvas de nível são dadas por:

$$L(c)=\left\{(x,y)\in\mathbb{R}^2 : c=g(x,y)\right\}=\left\{(x,y)\in\mathbb{R}^2 : c=|x-y|\right\}$$

Assim a curva de nível associada

- ao valor 0:

$$L(0)=\left\{(x,y)\in\mathbb{R}^2 : 0=|x-y|\right\}=\left\{(x,y)\in\mathbb{R}^2 : 0=x-y\right\}=\left\{(x,y)\in\mathbb{R}^2 : y=x\right\}$$

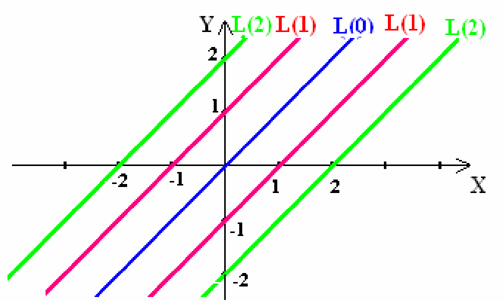
- ao valor 1:

$$L(1)=\left\{(x,y)\in\mathbb{R}^2 : 1=|x-y|\right\}=\left\{(x,y)\in\mathbb{R}^2 : |x-y|=1\right\}=\left\{(x,y)\in\mathbb{R}^2 : x-y=1\vee x-y=-1\right\}$$
$$=\left\{(x,y)\in\mathbb{R}^2 : -y=1-x\vee -y=-1-x\right\}=\left\{(x,y)\in\mathbb{R}^2 : y=x-1\vee y=x+1\right\}$$

- ao valor 2:

$$L(2)=\left\{(x,y)\in\mathbb{R}^2 : 2=|x-y|\right\}=\left\{(x,y)\in\mathbb{R}^2 : |x-y|=2\right\}=\left\{(x,y)\in\mathbb{R}^2 : x-y=2\vee x-y=-2\right\}$$
$$=\left\{(x,y)\in\mathbb{R}^2 : -y=2-x\vee -y=-2-x\right\}=\left\{(x,y)\in\mathbb{R}^2 : y=x-2\vee y=x+2\right\}$$

Representação gráfica destas curvas de nível:

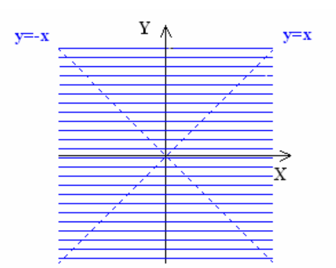


III. 4 Sejam $f(x,y)=\dfrac{\ln|x-y|}{x+y}+\dfrac{\sqrt[3]{xy+2}}{x^2+y^2+2}$ e $g(x,y)=(x-1)^2+(y+1)^2$.

Resolução:

$$D_f=\left\{(x,y)\in\mathbb{R}^2 : |x-y|>0\wedge x+y\neq 0\right\}=\left\{(x,y)\in\mathbb{R}^2 : x-y\neq 0\wedge x+y\neq 0\right\}$$
$$=\left\{(x,y)\in\mathbb{R}^2 : x\neq y\wedge x\neq -y\right\}$$

Representação gráfica do domínio:



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b) Represente graficamente algumas curvas de nível de $g(x,y)$.

<u>Resolução:</u>

As curvas de nível são dadas por:

$$L(c)=\left\{(x,y)\in\mathbb{R}^2 : c=g(x,y)\right\}=\left\{(x,y)\in\mathbb{R}^2 : c=(x-1)^2+(y+1)^2\right\}$$

<u>Assim a curva de nível associado</u>

- <u>ao valor 0:</u>

$$L(0)=\left\{(x,y)\in\mathbb{R}^2 : 0=g(x,y)\right\}=\left\{(x,y)\in\mathbb{R}^2 : 0=(x-1)^2+(y+1)^2\right\}$$
$$=\left\{(x,y)\in\mathbb{R}^2 : x=1\wedge y=-1\right\}=\left\{(1,-1)\right\}$$

- <u>ao valor 1:</u>

$$L(1)=\left\{(x,y)\in\mathbb{R}^2 : 1=(x-1)^2+(y+1)^2\right\}=\left\{(x,y)\in\mathbb{R}^2 : (x-1)^2+(y-(-1))^2=1\right\}$$

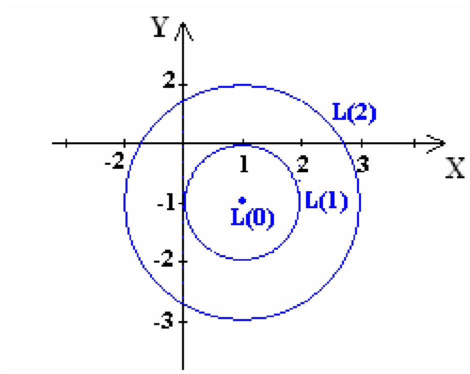
Representa uma circunferência de centro (1, -1) e de raio 1

- <u>ao valor 4:</u>

$$L(4)=\left\{(x,y)\in\mathbb{R}^2 : 4=(x-1)^2+(y+1)^2\right\}=\left\{(x,y)\in\mathbb{R}^2 : (x-1)^2+(y-(-1))^2=2^2\right\}$$

Representa uma circunferência de centro (1, -1) e de raio 2

<u>Representação gráfica destas curvas de nível:</u>



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