finalizar da ficha nº9

2a) $\int_0^1 \sqrt{4-x^2}\,dx = \int_0^1 2\sqrt{1-\left(\frac{x}{2}\right)^2}dx =$

$x = 2\,\text{sen}\,t = \varphi(t)$, $\varphi'(t) = 2\cos t$

$x=0 \Rightarrow t=0$

$x=1 \Rightarrow t=\frac{\pi}{6}$

$= \int_0^{\pi/6} 2\sqrt{1-\text{sen}^2 t}\,.\,2\cos t\,dt = 4\int_0^{\pi/6} \cos^2 t\,dt = 4\int_0^{\pi/6} \frac{1+\cos 2t}{2}dt =$

$= \int_0^{\pi/6} (2+2\cos 2t)\,dt \overset{\text{Barrow}}{=} \left[2t + \text{sen}\,2t\right]_0^{\pi/6} = \frac{\pi}{3} + \text{sen}\frac{\pi}{3} = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$

2b) $\int_0^{1/2} \sqrt{1+4x^2}\,dx = \int_0^{\pi/4} \sqrt{1+tg^2(t)}\,.\,\frac{1}{2\cos^2 t}\,dt =$

$x = \frac{tg\,t}{2} = \varphi(t)$, $\varphi'(t) = \frac{1}{2\cos^2 t}$; $x = 1/2 \Rightarrow t = \pi/4$

$$= \frac{1}{2}\int_0^{\pi/4} \frac{1}{\cos^3 t}\,dt \overset{\text{Barrow}}{=} \frac{1}{2}\left[\frac{\operatorname{tg} t}{2\cos t} + \frac{1}{2}\ln\left|\operatorname{tg} t + \frac{1}{\cos t}\right|\right]_0^{\pi/4} = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}\ln\left(1 + \frac{2}{\sqrt{2}}\right)\right)$$

$$P\left(\frac{1}{\cos^2 t}\,\frac{1}{\cos t}\right) \overset{P.P}{=} \frac{\operatorname{tg} t}{\cos t} - P\,\operatorname{tg} t \cdot \frac{\operatorname{sen} t}{\cos^2 t} = \frac{\operatorname{tg} t}{\cos t} - P\,\frac{\operatorname{tg}^2 t}{\cos t}$$

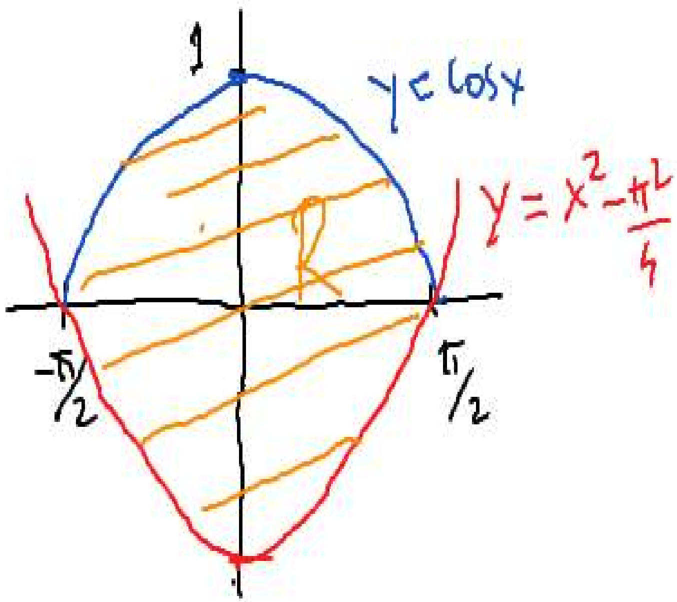
$$P\,\frac{1}{\cos^3 t} = \frac{\operatorname{tg} t}{\cos t} - P\left(\frac{1}{\cos^2} - 1\right)\frac{1}{\cos t}$$

$$P\,\frac{1}{\cos^3 t} = \frac{\operatorname{tg} t}{\cos t} - P\,\frac{1}{\cos^3 t} + P\,\frac{1}{\cos t}$$

$$\boxed{P\,\frac{1}{\cos t} = P\,\frac{1}{\cos t}\,\frac{\operatorname{tg} t + \frac{1}{\cos t}}{\operatorname{tg} t + \frac{1}{\cos t}} = \ln\left|\operatorname{tg} t + \frac{1}{\cos t}\right|}$$

$$2\,P\,\frac{1}{\cos^3 t} = \frac{\operatorname{tg} t}{\cos t} + P\,\frac{1}{\cos t} \Rightarrow P\,\frac{1}{\cos^3 t} = \frac{\operatorname{tg} t}{2\cos t} + \frac{1}{2}\ln\left|\operatorname{tg} t + \frac{1}{\cos t}\right|$$

3ii) Determine a área da região plana limitada pelas curvas
$y = x^2 - \frac{\pi^2}{4}$ e $y = \cos x$



$$\text{área } R = 2\int_0^{\pi/2} \left(\cos x - \left(x^2 - \frac{\pi^2}{4}\right)\right) dx \overset{\text{Barrow}}{=}$$

$$= 2\left[\operatorname{sen} x - \frac{x^3}{3} + \frac{\pi^2}{4}\cdot x\right]_0^{\pi/2} =$$

$$= 2\left(1 - \frac{\pi^3}{24} + \frac{\pi^3}{8}\right) = 2 + \frac{\pi^3}{6}$$

4i) $R = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq \pi/2 \text{ e } 0 \leq y \leq x \cos x\}$

$$\text{área de } R = \int_0^{\pi/2} x \cos x \, dx \overset{\text{I.P.P.}}{=} \left[x \,\text{sen}\, x\right]_0^{\pi/2} - \int_0^{\pi/2} \text{sen}\, x \, dx =$$

$$= \pi/2 - \left[-\cos x\right]_0^{\pi/2} = \pi/2 + \left(\cos \pi/2 - \cos(0)\right) = \pi/2 - 1$$

ficha nº 10

3. $f : [a,b] \to \mathbb{R}$ integrável tal que $f(a+b-x) = f(x)$, $x \in [a,b]$

$$\int_a^b x f(x)\,dx = \int_a^b x . f(a+b-x)\,dx \underset{x = a+b-t = \varphi(t)}{=} \int_b^a (a+b-t)\, f(t)(-1)\,dt$$

$x = a \Rightarrow t = b$ $\quad \varphi'(t) = -1$

$x = b \Rightarrow t = a$

$$\int_a^b x f(x) = -\int_a^b x f(x)\,dx + (a+b)\int_a^b f(x)\,dx$$

$(*)$

ou seja tem-se a relação em $(*)$

5. Seja $f: \mathbb{R}^+ \longrightarrow \mathbb{R}$, $f(x) = \int_0^{\ln x} x e^{t^2} dt - x$

Verifique que f tem um mínimo em 1.

$F(x) = \int_0^x e^{t^2} dt$ é um integral indefinido e como o integrando e^{t^2} é uma função contínua em $\mathbb{R}$ do teorema fundamental do cálculo integral (TFC), F é diferenciável em $\mathbb{R}$ e $F'(x) = e^{x^2}$, $x \in \mathbb{R}$

$f(x) = x \cdot F(\ln x) - x$, é diferenciável em $\mathbb{R}$ (resultante do produto, composição e adição de funções diferenciáveis)

e sendo f pelo menos 2 vezes diferenciável tem-se

$f'(x) = F(\ln x) + x e^{(\ln x)^2} - 1$ e $f''(x) = e^{(\ln x)^2} + e^{(\ln x)^2} + 2 \ln x \, e^{(\ln x)^2}$, com $f'(1) = 0$, $f''(1) = 2 > 0$ $\Rightarrow f(1)$ mínimo