Instituto Superior Técnico - 1^o Semestre 2006/2007

Cálculo Diferencial e Integral I

LEA-pB, LEM-pB, LEN-pB, LEAN, MEAer e MEMec

2^a Ficha de exercícios para as aulas práticas: 2 - 6 Outubro de 2006

1. Verifique que, para qualquer $x \in \mathbb{R}$, se tem:

   (a) $|x-1|+|x-2| \geq 1$, **(b)** $|x-1|+|x-2|+|x-3| \geq 2$.

2. Relativamente a cada um dos seguintes subconjuntos de $\mathbb{R}$, determine o conjunto dos majorantes, o conjunto dos minorantes e, no caso de existirem (em $\mathbb{R}$), o supremo, o ínfimo, o máximo e o mínimo:

   (1) $\{x \in \mathbb{R} : |x+1|+|x-2| = 7\}$, **(2)** $\{x \in \mathbb{R} : x^2 \geq 3x-2\}$, **(3)** $\{x \in \mathbb{R} : 1 < x^2 < 4\}$,

   (4) $\left\{x \in \mathbb{R} : \frac{1}{x} < x\right\}$, **(5)** $\left\{x \in \mathbb{R} : \frac{1}{x} < x^2\right\}$, **(6)** $\left\{x \in \mathbb{R} : \sqrt{2x+1} \leq \sqrt{x}+1\right\}$,

   (7) $\{x \in \mathbb{R} : |3-2x| > 1\}$, **(8)** $\{x \in \mathbb{R} : |3-2x| = 1\}$, **(9)** $\{x \in \mathbb{R} : |3-2x| < 1\}$,

   (10) $\{x \in \mathbb{R} : |1-x|-x \geq 0\}$, **(11)** $\left\{x \in \mathbb{R} : \left|1+\frac{1}{x}\right| < 6\right\}$, **(12)** $\left\{x \in \mathbb{R} : \left|\frac{x^2-x}{1+x}\right| > x\right\}$,

   (13) $\{x \in \mathbb{R} : |1-4x^{-1}| > 1\}$, **(14)** $\{x \in \mathbb{R} : |x^2-1| \leq 3\}$, **(15)** $\{x \in \mathbb{R} : |x^2-2x-15| \geq 9\}$,

   (16) $\{x \in \mathbb{R} : x\,|x-1| \leq 2\}$, **(17)** $\{x \in \mathbb{R} : 4 < |x+2|+|x-1| < 5\}$, **(18)** $\left\{x \in \mathbb{R} : \frac{4}{|x|} < 2\right\}$,

   (19) $\left\{x \in \mathbb{R} : (2x+3)^6\,(x-2) \geq 0\right\}$, **(20)** $\{x \in \mathbb{R} : |x+4| < |x-3|\}$,

   (21) $\{x \in \mathbb{R} : |x\,(x-3)| > |1-3x|\}$, **(22)** $\left\{x \in \mathbb{R} : \log\frac{x}{2} \leq 0\right\} \cap \left\{x \in \mathbb{R} : \operatorname{sen}^2\frac{\pi}{x} > 0\right\}$,

   (23) $\{x \in \mathbb{R} : |x-3| = 2\,|x|\}$, **(24)** $\{x \in \mathbb{R} : x+|x| < 1\} \cup \{0\}$, **(25)** $\{x : x \in \mathbb{R}\backslash\mathbb{Q} \wedge x > 0\}$,

   (26) $\left\{x \in \mathbb{R} : \frac{x+1}{x^3+2x} \leq 0\right\}$, **(27)** $\left\{x \in \mathbb{R}^+\backslash\mathbb{Q} : \frac{x+1}{x^3+2x} \leq 0\right\}$, **(28)** $\{x \in \mathbb{R} : e^x \geq e^{-x}\}$,

   (29) $\{x \in \mathbb{R} : x^4-3x^3+2x^2 \leq 0\}$, **(30)** $\{x \in \mathbb{R}^-\backslash\mathbb{Q} : |x-2| \geq 2\,|x+4|\}$,

   (31) $\left\{x \in \mathbb{R} : \frac{x^2-9}{\log(x-1)} \leq 0\right\}$, **(32)** $\left\{x \in \mathbb{R} : \log\frac{1}{x} \geq 1\right\}$, **(33)** $\left\{x \in \mathbb{R} : \frac{x}{e^x\,(x+1)} \leq 0\right\}$,

   (34) $\left\{x \in \mathbb{R} : \frac{x^2-2x+1}{x^4-x^2} \geq 0\right\}$, **(35)** $\left\{x \in \mathbb{R} : \frac{1}{\log x} \geq 1\right\}$, **(36)** $\left\{1-\frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$,

   (37) $\{x \in \mathbb{R} : x = 1+e^{-n}, n \in \mathbb{N}\}$, **(38)** $\{x \in \mathbb{R} : |3-2x| < 2x\} \cap [0,2]$,

   (39) $\{x \in \mathbb{R} : x^2\,(2\,|x+2|-|x-1|) \leq 0\}$, **(40)** $\{x : \operatorname{sen} x \geq 0\}$, **(41)** $\{x : |x| < 2\pi\}$,

   (42) $\{x : \operatorname{sen} x \geq 0\} \cap \left]-2\pi, 2\pi\right[$, **(43)** $\left\{x \in \mathbb{R} : x = \frac{1}{m}+\frac{1}{n};\, m,n \in \mathbb{N}\right\}$,

   (44) $\left\{m+\frac{1}{n} : m,n \in \mathbb{N}\right\}$, **(45)** $\left\{n^{(-1)^m} : m,n \in \mathbb{N}\right\}$, **(46)** $\left\{\frac{n}{n+1} : n \in \mathbb{N}\right\}$,

(47) $\left\{x \in \left]-2\pi, 2\pi\right[: \dfrac{(x-\pi)\cos\frac{x}{2}}{x} \leq 0\right\} \cap \mathbb{Q}$, **(48)** $\{x \in [0, 2\pi] : |\operatorname{sen} x| = |\cos x|\}$

(49) $\left\{x \in \mathbb{R} : (x+2)^2 \log \dfrac{2x-4}{x+1} \leq 0\right\}$, **(50)** $\left\{x \in \mathbb{R} : \left(\dfrac{\pi}{2} - \operatorname{arcsen}\dfrac{x}{2}\right)^2 \log \dfrac{x-1}{2} < 0\right\}$,

(51) $\left\{x \in \mathbb{R} : x\left(e^{2x} + e^x - 2\right)\left(\dfrac{\pi}{3} - \arccos\dfrac{1}{1+x^2}\right) > 0\right\}$, **(52)** $\left\{y : \dfrac{y}{y-1} < \dfrac{y-1}{y}\right\}$,

(53) $\left\{x \in \mathbb{R} : x^2\left(e^{x-1} - 1\right)\log(x+2)\arccos\dfrac{1}{\operatorname{ch} x} < 0\right\}$, **(54)** $\left\{x \in \mathbb{R} : \dfrac{|x+1|-1}{x-1} \leq 0\right\}$,

(55) $\{x \in \mathbb{R} : (\operatorname{arctg} x - \pi)\, x^2 \log(2+x) \geq 0\}$, **(56)** $\left\{x \in \mathbb{R} : |\operatorname{sen} x| < \dfrac{1}{2} \text{ e } x(2x-\pi) \leq 0\right\}$.

3. Considere os seguintes subconjuntos de $\mathbb{R}$:

$$A = \left\{x \in \mathbb{R} : \frac{x-1}{x\log x} > 0\right\}, \qquad B = \left\{x \in \mathbb{R} : x = -\frac{1}{n}, n \in \mathbb{N}\right\}.$$

Verifique que $A = \mathbb{R}^+ \setminus \{1\}$. Relativamente aos conjuntos A e $A \cup B$, determine, se existirem em $\mathbb{R}$, os respectivos: supremo, ínfimo, máximo e mínimo.

4. Considere os seguintes subconjuntos de $\mathbb{R}$:

$$A = \left\{x \in \mathbb{R} : |x-1| < x^2 - 1\right\}, \qquad B = [-2, 2].$$

(a) Verifique que $A = \left]-\infty, -2\right[\cup \left]1, +\infty\right[$.

(b) Determine, se existirem em $\mathbb{R}$, o máximo e o mínimo de $A \cap B$ e o supremo, ínfimo, máximo e mínimo de $A \cap B \cap (\mathbb{R} \setminus \mathbb{Q})$.

5. Considere os seguintes subconjuntos de $\mathbb{R}$:

$$A = \left\{x \in \mathbb{R} : \frac{x^2-1}{x} \geq |x-1|\right\}, \qquad B = \{x : \operatorname{sen} x = 0\}, \qquad C = \mathbb{Q}.$$

(a) Verifique que $A = \left[-\dfrac{1}{2}, 0\right[\cup \left[1, +\infty\right[$.

(b) Determine o conjunto dos majorantes e o conjunto dos minorantes de $A \cap C$ e de $B \cap C$. Determine, se existirem em $\mathbb{R}$, o $\sup A$, $\inf(A \cap C)$, $\min(A \cap C)$, $\min B$ e $\sup(B \cap C)$.

6. Considere os seguintes subconjuntos de $\mathbb{R}$:

$$A = \left\{x \in \mathbb{R} : |x| \geq \frac{1}{2}x + 2\right\}, \qquad B = [-3, 4], \qquad C = \mathbb{R} \setminus \mathbb{Q}.$$

(a) Verifique que $A \cap B = \left[-3, -\dfrac{4}{3}\right] \cup \{4\}$.

(b) Determine, se existirem em $\mathbb{R}$, o $\sup A$, $\min(A \cap B)$, $\max(A \cap B)$, $\inf(A \cap B \cap C)$, $\sup(A \cap B \cap C)$ e o $\min(A \cap B \cap C)$.

7. Considere os seguintes subconjuntos de $\mathbb{R}$:

$$A = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}, \qquad B = \mathbb{R} \backslash \mathbb{Q}, \qquad C = \{ x \in \mathbb{R} : \log x \geq 0 \}.$$

Determine, se existirem em $\mathbb{R}$, o $\inf A$, $\min(A \cup C)$, $\sup(A \cup C)$, $\inf(A \cap C)$, $\min(B \cap C)$ e o $\sup(A \cap B)$.