Exercícios de revisão

Determine a solução geral da EDO linear escalar:

$$(1+t^2)\frac{dy}{dt} = \mathsf{arctg}\,(t) - y$$

Resposta: $\quad y(t) = Ce^{-\mathsf{arctg}\,(t)} + \mathsf{arctg}\,(t) - 1, \quad$ onde C é uma constante real.

Considere a EDO separável e resolva o PVI:

$$y^2\left(\frac{1}{t} + \log t\right) + 2y\log t\,\frac{dy}{dt} = 0, \qquad y(e) = -1.$$

Resposta: $\quad y(t) = -\sqrt{\frac{\exp\,(e-t)}{\log t}}, \quad$ para $t > 1$.

Determine um factor integrante $\mu = \mu(t)$ e resolva o PVI:

$$y - 2t^2 + (2ty + t\log t)\,y' = 0, \qquad y(1) = 2.$$

Resposta: $\quad \mu(t) = \frac{1}{t}, \quad y(t) = \frac{-\log t + \sqrt{\log^2 t + 4(t^2+3)}}{2}, \quad$ para $t > 0$.

Exercícios de revisão

Calcule e^{At} para a matriz:

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Resposta:

$$e^{At} = e^{3t} \begin{bmatrix} 1 & 0 & 0 \\ t & 1 & 0 \\ \frac{t^2}{2} & t & 1 \end{bmatrix}$$

Calcule e^{At} para a matriz:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

Resposta: $e^{At} = \begin{bmatrix} e^t & 0 & 0 & 0 & 0 & 0 \\ te^t & e^t & 0 & 0 & 0 & 0 \\ \frac{t^2}{2}e^t & te^t & e^t & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-t} & 0 & 0 \\ 0 & 0 & 0 & te^{-t} & e^{-t} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{t/2} \end{bmatrix}$

Calcule e^{At} para a matriz:

$$A = \left[\begin{array}{cc} 1 & 1 \\ 4 & 1 \end{array} \right]$$

e resolva o PVI

$$X' = AX, \qquad X(0) = \left[\begin{array}{cc} 1 & 2 \end{array} \right]^T$$

Resposta:

$$e^{At} = \left[\begin{array}{cc} \dfrac{e^{3t} + e^{-t}}{2} & \dfrac{e^{3t} - e^{-t}}{4} \\ e^{3t} - e^{-t} & \dfrac{e^{3t} + e^{-t}}{2} \end{array} \right],$$

$$X(t) = \left[\begin{array}{c} e^{3t} \\ 2\,e^{3t} \end{array} \right]$$