Seja $f: I \to \mathbb{R}$, I intervalo real não degenerado

Se f é primitivável em I, dados $x_1 \in I$, $\alpha \in \mathbb{R}$

existe uma e uma só primitiva F_0 de f

tal que $F_0(x_1) = \alpha$

Sendo F uma primitiva de f, outra primitiva será

$F_0(x) = F(x) + C$. Para $C = \alpha - F(x_1)$

tem-se $F_0(x_1) = \alpha$ ∎

ficha nº7 : exercício 2) Determine a função $f: \mathbb{R}\setminus\{1\} \to \mathbb{R}$ q satisfaz

as condições : $f''(x) = \frac{1}{(1-x)^2}$; $f'(0)=0$, $\lim_{x\to+\infty} f'(x) = 1$
$x \neq 1$

$f(0)=0$, $f(e+1)=0$

• $P\frac{1}{(1-x)^2} = -P\underbrace{(-1)}_{u'}.\underbrace{(1-x)^{-2}}_{u^{-2}} = -\frac{(1-x)^{-1}}{-1} = \frac{1}{1-x}$; $f'_{C_1,C_2}(x) = \begin{cases} \frac{1}{1-x} + C_1 , & x>1 \\ \frac{1}{1-x} + C_2 , & x<1 \end{cases}$

$P\, u' u^{\alpha} = \frac{u^{\alpha+1}}{\alpha+1}$.
$\alpha \neq -1$

$0 = f'(0) = \frac{1}{1-0} + C_2 \Rightarrow C_2 = -1$

$1 = \lim_{x\to+\infty} f'(x) = \lim_{x\to+\infty} \frac{1}{1-x} + C_1$

$\Rightarrow C_1 = 1$

$P\left(\frac{1}{1-x} + C\right) = -P\frac{-1}{1-x} + C.P(1) = -\ln|1-x| + C.x$

$f_{C_1,C_2,C_3,C_4}(x) = \begin{cases} -\ln|1-x| + C_1 x + C_3 , & x>1 \\ -\ln|1-x| + C_2 x + C_4 \end{cases}$

f é única

$f(x) = \begin{cases} -\ln|1-x| + x - e, & x>1 \\ -\ln|1-x| - x , & x<1 \end{cases}$

$C_2 = -1$, $0 = f(0) = -\ln|1-0| - 0 + C_4 \Rightarrow C_4 = 0$

$C_1 = 1$, $0 = f(e+1) = -\ln|1-(e+1)| + (e+1) + C_3 \Rightarrow C_3 = -e$

Primitivação por partes: $P(f'g) = fg - P(fg')$

3a) $P\underset{g}{x}\,\underset{f'}{\cos 2x} \overset{P.P}{=} \underset{f}{\frac{\operatorname{sen} 2x}{2}}\cdot \underset{g}{x} - P\underset{f}{\frac{\operatorname{sen} 2x}{2}}\cdot \underset{g'}{1} = \frac{x}{2}\operatorname{sen}(2x) + \frac{\cos(2x)}{4}$

$P\cos 2x = \frac{\operatorname{sen} 2x}{2} = f$

$\cancel{Px = \frac{x^2}{2}}$

$(\ln x)' = \frac{1}{x}$, $P\frac{1}{x} = \ln x$, $P\ln x$ ñ é imediata

3b) $P\,1.\ln(2x) \overset{P.P.}{=} x.\ln(2x) - Px.\frac{2}{2x} = x.\ln(2x) - P1 = x\ln(2x) - x$

3c) análoga a b) $P\operatorname{arctg} x \overset{P.P}{=} x\operatorname{arctg} x - Px.\frac{1}{1+x^2} = x\operatorname{arctg} x - \frac{1}{2}P\frac{2x}{1+x^2} =$

$= x\operatorname{arctg} x - 1/2\ln(1+x^2)$

d) $P\,\underset{g}{x^3}\,\underset{f'}{\operatorname{ch} x} \overset{P.P}{=} \underset{g}{x^3}\,\underset{f}{\operatorname{sh} x} - P\,\underline{\underset{g'}{3x^2} \cdot \underset{f}{\operatorname{sh} x}} \overset{P.P}{=}$

$f = P \operatorname{ch} x = \operatorname{sh} x$ $\quad = x^3 \operatorname{sh} x - \left(3x^2 \operatorname{ch} x - P(\operatorname{ch} x \cdot 6x)\right) =$

$\overset{PP}{=} x^3 \operatorname{sh} x - 3x^2 \operatorname{ch} x + \left(\operatorname{sh} x \cdot 6x - P(\operatorname{sh} x \cdot 6)\right)$

$= \underline{x^3 \operatorname{sh} x - 3x^2 \operatorname{ch} x + 6x \operatorname{sh} x - 6 \operatorname{ch} x}$

$\operatorname{sh} x = \dfrac{e^x - e^{-x}}{2}$

$\operatorname{ch} x = \dfrac{e^x + e^{-x}}{2}$

$(\operatorname{sh} x)' = \operatorname{ch} x$

$(\operatorname{ch} x)' = \operatorname{sh} x$

$P \dfrac{1}{\sqrt{1-x^2}} = \operatorname{arcsen} x$

e) $P\,\underset{f'}{1}\,\underset{g}{(\operatorname{arcsen}^2 x)} = \underset{f}{x} \cdot \underset{g}{\operatorname{arcsen}^2 x} - P\,\overset{f}{x}\,\overset{g'}{\dfrac{2 \cdot \operatorname{arcsen} x}{\sqrt{1-x^2}}}$

$\overset{CA}{=} x \cdot \operatorname{arcsen}^2 x - 2\sqrt{1-x^2} \cdot \operatorname{arcsen} x - 2x$

CA: $P\left(\dfrac{2x}{\sqrt{1-x^2}} \cdot \operatorname{arcsen} x\right) \overset{PP}{=} \underset{f}{-2\sqrt{1-x^2}} \cdot \underset{g}{\operatorname{arcsen} x} + 2\,P\,\cancel{\sqrt{1-x^2}} \cdot \underset{g'}{\dfrac{1}{\cancel{\sqrt{1-x^2}}}} = -2\sqrt{1-x^2}\operatorname{arcsen} x + 2x$

3h) $P\cos(\ln x) \overset{PP}{=} x\cos(\ln x) - P\left[x \cdot \left(-\frac{\operatorname{sen}(\ln x)}{x}\right)\right] = x\cos(\ln x) + P\operatorname{sen}(\ln(x))$

$P\frac{\cos(\ln x)}{x} = \operatorname{sen}(\ln x)$

CA: $P\operatorname{sen}(\ln x) \overset{PP}{=} x\operatorname{sen}(\ln x) - P\left(x \cdot \frac{1}{x}\cos(\ln x)\right)$

$\underline{P\cos(\ln x)} \overset{2\text{ vezes PP}}{=} x\cos(\ln x) + x\operatorname{sen}(\ln x) - \underline{P\cos(\ln x)} \Leftrightarrow$

$\underline{2P(\cos(\ln x)} = x(\cos(\ln x) + \operatorname{sen}(\ln x))$

$P\cos(\ln x) = \frac{x}{2}(\cos(\ln x) + \operatorname{sen}(\ln x))$

TPC: análogo para $P\operatorname{sen}x \cdot e^x$

3k) $P\dfrac{\ln 2x}{\sqrt{x}} = P\left(\dfrac{1}{\sqrt{x}} \cdot \ln 2x\right) \overset{P.P}{=} 2\sqrt{x} \cdot \ln 2x - P\, 2\sqrt{x} \cdot \dfrac{1}{x}$

$= 2\sqrt{x}\ln 2x - 2P\dfrac{1}{\sqrt{x}} = 2\sqrt{x}\ln 2x - 4\sqrt{x}$

$P\dfrac{1}{\sqrt{x}} = P\, x^{-1/2}$

$= P\, \dfrac{x^{1/2}}{1/2}$

Ficha nº 8 Fórmula Barrow

F primitiva de f en $[a,b]$ $\displaystyle\int_a^b f(x)\,dx = F(b) - F(a)$

1a) $\displaystyle\int_{\pi^2/36}^{\pi^2/16} \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx \overset{\text{Barrow}}{=} \left[2\,\text{sen}\sqrt{x}\right]_{\pi^2/36}^{\pi^2/16} = 2\,\text{sen}\frac{\pi}{4} - 2\,\text{sen}\frac{\pi}{6} = \sqrt{2} - 1$

$P\dfrac{\cos\sqrt{x}}{\sqrt{x}} = 2\,\text{sen}\sqrt{x}$ (primitiva inmediata)

$(2\,\text{sen}\sqrt{x})' = \dfrac{\cos\sqrt{x}}{\sqrt{x}}$

$(\sqrt{x})' = \dfrac{1}{2\sqrt{x}}$

2a) $\displaystyle\int_0^1 x e^{2x}\,dx \overset{\text{Barrow}}{=} \left[\frac{e^{2x}}{2}\left(x-\tfrac{1}{2}\right)\right]_0^1 = \frac{e^2}{2}\cdot\frac{1}{2} - \frac{1}{2}\left(-\tfrac{1}{2}\right) = \frac{e^2+1}{4}$

PRIMITIVAÇÃO POR PARTES

$\displaystyle P e^{2x} = \frac{e^{2x}}{2}\ ,\quad P\, x e^{2x} \overset{P.P.}{=} \frac{e^{2x}}{2}\cdot x - P\frac{e^{2x}}{2} = \frac{e^{2x}}{2}\left(x-\frac{1}{2}\right)$

2c) $\displaystyle\int_1^e \ln^2 x\,dx$

Integração por partes

$\displaystyle\int_a^b f'(x)\cdot g(x)\,dx = [f\cdot g]_a^b - \int_a^b f(x)g'(x)\,dx$

$\displaystyle \overset{\text{I.P.P.}}{=} \left[x\ln^2 x\right]_1^e - \int_1^e x\cdot 2\cdot\frac{1}{x}\ln x\,dx = e - 2\int_1^e \ln(x)\,dx \overset{\text{I.P.P.}}{=}$

$\displaystyle = e - 2\left(\left[x\ln x\right]_1^e - \underbrace{\int_1^e x\cdot\frac{1}{x}\,dt}_{=[x]_1^e}\right) = e-2\left(e-(e-1)\right) =$

$= e-2$

Conclusão de 1. da ficha nº 7

1 o) $P \frac{2x^4-3x^2+1}{3x^2} = \frac{2}{3} P x^2 - P 1 + \frac{1}{3} P \frac{1}{x^2} = \frac{2x^3}{9} - x - \frac{1}{3x}$

q) $P \frac{2x+3}{2x+1} = P \frac{2x+1}{2x+1} + P \frac{2}{2x+1} = x + \ln|2x+1|$

s) $P \frac{4x}{x^4+1} = 2 P \frac{2x}{1+(x^2)^2} = 2 \operatorname{arctg}(x^2)$

1 j) $P\, tg^5 x = P\left(\frac{1}{\cos^2 x} - 1\right) tg^3 x = P \frac{1}{\cos^2 x} \cdot tg^3 x - P\left(\frac{1}{\cos^2 x} - 1\right) tg\, x =$

$= \frac{tg^4 x}{4} - \frac{tg^2 x}{2} - \ln|\cos x|$

l) $P \operatorname{sen} x \sqrt{1-\cos x} = P \operatorname{sen} x (1-\cos x)^{1/2} = \frac{2}{3}(1-\cos x)^{3/2}$

x) $P \operatorname{sen} 2x\, e^{\cos^2 x} = -P -2 \operatorname{sen} x \cos x\, e^{\cos^2 x} = -e^{\cos^2 x}$

1a), b) $P\left(\frac{2}{\sqrt{x}} + \frac{x\sqrt{x}}{2}\right) = 2P\, x^{-1/2} + \frac{1}{2}P\, x^{3/2} = 4\sqrt{x} + \frac{\sqrt{x^5}}{5}$

1m) $P\, \frac{1}{1+x^2}\cdot \operatorname{arctg}^4(x) = \frac{\operatorname{arctg}^5(x)}{5}$; 1t) $P\, \frac{1}{x \ln x^2} = \frac{\ln|\ln x^2|}{2}$

1u) $P\, x\sqrt{1+x^2} = \frac{(1+x^2)^{3/2}}{3}$ v) $P\, \frac{e^x}{1+e^x} = \ln(1+e^x)$

1y) $\frac{1}{2}P\, \frac{\frac{e^x}{2}}{1+\left(\frac{e^x}{2}\right)^2} = \frac{\operatorname{arctg}\left(\frac{e^x}{2}\right)}{2}$ z) $P\, \frac{x}{\sqrt{1-2x^4}} = \frac{1}{\sqrt{2^3}}P\, \frac{\sqrt{2^3}\,x}{\sqrt{1-\left(\sqrt{2}x^2\right)^2}} = \frac{\operatorname{arcsen}(\sqrt{2}x^2)}{2\sqrt{2}}$

Domínios: 1a) $\mathbb{R}^+$, b) $\mathbb{R}^+_0$, c) $]-1,2[$, d) $\mathbb{R}\setminus\{\frac{1}{2}\}$, e) $\mathbb{R}$, f) $\mathbb{R}$, g) $\bigcup_{k\in\mathbb{Z}} \left]\frac{k\pi}{2}, \frac{(k+1)\pi}{2}\right[$, h) $\mathbb{R}\setminus\{\frac{1}{2}\}$
i) $\bigcup_{k\in\mathbb{Z}}]k\pi, (k+1)\pi[$, j) $\bigcup_{k\in\mathbb{Z}} \left]\frac{\pi}{2}+k\pi, \frac{3\pi}{2}+k\pi\right[$, k) $\mathbb{R}$, l) $\mathbb{R}$, m) $\mathbb{R}$, n) $\mathbb{R}$, o) $\mathbb{R}\setminus\{0\}$
q) $\mathbb{R}\setminus\{-\frac{1}{2}\}$, r) $\mathbb{R}\setminus\{0\}$, s) $\mathbb{R}$, t) $\mathbb{R}\setminus\{-1,0,1\}$, u) $\mathbb{R}$, v) $\mathbb{R}$
x) $\mathbb{R}$, y) $\mathbb{R}$, z) $\left]-\frac{1}{\sqrt[4]{2}}, \frac{1}{\sqrt[4]{2}}\right[$. Funções primitiváveis em qualquer intervalo contido no seu domínio.

Concluindo 3 da ficha 7:

1g) $P\left(\frac{1}{x^2+1}\right)^2 = P\frac{1+x^2}{(x^2+1)^2} - P\frac{x^2}{(x^2+1)^2} = \operatorname{arctg} x - \frac{1}{2}P\, x.2x(x^2+1)^{-2} \overset{P.P}{=}$

$\overset{P.P}{=} \operatorname{arctg} x - \frac{1}{2}\left(x\frac{(x^2+1)^{-1}}{-1} - P - \frac{1}{x^2+1}\right) = \frac{\operatorname{arctg} x}{2} + \frac{x}{2(x^2+1)}$

1i) $P\, x^2 \ln x \overset{P.P}{=} \frac{x^3}{3}\ln x - P\frac{x^3}{3}\cdot\frac{1}{x} = \frac{x^3}{3}\left(\ln x - \frac{1}{3}\right)$

j) $P\, x^2 e^{2x} \overset{PP}{=} \frac{e^{2x}}{2}.x^2 - P\frac{e^{2x}}{2}.2x \overset{P.P.}{=} \frac{e^{2x}}{2}.x^2 - \left(\frac{e^{2x}}{2}.x - P\frac{e^{2x}}{2}.1\right) = \frac{e^{2x}}{2}\left(x^2 - x + \frac{1}{2}\right)$

l) $P\, 2x \operatorname{arctg} x \overset{P.P}{=} x^2 \operatorname{arctg} x - P\frac{x^2}{1+x^2} = x^2 \operatorname{arctg} x - P\frac{x^2+1-1}{1+x^2} =$

$= x^2 \operatorname{arctg} x - P\,1 + P\frac{1}{1+x^2} = x^2 \operatorname{arctg} x - x + \operatorname{arctg} x$