Universidade Lusófona de Humanidades e Tecnologias

Faculdade de Engenharia e Ciências Naturais

Cálculo II

Licenciaturas em
Biologia, Ciências do Mar, Engenharia do Ambiente, Engenharia Biotecnológica, Engenharia Civil, Engenharia Electrotécnica, Engenharia e Gestão Industrial e Química
2º Semestre 2008/2009

Ficha 8 – Domínios de funções e curvas de nível.

Parte I – Exercícios Propostos

Esboços no plano

I.1 Represente os seguintes domínios no plano:

a) $D=\{(x,y)\in\mathbb{R}^2 : x\geq y\}$

b) $D=\{(x,y)\in\mathbb{R}^2 : x^2+y^2\leq 1\}$

c) $D=\{(x,y)\in\mathbb{R}^2 : 2x-y>0\}$

d) $D=\{(x,y)\in\mathbb{R}^2 : x^2+y^2-2>0\}$

e) $D_f=\{(x,y)\in\mathbb{R}^2 : x+y>0\}$

f) $D=\{(x,y)\in\mathbb{R}^2 : y>0\wedge 1-x^2\geq 0\}$

Domínios de funções

I.2 Determine os domínios das seguintes funções e represente-os graficamente:

a) $f(x,y)=\sqrt[5]{x+y}-\sqrt[4]{x-y}$

b) $f(x,y)=\operatorname{sen}\left(\frac{1}{x^2-y^2}\right)+\cos(x^2-y^2)$

c) $f(x,y)=2^{4-x^2-y^2}+\sqrt{1-x^2-y^2}$

d) $f(x,y)=\frac{x+y}{\sqrt{x^2-y^2}}+\sqrt[6]{4-x^2}$

Curvas de Nível

I.3 Represente, sempre que possível, graficamente as curvas de nível -1,0,1 das funções seguintes:

a) $f(x,y)=x$

b) $f(x,y)=y$

c) $f(x,y)=x+y$

d) $f(x,y)=x^2-y$

e) $f(x,y)=y^2-x$

f) $f(x,y)=x^2+y^2$

g) $f(x,y)=\ln(x^2+y^2)$

Parte II – Exercícios Resolvidos

II.1 Determine os domínios das seguintes funções e represente-os graficamente:

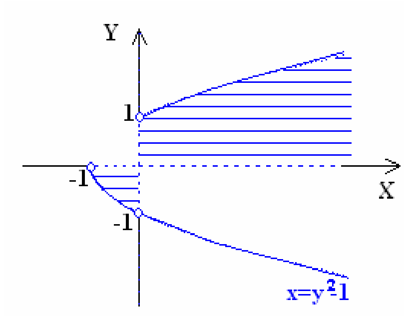
a) $f(x,y)=\cosh\left(1+x-y^2\right)-\ln\left(\dfrac{x}{y}\right)+\sqrt[4]{1+x-y^2}+\operatorname{sen}\left(x-y^2\right)$

Resolução:

Domínio:

$$D_f=\left\{(x,y)\in\mathbb{R}^2:\frac{x}{y}>0\wedge y\neq 0\wedge 1+x-y^2\geq 0\right\}$$
$$=\left\{(x,y)\in\mathbb{R}^2:\left[(x>0\wedge y>0)\vee(x<0\wedge y<0)\right]\wedge y\neq 0\wedge x\geq y^2-1\right\}$$
$$=\left\{(x,y)\in\mathbb{R}^2:\left[(x>0\wedge y>0)\vee(x<0\wedge y<0)\right]\wedge x\geq y^2-1\right\}$$

Representação gráfica do domínio:



b) $f(x,y)=\operatorname{senh}\left(9+x^2-y^2\right)+\sqrt[7]{1-x-y^2}+\cos\left(xy^2\right)+\operatorname{arc\,sen}\left(\dfrac{y-1}{x}\right)$

Resolução:

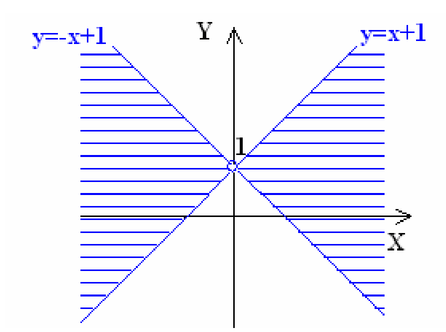
Domínio:

$$D_f=\left\{(x,y)\in\mathbb{R}^2:-1\leq\frac{y-1}{x}\leq 1\wedge x\neq 0\right\}=$$
$$\underset{(*)}{=}\left\{(x,y)\in\mathbb{R}^2:\left[(y\leq 1+x\wedge x>0)\vee(y\geq 1+x\wedge x<0)\right]\wedge\left[(y\leq 1-x\wedge x<0)\vee(y\geq 1-x\wedge x>0)\right]\right\}$$

Cálculos auxiliares: $(*)$

$$-1\leq\frac{y-1}{x}\leq 1\Leftrightarrow\frac{y-1}{x}\leq 1\wedge\frac{y-1}{x}\geq -1\Leftrightarrow\frac{y-1}{x}-1\leq 0\wedge\frac{y-1}{x}+1\geq 0\Leftrightarrow\frac{y-1-x}{x}\leq 0\wedge\frac{y-1+x}{x}\geq 0$$
$$\Leftrightarrow\left[(y-1-x\leq 0\wedge x>0)\vee(y-1-x\geq 0\wedge x<0)\right]\wedge\left[(y-1+x\leq 0\wedge x<0)\vee(y-1+x\geq 0\wedge x>0)\right]$$
$$\Leftrightarrow\left[(y\leq x+1\wedge x>0)\vee(y\geq x+1\wedge x<0)\right]\wedge\left[(y\leq -x+1\wedge x<0)\vee(y\geq -x+1\wedge x>0)\right]$$

Representação gráfica do domínio:



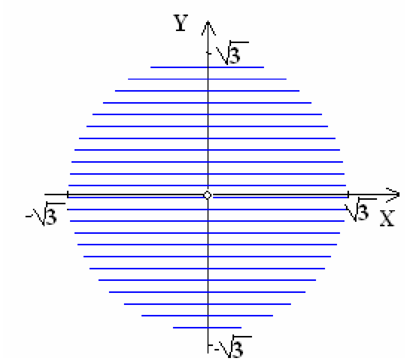
c) $f(x,y)=\dfrac{\ln\left(3-x^2-y^2\right)}{x^2+y^2}+\cosh\left(3-x^2y^2\right)+\sqrt[11]{5-xy^2}+\operatorname{sen}\left(xy^2\right)$

Resolução:

<u>Domínio:</u>

$$
\begin{aligned}
D_f &=\left\{(x,y)\in\mathbb{R}^2: 3-x^2-y^2>0 \wedge x^2+y^2\neq 0\right\}=\left\{(x,y)\in\mathbb{R}^2: -x^2-y^2>-3 \wedge (x,y)\neq(0,0)\right\}\\
&=\left\{(x,y)\in\mathbb{R}^2: x^2+y^2<3 \wedge (x,y)\neq(0,0)\right\}
\end{aligned}
$$

<u>Representação gráfica do domínio:</u>



d) $f(x,y)=\sqrt{y^2-y+x-x^2}+\log_5\left(4-x^2-y^2\right)+\cosh\left(x^2y^2\right)+\left|x-y^2\right|$

Resolução:

$$
\begin{aligned}
D_f &=\left\{(x,y)\in\mathbb{R}^2: y^2-y+x-x^2\geq 0 \wedge 4-x^2-y^2>0\right\}=\left\{(x,y)\in\mathbb{R}^2: x^2-x-y^2+y\leq 0 \wedge -x^2-y^2>-4\right\}\\
&=\left\{(x,y)\in\mathbb{R}^2: x^2-x+\frac{1}{4}-\frac{1}{4}-\left(y^2-y+\frac{1}{4}-\frac{1}{4}\right)\leq 0 \wedge x^2+y^2<4\right\}\\
&=\left\{(x,y)\in\mathbb{R}^2: \left(x^2-x+\frac{1}{4}\right)-\frac{1}{4}-\left(y^2-y+\frac{1}{4}\right)+\frac{1}{4}\leq 0 \wedge x^2+y^2<4\right\}\\
&=\left\{(x,y)\in\mathbb{R}^2: \left(x-\frac{1}{2}\right)^2-\left(y-\frac{1}{2}\right)^2\leq 0 \wedge x^2+y^2<4\right\}\\
&=\left\{(x,y)\in\mathbb{R}^2: \left(y-\frac{1}{2}\right)^2-\left(x-\frac{1}{2}\right)^2\geq 0 \wedge x^2+y^2<4\right\}\\
&\underset{(*)}{=}\left\{(x,y)\in\mathbb{R}^2: (y+x-1)(y-x)\geq 0 \wedge x^2+y^2<4\right\}\\
&=\left\{(x,y)\in\mathbb{R}^2: (y+x-1\geq 0 \wedge y-x\geq 0)\vee(y+x-1\leq 0 \wedge y-x\leq 0)\wedge x^2+y^2<4\right\}\\
&=\left\{(x,y)\in\mathbb{R}^2: (y\geq -x+1 \wedge y\geq x)\vee(y\leq -x+1 \wedge y\leq x)\wedge x^2+y^2<4\right\}
\end{aligned}
$$

Cálculos auxiliares: $(*)$

$$
\left(y-\frac{1}{2}\right)^2-\left(x-\frac{1}{2}\right)^2\underset{\substack{\uparrow\\ \text{Caso notável da multiplicação:}\\ a^2-b^2=(a-b)\cdot(a+b)\\ \text{(Folhas de apoio de Mat.0 – pág.7)}}}{=}\left[y-\frac{1}{2}-\left(x-\frac{1}{2}\right)\right]\left(y-\frac{1}{2}+x-\frac{1}{2}\right)=\left(y-\frac{1}{2}-x+\frac{1}{2}\right)(y+x-1)=(y-x)(y+x-1)
$$

Outra forma de raciocínio:

$$\left(y-\frac{1}{2}\right)^2=\left(x-\frac{1}{2}\right)^2 \underset{\substack{\uparrow \\ \text{Pela propriedade} \\ x^2=y^2 \Leftrightarrow x=y \vee x=-y}}{\Leftrightarrow} y-\frac{1}{2}=x-\frac{1}{2} \vee y-\frac{1}{2}=-\left(x-\frac{1}{2}\right)$$

$$\Leftrightarrow y=x-\frac{1}{2}+\frac{1}{2} \vee y=-x+\frac{1}{2}+\frac{1}{2} \Leftrightarrow y=x \vee y=-x+1$$

Assim,

$$\left(x-\frac{1}{2}\right)^2-\left(y-\frac{1}{2}\right)^2=\left(y-(-x+1)\right)\left(y-x\right)=\left(y+x-1\right)\left(y-x\right).$$

Outra forma de resolver:

$$\left(y-\frac{1}{2}\right)^2=\left(x-\frac{1}{2}\right)^2 \underset{\substack{\uparrow \\ \text{Ambos os membros} \\ \text{são positivos}}}{\Leftrightarrow} \sqrt{\left(y-\frac{1}{2}\right)^2}=\sqrt{\left(x-\frac{1}{2}\right)^2} \underset{\substack{\uparrow \\ \text{Se n é par e } x\in\mathbb{R}^- \\ \text{então } \sqrt[n]{x^n\cdot a}=|x|\cdot\sqrt[n]{a}}}{\Leftrightarrow} \left|y-\frac{1}{2}\right|=\left|x-\frac{1}{2}\right|$$

$$\underset{\substack{\uparrow \\ \text{Pela propriedade:} \\ |x|=|y| \Leftrightarrow x=y \ \vee \ x=-y}}{\Leftrightarrow} y-\frac{1}{2}=x-\frac{1}{2} \vee y-\frac{1}{2}=-\left(x-\frac{1}{2}\right)$$

$$\Leftrightarrow y=x-\frac{1}{2}+\frac{1}{2} \vee y=-x+\frac{1}{2}+\frac{1}{2} \Leftrightarrow y=x \vee y=-x+1$$

Assim,

$$\left(x-\frac{1}{2}\right)^2-\left(y-\frac{1}{2}\right)^2=\left(y-(-x+1)\right)\left(y-x\right)=\left(y+x-1\right)\left(y-x\right).$$

Outra forma de resolver:

$$\left(y-\frac{1}{2}\right)^2=\left(x-\frac{1}{2}\right)^2 \Leftrightarrow y-\frac{1}{2}=\pm\sqrt{\left(x-\frac{1}{2}\right)^2} \underset{\substack{\uparrow \\ \text{Se n é par e } x\in\mathbb{R}^- \\ \text{então } \sqrt[n]{x^n\cdot a}=|x|\cdot\sqrt[n]{a}}}{\Leftrightarrow} y-\frac{1}{2}=\pm\left|x-\frac{1}{2}\right|$$

- Se $x-\frac{1}{2}\geq 0$ então $\left|x-\frac{1}{2}\right|=x-\frac{1}{2}$, e temos

$$y-\frac{1}{2}=\pm\left(x-\frac{1}{2}\right) \Leftrightarrow y=\frac{1}{2}\pm\left(x-\frac{1}{2}\right) \Leftrightarrow y=\frac{1}{2}+\left(x-\frac{1}{2}\right) \vee y=\frac{1}{2}-\left(x-\frac{1}{2}\right)$$

$$\Leftrightarrow y=\frac{1}{2}+x-\frac{1}{2} \vee y=\frac{1}{2}-x+\frac{1}{2} \Leftrightarrow y=x \vee y=-x+1$$

- Se $x-\frac{1}{2}<0$ então $\left|x-\frac{1}{2}\right|=-\left(x-\frac{1}{2}\right)=-x+\frac{1}{2}$, e temos

$$y-\frac{1}{2}=\pm\left(-x+\frac{1}{2}\right) \Leftrightarrow y=\frac{1}{2}\pm\left(-x+\frac{1}{2}\right) \Leftrightarrow y=\frac{1}{2}+\left(-x+\frac{1}{2}\right) \vee y=\frac{1}{2}-\left(-x+\frac{1}{2}\right)$$

$$\Leftrightarrow y=\frac{1}{2}-x+\frac{1}{2} \vee y=\frac{1}{2}+x-\frac{1}{2} \Leftrightarrow y=-x+1 \vee y=x$$

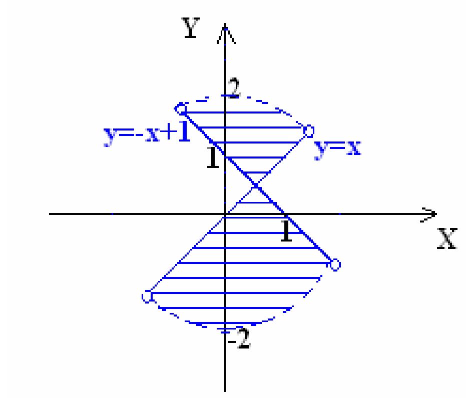
Logo em qualquer dos casos tem-se:

$$y=-x+1 \vee y=x\ .$$

Assim,

$$\left(x-\frac{1}{2}\right)^2-\left(y-\frac{1}{2}\right)^2=\left(y-(-x+1)\right)\left(y-x\right)=\left(y+x-1\right)\left(y-x\right).$$

Representação gráfica do domínio:



II .2 Determine os domínios das seguintes funções e represente-os graficamente:

a) $f(x,y) = \cosh(x^2y^2) + |x\text{-}y^2| - \sqrt[6]{x-\sqrt{y}}$

Resolução:

Domínio:

$$D_f = \left\{(x,y)\in \mathbb{R}^2 : x-\sqrt{y}\geq 0 \wedge y\geq 0\right\} = \left\{(x,y)\in \mathbb{R}^2 : x\geq \sqrt{y} \wedge y\geq 0\right\}$$
$$\underset{(*)}{=} \left\{(x,y)\in \mathbb{R}^2 : y\leq x^2 \wedge x\geq 0 \wedge y\geq 0\right\}$$

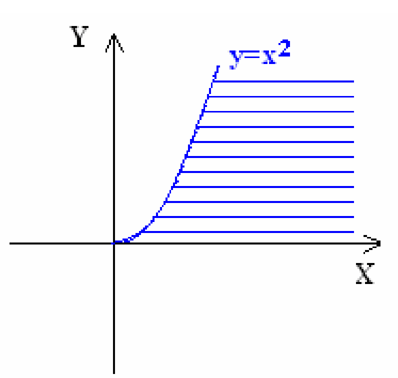
Cálculos auxiliares: $(*)$

$$x\geq \sqrt{y} \wedge y\geq 0 \Leftrightarrow \left[\left(x\geq \sqrt{y} \wedge x\geq 0\right) \vee \underbrace{\left(x\geq \sqrt{y} \wedge x<0\right)}_{\text{Condição impossível}}\right] \wedge y\geq 0 \Leftrightarrow x\geq \sqrt{y} \wedge x\geq 0 \wedge y\geq 0$$

$$\underset{\substack{\text{Ambos os membros}\\ \text{são positivos}}}{\Leftrightarrow} (x)^2 \geq \left(\sqrt{y}\right)^2 \wedge x\geq 0 \wedge y\geq 0 \Leftrightarrow x^2\geq \sqrt{y^2} \wedge x\geq 0 \wedge y\geq 0 \underset{\substack{\uparrow\\ \text{Como } y\geq 0\\ \text{então } \sqrt{y^2}=y}}{\Leftrightarrow} x^2\geq y \wedge x\geq 0 \wedge y\geq 0$$

$$\Leftrightarrow y\leq x^2 \wedge x\geq 0 \wedge y\geq 0$$

Representação gráfica do domínio:



b) $f(x,y)=\sqrt{x^2-y+4}+\log_9\left(y-\ln\frac{x}{2}\right)-\cos\left(3-x^2y^2\right)+\sqrt[3]{2\text{-}x\text{-}y^2}+\text{senh}\left(2x\text{-}3y^2\right)$

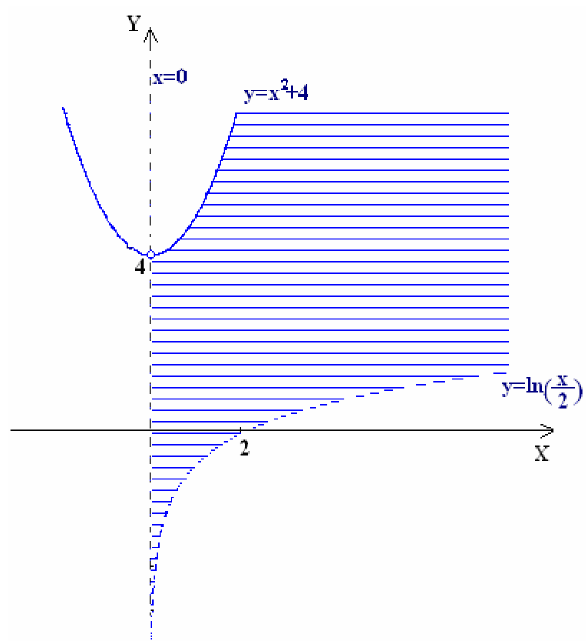
Resolução:

Domínio:

$$D_f=\left\{(x,y)\in\mathbb{R}^2: x^2-y+4\geq 0\wedge y-\ln\frac{x}{2}>0\wedge\frac{x}{2}>0\right\}=\left\{(x,y)\in\mathbb{R}^2:-y\geq -x^2-4\wedge y>\ln\frac{x}{2}\wedge x>0\right\}$$

$$=\left\{(x,y)\in\mathbb{R}^2: y\leq x^2+4\wedge y>\ln\frac{x}{2}\wedge x>0\right\}$$

Representação gráfica do domínio:



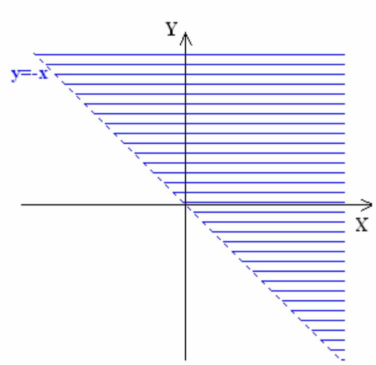
c) $f(x,y)=2\cos\left(3-x^2y^2\right)+5\log_4(x+y)-\sqrt[3]{2\text{-}3x\text{-}5y^2}+3\text{senh}\left(10x\text{-}3y^2\right)$

Resolução:

Domínio:

$$D_f=\left\{(x,y)\in\mathbb{R}^2: x+y>0\right\}=\left\{(x,y)\in\mathbb{R}^2: y>-x\right\}$$

Representação gráfica do domínio:



d) $f(x,y)=2\cos\left(3-x^2y^2\right)+7\sqrt[3]{2-x-y^2}+6\operatorname{senh}\left(2x-3y^2\right)-5\log_2(y)+\sqrt[10]{1-x^2}$

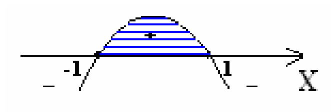
Resolução:

Domínio:

$$D_f=\left\{(x,y)\in\mathbb{R}^2: y>0\wedge 1-x^2\geq 0\right\}\underset{(*)}{=}\left\{(x,y)\in\mathbb{R}^2: y>0\wedge -1\leq x\leq 1\right\}$$

Cálculos auxiliares: $(*)$

$1-x^2=0\Leftrightarrow(1-x)(1+x)=0\Leftrightarrow 1-x=0\vee 1+x=0\Leftrightarrow -x=-1\vee x=-1\Leftrightarrow x=1\vee x=-1$



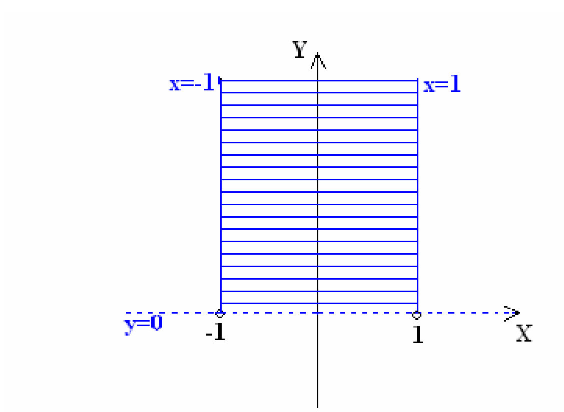
Outra forma de racionar:

$$\begin{aligned}
1-x^2\geq 0&\Leftrightarrow\left(1-x^2>0\vee 1-x^2=0\right)\Leftrightarrow\left[(1-x)(1+x)>0\vee -x^2=-1\right]\\
&\Leftrightarrow\left[\left((1-x>0\wedge 1+x>0)\vee(1-x<0\wedge 1+x<0)\right)\vee x^2=1\right]\\
&\Leftrightarrow\left[\left((-x>-1\wedge x>-1)\vee(-x<-1\wedge x<-1)\right)\vee x=\pm 1\right]\\
&\Leftrightarrow\left[\left((x<1\wedge x>-1)\vee(x>1\wedge x<-1)\right)\vee x=\pm 1\right]\\
&\Leftrightarrow\left[\left((-1<x<1)\vee(x\in\varnothing)\right)\vee x=\pm 1\right]\Leftrightarrow -1<x<1\vee x=\pm 1\Leftrightarrow -1\leq x\leq 1
\end{aligned}$$

Outra forma de resolver:

$$\begin{aligned}
1-x^2\geq 0&\Leftrightarrow 1-x^2>0\vee 1-x^2=0\Leftrightarrow -x^2>-1\vee -x^2=-1\\
&\Leftrightarrow x^2<1\vee x^2=1\Leftrightarrow\underset{x^2<y^2\Leftrightarrow|x|<|y|}{\underset{\uparrow}{|x|}}<1\vee x=\pm 1\Leftrightarrow -1<x<1\vee x=\pm 1\Leftrightarrow -1\leq x\leq 1
\end{aligned}$$

Representação gráfica do domínio:



e) $f(x,y) = \log_2\left(5x - x^2 - 6\right) + \ln\left(1 - y^2\right) - \cos(x) + \sqrt[3]{2-x-y^2} + \operatorname{senh}\left(3y^2\right)$

Resolução:

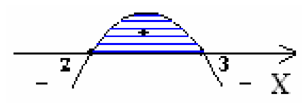
<u>Domínio:</u>

$$D_f = \left\{(x,y) \in \mathbb{R}^2 : 5x - x^2 - 6 > 0 \wedge 1 - y^2 > 0\right\} \underset{(*)}{=} \left\{(x,y) \in \mathbb{R}^2 : x \in \left]2,3\right[\wedge y \in \left]-1,1\right[\right\} = \left]2,3\right[\times \left]-1,1\right[$$

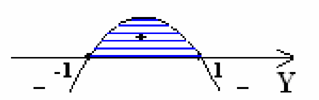
Cálculos auxiliares: $(*)$

• $5x - x^2 - 6 = 0 \Leftrightarrow -x^2 + 5x - 6 = 0 \Leftrightarrow x = \dfrac{-5 \pm \sqrt{5^2 - 4\left(-1 \times (-6)\right)}}{2 \times (-1)} \Leftrightarrow x = \dfrac{-5 \pm \sqrt{1}}{-2} \Leftrightarrow x = \dfrac{-5 \pm 1}{-2}$

$$\Leftrightarrow x = \frac{-6}{-2} \vee x = \frac{-4}{-2} \Leftrightarrow x = 3 \vee x = 2$$



• $1 - y^2 = 0 \Leftrightarrow (1-y)(1+y) = 0 \Leftrightarrow 1 - y = 0 \vee 1 + y = 0 \Leftrightarrow -y = -1 \vee y = -1 \Leftrightarrow y = 1 \vee y = -1$



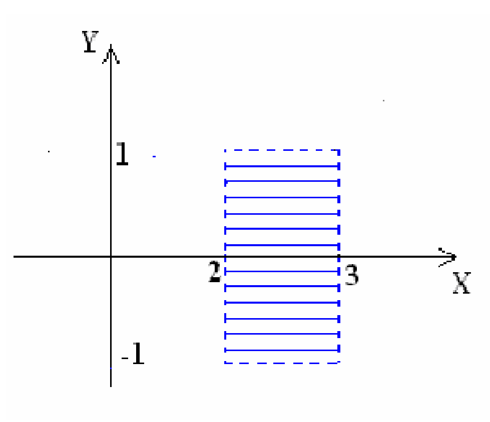
Outra forma de racionar:

$$1 - y^2 > 0 \Leftrightarrow (1-y)(1+y) > 0 \Leftrightarrow (1 - y > 0 \wedge 1 + y > 0) \vee (1 - y < 0 \wedge 1 + y < 0)$$
$$\Leftrightarrow (-y > -1 \wedge y > -1) \vee (-y < -1 \wedge y < -1) \Leftrightarrow (y < 1 \wedge y > -1) \vee (y > 1 \wedge y < -1)$$
$$\Leftrightarrow (-1 < y < 1) \vee (y \in \varnothing) \Leftrightarrow -1 < y < 1$$

Outra forma de resolver:

$$1 - y^2 > 0 \Leftrightarrow -y^2 > -1 \Leftrightarrow y^2 < 1 \underset{\substack{\uparrow \\ x^2 < y^2 \Leftrightarrow |x| < |y|}}{\Leftrightarrow} |y| < 1 \Leftrightarrow -1 < y < 1$$

<u>Representação gráfica do domínio:</u>



II.3 Sempre que possível represente graficamente algumas curvas de nível da função
$$f(x,y)=(x+1)^2+(y-2)^2 .$$

Resolução:
As curvas de nível são dadas por:
$$L(c)=\{(x,y)\in \mathbb{R}^2 : c=f(x,y)\}=\{(x,y)\in \mathbb{R}^2 : c=(x+1)^2+(y-2)^2\}$$

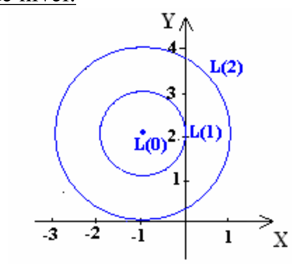
Assim a curva de nível associado

- ao valor 0:
$$L(0)=\{(x,y)\in \mathbb{R}^2 : 0=f(x,y)\}=\{(x,y)\in \mathbb{R}^2 : 0=(x+1)^2+(y-2)^2\}$$
$$=\{(x,y)\in \mathbb{R}^2 : x=-1\wedge y=2\}=\{(-1,2)\}$$
Representa o ponto (-1, 2)

- ao valor 1:
$$L(1)=\{(x,y)\in \mathbb{R}^2 : 1=(x+1)^2+(y-2)^2\}=\{(x,y)\in \mathbb{R}^2 : (x-(-1))^2+(y-2)^2=1\}$$
Representa uma circunferência de centro (-1, 2) e de raio 1

- ao valor 4:
$$L(4)=\{(x,y)\in \mathbb{R}^2 : 4=(x+1)^2+(y-2)^2\}=\{(x,y)\in \mathbb{R}^2 : (x-(-1))^2+(y-2)^2=2^2\}$$
Representa uma circunferência de centro (-1, 2) e de raio 2

Representação gráfica destas curvas de nível:



II.4 Sempre que possível represente graficamente algumas curvas de nível da função
$$f(x,y)=|y-x| .$$

Resolução:
As curvas de nível são dadas por:
$$L(c)=\{(x,y)\in \mathbb{R}^2 : c=f(x,y)\}=\{(x,y)\in \mathbb{R}^2 : c=|y-x|\}$$

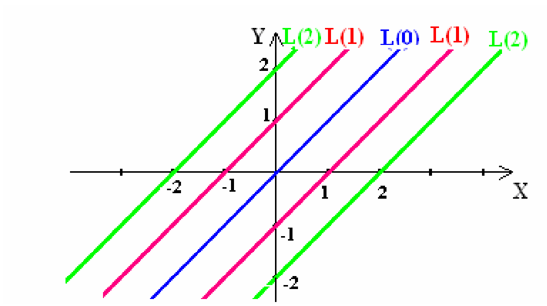
Assim a curva de nível associada

- ao valor 0:
$$L(0)=\{(x,y)\in \mathbb{R}^2 : 0=|y-x|\}=\{(x,y)\in \mathbb{R}^2 : 0=y-x\}=\{(x,y)\in \mathbb{R}^2 : y=x\}$$

- ao valor 1:
$$L(1)=\{(x,y)\in \mathbb{R}^2 : 1=|y-x|\}=\{(x,y)\in \mathbb{R}^2 : |y-x|=1\}=\{(x,y)\in \mathbb{R}^2 : y-x=1\vee y-x=-1\}$$
$$=\{(x,y)\in \mathbb{R}^2 : -x=1-y\vee -x=-1-y\}=\{(x,y)\in \mathbb{R}^2 : y=x+1\vee y=x-1\}$$

- ao valor 2:
$$L(2)=\{(x,y)\in \mathbb{R}^2 : 2=|y-x|\}=\{(x,y)\in \mathbb{R}^2 : |y-x|=2\}=\{(x,y)\in \mathbb{R}^2 : y-x=2\vee y-x=-2\}$$
$$=\{(x,y)\in \mathbb{R}^2 : y=2+x\vee y=-2+x\}=\{(x,y)\in \mathbb{R}^2 : y=x+2\vee y=x-2\}$$

Representação gráfica destas curvas de nível:



II.5 Sempre que possível represente graficamente algumas curvas de nível da função

$$f(x,y) = x^2 + y^2 - 2x + 2y + 2.$$

Resolução:

$$L(c) = \{(x,y) \in \mathbb{R}^2 : c = f(x,y)\} = \{(x,y) \in \mathbb{R}^2 : c = x^2 + y^2 - 2x + 2y + 2\}$$
$$= \{(x,y) \in \mathbb{R}^2 : c = x^2 - 2x + 1 + y^2 + 2y + 1\} = \{(x,y) \in \mathbb{R}^2 : c = (x-1)^2 + (y+1)^2\}$$

As curvas de nível são dadas por:

$$L(c) = \{(x,y) \in \mathbb{R}^2 : c = f(x,y)\} = \{(x,y) \in \mathbb{R}^2 : c = (x-1)^2 + (y+1)^2\}$$

Assim a curva de nível associado

- ao valor 0:

$$L(0) = \{(x,y) \in \mathbb{R}^2 : 0 = g(x,y)\} = \{(x,y) \in \mathbb{R}^2 : 0 = (x-1)^2 + (y+1)^2\}$$
$$= \{(x,y) \in \mathbb{R}^2 : x = 1 \wedge y = -1\} = \{(1,-1)\}$$

- ao valor 1:

$$L(1) = \{(x,y) \in \mathbb{R}^2 : 1 = (x-1)^2 + (y+1)^2\} = \{(x,y) \in \mathbb{R}^2 : (x-1)^2 + (y-(-1))^2 = 1\}$$

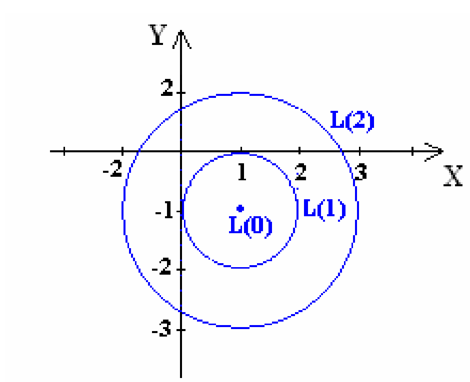
Representa uma circunferência de centro (1, -1) e de raio 1

- ao valor 4:

$$L(4) = \{(x,y) \in \mathbb{R}^2 : 4 = (x-1)^2 + (y+1)^2\} = \{(x,y) \in \mathbb{R}^2 : (x-1)^2 + (y-(-1))^2 = 2^2\}$$

Representa uma circunferência de centro (1, -1) e de raio 2

Representação gráfica destas curvas de nível:



Parte III – Exercícios de Auto-Avaliação

III.1 Determine os domínios das seguintes funções e represente-os graficamente:

a) $f(x,y)=\sqrt[4]{x\cdot \text{sen}(y)}-e^{2-xy^2}+\sqrt[5]{2\text{-}x\text{-}y^2}$

b) $f(x,y)=\text{sen}(3x-y)+\sqrt[7]{2\text{-}x\text{-}y^2}-\dfrac{1}{x^2+y-2x}$

c) $f(x,y)=\text{arc sen}\left(2y\left(1+x^2\right)\text{-}1\right)-e^{3-x^2y^2}+\sqrt[9]{10\text{-}x\text{-}y^2}$

d) $f(x,y)=e^{3-y^2}+\sqrt[9]{2\text{-}x\text{-}y^2}+\text{sen}\left(2xy^2\right)-\sqrt{\left(e^y-e^{-y}\right)\cdot\cos(x)}$

e) $f(x,y)=\sqrt[3]{2\text{-}x\text{-}y^2}+\cos(xy)+\log_3\left(\text{sen}(x)\right)$

III 2. Sejam $f(x,y)=\dfrac{\log_2\left(4-x^2-y^2\right)}{\sqrt[6]{y-|x|}}$ e $g(x,y)=x^2+2x+y^2-4y+5$.

a) Determine e esboce o domínio de $f(x,y)$.

b) Sempre que possível represente as curvas de nível 0,1 e 2.

III 3. Sejam $f(x,y)=\dfrac{\log_3\left[4-(x+1)^2-y^2\right]}{\sqrt{4-(x-1)^2-y^2}}$ e $g(x,y)=|x-y|$.

a) Determine e esboce o domínio de $f(x,y)$.

b) Represente graficamente as curvas de nível 0,1 e 2 de $g(x,y)$.

III. 4 Sejam $f(x,y)=\dfrac{\ln|x-y|}{x+y}+\dfrac{\sqrt[3]{xy+2}}{x^2+y^2+2}$ e $g(x,y)=(x-1)^2+(y+1)^2$.

a) Determine e esboce o domínio de $f(x,y)$.

b) Represente graficamente algumas curvas de nível de $g(x,y)$.