Ficha 1
Resolução dos exercícios de auto-avaliação

III.1 Calcule as derivadas das funções seguintes:

a) $\left(5x-\frac{1}{3}\right)^4$

Resolução:

$$\left(\left(5x-\frac{1}{3}\right)^4\right)' = 4\left(5x-\frac{1}{3}\right)^3\left(5x-\frac{1}{3}\right)' = 4\left(5x-\frac{1}{3}\right)^3 5 = 20\left(5x-\frac{1}{3}\right)^3$$

b) $\frac{x^2-16}{x+5}$

Resolução:

$$\left(\frac{x^2-16}{x+5}\right)' = \frac{\left(x^2-16\right)'(x+5)-\left(x^2-16\right)(x+5)'}{(x+5)^2} = \frac{(2x)(x+5)-\left(x^2-16\right).1}{(x+5)^2}$$

$$= \frac{2x^2+10x-\left(x^2-16\right)}{(x+5)^2} = \frac{2x^2+10x-x^2+16}{(x+5)^2} = \frac{x^2+10x+16}{(x+5)^2}$$

c) $x+1+\frac{1}{x-1}$

Resolução:

$$\left(x+1+\frac{1}{x-1}\right)' = (x+1)' + \left(\frac{1}{x-1}\right)' = 1+\left((x-1)^{-1}\right)' = 1+(-1)(x-1)^{-2}(x-1)' = 1-\frac{1}{(x-1)^2}$$

d) $3-\frac{1}{(x-1)^2}$

Resolução:

$$\left(3-\frac{1}{(x-1)^2}\right)' = (3)' - \left(\frac{1}{(x-1)^2}\right)' = 0-\left((x-1)^{-2}\right)' = -(-2)(x-1)^{-3}(x-1)' = \frac{2}{(x-1)^3}$$

e) $(1-2x^2)e^{-x^2}$

Resolução:

$$\left((1-2x^2)e^{-x^2}\right)' = \left(1-2x^2\right)' e^{-x^2} + \left(1-2x^2\right)\left(e^{-x^2}\right)' = -4x e^{-x^2} + \left(1-2x^2\right)\left(-2xe^{-x^2}\right) = -4xe^{-x^2} - 2xe^{-x^2} + 4x^3e^{-x^2}$$

f) $\frac{\ln(x)}{x}$

Resolução:

$$\left(\frac{\ln(x)}{x}\right)' = \frac{(\ln(x))' x - \ln(x)(x)'}{x^2} = \frac{\frac{1}{x}x - \ln(x)1}{x^2} = \frac{1-\ln(x)}{x^2}$$

III.2 Calcule as derivadas das funções seguintes:

a) $f(x)=\operatorname{sen}(2x)\cos(3x)$

Resolução:

$$f'(x)=\left(\operatorname{sen}(2x)\cos(3x)\right)'=\left(\operatorname{sen}(2x)\right)'\cos(3x)+\operatorname{sen}(2x)\left(\cos(3x)\right)'=2\cos(2x)\cos(3x)+\operatorname{sen}(2x)\left(-3\right)\operatorname{sen}(3x)$$
$$=2\cos(2x)\cos(3x)-3\operatorname{sen}(2x)\operatorname{sen}(3x)$$

b) $g(x)=\dfrac{1}{2\,\mathrm{tg}^2 x}$

Resolução:

$$g'(x)=\left(\frac{1}{2\,\mathrm{tg}^2x}\right)'=\frac{1}{2}\left(\frac{1}{\mathrm{tg}^2x}\right)'=\frac{1}{2}\left(\mathrm{tg}^{-2}x\right)'=\frac{1}{2}(-2)\left(\mathrm{tg}^{-2-1}x\right)\left(\mathrm{tg}\,x\right)'=-\left(\mathrm{tg}^{-3}x\right)\frac{1}{1+x^2}=-\frac{1}{\mathrm{tg}^3x\left(1+x^2\right)}$$

III.3 Determine as derivadas das seguintes funções:

a) $a(x)=\ln\left(\operatorname{arc\,sen}\left(\sqrt{x}\right)\right)$

Resolução:

$$a'(x)=\left(\ln\left(\operatorname{arc\,sen}\left(\sqrt{x}\right)\right)\right)'=\frac{\left(\operatorname{arc\,sen}\left(\sqrt{x}\right)\right)'}{\operatorname{arc\,sen}\left(\sqrt{x}\right)}=\frac{\dfrac{\left(\sqrt{x}\right)'}{\sqrt{1-\left(\sqrt{x}\right)^2}}}{\operatorname{arc\,sen}\left(\sqrt{x}\right)}=\frac{\left(\sqrt{x}\right)'}{\operatorname{arc\,sen}\left(\sqrt{x}\right)\sqrt{1-x}}=\frac{\dfrac{1}{2\sqrt{x}}}{\operatorname{arc\,sen}\left(\sqrt{x}\right)\sqrt{1-x}}$$
$$=\frac{1}{2\sqrt{x}\ \operatorname{arc\,sen}\left(\sqrt{x}\right)\sqrt{1-x}}$$

b) $b(x)=\dfrac{1}{e^{\cos x}}$

Resolução:

$$b'(x)=\left(\frac{1}{e^{\cos(x)}}\right)'=\left(e^{-\cos(x)}\right)'=\left(-\cos(x)\right)'e^{-\cos(x)}=-\left(-\operatorname{sen}(x)\right)e^{-\cos(x)}=\operatorname{sen}(x)\,e^{-\cos(x)}$$

c) $c(x)=\ln\left(\operatorname{arc\,tg}\left(\dfrac{1}{x}\right)\right)$

Resolução:

$$c'(x)=\left(\ln\left(\operatorname{arc\,tg}\left(\frac{1}{x}\right)\right)\right)'=\frac{\left(\operatorname{arc\,tg}\left(\frac{1}{x}\right)\right)'}{\operatorname{arc\,tg}\left(\frac{1}{x}\right)}=\frac{\dfrac{\left(\frac{1}{x}\right)'}{1+\left(\frac{1}{x}\right)^2}}{\operatorname{arc\,tg}\left(\frac{1}{x}\right)}=\frac{\dfrac{\left(x^{-1}\right)'}{1+\frac{1}{x^2}}}{\operatorname{arc\,tg}\left(\frac{1}{x}\right)}=\frac{\dfrac{-x^{-2}}{1+\frac{1}{x^2}}}{\operatorname{arc\,tg}\left(\frac{1}{x}\right)}=\frac{\dfrac{-x^{-2}}{\frac{x^2+1}{x^2}}}{\operatorname{arc\,tg}\left(\frac{1}{x}\right)}=\frac{-x^{-2}\,\dfrac{x^2}{x^2+1}}{\operatorname{arc\,tg}\left(\frac{1}{x}\right)}$$
$$=\frac{-\dfrac{x^2}{x^2\left(x^2+1\right)}}{\operatorname{arc\,tg}\left(\frac{1}{x}\right)}=\frac{-\dfrac{1}{x^2+1}}{\operatorname{arc\,tg}\left(\frac{1}{x}\right)}=-\frac{1}{\left(x^2+1\right)\operatorname{arc\,tg}\left(\frac{1}{x}\right)}$$

d) $e(x)=\dfrac{\cos^2(x)}{2\operatorname{sen}^2(x)}$

Resolução:

$$e'(x)=\left(\frac{\cos^2(x)}{2\operatorname{sen}^2(x)}\right)'=\left(\frac{1}{2}\left(\frac{\cos(x)}{\operatorname{sen}(x)}\right)^2\right)'=\frac{1}{2}\left(\left(\frac{\cos(x)}{\operatorname{sen}(x)}\right)^2\right)'=\frac{1}{2}2\frac{\cos(x)}{\operatorname{sen}(x)}\left(\frac{\cos(x)}{\operatorname{sen}(x)}\right)'$$

$$=\frac{\cos(x)}{\operatorname{sen}(x)}\frac{(\cos(x))'\operatorname{sen}(x)-\cos(x)(\operatorname{sen}(x))'}{(\operatorname{sen}(x))^2}=\frac{\cos(x)}{\operatorname{sen}(x)}\frac{-\operatorname{sen}(x)\operatorname{sen}(x)-\cos(x)\cos(x)}{\operatorname{sen}^2(x)}$$

$$=-\frac{\cos(x)}{\operatorname{sen}(x)}\frac{\operatorname{sen}^2(x)+\cos^2(x)}{\operatorname{sen}^2(x)}=-\frac{\cos(x)}{\operatorname{sen}^3(x)}$$

e) $f(x)=\ln(x\text{-}1)-3\ln(x)$

Resolução:

$$f'(x)=(\ln(x\text{-}1)-3\ln(x))'=(\ln(x\text{-}1))'-3(\ln(x))'=\frac{(x\text{-}1)'}{x\text{-}1}-3\frac{(x)'}{x}=\frac{1}{x\text{-}1}-3\frac{1}{x}=\frac{x-3(x\text{-}1)}{x(x\text{-}1)}$$

$$=\frac{x-3x+3}{x(x\text{-}1)}=-\frac{2x-3}{x(x\text{-}1)}$$

f) $g(x)=\operatorname{arcsen}\left(\dfrac{x+1}{x\text{-}1}\right)$

Resolução:

$$g'(x)=\left(\operatorname{arcsen}\left(\frac{x+1}{x\text{-}1}\right)\right)'=\frac{\left(\frac{x+1}{x\text{-}1}\right)'}{\sqrt{1-\left(\frac{x+1}{x\text{-}1}\right)^2}}=\frac{\frac{(x+1)'(x\text{-}1)-(x+1)(x\text{-}1)'}{(x\text{-}1)^2}}{\sqrt{1-\left(\frac{x+1}{x\text{-}1}\right)^2}}=\frac{\frac{x-1-(x+1)}{(x\text{-}1)^2}}{\sqrt{1-\frac{(x+1)^2}{(x\text{-}1)^2}}}=\frac{\frac{x-1-x-1}{(x\text{-}1)^2}}{\sqrt{\frac{(x\text{-}1)^2-(x+1)^2}{(x\text{-}1)^2}}}$$

$$=\frac{\frac{-2}{(x\text{-}1)^2}}{\frac{\sqrt{x^2-2x+1-(x^2+2x+1)}}{|x\text{-}1|}}=\frac{-2}{(x\text{-}1)^2}\frac{|x\text{-}1|}{\sqrt{-4x}}=\frac{|x\text{-}1|}{\left|(x\text{-}1)^2\right|}\frac{-2}{\sqrt{-4x}}=\left|\frac{x\text{-}1}{(x\text{-}1)^2}\right|\frac{-2}{2\sqrt{-x}}=-\frac{1}{|x\text{-}1|\sqrt{-x}}$$