1. Mostre que :

b) $$\int_{\sqrt{e}}^{e} \frac{\ln x}{x^2}\,dx = \frac{3\sqrt{e}-4}{2e}$$

Vamos calcular a primitiva (por partes):

$$P\left(\frac{\ln x}{x^2}\right) = P\left(\frac{1}{x^2}\ln x\right)$$

$$u' = \frac{1}{x^2} \qquad \text{então :} \qquad u = -\frac{1}{x}$$

$$v = \ln x \qquad\qquad v' = \frac{1}{x}$$

$$P\left(\frac{1}{x^2}\ln x\right) = -\frac{1}{x}\ln x - P\left(-\frac{1}{x^2}\right) = -\frac{1}{x}\ln x - \frac{1}{x} = -\frac{1}{x}(\ln x + 1)$$

O Integral vem dado por :

$$\int_{\sqrt{e}}^{e} \frac{\ln x}{x^2}\,dx = \left[-\frac{1}{x}(\ln x + 1)\right]_{\sqrt{e}}^{e} = -\frac{1}{e}(\ln e + 1) + \frac{1}{\sqrt{e}}(\ln(\sqrt{e}) + 1) = -\frac{2}{e} + \frac{1}{\sqrt{e}}\left(\frac{1}{2}\ln e + 1\right) =$$

$$= -\frac{2}{e} + \frac{1}{\sqrt{e}}\left(\frac{1}{2} + 1\right) = -\frac{2}{e} + \frac{3}{2\sqrt{e}} = \frac{-4 + 3\sqrt{e}}{2e} = \frac{3\sqrt{e}-4}{2e}$$

Nota :

$$\ln(e) = 1$$

$$\ln(\sqrt{e}) = \ln(e^{\frac{1}{2}}) = \frac{1}{2}\ln(e) = \frac{1}{2}$$