ficha 8: exerc. 3

a) $\displaystyle\int_0^{1/2} \frac{x^4}{x^2-1}\,dx = \int_0^{1/2}\left(x^2+1+\frac{-1/2}{x+1}+\frac{1/2}{x-1}\right)dx \overset{\text{Barrow}}{=} \left[\frac{x^3}{3}+x+\ln\sqrt{\left|\frac{x-1}{x+1}\right|}\right]_0^{1/2} = \frac{1}{24}+\frac{1}{2}+\ln\sqrt{\frac{1}{3}}$

f. racional imprópria

$\Downarrow$

$$\begin{array}{r|l} x^4 & x^2-1 \\ \cline{2-2} -x^4+x^2 & x^2+1 \\ \hline 0+x^2 & \\ -x^2+1 & \\ \hline 0+1 & \end{array}$$

$$P\frac{1}{x^2-1} = P\frac{-1/2}{x+1} + P\frac{1/2}{x-1} = -\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| = \ln\sqrt{\left|\frac{x-1}{x+1}\right|}$$

$A = -1/2,\ B = 1/2$

$$\frac{1}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1} \Rightarrow 0x+1 = A(x-1)+B(x+1) \Rightarrow \begin{cases} A+B=0 \\ -A+B=1 \end{cases}$$

$x^2-1 = (x+1)(x-1)$ $\quad x=1 \quad 1 = B.2$

2 fatores irredutíveis $\quad x=-1 \quad 1 = A(-2)$

de 1º grau simples (não têm multiplicidade)

$$x^4 = (x^2+1)(x^2-1)+1$$

$$\Rightarrow \frac{x^4}{x^2-1} = x^2+1+\underbrace{\frac{1}{x^2-1}}_{\text{função racional própria}}$$

b) $\int_1^2 \frac{x+1}{x^3+2x^2}dx \overset{\text{Barrow}}{=} \left[-\frac{1}{2x}+\frac{1}{4}\left(\ln|x|-\ln|x+2|\right)\right]_1^2 = \frac{1}{2}\left[-\frac{1}{x}+\ln\sqrt{\frac{x}{x+2}}\right]_1^2 =$

f. racional própria

$= \frac{1}{2}\left(-\frac{1}{2}+\ln\sqrt{1/2}-\left(-1+\ln\sqrt{\frac{1}{3}}\right)\right)$

$= \frac{1}{4}+\frac{1}{2}\ln\sqrt{3/2}$.

$x^3+2x^2 = \underline{x^2}(x+2)$

2 fatores irredutíveis de 1º grau, mas um deles tem $\underline{\text{multiplicidade 2}}$

$\frac{x+1}{x^2(x+2)} = \frac{A}{x^2}+\frac{B}{x}+\frac{C}{x+2}$

$\Rightarrow \quad x+1 = A(x+2)+Bx(x+2)+Cx^2$

$$\begin{array}{l} x^2 \\ x \\ x^0 \end{array}\left\{\begin{array}{l} B+C=0 \\ 2A+2B=1 \\ 2A=1 \end{array}\right.$$

também é possível usar ambos os métodos

$x=-2 \quad -2+1=0+0+4C$

$x=0 \quad 0+1=2A+0+0$

$B=1/4 \Leftarrow \quad C=-\frac{1}{4}$, .

$P\frac{A}{x^2} = AP\frac{1}{x^2} = A\left(-\frac{1}{x}\right)$

$P\frac{B}{x} = B\ln|x|$

...

c) $\int_0^{1/2} \frac{2}{x^3-1}\,dx = \int_0^{1/2} \left(\frac{2/3}{x-1} + \frac{-\frac{2}{3}x - \frac{4}{3}}{x^2+x+1}\right) dx = \left[\frac{2}{3}\ln|x-1|\right]_0^{1/2} - \frac{1}{3}\int_0^{1/2}\left(\frac{2x+1}{x^2+x+1} + \frac{3}{x^2+x+1}\right) dx =$

$= \frac{2}{3}\ln\frac{1}{2} - \frac{1}{3}\left(\left[\ln|x^2+x+1|\right]_0^{1/2} + 3\int_0^{1/2} \frac{1}{\frac{3}{4}\left(\left(\frac{2x+1}{\sqrt{3}}\right)^2+1\right)}\,dx\right) =$

$= -\frac{2}{3}\ln 2 - \frac{1}{3}\ln\frac{7}{4} - \frac{2}{\sqrt{3}}\int_0^{1/2} \frac{\frac{2}{\sqrt{3}}}{1+\left(\frac{2x+1}{\sqrt{3}}\right)^2}\,dx = -\frac{1}{3}\ln\left(\frac{7}{2}\right) - \frac{2}{\sqrt{3}}\left[\operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)\right]_0^{1/2}$

$= -\frac{1}{3}\ln\frac{7}{2} - \frac{2}{\sqrt{3}}\left(\operatorname{arctg}\frac{2}{\sqrt{3}} - \operatorname{arctg}\left(\frac{1}{\sqrt{3}}\right)\right)$

$P\, u' u^{\alpha} = \frac{u^{\alpha+1}}{\alpha+1}$

$P\,\frac{u'}{u} = \ln|u|$

$P\,\frac{u'}{1+u^2} = \operatorname{arctg} u$

$x = 1$

$x^3 - 1 = 0$

Ruffini

$$\begin{array}{c|cccc} & 1 & 0 & 0 & -1 \\ 1 & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & \boxed{0} \end{array}$$

$x^3 - 1 = (x-1)(x^2+x+1)$

$\left(x^2 + x + \frac{1}{4} + \frac{3}{4} = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}\right)$ ⊛

ser 2 fatores irredutíveis de 1º e 2º grau simples

$\frac{2}{x^3-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$

$2 = A(x^2+x+1) + (Bx+C)(x-1)$

$x = 1 \Rightarrow 2 = 3A$

$A = \frac{2}{3}$

$$\begin{array}{l} (x^2) \\ (x) \\ (x^0) \end{array} \begin{cases} A + B = 0 \\ \text{—} \\ A - C = 2 \end{cases}$$

$B = -\frac{2}{3}$

$C = -\frac{4}{3}$

ficha nº 9

1a) $\displaystyle\int_0^3 \frac{\sqrt{x+1}+2}{8+\sqrt{(x+1)^3}}\,dx = \int_1^2 \frac{t+2}{8+t^3}\cdot 2t\,dt \overset{*}{=}$

$\displaystyle\int_a^b f(x)\,dx = \int_\alpha^\beta f(\varphi(t))\underbrace{\varphi'(t)\,dt}$

$\varphi(\alpha)=a \quad x=\varphi(t)$

$\varphi(\beta)=b$

$\sqrt{x+1}=t$

$x=t^2-1=\varphi(t) \qquad x=0 \ \sqrt{0+1}=t$

$\varphi'(t)=2t \qquad x=3 \ \sqrt{3+1}=t$

$\displaystyle P\frac{u'}{1+u^2}=\operatorname{arctg} u$

$\displaystyle P\frac{u'}{u}=\ln|u|$

Ruffini

$$\begin{array}{r|rrrr} & 1 & 0 & 0 & 8 \\ -2 & & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & \boxed{0} \end{array}$$

$8+t^3=(t+2)(t^2-2t+4)$

$\displaystyle\frac{\cancel{(t+2)}2t}{\cancel{(t+2)}(t^2-2t+4)}=\frac{2t}{t^2-2t+4}$

irredutível 2º grau

$\displaystyle=\frac{2t-2+2}{t^2-2t+4}=\frac{2t-2}{t^2-2t+4}+\frac{2}{(t-1)^2+3}$ **

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$\displaystyle(t-1)^2+3=3\left(\frac{(t-1)^2}{3}+1\right)$

$\displaystyle=3\left(\left(\frac{t-1}{\sqrt3}\right)^2+1\right)$

*

$\displaystyle=\int_1^2\frac{2t-2}{t^2-2t+4}\,dt+\frac{2}{\sqrt3}\int_1^2\frac{1/\sqrt3}{1+\left(\frac{t-1}{\sqrt3}\right)^2}\,dt \overset{\text{Barrow}}{=} \Big[\ln|t^2-2t+4|\Big]_1^2+\frac{2}{\sqrt3}\left[\operatorname{arctg}\left(\frac{t-1}{\sqrt3}\right)\right]_1^2=$

$\displaystyle=\ln4-\ln3+\frac{2}{\sqrt3}\frac{\pi}{6}=\ln\frac{4}{3}+\frac{\pi}{3\sqrt3}$

Nota:

$$\frac{1}{(x+2)^2(x^2+1)^2} = \frac{A_1}{(x+2)^2} + \frac{A_2}{x+2} + \underbrace{\frac{B_1x+C_1}{(x^2+1)^2}} + \frac{B_2x+C_2}{x^2+1}$$

$R = \frac{P}{Q}$, grau $Q = 6$

A decomposição em frações simples tem <u>6 constantes a determinar</u> !!

Q, tem 2 fatores irredutíveis de 1º e de 2º grau e ambos com multiplicidade 2

$\frac{B_1 x}{(x^2+1)^2}$ a sua primitiva é da forma $P_u u^\alpha$

$\alpha \neq -1$

$\frac{C_1}{(x^2+1)^2}$ a determinação da primitiva desta fração simples usa o método por primitivação por partes vejam na ficha 7.

1b) $\displaystyle\int_1^2 \frac{1}{e^{2x}-1}\,dx = \int_e^{e^2} \frac{1}{t^2-1}\cdot\frac{1}{t}\,dt \overset{\text{TPC}}{=}$

$e^x = t \Rightarrow x = \ln t = \varphi(t)\,,\ \varphi'(t) = \frac{1}{t}$

$x = 1 \Rightarrow t = e$

$x = 2 \Rightarrow t = e^2$

1c) $\displaystyle\int_1^e \frac{\ln x}{x(\ln^2 x + 3\ln x + 2)}\,dx = \int_0^1 \frac{t}{\cancel{e^t}(t^2+3t+2)}\cdot\cancel{e^t}\,dt \overset{\text{TPC}}{=}$

$\ln x = t \Rightarrow x = e^t = \varphi(t)\,,\ \varphi'(t) = e^t$

$x = 1 \Rightarrow t = 0$

$x = e \Rightarrow t = 1$

$(t^2+3t+2) = (t+1)(t+2)$