Universidade Lusófona de Humanidades e Tecnologias

Faculdade de Engenharia e Ciências Naturais

Cálculo II

Licenciaturas em
Biologia, Ciências do Mar, Engenharia do Ambiente, Engenharia Biotecnológica,
Engenharia Civil, Engenharia Electrotécnica, Engenharia e Gestão Industrial e Química
2º Semestre 2008/2009

Ficha 1 – Revisões do cálculo de derivadas em $\mathbb{R}$

Parte I – Exercícios Propostos

I.1 Calcule as derivadas das seguintes funções:

a) x^5-e **b)** x^4-x^{-4} **c)** $x\sqrt{x}$ **d)** $\dfrac{x^2}{\sqrt{x}}$

I.2 Calcule as derivadas das seguintes funções:

a) $e^{x-1}-1$ **b)** $e^{2x}+1$ **c)** 4^x-2^x **d)** xe^{-x^2}

e) $\ln\sqrt{x}$ **f)** $\sqrt{\ln x}$ **g)** $\ln\dfrac{x-1}{x^3}$ **h)** $\sqrt{e^x}$

i) $\sqrt{\dfrac{x-1}{x}}$ **j)** $x\ln(1+x)$ **k)** $\ln(1+x)+\dfrac{x}{1+x}$ **l)** $\ln x+\ln\left(\operatorname{senh}(x)\right)$

I.3 Calcule as derivadas das seguintes funções:

a) $f(x)=\operatorname{sen}\sqrt{\ln(x)}$ **b)** $f(x)=\operatorname{arctg}\left(2e^{x+1}\right)$

c) $f(x)=\cos\left(\operatorname{sen}(3x)\right)$ **d)** $f(x)=\arccos\left(\ln(3x)\right)$

Parte II – Exercícios Resolvidos

II.1 Calcule a derivada de cada uma das seguintes funções:

1) $f(x)=(3x-5)+x$

Resolução

$$f'(x)=(f(x))'=((3x-5)+x)'=(3x-5)'+(x)'=(3x)'-(5)'+1=3(x)'-0+1=3\times 1+1=4$$

2) $f(x)=(x-1)(x-3)$

Resolução

$$\begin{aligned}f'(x)=(f(x))'=((x-1)(x-3))'&=(x-1)'(x-3)+(x-1)(x-3)'\\&=\left((x)'-(1)'\right)(x-3)+(x-1)\left((x)'-(3)'\right)\\&=(1-0)(x-3)+(x-1)(1-0)\\&=1\cdot(x-3)+(x-1)\cdot 1=x-3+x-1=2x-4\end{aligned}$$

3) $f(x)=\left(\dfrac{3x^2+4}{x^2+9}\right)$

Resolução

$$f'(x)=(f(x))'=\left(\frac{3x^2+4}{x^2+9}\right)'=\frac{(3x^2+4)'(x^2+9)-(3x^2+4)(x^2+9)'}{(x^2+9)^2}=\frac{(6x)(x^2+9)-(3x^2+4)(2x)}{(x^2+9)^2}$$

$$=\frac{6x^3+54x-(6x^3+8x)}{(x^2+9)^2}=\frac{6x^3+54x-6x^3-8x}{(x^2+9)^2}=\frac{46x}{(x^2+9)^2}$$

4) $f(x)=(x+3)^5$

Resolução

$$f'(x)=(f(x))'=\left((x+3)^5\right)'=5(x+3)^4(x+3)'=5(x+3)^4\cdot 1=5(x+3)^4$$

5) $f(x)=\left(\dfrac{x-1}{x+2}\right)^2$

Resolução

$$f'(x)=(f(x))'=\left(\left(\frac{x-1}{x+2}\right)^2\right)'=2\left(\frac{x-1}{x+2}\right)\left(\frac{x-1}{x+2}\right)'=2\left(\frac{x-1}{x+2}\right)\frac{(x-1)'(x+2)-(x-1)(x+2)'}{(x+2)^2}$$

$$=2\left(\frac{x-1}{x+2}\right)\frac{1\cdot(x+2)-(x-1)\cdot 1}{(x+2)^2}=2\left(\frac{x-1}{x+2}\right)\frac{x+2-x+1}{(x+2)^2}$$

$$=2\left(\frac{x-1}{x+2}\right)\frac{3}{(x+2)^2}=\frac{6(x-1)}{(x+2)^3}$$

6) $f(x)=\sqrt{x-3}$

Resolução

$$f'(x)=(f(x))'=\left(\sqrt{x-3}\right)'=\frac{(x-3)'}{2\sqrt{(x-3)^{2-1}}}=\frac{1}{2\sqrt{x-3}}$$

7) $f(x)=\sqrt[3]{\dfrac{3-x}{x-1}}$

Resolução

$$f'(x)=(f(x))'=\left(\sqrt[3]{\frac{3-x}{x-1}}\right)'=\frac{\left(\frac{3-x}{x-1}\right)'}{3\sqrt[3]{\left(\frac{3-x}{x-1}\right)^{3-1}}}=\frac{1}{3\sqrt[3]{\left(\frac{3-x}{x-1}\right)^{2}}}\left(\frac{3-x}{x-1}\right)'$$

$$=\frac{1}{3\sqrt[3]{\left(\frac{3-x}{x-1}\right)^{2}}}\frac{(3-x)'(x-1)-(3-x)(x-1)'}{(x-1)^2}=\frac{1}{3\sqrt[3]{\left(\frac{3-x}{x-1}\right)^{2}}}\frac{(-1)(x-1)-(3-x)\cdot 1}{(x-1)^2}$$

$$=\frac{1}{3\sqrt[3]{\left(\frac{3-x}{x-1}\right)^{2}}}\frac{-x+1-3+x}{(x-1)^2}=\frac{1}{3\sqrt[3]{\left(\frac{3-x}{x-1}\right)^{2}}}\frac{-2}{(x-1)^2}=-\frac{2}{3}\frac{1}{\sqrt[3]{\left(\frac{3-x}{x-1}\right)^{2}\left((x-1)^2\right)^3}}$$

$$=-\frac{2}{3}\frac{1}{\sqrt[3]{\frac{(3-x)^2}{(x-1)^2}(x-1)^6}}=-\frac{2}{3}\frac{1}{\sqrt[3]{(3-x)^2\frac{(x-1)^6}{(x-1)^2}}}=-\frac{2}{3}\frac{1}{\sqrt[3]{(3-x)^2(x-1)^{6-2}}}$$

$$=-\frac{2}{3}\frac{1}{\sqrt[3]{(3-x)^2(x-1)^4}}=-\frac{2}{3}\frac{1}{\sqrt[3]{(3-x)^2(x-1)^3(x-1)}}=-\frac{2}{3}\frac{1}{(x-1)\sqrt[3]{(3-x)^2(x-1)}}$$

8) $f(x)=\operatorname{sen}(2x+1)$

Resolução

$$f'(x)=(f(x))'=(\operatorname{sen}(2x+1))'=(2x+1)'\cos(2x+1)=2\cos(2x+1)$$

9) $f(x)=\operatorname{sen}^5(5x)$

Resolução

$$f'(x)=(f(x))'=\left(\operatorname{sen}^5(5x)\right)'=5(\operatorname{sen}(5x))^{5-1}(\operatorname{sen}(5x))'=5(\operatorname{sen}(5x))^4(\operatorname{sen}(5x))'$$

$$=5(\operatorname{sen}(5x))^4(5x)'\cos(5x)=5(\operatorname{sen}(5x))^4\,5\cos(5x)=25\operatorname{sen}^4(5x)\cos(5x)$$

10) $f(x)=x\operatorname{sen}(x^2)+3\operatorname{sen}(2x)$

Resolução

$$f'(x)=(f(x))'=\left(x\operatorname{sen}(x^2)+3\operatorname{sen}(2x)\right)'=(x)'\operatorname{sen}(x^2)+x\left(\operatorname{sen}(x^2)\right)'+3(2x)'\cos(2x)$$

$$=\operatorname{sen}(x^2)+x(x^2)'\cos(x^2)+6\cos(2x)=\operatorname{sen}(x^2)+2x^2\cos(x^2)+6\cos(2x)$$

11) $f(x)=2\cos^3(1-x)$

Resolução

$$f'(x)=(f(x))'=\left(2\cos^3(1-x)\right)'=2\times 3(\cos(1-x))^{3-1}(\cos(1-x))'$$

$$=2\cdot 3(\cos(1-x))^2(-1)(1-x)'\operatorname{sen}(1-x)=6(\cos(1-x))^2(-1)(-1)\operatorname{sen}(1-x)$$

$$=6\cos^2(1-x)\operatorname{sen}(1-x)$$

12) $f(x)=tg^2(x^2+1)$

Resolução

$$f'(x)=\left(tg^2(x^2+1)\right)'=2tg(x^2+1)\left(tg(x^2+1)\right)'=2tg(x^2+1)\frac{(x^2+1)'}{\cos^2(x^2+1)}$$

$$=2tg(x^2+1)\frac{2x}{\cos^2(x^2+1)}\underset{\frac{1}{\cos^2\alpha}=1+tg^2\alpha}{\underset{\uparrow}{=}}4x\,tg(x^2+1)\left[1+tg^2(x^2+1)\right]$$

13) $f(x)=\cos^2 x+tg(x\,sen^2x)$

Resolução

$$f'(x)=\left(\cos^2 x+tg(x\,sen^2x)\right)'=2\cos x(\cos x)'+\frac{(x\,sen^2x)'}{\cos^2(x\,sen^2x)}=2\cos x(-senx)+\frac{(x)'\,sen^2x+x(sen^2x)'}{\cos^2(x\,sen^2x)}$$

$$=-2\cos x\,senx+\frac{1\cdot sen^2x+x\cdot 2\,sen\,x\,(sen\,x)'}{\cos^2(x\,sen^2x)}\underset{sen(2a)=2\,sen\,a\cdot\cos a}{\underset{\uparrow}{=}}-sen(2x)+\frac{sen^2x+2x\,sen\,x\cos x}{\cos^2(x\,sen^2x)}$$

$$\underset{sen(2a)=2\,sen\,a\cdot\cos a}{\underset{\uparrow}{=}}-sen(2x)+\frac{sen^2x+x\,sen(2x)}{\cos^2(x\,sen^2x)}$$

14) $f(x)=cotg(3x^3+2x)$

Resolução

$$f'(x)=\left(cotg(3x^3+2x)\right)'=-\frac{(3x^3+2x)'}{sen^2(3x^3+2x)}=-\frac{9x^2+2}{sen^2(3x^3+2x)}$$

15) $f(x)=arc\,sen(x^2)$

Resolução

$$f'(x)=\left(arc\,sen(x^2)\right)'=\frac{(x^2)'}{\sqrt{1-(x^2)^2}}=\frac{2x}{\sqrt{1-x^4}}$$

16) $f(x)=2+arc\,sen^2(\cos^2 x)$

Resolução

$$f'(x)=\left(2+arc\,sen^2(\cos^2 x)\right)'=2arc\,sen(\cos^2 x)\left(arc\,sen(\cos^2 x)\right)'=2arc\,sen(\cos^2 x)\frac{(\cos^2 x)'}{\sqrt{1-(\cos^2 x)^2}}$$

$$=2arc\,sen(\cos^2 x)\frac{2\cos x(\cos x)'}{\sqrt{1-\cos^4 x}}=2arc\,sen(\cos^2 x)\frac{2\cos x(-sen\,x)}{\sqrt{1-\cos^4 x}}=-2arc\,sen(\cos^2 x)\frac{2\cos x\,sen\,x}{\sqrt{1-\cos^4 x}}$$

$$\underset{sen(2a)=2sen\,a\cdot\cos a}{=}-\frac{2sen(2x)\,arc\,sen(\cos^2 x)}{\sqrt{1-\cos^4 x}}$$

17) $f(x)=\dfrac{arc\cos x}{x}$

Resolução

$$f'(x)=\left(\frac{arc\cos x}{x}\right)'=\frac{(arc\cos x)'\,x-(arc\cos x)(x)'}{x^2}=\frac{-\frac{1}{\sqrt{1-x^2}}\cdot x-arc\cos x}{x^2}=\frac{-\frac{x}{\sqrt{1-x^2}}-arc\cos x}{x^2}$$

$$=\frac{-x-\sqrt{1-x^2}\,arc\cos x}{x^2\sqrt{1-x^2}}$$

18) $f(x)=\operatorname{sen} x+\operatorname{arc\,cotg}(x^2)$

Resolução

$$f'(x)=\left(\operatorname{sen} x+\operatorname{arc\,cotg}(x^2)\right)'=\cos x-\frac{(x^2)'}{1+(x^2)^2}=\cos x-\frac{2x}{1+x^4}$$

19) $f(x)=e^{-\frac{x}{2}}$

Resolução

$$f'(x)=\left(e^{-\frac{x}{2}}\right)'=\left(-\frac{x}{2}\right)'e^{-\frac{x}{2}}=-\frac{1}{2}e^{-\frac{x}{2}}$$

20) $f(x)=(x-1)^2.e^{-x}$

Resolução

$$f'(x)=\left((x-1)^2\cdot e^{-x}\right)'=\left((x-1)^2\right)'e^{-x}+(x-1)^2\left(e^{-x}\right)'=2(x-1)e^{-x}+(x-1)^2(-x)'\left(e^{-x}\right)$$

$$=2(x-1)e^{-x}+(x-1)^2(-1)\left(e^{-x}\right)=2(x-1)e^{-x}-(x-1)^2e^{-x}=\left(2(x-1)-(x-1)^2\right)e^{-x}$$

$$=\left(2x-2-(x^2-2x+1)\right)e^{-x}=\left(2x-2-x^2+2x-1\right)e^{-x}=\left(-x^2+4x-3\right)e^{-x}$$

21) $f(x)=e^{\operatorname{arc\,sen} x}$

Resolução

$$f'(x)=\left(e^{\operatorname{arc\,sen} x}\right)'=(\operatorname{arc\,sen} x)'e^{\operatorname{arc\,sen} x}=\frac{1}{\sqrt{1-x^2}}e^{\operatorname{arc\,sen} x}$$

22) $f(x)=2^{x^2+3x}$

Resolução

$$f'(x)=\left(2^{x^2+3x}\right)'=(x^2+3x)'\,2^{x^2+3x}\ln 2=(2x+3)\,2^{x^2+3x}\ln 2$$

23) $f(x)=\dfrac{1-3^x}{\cos x}$

Resolução

$$f'(x)=\left(\frac{1-3^x}{\cos x}\right)'=\frac{(1-3^x)'\cdot\cos x-(1-3^x)(\cos x)'}{(\cos x)^2}=\frac{(-3^x\ln 3)\cdot\cos x-(1-3^x)(-\operatorname{sen} x)}{(\cos x)^2}$$

$$=\frac{-3^x\cos x\ln 3+(1-3^x)\cdot\operatorname{sen} x}{(\cos x)^2}=\frac{-3^x\cos x\ln 3+\operatorname{sen} x-3^x\operatorname{sen} x}{(\cos x)^2}$$

24) $f(x)=\ln\left(e^{3x}+x^2\right)$

Resolução

$$f'(x)=\left(\ln\left(e^{3x}+x^2\right)\right)'=\frac{\left(e^{3x}+x^2\right)'}{e^{3x}+x^2}=\frac{3e^{3x}+2x}{e^{3x}+x^2}$$

25) $f(x)=\log_3\left(x^2+1\right)$

Resolução

$$f'(x)=\left(\log_3\left(x^2+1\right)\right)'=\frac{\left(x^2+1\right)'}{\left(x^2+1\right)\ln 3}=\frac{2x}{\left(x^2+1\right)\ln 3}$$

26) $f(x)=\log_7\left(\operatorname{sen}\left(x^2\right)\right)$

Resolução

$$f'(x)=\left(\log_7\left(\operatorname{sen}\left(x^2\right)\right)\right)'=\frac{\left(\operatorname{sen}\left(x^2\right)\right)'}{\operatorname{sen}\left(x^2\right)\ln 7}=\frac{2x\cos\left(x^2\right)}{\operatorname{sen}\left(x^2\right)\ln 7}$$

Parte III – Exercícios de Auto-Avaliação

III.1 Calcule as derivadas das funções seguintes:

a) $\left(5x-\frac{1}{3}\right)^4$ **b)** $\frac{x^2-16}{x+5}$ **c)** $x+1+\frac{1}{x-1}$

d) $3-\frac{1}{(x-1)^2}$ **e)** $(1-2x^2)e^{-x^2}$ **f)** $\frac{\ln(x)}{x}$

III.2 Calcule as derivadas das funções seguintes:

a) $f(x)=\operatorname{sen}(2x)\cos(3x)$ **b)** $g(x)=\frac{1}{2\,\operatorname{tg}^2 x}$

III.3 Determine as derivadas das seguintes funções:

a) $a(x)=\ln\left(\operatorname{arc\,sen}\left(\sqrt{x}\right)\right)$ **b)** $b(x)=\frac{1}{e^{\cos x}}$ **c)** $c(x)=\ln\left(\operatorname{arc\,tg}\left(\frac{1}{x}\right)\right)$

d) $e(x)=\frac{\cos^2(x)}{2\operatorname{sen}^2(x)}$ **e)** $f(x)=\ln(x-1)-3\ln(x)$ **f)** $g(x)=\operatorname{arcsen}\left(\frac{x+1}{x-1}\right)$