

3ª Ficha de exercícios para as aulas práticas: 9 - 13 Outubro de 2006

1. Indique quais são majoradas, minoradas, limitadas, de entre as sucessões definidas por:

$$\begin{aligned} (1) \ x_n &= \frac{n + (-1)^n}{n} & (2) \ x_n &= (-1)^n n^3 & (3) \ x_n &= 1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^n} \\ (4) \ x_n &= [1 + (-1)^n] n & (5) \ x_n &= n^{(-1)^n} & (6) \ x_n &= 1, \quad x_{n+1} = \frac{2x_n + 3}{4} \end{aligned}$$

2. Determine, se existirem em \mathbb{R} , os limites das sucessões que têm por termo de ordem n :

$$\begin{aligned} (1) \ \frac{2n+3}{3n-1} \quad (2) \ \frac{n^2-1}{n^4+3} \quad (3) \ \frac{(n+1000)^5}{n^6+1} \quad (4) \ \frac{2^n+1}{2^{n+1}-1} \quad (5) \ \frac{2^{2n}-3^n}{2^n-3^{2n}} \quad (6) \ \frac{(3^n)^2}{1+2^{3n}} \\ (7) \ \frac{2^n+4^n}{3^{n+1}} \quad (8) \ \frac{\sqrt{n+1}}{2n+1} \quad (9) \ n - \frac{n^2}{n+2} \quad (10) \ \frac{n+\cos n}{2n-1} \quad (11) \ \frac{n^2-2}{5n^2} \quad (12) \ \frac{n-1}{\sqrt[3]{n^2+1}} \\ (13) \ \sqrt{n} - \frac{n}{\sqrt{n}+2} \quad (14) \ \frac{\sqrt{n}+\sqrt[3]{n}}{\sqrt{2n}+1} \quad (15) \ \frac{\sqrt{2n^4-1}}{2-3n^2} \quad (16) \ \frac{(-1)^n n}{1+n^2} \quad (17) \ \frac{\sqrt[3]{n^2}-2n^2+3}{3n^2-\sqrt{n^3}+(-1)^n} \\ (18) \ \frac{\sqrt{n^3-n}+2n^2-1}{\sqrt{3n^4+2n^3+1}} \quad (19) \ \frac{\sqrt{2^n}}{\sqrt[3]{3^n}} \quad (20) \ \frac{n}{n+1} - \frac{n+1}{n} \quad (21) \ \frac{n^2}{n+1} - \frac{n^2+1}{n} \\ (22) \ \frac{n-\sqrt{n^3}}{\sqrt{2n^2+3n^3}} \quad (23) \ \sqrt{n+1}-\sqrt{n} \quad (24) \ \sqrt{n(n+1)}-\sqrt{n(n-1)} \quad (25) \ n(\sqrt{n^2+1}-n) \\ (26) \ (\sqrt{n+1}-\sqrt{n})\sqrt{n+3} \quad (27) \ \frac{\sqrt{n^2+1}-\sqrt{n}}{n+1} \quad (28) \ \frac{(n-1)(n-2)}{n(n+1)(n+2)} \\ (29) \ \frac{(n-1)(n-2)\dots(n-p)}{n(n+1)(n+2)\dots(n+q)}, \ (p, q \in \mathbb{N}) \quad (30) \ \frac{a^{n+1}+b^n}{a^n+b^{n+1}}, \ (a, b \in \mathbb{R}^+) \\ (31) \ \frac{a^n b^n}{a^n+b^n}, \ (a, b \in \mathbb{R}^+) \quad (32) \ \frac{2n}{3n^2+1} + \cdots + \frac{2n}{3n^2+n} \quad (33) \ \cos(n!\pi) \\ (34) \ \frac{1+a^n}{1+a^{2n}} \ (a \in \mathbb{R}) \quad (35) \ \frac{a^n}{2^{2n+1}} \ (a \in \mathbb{R}) \quad (36) \ \frac{2^n+2}{2^{n+1}-2} \quad (37) \ \frac{2^{n+3}+4^{n+2}}{3^{n+1}+5^n} \end{aligned}$$

3. Determine, se existirem em $\overline{\mathbb{R}}$, os limites das sucessões que têm por termo de ordem n :

$$\begin{aligned} (1) \ \frac{n^3+1}{n^2+2n-1} \quad (2) \ \frac{(-1)^n n^3+1}{n^2+2} \quad (3) \ \frac{n!}{n^{1000}} \quad (4) \ \frac{2^{2n}+6n}{3^n+4^{n+2}} \quad (5) \ \frac{n!}{5^n+(n+1)^2} \\ (6) \ \frac{n^{100}}{3^n+n!} \quad (7) \ \frac{2^n+n^{10}}{\log^2 n-2^n} \quad (8) \ \frac{2^n}{n^2} \quad (9) \ \frac{n^{2n}}{3^n} \quad (10) \ \sin \frac{-1-n}{e^n+n} \cos \frac{e^n+1+n}{n} \\ (11) \ \frac{a^n}{n} \ (a \in \mathbb{R}) \quad (12) \ n^4 e^{-n^2} \quad (13) \ \frac{n^2+e^n}{3^n+n^5} \quad (14) \ \frac{3^n+n^5+4^n}{n!+2^n+7\log^2 n} \quad (15) \ \frac{3\cos n+5\sin n}{2^n} n^2 \\ (16) \ \frac{2^n+(n+1)!}{n!+3^n} \quad (17) \ \frac{5^n-n!}{3^n+2n!} \quad (18) \ \frac{(n+1)^n-n!}{7^n-n^n} \quad (19) \ \sin\left(2n+\frac{1}{n}\right) - \sin\left(2n+\frac{1}{n^2}\right) \end{aligned}$$

$$\begin{aligned}
(20) \frac{n! + 30^n + n^{1000}}{3n! + \log^{10} n^2 + n^n} \quad (21) \frac{(-1)^{n+1} \cos(n\pi) + \sqrt{n}}{n^2 + 1} \quad (22) \frac{n! + \log n}{\left(\frac{1}{2}\right)^n + n^n} \quad (23) \frac{\log(e^n + n^2)}{2n} \\
(24) \frac{n! + n^3 \cos(n^5 + n^n)}{(n!)^2 + n^{2n}} \quad (25) \frac{\sqrt[3]{n} + \log n(\cos n^3)}{\sqrt{n} + 1} \quad (26) \log(n+1)! - \log[(n+2)! - n!] \\
(27) \frac{((-1)^n + 3)^n}{(2n)!} \quad (28) \frac{n^2 \log^3 n + \sqrt[4]{n^9}}{\sqrt{n^5} + e^{-n}} \quad (29) \frac{1}{\sqrt[n]{n^2}} \quad (30) \sqrt[n]{1 + \frac{1}{n}} \quad (31) \sqrt[n]{\frac{10}{n} - \frac{1}{(1.2)^n}} \\
(32) \frac{\sqrt[n]{3^n n! + 1}}{n} \quad (33) \sqrt[n]{\frac{n! + 1}{(2n)!} \log n^5} \quad (34) \sqrt[n]{\frac{n^2 + n - 1}{n + 3}} \quad (35) \sqrt[n]{(n+1)! - n!} \\
(36) \sqrt[n]{\frac{3^n + n^2}{n + 1}} \quad (37) \sqrt[n]{n! + 2^{2n} + 3^n} \quad (38) \left(\frac{n-1}{2n^2 + 1}\right)^{\frac{2}{n}} \quad (39) \left(\frac{3^n + 2}{n!}\right)^{\frac{2}{n}} \\
(40) \left(1 - \frac{n}{n+1}\right)^{\frac{1}{n}} \quad (41) \left(\frac{2^n}{n+1}\right)^{\frac{1}{2n}} \quad (42) (\sqrt{n+2} - \sqrt{n})^{\frac{1}{n}} \quad (43) \sqrt[n]{n!} \quad (44) n^{\frac{1}{n}} \\
(45) \left(\frac{1}{n}\right)^{\frac{1}{n}} \quad (46) \left(\frac{1}{n}\right)^n \quad (47) \left(1 + \frac{1}{2n}\right)^n \quad (48) \left(1 + \frac{1}{n}\right)^{n+5} \quad (49) \left(1 - \frac{2}{n}\right)^{3n} \\
(50) \left(2 - \frac{1}{n}\right)^n \quad (51) \left(1 + \frac{2}{n^2}\right)^{n^3} \quad (52) \left(1 + \frac{3}{n^3}\right)^{n^2} \quad (53) \left(1 - \frac{1}{n!}\right)^{n!} \\
(54) \left(\frac{n! + 3n}{n!}\right)^{(n-1)!} \quad (55) e^{(1-\frac{1}{n})^n} \quad (56) n \left(\frac{n!}{(2n)!}\right)^{\frac{1}{n}} \quad (57) \left(1 - \frac{1}{n + \log n}\right)^n \\
(58) \left(\frac{n-1}{n}\right)^{n!} \quad (59) \left(\frac{2^n + 2}{2^n}\right)^{n^2} \quad (60) \left(\frac{3n-2}{2n+5}\right)^{n-1} \quad (61) \left(\frac{n-2}{n+2}\right)^{2n+3} \\
(62) \left(\frac{n-1}{n+2}\right)^{1-n} \quad (63) \left(\frac{n^n - n!}{n^n}\right)^{3^n} \quad (64) \left(\frac{3n+2}{3n-1}\right)^{n/2} \quad (65) \left(\frac{n-1}{n+3}\right)^{n^2} \\
(66) \left(\frac{2n}{2n+1}\right)^{2n-1} \quad (67) \left(\frac{2n}{n+1} - 1\right)^n \quad (68) \left(\frac{n^2-1}{n^2+1}\right)^{n^2+6} \\
(69) (1 + \sqrt{n+2} - \sqrt{n})^{\sqrt{n+1}} \quad (70) n \frac{2^{n+2} + (n+1)!}{n^{n+2} + (n+2)^n} \quad (71) \frac{2^n n! + n^n}{n^n + n^2 \log n} \\
(72) \frac{3^n n! + n^n}{4n^n + 1000^n} \quad (73) 2^{n2^n} (1 + 2^n)^{-2^n} \quad (74) \sqrt{25n^2 + 6n + 7} - 5n \\
(75) \left(\frac{n^2-2}{n^2}\right)^{2^n} \quad (76) \sqrt{\frac{2n^{1/2} + 2}{3n^{1/2} + 3}}
\end{aligned}$$