$$\boxed{\int_0^{+\infty} \frac{\pi}{2} - arctg(x)dx}$$

Integral impróprio de 1ª espécie

$$\int_0^{+\infty} \frac{\pi}{2} - arctg(x)dx = \lim_{b\to+\infty} \int_0^{b} \frac{\pi}{2} - arctg(x)dx = *$$

Vamos calcular a primitiva de arctg(x) por partes :

$$u' = 1 \quad u = x$$

$$v = arctg(x) \quad v' = \frac{1}{1+x^2}$$

$$P(arctg(x)) = x \quad arctg(x) - P(\frac{x}{1+x^2}) = xarctg(x) - \frac{1}{2}P(\frac{2x}{1+x^2}) =$$

$$= xarctg(x) - \frac{1}{2}\ln(1+x^2)$$

$$* = \lim_{b\to+\infty}\left[\frac{\pi}{2}x - xarctg(x) + \frac{1}{2}\ln(1+x^2)\right]_0^b = \lim_{b\to+\infty}(\frac{\pi}{2}b - barctg(b) + \frac{1}{2}\ln(1+b^2) =$$

$$= \lim_{b\to+\infty}(\frac{\pi}{2}b - barctg(b)) + \lim_{b\to+\infty}\frac{1}{2}\ln(1+b^2) = \lim_{b\to+\infty} b(\frac{\pi}{2} - arctg(b)) + \lim_{b\to+\infty}\frac{1}{2}\ln(1+b^2) =$$

$$= \lim_{b\to+\infty}\frac{\frac{\pi}{2} - arctg(b)}{\frac{1}{b}} + \lim_{b\to+\infty}\frac{1}{2}\ln(1+b^2) \underset{\substack{R.Cauchy \\ parao1^{o}\lim ite}}{=} \lim_{b\to+\infty}\frac{-\frac{1}{1+b^2}}{-\frac{1}{b^2}} + \lim_{b\to+\infty}\frac{1}{2}\ln(1+b^2) =$$

$$= \lim_{b\to+\infty}\frac{b^2}{1+b^2} + \lim_{b\to+\infty}\frac{1}{2}\ln(1+b^2) \underset{\substack{R.Cauchy \\ parao1^{o}\lim ite}}{=} \lim_{b\to+\infty}\frac{2b}{2b} + \lim_{b\to+\infty}\frac{1}{2}\ln(1+b^2) = 1 + \frac{1}{2}\ln(+\infty) =$$

$$= 1 + (+\infty) = +\infty$$

O integral é divergente.