XGBoost: Mathematical Foundations
Genetic Algorithm
Code Implementation

# Cómputo evolutivo para la optimización de hiperparámetros en algoritmos de Aprendizaje Automático

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## **Definitions**

#### **Evolutionary Computation**

Bio-inspired optimization paradigm using:

- Natural selection
- Genetic recombination
- Random mutation

to evolve solutions iteratively.



#### **Machine Learning**

Algorithms that learn patterns from data through:

- Mathematical optimization
- Statistical generalization
- Parameter adjustment



# Hyperparameter Optimization

### What are Hyperparameters?

Parameters that are **not learned** during training but are **set before** the learning process begins. They control the model's architecture and learning behavior.

#### Some algorithms:

- XGBoost:
  - max\_depth (tree complexity)
  - learning\_rate (step size)
  - n\_estimators (number of trees)
- Neural Networks:
  - learning rate

#### SVM:

- C (regularization)
- kernel type
- gamma (RBF influence)
- Random Forest:
  - n\_estimators
  - max\_features



# Hyperparameter Optimization

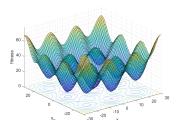
#### **Baking the Perfect Cake**

- Like perfecting a recipe:
  - Oven temperature
  - Baking time
  - Ingredients ratio
- Poor ratios ⇒ Bad results



#### **Optimization Techniques**

- Grid Search: Exhaustive combinatorial
- Random Search: Stochastic sampling
- Bayesian Opt.: Gaussian processes
- Genetic Algorithms: Evolutionary approach (as implemented)





# XGBoost: Overview

- Ensemble learning algorithm based on decision trees
- Gradient Boosting: builds models sequentially by correcting errors
- Regularized optimization to prevent overfitting
- Highly efficient and parallelizable

# Objective Function in XGBoost

The general objective function:

$$\mathcal{L}(\phi) = \sum_{i=1}^{n} I(y_i, \hat{y}_i) + \sum_{k=1}^{K} \Omega(f_k)$$

where:

- I: loss function (log-loss for classification)
- Ω: regularization term
- $\bullet$   $f_k$ : k-th tree

For binary classification:

$$I(y_i, \hat{y}_i) = -[y_i \log(p_i) + (1 - y_i) \log(1 - p_i)]$$

with 
$$p_i = \frac{1}{1+e^{-\hat{y}_i}}$$



# Tree Construction

XGBoost uses second-order Taylor expansion:

$$\mathcal{L}^{(t)} \approx \sum_{i=1}^{n} [g_i f_t(x_i) + \frac{1}{2} h_i f_t^2(x_i)] + \Omega(f_t)$$

where:

$$g_i = \partial_{\hat{\mathbf{v}}^{(t-1)}} I(y_i, \hat{\mathbf{y}}^{(t-1)})$$

$$h_i = \partial_{\hat{v}^{(t-1)}}^2 I(y_i, \hat{y}^{(t-1)})$$

The regularization term:

$$\Omega(f) = \gamma T + \frac{1}{2} \lambda ||w||^2$$

# **Optimized Parameters**

Parameters optimized by the GA and their mathematical meaning:

| Parameter                                                  | Mathematical Meaning                                                                                                                           |
|------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------|
| max_depth<br>learning_rate<br>n_estimators                 | Maximum depth $d$ of each tree $\eta$ : weight of each new tree $K$ : total number of trees                                                    |
| gamma<br>min_child_weight<br>subsample<br>colsample_bytree | $\gamma$ : minimum loss reduction $w_{min}$ : minimum child node weight Fraction of data to train each tree Fraction of features for each tree |

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# Exercise

#### Manual hyperparameter adjustment

Adjust XGBoost hyperparameter manually using the code given in the next address:

https://github.com/JARF1095/Delfines-2025-U-de-G

# **Basic Concepts**

- Biological evolution inspiration (natural selection)
- Population algorithm (several vectors)
- Each individual contains a fitness that measures the solution quality
- Genetic operators:
  - **Selection**: Survival of the fittest
  - Crossover: Combine parent information
  - Mutation: Introduce variations within the individual

# Mathematical Representation for Hyperparameter Optimization

#### In our code:

■ Each individual is a 7-parameter vector:

$$\mathbf{x} = [d, \eta, n, \gamma, w, s_s, s_c]$$

#### where:

- d: max\_depth (maximum tree depth)
- η: learning\_rate (learning rate)
- n: n\_estimators (number of trees)
- γ: min\_split\_loss (minimum loss reduction for split)
- w: min\_child\_weight (minimum child node weight)
- $\bullet$   $s_s$ : subsample (fraction of samples used)
- s<sub>c</sub>: colsample\_bytree (fraction of features used)



# Fitness Function

The evaluation function in the code:

$$f(\mathbf{x}) = \operatorname{accuracy}(M(\mathbf{x}), X_{val}, y_{val})$$

where:

- M(x) is the XGBoost model with parameters x
- $X_{val}, y_{val}$  are validation data

• accuracy = 
$$\frac{TP+TN}{TP+TN+FP+FN}$$

Implemented constraints:

- $\gamma \geq 0$
- $\eta \ge 0.001$
- $s_s, s_c \in [0,5,1,0]$



# Genetic Operators Used

- Selection: Tournament (selTournament)
- Crossover: Simulated Binary Crossover (cxSimulatedBinaryBounded)

$$c_1 = \frac{1}{2}[(1+\beta)p_1 + (1-\beta)p_2]$$

$$c_2 = \frac{1}{2}[(1-\beta)p_1 + (1+\beta)p_2]$$

Mutation: Polynomial Bounded Mutation (mutPolynomialBounded)

$$x' = x + \delta(\bar{x} - x)$$



# **Optimization Strategy**

- Random initial population (20 individuals)
- 15 generations
- Crossover probability: 70 %Mutation probability: 30 %
- Elitism (HallOfFame) preserves the best individual

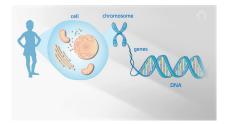


Figura: Fitness evolution across generations

# Program Flow

- Data preprocessing (normalization, encoding)
- 2 Train/val/test split (60 %/20 %/20 %)
- Genetic algorithm configuration:
  - Parameter search space
  - Genetic operators
- 4 Hyperparameter optimization
- Final model training
- 6 Evaluation and metrics

# **Evaluation Metrics**

The code calculates multiple metrics:

■ AUC-ROC: Area Under the ROC Curve

$$AUC = \int_0^1 TPR(FPR^{-1}(x))dx$$

■ Gini: Related to AUC

$$Gini = 2 \times AUC - 1$$

Accuracy: Overall correctness

■ Precision: <sup>TP</sup>/<sub>TP+FP</sub>

**Recall/Sensitivity**:  $\frac{TP}{TP+FN}$ 

**Specificity**:  $\frac{TN}{TN+FP}$ 

■ **F1-score**: Harmonic mean of precision\_and\_recall\_



# Exercise

#### Genetic algorithm configuration

Play with the next GA parameters:

- population size
- max generations
- cross probability
- mutation probability

using the code on the next address:

https://github.com/JARF1095/Delfines-2025-U-de-G

# Key Takeaways

- The genetic algorithm provides systematic hyperparameter search
- XGBoost is a powerful model combining multiple trees with regularization
- The combination leverages both strengths:
  - GA for parameter space exploration
  - XGBoost for precise modeling with overfitting protection
- Multiple metrics provide comprehensive performance insight

# Recommendations

- For further improvement:
  - Increase GA population and generations (with computational cost)
  - Consider alternative fitness functions (e.g., AUC instead of accuracy)
  - Add more parameters to optimize (e.g., reg\_lambda, reg\_alpha)
- Validate with additional medical datasets
- Clinically interpret the most important variables

# References

- Chen, T., & Guestrin, C. (2016). XGBoost: A Scalable Tree Boosting System.
- Goldberg, D. E. (1989). Genetic Algorithms in Search, Optimization, and Machine Learning.
- DEAP documentation: https://deap.readthedocs.io
- XGBoost documentation: https://xgboost.readthedocs.io