In [91]:

```
from sympy.interactive import printing
printing.init_printing(use_latex=True)
from sympy import *
import sympy as sp
from sympy.physics.quantum import Bra,Ket
mv1,mv2,mv3,eq=sp.symbols('||u1|| ||u2|| ||u3|| =')
su1,su2,su3,sv1,sv2,sv3=sp.symbols('a1 a2 a3 u1 u2 u3')
bv1,bv2,bv3=Bra('u1'),Bra('u2'),Bra('u3')
ku1,ku2,ku3=Ket('a1'),Ket('a2'),Ket('a3')
kv1,kv2,kv3=Ket('u1'),Ket('u2'),Ket('u3')

u1=list(map(int,input('coloumn 1 :').split()))
u2=list(map(int,input('coloumn 2 :').split()))
u3=list(map(int,input('coloumn 3 :').split()))
amat=transpose(Matrix((u1,u2,u3)))
display(amat)

miu1,miu2,miu3=Matrix(u1),Matrix(u2),Matrix(u3)
print('Let the given vectors be {a1,a2,a3}')
display((u1,u2,u3))

eq1=Eq(su1,sv1)
display(eq1)
display((sv1,eq,miu1))
v1=miu1
print('For our convinience we first find self inner products of v1')
v1s=v1.dot(v1)
ev1s=Bra(v1)*Ket(v1)
display((mv1**2,eq,bv1*kv1,eq,ev1s,eq,v1s))
eq2=Eq(((su2-((bv2*ku1)/(mv1**2))*su1)),sv2)
display(eq2)
u2v1=miu2.dot(v1)
display((bv1*ku2,eq,Bra(u2)*Ket(v1),eq,u2v1))
div1=u2v1/v1s
v2=miu2-(div1*v1)
display((sv2,eq,v2))
v2s=v2.dot(v2)
display((bv2*kv2,eq,Bra(v2)*Ket(v2),eq,v2s))
eq3=((su3-(((bv3*ku1)/(mv1**2))*sv1)-((bv3*ku2)/(mv2**2))*sv2))
display((eq3,eq,sv3))
u3v1=miu3.dot(v1)
u3v2=miu3.dot(v2)
display((bv3*ku1,eq,Bra(u3)*Ket(v1),eq,u3v1))
display((bv3*ku2,eq,Bra(u3)*Ket(v2),eq,u3v2))
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```
v2s=v2.dot(v2)
ev2s=Bra(v2)*Ket(v2)
display((mv2**2,eq,bv2*kv2,eq,ev2s,eq,v2s))
div2=u3v1/v1s
div3=u3v2/v2s
v3=miu3-(div2*v1)-(div3*v2)
display((sv3,eq,v3))
v3s=v3.dot(v3)
ev3s=Bra(v3)*Ket(v3)
display((mv3**2,eq,bv3*kv3,eq,ev3s,eq,v3s))
print('Orthogonal Vectors We Got')
display((v1,v2,v3))
print('Orthonormal Basis')

w1,w2,w3,mul,div=sp.symbols('w1 w2 w3 * /')
display((w1,eq,(sv1/mv1)))
display((w2,eq,(sv2/mv2)))
display((w3,eq,(sv3/mv3)))
W1=v1/sqrt(v1s)
W2=v2/sqrt(v2s)
W3=v3/sqrt(v3s)
display((w1,eq,v1,div,sqrt(v1s)))
display((w1,eq,W1))
display((w2,eq,v2,div,sqrt(v2s)))
display((w2,eq,W2))
display((w3,eq,v3,div,sqrt(v3s)))
display((w3,eq,W3))
display((W1,W2,W3))
sq,sr,sa,st=symbols('Q R A T')
display((sq,eq,(((sv1/mv1)),((sv2/mv2)),((sv3/mv3)))))
display((sq,eq,transpose(Matrix((list(W1),list(W2),list(W3))))))
```

```
coloumn 1 :1 1 1 1
coloumn 2 :-1 4 4 -1
coloumn 3 :4 -2 2 0
```

$$\begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

```
Let the given vectors be {a1,a2,a3}
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$$([1,\ 1,\ 1,\ 1],\ [-1,\ 4,\ 4,\ -1],\ [4,\ -2,\ 2,\ 0])$$

$$a_1 = u_1$$

$$\left(u_1,\ =,\ \begin{bmatrix}1\\1\\1\\1\end{bmatrix}\right)$$

```
For our convinience we first find self inner products of v1
```

$$\left(||u1||^2,\ =,\ \langle u_1\ |u_1\rangle,\ =,\ \langle(1,\ 1,\ 1,\ 1)\ |(1,\ 1,\ 1,\ 1)\rangle,\ =,\ 4\right)$$

$$-\frac{a_1\,\langle u_2\ |a_1\rangle}{||u1||^2}+a_2=u_2$$

$$(\langle u_1\ |a_2\rangle,\ =,\ \langle(-1,\ 4,\ 4,\ -1)\ |(1,\ 1,\ 1,\ 1)\rangle,\ =,\ 6)$$

$$\left(u_2,\ =,\ \begin{bmatrix}-\frac{5}{2}\\\frac{5}{2}\\\frac{5}{2}\\-\frac{5}{2}\end{bmatrix}\right)$$

$$\left(\langle u_2\ |u_2\rangle,\ =,\ \left\langle\left(-\frac{5}{2},\ \frac{5}{2},\ \frac{5}{2},\ -\frac{5}{2}\right)\ \middle|\left(-\frac{5}{2},\ \frac{5}{2},\ \frac{5}{2},\ -\frac{5}{2}\right)\right\rangle,\ =,\ 25\right)$$

$$\left(a_3-\frac{u_1\,\langle u_3\ |a_1\rangle}{||u1||^2}-\frac{u_2\,\langle u_3\ |a_2\rangle}{||u2||^2},\ =,\ u_3\right)$$

$$(\langle u_3\ |a_1\rangle,\ =,\ \langle(4,\ -2,\ 2,\ 0)\ |(1,\ 1,\ 1,\ 1)\rangle,\ =,\ 4)$$

$$\left(\langle u_3\ |a_2\rangle,\ =,\ \left\langle(4,\ -2,\ 2,\ 0)\ \middle|\left(-\frac{5}{2},\ \frac{5}{2},\ \frac{5}{2},\ -\frac{5}{2}\right)\right\rangle,\ =,\ -10\right)$$

$$\left(||u2||^2,\ =,\ \langle u_2\ |u_2\rangle,\ =,\ \left\langle\left(-\frac{5}{2},\ \frac{5}{2},\ \frac{5}{2},\ -\frac{5}{2}\right)\ \middle|\left(-\frac{5}{2},\ \frac{5}{2},\ \frac{5}{2},\ -\frac{5}{2}\right)\right\rangle,\ =,\ 25\right)$$

$$\left(u_3,\ =,\ \begin{bmatrix}2\\-2\\2\\-2\end{bmatrix}\right)$$

$$\left(||u3||^2,\ =,\ \langle u_3\ |u_3\rangle,\ =,\ \langle(2,\ -2,\ 2,\ -2)\ |(2,\ -2,\ 2,\ -2)\rangle,\ =,\ 16\right)$$

```
Orthogonal Vectors We Got
```

$$\left(\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}-\frac{5}{2}\\\frac{5}{2}\\\frac{5}{2}\\-\frac{5}{2}\end{bmatrix}, \begin{bmatrix}2\\-2\\2\\-2\end{bmatrix}\right)$$

```
Orthonormal Basis
```

$$\left(w_1, \ =, \ \frac{u_1}{||u1||}\right)$$

$$\left(w_2, \ =, \ \frac{u_2}{||u2||}\right)$$

$$\left(w_3, \ =, \ \frac{u_3}{||u3||}\right)$$

$$\left(w_1, \ =, \ \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \ /, \ 2\right)$$

$$\left(w_1, \ =, \ \begin{bmatrix}\frac{1}{2}\\\frac{1}{2}\\\frac{1}{2}\\\frac{1}{2}\end{bmatrix}\right)$$

$$\left(w_2, \ =, \ \begin{bmatrix}-\frac{5}{2}\\\frac{5}{2}\\\frac{5}{2}\\-\frac{5}{2}\end{bmatrix}, \ /, \ 5\right)$$

$$\left(w_2, \ =, \ \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix} \right)$$

$$\left(w_3, \ =, \ \begin{bmatrix} 2 \\ -2 \\ 2 \\ -2 \end{bmatrix}, \ /, \ 4 \right)$$

$$\left(w_3, \ =, \ \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix} \right)$$

$$\left(\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}, \ \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}, \ \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix} \right)$$

$$\left(Q, \ =, \ \left(\frac{u_1}{||u1||}, \ \frac{u_2}{||u2||}, \ \frac{u_3}{||u3||} \right) \right)$$

$$\left(Q, \ =, \ \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \right)$$

In [98]:

```
bw1,bw2,bw3=Bra('w1'),Bra('w2'),Bra('w3')
rmmat=Matrix(([bw1*ku1,bw1*ku2,bw1*ku3],[0,bw2*ku2,bw2*ku3],[0,0,bw3*ku3]))
display((sr,eq,rmmat))
a1w1=miu1.dot(W1)
display((bw1*ku1,eq,Bra(W1)*Ket(miu1),eq,a1w1))
a2w1=miu2.dot(W1)
```

```
display((bw1*ku2,eq,Bra(W1)*Ket(miu2),eq,a2w1))
a3w1=miu3.dot(W1)
display((bw1*ku3,eq,Bra(W1)*Ket(miu3),eq,a3w1))
a2w2=miu2.dot(W2)
display((bw2*ku2,eq,Bra(W2)*Ket(miu2),eq,a2w2))
a3w2=miu3.dot(W2)
display((bw2*ku3,eq,Bra(W2)*Ket(miu3),eq,a3w2))
a3w3=miu3.dot(W3)
display((bw3*ku3,eq,Bra(miu3)*Ket(W3),eq,a3w3))
rmat=Matrix(([a1w1,a2w1,a3w1],[0,a2w2,a3w2],[0,0,a3w3]))
```

$$\left(R,\ =,\ \begin{bmatrix} \langle w_1\ |a_1\rangle & \langle w_1\ |a_2\rangle & \langle w_1\ |a_3\rangle \\ 0 & \langle w_2\ |a_2\rangle & \langle w_2\ |a_3\rangle \\ 0 & 0 & \langle w_3\ |a_3\rangle \end{bmatrix}\right)$$

$$\left(\langle w_1\ |a_1\rangle,\ =,\ \left\langle\left(\frac{1}{2},\ \frac{1}{2},\ \frac{1}{2},\ \frac{1}{2}\right)\ |(1,\ 1,\ 1,\ 1)\right\rangle,\ =,\ 2\right)$$

$$\left(\langle w_1\ |a_2\rangle,\ =,\ \left\langle\left(\frac{1}{2},\ \frac{1}{2},\ \frac{1}{2},\ \frac{1}{2}\right)\ |(-1,\ 4,\ 4,\ -1)\right\rangle,\ =,\ 3\right)$$

$$\left(\langle w_1\ |a_3\rangle,\ =,\ \left\langle\left(\frac{1}{2},\ \frac{1}{2},\ \frac{1}{2},\ \frac{1}{2}\right)\ |(4,\ -2,\ 2,\ 0)\right\rangle,\ =,\ 2\right)$$

$$\left(\langle w_2\ |a_2\rangle,\ =,\ \left\langle\left(-\frac{1}{2},\ \frac{1}{2},\ \frac{1}{2},\ -\frac{1}{2}\right)\ |(-1,\ 4,\ 4,\ -1)\right\rangle,\ =,\ 5\right)$$

$$\left(\langle w_2\ |a_3\rangle,\ =,\ \left\langle\left(-\frac{1}{2},\ \frac{1}{2},\ \frac{1}{2},\ -\frac{1}{2}\right)\ |(4,\ -2,\ 2,\ 0)\right\rangle,\ =,\ -2\right)$$

$$\left(\langle w_3\ |a_3\rangle,\ =,\ \left\langle(4,\ -2,\ 2,\ 0)\ \middle|\left(\frac{1}{2},\ -\frac{1}{2},\ \frac{1}{2},\ -\frac{1}{2}\right)\right\rangle,\ =,\ 4\right)$$

In [93]:

```
sr,eq,rmat
```

Out[93]:

$$\left(R,\ =,\ \begin{bmatrix} 2 & 3 & 2 \\ 0 & 5 & -2 \\ 0 & 0 & 4 \end{bmatrix}\right)$$

In [94]:

```
display((sa,eq,sq*sr,eq,transpose(Matrix((list(W1),list(W2),list(W3)))),mul,rmat,eq,transpose(Matrix((list(W1),list(W2),l
```

$$\left(A, \ =, \ QR, \ =, \ \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}, \ *, \ \begin{bmatrix} 2 & 3 & 2 \\ 0 & 5 & -2 \\ 0 & 0 & 4 \end{bmatrix}, \ =, \ \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix} \right)$$

In [ ]: