Draw the flow chart, write the algorithm and create a program to execute and generate the result using any programming language for to find orthogonal basis and orthonormal basis from the given vectors (use Gram Schmidt Orthogonalization Process or any other idea)

In [95]:

```
from sympy.interactive import printing
printing.init_printing(use_latex=True)
from sympy import *
import sympy as sp
from sympy.physics.quantum import Bra,Ket
mv1,mv2,mv3,eq=sp.symbols('||v1|| ||v2|| ||v3|| =')
su1,su2,su3,sv1,sv2,sv3=sp.symbols('U1 U2 U3 V1 V2 V3')
bv1,bv2,bv3=Bra('V1'),Bra('V2'),Bra('V3')
ku1,ku2,ku3=Ket('U1'),Ket('U2'),Ket('U3')
kv1,kv2,kv3=Ket('V1'),Ket('V2'),Ket('V3')

u1=Matrix(list(map(int,input('Vector 1 :').split())))
u2=Matrix(list(map(int,input('Vector 2 :').split())))
u3=Matrix(list(map(int,input('Vector 3 :').split())))

print('Let the given vectors be {u1,u2,u3}')
display((u1,u2,u3))

eq1=Eq(su1,sv1)
display(eq1)
display(u1)
v1=u1
print('For our convinience we first find self inner products of v1')
v1s=v1.dot(v1)
ev1s=Bra(v1)*Ket(v1)
display((mv1**2,eq,bv1*kv1,eq,ev1s,eq,v1s))
eq2=Eq(((su2-((bv2*ku1)/(mu1**2))*su1)),sv2)
display(eq2)
u2v1=u2.dot(v1)
display((bv1*ku2,eq,Bra(u2)*Ket(v1),eq,u2v1))
div1=u2v1/v1s
```

```
v2=u2-(div1*v1)
display(v2)
v2s=v2.dot(v2)
display((bv2*kv2,eq,Bra(v2)*Ket(v2),eq,v2s))
eq3=((su3-(((bv3*ku1)/(mu1**2))*su1)-((bv3*ku2)/(mu2**2))*su2))
display((eq3,eq,sv3))
u3v1=u3.dot(v1)
u3v2=u3.dot(v2)
display((bv3*ku1,eq,Bra(u3)*Ket(v1),eq,u3v1))
display((bv3*ku2,eq,Bra(u3)*Ket(v2),eq,u3v2))
v2s=v2.dot(v2)
ev2s=Bra(v2)*Ket(v2)
display((mv2**2,eq,bv2*kv2,eq,ev2s,eq,v2s))
div2=u3v1/v1s
div3=u2v1/v2s
v3=u3-(div2*v1)-(div3*v2)
display(v3)
v3s=v3.dot(v3)
ev3s=Bra(v3)*Ket(v3)
display((mv3**2,eq,bv3*kv3,eq,ev3s,eq,v3s))
print('Orthogonal Vectors We Got')
display((v1,v2,v3))
print('Orthonormal Basis')

w1,w2,w3=sp.symbols('w1 w2 w3')
display((w1,eq,(sv1/mv1)))
display((w2,eq,(sv2/mv2)))
display((w3,eq,(sv3/mv3)))
W1=v1/sqrt(v1s)
W2=v2/sqrt(v2s)
W3=v3/sqrt(v3s)
display((w1,eq,W1))
display((w2,eq,W2))
display((w3,eq,W3))
print('The obtained Orthonormal Basis is ')
display((W1,W2,W3))
```

```
Vector 1 :1 0 1 0
Vector 2 :1 1 1 1
Vector 3 :0 1 2 1
Let the given vectors be {u1,u2,u3}
```

$$\left(\begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\2\\1\end{bmatrix}\right)$$

$U_1 = V_1$

$$\begin{bmatrix}1\\0\\1\\0\end{bmatrix}$$

```
For our convinience we first find self inner products of v1
```

$$\left(||v1||^2,\ =,\ \langle V_1\ |V_1\rangle,\ =,\ \langle(1,\ 0,\ 1,\ 0)\ |(1,\ 0,\ 1,\ 0)\rangle,\ =,\ 2\right)$$

$$-\frac{U_1\left\langle V_2\ |U_1\right\rangle}{||U1||^2} + U_2 = V_2$$

$$(\langle V_1\ |U_2\rangle,\ =,\ \langle(1,\ 1,\ 1,\ 1)\ |(1,\ 0,\ 1,\ 0)\rangle,\ =,\ 2)$$

$$\begin{bmatrix}0\\1\\0\\1\end{bmatrix}$$

$$(\langle V_2\ |V_2\rangle,\ =,\ \langle(0,\ 1,\ 0,\ 1)\ |(0,\ 1,\ 0,\ 1)\rangle,\ =,\ 2)$$

$$\left(-\frac{U_1\left\langle V_3\ |U_1\right\rangle}{||U1||^2} - \frac{U_2\left\langle V_3\ |U_2\right\rangle}{||U2||^2} + U_3,\ =,\ V_3\right)$$

$$(\langle V_3\ |U_1\rangle,\ =,\ \langle(0,\ 1,\ 2,\ 1)\ |(1,\ 0,\ 1,\ 0)\rangle,\ =,\ 2)$$

$$(\langle V_3\ |U_2\rangle,\ =,\ \langle(0,\ 1,\ 2,\ 1)\ |(0,\ 1,\ 0,\ 1)\rangle,\ =,\ 2)$$

$$\left(||v2||^2,\ =,\ \langle V_2\ |V_2\rangle,\ =,\ \langle(0,\ 1,\ 0,\ 1)\ |(0,\ 1,\ 0,\ 1)\rangle,\ =,\ 2\right)$$

$$\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\left(||v3||^2, \ =, \ \langle V_3 \, | V_3 \rangle, \ =, \ \langle (-1, \, 0, \, 1, \, 0) \, | (-1, \, 0, \, 1, \, 0) \rangle, \ =, \ 2\right)$$

```
Orthogonal Vectors We Got
```

$$\left(\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right)$$

```
Orthonormal Basis
```

$$\left(w_1, \ =, \ \frac{V_1}{||v1||}\right)$$

$$\left(w_2, \ =, \ \frac{V_2}{||v2||}\right)$$

$$\left(w_3, \ =, \ \frac{V_3}{||v3||}\right)$$

$$\left(w_1, \ =, \ \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}\right)$$

$$\left(w_2, \ =, \ \begin{bmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{bmatrix}\right)$$

$$\left(w_3, \ =, \ \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix} \right)$$

```
The obtained Orthonormal Basis is
```

$$\left(\begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}, \ \begin{bmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{bmatrix}, \ \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix} \right)$$

In [ ]: