In [1]:

```python
from sympy.interactive import printing
printing.init_printing(use_latex=True)
from sympy import Eq,solve_linear_system,Matrix,det,transpose,Transpose,symbols,trace,Trace
from sympy.solvers import solve
from sympy import simplify,sqrt
from sympy.physics.quantum import Bra,Ket
from time import time
import sympy as sp

print('________________________----Welcome to Linear Algebra Calculator Material----________________________')
print('\n\n\n')
print('1 -- UNIT-I   -- SOLVING SYSTEM OF LINEAR EQUATIONS')
print('2 -- UNIT-II  -- LINEAR DEPENDANCY AND BASIS FORMATION CHECKING')
print('3 -- UNIT-III -- FINDING EIGEN VALUE AND EIGEN VECTOR')
print('4 -- UNIT-IV  -- ORTHOGONALIZATION')
print('5 -- UNIT-V   -- QR DECOMPOSITION')

inum=int(input('Enter Your Choice of Chapter : '))
if inum==1:
    print("-------------Welcome! To Solving system Of Linear Equations---------------")


    print("2.2X2 Matrix")
    print("3.3X3 Matrix")
    print("4.4X4 Matrix")

    u=int(input("which nXn matrix you want : "))


    if u==2:
        display("--------------------------------------------------------------------")
        eq1=sp.Function("eq1")
        eq2=sp.Function("eq2")
        x,y=sp.symbols("x y")

        x1=int(input("Enter your X1 coefficient : "))
        y1=int(input("Enter your Y1 coefficient : "))
        display("--------------------------------------------------------------------")
        x2=int(input("Enter your X2 coefficient : "))
        y2=int(input("Enter your Y2 coefficient : "))
        display("--------------------------------------------------------------------")
        c1=int(input("Enter your constant 1 : "))
        c2=int(input("Enter your constant 2 : "))
        display("--------------------------------------------------------------------")
        display("Using Sympy Module")
```

```
    eq1=Eq(x1*x+y1*y,c1)
    eq2=Eq(x2*x+y2*y,c2)

    display(eq1)
    display(eq2)

    amatr1=[x1,y1]
    amatr2=[x2,y2]
    amat=Matrix((amatr1,amatr2))
    print("A Matrix")
    display(amat)
    xm1=[x]
    xm2=[y]
    xmat=Matrix((xm1,xm2))
    print("Variables")
    display(xmat)

    cm1=[c1]
    cm2=[c2]
    cmat=Matrix((cm1,cm2))
    print("Constant Matrix")
    display(cmat)
    row1=[x1,y1,c1]
    row2=[x2,y2,c2]
    system=Matrix((row1,row2))
    print("Augmented Form")
    display(system)
    start=time()
    if(solve_linear_system(system,x,y)==None):
        display("System is Inconsistent")
    else:
        display("System is Consistent")
        display(solve_linear_system(system,x,y))
    end=time()
    display("Execution Time : ",end-start)


if u==3:
    display("--------------------------------------------------------------------")

    x1=int(input("Enter your X1 coefficient : "))
    y1=int(input("Enter your Y1 coefficient : "))
    z1=int(input("Enter your z1 coefficient : "))

    display("--------------------------------------------------------------------")
```

```python
x2=int(input("Enter your X2 coefficient : "))
y2=int(input("Enter your Y2 coefficient : "))
z2=int(input("Enter your z2 coefficient : "))

display("--------------------------------------------------------------------")

x3=int(input("Enter your X3 coefficient : "))
y3=int(input("Enter your Y3 coefficient : "))
z3=int(input("Enter your z3 coefficient : "))

display("--------------------------------------------------------------------")

c1=int(input("Enter your constant 1 : "))
c2=int(input("Enter your constant 2 : "))
c3=int(input("Enter your constant 3 : "))

display("--------------------------------------------------------------------")
display("Using Sympy Module")

eq1=sp.Function("eq1")
eq2=sp.Function("eq2")
eq3=sp.Function("eq3")

x,y,z=sp.symbols('x y z')

eq1=Eq(x1*x+y1*y+z1*z,c1)
eq2=Eq(x2*x+y2*y+z2*z,c2)
eq3=Eq(x3*x+y3*y+z3*z,c3)

display(eq1)
display(eq2)
display(eq3)

amatr1=[x1,y1,z1]
amatr2=[x2,y2,z2]
amatr3=[x3,y3,z3]
amat=Matrix((amatr1,amatr2,amatr3))
print("A matrix")
display(amat)
xm1=[x]
xm2=[y]
xm3=[z]
xmat=Matrix((xm1,xm2,xm3))
print("Variables")
display(xmat)
cm1=[c1]
```

```
    cm2=[c2]
    cm3=[c3]
    print("Constant Matrix")
    cmat=Matrix((cm1,cm2,cm3))
    display(cmat)

    row1=[x1,y1,z1,c1]
    row2=[x2,y2,z2,c2]
    row3=[x3,y3,z3,c3]

    system=Matrix((row1,row2,row3))
    print("Augmented Form")
    display(system)
    start=time()
    if(solve_linear_system(system,x,y,z)==None):
        display("System is Inconsistent")
    else:
        display("System is Consistent")
        display(solve_linear_system(system,x,y,z))
    end=time()
    display("Execution Time : ",end-start)

if u==4:
    display("--------------------------------------------------------------------")
    w1=int(input("Enter your w1 coefficient : "))
    x1=int(input("Enter your X1 coefficient : "))
    y1=int(input("Enter your Y1 coefficient : "))
    z1=int(input("Enter your z1 coefficient : "))
    display("--------------------------------------------------------------------")
    w2=int(input("Enter your w2 coefficient : "))
    x2=int(input("Enter your X2 coefficient : "))
    y2=int(input("Enter your Y2 coefficient : "))
    z2=int(input("Enter your z2 coefficient : "))
    display("--------------------------------------------------------------------")
    w3=int(input("Enter your w3 coefficient : "))
    x3=int(input("Enter your X3 coefficient : "))
    y3=int(input("Enter your Y3 coefficient : "))
    z3=int(input("Enter your z3 coefficient : "))
    display("--------------------------------------------------------------------")

    w4=int(input("Enter your w4 coefficient : "))
    x4=int(input("Enter your X4 coefficient : "))
    y4=int(input("Enter your Y4 coefficient : "))
    z4=int(input("Enter your z4 coefficient : "))

    display("--------------------------------------------------------------------")
```

```python
c1=int(input("Enter your constant 1 : "))
c2=int(input("Enter your constant 2 : "))
c3=(input("Enter your constant 3 : "))
c4=int(input("Enter your constant 4 : "))

display("--------------------------------------------------------------------")
display("Using Sympy Module")

eq1=sp.Function("eq1")
eq2=sp.Function("eq2")
eq3=sp.Function("eq3")
eq4=sp.Function("eq4")
w,x,y,z=sp.symbols('w x y z')


eq1=Eq(w1*w+x1*x+y1*y+z1*z,c1)
eq2=Eq(w2*w+x2*x+y2*y+z2*z,c2)
eq3=Eq(w3*w+x3*x+y3*y+z3*z,c3)
eq4=Eq(w4*w+x4*x+y4*y+z4*z,c4)


display(eq1)
display(eq2)
display(eq3)
display(eq4)

amatr1=[w1,x1,y1,z1]
amatr2=[w2,x2,y2,z2]
amatr3=[w3,x3,y3,z3]
amatr4=[w4,x4,y4,z4]
amat=Matrix((amatr1,amatr2,amatr3,amatr4))
print("A matrix")
display(amat)

xm1=[w]
xm2=[x]
xm3=[y]
xm4=[z]
xmat=Matrix((xm1,xm2,xm3,xm4))
print("Variables")
display(xmat)

cm1=[c1]
cm2=[c2]
cm3=[c3]
cm4=[c4]
print("Constant Matrix")
```

```
        cmat=Matrix((cm1,cm2,cm3,cm4))
        display(cmat)


        row1=[w1,x1,y1,z1,c1]
        row2=[w2,x2,y2,z2,c2]
        row3=[w3,x3,y3,z3,c3]
        row4=[w4,x4,y4,z4,c4]


        system=Matrix((row1,row2,row3,row4))
        print("Augmented Form")
        display(system)
        start=time()
        if(solve_linear_system(system,w,x,y,z)==None):
            display("System is Inconsistent")
        else:
            display("System is Consistent")
            display(solve_linear_system(system,w,x,y,z))

        end=time()
        display("Execution Time : ",end-start)



    print("-------------------------------------------------------------------")
elif inum==2:
    print("------------- Welcome! To Find linear combination dependancy and basis formation checking ---------------")
    no=int(input("No of Times you going to execute the program : "))
    for j in range(0,no):
        n=int(input('Number of vectors : '))
        m,a,e,g=sp.symbols('* + = -->')
        if n==2:
                x,y,m,a,e,g=sp.symbols("x1 x2 * + = -->")
                a11,a12=map(int,input("Enter 1st row : ").split())
                a21,a22=map(int,input("Enter 2nd row : ").split())
                vect1,vect2=[a11,a12],[a21,a22]
                amat=Matrix((vect1,vect2))
                print("Given Matrix")
                display(amat)
                print('now')
                display(((x,m,vect1),a,(y,m,vect2),e,(0,0),g))
                r1,r2=[a11,a21,0],[a12,a22,0]

                eq1,eq2,eqn,eqm=sp.Function("eq1"),sp.Function("eq2"),sp.Function("eqn"),sp.Function("eqm")
                eqm=(a11*x,a12*x)
                eqn=(a21*y,a22*y)
```

```
            display((eqm,a,eqn,e,(0,0)))
            eq1,eq2=Eq(a11*x+a21*y,0),Eq(a12*x+a22*y,0)
            display(eq1,eq2)
            print("arranged matrix")
            display((transpose(amat),Matrix((x,y)),e,Matrix((0,0))))
            system=Matrix((r1,r2))
            print("augmented form")
            display(system)

            print("\n\n\nThe two vectors are ")
            if(det(amat)==0):
                print("linearly dependant")
                display(solve_linear_system(system,x,y))
                print("dim R2 != 2, The given vectors NOT form a basis of R^2")
            else:
                print("linearly independant")
                display(solve_linear_system(system,x,y))
                print("The given vectors does form a basis of R^2")
        if n==3:
            x,y,z=sp.symbols("x1 x2 x3")
            a11,a12,a13=map(int,input("Enter 1st row : ").split())
            a21,a22,a23=map(int,input("Enter 2nd row : ").split())
            a31,a32,a33=map(int,input("Enter 3rd row : ").split())
            vect1,vect2,vect3=[a11,a12,a13],[a21,a22,a23],[a31,a32,a33]
            amat=Matrix((vect1,vect2,vect3))
            print("Given Matrix")
            display(amat)
            print('now')
            display(((x,m,vect1),a,(y,m,vect2),a,(z,m,vect3),e,(0,0,0),g))
            r1,r2,r3=[a11,a21,a31,0],[a12,a22,a32,0],[a13,a23,a33,0]
            eq1,eq2,eq3,eqn,eqm,eqo=sp.Function("eq1"),sp.Function("eq2"),sp.Function("eq3"),sp.Function("eqn"),sp.Function("eqm"),sp.Fur
            eqm=(a11*x,a12*x,a13*x)
            eqn=(a21*y,a22*y,a23*y)
            eqo=(a31*z,a32*z,a33*z)
            display((eqm,a,eqn,a,eqo,e,(0,0,0),g))
            eq1,eq2,eq3=Eq(a11*x+a21*y+a31*z,0),Eq(a12*x+a22*y+a32*z,0),Eq(a13*x+a23*y+a33*z,0)
            display(eq1,eq2,eq3)
            print("arranged matrix")
            display((transpose(amat),Matrix((x,y,z)),e,Matrix((0,0,0))))
            system=Matrix((r1,r2,r3))
            print("augmented form")
            display(system)

            print("\n\n\nThe two vectors are ")
            if(det(transpose(amat))==0):
                print("linearly dependant")
                display(solve_linear_system(system,x,y,z))
```

```
                    print("dim R3 != 3, The given vectors NOT form a basis of R^3")
                else:
                    print("linearly independant")
                    display(solve_linear_system(system,x,y,z))
                    print("The given vectors does form a basis of R^3")
        if n==4:
                x,y,z,w=sp.symbols("x1 x2 x3 x4")
                a11,a12,a13,a14=map(int,input("Enter 1st row : ").split())
                a21,a22,a23,a24=map(int,input("Enter 2nd row : ").split())
                a31,a32,a33,a34=map(int,input("Enter 3rd row : ").split())
                a41,a42,a43,a44=map(int,input("Enter 4th row : ").split())
                vect1,vect2,vect3,vect4=[a11,a12,a13,a14],[a21,a22,a23,a24],[a31,a32,a33,a34],[a41,a42,a43,a44]
                amat=Matrix((vect1,vect2,vect3,vect4))
                print("Given Matrix")
                display(amat)
                print('now')
                display(((x,m,vect1),a,(y,m,vect2),a,(z,m,vect3),a,(w,m,vect4),e,(0,0,0,0),g))
                r1,r2,r3,r4=[a11,a21,a31,a41,0],[a12,a22,a32,a42,0],[a13,a23,a33,a43,0],[a14,a24,a34,a44,0]
                eq1,eq2,eq3,eq4=sp.Function("eq1"),sp.Function("eq2"),sp.Function("eq3"),sp.Function("eq4")
                eqm=(a11*x,a12*x,a13*x,a14*x)
                eqn=(a21*y,a22*y,a23*y,a24*y)
                eqo=(a31*z,a32*z,a33*z,a34*z)
                eqp=(a41*w,a42*w,a43*w,a44*w)
                display((eqm,a,eqn,a,eqo,a,eqp,e,(0,0,0,0),g))
                eq1,eq2,eq3,eq4=Eq(a11*x+a21*y+a31*z+a41*w,0),Eq(a12*x+a22*y+a32*z+a42*w,0),Eq(a13*x+a23*y+a33*z+a43*w,0),Eq(a14*x+a24*y+a34*
                display(eq1,eq2,eq3,eq4)
                system=Matrix((r1,r2,r3,r4))
                print('rearranged matrix')
                display((transpose(amat),Matrix((x,y,z,w)),e,Matrix((0,0,0,0))))
                print("augmented form")
                display(system)

                print("\n\n\nThe two vectors are ")
                if(det(amat)==0):
                    print("linearly dependant")
                    display(solve_linear_system(system,x,y,z,w))
                    print("dim R4 != 4, The given vectors NOT form a basis of R^4")
                else:
                    print("linearly independant")
                    display(solve_linear_system(system,x,y,z,w))
                    print("The given vectors form a basis of R^4")
    print('_______________________________________________')
elif inum==3:
    a11,a12,a13,a21,a22,a23,a31,a32,a33=map(int,input("Enter the matrix coefficients : ").split())
    r1=[a11,a12,a13]
    r2=[a21,a22,a23]
    r3=[a31,a32,a33]
```

```
    amat=sp.Matrix((r1,r2,r3))
    print("Input matrix")
    display(amat)
    s1=a11+a22+a33
    display('S1={0}+{1}+{2}'.format(a11,a22,a33))
    display('s1={0}'.format(s1))
    s2=(((a22*a33)-(a32*a23))+((a11*a33)-(a31*a13))+((a11*a22)-(a21*a12)))
    display('s2=((({0}*{1})-({2}*{3}))+(({4}*{5})-({6}*{7}))+(({8}*{9})-({10}*{11}))'.format(a22,a33,a32,a23,a11,a33,a31,a13,a11,a22,a21,a12)
    display('s2=({0}+{1}+{2})'.format(((a22*a33)-(a32*a23)),((a11*a33)-(a31*a13)),((a11*a22)-(a21*a12))))
    s3=det(amat)
    display('s3=det(amat)')
    display("s3={0}".format(s3))
    display('s1={0} s2={1} s3={2}'.format(s1,s2,s3))
    x=sp.symbols('λ')
    eq1=sp.Function("eq1")
    eq1=Eq((x**3 - (s1)*x**2 +(s2)*x- s3))
    display(eq1)
    k=sp.factor(eq1)
    display(k)
    K=solve(k)
    print("Eigen values are : ",K)
    a,b,c,d,e,f=sp.symbols('a b c * = ->')
    b1=[a11-x,a12,a13]
    b2=[a21,a22-x,a23]
    b3=[a31,a32,a33-x]
    bmat=sp.Matrix((b1,b2,b3))
    xmat=sp.Matrix((a,b,c))
    print("To find eigen vector")
    display(bmat,d,xmat)
    cmat=sp.Matrix((0,0,0))
    if(len(K)==2):
        print("If λ={0}".format(K[0]))
        b1=[a11-K[0],a12,a13]
        b2=[a21,a22-K[0],a23]
        b3=[a31,a32,a33-K[0]]
        bmat=sp.Matrix((b1,b2,b3))
        display((bmat,d,xmat,e,cmat,f))
        bm1=[a11-K[0],a12,a13,0]
        bm2=[a21,a22-K[0],a23,0]
        bm3=[a31,a32,a33-K[0],0]
        eq2=sp.Function("eq2")
        eq3=sp.Function("eq3")
        eq4=sp.Function("eq4")
        eq2=Eq((a11-K[0])*a+a12*b+a13*c)
        eq3=Eq(a21*a+(a22-K[0])*b+a23*c)
        eq4=Eq(a31*a+a32*b+(a33-K[0])*c)
        display(eq2,eq3,eq4)
```

```python
        system=Matrix((bm1,bm2,bm3))
        n=sp.solve_linear_system(system,a,b,c)
        display(n)

        print("If λ={0}".format(K[1]))
        b1=[a11-K[1],a12,a13]
        b2=[a21,a22-K[1],a23]
        b3=[a31,a32,a33-K[1]]
        bmat=sp.Matrix((b1,b2,b3))
        display((bmat,d,xmat,e,cmat,f))
        bm1=[a11-K[1],a12,a13,0]
        bm2=[a21,a22-K[1],a23,0]
        bm3=[a31,a32,a33-K[1],0]
        eq2=sp.Function("eq2")
        eq3=sp.Function("eq3")
        eq4=sp.Function("eq4")
        eq2=Eq((a11-K[1])*a+a12*b+a13*c)
        eq3=Eq(a21*a+(a22-K[1])*b+a23*c)
        eq4=Eq(a31*a+a32*b+(a33-K[1])*c)
        display(eq2,eq3,eq4)
        system=Matrix((bm1,bm2,bm3))
        n=sp.solve_linear_system(system,a,b,c)
        display(n)

        print("Eigen vector")
        j=amat.eigenvects()
        display(j)

    if(len(K)==3):
        print("If λ={0}".format(K[0]))
        b1=[a11-K[0],a12,a13]
        b2=[a21,a22-K[0],a23]
        b3=[a31,a32,a33-K[0]]
        bmat=sp.Matrix((b1,b2,b3))
        display((bmat,d,xmat,e,cmat,f))
        bm1=[a11-K[0],a12,a13,0]
        bm2=[a21,a22-K[0],a23,0]
        bm3=[a31,a32,a33-K[0],0]
        eq2=sp.Function("eq2")
        eq3=sp.Function("eq3")
        eq4=sp.Function("eq4")
        eq2=Eq((a11-K[0])*a+a12*b+a13*c)
        eq3=Eq(a21*a+(a22-K[0])*b+a23*c)
        eq4=Eq(a31*a+a32*b+(a33-K[0])*c)
        display(eq2,eq3,eq4)
        system=Matrix((bm1,bm2,bm3))
        n=sp.solve_linear_system(system,a,b,c)
```

```
            display(n)
            print("If λ={0}".format(K[1]))
            b1=[a11-K[1],a12,a13]
            b2=[a21,a22-K[1],a23]
            b3=[a31,a32,a33-K[1]]
            bmat=sp.Matrix((b1,b2,b3))
            display((bmat,d,xmat,e,cmat,f))
            bm1=[a11-K[1],a12,a13,0]
            bm2=[a21,a22-K[1],a23,0]
            bm3=[a31,a32,a33-K[1],0]
            eq2=sp.Function("eq2")
            eq3=sp.Function("eq3")
            eq4=sp.Function("eq4")
            eq2=Eq((a11-K[1])*a+a12*b+a13*c)
            eq3=Eq(a21*a+(a22-K[1])*b+a23*c)
            eq4=Eq(a31*a+a32*b+(a33-K[1])*c)
            display(eq2,eq3,eq4)
            system=Matrix((bm1,bm2,bm3))
            n=sp.solve_linear_system(system,a,b,c)
            display(n)
            print("If λ={0}".format(K[2]))
            b1=[a11-K[2],a12,a13]
            b2=[a21,a22-K[2],a23]
            b3=[a31,a32,a33-K[2]]
            bmat=sp.Matrix((b1,b2,b3))
            display((bmat,d,xmat,e,cmat,f))
            bm1=[a11-K[2],a12,a13,0]
            bm2=[a21,a22-K[2],a23,0]
            bm3=[a31,a32,a33-K[2],0]
            eq2=sp.Function("eq2")
            eq3=sp.Function("eq3")
            eq4=sp.Function("eq4")
            eq2=Eq((a11-K[2])*a+a12*b+a13*c)
            eq3=Eq(a21*a+(a22-K[2])*b+a23*c)
            eq4=Eq(a31*a+a32*b+(a33-K[2])*c)
            display(eq2,eq3,eq4)
            system=Matrix((bm1,bm2,bm3))

            n=sp.solve_linear_system(system,a,b,c)
            display(n)

            print("Eigen vector")

            j=amat.eigenvects()
            display(j)
elif inum==4:
    print('___________________----Welcome to Eigen value and Eigen vector Calculation----___________________')
```

```python
    print('1. Vector method\n2. Matrix method')
    ornum=int(input('Enter method to do Orthogonalization'))
    if ornum==1:
        mv1,mv2,mv3,eq=sp.symbols('||V1|| ||V2|| ||V3|| =')
        su1,su2,su3,sv1,sv2,sv3=sp.symbols('U1 U2 U3 V1 V2 V3')
        bu1,bu2,bu3=Bra('U1'),Bra('U2'),Bra('U3')
        bv1,bv2,bv3=Bra('V1'),Bra('V2'),Bra('V3')
        ku1,ku2,ku3=Ket('U1'),Ket('U2'),Ket('U3')
        kv1,kv2,kv3=Ket('V1'),Ket('V2'),Ket('V3')

        u1=Matrix(list(map(int,input('Vector 1 :').split())))
        u2=Matrix(list(map(int,input('Vector 2 :').split())))
        u3=Matrix(list(map(int,input('Vector 3 :').split())))

        print('Let the given vectors be {u1,u2,u3}')
        display((u1,u2,u3))

        eq1=Eq(su1,sv1)
        display(eq1)
        display((sv1,eq,u1))
        v1=u1
        print('For our convinience we first find self inner products of v1')
        v1s=v1.dot(v1)
        ev1s=Bra(v1)*Ket(v1)
        display((mv1**2,eq,bv1*kv1,eq,ev1s,eq,v1s))
        eq2=(((su2-((bu2*kv1)/(mv1**2))*su1)))
        display((sv2,eq,eq2))
        u2v1=u2.dot(v1)
        display((bu2*kv1,eq,Bra(u2)*Ket(v1),eq,u2v1))
        div1=u2v1/v1s
        v2=u2-(div1*v1)
        display((sv2,eq,v2))
        v2s=v2.dot(v2)
        display((bv2*kv2,eq,Bra(v2)*Ket(v2),eq,v2s))
        eq3=((su3-(((bu3*kv1/(mv1**2))*sv1)-((bu3*kv2)/(mv2**2))*sv2)))
        display((sv3,eq,eq3))
        u3v1=u3.dot(v1)
        u3v2=u3.dot(v2)
        display((bu3*kv1,eq,Bra(u3)*Ket(v1),eq,u3v1))
        display((bu3*kv2,eq,Bra(u3)*Ket(v2),eq,u3v2))
        v2s=v2.dot(v2)
        ev2s=Bra(v2)*Ket(v2)
        display((mv2**2,eq,bv2*kv2,eq,ev2s,eq,v2s))
        div2=u3v1/v1s
        div3=u2v1/v2s
        v3=u3-(div2*v1)-(div3*v2)
        display((sv3,eq,v3))
```

```
        v3s=v3.dot(v3)
        ev3s=Bra(v3)*Ket(v3)
        display((mv3**2,eq,bv3*kv3,eq,ev3s,eq,v3s))
        print('Orthogonal Vectors We Got')
        display((v1,v2,v3))
        print('Orthonormal Basis')

        w1,w2,w3,mul,div=sp.symbols('w1 w2 w3 * /')
        display((w1,eq,(sv1/mv1)))
        display((w2,eq,(sv2/mv2)))
        display((w3,eq,(sv3/mv3)))
        W1=v1/sqrt(v1s)
        W2=v2/sqrt(v2s)
        W3=v3/sqrt(v3s)
        display((w1,eq,v1,div,sqrt(v1s)))
        display((w1,eq,W1))
        display((w2,eq,v2,div,sqrt(v2s)))
        display((w2,eq,W2))
        display((w3,eq,v3,div,sqrt(v3s)))
        display((w3,eq,W3))
        print('The obtained Orthonormal Basis is ')
        display((W1,W2,W3))
        quit()
    elif ornum==2:
        mv1,mv2,mv3,eq=sp.symbols('||V1|| ||V2|| ||V3|| =')
        su1,su2,su3,sv1,sv2,sv3=sp.symbols('U1 U2 U3 V1 V2 V3')
        bv1,bv2,bv3=Bra('V1'),Bra('V2'),Bra('V3')
        bu1,bu2,bu3=Bra('U1'),Bra('U2'),Bra('U3')
        ku1,ku2,ku3=Ket('U1'),Ket('U2'),Ket('U3')
        kv1,kv2,kv3=Ket('V1'),Ket('V2'),Ket('V3')
        w1,w2,w3,mul,div=sp.symbols('w1 w2 w3 * /')

        a11,a12,a21,a22=map(int,input('Enter the matrix elements of A : ').split())
        b11,b12,b21,b22=map(int,input('Enter the matrix elements of A : ').split())
        c11,c12,c21,c22=map(int,input('Enter the matrix elements of A : ').split())
        ar1,ar2=[a11,a12],[a21,a22]
        br1,br2=[b11,b12],[b21,b22]
        cr1,cr2=[c11,c12],[c21,c22]
        print('Let u1 , u2 and u3 be given basis')
        matu1=Matrix((ar1,ar2))
        display(matu1)
        matu2=Matrix((br1,br2))
        display(matu2)
        matu3=Matrix((cr1,cr2))
        display(matu3)
        matv1=matu1
        display((sv1,eq,su1))
```

```
        display((sv1,eq,matv1))
        mev1s=Bra(Transpose(matv1))*Ket(Transpose(Transpose(matv1)))
        mv1s=trace(Transpose(matv1)*matv1)
        display((mv1**2,eq,bv1*kv1,eq,mev1s,eq,Trace(Transpose(matv1)*matv1),eq,mv1s))
        eq2=Eq(((su2-((bu2*kv1)/(mv1**2))*su1)),sv2)
        display(eq2)
        mu2mv1=trace(Transpose(matu2)*matv1)
        display((bu2*kv1,eq,Bra(Transpose(matv1))*Ket(Transpose(Transpose(matu2))),eq,Trace(Transpose(matv1)*matu2),eq,mu2mv1))
        mdiv1=mu2mv1/mv1s
        matv2=matu2-(matu1*mdiv1)
        display((sv2,eq,matv2))
        mev2s=Bra(Transpose(matv2))*Ket(Transpose(Transpose(matv2)))
        mv2s=trace(Transpose(matv2)*matv2)
        display((mv2**2,eq,bv2*kv2,eq,mev2s,eq,Trace(Transpose(matv2)*matv2),eq,mv2s))
        eq3=((su3-(((bu3*kv1)/(mv1**2))*sv1)-((bu3*kv2)/(mv2**2))*sv2))
        display((eq3,eq,sv3))
        mu3mv1=trace(Transpose(matv1)*matu3)
        mu3mv2=trace(Transpose(matv2)*matu3)
        display((bu3*kv1,eq,Bra(Transpose(matv1))*Ket(Transpose(Transpose(matu3))),eq,Trace(Transpose(matv1)*matu3),eq,mu3mv1))
        display((bu3*kv2,eq,Bra(Transpose(matv2))*Ket(Transpose(Transpose(matu3))),eq,Trace(Transpose(matv2)*matu3),eq,mu3mv2))
        mdiv2=mu3mv1/mv1s
        mdiv3=mu3mv2/mv2s
        matv3=matu3-(mdiv2*matv1)-(mdiv3*matv2)
        display((sv3,eq,matv3))
        mev3s=Bra(Transpose(matv3))*Ket(Transpose(Transpose(matv3)))
        mv3s=trace(transpose(matv3)*matv3)
        display((mv3**2,eq,bv3*kv3,eq,mev3s,eq,Trace(Transpose(matv3)*matv3),eq,mv3s))
        print('Orthogonal basis We Got')
        display((matv1,matv2,matv3))
        print('Orthonormal Basis')
        display((w1,eq,(sv1/mv1)))
        display((w2,eq,(sv2/mv2)))
        display((w3,eq,(sv3/mv3)))
        matW1=matv1/sqrt(mv1s)
        matW2=matv2/sqrt(mv2s)
        matW3=matv3/sqrt(mv3s)
        display((w1,eq,matv1,div,sqrt(mv1s)))
        display((w1,eq,matW1))
        display((w2,eq,matv2,div,sqrt(mv2s)))
        display((w2,eq,matW2))
        display((w3,eq,matv3,div,sqrt(mv3s)))
        display((w3,eq,matW3))
        print('The obtained Orthonormal Basis is ')
        display((matW1,matW2,matW3))
        quit()
elif inum==5:
    print('_______________________----Welcome To Calculate QR decomposition----_______________________')
```

```python
mv1,mv2,mv3,eq=sp.symbols('||u1|| ||u2|| ||u3|| =')
su1,su2,su3,sv1,sv2,sv3=sp.symbols('a1 a2 a3 u1 u2 u3')
bv1,bv2,bv3=Bra('u1'),Bra('u2'),Bra('u3')
ku1,ku2,ku3=Ket('a1'),Ket('a2'),Ket('a3')
kv1,kv2,kv3=Ket('u1'),Ket('u2'),Ket('u3')

u1=list(map(int,input('coloumn 1 :').split()))
u2=list(map(int,input('coloumn 2 :').split()))
u3=list(map(int,input('coloumn 3 :').split()))
amat=transpose(Matrix((u1,u2,u3)))
display(amat)

miu1,miu2,miu3=Matrix(u1),Matrix(u2),Matrix(u3)
print('Let the given vectors be {a1,a2,a3}')
display((u1,u2,u3))

eq1=Eq(su1,sv1)
display(eq1)
display((sv1,eq,miu1))
v1=miu1
print('For our convinience we first find self inner products of v1')
v1s=simplify(v1.dot(v1))
ev1s=Bra(v1)*Ket(v1)
display((mv1**2,eq,bv1*kv1,eq,ev1s,eq,v1s))
eq2=Eq(((su2-((bv2*ku1)/(mv1**2))*su1)),sv2)
display(eq2)
u2v1=simplify(miu2.dot(v1))
display((bv1*ku2,eq,Bra(u2)*Ket(v1),eq,u2v1))
div1=simplify((u2v1/v1s))
v2=simplify(miu2-(div1*v1))
display((sv2,eq,v2))
v2s=simplify(v2.dot(v2))
display((bv2*kv2,eq,Bra(v2)*Ket(v2),eq,v2s))
eq3=((su3-(((bv3*ku1)/(mv1**2))*sv1)-((bv3*ku2)/(mv2**2))*sv2))
display((eq3,eq,sv3))
u3v1=simplify(miu3.dot(v1))
u3v2=simplify(miu3.dot(v2))
display((bv3*ku1,eq,Bra(u3)*Ket(v1),eq,u3v1))
display((bv3*ku2,eq,Bra(u3)*Ket(v2),eq,u3v2))
v2s=simplify(v2.dot(v2))
ev2s=Bra(v2)*Ket(v2)
display((mv2**2,eq,bv2*kv2,eq,ev2s,eq,v2s))
div2=simplify((u3v1/v1s))
div3=simplify((u3v2/v2s))
v3=simplify((miu3-(div2*v1)-(div3*v2)))
display((sv3,eq,v3))
v3s=simplify(v3.dot(v3))
```

```
ev3s=Bra(v3)*Ket(v3)
display((mv3**2,eq,bv3*kv3,eq,ev3s,eq,v3s))
print('Orthogonal Vectors We Got')
display((v1,v2,v3))
print('Orthonormal Basis')

w1,w2,w3,mul,div=sp.symbols('w1 w2 w3 * /')
display((w1,eq,(sv1/mv1)))
display((w2,eq,(sv2/mv2)))
display((w3,eq,(sv3/mv3)))
W1=simplify(v1/sqrt(v1s))
W2=simplify(v2/sqrt(v2s))
W3=simplify(v3/sqrt(v3s))
display((w1,eq,v1,div,simplify(sqrt(v1s))))
display((w1,eq,W1))
display((w2,eq,v2,div,simplify(sqrt(v2s))))
display((w2,eq,W2))
display((w3,eq,v3,div,simplify(sqrt(v3s))))
display((w3,eq,W3))
display((W1,W2,W3))
sq,sr,sa,st=symbols('Q R A T')
display((sq,eq,(((sv1/mv1)),((sv2/mv2)),((sv3/mv3)))))
display((sq,eq,transpose(Matrix((list(W1),list(W2),list(W3))))))
bw1,bw2,bw3=Bra('w1'),Bra('w2'),Bra('w3')
rmmat=Matrix(([bw1*ku1,bw1*ku2,bw1*ku3],[0,bw2*ku2,bw2*ku3],[0,0,bw3*ku3]))
display((sr,eq,rmmat))
a1w1=simplify(miu1.dot(W1))
display((bw1*ku1,eq,Bra(W1)*Ket(miu1),eq,a1w1))
a2w1=simplify(miu2.dot(W1))
display((bw1*ku2,eq,Bra(W1)*Ket(miu2),eq,a2w1))
a3w1=simplify(miu3.dot(W1))
display((bw1*ku3,eq,Bra(W1)*Ket(miu3),eq,a3w1))
a2w2=simplify(miu2.dot(W2))
display((bw2*ku2,eq,Bra(W2)*Ket(miu2),eq,a2w2))
a3w2=simplify(miu3.dot(W2))
display((bw2*ku3,eq,Bra(W2)*Ket(miu3),eq,a3w2))
a3w3=simplify(miu3.dot(W3))
display((bw3*ku3,eq,Bra(miu3)*Ket(W3),eq,a3w3))
rmat=Matrix(([a1w1,a2w1,a3w1],[0,a2w2,a3w2],[0,0,a3w3]))
print('One Last Check')
display((sa,eq,sq*sr,eq,transpose(Matrix((list(W1),list(W2),list(W3)))),mul,rmat,eq,transpose(Matrix((list(W1),list(W2),list(W3))))*rmat)
quit()
```

```
1 -- UNIT-I   -- SOLVING SYSTEM OF LINEAR EQUATIONS
2 -- UNIT-II  -- LINEAR DEPENDANCY AND BASIS FORMATION CHECKING
3 -- UNIT-III -- FINDING EIGEN VALUE AND EIGEN VECTOR
4 -- UNIT-IV  -- ORTHOGONALIZATION
5 -- UNIT-V   -- QR DECOMPOSITION
Enter Your Choice of Chapter : 5
_______________________----Welcome To Calculate QR decomposition----_______________________
coloumn 1 :3 -1 1
coloumn 2 :-1 5 -1
coloumn 3 :1 -1 3
```

```
c:\users\elcot\appdata\local\programs\python\python39\lib\site-packages\IPython\lib\latextools.py:126: MatplotlibDeprecationWarning:
The to_png function was deprecated in Matplotlib 3.4 and will be removed two minor releases later. Use mathtext.math_to_image instead.
  mt.to_png(f, s, fontsize=12, dpi=dpi, color=color)
c:\users\elcot\appdata\local\programs\python\python39\lib\site-packages\IPython\lib\latextools.py:126: MatplotlibDeprecationWarning:
The to_rgba function was deprecated in Matplotlib 3.4 and will be removed two minor releases later. Use mathtext.math_to_image instead.
  mt.to_png(f, s, fontsize=12, dpi=dpi, color=color)
c:\users\elcot\appdata\local\programs\python\python39\lib\site-packages\IPython\lib\latextools.py:126: MatplotlibDeprecationWarning:
The to_mask function was deprecated in Matplotlib 3.4 and will be removed two minor releases later. Use mathtext.math_to_image instead.
  mt.to_png(f, s, fontsize=12, dpi=dpi, color=color)
c:\users\elcot\appdata\local\programs\python\python39\lib\site-packages\IPython\lib\latextools.py:126: MatplotlibDeprecationWarning:
The MathtextBackendBitmap class was deprecated in Matplotlib 3.4 and will be removed two minor releases later. Use mathtext.math_to_image ins
tead.
  mt.to_png(f, s, fontsize=12, dpi=dpi, color=color)
```

$$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

Let the given vectors be {a1,a2,a3}

$$([3,\ -1,\ 1],\ [-1,\ 5,\ -1],\ [1,\ -1,\ 3])$$

$$a_1 = u_1$$

$$\left(u_1,\ =,\ \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}\right)$$

For our convinience we first find self inner products of v1

$$\left(||u1||^2,\ =,\ \langle u_1\ | u_1\rangle,\ =,\ \langle(3,\ -1,\ 1)\ |(3,\ -1,\ 1)\rangle,\ =,\ 11\right)$$

$$-\frac{a_1 \langle u_2\ | a_1\rangle}{||u1||^2} + a_2 = u_2$$

$$(\langle u_1\ | a_2\rangle,\ =,\ \langle(-1,\ 5,\ -1)\ |(3,\ -1,\ 1)\rangle,\ =,\ -9)$$

$$\left(u_2, \ =, \ \begin{bmatrix} \frac{16}{11} \\ \frac{46}{11} \\ -\frac{2}{11} \end{bmatrix}\right)$$

$$\left(\langle u_2 \,|u_2\rangle, \ =, \ \left\langle\left(\frac{16}{11}, \frac{46}{11}, \ -\frac{2}{11}\right)\middle|\left(\frac{16}{11}, \frac{46}{11}, \ -\frac{2}{11}\right)\right\rangle, \ =, \ \frac{216}{11}\right)$$

$$\left(a_3 - \frac{u_1 \langle u_3 \,|a_1\rangle}{||u1||^2} - \frac{u_2 \langle u_3 \,|a_2\rangle}{||u2||^2}, \ =, \ u_3\right)$$

$$(\langle u_3 \,|a_1\rangle, \ =, \ \langle(1, \ -1, \ 3) \,|(3, \ -1, \ 1)\rangle, \ =, \ 7)$$

$$\left(\langle u_3 \,|a_2\rangle, \ =, \ \left\langle(1, \ -1, \ 3)\middle|\left(\frac{16}{11}, \frac{46}{11}, \ -\frac{2}{11}\right)\right\rangle, \ =, \ -\frac{36}{11}\right)$$

$$\left(||u2||^2, \ =, \ \langle u_2 \,|u_2\rangle, \ =, \ \left\langle\left(\frac{16}{11}, \frac{46}{11}, \ -\frac{2}{11}\right)\middle|\left(\frac{16}{11}, \frac{46}{11}, \ -\frac{2}{11}\right)\right\rangle, \ =, \ \frac{216}{11}\right)$$

$$\left(u_3, \ =, \ \begin{bmatrix} -\frac{2}{3} \\ \frac{1}{3} \\ \frac{7}{3} \end{bmatrix}\right)$$

$$\left(||u3||^2, \ =, \ \langle u_3 \,|u_3\rangle, \ =, \ \left\langle\left(-\frac{2}{3}, \frac{1}{3}, \frac{7}{3}\right)\middle|\left(-\frac{2}{3}, \frac{1}{3}, \frac{7}{3}\right)\right\rangle, \ =, \ 6\right)$$

Orthogonal Vectors We Got

$$\left(\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{16}{11} \\ \frac{46}{11} \\ -\frac{2}{11} \end{bmatrix}, \begin{bmatrix} -\frac{2}{3} \\ \frac{1}{3} \\ \frac{7}{3} \end{bmatrix}\right)$$

Orthonormal Basis

$$\left(w_1, \ =, \ \frac{u_1}{||u1||}\right)$$

$$\left(w_2, \ =, \ \frac{u_2}{||u2||}\right)$$

$$\left(w_3, \ =, \ \frac{u_3}{||u3||}\right)$$

$$\left(w_1, \ =, \ \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, /, \sqrt{11}\right)$$

$$\left(w_1, \ =, \ \begin{bmatrix} \frac{3\sqrt{11}}{11} \\ -\frac{\sqrt{11}}{11} \\ \frac{\sqrt{11}}{11} \end{bmatrix}\right)$$

$$\left(w_2, \ =, \ \begin{bmatrix} \frac{16}{11} \\ \frac{46}{11} \\ -\frac{2}{11} \end{bmatrix}, /, \frac{6\sqrt{66}}{11}\right)$$

$$\left(w_2, \ =, \ \begin{bmatrix} \frac{4\sqrt{66}}{99} \\ \frac{23\sqrt{66}}{198} \\ -\frac{\sqrt{66}}{198} \end{bmatrix}\right)$$

$$\left(w_3, \ =, \ \begin{bmatrix} -\frac{2}{3} \\ \frac{1}{3} \\ \frac{7}{3} \end{bmatrix}, /, \sqrt{6}\right)$$

$$\left(w_3, \ =, \ \begin{bmatrix} -\frac{\sqrt{6}}{9} \\ \frac{\sqrt{6}}{18} \\ \frac{7\sqrt{6}}{18} \end{bmatrix}\right)$$

$$\left(\begin{bmatrix} \frac{3\sqrt{11}}{11} \\ -\frac{\sqrt{11}}{11} \\ \frac{\sqrt{11}}{11} \end{bmatrix}, \begin{bmatrix} \frac{4\sqrt{66}}{99} \\ \frac{23\sqrt{66}}{198} \\ -\frac{\sqrt{66}}{198} \end{bmatrix}, \begin{bmatrix} -\frac{\sqrt{6}}{9} \\ \frac{\sqrt{6}}{18} \\ \frac{7\sqrt{6}}{18} \end{bmatrix}\right)$$

$$\left(Q, \ =, \ \left(\frac{u_1}{||u1||}, \frac{u_2}{||u2||}, \frac{u_3}{||u3||}\right)\right)$$

$$\left(Q, \quad =, \quad \begin{bmatrix} \frac{3\sqrt{11}}{11} & \frac{4\sqrt{66}}{99} & -\frac{\sqrt{6}}{9} \\ -\frac{\sqrt{11}}{11} & \frac{23\sqrt{66}}{198} & \frac{\sqrt{6}}{18} \\ \frac{\sqrt{11}}{11} & -\frac{\sqrt{66}}{198} & \frac{7\sqrt{6}}{18} \end{bmatrix}\right)$$

$$\left(R, \quad =, \quad \begin{bmatrix} \langle w_1\ |a_1\rangle & \langle w_1\ |a_2\rangle & \langle w_1\ |a_3\rangle \\ 0 & \langle w_2\ |a_2\rangle & \langle w_2\ |a_3\rangle \\ 0 & 0 & \langle w_3\ |a_3\rangle \end{bmatrix}\right)$$

$$\left(\langle w_1\ |a_1\rangle, \quad =, \quad \left\langle \left(\frac{3\sqrt{11}}{11}, \ -\frac{\sqrt{11}}{11}, \ \frac{\sqrt{11}}{11}\right) |(3, \ -1, \ 1)\right\rangle, \quad =, \ \sqrt{11}\right)$$

$$\left(\langle w_1\ |a_2\rangle, \quad =, \quad \left\langle \left(\frac{3\sqrt{11}}{11}, \ -\frac{\sqrt{11}}{11}, \ \frac{\sqrt{11}}{11}\right) |(-1, \ 5, \ -1)\right\rangle, \quad =, \ -\frac{9\sqrt{11}}{11}\right)$$

$$\left(\langle w_1\ |a_3\rangle, \quad =, \quad \left\langle \left(\frac{3\sqrt{11}}{11}, \ -\frac{\sqrt{11}}{11}, \ \frac{\sqrt{11}}{11}\right) |(1, \ -1, \ 3)\right\rangle, \quad =, \ \frac{7\sqrt{11}}{11}\right)$$

$$\left(\langle w_2\ |a_2\rangle, \quad =, \quad \left\langle \left(\frac{4\sqrt{66}}{99}, \ \frac{23\sqrt{66}}{198}, \ -\frac{\sqrt{66}}{198}\right) |(-1, \ 5, \ -1)\right\rangle, \quad =, \ \frac{6\sqrt{66}}{11}\right)$$

$$\left(\langle w_2\ |a_3\rangle, \quad =, \quad \left\langle \left(\frac{4\sqrt{66}}{99}, \ \frac{23\sqrt{66}}{198}, \ -\frac{\sqrt{66}}{198}\right) |(1, \ -1, \ 3)\right\rangle, \quad =, \ -\frac{\sqrt{66}}{11}\right)$$

$$\left(\langle w_3\ |a_3\rangle, \quad =, \quad \left\langle (1, \ -1, \ 3) \middle| \left(-\frac{\sqrt{6}}{9}, \ \frac{\sqrt{6}}{18}, \ \frac{7\sqrt{6}}{18}\right)\right\rangle, \quad =, \ \sqrt{6}\right)$$

One Last Check

$$\left(A, \quad =, \ QR, \quad =, \quad \begin{bmatrix} \frac{3\sqrt{11}}{11} & \frac{4\sqrt{66}}{99} & -\frac{\sqrt{6}}{9} \\ -\frac{\sqrt{11}}{11} & \frac{23\sqrt{66}}{198} & \frac{\sqrt{6}}{18} \\ \frac{\sqrt{11}}{11} & -\frac{\sqrt{66}}{198} & \frac{7\sqrt{6}}{18} \end{bmatrix}, \ *, \quad \begin{bmatrix} \sqrt{11} & -\frac{9\sqrt{11}}{11} & \frac{7\sqrt{11}}{11} \\ 0 & \frac{6\sqrt{66}}{11} & -\frac{\sqrt{66}}{11} \\ 0 & 0 & \sqrt{6} \end{bmatrix}, \quad =, \quad \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}\right)$$

```
In [ ]:
```