Assignment 1

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1) Find the equation of a circle with centre $(2, 2)$ and passes through the point $(4, 5)$.

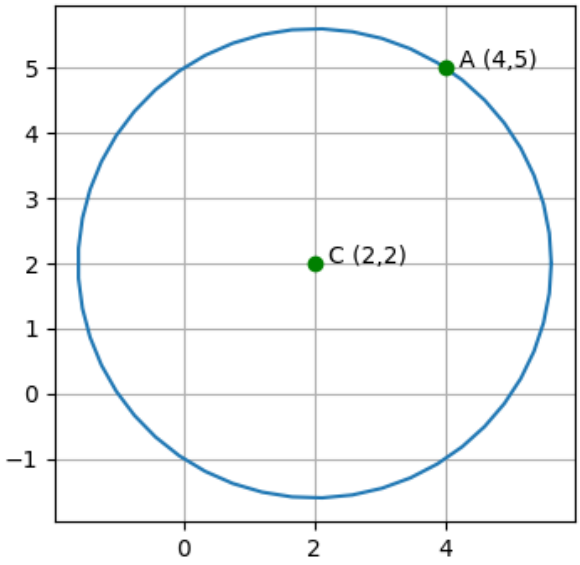

Fig. 1: Graph

Solution: We know that the equation to the circle is given as

$$\|\mathbf{x}\|^2 + 2\mathbf{x}^\top \mathbf{u} + f = 0 \tag{0.0.1}$$

Given the centre is $(2, 2)$ and a point $(4, 5)$ lies on circle

$$\mathbf{u} = -\begin{pmatrix}2\\2\end{pmatrix},\ \mathbf{A} = \begin{pmatrix}4\\5\end{pmatrix} \tag{0.0.2}$$

$$\|\mathbf{A}\|^2 + 2\mathbf{A}^\top \mathbf{u} + f = 0 \tag{0.0.3}$$

$$f = -\|\mathbf{A}\|^2 - 2\mathbf{A}^\top \mathbf{u} \tag{0.0.4}$$

$$f = -(4^2 + 5^2) - 2\begin{pmatrix}4 & 5\end{pmatrix}\begin{pmatrix}-2\\-2\end{pmatrix} \tag{0.0.5}$$

$$f = -41 - 2\,(4 \times -2 + 5 \times -2) \tag{0.0.6}$$

$$f = -5 \tag{0.0.7}$$

Hence the equation of circles is given by,

$$\|\mathbf{x}\|^2 + 2\mathbf{x}^\top \begin{pmatrix}-2\\-2\end{pmatrix} - 5 = 0 \tag{0.0.8}$$