Assignment 1

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1) In the triangle ABC with vertices $\mathbf{A}(2,3)$, $\mathbf{B}(4,-1)$ and $\mathbf{C}(1,2)$, find the equation and length of altitude from the vertex $\mathbf{A}$.

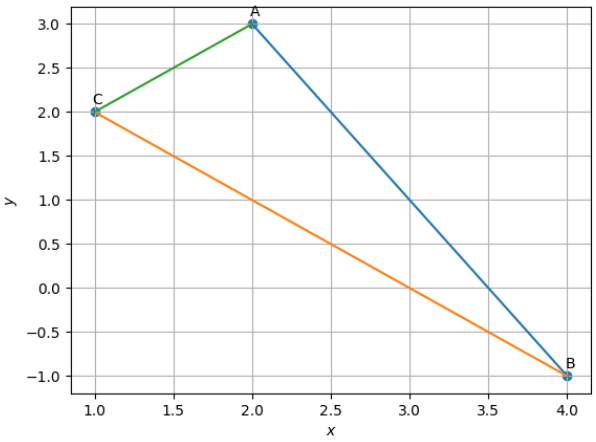

Fig. 1: Graph

Solution:

a) Given points are,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{0.0.1}$$

The normal vector for the altitude from vertex $\mathbf{A}$ is the direction vector of the line BC. The direction vector of the line BC is given by,

$$\mathbf{m}_{BC} = \mathbf{B} - \mathbf{C} \tag{0.0.2}$$

$$= \begin{pmatrix} 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{0.0.3}$$

$$= \begin{pmatrix} 3 \\ -3 \end{pmatrix} \tag{0.0.4}$$

$$\mathbf{m}_{BC} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{0.0.5}$$

$$\mathbf{n}_{BC} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.0.6}$$

The equation of line BC is given by,

$$\mathbf{n}_{BC}^{\top}\mathbf{x} = \mathbf{n}_{BC}^{\top}\mathbf{B} \tag{0.0.7}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ -1 \end{pmatrix} \tag{0.0.8}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 3 \tag{0.0.9}$$

The length of the altitude from a point $\mathbf{A}$ to the line $\mathbf{n}^{\top}\mathbf{x} = c$ is given by

$$d = \frac{|\mathbf{n}^{\top}\mathbf{A} - c|}{\|\mathbf{n}\|} \tag{0.0.10}$$

$$d = \frac{\left|\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} - 3\right|}{\left\|\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right\|} \tag{0.0.11}$$

$$d = \sqrt{2} \tag{0.0.12}$$

The length of the altitude from the vertex $\mathbf{A}$ to BC is $\sqrt{2}$ units.

b) The normal vector of the altitude from $\mathbf{A}$ is,

$$\mathbf{n} = \mathbf{m}_{BC} \tag{0.0.13}$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{0.0.14}$$

The equation of the altitide from vertex $\mathbf{A}$ is given by,

$$\mathbf{n}^{\top}\mathbf{x} = \mathbf{n}^{\top}\mathbf{A} \tag{0.0.15}$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{0.0.16}$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x} = -1 \tag{0.0.17}$$

The equation of the altitide from vertex $\mathbf{A}$ to the line BC is given by,

$$\begin{pmatrix} 1 & -1 \end{pmatrix}\mathbf{x} = -1 \tag{0.0.18}$$