Assignment 5

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1) Find the perpendicular distance from the origin to the line $x–y = 4$ and angle between perpendicular and the positive x-axis.
Solution: The given problem can be expressed as a constrained optimization problem as

$$\min_{\mathbf{x}} g(\mathbf{x}) = \|\mathbf{x} - \mathbf{P}\|^2 \tag{0.0.1}$$
$$\text{s.t.} \quad \mathbf{n}^T \mathbf{x} = c \tag{0.0.2}$$

where

$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.0.3}$$
$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{0.0.4}$$
$$c = 4 \tag{0.0.5}$$

The problem (0.0.1) can be modified into an unconstrained optimized problem as follows

$$\min_{\lambda} f(\lambda) = \|\mathbf{A} + \lambda\mathbf{m} - \mathbf{P}\|^2 \tag{0.0.6}$$

where $\mathbf{m}$ is the direction vector of the given line and $\mathbf{A}$ is any point on the line.

$$\mathbf{m} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{0.0.7}$$
$$\mathbf{A} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{0.0.8}$$

$$f(\lambda) = (\lambda\mathbf{m} + \mathbf{A} - \mathbf{P})^T (\lambda\mathbf{m} + \mathbf{A} - \mathbf{P}) \tag{0.0.9}$$
$$= \lambda^2 \|\mathbf{m}\|^2 + 2\lambda\mathbf{m}^T (\mathbf{A} - \mathbf{P}) + \|\mathbf{A} - \mathbf{P}\|^2 \tag{0.0.10}$$

Here we have,

$$f(\lambda) = 2\lambda^2 - 8\lambda + 16 \tag{0.0.11}$$

A numerical solution for (0.0.6) is obtained as

$$\lambda_{n+1} = \lambda_n - \alpha f'(\lambda_n) \tag{0.0.12}$$

where λ_0 is an inital guess and μ is a variable parameter. These parameters decide how fast the algorithm converges.

From (0.0.11) we get

$$\lambda_{n+1} = \lambda_n - \alpha (4\lambda_n - 8) \tag{0.0.13}$$

By taking the parameters as listed in the below table

Parameter	Description	Value
λ_0	Initial guess	-1
α	Variable parameter	0.01
N	Number of iterations	10000
ϵ	Tolerance in λ	10^{-6}

TABLE 1

λ obtained is

$$\lambda = 2 \tag{0.0.14}$$

Hence, from equation (0.0.11)

$$f(2) = 8 \tag{0.0.15}$$

Hence the perpendicular distance is $2\sqrt{2}$