Assignment 1

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1) A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that
   a) all will be blue?
   b) atleast one will be green?

   Solution:

   Lemma-1: The probability of the event where n marbles drawn such that there are - r red, b blue, g green marbles from the box which contains total of N marbles - R red, B blue, G green is $\frac{{}^{R}C_r\,{}^{B}C_b\,{}^{G}C_g}{{}^{N}C_n}$

   Proof:
   Total ways of choosing r red balls - ${}^{R}C_r$
   Total ways of choosing b blue balls - ${}^{B}C_b$
   Total ways of choosing g green balls - ${}^{G}C_g$
   The total number of ways of choosing n marbles such that there are r red, b blue, g green is ${}^{R}C_r\,{}^{B}C_b\,{}^{G}C_g$
   The total number of ways of choosing n marbles out of N marbles is ${}^{N}C_n$
   Hence the required probability is given by,

$$= \frac{{}^{P}C_p \times^{Q} C_q \times^{R} C_r}{{}^{N}C_n} \tag{0.0.1}$$

   Lemma-2:

$${}^{R}C_0\,{}^{B}C_n + {}^{R}C_1\,{}^{B}C_{n-1} + \cdots + {}^{R}C_n\,{}^{B}C_0 = {}^{R+B}C_n \tag{0.0.2}$$

   Proof:
   We solve this by relating the LHS of the equation (0.0.2) to some coefficient in a Binomial expansion. From Binomial theorem we have,

$$(a+b)^n = \sum_{k=0}^{n} {}^{n}C_k\, a^r\, b^{n-r} \tag{0.0.3}$$

   Consider the following Binomial expansions,

$$(x+1)^R = \sum_{k=0}^{R} {}^{R}C_k\, x^k \tag{0.0.4}$$

$$(x+1)^B = \sum_{m=0}^{B} {}^{B}C_m\, x^m \tag{0.0.5}$$

   Now lets take the product of the eqations (0.0.4), (0.0.5)

$$(x+1)^R\,(x+1)^B = \sum_{k=0}^{R}\sum_{m=0}^{B} {}^{R}C_k{}^{B}C_m\, x^{k+m} \tag{0.0.6}$$

$$\implies (x+1)^{R+B} = \sum_{k=0}^{R}\sum_{m=0}^{B} {}^{R}C_k{}^{B}C_m\, x^{k+m} \tag{0.0.7}$$

   From the RHS of (0.0.7), the required expression, LHS of (0.0.2) is the coefficient of x^n in the above equation. The coefficient of x^n in LHS of (0.0.7) is ${}^{R+B}C_n$.

$$\implies {}^{R}C_0\,{}^{B}C_n + {}^{R}C_1\,{}^{B}C_{n-1} + \cdots + {}^{R}C_n\,{}^{B}C_0 = {}^{R+B}C_n \tag{0.0.8}$$

   In this question, total marbles in the box are 60 - 10 red, 20 blue, 30 green. Out of which 5 balls are drawn. Hence we have,

$$N = 60,\ R = 10,\ B = 20,\ G = 30,\ n = 5 \tag{0.0.9}$$

   a) The probability that all drawn marbles are blue implies

$$r = 0,\ b = 5,\ G = 0 \tag{0.0.10}$$

   From (0.0.1) we get the probability as,

$$= \frac{{}^{20}C_5}{{}^{60}C_5} \tag{0.0.11}$$

   b) The probability that the drawn marble contains atleast 1 green, It is complement to the event where no marble drawn is green, For this event

$$g = 0 \tag{0.0.12}$$

From (0.0.1) its probability is given by,

$$= \frac{{}^{10}C_0\,{}^{20}C_5 + {}^{10}C_1\,{}^{20}C_4 + \cdots + {}^{10}C_5\,{}^{20}C_0}{{}^{60}C_5} \tag{0.0.13}$$

From (0.0.2) we get this as,

$$= \frac{{}^{10+20}C_5}{{}^{60}C_5} \tag{0.0.14}$$

$$= \frac{{}^{30}C_5}{{}^{60}C_5} \tag{0.0.15}$$

Hence the required probability is given by,

$$1 - \frac{{}^{30}C_5}{{}^{60}C_5} \tag{0.0.16}$$