Assignment 1

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1) A coin is tossed three times, where. Determine $\Pr(E \mid F)$ where
   a) E : head on third toss, F : heads on first two tosses
   b) E : at least two heads, F : at most two heads
   c) E : at most two tails, F : at least one tail

   Solution: Consider the random variables X_1, X_2, X_3, X, which denotes the first, second, third toss and number of heads in the 3 tosses respectively as described in table 1.

RV	Values	Description
X	$\{0,1,2,3\}$	Number of heads in 3 tosses
X_1	$\{0,1\}$	0: Heads , 1: Tails
X_2	$\{0,1\}$	0: Heads , 1: Tails
X_3	$\{0,1\}$	0: Heads , 1: Tails

TABLE 1: Random variables X_1, X_2, X_3, X

The random variable X follows binomial distribution

$$X = X_1 + X_2 + X_3 \tag{0.0.1}$$

The PMF of the random variable X is given by,

$$P_X(n) = \begin{cases} {}^3C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^3 = \frac{1}{8} & n = 0 \\ {}^3C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2 = \frac{3}{8} & n = 1 \\ {}^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 = \frac{3}{8} & n = 2 \\ {}^3C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0 = \frac{1}{8} & n = 3 \end{cases} \tag{0.0.2}$$

The CDF of the random variable X is given by,

$$F_X(n) = \Pr(X \le n) = \sum_{i=0}^{n} {}^NC_i p^i (1-p)^{N-i} \tag{0.0.3}$$

a) The events E, F can be described by the RV as

$$E : X_3 = 0 \tag{0.0.4}$$
$$F : X_1 + X_2 = 0 \tag{0.0.5}$$

The required probability is given by,

$$\Pr(X_3 = 0 \mid X_1 + X_2 = 0) \tag{0.0.6}$$
$$= \frac{\Pr(X = 0)}{\Pr(X_1 + X_2 = 0)} \tag{0.0.7}$$
$$= \frac{1}{2} \tag{0.0.8}$$

b) The events E, F, F' can be described by the RV as

$$E : X \le 1 \tag{0.0.9}$$
$$F : X \ge 1 \tag{0.0.10}$$
$$F' : X = 0 \tag{0.0.11}$$

The required probability is given by,

$$= \frac{\Pr(X = 1)}{1 - \Pr(X = 0)} \tag{0.0.12}$$
$$= \frac{\frac{3}{8}}{1 - \frac{1}{8}} \tag{0.0.13}$$
$$= \frac{3}{7} \tag{0.0.14}$$

c) For the events E, F, their complements are E' : all 3 tails, F' : zero tails. The events E', F' can be described by the RV as

$$E' : X = 3 \tag{0.0.15}$$
$$F' : X = 0 \tag{0.0.16}$$

By using property of conditional probability we have,

$$\Pr(E \mid F) = \frac{\Pr(EF)}{\Pr(F)} \tag{0.0.17}$$
$$= \frac{1 - \Pr(E' \text{ or } F')}{\Pr(F)} \tag{0.0.18}$$

The required probability is given by,

$$= \frac{1 - \Pr\left(X = 0 \text{ or } 3\right)}{1 - \Pr\left(X = 0\right)} \tag{0.0.19}$$

$$= \frac{1 - \left(\frac{1}{8} + \frac{1}{8}\right)}{1 - \frac{1}{8}} \tag{0.0.20}$$

$$= \frac{6}{7} \tag{0.0.21}$$