Assignment 5

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1) A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.
Solution: Let the given side of tin be a.

$$a = 18cm \tag{0.0.1}$$

Lets cut a square of side x from each corner then the box formed folding up the flaps has dimensions as

$$l = a - 2x,\ b = a - 2x,\ h = x \tag{0.0.2}$$

The length, breadth, height are positive. These give the constraints on x

$$a - 2x > 0,\ x > 0 \tag{0.0.3}$$
$$\implies 0 < x < \frac{a}{2} \tag{0.0.4}$$

Volume of the box is given by

$$V(x) = x(a - 2x)^2 \tag{0.0.5}$$

a) We now check the convexity of the function $V(x)$ under the constraints given by (**??**)

$$V'(x) = (a - 2x)(a - 6x) \tag{0.0.6}$$
$$V''(x) = 8(3x - a) \tag{0.0.7}$$

For $x > \frac{a}{3}$ the function $V(x)$ is convex. The function convexity is changing under the constraints. So the given problem cannot be expressed as a Convex optimization problem.

b) The problem can be solved using the Gradient Descent Algorithm. It can be modified as

$$\min_x f(x) = -V(x) \tag{0.0.8}$$

Here we have,

$$f(x) = -x(a - 2x)^2 \tag{0.0.9}$$
$$f'(x) = -(a - 2x)(a - 6x) \tag{0.0.10}$$

A numerical solution for (**??**) is obtained as

$$\lambda_{n+1} = \lambda_n - \alpha f'(x) \tag{0.0.11}$$

where λ_0 is an inital guess and μ is a variable parameter. These parameters decide how fast the algorithm converges. By taking the parameters as listed in the below table

Parameter	**Description**	**Value**
λ_0	Initial guess	8.5
α	Variable parameter	0.01
N	Number of iterations	10000
ϵ	Tolerance in λ	10^{-6}

TABLE 1

The value of x obtained is

$$x = 3 \tag{0.0.12}$$

From equation (**??**), the minimum value of $f(x)$ is equivalent to maximum value of $V(x)$. Hence the maximum value of $V(x)$ occurs at $x = 3$.

$$V(3) = 432 \tag{0.0.13}$$