

Assignment 1

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- 1) Find the area of the triangle with vertices $\mathbf{A}(1,1,2)$, $\mathbf{B}(2,3,5)$, $\mathbf{C}(1,5,5)$.

Solution: The area of the triangle ABC is given by

$$ar(ABC) = \frac{1}{2} \|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A})\| \quad (0.0.1)$$

given points are $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 \\ 5 \\ 5 \end{pmatrix}$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad (0.0.2)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} \quad (0.0.3)$$

$$\text{area} = \left\| \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} \right\| \quad (0.0.4)$$

$$(0.0.5)$$

The cross product or vector product of \mathbf{P}, \mathbf{Q} is defined as

$$\mathbf{P} \times \mathbf{Q} = \begin{pmatrix} \mathbf{P}_{23} & \mathbf{Q}_{23} \\ \mathbf{P}_{31} & \mathbf{Q}_{31} \\ \mathbf{P}_{12} & \mathbf{Q}_{12} \end{pmatrix} \quad (0.0.6)$$

$$|\mathbf{P}_{23} \quad \mathbf{Q}_{23}| = \begin{vmatrix} 2 & 4 \\ 3 & 3 \end{vmatrix} = -6 \quad (0.0.7)$$

$$|\mathbf{P}_{31} \quad \mathbf{Q}_{31}| = \begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} = 3 \quad (0.0.8)$$

$$|\mathbf{P}_{12} \quad \mathbf{Q}_{12}| = \begin{vmatrix} 1 & 0 \\ 2 & 4 \end{vmatrix} = 4 \quad (0.0.9)$$

Hence

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \\ 4 \end{pmatrix} \quad (0.0.10)$$

The area of the triangle is given by,

$$ar(ABC) = \frac{1}{2} \left\| \begin{pmatrix} -6 \\ 3 \\ 4 \end{pmatrix} \right\| \quad (0.0.11)$$

$$= \frac{1}{2} \sqrt{36 + 9 + 16} \quad (0.0.12)$$

$$= \frac{\sqrt{61}}{2} \quad (0.0.13)$$

The area of the triangle is $\frac{\sqrt{61}}{2}$ square units.