Assignment 5

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1) A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.
Solution: Let the given side of tin be a.

$$a = 18cm \tag{0.0.1}$$

Lets cut a square of side x from each corner then the box formed folding up the flaps has dimensions as

$$l = a - 2x,\ b = a - 2x,\ h = x \tag{0.0.2}$$

The length, breadth, height are positive. These give the constraints on x

$$a - 2x > 0,\ x > 0 \tag{0.0.3}$$

$$\implies 0 < x < \frac{a}{2} \tag{0.0.4}$$

Volume of the box is given by

$$V(x) = x(a - 2x)^2 \tag{0.0.5}$$

The given problem can be expressed as a constrained optimization problem as

$$\max_x f(x) \triangleq V(x) \tag{0.0.6}$$

$$\text{s.t.} \quad g_1(x) \triangleq -x \le 0 \tag{0.0.7}$$

$$g_2(x) \triangleq x \le \frac{a}{2} \tag{0.0.8}$$

Define

$$L(x, \lambda, \mu) = f(x) + \lambda g_1(x) + \mu g_2(x) \tag{0.0.9}$$

Using the KKT conditions for finding the optimal point we have,

$$\implies \nabla L(x, \lambda, \mu) = 0 \tag{0.0.10}$$

$$\text{subject to} \quad \lambda g_1(x) = 0 \tag{0.0.11}$$

$$\mu g_2(x) = 0 \tag{0.0.12}$$

$$\lambda \ge 0, \mu \ge 0 \tag{0.0.13}$$

Here we get

$$\nabla f(x) = (a - 2x)(a - 6x) \tag{0.0.14}$$

$$\nabla g_1(x) = -1 \tag{0.0.15}$$

$$\nabla g_2(x) = 1 \tag{0.0.16}$$

By using the equations (0.0.14), (0.0.15), (0.0.16) we get

$$(a - 2x)(a - 6x) - \lambda + \mu = 0 \tag{0.0.17}$$

$$\lambda = 0 \tag{0.0.18}$$

$$\mu = 0 \tag{0.0.19}$$

Hence the value of x is given by,

$$x = \frac{a}{2}, \frac{a}{6} \tag{0.0.20}$$

For the value of $x = \frac{a}{2}$, is a boundary point for which the volume is zero. Hence the maximum volume is given by

$$\max V(x) = V\left(\frac{a}{6}\right) \tag{0.0.21}$$

$$= \frac{2}{27}a^3 \tag{0.0.22}$$

$$= 432 \tag{0.0.23}$$