Assignment 1

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1) Let $\overrightarrow{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\overrightarrow{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\overrightarrow{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\overrightarrow{d}$ which is perpendicular to both $\overrightarrow{a}$ and $\overrightarrow{b}$, and $\overrightarrow{c}.\overrightarrow{d} = 15$.
Solution: The vector perpendicular to both **A** and **B** has the direction that of $\mathbf{A} \times \mathbf{B}$.
Here we have

$$\mathbf{A} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \tag{0.0.1}$$

Given that vector $\overrightarrow{d}$ is perpendicular to both $\overrightarrow{a}$ and $\overrightarrow{b}$, and $\overrightarrow{c}.\overrightarrow{d} = 15$

$$\mathbf{A}^\top \mathbf{D} = 0 \tag{0.0.2}$$

$$\mathbf{B}^\top \mathbf{D} = 0 \tag{0.0.3}$$

$$\mathbf{C}^\top \mathbf{D} = 15 \tag{0.0.4}$$

Joining all the equations in matrix form gives,

$$\begin{pmatrix} \mathbf{A}^\top \\ \mathbf{B}^\top \\ \mathbf{C}^\top \end{pmatrix} \mathbf{D} = \begin{pmatrix} 0 \\ 0 \\ 15 \end{pmatrix} \tag{0.0.5}$$

$$\begin{pmatrix} 1 & 4 & 2 \\ 3 & -2 & 7 \\ 2 & -1 & 4 \end{pmatrix} \mathbf{D} = \begin{pmatrix} 0 \\ 0 \\ 15 \end{pmatrix} \tag{0.0.6}$$

The augmented matrix for the system equations in (0.0.6) is expressed as

$$\left(\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 3 & -2 & 7 & 0 \\ 2 & -1 & 4 & 15 \end{array}\right) \tag{0.0.7}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - 2R_1]{R_2 \leftarrow R_2 - 3R_1} \tag{0.0.8}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & -14 & 1 & 0 \\ 0 & -9 & 0 & 15 \end{array}\right) \tag{0.0.9}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - \frac{9}{14} R_2} \tag{0.0.10}$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & -14 & 1 & 0 \\ 0 & 0 & -\dfrac{9}{14} & 15 \end{array}\right) \tag{0.0.11}$$

The augmented matrix for the system equations is reduced to Row echelon form, From the above equation 0.0.11 we get the vector **D** as

$$\mathbf{D} = \begin{pmatrix} \frac{160}{3} \\ -\frac{5}{3} \\ -\frac{70}{3} \end{pmatrix} \tag{0.0.12}$$