Assignment 1

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1) A couple has two children,

  a) Find the probability that both children are males, if it is known that at least one of the children is male.

  b) Find the probability that both children are females, if it is known that the elder child is a female.

Solution: Consider the random variables X, Y, which denotes the first child, second child gender respectively as described in table 1.

RV	Values	Description
X	$\{0, 1\}$	0: Male , 1: Female
Y	$\{0, 1\}$	0: Male, 1: Female

TABLE 1: Random variables X

The probabilities for the random variables X, Y is listed in table 1.

Event	Probability
$\Pr(X = 0)$	$\frac{1}{2}$
$\Pr(X = 1)$	$\frac{1}{2}$
$\Pr(Y = 0)$	$\frac{1}{2}$
$\Pr(Y = 1)$	$\frac{1}{2}$
$\Pr(X + Y = 0)$	$\frac{1}{4}$
$\Pr(X + Y = 2)$	$\frac{1}{4}$
$\Pr(X = 0 \text{ or } Y = 0)$	$\frac{3}{4}$

TABLE 1: Probabilities

The probability $\Pr(X = 0 \text{ or } Y = 0)$ is given by

$$= \Pr(X = 0) + \Pr(Y = 0) - \Pr(X + Y = 0) \tag{0.0.1}$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{4} \tag{0.0.2}$$

$$= \frac{3}{4} \tag{0.0.3}$$

a) The event of both children being Male is when $X + Y = 0$. The event of atleast one of the children being Male is when $X = 0$ or $Y = 0$.

$$\{X + Y = 0\} \cap \{X = 0 \text{ or } Y = 0\} \equiv \{X + Y = 0\} \tag{0.0.4}$$

The required probability is given by,

$$\Pr(X + Y = 0 \mid X = 0 \text{ or } Y = 0) \tag{0.0.5}$$

$$= \frac{\Pr(X + Y = 0)}{\Pr(X = 0 \text{ or } Y = 0)} \tag{0.0.6}$$

$$= \frac{1}{3} \tag{0.0.7}$$

b) The event of both children being Female is when $X + Y = 2$. The event of elder child being Female is when $X = 1$.

$$\{X + Y = 2\} \cap \{X = 1\} \equiv \{X + Y = 2\} \tag{0.0.8}$$

The required probability is given by,

$$\Pr(X + Y = 2 \mid X = 1) \tag{0.0.9}$$

$$= \frac{\Pr(X + Y = 2)}{\Pr(X = 1)} \tag{0.0.10}$$

$$= \frac{1}{2} \tag{0.0.11}$$