Assignment 5

Jaswanth Chowdary Madala

1) Find the perpendicular distance from the origin to the line x–y = 4 and angle between perpendicular and the positive x-axis.
Solution: The given problem can be expressed as a constrained optimization problem as

$$\min_{\mathbf{x}} f(\mathbf{x}) \triangleq \|\mathbf{x} - \mathbf{P}\|^2 \tag{0.0.1}$$

$$\text{s.t.} \quad g(\mathbf{x}) \triangleq \mathbf{n}^T \mathbf{x} - c = 0 \tag{0.0.2}$$

where

$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, c = 4 \tag{0.0.3}$$

Define

$$L(\mathbf{x}, \lambda) = f(\mathbf{x}) - \lambda g(\mathbf{x}) \tag{0.0.4}$$

Here we find the optimal point, $\mathbf{Q}$ that is closest to the point $\mathbf{P}$, by finding λ using the following equation

$$\nabla L(\mathbf{x}, \lambda) = 0 \tag{0.0.5}$$

Here we have,

$$\nabla f(\mathbf{x}) = 2(\mathbf{x} - \mathbf{P}) \tag{0.0.6}$$

$$\nabla g(\mathbf{x}) = \mathbf{n} \tag{0.0.7}$$

From (0.0.5), (0.0.6), (0.0.7) we get

$$2(\mathbf{x} - \mathbf{P}) - \lambda \mathbf{n} = 0 \tag{0.0.8}$$

$$\implies \mathbf{x} = \frac{\lambda}{2}\mathbf{n} + \mathbf{P} \tag{0.0.9}$$

The point $\mathbf{x}$ lies on the given line (0.0.2)

$$\mathbf{n}^\top \left(\frac{\lambda}{2}\mathbf{n} + \mathbf{P}\right) - c = 0 \tag{0.0.10}$$

$$\implies \lambda = -\frac{2(\mathbf{n}^\top \mathbf{P} - c)}{\|\mathbf{n}\|^2} \tag{0.0.11}$$

Substituting (0.0.11) in (0.0.9), we get the optimal point $\mathbf{Q}$ as

$$\mathbf{Q} = \mathbf{P} - \frac{\mathbf{n}^\top \mathbf{P} - c}{\|\mathbf{n}\|^2}\mathbf{n} \tag{0.0.12}$$

Substituting (0.0.3) in (0.0.11), (0.0.12) gives

$$\lambda = 4 \tag{0.0.13}$$

$$\mathbf{Q} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} \tag{0.0.14}$$

Hence the perpendicular distance is given by

$$d = \|\mathbf{Q} - \mathbf{P}\| \tag{0.0.15}$$

$$= 2\sqrt{2} \tag{0.0.16}$$