Assignment 1

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1) A coin is tossed three times, where. Determine $\Pr(E \mid F)$ where
   a) E : head on third toss, F : heads on first two tosses
   b) E : at least two heads, F : at most two heads
   c) E : at most two tails, F : at least one tail

   Solution: Consider the random variables X_1, X_2, X_3, which denotes the first, second and third toss respectively as described in table 1.

RV	Values	Description
X_1	$\{0, 1\}$	0: Heads , 1: Tails
X_2	$\{0, 1\}$	0: Heads , 1: Tails
X_3	$\{0, 1\}$	0: Heads , 1: Tails

TABLE 1: Random variables X_1, X_2, X_3

The probabilities for the random variables X_1, X_2, X_3 is listed in table 1.

Event	Probability
$\Pr(X_1 + X_2 + X_3 = 0)$	$\frac{1}{8}$
$\Pr(X_1 + X_2 + X_3 = 3)$	$\frac{1}{8}$
$\Pr(X_1 + X_2 + X_3 = 1)$	$\frac{3}{8}$
$\Pr(X_1 + X_2 = 0)$	$\frac{1}{4}$

TABLE 1: Probabilities

By using property of conditional probability we have,

$$\Pr(E \mid F) = \frac{\Pr(EF)}{\Pr(F)} \tag{0.0.1}$$

$$= \frac{1 - \Pr(E' \text{ or } F')}{\Pr(F)} \tag{0.0.2}$$

a) The events E, F can be described by the RV as

$$E : X_3 = 0 \tag{0.0.3}$$

$$F : X_1 + X_2 = 0 \tag{0.0.4}$$

The required probability is given by,

$$\Pr(X_3 = 0 \mid X_1 + X_2 = 0) \tag{0.0.5}$$

$$= \frac{\Pr(X_1 + X_2 + X_3 = 0)}{\Pr(X_1 + X_2 = 0)} \tag{0.0.6}$$

$$= \frac{1}{2} \tag{0.0.7}$$

b) The events E, F, F' can be described by the RV as

$$E : X_1 + X_2 + X_3 \le 1 \tag{0.0.8}$$

$$F : X_1 + X_2 + X_3 \ge 1 \tag{0.0.9}$$

$$F' : X_1 + X_2 + X_3 = 0 \tag{0.0.10}$$

The required probability is given by,

$$= \frac{\Pr(X_1 + X_2 + X_3 = 1)}{1 - \Pr(X_1 + X_2 + X_3 = 0)} \tag{0.0.11}$$

$$= \frac{\frac{3}{8}}{1 - \frac{1}{8}} \tag{0.0.12}$$

$$= \frac{3}{7} \tag{0.0.13}$$

c) For the events E, F, their complements are E' : all 3 tails, F' : zero tails.
The events E', F' can be described by the RV as

$$E' : X_1 + X_2 + X_3 = 3 \tag{0.0.14}$$

$$F' : X_1 + X_2 + X_3 = 0 \tag{0.0.15}$$

The required probability is given by,

$$= \frac{1 - \Pr(X_1 + X_2 + X_3 = 0 \text{ or } 3)}{1 - \Pr(X_1 + X_2 + X_3 = 0)} \tag{0.0.16}$$

$$= \frac{1 - \left(\frac{1}{8} + \frac{1}{8}\right)}{1 - \frac{1}{8}} \tag{0.0.17}$$

$$= \frac{6}{7} \tag{0.0.18}$$