

# Assignment 1

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1) A coin is tossed three times, where. Determine  $\Pr(E | F)$  where

a)  $E$  : head on third toss,  $F$  : heads on first two tosses

b)  $E$  : at least two heads,  $F$  : at most two heads

c)  $E$  : at most two tails,  $F$  : at least one tail

**Solution:** Consider the random variables  $X_1, X_2, X_3, X$ , which denotes the first, second, third toss and number of heads in the 3 tosses respectively as described in table 1.

RV	Values	Description
$X$	$\{0, 1, 2, 3\}$	Number of heads in 3 tosses
$X_1$	$\{0, 1\}$	0: Heads , 1: Tails
$X_2$	$\{0, 1\}$	0: Heads , 1: Tails
$X_3$	$\{0, 1\}$	0: Heads , 1: Tails

TABLE 1: Random variables  $X_1, X_2, X_3, X$

The random variable  $X$  follows binomial distribution

$$X = X_1 + X_2 + X_3 \quad (0.0.1)$$

The PMF of the random variable  $X$  is given by,

$$P_X(n) = {}^N C_n p^n (1-p)^{N-n} \quad (0.0.2)$$

Here we have

$$N = 3, p = \frac{1}{2} \quad (0.0.3)$$

Hence we get the PMF as,

$$P_X(n) = \begin{cases} {}^3 C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^3 = \frac{1}{8} & n = 0 \\ {}^3 C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2 = \frac{3}{8} & n = 1 \\ {}^3 C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 = \frac{3}{8} & n = 2 \\ {}^3 C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0 = \frac{1}{8} & n = 3 \end{cases} \quad (0.0.4)$$

The CDF of the random variable  $X$  is given by,

$$F_X(n) = \Pr(X \leq n) = \sum_{i=0}^n {}^N C_i p^i (1-p)^{N-i} \quad (0.0.5)$$

a) The events  $E, F$  can be described by the RV as

$$E : X_3 = 0 \quad (0.0.6)$$

$$F : X_1 + X_2 = 0 \quad (0.0.7)$$

The required probability is given by,

$$\Pr(X_3 = 0 | X_1 + X_2 = 0) \quad (0.0.8)$$

$$= \frac{\Pr(X = 0)}{\Pr(X_1 + X_2 = 0)} \quad (0.0.9)$$

$$= \frac{1}{2} \quad (0.0.10)$$

b) The events  $E, F, F'$  can be described by the RV as

$$E : X \leq 1 \quad (0.0.11)$$

$$F : X \geq 1 \quad (0.0.12)$$

$$F' : X = 0 \quad (0.0.13)$$

The required probability is given by,

$$= \frac{\Pr(X = 1)}{1 - \Pr(X = 0)} \quad (0.0.14)$$

$$= \frac{\frac{3}{8}}{1 - \frac{1}{8}} \quad (0.0.15)$$

$$= \frac{3}{7} \quad (0.0.16)$$

c) For the events  $E, F$ , their complements are  $E'$  : all 3 tails,  $F'$  : zero tails. The events  $E', F'$  can be described by the RV as

$$E' : X = 3 \quad (0.0.17)$$

$$F' : X = 0 \quad (0.0.18)$$

By using property of conditional probability we have,

$$\Pr(E | F) = \frac{\Pr(EF)}{\Pr(F)} \quad (0.0.19)$$

$$= \frac{1 - \Pr(E' \text{ or } F')}{\Pr(F)} \quad (0.0.20)$$

The required probability is given by,

$$= \frac{1 - \Pr(X = 0 \text{ or } 3)}{1 - \Pr(X = 0)} \quad (0.0.21)$$

$$= \frac{1 - \left(\frac{1}{8} + \frac{1}{8}\right)}{1 - \frac{1}{8}} \quad (0.0.22)$$

$$= \frac{6}{7} \quad (0.0.23)$$