

Assignment 1

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- 1) If a line intersects two concentric circles (circles with the same centre) with centre **O** at **A**, **B**, **C** and **D**, prove that $AB = CD$.

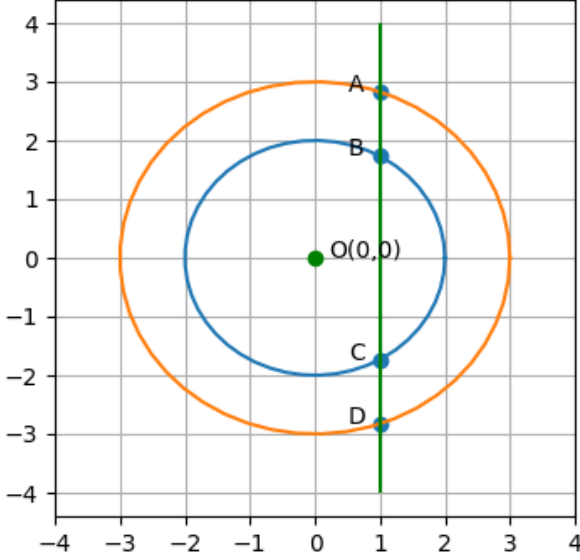


Fig. 1: Graph

Solution: Let the equations of two concentric circles be,

$$\|\mathbf{x}\|^2 = 4 \quad (0.0.1)$$

$$\|\mathbf{x}\|^2 = 9 \quad (0.0.2)$$

Let the equation of the line be,

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (0.0.3)$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ \lambda \end{pmatrix} \quad (0.0.4)$$

The points of intersection of circle (0.0.1) and

the line (0.0.4) **B**, **C** are given by,

$$\|\mathbf{x}\|^2 = 4 \quad (0.0.5)$$

$$1^2 + \lambda^2 = 4 \quad (0.0.6)$$

$$\lambda^2 = 3 \quad (0.0.7)$$

$$\lambda = \pm \sqrt{3} \quad (0.0.8)$$

$$\mathbf{B} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \quad (0.0.9)$$

The points of intersection of circle (0.0.2) and the line (0.0.4) **A**, **D** are given by,

$$\|\mathbf{x}\|^2 = 9 \quad (0.0.10)$$

$$1^2 + \lambda^2 = 9 \quad (0.0.11)$$

$$\lambda^2 = 8 \quad (0.0.12)$$

$$\lambda = \pm 2\sqrt{2} \quad (0.0.13)$$

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2\sqrt{2} \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 1 \\ -2\sqrt{2} \end{pmatrix} \quad (0.0.14)$$

$$\|\mathbf{AB}\| = \|\mathbf{A} - \mathbf{B}\| \quad (0.0.15)$$

$$= \left\| \begin{pmatrix} 0 \\ 2\sqrt{2} - \sqrt{3} \end{pmatrix} \right\| \quad (0.0.16)$$

$$= 2\sqrt{2} - \sqrt{3} \quad (0.0.17)$$

$$\|\mathbf{CD}\| = \|\mathbf{C} - \mathbf{D}\| \quad (0.0.18)$$

$$= \left\| \begin{pmatrix} 0 \\ 2\sqrt{2} - \sqrt{3} \end{pmatrix} \right\| \quad (0.0.19)$$

$$= 2\sqrt{2} - \sqrt{3} \quad (0.0.20)$$

Hence $AB = CD$.