Assignment 1

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1) Show that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ form the vertices of a right angled triangle.
Solution: Let us first check whether the given points form a triangle. Let us consider,

$$\mathbf{A}\begin{pmatrix}2\\-1\\1\end{pmatrix}, \mathbf{B}\begin{pmatrix}1\\-3\\-5\end{pmatrix}, \mathbf{C}\begin{pmatrix}3\\-4\\-4\end{pmatrix} \tag{0.0.1}$$

To check whether the points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ form a triangle, we find the rank of the matrix $\begin{pmatrix}\mathbf{A} & \mathbf{B} & \mathbf{C}\end{pmatrix}$

$$\begin{pmatrix}2 & 1 & 3\\-1 & -3 & -4\\1 & -5 & -4\end{pmatrix} \tag{0.0.2}$$

$$\xleftrightarrow[R_2\leftarrow R_2+\frac{1}{2}R_1]{R_3\leftarrow R_3-\frac{1}{2}R_1} \tag{0.0.3}$$

$$\begin{pmatrix}2 & 1 & 3\\0 & -\frac{5}{2} & -\frac{5}{2}\\0 & -\frac{11}{2} & -\frac{11}{2}\end{pmatrix} \tag{0.0.4}$$

$$\xleftrightarrow{R_3\leftarrow R_3-\frac{11}{5}R_2} \tag{0.0.5}$$

$$\begin{pmatrix}2 & 1 & 3\\0 & -\frac{5}{2} & -\frac{5}{2}\\0 & 0 & 0\end{pmatrix} \tag{0.0.6}$$

The rank of the matrix is 2 and the points are in 3-Dimensional space, So the points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ form a triangle.

Now, we check whether the triangle is right angled at any of the vertices - $\mathbf{A}, \mathbf{B}, \mathbf{C}$.
For a right angled triangle ABC which is right angled at $\mathbf{A}$

$$(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A}) = 0 \tag{0.0.7}$$

a) checking whether the triangle is right angled at $\mathbf{A}$

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix}-1\\-2\\-6\end{pmatrix} \tag{0.0.8}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix}1\\-3\\-5\end{pmatrix} \tag{0.0.9}$$

$$(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A}) = \begin{pmatrix}-1 & -2 & -6\end{pmatrix}\begin{pmatrix}1\\-3\\-5\end{pmatrix} = 35 \tag{0.0.10}$$

$$(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A}) \neq 0 \tag{0.0.11}$$

The triangle is not right angled at $\mathbf{A}$.

b) checking whether the triangle is right angled at $\mathbf{B}$

$$\mathbf{A}-\mathbf{B} = \begin{pmatrix}1\\2\\6\end{pmatrix} \tag{0.0.12}$$

$$\mathbf{C}-\mathbf{B} = \begin{pmatrix}2\\-1\\1\end{pmatrix} \tag{0.0.13}$$

$$(\mathbf{A}-\mathbf{B})^\top(\mathbf{C}-\mathbf{B}) = \begin{pmatrix}1 & 2 & 6\end{pmatrix}\begin{pmatrix}2\\-1\\1\end{pmatrix} = 6 \tag{0.0.14}$$

$$(\mathbf{A}-\mathbf{B})^\top(\mathbf{C}-\mathbf{B}) \neq 0 \tag{0.0.15}$$

The triangle is not right angled at $\mathbf{B}$.

c) checking whether the triangle is right angled at $\mathbf{C}$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \tag{0.0.16}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} \tag{0.0.17}$$

$$(\mathbf{A} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}) = \begin{pmatrix} -1 & 3 & 5 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} = 0 \tag{0.0.18}$$

$$(\mathbf{A} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}) = 0 \tag{0.0.19}$$

Hence the triangle is right angled at $\mathbf{C}$.