Assignment 1

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1) Point $\mathbf{R}(h, k)$ divides a line segment between the axes in the ratio $1: 2$. Find equation of the line.

Solution: Let the line segment between the axes be AB, with point $\mathbf{A}$ on X-axis, $\mathbf{B}$ on Y-axis. Let the points $\mathbf{A}$, $\mathbf{B}$ be

$$\mathbf{A}\begin{pmatrix} \alpha \\ 0 \end{pmatrix} \tag{0.0.1}$$

$$\mathbf{B}\begin{pmatrix} 0 \\ \beta \end{pmatrix} \tag{0.0.2}$$

Given that $\dfrac{AR}{RB} = \dfrac{1}{2}$. By using section formula, we get

$$\mathbf{R} = \frac{2\mathbf{A} + \mathbf{B}}{3} \tag{0.0.3}$$

$$\begin{pmatrix} h \\ k \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2\alpha \\ \beta \end{pmatrix} \tag{0.0.4}$$

$$h = \frac{2\alpha}{3} \tag{0.0.5}$$

$$k = \frac{\beta}{3} \tag{0.0.6}$$

The points $\mathbf{A}\begin{pmatrix} \frac{3h}{2} \\ 0 \end{pmatrix}$, $\mathbf{B}\begin{pmatrix} 0 \\ 3k \end{pmatrix}$.

The direction vector of the line is given by,

$$\mathbf{m} = \mathbf{R} - \mathbf{B} \tag{0.0.7}$$

$$\mathbf{m} = \begin{pmatrix} h \\ -2k \end{pmatrix} \tag{0.0.8}$$

The normal vector to the line is given by,

$$\mathbf{n} = \begin{pmatrix} 2k \\ h \end{pmatrix} \tag{0.0.9}$$

The equation of line is given by,

$$\mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{B} \tag{0.0.10}$$

$$\begin{pmatrix} 2k & h \end{pmatrix}\mathbf{x} = \begin{pmatrix} 2k & h \end{pmatrix}\begin{pmatrix} 0 \\ 3k \end{pmatrix} \tag{0.0.11}$$

$$\begin{pmatrix} 2k & h \end{pmatrix}\mathbf{x} = 3hk \tag{0.0.12}$$

The equation of the line is given by,
$$\begin{pmatrix} 2k & h \end{pmatrix}\mathbf{x} = 3hk$$