

Assignment 1

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- 1) Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right angled triangle.

Solution: Let us first check whether the given points form a triangle. Let us consider,

$$\mathbf{A} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \mathbf{B} \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix} \mathbf{C} \begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix} \quad (0.0.1)$$

To check whether the points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ form a triangle, we find the rank of the matrix $\begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$

$$\begin{pmatrix} 2 & 1 & 3 \\ -1 & -3 & -4 \\ 1 & -5 & -4 \end{pmatrix} \quad (0.0.2)$$

$$R_2 \longrightarrow R_2 + \frac{1}{2}R_1, R_3 \longrightarrow R_3 - \frac{1}{2}R_1$$

$$\begin{pmatrix} 2 & 1 & 3 \\ 0 & -\frac{5}{2} & -\frac{5}{2} \\ 0 & -\frac{11}{2} & -\frac{11}{2} \end{pmatrix} \quad (0.0.3)$$

For a right angled triangle ABC which is right angled at \mathbf{A}

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{A}) = 0 \quad (0.0.4)$$

$$R_3 \longrightarrow R_3 - \frac{11}{5}R_2$$

$$\begin{pmatrix} 2 & 1 & 3 \\ 0 & -\frac{5}{2} & -\frac{5}{2} \\ 0 & 0 & 0 \end{pmatrix} \quad (0.0.5)$$

The rank of the matrix is 2 and the points are in 3-Dimensional space, So the points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ form a triangle.

Now, we check whether the triangle is right angled at any of the vertices - $\mathbf{A}, \mathbf{B}, \mathbf{C}$

- a) checking whether the triangle is right angled at \mathbf{A}

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -1 \\ -2 \\ -6 \end{pmatrix} \quad (0.0.6)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix} \quad (0.0.7)$$

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} -1 & -2 & -6 \end{pmatrix} \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix} = 35 \quad (0.0.8)$$

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{A}) \neq 0 \quad (0.0.9)$$

The triangle is not right angled at \mathbf{A} .

- b) checking whether the triangle is right angled at \mathbf{B}

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix} \quad (0.0.10)$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \quad (0.0.11)$$

$$(\mathbf{A} - \mathbf{B})^\top (\mathbf{C} - \mathbf{B}) = \begin{pmatrix} 1 & 2 & 6 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 6 \quad (0.0.12)$$

$$(\mathbf{A} - \mathbf{B})^\top (\mathbf{C} - \mathbf{B}) \neq 0 \quad (0.0.13)$$

The triangle is not right angled at \mathbf{B} .

- c) checking whether the triangle is right angled

at **C**

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \quad (0.0.14)$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} \quad (0.0.15)$$

$$(\mathbf{A} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}) = \begin{pmatrix} -1 & 3 & 5 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} = 0 \quad (0.0.16)$$

$$(\mathbf{A} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}) = 0 \quad (0.0.17)$$

Hence the triangle is right angled at **C**.