

# Assignment 1

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- 1) Find the shortest distance between the lines  $l_1$  and  $l_2$  whose vector equations are  $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$  and  $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

**Solution:** The lines  $l_1$  and  $l_2$  in vector form can be written as

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \quad (0.0.1)$$

$$\mathbf{x} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} \quad (0.0.2)$$

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \mathbf{m}_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \mathbf{m}_2 = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} \quad (0.0.3)$$

The distance between the lines is given by,

$$d = \|(\mathbf{x}_2 + \lambda_2 \mathbf{m}_2) - (\mathbf{x}_1 + \lambda_1 \mathbf{m}_1)\| \quad (0.0.4)$$

$$\Rightarrow d = \|\mathbf{x}_2 - \mathbf{x}_1 - \lambda_1 \mathbf{m}_1 + \lambda_2 \mathbf{m}_2\| \quad (0.0.5)$$

Consider the following definitions

$$\mathbf{A} \triangleq \mathbf{x}_2 - \mathbf{x}_1 \quad (0.0.6)$$

$$\mathbf{M} \triangleq \begin{pmatrix} \mathbf{m}_1 & \mathbf{m}_2 \end{pmatrix} \quad (0.0.7)$$

$$\lambda \triangleq \begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix} \quad (0.0.8)$$

From the above definitions (0.0.6), (0.0.7), (0.0.8) we get,

$$d = \|\mathbf{A} - \mathbf{M}\lambda\| \quad (0.0.9)$$

Here we have the values of  $\mathbf{A}, \mathbf{M}$  as

$$\mathbf{M} = \begin{pmatrix} 2 & 3 \\ -1 & -5 \\ 1 & 2 \end{pmatrix} \quad (0.0.10)$$

$$\mathbf{A} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad (0.0.11)$$

The given problem can be formulated as

$$\min_{\lambda} d = \|\mathbf{A} - \mathbf{M}\lambda\| \quad (0.0.12)$$

$$\text{s.t. } \lambda \in \mathbb{R}^2 \quad (0.0.13)$$

By solving using cvxpy, we get

$$\min_{\lambda} d = 1.3019 \quad (0.0.14)$$

$$\lambda = \begin{pmatrix} 0.4237 \\ -0.1186 \end{pmatrix} \quad (0.0.15)$$

The shortest distance between the given lines is 1.3019 units, and this answer matches with the other methods.

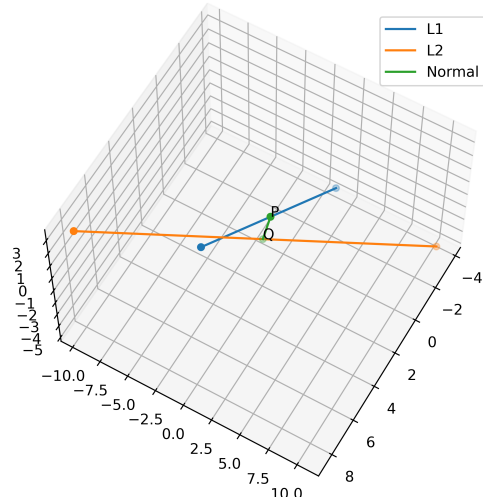


Fig. 1:  $PQ$  is the required shortest distance.