Assignment 1

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1) An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

Solution: Consider the random variables X, Y. X denotes the first random draw, Y denotes the random draw after 2 additional balls are put back to the urn. Refer the table 1.

RV	Values	Description
X	$\{0, 1\}$	1st draw - 0: Red, 1: Black
Y	$\{0, 1\}$	2nd draw - 0: Red, 1: Black

TABLE 1: Random variables X,Y

The probabilities of random variable X, Y is listed in table 1.

Event	Probability
$\Pr(X = 0)$	$\frac{5}{10}$
$\Pr(X = 1)$	$\frac{5}{10}$
$\Pr(Y = 1 \mid X = 0)$	$\frac{7}{12}$
$\Pr(Y = 1 \mid X = 1)$	$\frac{5}{12}$

TABLE 1: Probabilities

The random variable Y is the event that occurs after the event X. The required probability is given by

$$\Pr(Y = 0) = \Pr(X = 0)\Pr(Y = 0 \mid X = 0) + \Pr(X = 1)\Pr(Y = 0 \mid X = 1) \tag{0.0.1}$$

$$= \left(\frac{5}{10} \times \frac{7}{12}\right) + \left(\frac{5}{10} \times \frac{5}{12}\right) \tag{0.0.2}$$

$$= \frac{1}{2} \tag{0.0.3}$$