Assignment 1

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1) A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to maximise the amount of vitamin A in the diet? What is the maximum amount of vitamin A in the diet?

Solution: Let x and y be the number of packets of food P and Q respectively

$$x \ge 0 \tag{0.0.1}$$

$$y \ge 0 \tag{0.0.2}$$

From the constraint on Calcium we have,

$$12x + 3y \ge 240 \tag{0.0.3}$$

$$\implies 4x + y \ge 80 \tag{0.0.4}$$

From the constraint on Iron we have,

$$4x + 20y \ge 460 \tag{0.0.5}$$

$$\implies x + 5y \ge 115 \tag{0.0.6}$$

From the constraint on cholesterol we have,

$$6x + 4y \le 300 \tag{0.0.7}$$

$$\implies 3x + 2y \le 150 \tag{0.0.8}$$

Vitamin A in the diet is given by,

$$Z = 6x + 3y (0.0.9)$$

So, the given problem can be formulated as,

$$\min_{\mathbf{x}} -Z = \begin{pmatrix} -6 & -3 \end{pmatrix} \mathbf{x}$$
 (0.0.10)
s.t. $\mathbf{A}\mathbf{x} \le B$ (0.0.11)

$$s.t. \quad \mathbf{A}\mathbf{x} \le B \tag{0.0.11}$$

where

$$A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \\ -4 & -1 \\ -1 & -5 \\ 3 & 2 \end{pmatrix} \tag{0.0.12}$$

$$B = \begin{pmatrix} 0\\0\\-80\\-115\\150 \end{pmatrix} \tag{0.0.13}$$

By solving using cvxpy, we get

$$\min_{\mathbf{z}} Z = 285 \tag{0.0.14}$$

$$\min_{\mathbf{x}} Z = 285 \qquad (0.0.14)$$

$$\mathbf{x} = \begin{pmatrix} 40 \\ 15 \end{pmatrix} \qquad (0.0.15)$$