Assignment 1

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1) The base of an equilateral triangle with side $2a$ lies along the y-axis such that the mid-point of the base is at the origin. Find vertices of the triangle.
Solution: Let the points of triangle be $\mathbf{A}, \mathbf{B}, \mathbf{C}$, the base BC be along the y-axis.

$$\mathbf{B} = \begin{pmatrix} 0 \\ y_1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ y_2 \end{pmatrix} \tag{0.0.1}$$

Given that mid point of the base is at origin, this gives

$$\frac{\mathbf{B} + \mathbf{C}}{2} = 0 \tag{0.0.2}$$

$$\mathbf{B} = -\mathbf{C} \tag{0.0.3}$$

$$y_1 = -y_2 \tag{0.0.4}$$

Given the side length is $2a$, this gives

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{C} - \mathbf{A}\| = 2a \tag{0.0.5}$$

$$\|\mathbf{B} - \mathbf{C}\| = 2a \tag{0.0.6}$$

$$\|\mathbf{B} - (-\mathbf{B}))\| = 2a \tag{0.0.7}$$

$$\|\mathbf{B}\| = a \tag{0.0.8}$$

$$y_1 = a, \ y_2 = -a \tag{0.0.9}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ a \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ -a \end{pmatrix} \tag{0.0.10}$$

Let the point $\mathbf{A}$ be

$$\mathbf{A} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{0.0.11}$$

$$\|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A} - \mathbf{C}\|^2 \tag{0.0.12}$$

$$\begin{pmatrix} x & y - a \end{pmatrix} \begin{pmatrix} x \\ y - a \end{pmatrix} = \begin{pmatrix} x & y + a \end{pmatrix} \begin{pmatrix} x \\ y + a \end{pmatrix} \tag{0.0.13}$$

$$x^2 + (y - a)^2 = x^2 + (y + a)^2 \tag{0.0.14}$$

$$y = 0 \tag{0.0.15}$$

$$\|\mathbf{A} - \mathbf{B}\|^2 = (2a)^2 \tag{0.0.16}$$

$$\begin{pmatrix} x & -a \end{pmatrix} \begin{pmatrix} x \\ -a \end{pmatrix} = 4a^2 \tag{0.0.17}$$

$$x^2 + a^2 = 4a^2 \tag{0.0.18}$$

$$x = \pm \sqrt{3}a \tag{0.0.19}$$

The vertices of the triangle are either

$$\mathbf{A} = \begin{pmatrix} \sqrt{3}a \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ a \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ -a \end{pmatrix} \tag{0.0.20}$$

or

$$\mathbf{A} = \begin{pmatrix} -\sqrt{3}a \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ a \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ -a \end{pmatrix} \tag{0.0.21}$$