Assignment 1

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1) If a line intersects two concentric circles (circles with the same centre) with centre **O** at **A**, **B**, **C** and **D**, prove that $AB = CD$.

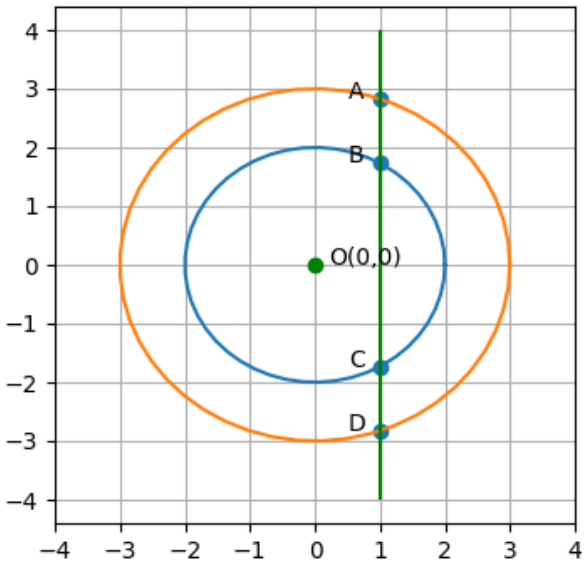

Fig. 1: Graph

Solution: Let the equations of two concentric circles be,

$$\|\mathbf{x}\|^2 = 4 \tag{0.0.1}$$

$$\|\mathbf{x}\|^2 = 9 \tag{0.0.2}$$

Let the equation of the line be,

$$\mathbf{x} = \begin{pmatrix}1\\0\end{pmatrix} + \lambda\begin{pmatrix}0\\1\end{pmatrix} \tag{0.0.3}$$

$$\mathbf{x} = \begin{pmatrix}1\\\lambda\end{pmatrix} \tag{0.0.4}$$

The points of intersection of circle (0.0.1) and the line (0.0.4) $\mathbf{B}, \mathbf{C}$ are given by,

$$\|\mathbf{x}\|^2 = 4 \tag{0.0.5}$$

$$1^2 + \lambda^2 = 4 \tag{0.0.6}$$

$$\lambda^2 = 3 \tag{0.0.7}$$

$$\lambda = \pm\sqrt{3} \tag{0.0.8}$$

$$\mathbf{B} = \begin{pmatrix}1\\\sqrt{3}\end{pmatrix}, \mathbf{C} = \begin{pmatrix}1\\-\sqrt{3}\end{pmatrix} \tag{0.0.9}$$

The points of intersection of circle (0.0.2) and the line (0.0.4) $\mathbf{A}, \mathbf{D}$ are given by,

$$\|\mathbf{x}\|^2 = 9 \tag{0.0.10}$$

$$1^2 + \lambda^2 = 9 \tag{0.0.11}$$

$$\lambda^2 = 8 \tag{0.0.12}$$

$$\lambda = \pm 2\sqrt{2} \tag{0.0.13}$$

$$\mathbf{A} = \begin{pmatrix}1\\2\sqrt{2}\end{pmatrix}, \mathbf{D} = \begin{pmatrix}1\\-2\sqrt{2}\end{pmatrix} \tag{0.0.14}$$

$$\|\mathbf{A} - \mathbf{B}\| = \left\|\begin{pmatrix}0\\2\sqrt{2} - \sqrt{3}\end{pmatrix}\right\| \tag{0.0.15}$$

$$= 2\sqrt{2} - \sqrt{3} \tag{0.0.16}$$

$$\|\mathbf{C} - \mathbf{D}\| = \left\|\begin{pmatrix}0\\2\sqrt{2} - \sqrt{3}\end{pmatrix}\right\| \tag{0.0.17}$$

$$= 2\sqrt{2} - \sqrt{3} \tag{0.0.18}$$

Hence $AB = CD$. The parameters used in the construction are shown in the below table 1

Parameter	Value
A	$\begin{pmatrix} 1 \\ 2\sqrt{2} \end{pmatrix}$
B	$\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$
C	$\begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$
D	$\begin{pmatrix} 1 \\ -2\sqrt{2} \end{pmatrix}$
O	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

TABLE 1