Assignment 1

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1) A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that
   a) all will be blue?
   b) atleast one will be green?

Solution:

Lemma: Consider the generalized problem, where there are total N marbles - P red, Q blue, R green. We need to find the probability of the event where n marbles drawn such that there are - p red, q blue, r green marbles.

Consider the random variable X_i, denoting the ith draw, $i \in \{1, 2, \cdots, n\}$ as shown in the table 1

RV	Values	Description
X_i	$\{0, 1, 2\}$	ith draw - 0: red, 1: blue, 2: green

TABLE 1: Random variables X_i

The probability that the first p draws are red, next q draws are blue, next r draws are green is given by the expression,

$$\Pr\left(X_1 = 0, \cdots, X_p = 0, X_{p+1} = 1, \cdots X_{p+q} = 1, X_{p+q+1} = 2, \cdots X_{p+q+r} = 2\right) \tag{0.0.1}$$

$$= \left(\frac{P}{N} \times \cdots \times \frac{P-(p-1)}{N-(p-1)}\right)\left(\frac{Q}{N-p} \times \cdots \times \frac{Q-(q-1)}{N-(p+q-1)}\right)\left(\frac{R}{N-(p+q)} \times \cdots \times \frac{R-(r-1)}{N-(p+q+r-1)}\right) \tag{0.0.2}$$

$$= \frac{P!Q!R!\,(N-(p+q+r))!}{N!\,(P-p)!\,(Q-q)!\,(R-r)!} \tag{0.0.3}$$

$$= \frac{P!Q!R!\,(N-n)!}{N!\,(P-p)!\,(Q-q)!\,(R-r)!} \tag{0.0.4}$$

The expression 0.0.4 is one possibility of draws such that there are - p red, q blue, r greem marbles. There are $\frac{n!}{p!q!r!}$ such terms, All have the same probability. Hence the required probability is given by,

$$= \frac{n!}{p!q!r!}\frac{P!Q!R!\,(N-n)!}{N!\,(P-p)!\,(Q-q)!\,(R-r)!} \tag{0.0.5}$$

$$= \frac{P!}{p!\,(P-p)!}\frac{Q!}{q!\,(Q-q)!}\frac{R!}{R!\,(R-r)!}\frac{n!\,(N-n)!}{N!} \tag{0.0.6}$$

$$= \frac{{}^{P}C_p \times {}^{Q}C_q \times {}^{R}C_r}{{}^{N}C_n} \tag{0.0.7}$$

The formula (0.0.7) is also verified through simulation

In this question, total marbles in the box are 60 - 10 red, 20 blue, 30 green. Out of which 5 balls are drawn. Hence we have,

$$N = 60,\ P = 10,\ Q = 20,\ R = 30,\ n = 5 \tag{0.0.8}$$

a) The probability that all drawn marbles are blue implies

$$p = 0,\ q = 5,\ r = 0 \tag{0.0.9}$$

From (0.0.7) we get the probability as,

$$= \frac{^{20}C_5}{^{60}C_5} \tag{0.0.10}$$

b) The probability that the drawn marble contains atleast 1 green. This event is complement to the event where no marble drawn is green, Its probability is given by,

$$= \frac{^{30}C_5}{^{60}C_5} \tag{0.0.11}$$

Hence the required probability is given by,

$$1 - \frac{^{30}C_5}{^{60}C_5} \tag{0.0.12}$$