

Assignment 1

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- 1) Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum of $y^2 = -8x$

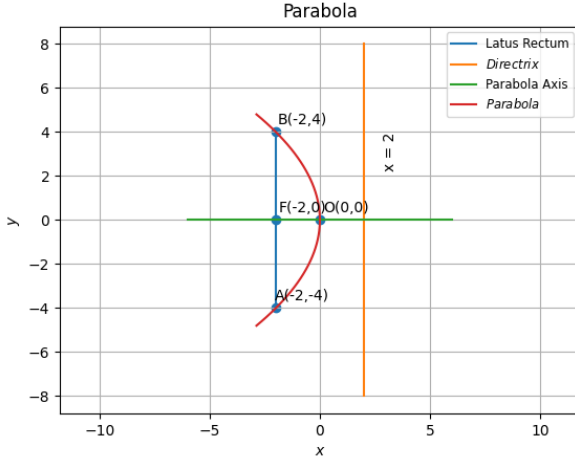


Fig. 1: Graph

Solution: The given equation of the parabola can be rearranged as

$$y^2 + 8x = 0 \quad (0.0.1)$$

The above equation can be equated to the generic equation of conic sections

$$g(\mathbf{x}) = \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (0.0.2)$$

Comparing coefficients of (0.0.1) and (0.0.2),

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (0.0.3)$$

$$\mathbf{u} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (0.0.4)$$

$$f = 0 \quad (0.0.5)$$

- a) From (0.0.3), since \mathbf{V} is already diagonalized, the Eigen values λ_1 and λ_2 are given as

$$\lambda_1 = 0 \quad (0.0.6)$$

$$\lambda_2 = 1 \quad (0.0.7)$$

and the eigenvector matrix

$$\mathbf{P} = \mathbf{I}. \quad (0.0.8)$$

$$\mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (0.0.9)$$

$$\mathbf{n} = \sqrt{\lambda_2} \mathbf{p}_1 \quad (0.0.10)$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (0.0.11)$$

Since

$$c = \frac{\|\mathbf{u}^2\| - \lambda_2 f}{2\mathbf{u}^T \mathbf{n}}, \quad (0.0.12)$$

Substituting values of $\mathbf{u}, \mathbf{n}, \lambda_2$ and f in (0.0.12)

$$c = \frac{4^2 - 1(0)}{2 \begin{pmatrix} 4 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}} = 2 \quad (0.0.13)$$

The focus \mathbf{F} of parabola is expressed as

$$\mathbf{F} = \frac{ce^2 \mathbf{n} - \mathbf{u}}{\lambda_2} \quad (0.0.14)$$

$$= \frac{2(1)^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix}}{1} \quad (0.0.15)$$

$$= \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (0.0.16)$$

- b) The directrix is given by

$$\mathbf{n}^T \mathbf{x} = c \quad (0.0.17)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 2 \quad (0.0.18)$$

- c) The equation for the axis of parabola, passing through \mathbf{F} and orthogonal to the directrix is given as

$$\mathbf{m}^T (\mathbf{x} - \mathbf{F}) = 0 \quad (0.0.19)$$

where \mathbf{m} is the normal vector to the axis and

also the slope of the directrix.

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (0.0.20)$$

$$\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (0.0.21)$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right) = 0 \quad (0.0.22)$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (0.0.23)$$

d) The latus rectum of a parabola is given by

$$l = \frac{\eta}{\lambda_2} = \frac{2\mathbf{u}^\top \mathbf{p}_1}{\lambda_2} \quad (0.0.24)$$

$$= \frac{2 \begin{pmatrix} 4 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{1} \quad (0.0.25)$$

$$= 8 \text{ units} \quad (0.0.26)$$