Assignment 1

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1) Let $\overrightarrow{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\overrightarrow{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\overrightarrow{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\overrightarrow{d}$ which is perpendicular to both $\overrightarrow{a}$ and $\overrightarrow{b}$, and $\overrightarrow{c}.\overrightarrow{d} = 15$.
Solution: The vector perpendicular to both **A** and **B** has the direction that of $\mathbf{A} \times \mathbf{B}$.
Here we have

$$\mathbf{A} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \tag{0.0.1}$$

The cross product or vector product of $\mathbf{A}, \mathbf{B}$ is defined as

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} \begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \\ \mathbf{A}_{31} & \mathbf{B}_{31} \end{vmatrix} \\ \begin{vmatrix} \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} \end{pmatrix} \tag{0.0.2}$$

$$\begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \end{vmatrix} = \begin{vmatrix} 4 & -2 \\ 2 & 7 \end{vmatrix} = 32 \tag{0.0.3}$$

$$\begin{vmatrix} \mathbf{A}_{31} & \mathbf{B}_{31} \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} = 1 \tag{0.0.4}$$

$$\begin{vmatrix} \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 4 & -2 \end{vmatrix} = -14 \tag{0.0.5}$$

Hence

$$\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \times \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix} = \begin{pmatrix} 32 \\ 1 \\ -14 \end{pmatrix} \tag{0.0.6}$$

As the vector **D** is in the direction of the $\mathbf{A} \times \mathbf{B}$, the vector **D** can be written as,

$$\mathbf{D} = \lambda \begin{pmatrix} 32 \\ 1 \\ -14 \end{pmatrix} \tag{0.0.7}$$

Given that $\mathbf{C}^{\top}\mathbf{D} = 15$.

$$\begin{pmatrix} 2 & -1 & 4 \end{pmatrix} \lambda \begin{pmatrix} 32 \\ 1 \\ -14 \end{pmatrix} = 15 \tag{0.0.8}$$

$$\lambda \begin{pmatrix} 2 & -1 & 4 \end{pmatrix} \begin{pmatrix} 32 \\ 1 \\ -14 \end{pmatrix} = 15 \tag{0.0.9}$$

$$\lambda \times 7 = 15 \tag{0.0.10}$$

$$\lambda = \frac{15}{7} \tag{0.0.11}$$

Hence the vector **D** is given by,

$$\mathbf{D} = \frac{15}{7} \begin{pmatrix} 32 \\ 1 \\ -14 \end{pmatrix} \tag{0.0.12}$$

$$\mathbf{D} = \begin{pmatrix} \frac{480}{7} \\ \frac{15}{7} \\ -30 \end{pmatrix} \tag{0.0.13}$$