Assignment 1

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1) Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum of $y^2 = -8x$

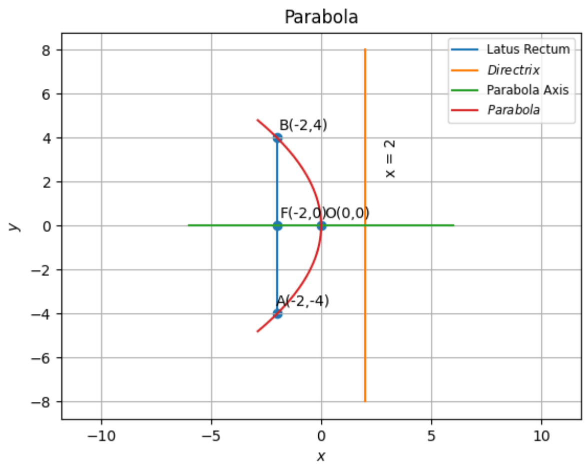

Fig. 1: Graph

Solution: The given equation of the parabola can be rearranged as

$$y^2 + 8x = 0 \tag{0.0.1}$$

The above equation can be equated to the generic equation of conic sections

$$g(\mathbf{x}) = \mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{0.0.2}$$

Comparing coefficients of (0.0.1) and (0.0.2),

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{0.0.3}$$

$$\mathbf{u} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{0.0.4}$$

$$f = 0 \tag{0.0.5}$$

a) Focus: From (0.0.3), since $\mathbf{V}$ is already diagonalized, the Eigen values λ_1 and λ_2 are given as

$$\lambda_1 = 0 \tag{0.0.6}$$

$$\lambda_2 = 1 \tag{0.0.7}$$

and the eigenvector matrix

$$\mathbf{P} = \mathbf{I}. \tag{0.0.8}$$

$$\mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.0.9}$$

$$\mathbf{n} = \sqrt{\lambda_2}\mathbf{p_1} \tag{0.0.10}$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.0.11}$$

Since

$$c = \frac{\left\|\mathbf{u}^2\right\| - \lambda_2 f}{2\mathbf{u}^\top\mathbf{n}}, \tag{0.0.12}$$

Substituting values of $\mathbf{u}, \mathbf{n}, \lambda_2$ and f in (0.0.12)

$$c = \frac{4^2 - 1(0)}{2\begin{pmatrix} 4 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}} = 2 \tag{0.0.13}$$

The focus $\mathbf{F}$ of parabola is expressed as

$$\mathbf{F} = \frac{ce^2\mathbf{n} - \mathbf{u}}{\lambda_2} \tag{0.0.14}$$

$$= \frac{2(1)^2\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix}}{1} \tag{0.0.15}$$

$$= \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{0.0.16}$$

b) Directrix: The directrix is given by

$$\mathbf{n}^\top\mathbf{x} = c \tag{0.0.17}$$

$$\implies \begin{pmatrix} 1 & 0 \end{pmatrix}\mathbf{x} = 2 \tag{0.0.18}$$

c) Axis: The equation for the axis of parabola, passing through $\mathbf{F}$ and orthogonal to the directrix is given as

$$\mathbf{m}^\top(\mathbf{x} - \mathbf{F}) = 0 \tag{0.0.19}$$

where $\mathbf{m}$ is the normal vector to the axis and also the slope of the directrix.

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.0.20}$$

$$\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{0.0.21}$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right) = 0 \tag{0.0.22}$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{0.0.23}$$

d) Latus rectum: The latus rectum of a parabola is given by

$$l = \frac{\eta}{\lambda_2} = \frac{2\mathbf{u}^\top \mathbf{p_1}}{\lambda_2} \tag{0.0.24}$$

$$= \frac{2\begin{pmatrix} 4 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}}{1} \tag{0.0.25}$$

$$= 8 \text{ units} \tag{0.0.26}$$