Assignment 1

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1) If a leap year is selected at random, what is the chance that it will contain 53 tuesdays? **Solution:** We know that, A leap year has 366 days - 52 weeks and 2 days. In 52 weeks, there are 52 Tuesdays. Define the random variable X for the first day of 2 consecutive days as in the following table 1

RV	Description	Probabilities
$X = 0$	Monday	$\frac{1}{7}$
$X = 1$	Tuesday	$\frac{1}{7}$
$X = 2$	Wednesday	$\frac{1}{7}$
$X = 3$	Thursday	$\frac{1}{7}$
$X = 4$	Friday	$\frac{1}{7}$
$X = 5$	Saturday	$\frac{1}{7}$
$X = 6$	Sunday	$\frac{1}{7}$

TABLE 1: Random variable X

The probability that the leap year will contain 53 Tuesdays is equal to the probability that the remaining 2 days will contain a tuesday. The first day should be either Monday or tuesday.

$$X = 0 \text{ or } X = 1 \equiv X \in \{0, 1\} \qquad (0.0.1)$$
$$X = 0 \text{ and } X = 1 \equiv X = \phi \qquad (0.0.2)$$

Hence, the required probability is given by,

$$\Pr(X \in \{0, 1\}) = \Pr(X = 0) + \Pr(X = 1) - \Pr(X = \phi) \qquad (0.0.3)$$

The random variable X takes the values from the set $\{0, 1, 2, 3, 4, 5, 6\}$ uniformly, so we have the probabilities as $\frac{1}{7}$ as in the table 1. Hence we have,

$$\Pr(X \in \{0, 1\}) = \frac{1}{7} + \frac{1}{7} \qquad (0.0.4)$$
$$= \frac{2}{7} \qquad (0.0.5)$$