Assignment 1

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1) A ray of light passing through the point $(1,2)$ reflects on the x-axis at point A and the reflected ray passes through the point $(5,3)$. Find the coordinates of A.

Solution:

a) Expression for reflection of a point $\mathbf{P}$ in the line $\mathbf{n}^\top\mathbf{x} = c$.

Let the reflected point be $\mathbf{Q}$. The point $\mathbf{Q}$ can be written in parametric form as,

$$\mathbf{Q} = \mathbf{P} + \lambda\mathbf{n} \tag{0.0.1}$$

The points $\mathbf{P}$,$\mathbf{Q}$ are both equidistant from the line $\mathbf{n}^\top\mathbf{x} = c$. The point $\frac{\mathbf{P}+\mathbf{Q}}{2}$ lies on the line.

$$\mathbf{n}^\top\left(\frac{\mathbf{P}+\mathbf{Q}}{2}\right) = c \tag{0.0.2}$$

$$\mathbf{n}^\top\left(\frac{2\mathbf{P}+\lambda\mathbf{n}}{2}\right) = c \tag{0.0.3}$$

$$2\mathbf{n}^\top\mathbf{P} + \lambda\mathbf{n}^\top\mathbf{n} = 2c \tag{0.0.4}$$

$$\lambda = -\frac{2\left(\mathbf{n}^\top\mathbf{P} - c\right)}{\|\mathbf{n}\|} \tag{0.0.5}$$

Hence, the point $\mathbf{Q}$ is given by,

$$\mathbf{Q} = \mathbf{P} - \frac{2\left(\mathbf{n}^\top\mathbf{P} - c\right)}{\|\mathbf{n}\|}\mathbf{n} \tag{0.0.6}$$

b) Let the points be,

$$\mathbf{P} = \begin{pmatrix}1\\2\end{pmatrix},\ \mathbf{Q} = \begin{pmatrix}5\\3\end{pmatrix} \tag{0.0.7}$$

The equation of x-axis is given by,

$$\begin{pmatrix}0 & 1\end{pmatrix}\mathbf{x} = 0 \tag{0.0.8}$$

Let the reflection of point $\mathbf{Q}$ in the x-axis be $\mathbf{R}$ is given by

$$\mathbf{R} = \mathbf{Q} - \frac{2\left(\mathbf{n}^\top\mathbf{Q} - c\right)}{\|\mathbf{n}\|}\mathbf{n} \tag{0.0.9}$$

$$= \begin{pmatrix}5\\3\end{pmatrix} - 6\begin{pmatrix}0\\1\end{pmatrix} \tag{0.0.10}$$

$$= \begin{pmatrix}5\\-3\end{pmatrix} \tag{0.0.11}$$

The point $\mathbf{A}$ is the point of intersection of the line PR and x-axis.

Direction vector of line PR is given by,

$$\mathbf{m} = \mathbf{R} - \mathbf{P} \tag{0.0.12}$$

$$= \begin{pmatrix}5\\-3\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix} \tag{0.0.13}$$

$$= \begin{pmatrix}4\\-5\end{pmatrix} \tag{0.0.14}$$

Normal vector $\mathbf{n}$ is given by,

$$\mathbf{n} = \begin{pmatrix}5\\4\end{pmatrix} \tag{0.0.15}$$

Equation of line PR is given by

$$\begin{pmatrix}5 & 4\end{pmatrix}\mathbf{x} = \begin{pmatrix}5 & 4\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix} \tag{0.0.16}$$

$$\begin{pmatrix}5 & 4\end{pmatrix}\mathbf{x} = 13 \tag{0.0.17}$$

$$\mathbf{A} = \begin{pmatrix}x\\0\end{pmatrix} \tag{0.0.18}$$

The point $\mathbf{A}$ satisfies the equation (0.0.17)

$$5 \times x = 13 \tag{0.0.19}$$

$$x\frac{13}{5} \tag{0.0.20}$$

Hence the point $\mathbf{A}$ is given by,

$$\mathbf{A} = \begin{pmatrix}\frac{13}{5}\\0\end{pmatrix} \tag{0.0.21}$$