

Assignment 1

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- 1) A ray of light passing through the point (1, 2) reflects on the x-axis at point A and the reflected ray passes through the point (5, 3). Find the coordinates of A.

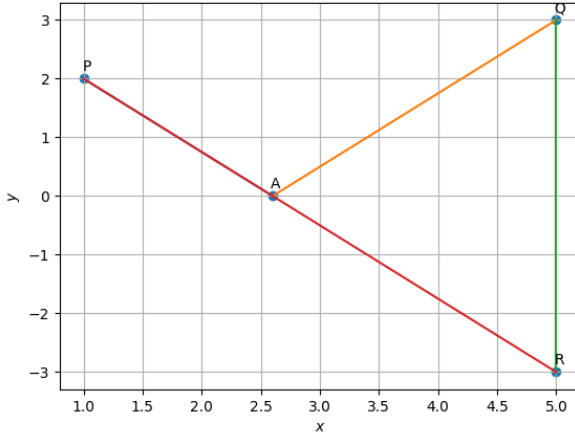


Fig. 1: Graph

Solution:

- a) Expression for reflection of a point \mathbf{P} in the line $\mathbf{n}^T \mathbf{x} = c$.
Let the reflected point be \mathbf{Q} . The point \mathbf{Q} can be written in parametric form as,

$$\mathbf{Q} = \mathbf{P} + \lambda \mathbf{n} \quad (0.0.1)$$

The points \mathbf{P}, \mathbf{Q} are both equidistant from the line $\mathbf{n}^T \mathbf{x} = c$. The point $\frac{\mathbf{P} + \mathbf{Q}}{2}$ lies on the line.

$$\mathbf{n}^T \left(\frac{\mathbf{P} + \mathbf{Q}}{2} \right) = c \quad (0.0.2)$$

$$\mathbf{n}^T \left(\frac{2\mathbf{P} + \lambda \mathbf{n}}{2} \right) = c \quad (0.0.3)$$

$$2\mathbf{n}^T \mathbf{P} + \lambda \mathbf{n}^T \mathbf{n} = 2c \quad (0.0.4)$$

$$\lambda = -\frac{2(\mathbf{n}^T \mathbf{P} - c)}{\|\mathbf{n}\|^2} \quad (0.0.5)$$

Hence, the point \mathbf{Q} is given by,

$$\mathbf{Q} = \mathbf{P} - \frac{2(\mathbf{n}^T \mathbf{P} - c)}{\|\mathbf{n}\|^2} \mathbf{n} \quad (0.0.6)$$

- b) Let the points be,

$$\mathbf{P} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \quad (0.0.7)$$

The equation of x-axis is given by,

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (0.0.8)$$

Let the reflection of point \mathbf{Q} in the x-axis be \mathbf{R} is given by

$$\mathbf{R} = \mathbf{Q} - \frac{2(\mathbf{n}^T \mathbf{Q} - c)}{\|\mathbf{n}\|^2} \mathbf{n} \quad (0.0.9)$$

$$= \begin{pmatrix} 5 \\ 3 \end{pmatrix} - 6 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (0.0.10)$$

$$= \begin{pmatrix} 5 \\ -3 \end{pmatrix} \quad (0.0.11)$$

The point \mathbf{A} is the point of intersection of the line PR and x-axis.

Direction vector of line PR is given by,

$$\mathbf{m} = \mathbf{R} - \mathbf{P} \quad (0.0.12)$$

$$= \begin{pmatrix} 5 \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (0.0.13)$$

$$= \begin{pmatrix} 4 \\ -5 \end{pmatrix} \quad (0.0.14)$$

Normal vector \mathbf{n} is given by,

$$\mathbf{n} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \quad (0.0.15)$$

Equation of line PR is given by

$$\begin{pmatrix} 5 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (0.0.16)$$

$$\begin{pmatrix} 5 & 4 \end{pmatrix} \mathbf{x} = 13 \quad (0.0.17)$$

$$\mathbf{A} = \begin{pmatrix} x \\ 0 \end{pmatrix} \quad (0.0.18)$$

The point \mathbf{A} satisfies the equation (0.0.17)

$$5 \times x = 13 \quad (0.0.19)$$

$$x = \frac{13}{5} \quad (0.0.20)$$

Hence the point \mathbf{A} is given by,

$$\mathbf{A} = \begin{pmatrix} \frac{13}{5} \\ 0 \end{pmatrix} \quad (0.0.21)$$