

Assignment 1

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- 1) If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D , prove that $AB = CD$.

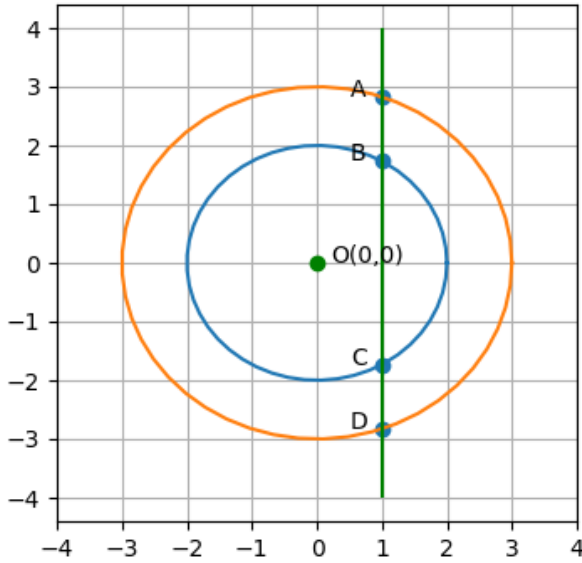


Fig. 1: Graph

Solution: Let the equations of two concentric circles be,

$$\|\mathbf{x}\|^2 = 4 \quad (0.0.1)$$

$$\|\mathbf{x}\|^2 = 9 \quad (0.0.2)$$

Let the line given by,

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 1 \quad (0.0.3)$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (0.0.4)$$

$$x = 1 \quad (0.0.5)$$

The points of intersection of circle (0.0.1) and

the line (0.0.5) B, C are given by,

$$x^2 + y^2 = 4 \quad (0.0.6)$$

$$x = 1 \quad (0.0.7)$$

$$y^2 = 3 \quad (0.0.8)$$

$$y = \pm \sqrt{3} \quad (0.0.9)$$

$$\mathbf{B} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \quad (0.0.10)$$

The points of intersection of circle (0.0.2) and the line (0.0.5) A, D are given by,

$$x^2 + y^2 = 9 \quad (0.0.11)$$

$$x = 1 \quad (0.0.12)$$

$$y^2 = 8 \quad (0.0.13)$$

$$y = \pm 2\sqrt{2} \quad (0.0.14)$$

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2\sqrt{2} \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 1 \\ -2\sqrt{2} \end{pmatrix} \quad (0.0.15)$$

$$\|\mathbf{AB}\| = \left\| \begin{pmatrix} 0 \\ 2\sqrt{2} - \sqrt{3} \end{pmatrix} \right\| \quad (0.0.16)$$

$$= 2\sqrt{2} - \sqrt{3} \quad (0.0.17)$$

$$\|\mathbf{CD}\| = \left\| \begin{pmatrix} 0 \\ 2\sqrt{2} - \sqrt{3} \end{pmatrix} \right\| \quad (0.0.18)$$

$$= 2\sqrt{2} - \sqrt{3} \quad (0.0.19)$$

Hence $AB = CD$.