

# Assignment 1

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- 1) A couple has two children,
- Find the probability that both children are males, if it is known that at least one of the children is male.
  - Find the probability that both children are females, if it is known that the elder child is a female.

**Solution:** Consider the random variables  $X, Y$ , which denotes the first child, second child gender respectively as described in table 1.

RV	Values	Description
$X$	$\{0, 1\}$	0: Male, 1: Female
$Y$	$\{0, 1\}$	0: Male, 1: Female

TABLE 1: Random variables  $X$

The probabilities for the random variables  $X, Y$  is listed in table 1.

Event	Probability
$\Pr(X = 0)$	$\frac{1}{2}$
$\Pr(X = 1)$	$\frac{1}{2}$
$\Pr(Y = 0)$	$\frac{1}{2}$
$\Pr(Y = 1)$	$\frac{1}{2}$
$\Pr(X + Y = 0)$	$\frac{1}{4}$
$\Pr(X + Y = 2)$	$\frac{1}{4}$
$\Pr(XY = 0)$	$\frac{3}{4}$

TABLE 1: Probabilities

The probability  $\Pr(XY = 0)$  is given by

$$= \Pr(X = 0) + \Pr(Y = 0) - \Pr(X + Y = 0) \quad (0.0.1)$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{4} \quad (0.0.2)$$

$$= \frac{3}{4} \quad (0.0.3)$$

- The event of both children being Male is when  $X + Y = 0$ . The event of atleast one of the children being Male is when  $XY = 0$ .

$$\{X + Y = 0\} \cap \{XY = 0\} \equiv \{X + Y = 0\} \quad (0.0.4)$$

The required probability is given by,

$$\Pr(X + Y = 0 \mid XY = 0) \quad (0.0.5)$$

$$= \frac{\Pr(X + Y = 0)}{\Pr(XY = 0)} \quad (0.0.6)$$

$$= \frac{1}{3} \quad (0.0.7)$$

- The event of both children being Female is when  $X + Y = 2$ . The event of elder child being Female is when  $X = 1$ .

$$\{X + Y = 2\} \cap \{X = 1\} \equiv \{X + Y = 2\} \quad (0.0.8)$$

The required probability is given by,

$$\Pr(X + Y = 2 \mid X = 1) \quad (0.0.9)$$

$$= \frac{\Pr(X + Y = 2)}{\Pr(X = 1)} \quad (0.0.10)$$

$$= \frac{1}{2} \quad (0.0.11)$$