CHAPTER 0

INTRODUCTION

In this chapter we will introduce some key concepts that will be used in later chapters.

> **Note 0.1 In case you are already familiar with the topics**
>
> It is recommended for readers who are familiar with the topics to at least gloss over this chapter and make sure they know and understand all the concept presented here.

0.1 MATHEMATICAL SYBOLS AND SETS

LOGICAL STATEMENTS AND THEIR TRUTH VALUE

We start our discussion with the simplest mathematical concept: a **proposition**. A proposition is simply a statement that might be either **true** or **false**.

Example 0.1 Truth of Propositions

- $3 > 1$ (**true**)
- $-2 = 5 - 7$ (**true**)
- $7 < 5$ (**false**)
- The radius of the earth is bigger than that of the moon. (**true**)
- The word 'House' starts with the letter 'G'. (**false**)

We can group together propositions using **logical operators**. Two of the most common logical operators are AND and OR.

The **AND** operator returns a **true** statement only if **both** the statements it groups are themselves **true**, otherwise it returns **false**.

Example 0.2 The AND operator

- $2+4=6$ is **true**, $4-2=2$ is **true**. ($2+4=6$ **AND** $4-2=2$) is therefore **true**.
- $2+4=6$ is **true**, $2>6$ is **false**. ($2+4=6$ **AND** $2>6$) is therefore **false**.
- $\frac{10}{2}=1$ is **false**, $2^4=16$ is **true**. $\left(\frac{10}{2}=1 \textbf{ AND } 2^4=16\right)$ is therefore **false**.
- $7<5$ is **false**, $10+2=13$ is **false**. ($7<5$ **AND** $10+2=13$) is therefore **false**.

The **OR** operator returns **true** if **at least** one of the statements it groups is true.

Example 0.3 The OR Operator

- $2+4=6$ is **true**, $4-2=2$ is **true**. ($2+4=6$ **OR** $4-2=2$) is therefore **true**.
- $2+4=6$ is **true**, $2>6$ is **false**. ($2+4=6$ **OR** $2>6$) is therefore **true**.
- $\frac{10}{2}=1$ is **false**, $2^4=16$ is **true**. $\left(\frac{10}{2}=1 \textbf{ OR } 2^4=16\right)$ is therefore **true**.
- $7<5$ is **false**, $10+2=13$ is **false**. ($7<5$ **OR** $10+2=13$) is therefore **false**.

The behaviour of both operators can be summarized using a **truth table** (see Table 1 below).

Table 1 The truth table for the operators **AND** and **OR**.

A	B	A AND B	A OR B
true	true	true	true
true	false	false	true
false	true	false	true
false	false	false	false

When writing, it is convinient to use **notations** to represent operators: the **AND** operator is denoted by $\wedge$, while the **OR** operator is denoted by $\vee$.

Example 0.4 Using the notations for AND and OR

$$\underbrace{2+2=5}_{\textbf{false}} \wedge \underbrace{1-1=0}_{\textbf{true}} \Rightarrow \textbf{false}$$
$$\underbrace{2+2=5}_{\textbf{false}} \vee \underbrace{1-1=0}_{\textbf{true}} \Rightarrow \textbf{true}$$

Several more common mathematical notations are given in Table 2 .

Table 2 Common Mathematical NotationsUsed in this Book.

Symbol	In words
$\neg a$	**not** a
$a \wedge b$	a **and** b
$a \vee b$	a **or** b
$a \Rightarrow b$	a **implies** b
$a \Leftrightarrow b$	a **is equivalent to** b
$\forall x$	**For all** x (...)
$\exists x$	**There exists** x **such that** (...)
$a := b$	a **is defined to be** b

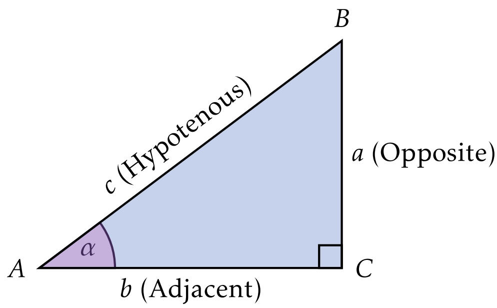
SETS

0.2 RELATIONS AND FUNCTIONS

0.3 TRIGONOMETRIC FUNCTIONS

BASIC DEFINITIONS

Consider a **right triangle** $\triangle ABC$ with sides a, b, and Hypotenous c, where the angle $\angle ACB$ is $90°$, and the angle $\angle BAC$ is denoted as α:



We use the ratios between the three sides of the triangle to define three functions of α:

Definition 0.1 The basic triginometric functions

1. The **sine** of the angle α is $\sin(\alpha) = \frac{a}{c}$,
2. the **cosine** of the angle α is $\cos(\alpha) = \frac{b}{c}$, and
3. the **tangent** of the angle α is $\tan(\alpha) = \frac{a}{b}$, which in turn is equal to $\frac{\sin(\alpha)}{\cos(\alpha)}$.

π

We can rearrange the above definitions to yield

$$\begin{aligned} a &= c\sin(\alpha), \\ b &= c\cos(\alpha). \end{aligned} \tag{1}$$

Normaly, the Hypotenous is the longest side of a right triangle. We will consider here the two edge cases where one of the sides a, b is equal to the Hypotenous (and the other side is thus 0):

- if $a = c$ then $\alpha = 90°$,
- if $b = c$ then $\alpha = 0$.

The posssible length of a is therefore in the range $0 \le a \le c$, which means that $0 \le \frac{a}{c} \le 1$, or since $\sin(\alpha) = \frac{a}{c}$,

$$0 \le \sin(\alpha) \le 1. \tag{2}$$

The same is of course true for b, and thus

$$0 \le \cos(\alpha) \le 1 \tag{3}$$

as well.

As a reminder, the **Pythagorean theorem**[1] states that for a right triangle like the one

[1] It's worth mentioning that no three positive integers a, b, and c satisfy the equation $a^n + b^n = c^n$ for any integer value of $n > 2$. This can be proven, however the proof is too large to fit in the footnotes.

Table 3 Common angles in radians.

Degrees	Radians
$0°$	0
$45°$	$\frac{\pi}{4}$
$90°$	$\frac{\pi}{2}$
$180°$	π
$270°$	$\frac{3\pi}{2}$
$360°$	2π

here,

$$a^2 + b^2 = c^2. \tag{4}$$

By substituting **Equation 1** into the above we get

$$c^2 = a^2 + b^2 = (c\sin(\alpha))^2 + (c\cos(\alpha))^2 = c^2\sin^2(\alpha) + c^2\cos^2(\alpha) = c^2\left[\sin^2(\alpha) + \cos^2(\alpha)\right],$$

and cancelling c^2 on both sides simply yields

$$\sin^2(\alpha) + \cos^2(\alpha) = 1. \tag{5}$$

THE UNIT CIRCLE

The range of the trigonometric functions can be extended by using the **unit circle**: a circle of radius $R = 1$ is placed such that its center lies at the origin of a 2-dimensional axis system, i.e. at the point $\boldsymbol{O} = (0, 0)$. A radius to a point $\boldsymbol{P} = (x, y)$ on the circle's circumference is the drawn. This radius has an angle θ to the x-axis. A line from $\boldsymbol{P}$ perpendicular to the x-axis intersecting at the point $\boldsymbol{D}$ is drawn (see **Figure 1**).

The triangle $\triangle OPD$ is a right triangle. Therefore, we can use the trigonometic functions to calculate the coordinates of the point $\boldsymbol{P} = (x, y)$:

$$\begin{aligned} x &= R\cos(\theta) = \cos(\theta), \\ y &= R\sin(\theta) = \sin(\theta). \end{aligned} \tag{6}$$

We can now define $\cos(\theta)$ and $\sin(\theta)$ as the values of x and y, respectively, as a function of θ.

We will switch to measuring angles in **Radians** instead of degrees: θ radians are equal to the length of an arc on a unit circle, which corresponds to the angle θ (**Figure 2**). This allows us to use the same units as x and y: for example, when length is measured in [meter], an angle in radians is measured in [meter] as well. The full circumference of a circle equals 2π radians, and therefore a single radian is equivalent to $\frac{180}{\pi} \approx 57.3°$. **Table 3** shows some common angles in radians.

Another advantage which we gain by defining the trigonometric functions using the unit circle is the extension of their domain to all of $\mathbb{R}$: an angle of size $2\frac{1}{2}\pi$ (equivalent to $450°$), for example is the same as an angle of size $\frac{1}{2}\pi$ ($90°$), and an angle of $-\frac{1}{6}\pi$ ($-30°$) is the same as $\frac{5}{6}\pi$ ($330°$).

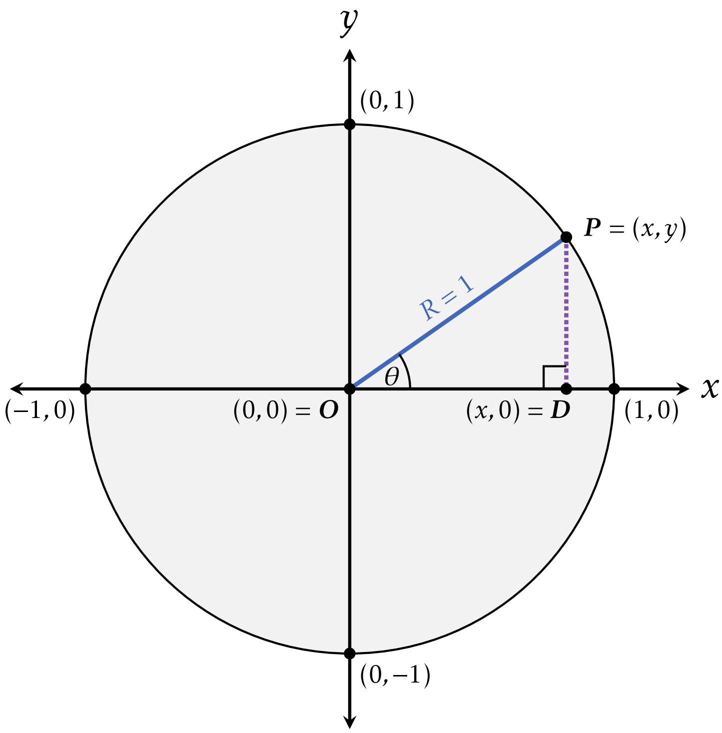


Figure 1 A unit circle with (...)

Table 4 Text

Quadrant	$\cos(\theta) = x$	$\sin(\theta) = y$
1	$[0, 1]$	$[0, 1]$
2	$[-1, 0]$	$[0, 1]$
3	$[-1, 0]$	$[-1, 0]$
4	$[0, 1]$	$[-1, 0]$

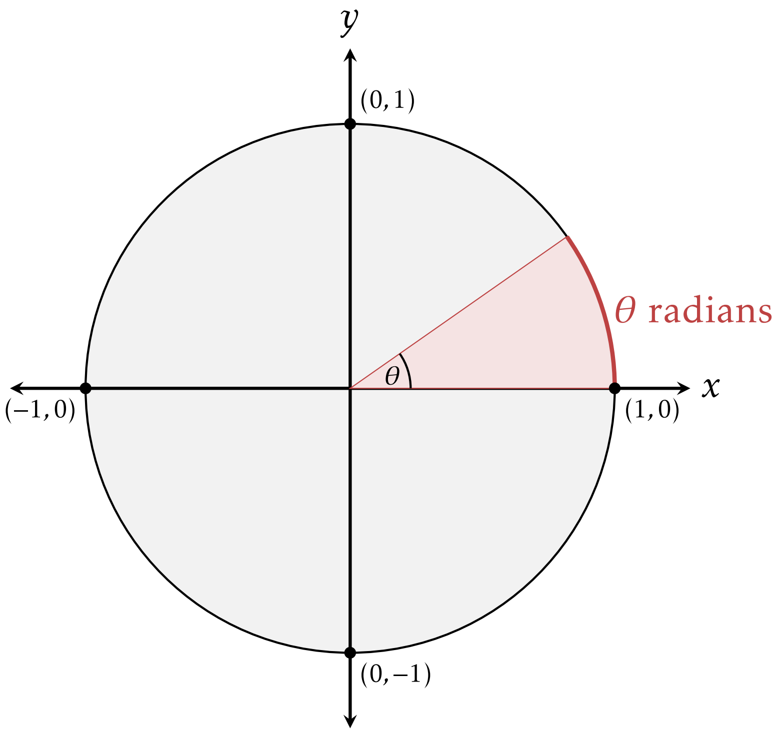


Figure 2 Radians

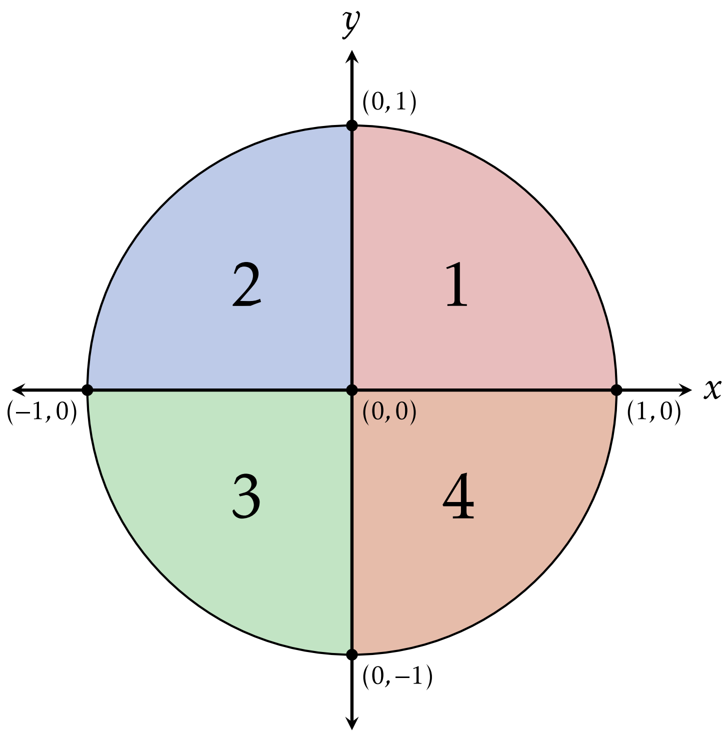


Figure 3 The different quadrants of the unit circle.

CHAPTER

1

THIS IS A TEST TITLE

1.1 THE FOO AND THE BAR

THE FOO; OR: WHY DOES A FISH?

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Definition 1.1 The Foo

A Foo is simply a thing, i.e.

$$e^{i\pi} = -e^{i\tau} = -1. \tag{1.1}$$

π

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Example 1.1 A Bla which is not a Foo

Is there even such a thing? Consider

$$f(x) = \sin(x) + 5x^2.$$

THE BAR, THE ONE AND ONLY

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Note 1.1 Something to Consider

Bla bla bla, yada yada yada. !

More lines.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

In the middle of this random Lorem Ipsum, there are some other words! This is refreshing, isn't it? Any way, this is used to test some **highlighting** and **indexing** of words. I hope it works. Well, it doesn't. I need to check how to make the marginnotes to go to the left side in odd pages.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat.

Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

Challange 1.1 Generalize this thing

In 2D, the distance between two points $\boldsymbol{A} = (A_x, A_y)$ and $\boldsymbol{B} = (B_x, B_y)$ is

$$|AB| = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2}. \tag{1.2}$$

Generalize this to 3D and to ND, where $N \in \mathbb{N}$.

?

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

As you probably learned in school, a **triangle** is a shape with three sides. An example of a triangle can be seen in **Figure** 1.1 .

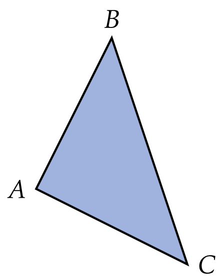


Figure 1.1 This is a triangle.

CHAPTER 1

Index