MATHEMATICS FOR SCIENCE STUDENTS

An open-source book

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$$a^b = e^{b\log(a)}$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$T\left(\alpha\vec{u} + \beta\vec{v}\right) = \alpha T\left(\vec{u}\right) + \beta T\left(\vec{v}\right)$$

$$A = Q\Lambda Q^{-1}$$

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\,\mathrm{d}t$$

$$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

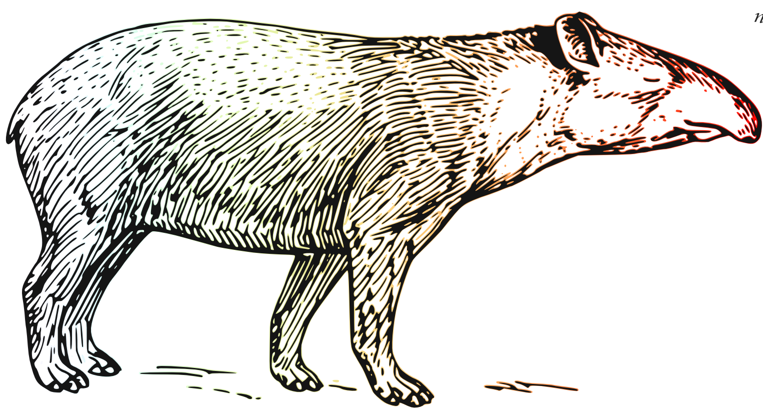
$$\langle \hat{e}_i, \hat{e}_j \rangle = \delta_{ij}$$

$$\vec{v} = \sum_{i=1}^{n} \alpha_i \hat{e}_i$$

$$e^{\pi i} + 1 = 0$$

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$



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! **To be written/to do**: Rights, lefts, etc. will be written here in the future !

HERE BE TABLE

CHAPTER 0

INTRODUCTION

In this chapter we introduce key concepts that will be used in later chapters. For this reason, unlike other chapters it contains many statements, sometimes given without thorough explanations or reasoning. While all of these statements are grounded in deep ideas and can be formulated in a rigorous manner, it is advised to first get an intuitive understanding of the ideas before diving into their more formal construction.

Note 0.1 In case you are already familiar with the topics

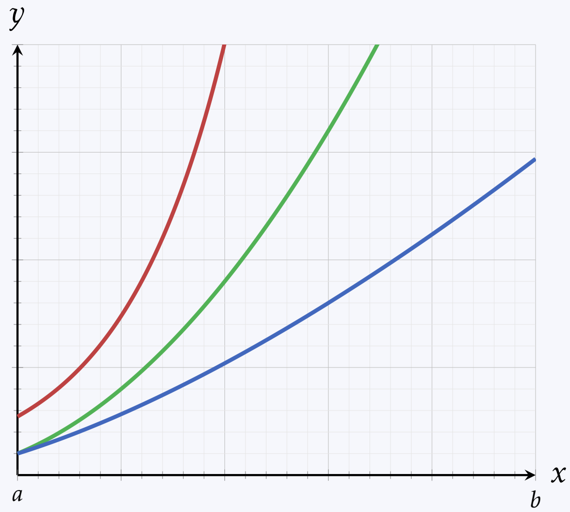
It is recommended for readers who are familiar with the topics to at least gloss over this chapter and make sure they know and understand all the concepts presented here.

0.1 DERIVATIVES

One of the most important tool in analyzing a function $f : \mathbb{R} \to \mathbb{R}$ is the ability to quantitatively describe the way it behaves as we change its argument x. At any given point a function can either increase in its value, decrease in its value, or stay constant. We would like to develop a method that is able to tell us exactly how such function changes at any given point.

Example 0.1 Quantitative measure of change

Compare the following three functions on the domain $x \in [a, b]$:



While all three functions are increasing on $[a, b]$ it is clear that the rate of increase between the functions is different: the red function increases faster than the green one, which in turn increases faster than the blue one. In fact, even within each function the increase is not uniform: the more x increases so does the rate of increase of the function.

To start developing a method to quantitatively measure the rate of change in a real function, we can first notice a property of some functions: if we zoom in on any point of the function we would see that the more we zoom in the more the function "straightens out", i.e. the function look more and more like a straight line (Figure 0.1).

This works for almost all point of each of the real fundamental functions and their compositions (except for some special points which we will discuss later). To quantify this "linearity", at some point x_0 on some function $f(x)$, we can measure the slope of the line we get when we zoom in enough on the function at x_0, i.e. the point $\left(x_0, f\left(x_0\right)\right)$. In Figure 0.1, for example, at $x = 1.5$ the slope would be approximately $m = -1$.

To calculate the slope we can start by marking another point a on the function, i.e.

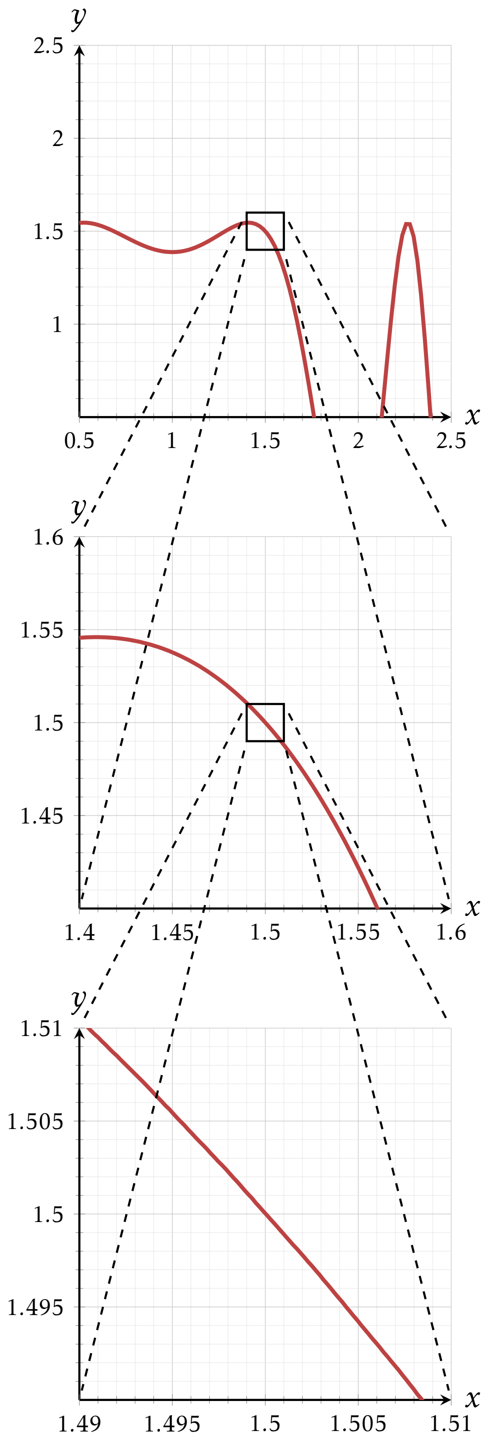


Figure 0.1 Zooming in on a real function at $x = 1.5$. Note how the more we zoom in, the more the function becomes "linear" and we can approximate its slope at $x = 1.5$ as being $m = -1$.

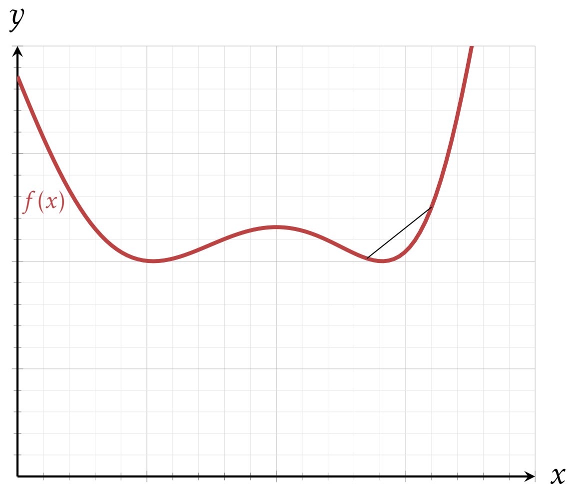


Figure 0.2

$\left(a, f(a)\right)$, such that a is a bit to the left of x_0: $a = x_0 + \Delta x$. We then connect the two points $\left(x_0, f(x_0)\right)$ and $\left(a, f_(a)\right)$ with a line. The slope of the line would be the difference in the y-values of the points divided by the difference in their x-values (see Figure 0.2).