CHAPTER 0

INTRODUCTION

In this chapter we introduce key concepts that will be used in later chapters. For this reason, unlike other chapters it contains many statements, sometimes given without thorough explainations or reasoning. While all of these statements are grounded in deep ideas and can be formulated in a rigorous manner, it is adviced to first get an intuitive understanding of the ideas before diving into their more formal construction.

Note 0.1 In case you are already familiar with the topics

It is recommended for readers who are familiar with the topics to at least gloss over this chapter and make sure they know and understand all the concept presented here. **!**

0.1 EXERCISES

0.1. Write the following sets explicitly:

(i) $\{x \in \mathbb{N} \mid 1 < x \leq 7\}$

(ii) $\{x \in \mathbb{Z} \mid x < 5\}$

(iii) $\left\{x \in \mathbb{R} \mid x^2 = -1\right\}$

(iv) $\{x \in \mathbb{N} \wedge x \in \mathbb{Q}\}$

(v) $\left\{x \in \mathbb{R} \mid x^2 - 3x - 4 = 0\right\}$

(vi) $\{x \in \mathbb{R} \mid x < 5 \wedge x \geq 2\}$

0.2. Determine the relation between the sets:

(i) $A = \{1,\ 2,\ 3\},\ B = \{1,\ 2\}$

(ii) $A = \varnothing,\ B = \{2,\ -5,\ \pi\}$

(iii) $A = \mathbb{Z},\ B = \{\pm x \mid x \in \mathbb{N} \cup \{0\}\}$

(iv) $A = \left\{\pi, e, \sqrt{2}\right\},\ B = \mathbb{Q}$

0.3. Write all elements in $S^2 \times W$, where $S = \{\alpha, \beta, \gamma\}$ and $W = \{x, y, z\}$. Find a condition that guarantees $S^2 \times W = W \times S^2$.

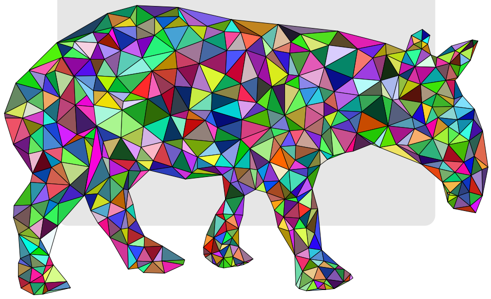
0.4. How many different injective functions $f : \{1,2\} \to \{1,2\}$ exist? How many injective functions $f : \{1,2,3\} \to \{1,2,3\}$ exist? How many inject functions $f : \{1,2,\ldots,n\} \to \{1,2,\ldots,n\}$ exist for a given $n \in \mathbb{N}$?

0.5. For each of the real functions below, find a set on which it is surjective (use a graphing calculator if your are not familiar with the shape of a function):

$$x^2,\ x^3 - 5,\ e^{-x^2/2},\ \sin(x),\ \sin(x) + \cos(x),\ xe^x.$$

0.6. Given two sets A, B such that $|B| = |A| - 1$, can a bijective function $f : A \to B$ exist? Explain your answer.

CHAPTER 1



LINEAR ALGEBRA
(INTUITIVE APPROACH)

Linear algebra is one of the most important and often used fields, both in theoretical and applied mathematics. It brings together the analysis of systems of linear equations and the analysis of linear functions (in this context usually called linear transformations), and is employed extensively in almost any modern mathematical field, e.g. approximation theory, vector analysis, signal analsys, error correction, 3-dimensional computer graphics and many, many more.

In this book, we divide our discussion of linear algebra into to chapters: the first (this chapter) deals with a wider, birds-eye view of the topic: it aims to give an intuitive understanding of the major ideas of the topic. For this reason, in this chapter we limit ourselves almost exclusively to discussing linear algebra using 2- and 3-dimensional analysis (and higher dimensions when relevant) using real numbers only. This allows us to first create an intuitive picture of what is linear algebra all about, and how to use correctly the tools it provides us with.

The next chapter takes the opposite approach: it builds all concepts from the ground-up, defining precisely (almost) all basic concepts and proving them rigorously, and only then using them to build the next steps. This approach has two major advantages: it guarntees that what we build has firm foundations and does not fall apart at any future point, and it also allows us to generalize the ideas constructed during the process to such extent that they can be used as foundation to build ever newer tools we can apply in a wide range of cases.

1.1 VECTORS

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1.2 LINEAR TRANSFORMATIONS

1.3 MATRICES

1.4 SYSTEMS OF LINEAR EQUATIONS

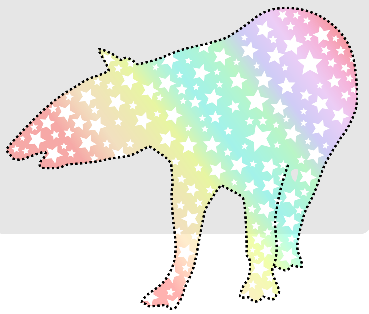
1.5 EIGENVECTORS AND EIGENVALUES

1.6 DECOMPOSITIONS

1.7 SOME REAL LIFE USES OF LINEAR ALGEBRA

1.8 EXERCISES

CHAPTER 2



LINEAR ALGEBRA

(RIGOROUS APPROACH)