MATHEMATICS FOR SCIENCE STUDENTS

An open-source book

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$a^b = e^{b \log(a)}$

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$T\left(\alpha \vec{u} + \beta \vec{v}\right) = \alpha T\left(\vec{u}\right) + \beta T\left(\vec{v}\right)$

$A = Q \Lambda Q^{-1}$

$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} \, dt$

$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$\langle \hat{e}_i, \hat{e}_j \rangle = \delta_{ij}$

$\vec{v} = \sum_{i=1}^{n} \alpha_i \hat{e}_i$

$e^{\pi i} + 1 = 0$

$\int_a^b f(x) \, dx = F(b) - F(a)$

$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$

PUBLISHED IN THE WILD

! **To be written**: Rights, lefts, etc. will be written here in the future **!**

HERE BE TABLE

CHAPTER

0

LINEAR ALGEBRA
(INTUITIVE APPROACH)

Linear algebra is one of the most important and often used fields, both in theoretical and applied mathematics. It brings together the analysis of systems of linear equations and the analysis of linear functions (in this context usually called linear transformations), and is employed extensively in almost any modern mathematical field, e.g. approximation theory, vector analysis, signal analysis, error correction, 3-dimensional computer graphics and many, many more.

In this book, we divide our discussion of linear algebra into to chapters: the first (this chapter) deals with a wider, birds-eye view of the topic: it aims to give an intuitive understanding of the major ideas of the topic. For this reason, in this chapter we limit ourselves almost exclusively to discussing linear algebra using 2- and 3-dimensional analysis (and higher dimensions when relevant) using real numbers only. This allows us to first create an intuitive picture of what is linear algebra all about, and how to use correctly the tools it provides us with.

The next chapter takes the opposite approach: it builds all concepts from the ground-up, defining precisely (almost) all basic concepts and proving them rigorously, and only

then using them to build the next steps. This approach has two major advantages: it guarantees that what we build has firm foundations and does not fall apart at any future point, and it also allows us to generalize the ideas constructed during the process to such extent that they can be used as foundation to build ever newer tools we can apply in a wide range of cases.

0.1 SOME REAL LIFE USES OF LINEAR ALGEBRA

! **To be written**: you guessed it - the entire section. **!**

0.1.1 Formulating plant nutrient solutions

Theoretical background

In order to grow and thrive, plants require nutrients. These nutrients are chemical elements which the plants use in different processes to create new roots, stems, leaves, fruits, etc. and to sustain those that already exist. Generaly speaking, there are 17 such elements, and they can be roughly divided in to three categories:

• **Nutrients from air**: carbon (C), oxygen (O) and hydrogen (H). Plants get these from water (H_2O), carbon dioxide (CO_2) and atmospheric oxygen (O_2), and use them together with light to build carbohydrates (such as glucose, fructose and cellulose), in a process known as photosynthesis.

• **Macronutrients from ground**: nitrogen (N), phosphorus (P), potassium (K), calcium (Ca), magnesium (Mg) and sulfur (S). These nutrients are absorbed mostly through the roots of the plant, and are needed in relatively large quantities.

• **Micronutrients from groud**: chlorine (Cl), iron (Fe), manganese (Mn), zinc (Zn), copper (Cu), molybdenum (Mo), boron (B) and nickel (Ni). These are also absorbed mainly via the roots, and are needed in relatively smaller quantities than the macronutrients.

Table 0.1 describes what each of the above elements is used for by plants.

! **To be written**: add data and format the table **!**

Different plants require different nutrient concentrations for optimal growth: not only do the actuall concentrations change, but the ratio between the different elements. See for example Table 0.2, which compares between the nutritional needs of a basil plant and a tomato plant - the ratio of the calcium vs. nitrogen requirements is 30% higher in tomatoes compared to basil (this is especially true during the fruiting stage of tomatoes).

Table 0.1 The 17 essential elements for plants and their uses. The column marked [ppm] shows the mass fraction of the element in dried plant material ($1[\text{ppm}] = 1 \times 10^{-6}$).

	Element	Symbol	[ppm]	Main uses
Air	Oxygen	O		Practically all organic mulecules
	Carbon	C		Carbohydrates and protein synthesis
	Hydrogen	H		Practically all organic mulecules
Macronutrients	Nitrogen	N	15,000	Proteins
	Phosphorus	P	2,000	Phospholipids, ATP
	Potassium	K	10,000	Ion balance
	Calcium	Ca	5,000	Ion balance
	Magnesium	Mg	2,000	Chlorophyl
	Sulfur	S	1,000	Proteins, vitamins, chloroplast
Micronutrients	Chlorine	Cl	100	Ion balance
	Iron	Fe	100	Enzyme cofactors, photosynthesis
	Manganese	Mn	50	Chloroplast
	Zinc	Zn	20	DNA transcriptase
	Copper	Cu	6	Photosynthesis
	Molybdenum	Mo	0.1	Enzyme cofactors
	Boron	B	20	Metabolism
	Nickel	Ni	0.1	Nitrogen metabolism

Table 0.2 Text text bla.

	Basil	Tomato
N	280	308
P	25	31
K	274	313
Ca	280	400
Mg	61	109
S	112	218
Cl	< 284	< 284
Fe	1.4	1.96
Mn	0.27	0.28
Zn	0.46	0.46
Cu	0.06	0.04
Mo	0.05	0.05
B	0.54	0.54

! **To be written**: rest of subsection !

0.1.2 Primary components analsis

0.1.3 Hunter-pray population growth