CHAPTER 0

INTRODUCTION

In this chapter we will introduce some key concepts that will be used in later chapters.

> **Note 0.1 In case you are already familiar with the topics** !
>
> It is recommended for readers who are familiar with the topics to at least gloss over this chapter and make sure they know and understand all the concept presented here.

0.1 MATHEMATICAL SYBOLS AND SETS

LOGICAL STATEMENTS AND THEIR TRUTH VALUE

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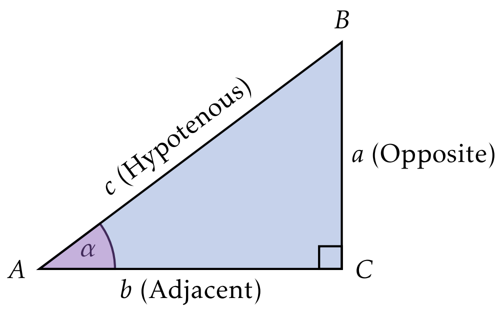
0.2 RELATIONS AND FUNCTIONS

0.3 TRIGONOMETRIC FUNCTIONS

BASIC DEFINITIONS

Consider a **right triangle** $\triangle ABC$ with sides a, b, and Hypotenous c, where the angle $\angle ACB$ is $90°$, and the angle $\angle BAC$ is denoted as α:



We use the ratios between the three sides of the triangle to define three functions of α:

Definition 0.1 The basic triginometric functions

1. The **sine** of the angle α is $\sin(\alpha) = \frac{a}{c}$,
2. the **cosine** of the angle α is $\cos(\alpha) = \frac{b}{c}$, and
3. the **tangent** of the angle α is $\tan(\alpha) = \frac{a}{b}$, which in turn is equal to $\frac{\sin(\alpha)}{\cos(\alpha)}$.

π

We can rearrange the above definitions to yield

$$\begin{aligned} a &= c\sin(\alpha), \\ b &= c\cos(\alpha). \end{aligned} \tag{1}$$

Normaly, the Hypotenous is the longest side of a right triangle. We will consider here the two edge cases where one of the sides a, b is equal to the Hypotenous (and the other side is thus 0):

- if $a = c$ then $\alpha = 90°$,
- if $b = c$ then $\alpha = 0$.

The posssible length of a is therefore in the range $0 \leq a \leq c$, which means that $0 \leq \frac{a}{c} \leq 1$, or since $\sin(\alpha) = \frac{a}{c}$,

$$0 \leq \sin(\alpha) \leq 1. \tag{2}$$

The same is of course true for b, and thus

$$0 \leq \cos(\alpha) \leq 1 \tag{3}$$

as well.

As a reminder, the **Pythagorean theorem**[1] states that for a right triangle like the one here,

$$a^2 + b^2 = c^2. \tag{4}$$

By substituting **Equation 1** into the above we get

$$c^2 = a^2 + b^2 = (c\sin(\alpha))^2 + (c\cos(\alpha))^2 = c^2\sin^2(\alpha) + c^2\cos^2(\alpha) = c^2\left[\sin^2(\alpha) + \cos^2(\alpha)\right],$$

and cancelling c^2 on both sides simply yields

$$\sin^2(\alpha) + \cos^2(\alpha) = 1. \tag{5}$$

THE UNIT CIRCLE

The range of the trigonometric functions can be extended by using the **unit circle**: a circle of radius $R = 1$ is placed such that its center lies at the origin of a 2-dimensional axis system, i.e. at the point $\boldsymbol{O} = (0, 0)$. A radius to a point $\boldsymbol{P} = (x, y)$ on the circle's circumference is the drawn. This radius has an angle θ to the x-axis. A line from $\boldsymbol{P}$ perpendicular to the x-axis intersecting at the point $\boldsymbol{D}$ is drawn (see **Figure 1**).

The triangle $\triangle OPD$ is a right triangle. Therefore, we can use the trigonometic functions to calculate the coordinates of the point $\boldsymbol{P} = (x, y)$:

$$\begin{aligned} x &= R\cos(\theta) = \cos(\theta), \\ y &= R\sin(\theta) = \sin(\theta). \end{aligned} \tag{6}$$

We can now define $\cos(\theta)$ and $\sin(\theta)$ as the values of x and y, respectively, as a function of θ.

We will switch to measuring angles in **Radians** instead of degrees: θ radians are equal to the length of an arc on a unit circle, which corresponds to the angle θ (**Figure 2**). This allows us to use the same units as x and y: for example, when length is measured in [meter], an angle in radians is measured in [meter] as well. The full circumference of a circle equals 2π radians, and therefore a single radian is equivalent to $\frac{180}{\pi} \approx 57.3°$. **Table 1** shows some common angles in radians.

Another advantage which we gain by defining the trigonometric functions using the unit circle is the extension of their domain to all of $\mathbb{R}$: an angle of size $2\frac{1}{2}\pi$ (equivalent to $450°$), for example is the same as an angle of size $\frac{1}{2}\pi$ ($90°$), and an angle of $-\frac{1}{6}\pi$ ($-30°$) is the same as $\frac{5}{6}\pi$ ($330°$).

[1] It's worth mentioning that no three positive integers a, b, and c satisfy the equation $a^n + b^n = c^n$ for any integer value of $n > 2$. This can be proven, however the proof is too large to fit in the footnotes.

Table 1 Common angles in radians.

Degrees	Radians
0°	0
45°	$\frac{\pi}{4}$
90°	$\frac{\pi}{2}$
180°	π
270°	$\frac{3\pi}{2}$
360°	2π

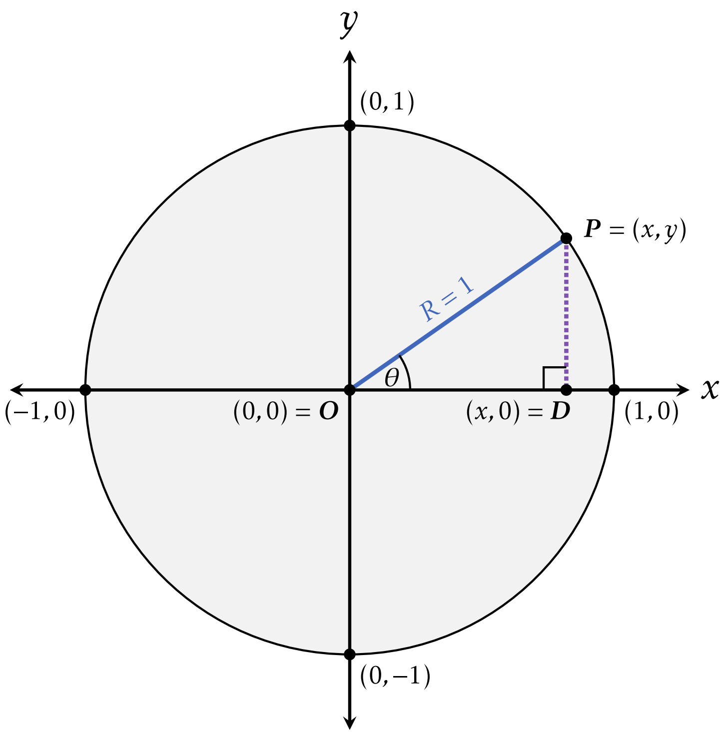


Figure 1 A unit circle with (...)

Table 2 Text

Quadrant	$\cos(\theta) = x$	$\sin(\theta) = y$
1	$[0,1]$	$[0,1]$
2	$[-1,0]$	$[0,1]$
3	$[-1,0]$	$[-1,0]$
4	$[0,1]$	$[-1,0]$

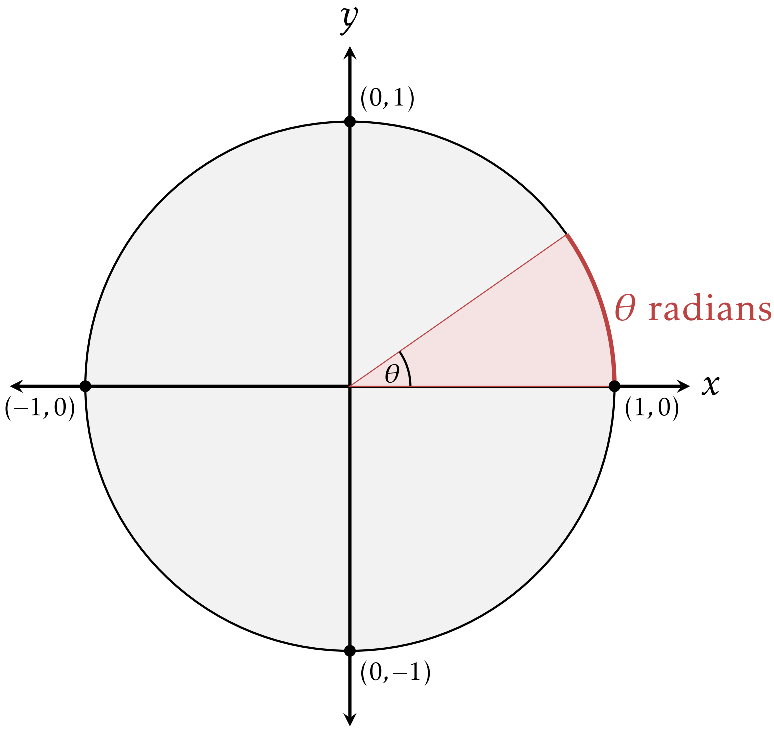


Figure 2 Radians

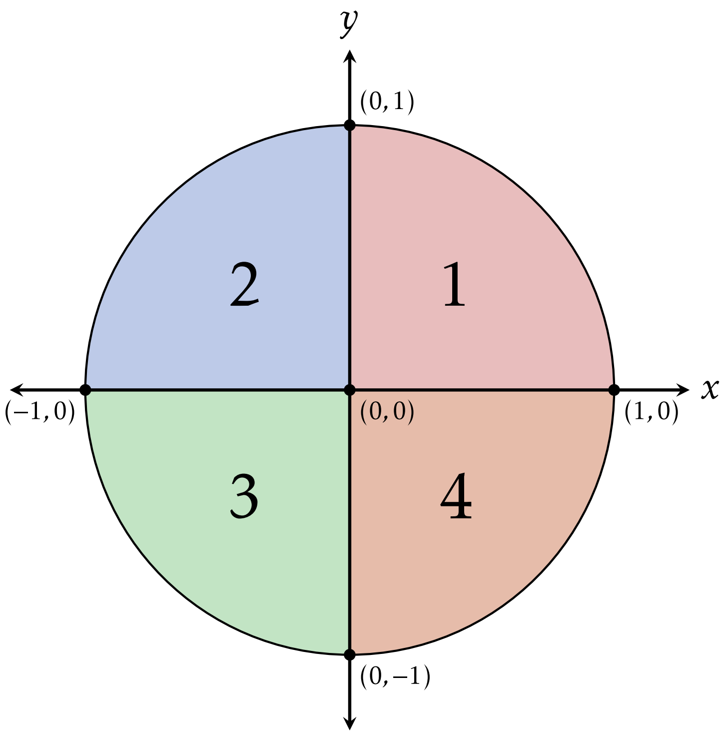


Figure 3 The different quadrants of the unit circle.

CHAPTER 1

THIS IS A TEST TITLE

1.1 THE FOO AND THE BAR

THE FOO; OR: WHY DOES A FISH?

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Definition 1.1 The Foo

A Foo is simply a thing, i.e.

$$e^{i\pi} = -e^{i\tau} = -1. \tag{1.1}$$

π

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Example 1.1 A Bla which is not a Foo

Is there even such a thing? Consider

$$f(x) = \sin(x) + 5x^2.$$

THE BAR, THE ONE AND ONLY

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Note 1.1 Something to Consider

Bla bla bla, yada yada yada.

More lines.

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In the middle of this random Lorem Ipsum, there are some other words! This is refreshing, isn't it? Any way, this is used to test some **highlighting** and **indexing** of words. I hope it works. Well, it doesn't. I need to check how to make the marginnotes to go to the left side in odd pages.

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Challange 1.1 Generalize this thing

In 2D, the distance between two points $\boldsymbol{A} = (A_x, A_y)$ and $\boldsymbol{B} = (B_x, B_y)$ is

$$|AB| = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2}. \tag{1.2}$$

Generalize this to 3D and to ND, where $N \in \mathbb{N}$.

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As you probably learned in school, a **triangle** is a shape with three sides. An example of a triangle can be seen in **Figure** 1.1 .

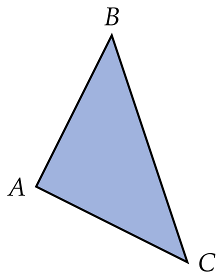

Figure 1.1 This is a triangle.

CHAPTER 1

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