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Lab 8: Momentum

Introduction:

The general form of Newton's Second Law states that the net external force $\vec{F}_{net}$ acting on an object is equal to the change in the object's linear momentum $\vec{p} = m\vec{v}$. If the mass of the object remains constant, Newton's Second Law reduces to the simplified form you are used to seeing in introductory physics. That is:

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} \tag{1}$$

Today you will study collisions between two objects of fixed mass, m_1 and m_2. You should assume the two objects form a system that is otherwise isolated in space. According to Newton's 3^{rd} Law, when the objects collide they exert *equal and opposite* forces on each other. Thus, the net change in momentum *of the system* of two masses due to the collision is zero, meaning $\vec{p}_{sys}$ remains constant. Mathematically,

$$\vec{F}_{1\to2} + \vec{F}_{2\to1} = \vec{F}_{sys} = 0 \quad \to \quad \frac{d\vec{p}_{sys}}{dt} = 0 \quad \to \quad \vec{p}_{sys} = constant. \tag{2}$$

There are two common types of collisions, *elastic* and *inelastic*. In both types of collisions the total momentum of the system is conserved, since the colliding objects exert equal and opposite forces on each other. But the total kinetic energy of the system is conserved only in purely elastic collisions. For the case of two colliding objects:

Pure Elastic Collision:

$$\vec{p}_{sys} = m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} = constant \tag{3}$$

$$(KE)_{sys} = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 = constant \tag{4}$$

Inelastic Collision:

$$\vec{p}_{sys} = m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} = constant \tag{5}$$

$$(KE)_{sys} \neq constant \tag{6}$$

One example of an elastic collision is when two hard objects bounce off each other. An example of an inelastic collision is when wet putty is thrown against a moving solid block and the two stick together. In that case, the momentum of the system (putty & block) is still conserved (the forces between the colliding putty and block are equal and opposite), but kinetic energy is not. A fraction of the initial kinetic energy of the system gets converted into other forms, such as heat, permanent shape deformation, sound, etc. In reality, there is often kinetic energy lost in all collisions configured in a lab. To quantify the effect, it is helpful to introduce the restitution coefficient:

$$e = \frac{\vec{v}_{1f} - \vec{v}_{2f}}{\vec{v}_{2i} - \vec{v}_{1i}}. \tag{7}$$

For a perfectly elastic collision, $e = 1$ and for a perfectly inelastic collision (starting with two bodies and ending with one), $e = 0$. For most real collisions $0 < e < 1$. It is convenient to note that Eqs. (3) and (4) can be combined to express the final velocity of each mass in a perfectly elastic collision in terms of initial velocities:

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2}\vec{v}_{1i} + \frac{2m_2}{m_1 + m_2}\vec{v}_{2i} \tag{8}$$

$$\vec{v}_{2f} = \frac{2m_1}{m_1 + m_2}\vec{v}_{1i} + \frac{m_2 - m_1}{m_1 + m_2}\vec{v}_{2i}. \tag{9}$$

Click here to access a java applet that simulates head-on elastic and inelastic collisions involving bumper carts. The user interface window is shown in Figure 1. Try playing with the interface for a few minutes. Notice how you can enter different velocity (has direction!) and mass values. Click the "set values and play" button to register your values and run the virtual simulation. The bar graph provided shows the instantaneous value of each cart's kinetic energy (alas, in arbitrary units). Position is given in meters and time is given in seconds.

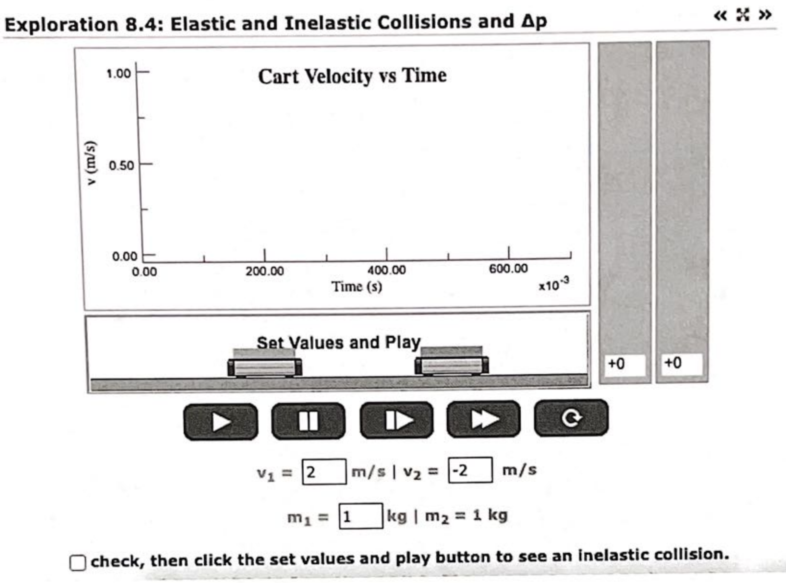


Figure 1: The user interface window for studying collisions between two objects. Use the check box at the bottom of the window to change the collision type (perfectly elastic vs. perfectly inelastic). You can find the coordinates of a point on the "Cart Velocity vs Time" graph by clicking on that point.

Part 1: Elastic Collisions

Here you will perform four experiments to observe and verify conservation of linear momentum and conservation of energy for <u>elastic</u> collisions. Use the table below to organize your data. The "Collision type" in the table correspond to the experiments described in parts 1-4 below. For each collision, you will calculate the initial and final momentum ($\vec{p}_i$ and $\vec{p}_f$) of the system. Remember, velocity and momentum are vectors, so you must assign them a direction! Add proper units at the top of each column of the table.

Collision type	m_1 kg	m_2 kg	$\vec{v}_i$ of m_1 m/s	$\vec{v}_i$ of m_2 m/s	$\vec{v}_f$ of m_1 m/s	$\vec{v}_f$ of m_2 m/s	$\vec{p}_i$	$\vec{p}_f$
1	1	1	2	0	0	2	2	2
2	2	1	2	0	0.6	2.7	4	3.9
3	0.5	1	2	0	-0.67	1.3	1	1.01
4	1	1	2	-1	-1	2	1	1

$P_1 = P_F$

$M_1V_1 + M_2V_2 = M_1V_1'^{2} + M_2V_2'$

1. **Collision between two carts of similar mass, one of them at rest:**

Set $m_1 = m_2$. With Cart 2 sitting at rest, "push" Cart 1 (with any speed) towards Cart 2. By default, the software will simulate an elastic collision. Read the initial and final velocities (in the x-direction) of the carts from the "Cart Velocity vs Time" graph and record the values both in the table and below. Estimate the uncertainties of the velocity values as read from the graph.

$\vec{v}_{1i} =$ 2 $\pm$ 0.05 $\quad\quad$ $\vec{v}_{1f} =$ 0 $\pm$ 0.05

$\vec{v}_{2i} =$ 0 $\pm$ 0.05 $\quad\quad$ $\vec{v}_{2f} =$ 2 $\pm$ 0.05

Calculate the total initial and final momenta and estimate their uncertainties using error propagation rules (see Appendix). Consider the mass uncertainty Δm to be ± 1 *gram*. Show your work clearly. Considering the experimental uncertainties, can you conclude that the total momentum was conserved in the collision?

$\vec{p}_i =$ 2 $\pm$ 0.032 $\quad\quad$ $\vec{p}_f =$ 2 $\pm$ 0.032

$$\frac{\Delta P}{p} = \sqrt{\left(\frac{\Delta v_1}{v_1}\right)^2 + \left(\frac{\Delta m}{m}\right)^2} \quad \sqrt{\left(\frac{0.001}{1}\right)^2 + \left(\frac{0.05}{2}\right)^2}$$

$$= \frac{\Delta P_1}{2} \quad \Delta p = 0.05$$

$$\Delta P = \sqrt{P_1^2 + P_2^2}$$

$$\Delta P_2 = 0.5$$

Was the total kinetic energy of the system conserved in the collision? What did you expect? What were some sources of experimental error? Show your work.

$KE_i =$ 2 $\pm$ 0.032 $\quad\quad$ $KE_f =$ 2 $\pm$ 0.032

$$\frac{1}{2} m v^2$$

$$\frac{\Delta P}{P} = \sqrt{\left(\frac{\Delta v_1}{v_1}\right)^2 + \left(\frac{\Delta m}{m}\right)^2}$$

Energy was conserved. Yes, because we aren't assuming friction.

2. **Collision between two carts of different masses, the lighter one at rest:**

Set $m_1 = 2$ kg, $v_{1i} = 2$ m/s and $v_{2i} = 0$ m/s and "push" Cart 1 towards Cart 2 (initially at rest). Use Equations 8 and 9 and your initial velocities to *predict* the magnitudes and directions of the cart velocities after the collision. Show all work.

$\vec{v}_{1f} =$ 0.66 $\vec{v}_{2f} =$ 2.66

$v_{2i} = 0$

$$v_i = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 \qquad v_F = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_2 + m_1} v_{2i}$$

$$\frac{2-1}{3}(2)$$

Now run the simulation, and measure the initial and final velocities. Are your measured values in agreement with your calculated (expected) ones? If not, what sources of errors could explain your discrepancies?

They are very similar to our calculations. Discrepancy is from the error in our measurement.

Confirm conservation of momentum. Show your work. Remember to write your values in the data table above.

$$P_i = P_F$$

$$m_1 v_1 + m_2 v_2 = m_1 v_i' + m_2 v_2'$$

$$2(0.66) + 1(2.66) = 3.98 \text{ JJ}$$

$$2(2) + 0 = 4$$

3. **Collision between two carts of different masses, the heavier one at rest:**

This time set $m_1 = 0.5$ kg, $v_{2i} = 0$ m/s, and the velocity of Cart 1 to a non-zero value and observe the collision. Check to see that total momentum was conserved in the collision. Show your work.

$$P_i = m_1 v_1 + m_2 v_2 = 1$$

$$P_f = m_1 v_i' + m_2 v_2' = 1.01$$

Determine if the collision was perfectly elastic by calculating the restitution coefficient: $e =$ 1.
(Did you get $e = 1$?)

$$e = \frac{v_1' - v_2'}{v_2 - v_1} \qquad \frac{2-0}{1.3 - 0.6} = 1$$

$\boxed{\text{Yes, I got } e = 1!}$

4. **Head-on collision between two carts of similar mass, both moving:**

   Set $m_1 = m_2$, v_{1i} = 2 m/s and v_{2i} = -1 m/s and run the simulation. Check to see that total momentum was conserved in the collision. Show your work.

$$P_i = m_1 v_1 + m_2 v_2 = 1(2) + 1(-1) = 1$$

J

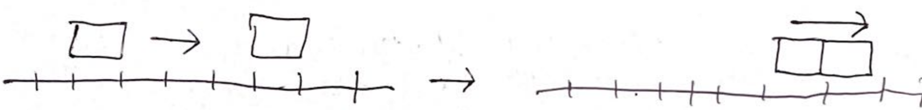
$$P_f = m_1 v_1' + m_2 v_2' = 1(-1) + 1(2) = 1$$

Part 2: Inelastic Collisions

Now you will conduct a series of "perfectly inelastic" collisions and check to see if momentum and / or kinetic energy are conserved in each case. To do this part of the experiment you must first check-the-box at the bottom of the applet window to simulate inelastic collisions.

1. **Sticky collision between two carts of similar masses, one of them at rest:**

   Derive a general expression for the final velocity of two "stuck" objects after they undergo a totally inelastic collision. Assume object m_1 is initially at rest. Sketch "before" and "after" pictures of the collision. Write your answer for $\vec{v}_f$ in terms of the two masses and initial velocities, noting your choice of coordinate axes.



$$v_f = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

Now set $m_1 = m_2$, v_{1i} = 0 m/s and v_{2i} = -2 m/s and calculate the expected final velocity (magnitude and direction) of your system. Then carry out the experiment. Does your experimental result agree with your prediction to within your experimental uncertainty? Comment.

$\vec{v}_f$ (predicted) = -1 $\qquad$ $\vec{v}_f$ (experiment) = -1 ± 0.5

$$\frac{0 + (-2)}{2} = -1$$

Use your experimental values to calculate the total initial and final momentum. Was momentum conserved in the collision? Comment.

$\vec{p}_i$ = -2 $\qquad$ $\vec{p}_f$ = -2

$m_1(0) + (1)(-2)$

$(m_1 + m_2) v_F \rightarrow (2)(-1)$

Calculate the change in total kinetic energy ΔKE of the system before and after the collision. Also estimate your uncertainty in this value. Show your work and comment on your result.

$KE_i =$ 2 J ± 0.05 $KE_f =$ 1 ± 0.05 $\Delta KE =$ 1 ± 0.005

$\frac{1}{2} m v^2$

$\frac{1}{2}(1)(-2)^2$

$\frac{1}{2}(2)(1)^2$

$\frac{\Delta h}{h} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta v}{v}\right)^2}$

$\Delta KE = (0.05)^2 + (0.05)^2$

$\Delta h = 2\sqrt{\left(\frac{0.001}{1}\right)^2 + \left(\frac{0.05}{-2}\right)^2}$

$= 0.05$

2. **Sticky head-on collision between two carts of similar masses, both moving:**

Suppose you push the carts towards each other, aiming for a "sticky" collision. Predict (explain in words) the relationship between the initial velocities of the two carts, in order for the system to come to rest directly after the collision. Then verify your prediction experimentally.

The initial ~~velocitys~~ velocities are opposite signs. The cart with the greater momentum will dictate what direction it goes in after the collision. If everything is equal in opposite direction then it's zero.

$\vec{v}_{1i} =$ 1 $\vec{v}_{1f} =$ 0 $\vec{v}_{2i} =$ -1 $\vec{v}_{2f} =$ 0

Calculate the total momentum of the system before and after the collision to determine if momentum was conserved in this process. Comment on your result.

$\vec{p}_i =$ 0 $\vec{p}_f =$ 0

$m_1 \quad v_1 \qquad m_2 \quad v_2$

$(1)(-1) + (1)(-1) = 0$ conserved

- Momentum was conserved in this process