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Lab Partners: ____________

Lab 9: Torque and Rotational Motion

Introduction

Today you will study rigid objects rotating about a fixed axis. Such motion is well described by Newton's 2nd Law in rotational form:

$$\vec{\tau}_{net} = I\vec{\alpha}, \tag{1}$$

where $\vec{\tau}$ is the "torque" (rotational analogue of force) acting on the body, I is the moment of inertia of the body relative to some fixed axis (rotational analog of mass), and $\vec{\alpha}$ is the angular acceleration of the body about that axis. Mathematically, the torque is defined by:

$$\vec{\tau} = \vec{r} \times \vec{F} = |r||F|sin(\theta)\hat{n} \tag{2}$$

The magnitude of $\vec{\tau}$ describes the amount of "twist" applied to the object. It depends on the external force $\vec{F}$ applied to the object, the location $\vec{r}$ of the applied force relative to the rotational axis, and the angle θ between $\vec{r}$ and $\vec{F}$. The direction $\hat{n}$ of the torque is perpendicular to both $\vec{r}$ and $\vec{F}$ and is given by the right-hand-rule. The moment of inertia I of an object depends on the mass of the object and how the mass is distributed about some rotational axis. The moment of inertia describes a body's resistance to being rotated about some axis.

For the simplest case of a point-like particle of mass m a distance r from the axis of rotation, the moment of inertia is simply $I = mr^2$. For a system of N point-like particles with individual masses m_i and locations r_i relative to a common axis of rotation, the moment of inertia (Eq. 1) is given by:

$$I = \sum_{i}^{N} m_i r_i^2. \tag{3}$$

Most real-world objects are comprised of a continuous distribution of mass. For those situations the discrete sum over masses given in Eq. 3 is replaced by an integral:

$$I = \int r^2 dm \tag{4}$$

Expressions for the moment of inertia of common objects such as uniform cylinders, spheres, hoops, rods, *etc.* rotated about a specific axis (not always passing through the center of mass of the object) are given in your textbook.

Part 1: Torque and Moment of Inertia

In the first part of this experiment you will use a simulator to apply a known torque to an ideal frictionless pulley wheel (mass m and radius R) and measure its resulting angular acceleration. From your data you will experimentally determine the moment of inertia of the wheel. You will then compare your experimental result for I to a calculated (theoretical) value. The applet you will use can be found here. The user interface window is shown in Fig. 1 below. Real pulley systems often have friction and massive connecting ropes but you will get to ignore these factors today.

1. Draw a free body diagram for the mass-pulley system shown in Fig. 1. Clearly label all the forces and indicate your established coordinate system. Show that the applied force that creates a torque on the disk, rotating it about its central axis, is due to the tension in the rope:

$$T = m(g - \alpha R), \tag{5}$$

where m is the mass of the hanging weight, α is the angular acceleration of the system (magnitude), R is the radius of the pulley wheel, and $g = 9.8\ m/s^2$. Recall: linear acceleration $a = R\alpha$.

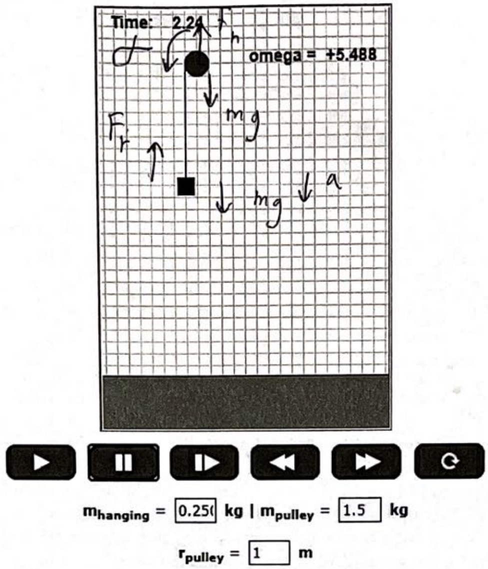

Figure 1: Side view of the pulley system that consists of a block hanging from a long (massless) string that is wrapped around a uniform pulley wheel of known mass and radius. Time is measured in seconds and angular speed ω in rad/sec. Each square on the grid is 1 m in length and height. Set the pulley (disk) mass to 1.5 kg. The coordinates (in meters) of any point on the grid screen can be found by clicking on the point.

2. Set the mass of the pulley (M) to 1.5 kg and its radius (R) to 1.0 m. You will keep these values fixed throughout the lab. Attach a 250 g "hanging mass" to the end of the pulley's string. Release the system from rest and wait at least 15 m before stopping it. Record the elapsed time and corresponding final angular velocity of the pulley wheel. Then use the expression $\omega(t) = \omega_o + \alpha t$ to determine the angular acceleration of the pulley.

$\omega =$ 10.241 rad/s, $\alpha =$ 2.45 rad/s^2.

3. Repeat the experiment until you have data for six different hanging mass values ranging from 250 g to 500 g. Organize your data in the table below. Make sure to add the appropriate units in the heading of each column. Determine the angular acceleration α for each experimental run. Calculate the applied tension force (Eq. 5) that creates the torque, and use Eq. 2 to calculate the magnitude of the applied torque You will use this data later to experimentally determine the moment of inertia of your pulley wheel.

Hanging Mass	Time s	ω rad/s	α rad/s^2	Tension N	$\lvert\vec{\tau}\rvert$ N·m
250	4.18	10.471	2.45	1.837	1.837
300	3.9	10.92	2.8	2.1	2.1
350	3.7	11.537	3.12	2.338	2.338
400	3.54	12.067	3.41	2.556	2.556
450	3.4	12.495	3.675	2.756	2.756
500	3.3	12.936	3.92	2.94	2.94

4. Show a sample calculation for each column in the table above.

250_g ; $t = 4.18$; $\omega = 10.421$

$10.421 = 0 + \alpha 4.18$

$\alpha = 2.45$

$T = m(g - \alpha R)$

$T = 0.25(9.8 - 2.45(1m))$

$= 1.837$

$\tau = Fl \sin\theta$

$\tau = 1.837(1) \sin 90^\circ = 1.837$

5. Plot the applied torque τ as a function of the angular acceleration α. Draw a best fit line through your data points. From the slope of line of best fit, determine the moment of inertia of the pulley wheel. Be sure to include units.

$I =$ 0.75 $kg \cdot m^2$.

- See last page of lab

6. Find an expression (in your textbook or notes) that approximates the moment of inertia I of your pulley wheel for the given conditions, including the axis of rotation. Use the known mass and radius of your pulley wheel to calculate its moment of inertia (in this configuration).

$I_{theoretical} =$ 0.75 $kg \cdot m^2$.

$= \frac{1}{2} mr^2$

$= \frac{1}{2}(1.5)(1)^2$

7. Compare your theoretical value of I with the moment of inertia you obtained from your slope calculation. Was it reasonable to approximate the moment of the pulley as $\frac{1}{2}MR^2$? What were your largest sources of uncertainty and do they account for the discrepancy between your two values for I?

The numbers are identical because there was no error in the data gathering because I waited until it hit the bottom for every run of the simulation.

$\theta = 19.92$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

Part 2: Rotational and Translational Motion

1. Use your experimental value for I and the standard rotational kinematic equations to predict: (a) the time it would take for a 150 gram hanging mass to fall 20 meters in your pulley system, (b) the angular speed of the pulley wheel at this time, and (c) the number of revolutions the pulley wheel made during this time interval. Show your work.

   $t =$ 4.94 s, $\omega =$ 8.067, # revolutions = 3.17

$m = 0.15$ kg

$x_0 = 0$

$x = 20$ m

$\omega_0 = 0$

$\omega_1 = ?$

$I = 0.75$

$\theta_0 = 0$

$\theta = \frac{\omega}{2\pi}$

$R\alpha = \frac{mg - T}{m}$

$T = \frac{mg}{1 + \left(\frac{mR^2}{I}\right)}$

$a = \frac{\frac{mg}{1 + \left(\frac{mR}{I}\right)^2} R^2}{I}$

$\omega = \omega_0 + \alpha t$

$\omega = 0 + 1.633(4.94)$

$0 + 0 + \frac{1}{2}(1.633)(4.94)^2$

$\frac{19.92}{2\pi}$

$\alpha = \frac{g}{r + \frac{I}{mR}}$

$\alpha = 1.633$

$1.633 - 1 = \alpha$

$a = \frac{g}{1 + \left(\frac{I}{mR^2}\right)}$

$\alpha = \frac{a}{R} \rightarrow \frac{1}{R} \cdot \frac{g}{1 + \left(\frac{I}{mR^2}\right)}$

$20 = 0 + 0 + \frac{1}{2}(1.633)t^2$

$t = 4.94$ s

2. Now do the experiment! Measure the time it takes for a 150 g hanging mass to fall a vertical distance of 20 m. Estimate your uncertainty in this value. Also count the number of revolutions and record the angular velocity of the wheel at this time. If you are unsure how to determine your uncertainties, consider running the experiment several times and comparing your results, etc.

   $t =$ 4.94 $\pm$ 0.05 s, # revolutions = 3.2

   $\omega =$ 8.064 $\pm$ 0.05 rad/s

$\alpha = \frac{8.069}{4.94} = 1.633$

3. How do your experimental results compare with your predicted values? Be quantitative. Briefly explain what you needed to do to get results as accurately as possible.

The experimental results were enormously close to our predicted results. We counted the grid to find out where we needed to stop the simulation as worked together to get data.