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Lab 6: Uniform Circular Motion

Introduction:

Uniform circular motion is characterized by an object moving in a circular path with constant speed $v = |\vec{v}|$. In this case, although there is no change in the object's speed, it still experiences a non-zero (vector) acceleration, because the direction of $\vec{v}$ is changing. The acceleration in this special case is called centripetal acceleration: $\vec{a}_R$ (or $\vec{a}_c$ in some textbooks). By definition the centripetal acceleration points toward the center of the circular path (see Figure 1) and has a magnitude given by:

$$|\vec{a}_R| = \frac{|\vec{v}|^2}{r}, \tag{1}$$

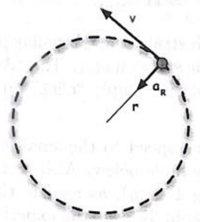


Figure 1: Particle moving with constant speed $|\vec{v}|$ along a circular path of radius r. The particle's instantaneous velocity $\vec{v}$ and centripetal acceleration $\vec{a}_R$ vectors are shown. There is no component of acceleration along the path of the particle because the particle's speed is constant in this example.

According to Newton's 2nd law, the centripetal acceleration acting on an object of mass m must originate from some net force that acts perpendicular to the object's path. We call this net force the centripetal (or radial) force:

$$|\vec{F}_R| = m|\vec{a}_R|. \tag{2}$$

Centripetal forces always have a physical origin, for example, tension, normal force, friction, etc. Today, you will explore the motion of a car rounding a curved road at constant speed ($|\vec{v}|$). You will study the centripetal acceleration of the car and identify the forces acting on the car that provide this acceleration. You will see how your car can maintain a circular path only if sufficient centripetal force is present. In the real world, the centripetal force on a car is most often provided by the combination of a banked road and friction between the tires and the road.

Part 1: Centripetal Acceleration in Uniform Circular Motion

The Physics Aviary app found here simulates the motion of a car traveling with a constant speed along a curve. Car's acceleration is monitored using an accelerometer and displayed in an "Acceleration vs Time" graph. The goal of this part of the lab is to determine the radius of curvature of the road from speed and acceleration measurements. You will use different techniques to estimate how the experimental uncertainties of speed and acceleration values propagate into the uncertainty of your final result for the radius of the road.

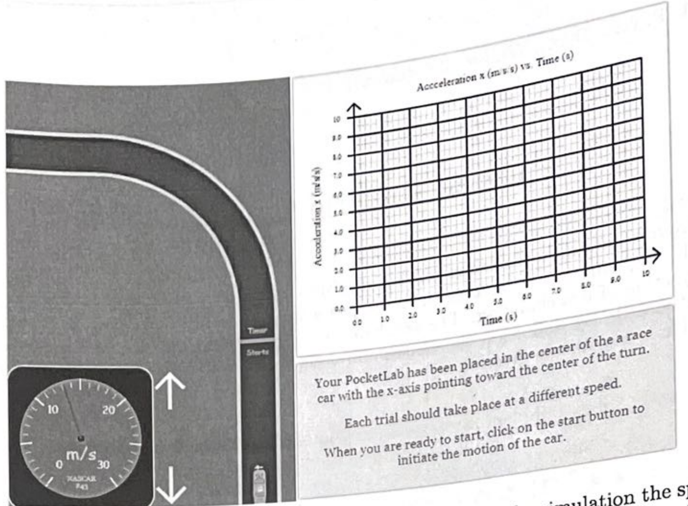


Figure 2: Car on a track that includes both straight and circular portions. In the simulation the speed of the car can be adjusted using the arrows adjacent to the speedometer. The "Acceleration vs Time" graph displays the magnitude of the acceleration of the car during its motion. Simply "click" on the car when you are ready to run the simulation.

1. Start the simulation and set the car's speed to the smallest possible value. Record the value of v and your experimental uncertainty $\pm\Delta v$ in the table below. Also record the corresponding magnitude of the centripetal acceleration with its uncertainty ($a_R \pm \Delta a_R$), as read of the "Acceleration vs Time" graph. Make sure you understand how to interpret that graph! Repeat the experiment for five other speeds. Record all data in the table and make sure to add the appropriate units to the top of each column.

Speed, v	$\pm\Delta v$	Acceleration, a_R	$\pm\Delta a_R$	v^2	$\pm\Delta(v^2)$
5 m/s	0.5 m/s	0.33 m/s^2	0.5 m/s^2	25 m/s^2	5 m/s^2
8.1 m/s	0.5 m/s	0.91 m/s^2	0.5 m/s^2	65.61 m/s^2	8.1 m/s^2
10.1 m/s	0.5 m/s	2.1 m/s^2	0.5 m/s^2	102.01 m/s^2	10.1 m/s^2
13.5 m/s	0.5 m/s	3.0 m/s^2	0.5 m/s^2	182.25 m/s^2	13.5 m/s^2
20.9 m/s	0.5 m/s	5.25 m/s^2	0.5 m/s^2	496.81 m/s^2	20.9 m/s^2
23.9 m/s	0.5 m/s	7 m/s^2	0.5 m/s^2	571.21 m/s^2	23.9 m/s^2

2. Explain how you estimated the uncertainties of your speed and acceleration measurements.

I estimated the uncertainties by taking half of the smallest increment of the instrument's scale. The smallest increment for the speed instrument is 1 m/s and the smallest increment on the acceleration axis of the graph is 0.5 m/s^2, so we take half of those values to estimate the uncertainties.

3. Complete the table using the rules for propagation of errors (see Appendix on Camino) to calculate the derived uncertainty $\Delta(v^2)$. Show a sample calculation.

Row 6: $\Delta v^2 = |2|(23.9)^{(2-1)}(0.5) = 23.9\ m/s^2$

4. Make a graph of the centripetal acceleration a_R as a function of v^2. Include horizontal and vertical error bars.

5. Now draw a line of best fit to your data and determine its slope, including units! What does the slope physically represent in this case? (Try to answer without reading below.)

Slope = 0.0108 1/m $\qquad f(x) = 0.0108x + 0.4916$

The slope physical represents the reciprocal of the radius.

6. In general, any experimental result is only as good as its error analysis allows. Today we want you to use a simple graphical error analysis technique to determine the quality of your results. Do the following: Carefully draw two additional straight-line fits to your data that focus on the error bars. One of the additional lines (the "max-slope" line) should have the largest reasonable slope that fits the majority of your error bars (i.e., not just your data points). The other (the "min-slope" line) should have the smallest reasonable slope that fits the majority of your error bars. Find the slopes of these two lines and use them to estimate the experimental uncertainty in your "best-fit" results. (Hint: Consider using $\Delta Slope = \frac{(slope_{max} - slope_{min})}{2}$.) Show your work (with units).

$Slope_{max}$ = 0.0199 1/m $\quad Slope_{min}$ = 0.01 1/m $\quad \Delta Slope = \pm$ 0.00495 1/m

$$\Delta\ slope = \frac{(0.0199 - 0.01)}{2} = \boxed{0.00495\ 1/m}$$

7. Use your experimental results to determine the radius r of the circular portion of your racetrack.

$r =$ 92.59 m $\pm$ 42.44 m

$slope = \frac{a_r}{v^2} = 0.0108\ 1/m$

$a_r = v^2/r$

$r = v^2/a_r = 1/0.0108 = 92.5926\ m$

Power uncertainty, since $0.0108^\wedge(-1) = 92.5926$:

$\Delta r = |-1|(0.0108)^\wedge(-1-1)(0.00495) = 42.44\ m$

Part 2: Car Rounding a Curve

In this part of the lab you will explore the conditions necessary for a car to safely maneuver a curved road. You will start by considering a flat road using the Physics Aviary simulation found here.

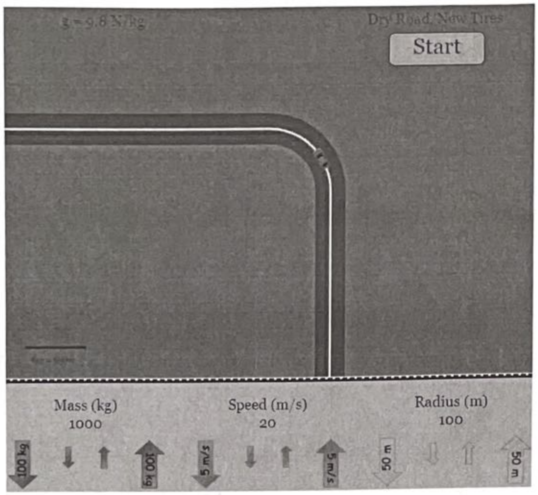


Figure 3: Simulation window for a car traveling on a flat, curved road. You can choose different road and tire conditions by clicking sequentially on "Dry Road/New Tires" (or similar text) above the "Start" button.

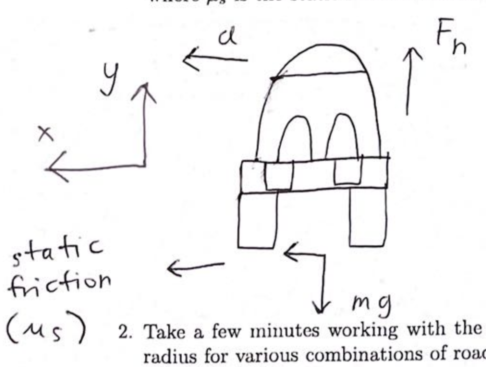
Unbanked curve:

For an unbanked curve under normal driving conditions, the only force responsible for keeping the car moving nicely on the curve is static friction $\mu_s|\vec{N}|$ between the tires and the road. The friction is static because the contact point between a car's rotating tires and the road is momentarily fixed in location. A car will skid or slip on the road if the rotating tires can not "grab" the road using static friction. In this case, the car will slide the way any object would, experiencing only kinetic friction ($\mu_k|\vec{N}|$), which might be close to zero on an icy road.

1. Draw a free body diagram for a car moving on a flat circular path of radius r. Clearly indicate your choice of coordinate axes. Then use Newton's 2^{nd} Law to show that maximum speed v_{max} at which the car can safely remain on the unbanked road is:

$$v_{max} = \sqrt{rg\mu_s}, \tag{3}$$

where μ_s is the static friction coefficient between the tires of the car and the road.



$a_R = \frac{v^2}{R}$

$F_N = mg$

$\Sigma F_x : F_{fr} = ma_R$

$\mu_s F_n = m \cdot \frac{v^2}{R}$

$r \cdot \mu_s mg = \frac{mv^2}{r} \cdot r$

$\sqrt{v^2} = \sqrt{rg\mu_s}$

$\boxed{v = \sqrt{rg\mu_s}}$

2. Take a few minutes working with the simulation to explore how fast a car can safely take a curve of known radius for various combinations of road/tire conditions.

3. Set the mass of the car to 1000 kg and the radius of the road curve to 200 m. Determine the maximum speed the car can safely take curve without skidding when on a dry road with new tires. Then repeat the exercise for at least three other road/tires conditions. Report all four of your values here.

- For mass of 1000 and radius of 200m, max speed is 35 m/s.
- For mass of 1000 and radius of 250m, max speed is 40 m/s.
- For mass of 1100 and radius of 250m, max speed is 40 m/s.
- For mass of 1200 and radius of 250m, max speed is 40 m/s.
- Looking at results, increasing radius increases max speed but mass has no effect.

4. Consider your result above for the new tires/dry road scenario. Compute the coefficient of static friction between the ties and the road in this case.

$\mu_s =$ 0.625

$$v = (r g \mu_s)^{(1/2)}$$

$$\mu_s = \frac{v^2}{rg} = \frac{(35)^2}{(200 \times 9.8)} = 0.625$$

Banked curve:

You may have noticed that curvy roads are often banked at an angle. The banking is there to keep cars from skidding out of the turn radius. Sufficiently large banking angles can even allow a car to safely make a turn even without static friction between the tires and the road. (Why is this?) Today you'll study the effects of banked roads only in the case where static friction is present, since the other case really is not safe for real-world driving.

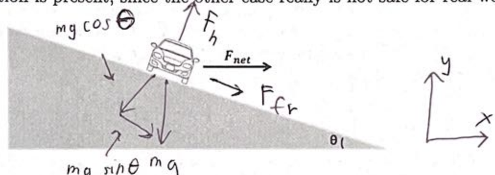


Figure 4: A car travels safely with speed v around a curve of radius r on a road with banking angle θ.

5. Figure 4 shows a car traveling safely with speed v around a curve of radius r on a road with banking angle θ. Draw a free body diagram that shows the forces acting on the car. You may draw directly on Fig. 4.

6. Use Newton's 2^{nd} Law and specify explicitly the relevant force equations for the car. (Hint: For convenience, make the x axis of your coordinate system parallel to the centripetal acceleration a_R.)

$$\Sigma F_y: F_h \cos\theta - F_{fr} \sin\theta - mg = \emptyset$$

$$\Sigma F_x: F_h \sin\theta + F_{fr} \cos\theta = m a_R = \frac{mv^2}{R}$$

7. Use your Newton's force equations to prove that the maximum speed at which the car can now safely maneuver the curved road is given by:

$$v_{max} = \sqrt{\frac{rg(sin\theta + \mu_s cos\theta)}{cos\theta - \mu_s sin\theta}}. \tag{4}$$

[From ΣF_x]

$$v^2 = \frac{r(F_n \sin\theta + F_{fr}\cos\theta)}{m}$$

[From ΣF_y]

$$m = \frac{F_n \cos\theta - F_{fr}\sin\theta}{g}$$

$$v = \sqrt{\frac{rg(F_n \sin\theta + F_{fr}\cos\theta}{F_n\cos\theta - F_{fr}\sin\theta}} = \sqrt{\frac{rg(F_n\sin\theta + \mu_s F_n\cos\theta)}{F_n\cos\theta - \mu_s F_n\sin\theta}} = \sqrt{\frac{rg(\sin\theta + \mu_s\cos\theta)}{\cos\theta - \mu_s\sin\theta}}$$

8. In part 2.2, you found the maximum safe speed for a car traveling on a curved (r = 200 m) but flat road under ideal road/tire conditions. Now use Eq. 4 and the coefficient of friction determined previously to calculate the new v_{max} if the road is banked at $\theta = 20^o$ and the road and tire conditions remain ideal. Compare your new result for v_{max} with the one you measured for the flat road.

$v_{banked} =$ 30.4 m/s

$$v = \sqrt{\frac{(200)(9.8)(\sin(20^\circ) + 0.625\cos(20^\circ))}{\cos(20^\circ) - 0.625\sin(20^\circ)}} \approx 50 \text{ m/s}$$

The new result for v_{max} which is 50 m/s is greater than the one measured for the flat road which was 35 m/s. I expected my result to have a higher max velocity because the banked curve is there to help keep cars from skidding out of the turn radius so thus the car would be allowed to drive at a faster speed before it skids on a banked road than on a flat road.