

Lab 4: Newton's 2nd LawPart 1: The Inclined Plane

Introduction: In 1608, Galileo Galilei wanted to investigate the motion of falling objects. However, the objects fell too fast for him to accurately measure their velocities and accelerations. His solution was to slide the objects down an inclined plane, thereby controlling the effective velocities, accelerations, transit times and forces. Our virtual experiment today will be similar to Galileo's, but will use a frictionless inclined ramp and a "crate" for the object. We will use an applet to simulate Galileo's experiments in order to study Newton's Second Law of Motion, $\Sigma \vec{F} = m\vec{a}$. A schematic view of your basic set-up is shown in Figure 1.

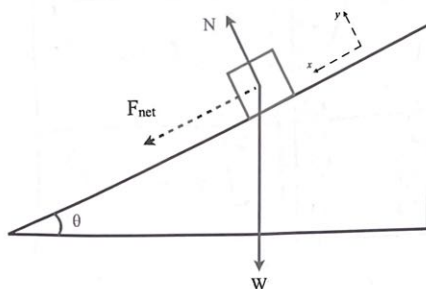


Figure 1: Under the influence of gravity, an object with weight $\vec{W} = m\vec{g}$ slides down a frictionless incline plane at an angle θ with respect to the horizontal. A normal force (by definition, perpendicular to the contact surface) also acts on the object. The two vector forces together (adding by components) results in a net force, as shown.

As shown in Fig. 1, there are two forces acting on the object: (i) its weight \vec{W} , pointing vertically downward and (ii) a normal force \vec{N} pointing perpendicular to the inclined plane. The weight of the box can be resolved into horizontal and vertical components with respect to the surface of the frictionless incline. Since only motion along the incline is permitted and we assume no friction is present, the component of the weight along the incline represents the net force (\vec{F}_{net}) applied to the box. Thus, knowing the angle of the incline and the mass of the object allows us to predict the acceleration of the object as it slides down the ramp. But even better, in this case where gravity alone provides the object's accelerating force, the mass cancels out of the equation so the acceleration of the sliding object varies *only* with ramp angle - independent of the object's mass!

$$|\vec{a}| = \frac{F_{net}}{m} = \frac{mg \sin(\theta)}{m} = g \sin(\theta) \quad (1)$$

You will use Eq. 1 to predict the acceleration corresponding to different ramp angles. You'll then perform kinematics experiments at these angles (and with objects of different masses) to measure the object's acceleration in each case. The motion can also be analyzed using simple kinematics equations since it's in one-dimension (along the incline) with constant acceleration. The speed v of the object moving down the ramp is thus given by:

$$v^2 - v_o^2 = 2a\Delta x \quad (2)$$

where v_o is the initial speed of the object ($v_o = 0$ here) and Δx is the distance the it travels along the incline.

The position of the object as it slides down the incline is given by the simple and by now familiar equation

$$x(t) = x_o + v_o t + \frac{1}{2} a t^2 \quad (3)$$

You will test out Newton's Second Law using PhET's "The Ramp" java applet found here (you can also download the program directly onto your computer and run it locally). Choose the "Introduction" tab. The user interface window is shown in Figure 2. Please study it carefully and enter/set the parameters shown in the figure into your own applet. Select "crate" from the list of objects and record its mass. The value of mass can be adjusted by selecting different objects ("file cabinet", "sleepy dog", etc) from the drop-down menu.

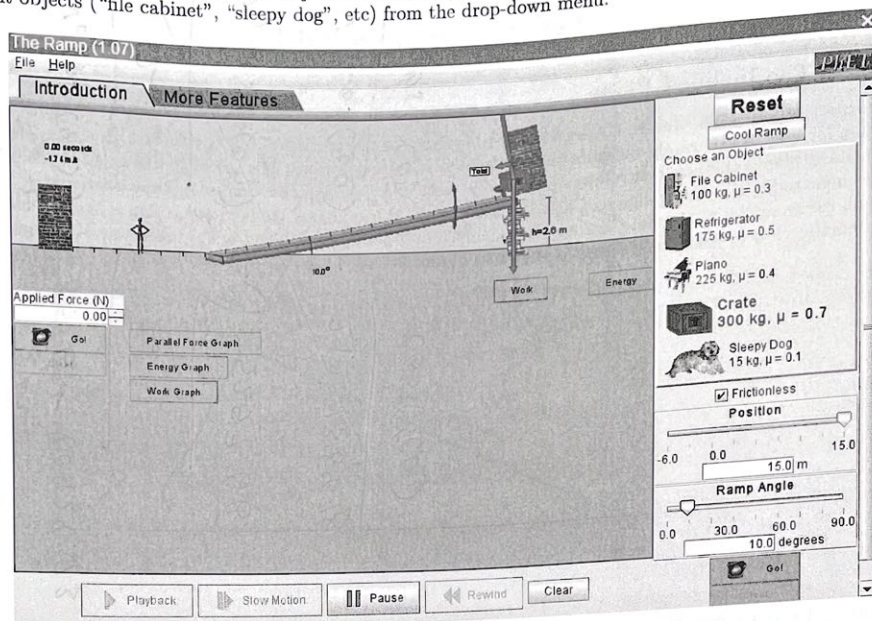


Figure 2: The user interface. The bottom of the incline is define as $x=0$ m. Be sure to check the "frictionless" box. (We will study friction in a future lab!) After each run, it's best to "Clear" the recorded data and start fresh.

1. Raise the ramp to $\theta=10^\circ$. Calculate the expected acceleration of the object using Equation 1.

$$a = \underline{1.7 \text{ m/s}^2} \quad a = g \sin \theta = (9.81) \sin 10^\circ = 1.7 \text{ m/s}^2$$

2. Release the crate from rest from the top of the incline and stop it before it reaches the bottom of the incline. Record the displacement of the crate from its initial location (Δx), the time it took for it to reach the final position, and its final speed. You might need to zoom in to read the data.

$$\Delta x = \underline{15.06 \text{ m}}, \quad t = \underline{0.6 \text{ s}}, \quad v = \underline{-6.6 \text{ m/s}}$$

Use your data to calculate the acceleration of the crate in two more ways, using Equations 2, and 3. Clearly show your work.

$$a_{\text{eqn. 2}} = \underline{1.627 \text{ m/s}^2}, \quad a_{\text{eqn. 3}} = \underline{\hspace{2cm}}$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

$$(-7)^2 - 0 = 2a(15.06)$$

$$a = 1.627 \text{ m/s}^2$$

Compare each of your experimental results for acceleration with your predicted value; comment.

My experimental result of 1.626 m/s^2 is close to my predicted value of 1.7 m/s^2 but is a little off by a small amount. The reason they're not the same is probably because of human error due to precision of

3. Perform the same experiment for two other masses (objects) at two other angles (15 and 20 degrees). You may share your work with each other, but make sure that each lab group member makes at least two sets of measurements. Tabulate these data and the previous ones in the table below. Calculate the acceleration in each situation using Equations 1 and 2. human time while gauging the

| | Mass 1 = Fridge | Mass 2 = Crate | Mass 3 = Dog |
|-----------------------|---|--|--|
| $\theta_1 = 10^\circ$ | $\Delta x = -14.96 \text{ m}$ $v = -7.1 \text{ m/s}$ $a_{\text{eqn. 1}} = 1.70 \text{ m/s}^2$ $a_{\text{eqn. 2}} = 1.68 \text{ m/s}^2$ | $\Delta x = 15.06 \text{ m}$ $v = -6.6 \text{ m/s}$ $a_{\text{eqn. 1}} = 9.8 \sin 10^\circ$ $a_{\text{eqn. 2}} = 1.627 \text{ m/s}^2$ | $\Delta x = 14.96 \text{ m}$ $v = -7.02 \text{ m/s}$ $a_{\text{eqn. 1}} = 1.7 \text{ m/s}^2$ $a_{\text{eqn. 2}} = 1.69 \text{ m/s}^2$ |
| $\theta_2 = 15^\circ$ | $\Delta x = 14.26 \text{ m}$ $v = -6.43 \text{ m/s}$ $a_{\text{eqn. 1}} = 2.54 \text{ m/s}^2$ $a_{\text{eqn. 2}} = 1.68 \text{ m/s}^2$ | $\Delta x = -14.43 \text{ m}$ $v = -8.43 \text{ m/s}$ $a_{\text{eqn. 1}} = 2.54 \text{ m/s}^2$ $a_{\text{eqn. 2}} = 2.46 \text{ m/s}^2$ | $\Delta x = -14.01 \text{ m}$ $v = -6.87 \text{ m/s}$ $a_{\text{eqn. 1}} = 2.54 \text{ m/s}^2$ $a_{\text{eqn. 2}} = 1.68 \text{ m/s}^2$ |
| $\theta_3 = 20^\circ$ | $\Delta x = 12.8 \text{ m}$ $v = -8.49 \text{ m/s}$ $a_{\text{eqn. 1}} = 3.35 \text{ m/s}^2$ $a_{\text{eqn. 2}} = 2.81 \text{ m/s}^2$ | $\Delta x = -14.72 \text{ m}$ $v = -9.79 \text{ m/s}$ $a_{\text{eqn. 1}} = 3.35 \text{ m/s}^2$ $a_{\text{eqn. 2}} = 3.26 \text{ m/s}^2$ | $\Delta x = 12.74 \text{ m}$ $v = -9 \text{ m/s}$ $a_{\text{eqn. 1}} = 3.35 \text{ m/s}^2$ $a_{\text{eqn. 2}} = 3.16 \text{ m/s}^2$ |

moment before it reaches down the incline.

4. Clearly explain your results and draw some (perceptive!) conclusions on how the acceleration varies as a function of mass and θ .

Looking at our data for mass 1, 2, 3, if the angle doesn't change, it seems like the acceleration stays the same. However, it seems like as the angle increases, the acceleration also increases. Thus, we can conclude that acceleration increases as the angle increases but stays the same no matter what.

Part 2: Newton's Laws with Simple Machines

Now we'll investigate Newton's 2nd Law of Motion using the pulley system shown in Figure 3, where two hanging weights are connected by a taut string of negligible mass that's draped over a frictionless pulley. This pulley system, known as an Atwood's machine, is classified as a simple machine, that is, an elementary device that more complicated and advanced machines are built around. An Atwood's machine (and any modified version) can be used to measure the gravitational acceleration g by allowing the user to control the rate at which the mass (m and M in Fig 3), move up and down. We will assume that the pulley is massless and frictionless, and that uniform tension runs through the string. With these assumptions, the acceleration of the two masses is the same (i.e. uniform). Convince yourself this is true!

In this part of the experiment you will use the applet found here to study the physics of an Atwood's Machine. The objectives are to experimentally measure the gravitational acceleration constant g , to test the theoretical acceleration obtained from Newton's 2nd law, and to study the relationship between mass and acceleration using a simple machine.

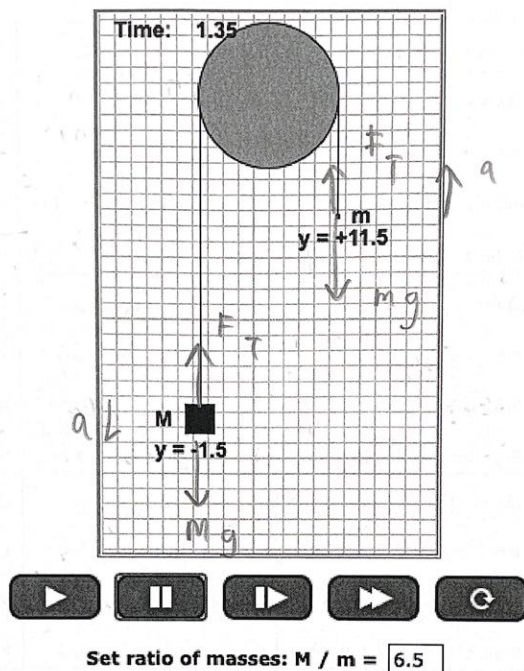


Figure 3: Atwood's Machine graphical interface window. Time is measured in seconds, and the vertical positions of the masses (y_M , y_m) are in given meters. The applet doesn't allow you to assign specific masses to the hanging objects, but instead relates the masses in terms of a "mass ratio" (M/m) which can be set to various values, ranging between $0.25 < \frac{M}{m} < 10$.

1. Draw free-body diagrams for the two hanging masses directly onto Figure 3, clearly labeling all relevant forces acting on each object. Indicate the direction of motion if $M > m$.

Apply Newton's 2nd Law $\sum \vec{F} = m\vec{a}$ to each mass and use the results to derive this simple expression for the magnitude of the acceleration in each case:

$$\sum F_{yM} = F_T - Mg = M(-a) \quad a = \frac{M-m}{M+m}g$$

$$F_T = Mg - Ma$$

$$\sum F_{ym} = F_T - mg = m(-a)$$

$$F_T = Mg - Ma$$

$$\sum F_{yM} = F_T - mg = ma \quad (4)$$

$$F_T = ma + mg$$

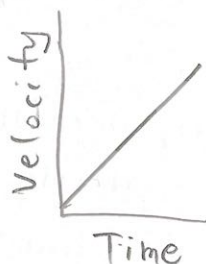
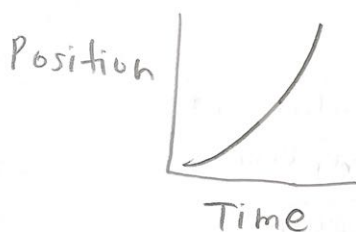
$$\sum F_{ym} = F_T - mg = ma$$

$$F_T = mg + ma$$

$$Mg - Ma = mg + ma \rightarrow ma + ma = mg - mg$$

2. Prediction: Once released, if the smaller mass (m) starts moving upwards from rest, make qualitative graphs showing how its position, velocity, and acceleration will behave as a function of time. Note: moving upwards is considered positive. Hint: Does the mass move in linear motion with constant acceleration?

$$a = \frac{M-m}{M+m}g$$



3. Record the time t for block m to move through a displacement Δy , for six different mass ratios. Use $M/m = 1.4$ for your first measurement and increase the ratio in 0.5 increments. Compute the acceleration of the blocks in each case and fill-in the table below. To get consistent data, you might want to let the masses move for more than 2 seconds, but be sure to keep track of their vertical displacements.

| M/m | t | Δy | a (m/s^2) |
|-------|--------|------------|-----------------|
| 1.4 | 2.15 s | 3.8 m | 1.63 |
| 1.9 | 2.05 s | 6.4 m | 3.04 |
| 2.4 | 2.05 s | 8.5 m | 4.04 |
| 2.9 | 2 s | 9.5 m | 4.77 |
| 3.4 | 2.05 s | 11.2 m | 5.35 |
| 3.9 | 2 s | 11.6 m | 5.8 |

$$M/m = 1.4$$

$$M = 1.4m$$

$$a = \frac{(M-m)}{M+m} \times 9.8$$

$$= \frac{(1.4m-m)}{(1.4m+m)} \times 9.8$$

$$= \left(\frac{.4}{2.4}\right) \times 9.8$$

$$= 1.63 m/s^2$$

4. You will now graph your data to experimentally determine the "unknown" value of $|g|$ using Eq. 4. Notice that Eq. 4 describes a straight line ($y=mx+b$) with slope g . (What is "y" and what is "x" in this case?) To make the graph, you will need values of $(M-m)/(M+m)$. Determine these values using your M/m data. Here is a table for you to keep your work tidy. Show one sample calculation for determining $(M-m)/(M+m)$.

| Team Member | g |
|-------------|----------------------|
| Jason | 9.83 m/s^2 |
| Emilio | 9.73 m/s^2 |
| Samuel | 9.96 m/s^2 |
| Ethan | 9.82 m/s^2 |

| M/m | $\frac{M-m}{M+m}$ | a |
|-------|-------------------|---------------------|
| 1.4 | .17 | 1.63 m/s^2 |
| 1.9 | .31 | 3.04 m/s^2 |
| 2.4 | .41 | 4.04 m/s^2 |
| 2.9 | .49 | 4.77 m/s^2 |
| 3.4 | .55 | 5.35 m/s^2 |
| 3.9 | .59 | 5.8 m/s^2 |

$$M/m = 1.4$$

$$M = 1.4m$$

$$\frac{(M-m)}{(M+m)} =$$

$$\frac{(1.4m - m)}{(1.4m + m)} = .17$$

Experimental value of
gravity for each
team member

5. Plot your data appropriately and draw a straight line of best fit to determine an experimental value of the acceleration due to gravity. Compare your result to the accepted value (at our location!) of 9.81 m/s^2 .

My experimental value of the acceleration due to gravity is 9.83 m/s^2 . My value is close to the accepted value of 9.81 m/s^2 but is a little off because my recorded times aren't all the same, since I have values ranging from 2 to 2.15 seconds. I feel like if I were to redo the experiment and make the times the same, my value would be closer to the accepted value.

Part 1: Question 2

| Team Member | Acceleration (Equation 2) |
|-------------|---------------------------|
| Jason | ~1.63 m/s^2 |
| Emilio | 1.65 m/s^2 |
| Samuel | 1.66 m/s^2 |
| Ethan | 1.68 m/s^2 |

Part 2: Question 4

| Team Member | g value |
|-------------|------------|
| Jason | 9.83 m/s^2 |
| Emilio | 9.73 m/s^2 |
| Samuel | 9.96 m/s^2 |
| Ethan | 9.82 m/s^2 |

Part 2: Question 5

| (M - m) / (M + m) | a (m/s^2) |
|-------------------|-----------|
| 0.17 | 1.63 |
| 0.31 | 3.04 |
| 0.41 | 4.04 |
| 0.49 | 4.77 |
| 0.55 | 5.35 |
| 0.59 | 5.8 |

SLOPE
 $y = 9.8333x + 0.025$

