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Lab 3: Projectile Motion

In this lab you will study two dimensional projectile motion of an object in *free fall* - that is, an object launched into the air that falls back to the ground under the influence of gravity only. To describe the trajectory of the projectile, we will use a coordinate system where the positive y-axis is vertically upward and the x-axis is horizontal and in the direction of the initial launch velocity (see Figure 1). We will we assume that the gravitational acceleration ($g = 9.81\ m/s^2$) is constant, so that $a_x = 0$ and $a_y = -g$, and we will ignore air resistance. The equations of motion in the x and y directions for such a projectile launched with initial velocity $\vec{v}_o = v_{ox}\hat{x} + v_{oy}\hat{y}$ at an angle θ are:

$$x(t) = x_o + v_{ox}t \tag{1}$$

$$v_x(t) = v_{ox} \tag{2}$$

$$y(t) = y_o + v_{oy}t - \frac{1}{2}gt^2 \tag{3}$$

$$v_y(t) = v_{oy} - gt \tag{4}$$

where t is time. The x and y components of the initial velocity $\vec{v}_o$ are $v_{ox} = v_o cos(\theta)$ and $v_{oy} = v_o sin(\theta)$ for the geometry shown in Fig. 1a. Remember that $v_x = v_{ox}$ here because there is no acceleration in the x direction.

Figure 1b shows the path of a projectile with initial speed v_o, launch angle θ and initial height H relative to the ground. Using Eqs. 1- 4 you should be able to show (optional!) that the projectile's path, *i.e.* its vertical position y as a function of its horizontal position x is described by:

$$y(x) = H + tan(\theta)x - \frac{gx^2}{2(v_o cos(\theta))^2} \tag{5}$$

where $x = x(t)$ and $g = 9.81 m/s^2$. Notice that Eq. 5 describes a parabola! Last but not least, notice that the range R of the projectile, *i.e.*, the total horizontal distance traveled by the projectile in the total flight time t_{tot} is:

$$R = v_{x0}t_{tot} = v_0 cos(\theta)t_{tot} \tag{6}$$

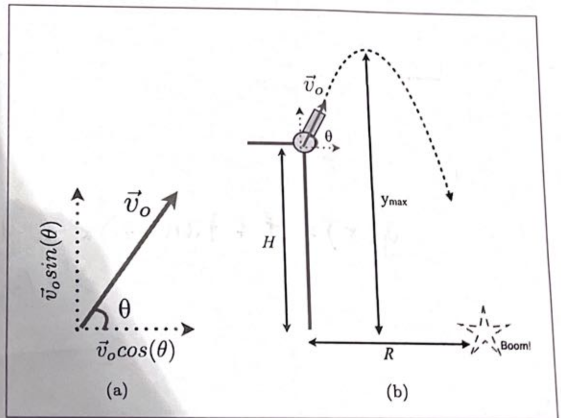


Figure 1: (a) Vector components of the initial velocity $\vec{v}_o$. (b) A projectile is launched at an angle θ off a platform with initial height H relative to the ground. The projectile strikes the ground a horizontal distance R away from the launch point.

Experimental (Simulation) Set-up

In this lab you will use a projectile launcher applet provided by PhET that can be found here (click the "Lab" icon). The user interface is shown in Figure 2. The projectile launcher can be positioned at different heights by clicking-and-dragging the platform, and can launch the ball with different initial velocities (magnitude and direction). The protractor on the side of the launcher allows you to set the desired launch angle, ranging from 0° to 90°. The built-in tape measure widget will be used to measure various distances. You will need an external stop-watch to record the amount of time t_{tot} the projectile is in the air. You can use your cellphone's stopwatch since it's quick and easy to use. With this virtual equipment, you will determine the initial launch velocity $\vec{v_o}$, an experimental value for $|\vec{g}|$, and the launch angle θ that provides the longest range when the projectile is launched from above ground level.

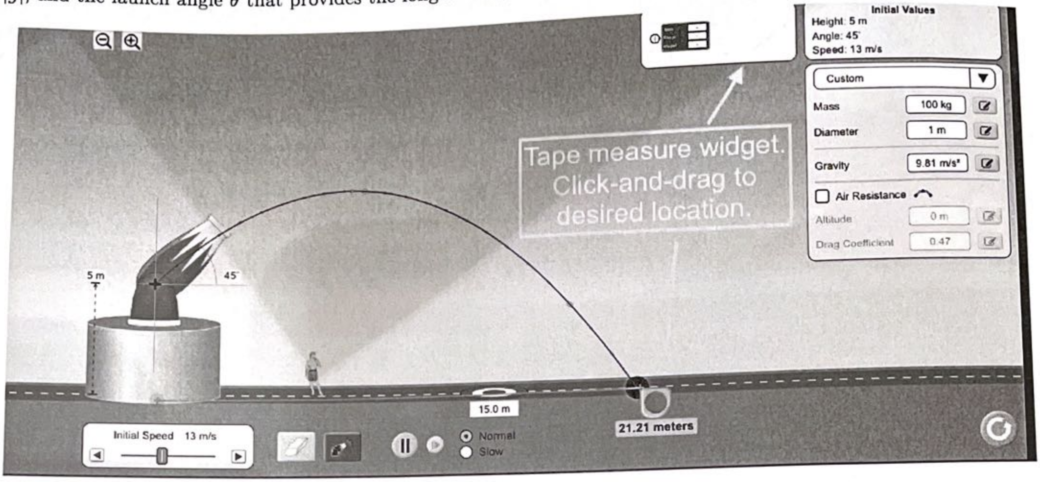


Figure 2: Use interface for PhET Projectile Motion applet.

Part 1: Finding the Initial Launch Velocity

You will use two different methods to determine (that is, confirm) the initial speed of the ball when launched from 5 meters above ground level, with an initial velocity $|\vec{v_o}|$ at an angle of 40 degrees.

1. Go to the PHET simulation and click the "Lab" icon. Position the launcher to 40° and 5 m above the ground. Set the initial velocity $|\vec{v_o}|$ to 30 m/s. Use the built-in feature of the simulator to zoom out enough to get a complete view of the ball trajectory (top left icon). In order to minimize random errors, it is very important that all of your measurements are made several times. Each lab group member must make at least two sets of time and range measurements and then share them with the group. Use your phone stopwatch to measure time and use the built-in tape measure widget to measure range. Use the eraser button to reset the simulation window before starting each trial.

$$y(x) = H + \tan(\theta)x - \frac{gx^2}{2(v_0\cos(\theta))^2}$$

Fire away! Record your data in the table below and calculate the average time of flight and range. Add the proper units at the top of each column.

Total time of flight	Time uncertainty, $\pm\Delta t$	Range	Range uncertainty, $\pm\Delta R$
4.35 s	0.1	96.19	0.05
4.29 s	0.1	96.21	0.05
4.16 s	0.1	96.06	0.05
4.09 s	0.1	95.94	0.05
4.28 s	0.1	95.69	0.05
4.31 s	0.1	96.08	0.05
4.28 s	0.1	96.14	0.05
4.24 s	0.1	96.14	0.05
$t_{tot-avg}$= 4.25 s	0.1	R_{avg}= 96.06	0.05

2. Clearly explain how you estimated your experimental uncertainties.

For time, I estimated my reaction time to be 0.1 s. as one of my other partners for the stopwatch. For range, I took the smallest unit on the tape measure and divided in half.

3. Calculate the standard error (standard deviation) for the average time-of-flight and range values. (Refer to the "Statistical Analysis of Data Set" section in the Appendix on errors and uncertainties posted on Camino).

- Time Uncertainty Standard Deviation: 0.0851889

- Range Uncertainty Standard Deviation: 0.170624

$$\sigma = \sqrt{\frac{\sum_{1}^{6}(x_i - x_{ave})^2}{6-1}}$$

$$\Delta x_{ave} = \frac{\sigma}{\sqrt{N}}$$

Used standard deviation equation to calculate time and range uncertainty

4. Getting the initial speed v_o from **range** measurements: Use Eq. 5 and your average range value to determine the initial launch speed of the projectile. Include units!

v_o = 30.24 m/s

$$y(x) = H + \tan(\theta)x - \frac{gx^2}{2(v_0\cos(\theta))^2}$$

$$0 = 5 + \tan(40°)(96.06) - \frac{(9.8)(96.06)^2}{2(v_0\cos(40))^2}$$

$$-85.6 = -\frac{45214.8656}{(v_0\cos(40))^2}$$

$$v_0\cos(40) = 22.9828424$$

$$v_0 \approx 30.24 \text{ m/s}$$

5. Getting the initial speed v_o from <u>time</u> measurements: Now use Eq. 1 to determine the initial launch speed of the projectile.

$v_o =$ 29.51 m/s

$x(t) = x_0 + v_{0x} t$

$96.06 = 0 + v_0 \cos(40°)(4.25)$

$v_0 \approx 96.06/3.2555 \approx 29.51$

6. Compare your two results for v_o with the value you used in the simulation. What were the biggest sources of error in your measurements?

My two results for v_0, 30.24 m/s and 29.51 m/s are close to the value of 30 m/s I used in the simulation. The biggest sources of error were human reaction time when using the stopwatch and the accuracy of one's hands and naked eye when measuring.

Part 2: Maximum Height and the Acceleration Due to Gravity on Earth

The goal of this part of the lab is to experimentally determine the magnitude of the acceleration due to gravity, g, near the surface of the Earth. As before, each student will collect at least two data points and share them with the rest of the group so that all rows of the data table below are filled. Eqs. 1-4 can be used to write the maximum height y_{max} reached by a projectile as:

$$y_{max} = \frac{v_o^2 sin^2(\theta)}{2g} \tag{7}$$

where v_o is the launch speed, θ is the launch angle, and g is the acceleration due to gravity. Now you will use this equation and maximum height measurements to determine an experimental value for g. Notice the simple linear relationship (i.e. y = mx+b) between y_{max} and v_o^2 in Eq. 7.

1. Equation 7 only holds when the projectile launch and landing heights are the same, so set $H = 0$ m in the simulation. Keep the launch angle fix to $\theta = 40°$. Launch the ball and measure its maximum height above the ground (use the measure tape) for each of the initial speeds in the table below. Record your maximum height values, calculate the corresponding v_o^2 values and add the proper units at the top of each column.

v_O	$y_{max} \pm y_{max}$ (m)	v_o^2 (m/s)^2
15	4.7 ± .05	225 m/s
16	5.38 ± .05	256 m/s
17	5.55 ± .05	289 m/s
18	6.79 ± .05	324 m/s
19	7.285 ± .05	361 m/s
20	8.44 ± .05	400 m/s
21	9.29 ± .05	441 m/s
22	10.14 ± .05	484 m/s

2. Plot y_{max} as a function of v_o^2. Include error bars $\pm y_{max}$ on your graph. Draw a line of best fit through your data (with error bars) and calculate its slope. Include units!

slope = 0.0217 s^2/m

3. Use Eq. 7 to derive the expression for g as a function of the slope of your graph.

$$y_{max} = \frac{v_0^2 \sin^2(\theta)}{2g}$$

$$\text{slope} = \frac{(\sin\theta)^2}{2g} \qquad g = \frac{(\sin\theta)^2}{2(\text{slope})}$$

$$\text{slope} \cdot 2g = (\sin\theta)^2$$

4. Use your slope to calculate g and compare your result with accepted value of 9.81 m/s^2. Comment on how your error bars can (or cannot) account for any discrepancy.

$$g = \frac{(\sin\theta)^2}{2(\text{slope})} = \frac{(\sin 40°)^2}{2(0.0217)} = 9.52\ m/s^2$$

Error bars can account for discrepancies by taking into account uncertainties of the measurements made like ymax. When taking into account uncertainty, the error bars we have now may not be able to account for the inaccuracy and may need to be larger. Error bars are necessary, because discrepancies can affect our other calculations such as how our calculated g of 9.52 is ~0.3 off from the accepted value of 9.81 m/s².

Part 3: Launch Angle and Range

1. If a projectile is launched from ground level, the launch angle of 45° would provide the largest possible range. *Predict* if the launch angle corresponding to maximum range should be larger or smaller than 45° if the projectile is launched from above the ground level ($H > 0$). State your reasoning.

The launch angle would be smaller than 45° because with a greater initial height, it would need to be angled less to account for the height increase as it travels. Since the projectile is being launched at a higher height, it'll spend more time in the air and has already covered more distance. If you decrease the launch angle, you'll affect the time in the air and horizontal velocity.

2. You will now experimentally test your prediction. Your lab instructor will tell you what initial velocity v_o and height H to use (record them below):

v_o= 10 m/s, H= 10 m

3. Measure the range of the projectile using the tape measure widget. As you perform the experiment, fill in the following table for θ and the corresponding range R values:

θ (deg)	0	20	30	40	45	50	60	70	80
Range (m)	14.28	17.09	17.54	17.05	16.41	15.48	12.81	9.16	4.77

4. Plot the measured Range as a function of θ (Range vs. θ). Draw a smooth curve through your data. From your plot, identify the *critical* angle that gives the largest range. Was your earlier prediction correct?

Part 3 - Question 1 continued

Starting at a higher height means an increase in air time, so you need to ~~increase~~ decrease the horizontal velocity and air time so you don't travel as fast and for as far/long, and you do this by simply decreasing the angle.

Yes, my earlier prediction is correct because while playing around with the simulator, using the given height of 10 m with initial given velocity of 10 m/s, while testing an angle of 50° and an angle of 40°, the angle of 40° gave us a greater range of 17.05 m when compared to an angle of 50° which gave us a lower range of 15.48 m.

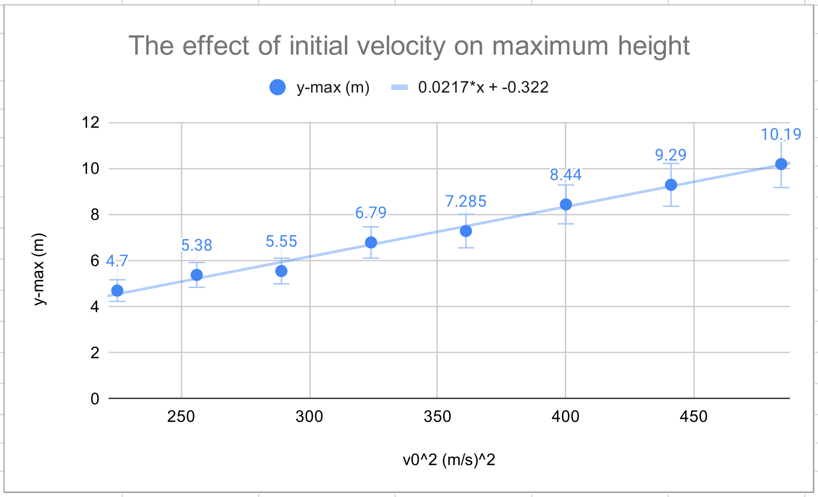
Part 1:

Total time of flight	Time uncertainty ±Δt	Range	Range uncertainty ±Δr	Lab Mates
4.35	0.1	96.19	0.1	LP
4.29	0.1	96.21	0.1	LP
4.16	0.1	96.06	0.1	JV
4.09	0.1	95.94	0.1	JV
4.28	0.1	95.69	0.1	LJ
4.31	0.1	96.08	0.1	LJ
4.28	0.1	96.14	0.1	KT
4.24	0.1	96.14	0.1	KT
tavg=4.25		Ravg=96.06		

Standard deviation for flight	Standard deviation for range
0.08518886581	0.1706238553

PART 2:

v0^2 (m/s)^2	y-max (m)
225	4.7
256	5.38
289	5.55
324	6.79
361	7.285
400	8.44
441	9.29
484	10.19



PART 3:

θ (degrees)	Range (m)
0	14.28
20	17.09
30	17.54
40	17.05
45	16.41
50	15.48
60	12.81
70	9.16
80	4.77