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Lab 7: Conservation of Energy

Introduction:

One of the most important concepts in physics is "Conservation of Energy". Today you will make measurements that demonstrate the conservation of total mechanical energy in conservative systems, where energy is continually transformed between kinetic and potential forms. You also will learn about the "Work-Energy Theorem" and the differences between conservative and non-conservative forces. By definition, a force $\vec{F}$ is *conservative* if: (i) it does no net work around a closed path so that $\int \vec{F}(r) \cdot \vec{r} = 0$, (ii) any work done by the force is path-independent, and (iii) the force can be described in terms of a potential energy function. Two common conservative forces are gravity ($\vec{F} = m\vec{g}$ near the Earth's surface) and the spring force $\vec{F} = -k\vec{x}$. By contrast, friction is a *non*-conservative force because a system loses mechanical energy when friction is present.

Potential energy (U or PE) is energy associated with the position and arrangement of particles in a system. It is a consequence of the work done by conservative forces to keep the system together. In a sense, potential energy represents storage of energy in a system. Two common potential energy terms you've already seen in lecture are $U_{gravity} = mgh$ (near the Earth's surface) and $U_{spring} = \frac{1}{2}k(\Delta x)^2$.

Kinetic energy (KE) is energy of motion. Translational kinetic energy is defined as $KE = \frac{1}{2}mv^2$, where m is the mass of an object moving with speed v. The total mechanical energy (E) of a system is the sum of its kinetic and potential energies: $E = KE + U$. If there is no friction or other non-conservative forces present, the total mechanical energy of a system remains constant and thus any change in the kinetic energy of the system must be balanced by a comparable change in potential energy:

$$\Delta E = \Delta KE + \Delta U = 0 \quad \rightarrow \quad KE_i + U_i = KE_f + U_f \tag{1}$$

If friction or another non-conservative force is present, the energy equation must be modified to account for energy loss due to non-conservative work. The result is the "Work-Energy Theorem" (shown below when friction is the non-conservative force):

$$KE_i + U_i = KE_f + U_f + \Delta W_{friction} \tag{2}$$

Part1: Loop-the-loop

Here, you will quantitatively observe how the energy of a snowboarder moving on the loop-the-loop track shown in Figure 1 gets converted back and forth between potential and kinetic energy. You will use the simulation found here to do your experiments. Please note that the simulation shows a skateboarder but you should pretend she is a snowboarder; this way you can ignore the kinetic energy of rotating skateboard wheels in your analysis. (The simulation software also ignores this rotational kinetic energy.)

Click on the "Playground" setting and then construct ramp and loop similar to the one shown in Figure 1. Click and drag the track components into the simulation window. You can also click and drag the red circles to lengthen or rotate the track. Try to make the center loop as perfect a circle as possible. The ramp has three key locations: the top of the ramp (A), the top of the loop (B), and the bottom of the track (C). You will analyze the snowboarder's total mechanical energy at each of these three stages. Set the mass of the snowboarder to 60 Kg and remove friction from the system.

1. Carefully measure the vertical coordinates of the top of the ramp ($y_A \pm \Delta y_A$), top of the loop ($y_B \pm \Delta y_B$) and bottom of the track ($y_C \pm \Delta y_C$) relative to the ground. Use the simulator's built-in tape measure tool.

$y_A =$ 8.0 m $\pm$ 0.05 m, $y_B =$ 7.0 m $\pm$ 0.05 m,

$y_C =$ 0 m $\pm$ 0.05 m.

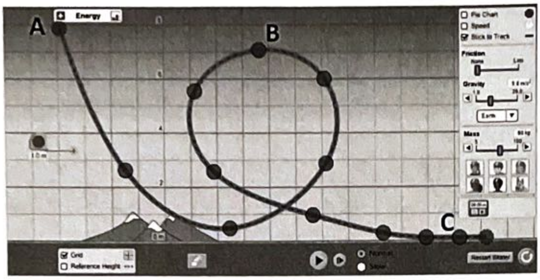


Figure 1: User interface for a loop-the-loop conservation of energy experiment. Try to make your loop as circular as possible. (Skateboarder Tony Hawk introduced the loop-the-loop maneuver in 1998.)

2. Release the snowboarder from rest from the top of the ramp (position A). Measure her speed at the top of the loop (v_B) and the bottom of the track (v_C). Perform three trials and tabulate your results below. Be sure to include appropriate units in the column headers. Then calculate the average speeds $\bar{v}_B$ and $\bar{v}_C$.

$U_c = mgh = (60)(9.8)(0) = 0\ J$

$KE_c = \frac{1}{2}mv^2 = \frac{1}{2}(60)(12.2)^2 = 4465\ J$

$E_c = U_c + KE_c = 4465\ J$

Trial	v_B	v_C
1	3.5 m/s	12.2 m/s
2	3.4 m/s	12.2 m/s
3	3.4 m/s	12.2 m/s
Average	3.43 m/s	12.2 m/s

3. Use your vertical position and average speed values to verify that the total mechanical energy (E) of the system is the same at points A, B and C. Show your work (may use a separate sheet of paper if needed). Thoughtfully discuss your results and comment on at least two sources of experimental errors.

$U_A =$ 4704 J $\quad KE_A =$ 0 J $\quad E_A =$ 4704 J

$U_B =$ 4116 J $\quad KE_B =$ 353 J $\quad E_B =$ 4469 J

$U_C =$ 0 J $\quad KE_C =$ 4465 J $\quad E_C =$ 4465 J

$U_A = mgh = (60)(9.8)(8) = 4704\ J$

$KE_A = \frac{1}{2}mv^2 = \frac{1}{2}(60)(0) = 0$

$E_A = U_A + KE_A = 4704\ J$

$U_B = mgh = (60)(9.8)(7) = 4116\ J$

$KE_B = \frac{1}{2}mv^2 = \frac{1}{2}(60)(3.43)^2 = 353\ J$

$E_B = U_B + KE_B = 4469\ J$

My results make sense, because due to conservation of energy, the mechanical energy stays the same throughout, and as seen in our results, the mechanical energy at points A, B, and C are more or less close to each other.

Q 3) continued →

Part 1

Question 3 contineued...

One source of error is not accurately constructing the loop shown in Figure 1 and making it as round as possible, which may change our values for height and velocity. Another source of error is not accurately measuring the height using the tape measure and not stopping the stopwatch exactly at point B due to inaccuracy of the human eye, which would change our values for velocity. The total mechanical energy should be the same at all points, but these sources of errors caused the values to deviate a little from each other.

Part 2: Conservative vs. Non-conservative Forces

You will now use the same simulation package as before to learn how friction affects the motion of our snowboarder. Begin by building an incline / ramp at least 16 m long, similar to the one shown in Figure 2. Set the mass of the snowboarder to 60 kg.

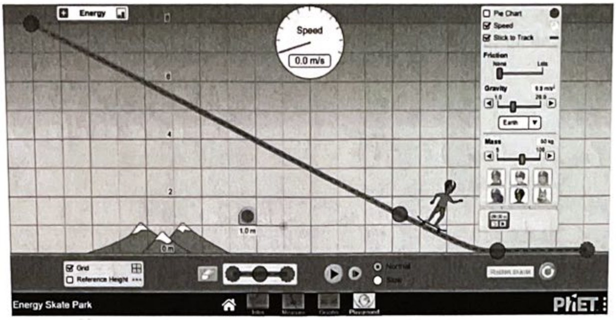


Figure 2: User interface window for studying the [non-conservative] force of friction in Part 2. Try to make your ramp as straight as possible. Join the bottom of the ramp to the horizontal portion of the track as gently and smoothly as possible.

1. Set the friction in the system to maximum ("Lots"). Release the person from rest at the top of the ramp and measure her speed at the bottom. Perform three trials and record your results in the table below. Note: It is very important that you measure the speed exactly at the bottom of the ramp, before the snowboarder starts to travel along the horizontal section of track. Repeat the experiment when there is no friction in the system.

Trial	v_{bottom} (with friction)	v_{bottom} (no friction)	KE_{bottom} (with friction)	KE_{bottom} (no friction)
1	10.9 m/s	12.5 m/s	3564 J	4688 J
2	10.9 m/s	12.5 m/s	3564 J	4688 J
3	10.9 m/s	12.5 m/s	3564 J	4688 J
Average	10.9 m/s	12.5 m/s	3564 J	4688 J

$$KE\text{ (with friction)} = \frac{1}{2}mv^2 = \frac{1}{2}(60)(10.9)^2 = 3564\text{ J}$$

$$KE\text{ (w/o friction)} = \frac{1}{2}mv^2 = \frac{1}{2}(60)(12.5)^2 = 4688\text{ J}$$

2. Use the Work-Energy Theorem and the definition of work $\int \vec{F} \cdot \vec{dl}$ to show that the difference ΔKE between the final KE at bottom of the ramp when friction is present vs. not present is given by:

$$\Delta KE = -\mu_k mgDcos(\theta), \tag{3}$$

where θ is the angle the ramp makes with the horizontal, μ_k is the coefficient of kinetic friction, and D is the distance traveled *along* the ramp.

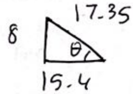
KE (w/o friction) $= -PE$ $\qquad$ KE (w/ friction) $= -PE + W_{NC}$

$$KE_f - KE_i = -W_{nc} = -F_{fr}\, d \cos\theta$$

$$\Delta KE = -\mu_k F_n \, d\cos\theta$$

$$\Delta KE = -\mu_k mgd\cos\theta$$

3. Use your data and Eq. 3 to compute the coefficient of kinetic friction μ_k between the ramp and the snowboarder. Show your work clearly. Comment on the meaning of your results.



$$d = \sqrt{8^2 + 15.4^2}$$
$$= 17.35$$

$$\Delta KE = -\mu_k mgd\cos\theta$$

$$4688 = -\mu_k (60)(9.8)(17.35)\cos 27.5°$$

$$\boxed{\mu_k = -0.051}$$

$$\theta = \tan^{-1}\left(\frac{8}{15.4}\right) = \boxed{27.5°}$$

The force of kinetic friction between the ramp and the snowboarder is -0.051.

Part 3: Energy Transformation: Spring Potential Energy to Kinetic Energy

In this part of the lab you will use the app found here to study how the potential energy of an ideal, massless spring launcher gets converted into kinetic energy of a person(!) Figure 3 shows the simulation window that includes a birds-eye view of a hockey rink. In the simulation, Trevor, a hockey player, gets propelled onto the ice by a large compressed spring mounted at the far right-end of the rink. The goal of this part of the lab is to use conservation of energy to determine the spring constant k of the launcher. You may ignore friction in this experiment.

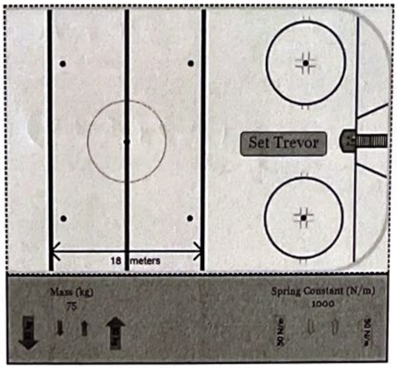


Figure 3: User interface for studying the motion of an object launched onto a frictionless surface using a compressed spring. The simulation window shows a birds-eye view of an ice-hockey rink. Trevor, a hockey player, gets propelled onto the frictionless ice by a compressed spring located just behind the goal (shown at the far right of the figure).

1. Start the simulation. Set Trevor's mass to 80 kg and the spring constant to 500 N/m. Click "Set Trevor" and then "Pull Back" to compress the spring. For your first data point, compress the spring by $\Delta x = 1$ m and then click "Fire Travis"! Using the built-in timer, measure the time it takes Trevor to travel the 18 m marked on the screen. Complete at least four trials to reduce the effect of random experimental errors. Record your data in the first empty row in the table below. Be sure to add units at the top of each column.

Spring compression Δx	Measured Time				Average time, t_{ave}	Speed v	Kinetic energy, KE
	Trial 1	Trial 2	Trial 3	Trial 4			161.6
1 m	9.08 s	8.76 s	8.9 s	9.08 s	8.95 s	2.01 m/s	~~258.1~~ J
1.2 m	7.12 s	6.92 s	7.24 s	7.12 s	7.1 s	2.54 m/s	258.1 J
2 m	3.56 s	3.6 s	3.58 s	3.62 s	3.59 s	5 m/s	1005.6 J
2.2 m	3.28 s	3.28 s	3.3 s	3.26 s	3.28 s	5.49 m/s	1265 J
2.6 m	2.82 s	2.9 s	2.78 s	2.8 s	2.825 s	6.4 m/s	1624 J
2.8 m	2.66 s	2.6 s	2.62 s	2.64 s	2.63 s	6.84 m/s	1874 J
3 m	2.36 s	2.34 s	2.46 s	2.38 s	2.39 s	7.55 m/s	2278 J
2.4 m	3 s	2.96 s	3 s	3.08 s	3.03 s	5.94 m/s	1411.3 J

Show one example of each calculation used to fill-in the columns of your table.

[From Row 6]

Average time $= (2.66 + 2.6 + 2.62 + 2.64) / 4 = 2.63$ s

Speed $= \dfrac{18 \text{ m}}{2.63 \text{ s}} = 6.84$ m/s

$KE = \frac{1}{2} m v^2 = \frac{1}{2} (80)(6.84)^2 = 1874$ J

2. Repeat the experiment for at least two other values of Δx (< 3 m) and include the data in your table.
3. Use your average measured times to calculate Trevor's speed and kinetic energy for each value of Δx. Remember, there is no friction between Trevor and the ice.
4. Now exchange data with your lab partners until you each have a full table of measurements.

Analysis: In this idealized, frictionless system all of the potential energy initially stored in the compressed spring gets converted into kinetic energy of Trevor gliding on the ice. Notice the simple $y = mx + b$ linear relationship between KE and $(\Delta x)^2$ in the corresponding conservation of energy equation:

$$KE = \frac{1}{2} k (\Delta x)^2 \qquad y = 258.17x - 84.293 \tag{4}$$

6. Plot your experimental values of KE as a function of $(\Delta x)^2$. Do not worry today about error bars. Fit your data with a line and calculate the slope of this line (including units).

slope = [redacted] J/m^2 = [redacted] (kg × m^2)/(m^2 × s^2) = [redacted] kg/s^2

258.17 258.17 258.17

7. Using Eq. 4 as a guide, clearly explain what the slope represents. Use your graph to experimentally determine the spring constant k of the launcher and compare your results to the value provided in the simulation.

$k =$ 501.36 kg/s^2

The slope represents half of the spring constant. My result of 501.36 kg/s^2 for the spring constant is close to the value of 500 kg/s^2 in the simulation.

$KE = \frac{1}{2} k (\Delta x)^2$

$\dfrac{KE}{\Delta x^2} = \frac{1}{2} k$

slope $= \dfrac{k}{2}$

$k =$ slope $\times 2 = 250.68 \times 2$

$= 501.36$ kg/s^2

Question 7) continued →

Part 3,

Question 7 continued...

I believe that the reason my value is a little bit off is because I may have measured the time wrong for one or a few of the trials which would affect my other values such as kinetic energy and speed.

PART 2

Team Member	u_k value
Liam	0.131
Patrick	0.13
Quoc Trong	0.157
Jason	-0.051

PART 3

Spring Compression (m)	Spring Compression (m^2)	Kinetic Energy (joules)
1	1	161.6
1.2	1.44	258.1
2	4	1005.6
2.2	4.84	1205
2.6	6.76	1624
2.8	7.84	1874
3	9	2278
2.4	5.76	1411.3