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Lab 10: Angular Momentum and Rolling Motion

Introduction:

In this part of the lab you will investigate the principle of conservation of angular momentum. The angular momentum, $\vec{L}$, of a particle of mass m moving with velocity $\vec{v}$ relative to some fixed point is:

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times (m\vec{v}) \tag{1}$$

where $\vec{r}$ is the instantaneous position vector from the fixed point to the particle. The magnitude of $\vec{L}$ can be written simply as $|\vec{L}| = m|\vec{v}||\vec{r}|sin\theta$, where θ is the angle between $\vec{r}$ and $\vec{p}$. The direction of $\vec{L}$ is in the direction along the axis of rotation and is specified by the right-hand-rule. The total angular momentum of a system of masses moving with respect to some rotation axis can be calculated by adding together the angular momentum vectors for all particles in the system. In the case of a rigid-body, a much easier way to compute the total angular momentum of the object about some fixed access is to use:

$$\vec{L} = I\vec{\omega}, \tag{2}$$

where I is the moment of inertia and $\vec{\omega}$ is the angular velocity (that points along the axis of rotation). Notice that when you take the derivative of Eq. 2 with respect to time you get (for a rigid body):

$$\frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt} = I\vec{\alpha} \tag{3}$$

Newton's Second Law can be expressed in rotational form by replacing force by torque ($\vec{\tau} = \vec{r} \times \vec{F}$) and linear momentum by angular momentum ($\vec{L} = \vec{r} \times \vec{p}$). Notice that if the net torque on a system is zero, the system conserves angular momentum ($\vec{L} =$ constant). This is analogous to the translational case, where if the net force on a system is zero, the system conserves linear momentum ($\vec{p} =$ constant).

$$\sum \vec{F}_{sys} = \frac{d\vec{p}_{sys}}{dt} = 0 \qquad \longrightarrow \qquad \vec{p}_{initial} = \vec{p}_{final} \tag{4}$$

$$\sum \vec{\tau}_{sys} = \frac{d\vec{L}_{sys}}{dt} = 0 \qquad \longrightarrow \qquad \vec{L}_{initial} = \vec{L}_{final} \tag{5}$$

For rigid bodies it is often most convenient to express angular momentum conservation (Eq. 5) using moment of inertia I and angular speed ω (Eq. 2):

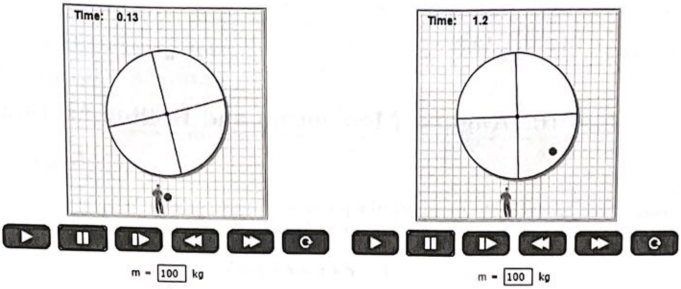
$$I_i\omega_i = I_f\omega_f \tag{6}$$

This is the mathematical form of the law of conservation of angular momentum that you will use today. It is important to remember that computing the moment of inertia of a compound object and/or for the case when the rotational axis does not pass through the center-of-mass of the object typically requires application of the "parallel axis theorem" and/or the superposition principle.

Part 1 - Conservation of Angular Momentum:

The applet linked here [1] simulates the collision between a solid, uniform disk and a small object modeled as a point mass. Using the app, you will measure the angular speed of the objects before and after the collision and verify that the angular momentum of the system is conserved. The simulation windows before and after the collision are shown in Figure 1.

[1] *https://www.compadre.org/Physlets/mechanics/ex11_5.cfm*



(a) (b)

Figure 1: (a) The simulation window representing a top view of a solid disk rotating with constant angular velocity and a person standing beside the disk and holding a heavy(!) red object. In the simulation, the person drops the object onto the rotating disk and it sticks. (b) The system then rotates with a new constant angular velocity. The simulation uses SI units for time (sec) and distance (m). The coordinates of any point on the grid space are displayed when clicking on the point.

1. Measure the radius R of the large yellow disk ($M = 150.0$ kg). Run the simulation and observe the small object being dropped on the large disk. Measure the distance r from the small red object to the center of rotation of the system. Estimate the experimental uncertainties of your two measurements and explain how you made these estimations.

$R =$ 1.5 m $\pm$ 0.02 m, $r =$ 1.25 m $\pm$ 0.02 m.

- Used marker to measure distance
- Marker is hard to move precisely hence the error.

2. Set the mass of the small object to 100 kg. Run the simulation and measure the time it takes for the system to complete one full rotation before and after the collision, t_i and t_f, respectively. Repeat the experiment 3 times and calculate the average initial and final time values for one full rotation. You may want to use the double arrows implemented in the simulation to get more accurate results.

	t_i	t_f
Trial 1	0.8	1.54
Trial 2	0.8	1.55
Trial 3	0.8	1.53
Average	0.8	1.54

Use your average time values and angular kinematic equations to calculate the initial angular speed ω_i of the disk before the collision and the angular velocity of the system ω_f after the collision. Show a sample calculation.

$\omega_i =$ 7.85 rad/s, $\omega_f =$ 4.07 rad/s.

$$\frac{2\pi}{0.8} \qquad \frac{2\pi}{1.54}$$

3. Calculate the moment of inertia of the system before the collision (*i.e.* solid disk) and after the collision (*i.e.* disk with red object). Clearly show your work.

$I_i =$ $168.75\ kg \cdot m^2$ $\quad I_f =$ $325\ kg \cdot m^2$

$\frac{1}{2} mr^2$ $\quad I_f = \frac{1}{2} mr^2 + mr^2$

4. Calculate the angular momentum of the system before and after the collision. Be sure to include units! Show your work. Was angular momentum conserved during this collision? Discuss at least two possible sources of experimental errors that could explain a discrepancy between your two values of $|\vec{L}|$.

$L_i =$ $1324.68\ kg \cdot m/s$ $\quad L_f =$ $1322.75\ kg \cdot m/s$

$I\omega$

$(168.75)(7.85)$ $\quad (325)(4.07)$

Since we are rounding and not able to take 100% accurate measurements, there's an acceptably small amount of error.

- It was conserved.

5. Calculate the ratio of the rotational kinetic energy of the system after the collision to that before the collision. Was the collision elastic or inelastic? How might energy have been dissipated from the system in a real collision?

$\frac{KE_{final}}{KE_{initial}} =$ 0.5177

- Collision was inelastic

$KE = \frac{1}{2} I\omega^2$

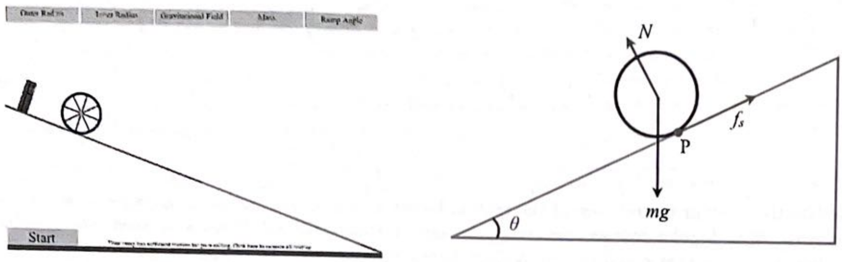
- Energy would be dissipated by an amount of thermal energy in a real collision

$KE_i = 10398.796$

$KE_f = 5383.5925$

Part 2 - Rolling Motion:

In this part of the lab you will use the concepts of torque and energy to examine the motion of rigid round objects rolling-without-slipping down a ramp. The motion will be simulated using the applet found here [2]. The motion will be monitored using a motion sensor placed at the top of the ramp (Fig. 2a). Fig. 2b shows a hollow cylinder (or it could be a hoop!) of mass m and outer radius R on a ramp of incline angle θ.



(a) (b)

Figure 2: (a) Simulation interface showing a hollow cylinder (or hoop) rolling down a ramp. The object's motion is monitored using a motion sensor place at the top of the ramp. (b) Free body diagram of a round object rolling-without-slipping down a ramp.

- Use Fig. 2b to convince yourself that the forces acting on the rolling cylinder in a direction along the ramp satisfy Newton's Second Law according to:

$$ma_{\text{CM}} = -f_s + mgsin\theta, \tag{7}$$

where f_s is the static friction between the ramp and the cylinder at their instantaneous point of contact.

- Convince yourself that the torque acting on the rolling cylinder (radius R) about an axis through its <u>center of mass</u> can be written in terms of the frictional force or (equivalently) rotational variables as:

$$\tau = Rf_s \qquad \tau = I_{\text{CM}}\alpha, \tag{8}$$

- Convince yourself that the translational acceleration of the rolling cylinder – that is, the linear acceleration of its center of mass (CM) – can be expressed as:

$$a_{\text{CM}} = \frac{gsin\theta}{1 + (I_{\text{CM}}/mR^2)} \tag{9}$$

Experiment - Rolling Motion of a Hollow Cylinder and Acceleration due to Gravity:

In this experiment you will explore how the wall-thickness of a hollow cylinder impacts the rolling motion of the cylinder (assuming constant mass). You will and experimentally determine the gravitational acceleration g using rolling motion measurements.

1. Start the simulation app referenced above. Your simulation page should look similar to Figure 2a. Set the ramp angle to 20 degrees, the cylinder mass to 100 g, the and the cylinder's inner and outer radii to 20 mm and 100 mm, respectively. Near the bottom of the simulation window you will see empty "Position vs. Time" and "Velocity vs. Time" graphs. These graphs will get populated once the simulation is started. Click "Start" to collect data and watch the cylinder (or hoop) roll down the incline. Sketch (or take a screenshot of) the two graphs on graph paper and be sure to label the axes, indicate units, and evenly label the graduation tick-marks.

[2] https://www.thephysicsaviary.com/Physics/Programs/Labs/DiskDownIncline/

2. Use your velocity graph to determine the (constant) acceleration of the cylinder (hoop) as it rolls down the ramp. Mark on the graph the data points used in your calculation and clearly show your work. Record the acceleration value in the table below.

3. Repeat the experiment until you have data for six different inner radius values ranging from 20 mm to 90 mm. Organize your data in the table below. Make sure to add the appropriate units in the heading of each column. Calculate the moment of inertia of your hollow cylinder in each case. The moment of inertia of a hollow cylinder of inner radius r and outer radius R rotating around an axis going through its center is $\frac{1}{2}M(R^2 + r^2)$. Use Eq. 9 to calculate the gravitational acceleration g for each data point. Be sure to show a sample calculation for the moment of inertia and gravitational acceleration values.

Inner Radius, r m	Acceleration, a_{CM} m/s	Moment of Inertia, I_{CM} kg · m/s	Gravitational Acceleration g m/s²
0.02	2.22	5.2×10^{-4}	9.87
0.03	2.14	5.45×10^{-4}	9.67
0.04	2.142	5.8×10^{-4}	9.895
0.05	2.07	6.25×10^{-4}	9.83
0.06	2	6.8×10^{-4}	9.823
0.07	1.93	7.45×10^{-4}	9.85

$r = 40 = \frac{1.5 - 0}{0.7 - 0} = 2.142$

$\frac{g \sin \theta}{1 + \frac{I}{mR^2}} = 9 \text{ cm}$

$\frac{g \sin(20)}{1 + \frac{(5.8 \times 10^{-4})}{(0.1)(0.1)^2}} = 2.142$

4. Calculate your average gravitational acceleration value and its standard error and compare your result with accepted value of 9.81 m/s^2. (Refer to the "Statistical Analysis of Data Set" section in the Appendix on errors and uncertainties posted on Camino).

$g_{ave} =$ 9.82 $\pm$ 0.036

$\frac{\text{Standard deviation}}{\sqrt{\text{total}}}$

The error is from us not being 100% exact on the measurement data.

5. Consider a solid disk that has the same mass and outer radius as a thin hoop. If both objects were released from rest from the top of the ramp, will they reach the bottom of the incline at the same time? Explain your prediction. Then test your prediction experimentally and discuss your observations.

- Smaller the mass, greater acceleration so hollow cylinder gets there first

- Solid = 0.68s
- Hollow = 0.75s

The hollow cylinder is faster and appears to have greater momentum.

Experiment - Rolling Motion of a Thin Hoop:

6. Use conservation of energy to prove that a thin hoop of mass M that rolls without slipping down a ramp of vertical height h reaches the bottom with a speed $v = \sqrt{gh}$. *Hint:* The hoop will have translational and rotational kinetic energy as it rolls down the ramp.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$
$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(\cancel{m}r^2)\frac{v^2}{r^2}$$
$$gh = \frac{1}{2}v^2 + \frac{1}{2}v^2$$
$$gh = v^2 \rightarrow \boxed{v = \sqrt{gh}}$$

7. Consider a thin hoop of radius 120 mm and mass 100 grams. Calculate the hoop's moment of inertia I about its center of mass. Compute the (constant) linear acceleration of the hoop a_{CM} as it rolls down a ramp with a 25° incline.

$I =$ 0.001 kg·m/s $a_{CM} =$ 2.07 m/s^2

$$\frac{(9.8)\sin 25^\circ}{1+1}$$

8. If the hoop is released from an initial location 50 cm up (along) the incline, predict its speed at the bottom of the ramp and the time it took to reach the bottom of the ramp.

$v_{final} =$ 2.21 $time =$ 1.06

$$V = \sqrt{gh}$$
$$V = \sqrt{(9.8)(0.5)}$$

$$V = V_0 + at$$
$$2.21 = 0 + 2.07\cancel{6}$$

9. Now use the simulation to test your predictions. Set the outer radius of the hoop to 120 mm, the inner radius to 115 mm and ramp angle to 25° degrees. Determine the experimental values for a_{CM}, the speed v at the bottom of the ramp, and the time it takes the hoop to reach the bottom of the ramp.

$a_{CM} =$ 2.2 $v_{final} =$ 2.38 $time =$ 1.06 s

?

Doesn't show full information on graph so we had to use model data

Compare your measured vales with your theoretical calculations. Is the theoretical model of a thin hoop appropriate in predicting the behavior of hoop you tested experimentally (i.e., having $R_{inner} \neq R_{outer}$)?

The data doesn't align super closely. The evaluation assumes R_{inner} and R_{outer} are being equal while the other simulation doesn't.