

Significance and Novelty

We present a highly scalable strategy for developing mesh-free neuro-symbolic partial differential equation solvers from existing mesh-based numerical discretization schemes found in scientific computing. This strategy is unique in that it can be used to efficiently train neural network models for the solution of partial differential equations with discontinuities, while retaining the accuracy and convergence properties of state-of-the-art numerical solvers. This neural bootstrapping method (NBM) is based on minimizing residuals of discretized differential systems on a set of random collocation points with respect to the trainable parameters of the neural network, achieving unprecedented resolution and optimal scaling for solving physical and biological systems.

We also present JAX-DIPS, an end-to-end differentiable implementation of NBM on the problem of elliptic PDEs with jump conditions across irregular discontinuities. As far as we know, this is the only existing differentiable framework for this class of problems. We demonstrate performance and scaling of JAX-DIPS in solving an elliptic problem with complex jump conditions over a highly irregular geometry (a dragon) using an NVIDIA DGX machine with 8 counts of A100 GPUs.