Statistical Inference

Computer Oriented Numerical and Statistical Methods

- **Statistical Inference** is branch of statistics which is concerned with using probability concept to deal with uncertainty in decision making
- Statistical inference treats two different classes of problems
  - **Hypothesis Testing :** To test some hypothesis about parent population from which the sample is drawn
  - **Estimation :** To use the statistics obtained from the sample as estimate of the unknown parameters of the population from which the sample is drawn

- Hypothesis Testing begins with an assumption, called a hypothesis
- A **hypothesis** in statistics is simply a quantitative statement about a population.
- In order to make statistical decisions, we make an certain assumptions about the population parameters to be tested.
- These ***assumptions*** are known as hypothesis

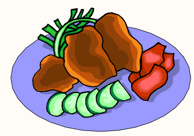
Hypothesis Testing

- There can be several types of hypotheses
- **For example** : The average marks of the 100 students of a class and may get the result as 65% we are now interested in testing the hypothesis that the sample has been drawn from a population with average marks 70%
- A coin may be tossed 100 times and we may get heads 75 time and tails 25 times, we are now interested in testing the hypothesis that the coin is unbiased

Hypothesis Testing

What is a Hypothesis?

A statement about the value of a population parameter developed for the purpose of testing.



The mean monthly income for systems analysts is $6,325.

Twenty percent of all customers at Bovine's Chop House return for another meal within a month.

What is a Hypothesis?

Hypothesis testing

Based on sample evidence and probability theory

Used to determine whether the hypothesis is a reasonable statement and should not be rejected, or is unreasonable and should be rejected

What is Hypothesis Testing?

Step 1: State null and alternate hypotheses

Step 2: Select a level of significance

Step 3: Identify the test statistic

Step 4: Formulate a decision rule

Step 5: Take a sample, arrive at a decision

Do not reject null | Reject null and accept alternate

Hypothesis Testing

Step One: State the null and alternate hypotheses

Null Hypothesis H_0

A statement about the value of a population parameter

Alternative Hypothesis H_1:

A statement that is accepted if the sample data provide evidence that the null hypothesis is false

Level of Significance

The probability of rejecting the null hypothesis when it is actually true; the level of risk in so doing.

Type I Error

Rejecting the null hypothesis when it is actually true (α).

Type II Error

Accepting the null hypothesis when it is actually false (β).

Step Two: Select a Level of Significance.

- **Defines the unlikely values of the sample statistic if the null hypothesis is true**
  - Defines rejection region of the sampling distribution
- Is designated by α , (level of significance)
  - Typical values are 0.01, 0.05, or 0.10
- Is selected by the researcher at the beginning
- Provides the critical value(s) of the test

Level of Significance, α

Step Two: Select a Level of Significance.

Null Hypothesis	Researcher Accepts H_o	Researcher Rejects H_o
H_o is true	Correct decision	Type I error (α)
H_o is false	Type II Error (β)	Correct decision

Risk table

Test statistic

A value, determined from sample information, used to determine whether or not to reject the null hypothesis.

Examples: z, t, F, χ^2

z Distribution as a test statistic

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

The *z* value is based on the sampling distribution of X, which is normally distributed when the sample is reasonably large (recall Central Limit Theorem).

Step Three: Select the test statistic.

Hypothesis Tests for the Mean

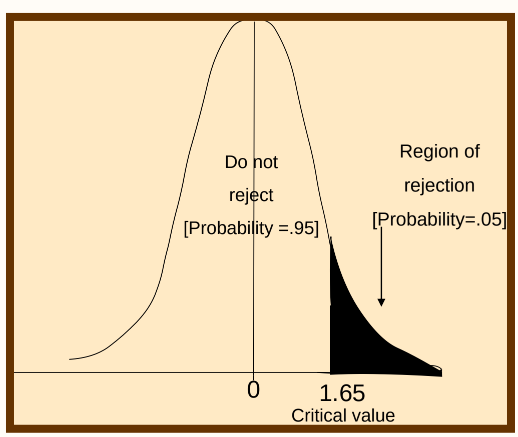
Hypothesis Tests for μ

- **σ Known (Z test)**
- **σ Unknown (t test)**

Step Four: Formulate the decision rule.

Critical value: The dividing point between the region where the null hypothesis is rejected and the region where it is not rejected.

Sampling Distribution Of the Statistic *z*, a Right-Tailed Test, .05 Level of Significance



p-Value

The probability, assuming that the null hypothesis is true, of finding a value of the test statistic at least as extreme as the computed value for the test

Decision Rule

If the *p*-Value is larger than or equal to the significance level, α, H_0 is not rejected.

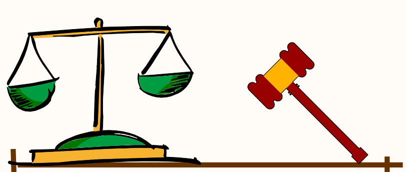
If the *p*-Value is smaller than the significance level, α, H_0 is rejected.

Calculated from the probability distribution function or by computer

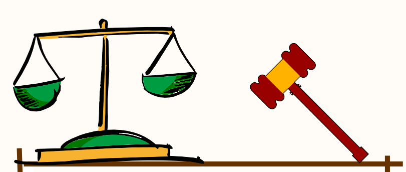
Using the p-Value in Hypothesis Testing

Interpreting p-values

SOME evidence H_o is not true

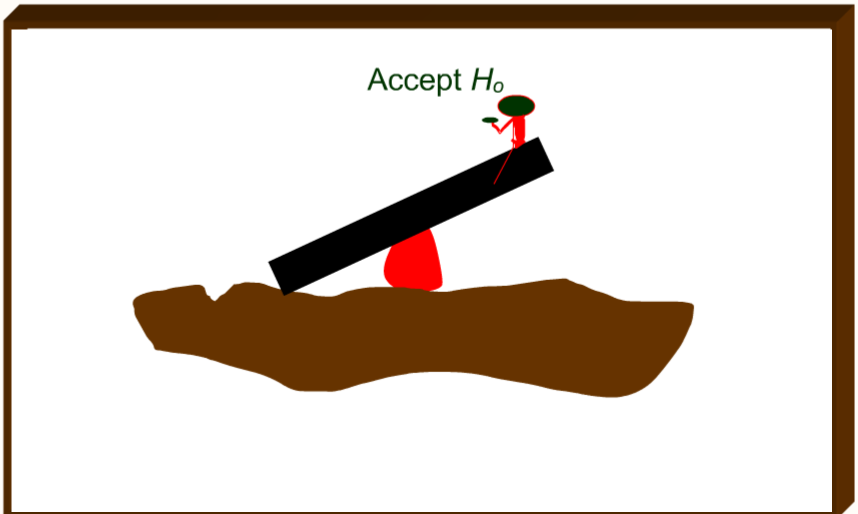


STRONG evidence H_o is not true



VERY STRONG evidence H_o is not true

Step Five: Make a decision.



Movie

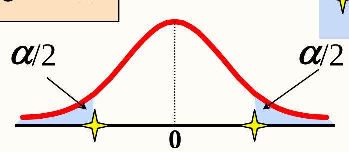
Level of Significance
and the Rejection Region

Level of significance = α

Represents critical value

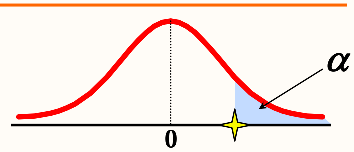
Rejection region is shaded

H_0: $\mu = 3$
H_1: $\mu \neq 3$



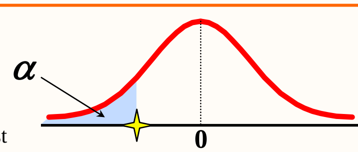
Two-tail test

H_0: $\mu \leq 3$ H_1:
$\mu > 3$



Upper-tail test

H_0: $\mu \geq 3$
H_1: $\mu < 3$



Lower-tail test

Test for the population mean from a large sample with population standard deviation known

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

Testing for the Population Mean: Large Sample, Population Standard Deviation Known

Hypothesis Testing Example

Test the claim that the true mean # of TV sets in US homes is equal to 3. (Assume $\sigma = 0.8$)

1. State the appropriate null and alternative hypotheses
   - H_0: $\mu = 3$ H_1: $\mu \neq 3$ (This is a two-tail test)
2. Specify the desired level of significance and the sample size
   - Suppose that $\alpha = 0.05$ and $n = 100$ are chosen for this test

Hypothesis Testing Example

3. Determine the appropriate technique
   - σ is known so this is a Z test.
4. Determine the critical values
   - For $\alpha = 0.05$ the critical Z values are ± 1.96
5. Collect the data and compute the test statistic
   - Suppose the sample results are

     $n = 100$, $\bar{X} = 2.84$ ($\sigma = 0.8$ is assumed known)

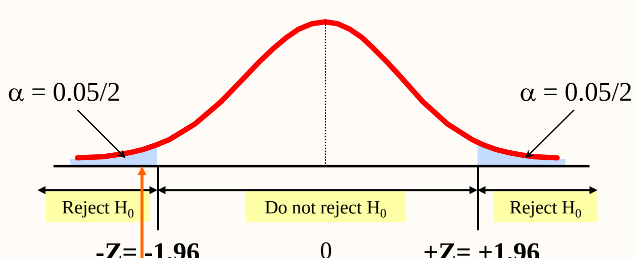
   So the test statistic is:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = -2.0$$

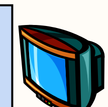
Hypothesis Testing Example

6. Is the test statistic in the rejection region?

Reject H_0 if $Z < -1.96$ or $Z > 1.96$; otherwise do not reject H_0



Here, $Z = -2.0 < -1.96$, so the test statistic is in the rejection region and conclude that there is sufficient evidence that the mean number of TVs in US homes is not equal to 3



Example: Upper-Tail Z Test for Mean (σ Known)

A phone industry manager thinks that customer monthly cell phone bills have increased, and now average over \$52 per month. The company wishes to test this claim. (Assume $\sigma = 10$ is known)

Form hypothesis test:

H_0: $\mu \leq 52$ the average is not over \$52 per month

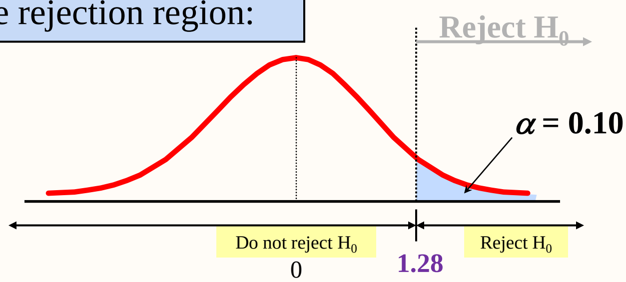
H_1: $\mu > 52$ the average is greater than \$52 per month (i.e., sufficient evidence exists to support the manager's claim)

Example: Find Rejection Region

(continued)

- Suppose that $\alpha = 0.10$ is chosen for this test

Find the rejection region:



Reject H_0 if $Z > 1.28$

Example: Test Statistic

Obtain sample and compute the test statistic

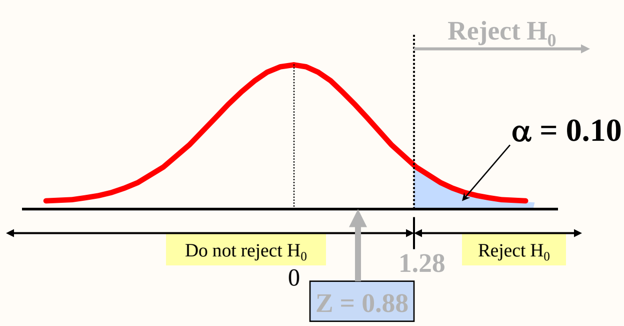
Suppose a sample is taken with the following results:
n = 64, $\bar{X}$ = 53.1 (σ=10 was assumed known)

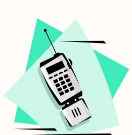
- Then the test statistic is:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{64}}} = 0.88$$

Example: Decision

Reach a decision and interpret the result:





Do not reject H_0 since Z = 0.88 ≤ 1.28

i.e.: there is not sufficient evidence that the mean bill is over $52

t Test of Hypothesis for the Mean (σ Unknown)

- Convert sample statistic ($\bar{X}$) to a t test statistic

Hypothesis Tests for μ

- **σ Known (Z test)**
- **σ Unknown (t test)**

The test statistic is:

$$t_{n-1} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

Example: Two-Tail Test (σ Unknown)

The average cost of a hotel room in New York is said to be \$168 per night. A random sample of 25 hotels resulted in $\bar{X}$ = \$172.50 and

S = \$15.40. Test at the

α = 0.05 level.

(Assume the population distribution is normal)

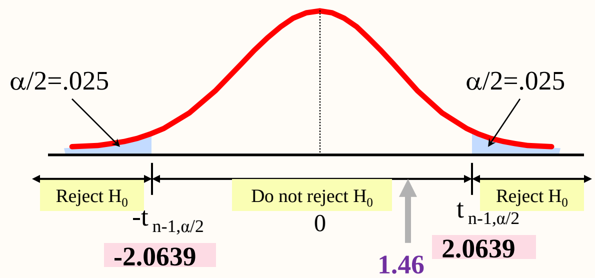
H_0: μ = 168
H_1: μ ≠ 168

Example Solution: Two-Tail Test

$H_0: \mu = 168$
$H_1: \mu \neq 168$

- $\alpha = 0.05$
- **n = 25**
- **σ is unknown, so use a t statistic**
- **Critical Value:**

$t_{24} = \pm 2.0639$



$$t_{n-1} = \frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}} = \frac{172.50-168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

Do not reject H_0: not sufficient evidence that true mean cost is different than $168