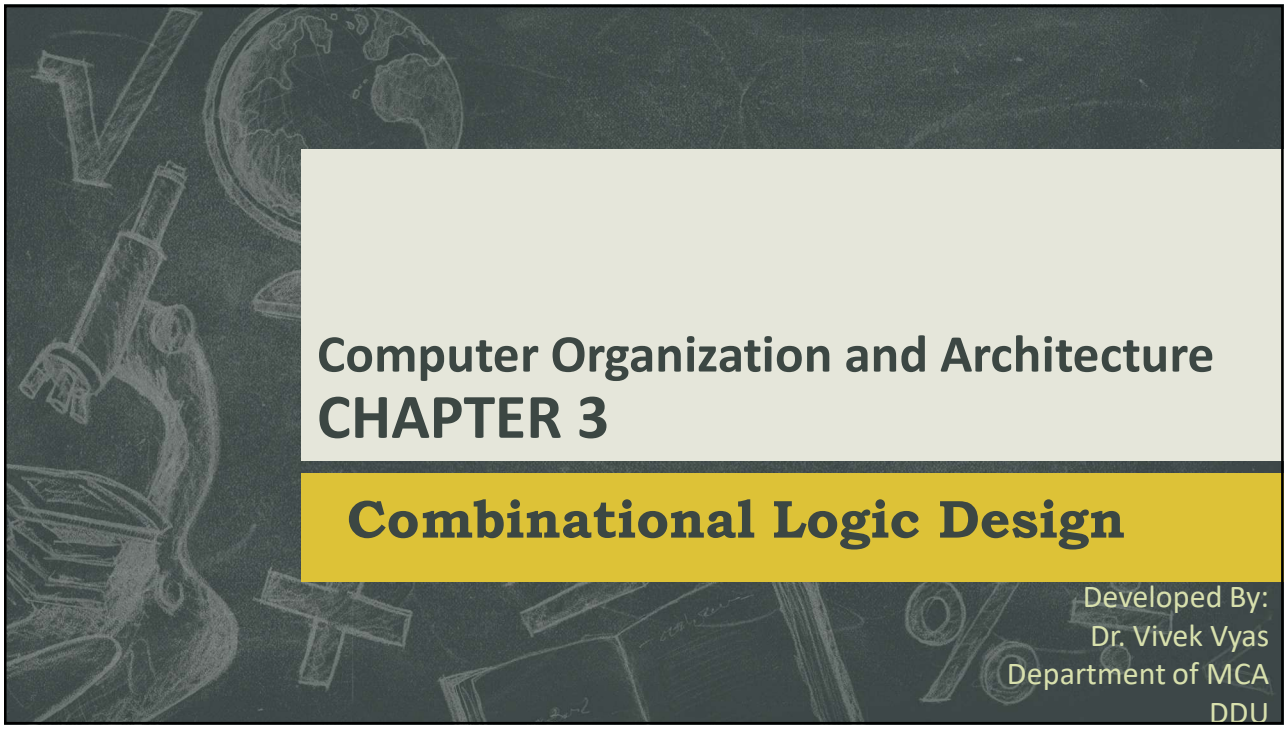

Computer Organization and Architecture
CHAPTER 3

Combinational Logic Design

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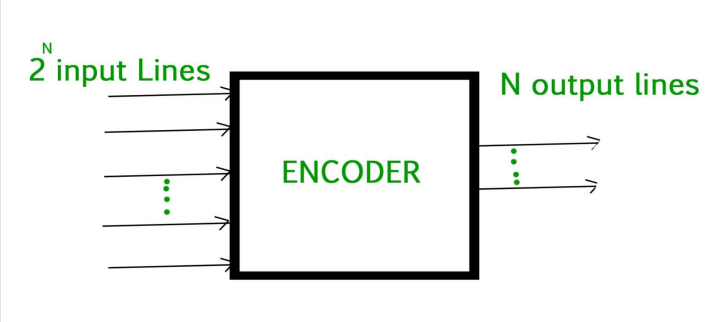
Department of MCA
DDU

❑ Encoder and Decoder

- **Encoder**
- An Encoder is a **combinational circuit** that performs the reverse operation of Decoder.
- It has maximum of **2^n input lines** and **'n' output lines**, hence it encodes the information from 2^n inputs into an n-bit code.
- It will produce a binary code equivalent to the input. Therefore, the encoder encodes 2^n input lines with 'n' bits.
- The encoders and decoders play an essential role in digital electronics projects; encoders & decoders are used to convert data from one form to another form.

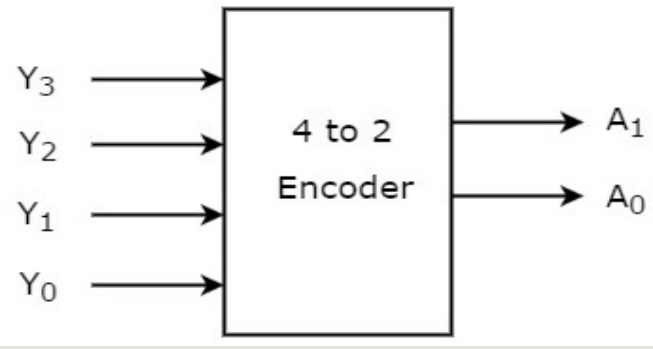
❑ Encoder and Decoder

- These are frequently used in communication system such as telecommunication, networking, etc..to transfer data from one end to the other end.



❑ Encoder and Decoder

- **4 to 2 Encoder**
- 4 to 2 Encoder has four inputs Y_3, Y_2, Y_1 & Y_0 and two outputs A_1 & A_0. The **block diagram** of 4 to 2 Encoder is shown in the following figure.



❑ Encoder and Decoder

- At any time, only one of these 4 inputs can be '1' in order to get the respective binary code at the output. The **Truth table** of 4 to 2 encoder is shown below.

Inputs				Outputs	
Y_3	Y_2	Y_1	Y_0	A_1	A_0
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	0	1	0
1	0	0	0	1	1

❑ Encoder and Decoder

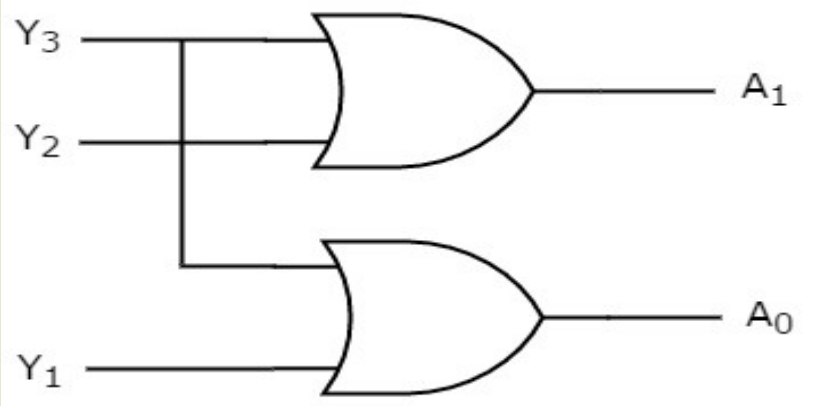
From Truth table, we can write the **Boolean functions** for each output as

$$A_1 = Y_3 + Y_2$$

$$A_0 = Y_3 + Y_1$$

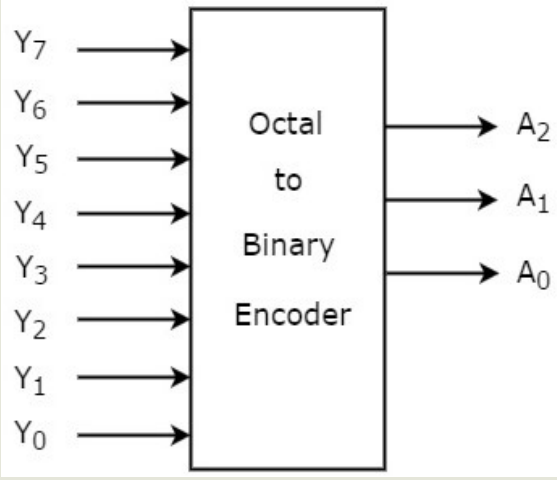
We can implement the above two Boolean functions by using two input OR gates. The **circuit diagram** of 4 to 2 encoder is shown in the following figure.

❑ Encoder and Decoder



❑ Encoder and Decoder

- **Octal to Binary Encoder**
- Octal to binary Encoder has eight inputs, Y_7 to Y_0 and three outputs A_2, A_1 & A_0. Octal to binary encoder is nothing but 8 to 3 encoder. The **block diagram** of octal to binary Encoder is shown in the following figure.



❑ Encoder and Decoder

- At any time, only one of these eight inputs can be '1' in order to get the respective binary code. The **Truth table** of octal to binary encoder is shown below.

Inputs								Outputs		
Y_7	Y_6	Y_5	Y_4	Y_3	Y_2	Y_1	Y_0	A_2	A_1	A_0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

❑ Encoder and Decoder

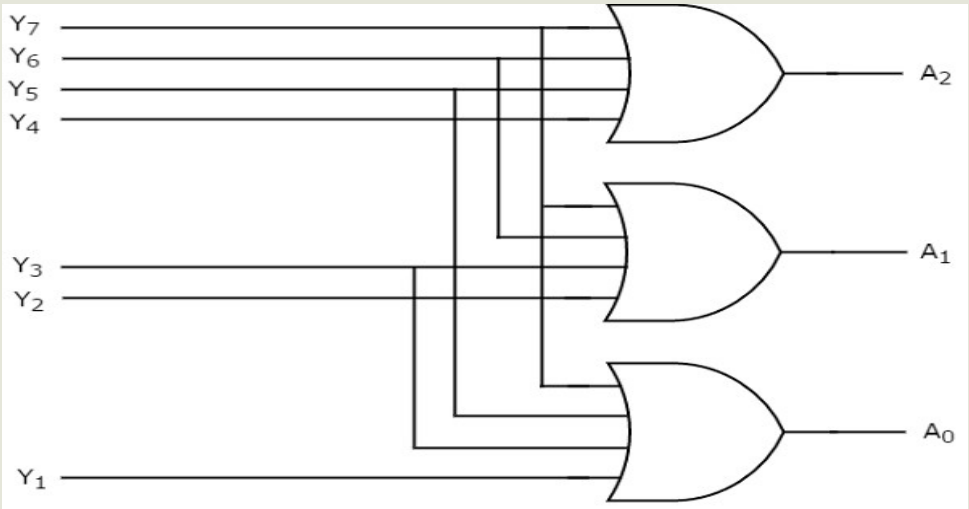
From Truth table, we can write the **Boolean functions** for each output as

$$A_2 = Y_7 + Y_6 + Y_5 + Y_4$$

$$A_1 = Y_7 + Y_6 + Y_3 + Y_2$$

$$A_0 = Y_7 + Y_5 + Y_3 + Y_1$$

❑ Encoder and Decoder



❑ Encoder and Decoder

- **Drawbacks of Encoder**
- There is an ambiguity, when all outputs of encoder are equal to zero. Because, it could be the code corresponding to the inputs, when only least significant input is one or when all inputs are zero.
- If more than one input is active High, then the encoder produces an output, which may not be the correct code. For **example**, if both Y_3 and Y_6 are '1', then the encoder produces 111 at the output. This is neither equivalent code corresponding to Y_3, when it is '1' nor the equivalent code corresponding to Y_6, when it is '1'.

❑ Encoder and Decoder

- So, to overcome these difficulties, we should assign priorities to each input of encoder. Then, the output of encoder will be the (binary) code corresponding to the active High input(s), which has higher priority. This encoder is called as **priority encoder**.

❑ Encoder and Decoder

- **Priority Encoder**
- A 4 to 2 priority encoder has four inputs Y_3, Y_2, Y_1 & Y_0 and two outputs A_1 & A_0. Here, the input, Y_3 has the highest priority, whereas the input, Y_0 has the lowest priority. In this case, even if more than one input is '1' at the same time, the output will be the (binary) code corresponding to the input, which is having **higher priority**.

❑ Encoder and Decoder

- We considered one more **output, V** in order to know, whether the code available at outputs is valid or not.
  - If at least one input of the encoder is '1', then the code available at outputs is a valid one. In this case, the output, V will be equal to 1.
  - If all the inputs of encoder are '0', then the code available at outputs is not a valid one. In this case, the output, V will be equal to 0.

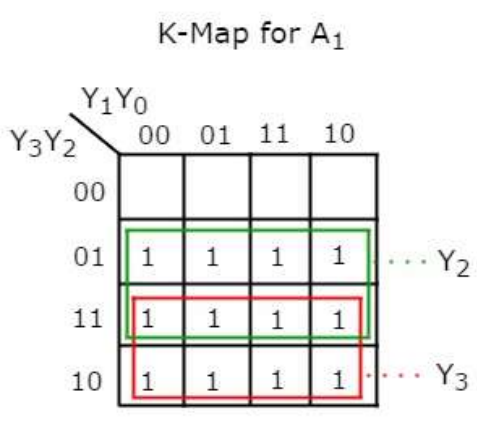
❑ Encoder and Decoder

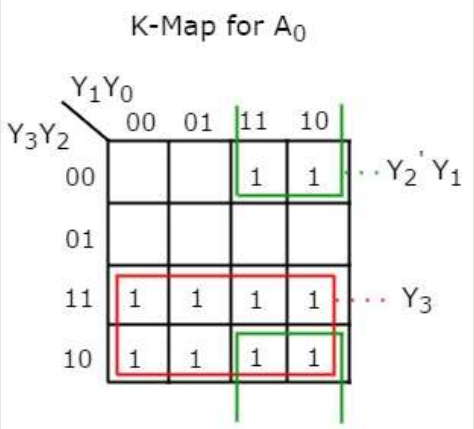
- **Truth table** of 4 to 2 priority encoder is shown below.

Inputs				Outputs		
Y_3	Y_2	Y_1	Y_0	A_1	A_0	V
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

❑ Encoder and Decoder

- Use **4 variable K-maps** for getting simplified expressions for each output.





❑ Encoder and Decoder

The simplified **Boolean functions** are

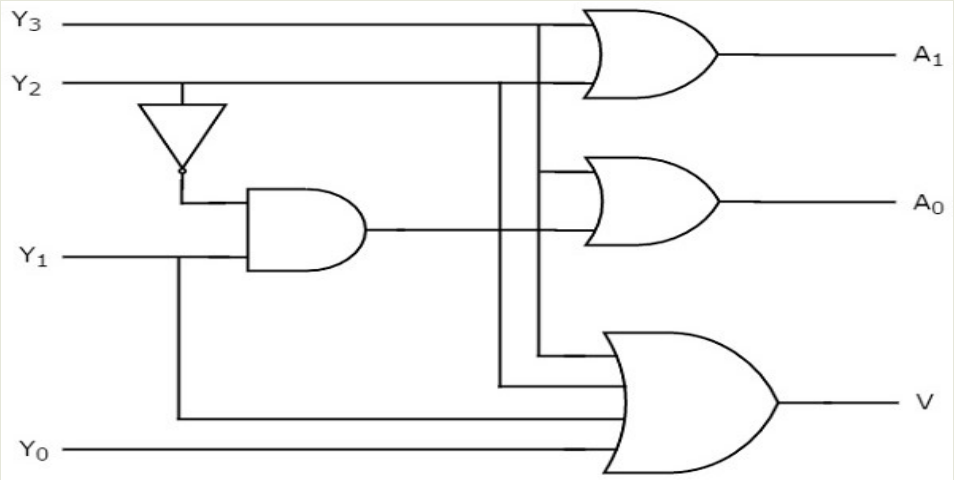
$$A_1 = Y_3 + Y_2$$

$$A_0 = Y_3 + Y_2'Y_1$$

Similarly, we will get the Boolean function of output, V as

$$V = Y_3 + Y_2 + Y_1 + Y_0$$

❑ Encoder and Decoder



❑ Encoder and Decoder

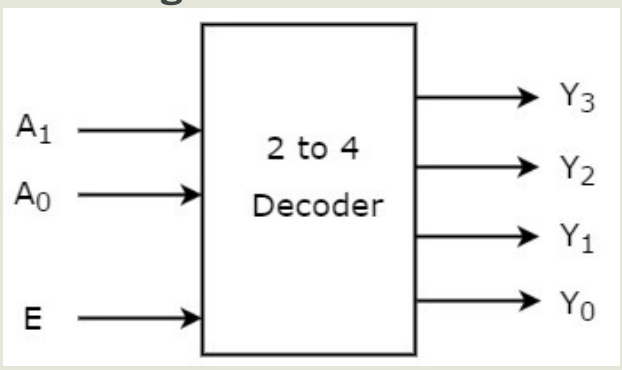
- The above circuit diagram contains two 2-input OR gates, one 4-input OR gate, one 2input AND gate & an inverter.
- Here AND gate & inverter combination are used for producing a valid code at the outputs, even when multiple inputs are equal to '1' at the same time.
- Hence, this circuit encodes the four inputs with two bits based on the **priority** assigned to each input.

❑ Encoder and Decoder

- **Decoder**
- **Decoder** is a combinational circuit that has 'n' input lines and maximum of 2^n output lines.
- One of these outputs will be active High based on the combination of inputs present, when the decoder is enabled. That means decoder detects a particular code.
- The outputs of the decoder are nothing but the **min terms** of 'n' input variables (lines), when it is enabled.

❑ Encoder and Decoder

- **2 to 4 Decoder**
- Let 2 to 4 Decoder has two inputs A_1 & A_0 and four outputs Y_3, Y_2, Y_1 & Y_0. The **block diagram** of 2 to 4 decoder is shown in the following figure.



❑ Encoder and Decoder

- One of these four outputs will be '1' for each combination of inputs when enable, E is '1'. The **Truth table** of 2 to 4 decoder is shown below.

Enable	Inputs		Outputs			
E	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	x	x	0	0	0	0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0

❑ Encoder and Decoder

From Truth table, we can write the **Boolean functions** for each output as

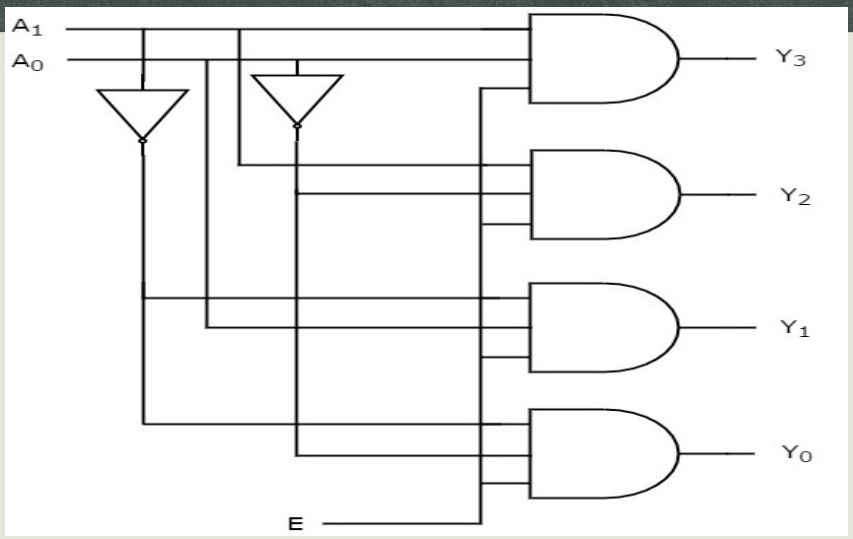
$$Y_3 = E.A_1.A_0$$

$$Y_2 = E.A_1.A_0'$$

$$Y_1 = E.A_1'.A_0$$

$$Y_0 = E.A_1'.A_0'$$

❑ Encoder and Decoder



❑ Encoder and Decoder

- Therefore, the outputs of 2 to 4 decoder are nothing but the **min terms** of two input variables A_1 & A_0, when enable, E is equal to one. If enable, E is zero, then all the outputs of decoder will be equal to zero.

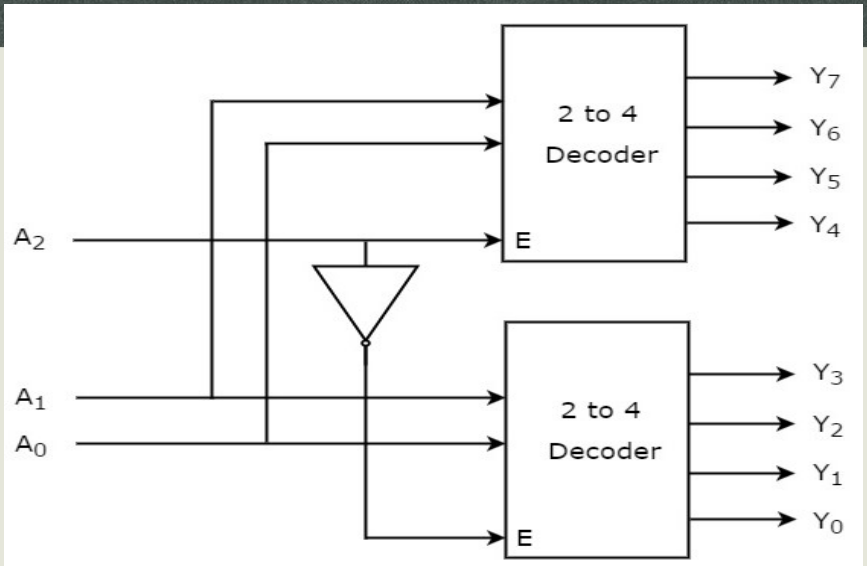
❑ Encoder and Decoder

- **3 to 8 Decoder**
- We know that 2 to 4 Decoder has two inputs, A_1 & A_0 and four outputs, Y_3 to Y_0. Whereas, 3 to 8 Decoder has three inputs A_2, A_1 & A_0 and eight outputs, Y_7 to Y_0.
- We can find the number of lower order decoders required for implementing higher order decoder using the following formula.
  - Required number of lower order decoders = m2/m1
- Where, m1 is the number of outputs of lower order decoder.
- m2 is the number of outputs of higher order decoder.

❑ Encoder and Decoder

- Here, m1= 4 and m2 = 8. Substitute, these two values in the above formula.
  - Required number of 2 to 4 decoders = 8/4 = 2
- Therefore, we require two 2 to 4 decoders for implementing one 3 to 8 decoder. The **block diagram** of 3 to 8 decoder using 2 to 4 decoders is shown in the following figure.

❑ Encoder and Decoder



❑ Encoder and Decoder

Inputs								Outputs		
Y_7	Y_6	Y_5	Y_4	Y_3	Y_2	Y_1	Y_0	A_2	A_1	A_0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

❑ Encoder and Decoder

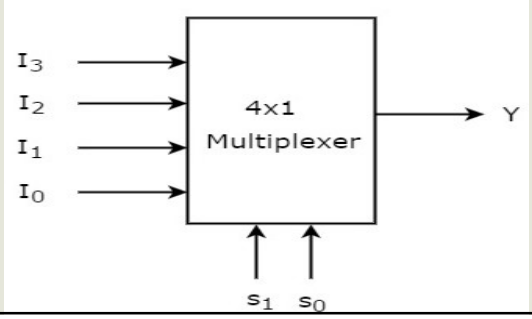
- The parallel inputs A_1 & A_0 are applied to each 2 to 4 decoder. The complement of input A_2 is connected to Enable, E of lower 2 to 4 decoder in order to get the outputs, Y_3 to Y_0.
- These are the **lower four min terms**. The input, A_2 is directly connected to Enable, E of upper 2 to 4 decoder in order to get the outputs, Y_7 to Y_4. These are the **higher four min terms**.

❑ Multiplexers and De-Multiplexers

- **Multiplexer** is a combinational circuit that has maximum of 2^n data inputs, 'n' selection lines and single output line. One of these data inputs will be connected to the output based on the values of selection lines.
- Since there are 'n' selection lines, there will be 2^n possible combinations of zeros and ones. So, each combination will select only one data input. Multiplexer is also called as **Mux**.

❑ Multiplexers and De-Multiplexers

- **4x1 Multiplexer**
- 4x1 Multiplexer has four data inputs I_3, I_2, I_1 & I_0, two selection lines s_1 & s_0 and one output Y. The **block diagram** of 4x1 Multiplexer is shown in the following diagram.



❑ Multiplexers and De-Multiplexers

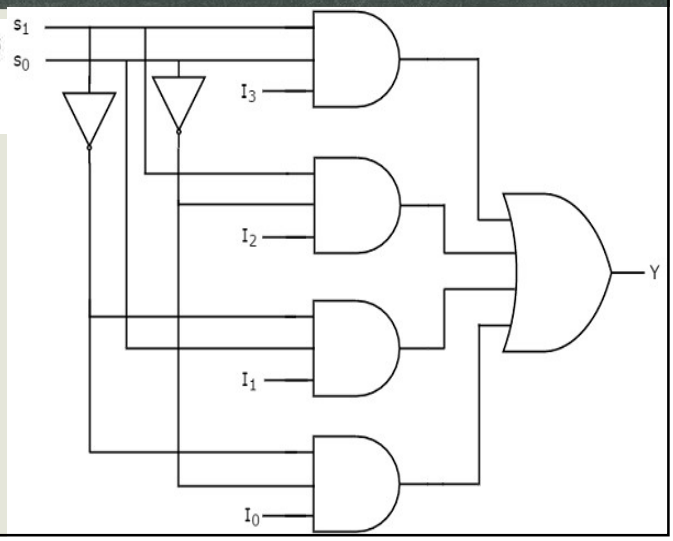
- One of these 4 inputs will be connected to the output based on the combination of inputs present at these two selection lines. **Truth table** of 4x1 Multiplexer is shown below.

Selection Lines		Output
S_1	S_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

❑ Multiplexers and De-Multiplexers

From Truth table, we can directly write the **Boolean function** for output, Y as

$$Y = S_1'S_0'I_0 + S_1'S_0I_1 + S_1S_0'I_2 + S_1S_0I_2$$



❑ Multiplexers and De-Multiplexers

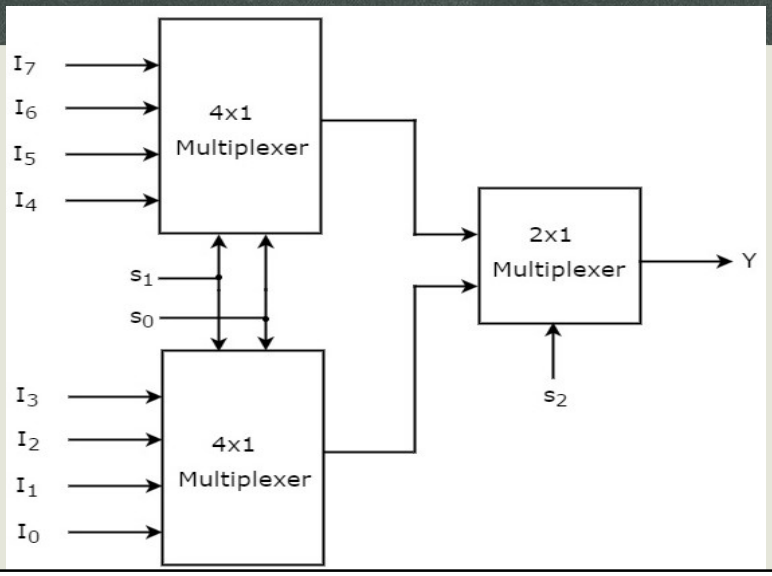
- **8x1 Multiplexer**
- We know that 4x1 Multiplexer has 4 data inputs, 2 selection lines and one output. Whereas, 8x1 Multiplexer has 8 data inputs, 3 selection lines and one output.
- So, we require two **4x1 Multiplexers** in first stage in order to get the 8 data inputs. Since, each 4x1 Multiplexer produces one output, we require a **2x1 Multiplexer** in second stage by considering the outputs of first stage as inputs and to produce the final output.

❑ Multiplexers and De-Multiplexers

- Let the 8x1 Multiplexer has eight data inputs I_7 to I_0, three selection lines s_2, s_1 & s0 and one output Y. The **Truth table** of 8x1 Multiplexer is shown below.

Selection Inputs			Output
S_2	S_1	S_0	Y
0	0	0	I_0
0	0	1	I_1
0	1	0	I_2
0	1	1	I_3
1	0	0	I_4
1	0	1	I_5
1	1	0	I_6
1	1	1	I_7

❑ Multiplexers and De-Multiplexers



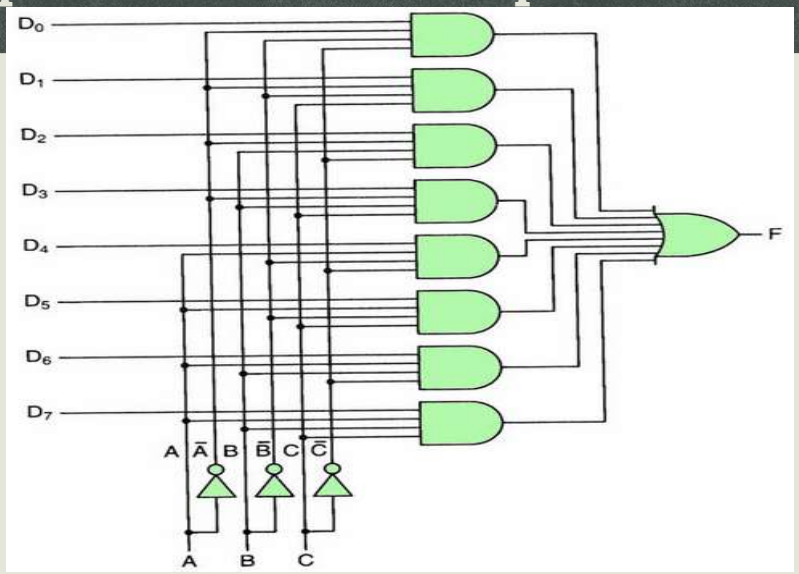
❑ Multiplexers and De-Multiplexers

- The same **selection lines, s_1 & s_0** are applied to both 4x1 Multiplexers. The data inputs of upper 4x1 Multiplexer are I_7 to I_4 and the data inputs of lower 4x1 Multiplexer are I_3 to I_0. Therefore, each 4x1 Multiplexer produces an output based on the values of selection lines, s_1 & s_0.
- The outputs of first stage 4x1 Multiplexers are applied as inputs of 2x1 Multiplexer that is present in second stage. The other **selection line, s_2** is applied to 2x1 Multiplexer.

❑ Multiplexers and De-Multiplexers

- If s_2 is zero, then the output of 2x1 Multiplexer will be one of the 4 inputs I_3 to I_0 based on the values of selection lines s_1 & s_0.
- If s_2 is one, then the output of 2x1 Multiplexer will be one of the 4 inputs I_7 to I_4 based on the values of selection lines s_1 & s_0.

❑ Multiplexers and De-Multiplexers



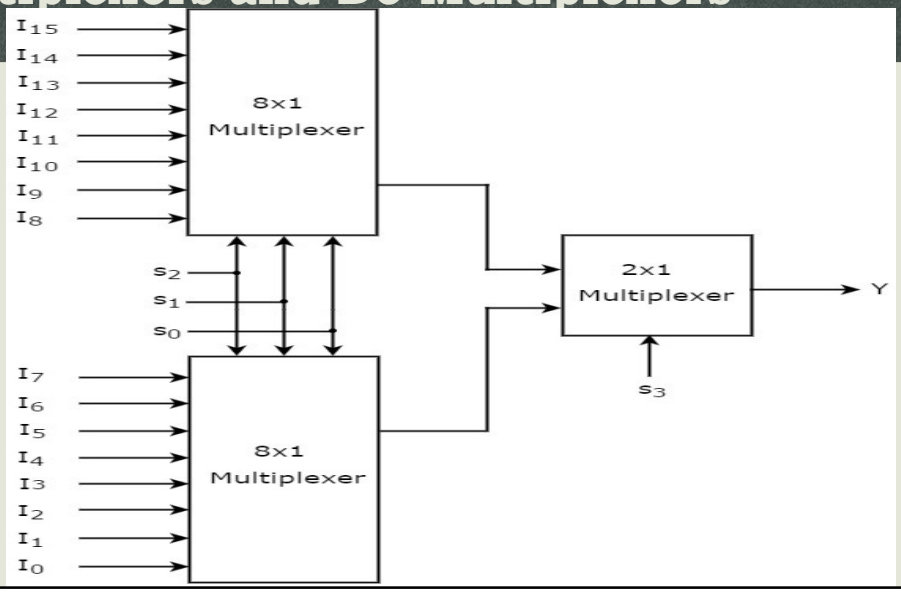
❑ Multiplexers and De-Multiplexers

- **16x1 Multiplexer**
- let us implement 16x1 Multiplexer using 8x1 Multiplexers and 2x1 Multiplexer. We know that 8x1 Multiplexer has 8 data inputs, 3 selection lines and one output. Whereas, 16x1 Multiplexer has 16 data inputs, 4 selection lines and one output.
- So, we require two **8x1 Multiplexers** in first stage in order to get the 16 data inputs. Since, each 8x1 Multiplexer produces one output, we require a 2x1 Multiplexer in second stage by considering the outputs of first stage as inputs and to produce the final output.
- Let the 16x1 Multiplexer has sixteen data inputs I_{15} to I_0, four selection lines s_3 to s_0 and one output Y. The **Truth table** of 16x1 Multiplexer is shown below.

❑ Multiplexers and De-Multiplexers

Selection Inputs				Output
S_3	S_2	S_1	S_0	Y
0	0	0	0	I_0
0	0	0	1	I_1
0	0	1	0	I_2
0	0	1	1	I_3
0	1	0	0	I_4
0	1	0	1	I_5
0	1	1	0	I_6
0	1	1	1	I_7
1	0	0	0	I_8
1	0	0	1	I_9
1	0	1	0	I_{10}
1	0	1	1	I_{11}
1	1	0	0	I_{12}
1	1	0	1	I_{13}
1	1	1	0	I_{14}
1	1	1	1	I_{15}

❑ Multiplexers and De-Multiplexers



❑ Multiplexers and De-Multiplexers

- The **same selection lines, s_2, s_1 & s_0** are applied to both 8x1 Multiplexers. The data inputs of upper 8x1 Multiplexer are I_{15} to I_8 and the data inputs of lower 8x1 Multiplexer are I_7 to I_0. Therefore, each 8x1 Multiplexer produces an output based on the values of selection lines, s_2, s_1 & s_0.
- The outputs of first stage 8x1 Multiplexers are applied as inputs of 2x1 Multiplexer that is present in second stage. The other **selection line, s_3** is applied to 2x1 Multiplexer.

❑ Multiplexers and De-Multiplexers

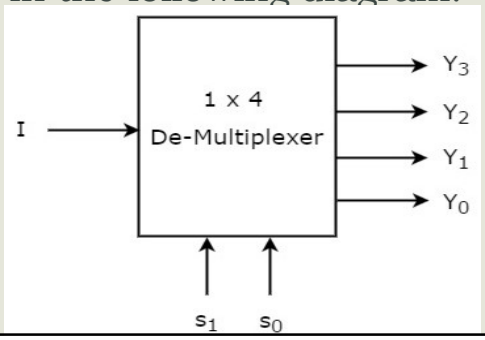
- If s_3 is zero, then the output of 2x1 Multiplexer will be one of the 8 inputs Is_7 to I_0 based on the values of selection lines s_2, s_1 & s_0.
- If s_3 is one, then the output of 2x1 Multiplexer will be one of the 8 inputs I_{15} to I_8 based on the values of selection lines s_2, s_1 & s_0.

❑ Multiplexers and De-Multiplexers

- **De-Multiplexers**
- **De-Multiplexer** is a combinational circuit that performs the reverse operation of Multiplexer. It has single input, 'n' selection lines and maximum of 2^n outputs. The input will be connected to one of these outputs based on the values of selection lines.
- Since there are 'n' selection lines, there will be 2^n possible combinations of zeros and ones. So, each combination can select only one output. De-Multiplexer is also called as **De-Mux**.

❑ Multiplexers and De-Multiplexers

- **1x4 De-Multiplexer**
- 1x4 De-Multiplexer has one input I, two selection lines, s_1 & s_0 and four outputs Y_3, Y_2, Y_1 &Y_0. The **block diagram** of 1x4 De-Multiplexer is shown in the following diagram.



❑ Multiplexers and De-Multiplexers

- The single input 'I' will be connected to one of the four outputs, Y_3 to Y_0 based on the values of selection lines s_1 & s0. The **Truth table** of 1x4 De-Multiplexer is shown below.

Selection Inputs		Outputs			
S_1	S_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	I
0	1	0	0	I	0
1	0	0	I	0	0
1	1	I	0	0	0

From the above Truth table, we can directly write the **Boolean functions** for each output as

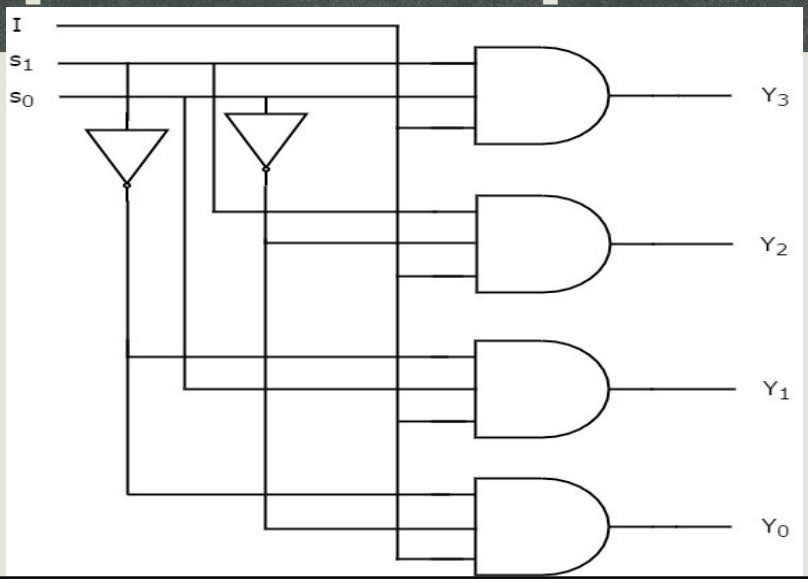
$$Y_3 = s_1 s_0 I$$

$$Y_2 = s_1 s_0' I$$

$$Y_1 = s_1' s_0 I$$

$$Y_0 = s_1' s_0' I$$

❑ Multiplexers and De-Multiplexers



❑ Multiplexers and De-Multiplexers

- **1x8 De-Multiplexer**
- let us implement 1x8 De-Multiplexer using 1x4 De-Multiplexers and 1x2 De-Multiplexer. We know that 1x4 De-Multiplexer has single input, two selection lines and four outputs. Whereas, 1x8 De-Multiplexer has single input, three selection lines and eight outputs.
- So, we require two **1x4 De-Multiplexers** in second stage in order to get the final eight outputs. Since, the number of inputs in second stage is two, we require **1x2 DeMultiplexer** in first stage so that the outputs of first stage will be the inputs of second stage. Input of this 1x2 De-Multiplexer will be the overall input of 1x8 De-Multiplexer.

❑ Multiplexers and De-Multiplexers

- Let the 1x8 De-Multiplexer has one input I, three selection lines s_2, s_1 & s_0 and outputs Y_7 to Y_0. The **Truth table** of 1x8 De-Multiplexer is shown below.

Selection Inputs			Outputs							
s_2	s_1	s_0	Y_7	Y_6	Y_5	Y_4	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0	0	0	I
0	0	1	0	0	0	0	0	0	I	0
0	1	0	0	0	0	0	0	I	0	0
0	1	1	0	0	0	0	I	0	0	0
1	0	0	0	0	0	I	0	0	0	0
1	0	1	0	0	I	0	0	0	0	0
1	1	0	0	I	0	0	0	0	0	0
1	1	1	I	0	0	0	0	0	0	0

❑ Multiplexers and De-Multiplexers

$Y0 = D\,\overline{S2}\,\overline{S1}\,\overline{S0}$

$Y1 = D\,\overline{S2}\,\overline{S1}\,S0$

$Y2 = D\,\overline{S2}\,S1\,\overline{S0}$

$Y3 = D\,\overline{S2}\,S1\,S0$

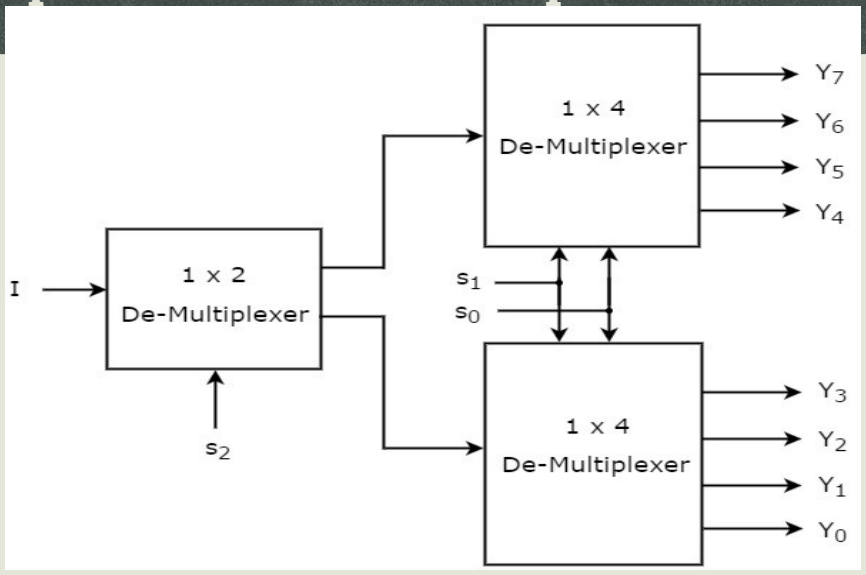
$Y4 = D\,S2\,\overline{S1}\,\overline{S0}$

$Y5 = D\,S2\,\overline{S1}\,S0$

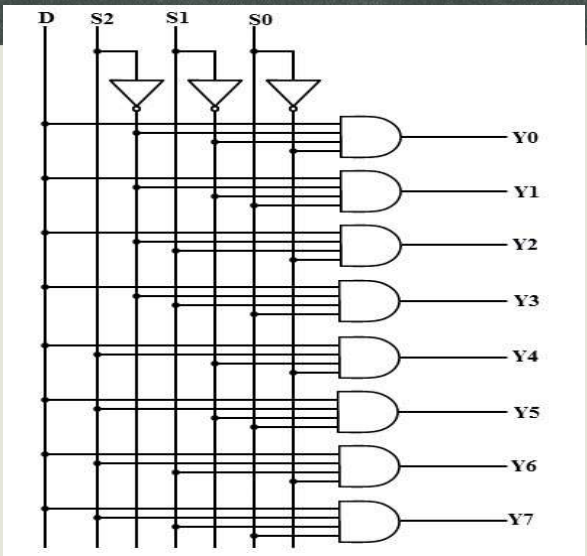
$Y6 = D\,S2\,S1\,\overline{S0}$

$Y7 = D\,S2\,S1\,S0$

❑ Multiplexers and De-Multiplexers



❑ Multiplexers and De-Multiplexers



❑ Multiplexers and De-Multiplexers

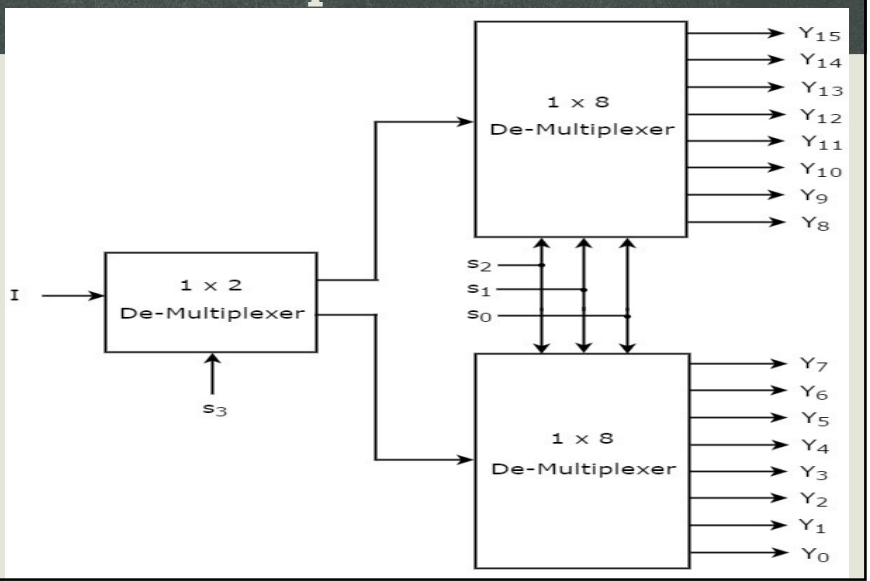
- The common **selection lines, s_1 & s_0** are applied to both 1x4 De-Multiplexers. The outputs of upper 1x4 De-Multiplexer are Y_7 to Y_4 and the outputs of lower 1x4 De-Multiplexer are Y_3 to Y_0.
- The other **selection line, s_2** is applied to 1x2 De-Multiplexer. If s_2 is zero, then one of the four outputs of lower 1x4 De-Multiplexer will be equal to input, I based on the values of selection lines s_1 & s_0. Similarly, if s_2 is one, then one of the four outputs of upper 1x4 DeMultiplexer will be equal to input, I based on the values of selection lines s_1 & s_0.

❑ Multiplexers and De-Multiplexers

- **1x16 De-Multiplexer**
- let us implement 1x16 De-Multiplexer using 1x8 De-Multiplexers and 1x2 De-Multiplexer. We know that 1x8 De-Multiplexer has single input, three selection lines and eight outputs. Whereas, 1x16 De-Multiplexer has single input, four selection lines and sixteen outputs.
- So, we require two **1x8 De-Multiplexers** in second stage in order to get the final sixteen outputs. Since, the number of inputs in second stage is two, we require **1x2 DeMultiplexer** in first stage so that the outputs of first stage will be the inputs of second stage. Input of this 1x2 De-Multiplexer will be the overall input of 1x16 De-Multiplexer.

❑ Multiplexers and De-Multiplexers

- Let the 1x16 De-Multiplexer has one input I, four selection lines s_3, s_2, s_1 & s_0 and outputs Y_{15} to Y_0. The **block diagram** of 1x16 De-Multiplexer using lower order Multiplexers is shown in the following figure.



❑ Multiplexers and De-Multiplexers

- The common **selection lines s_2, s_1 & s_0** are applied to both 1x8 De-Multiplexers. The outputs of upper 1x8 De-Multiplexer are Y_{15} to Y_8 and the outputs of lower 1x8 DeMultiplexer are Y_7 to Y_0.
- The other **selection line, s_3** is applied to 1x2 De-Multiplexer. If s_3 is zero, then one of the eight outputs of lower 1x8 De-Multiplexer will be equal to input, I based on the values of selection lines s_2, s_1 & s_0. Similarly, if s3 is one, then one of the 8 outputs of upper 1x8 De-Multiplexer will be equal to input, I based on the values of selection lines s_2, s_1 & s_0.