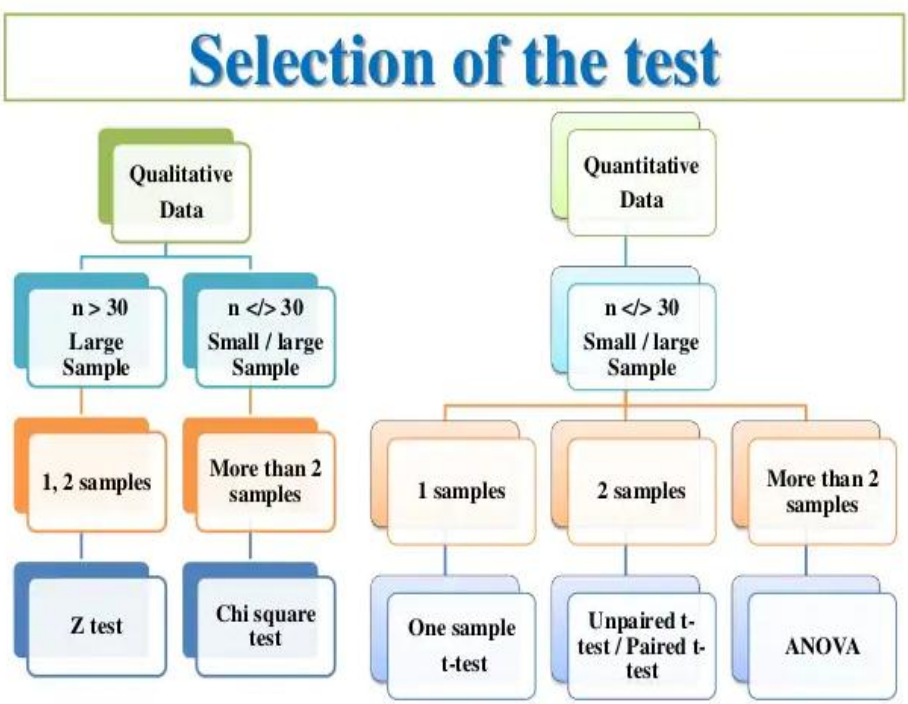
HYPOTESIS EXAMPLES

Computer Oriented Numerical and Statistical Methods



Test statistic	Associated test	Sample size	Information given	Distribution	Test question
z-score	z-test	Two populations or large samples (n > 30)	• Standard deviation of the population (this will be given as σ) • Population mean or proportion	Normal	Do these two populations differ?
t-statistic	t-test	Two small samples (n < 30)	• Standard deviation of the sample (this will be given as *s*) • Sample mean	Normal	Do these two samples differ?
f-statistic	ANOVA	Three or more samples	• Group sizes • Group means • Group standard deviations	Normal	Do any of these three or more samples differ from each other?
chi-squared	chi-squared test	Two samples	• Number of observations for each categorical variable	Any	Are these two categorical variables independent?

- A manufacturer supplies the rear axles for U.S. postal services mail trucks. These axles must be able to withstand 80,000 pounds per square inch in stress test, but an excessively strong axle raises production cost significantly. Long experience indicates that the S.D. of the strength of its axles is 4000 pounds per square inch. The manufacture selects a sample of 100 axles from production, tests them, and finds that the mean stress capacity of the sample is 79600 pounds per square inch. If the axle manufacturer uses a significance level (α) of 0.05 in testing, will the axles meet his stress requirements?

- $H_0 : \mu = 80,000 \rightarrow$ Null hypothesis
- $H_1 : \mu \neq 80,000 \rightarrow$ Alternative hypothesis
- $\alpha = 0.05 \rightarrow$ level of significance for testing this hypothesis.
- Calculate the standard error

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

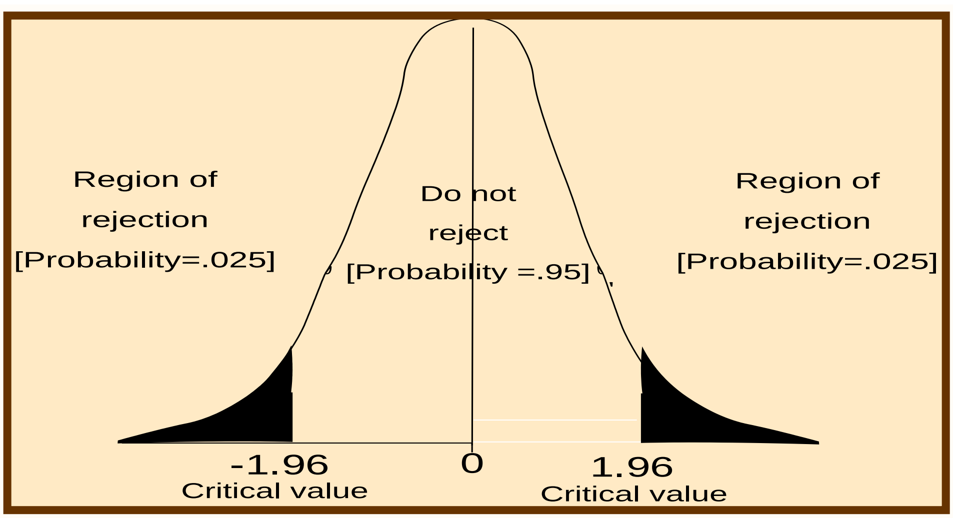
$$= \frac{4000}{\sqrt{100}}$$

$$= 400 \, pounds \; per \; square \; inch$$

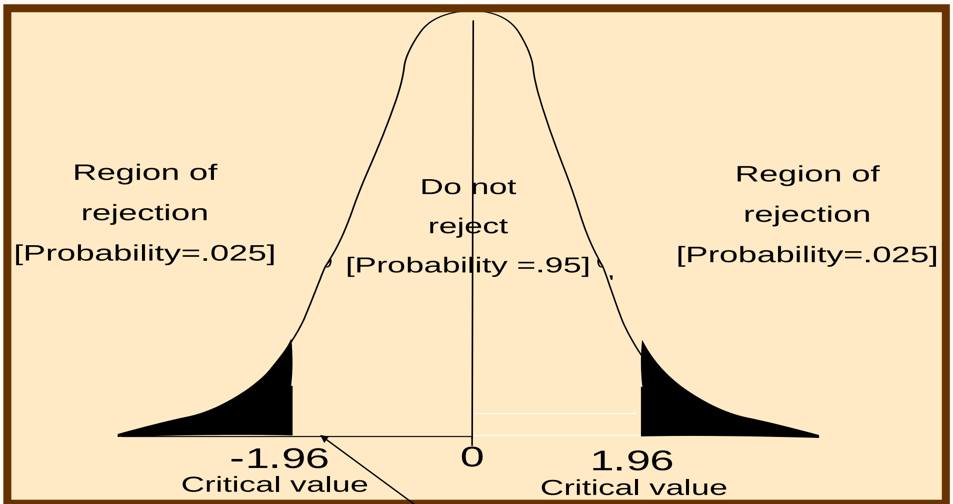
- Determining the limits of acceptance region :
  - 0.95 (1 – 0.05) acceptance region contains two equal area of 0.475 (0.95 / 2) each.
- From the normal distribution table we can see that the appropriate z value for 0.475 of the area under the curve is ±1.96.
- Now we can determine the limits of the acceptance region

TABLE A.1 AREAS OF THE NORMAL DISTRIBUTION

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997



$$Z = \frac{\bar{x} - \mu}{\sigma \bar{x}} = \frac{79600-80000}{400} = -1.00\ (\sigma \bar{x} = \sqrt{n})$$



$$Z = \frac{\bar{x} - \mu}{\sigma\bar{x}} = \frac{79600-80000}{400} = -1.00 \; (\sigma\bar{x} = \sqrt{n})$$

Manufacturer should accept the production run as meeting stress requirement.

- A Hospital uses large quantities of packaged doses of a particular drug. The individual dose of this drug is 100 cubic cm (100 cc). The action of the drug is such that the body will harmlessly pass off excessive doses on the other hand, insufficient doses so not produce the desired medical effect, and they interfere with patient treatment. The hospital has purchased this drug from the same manufacturer for a number of years and knows that the population S.D. is 2cc. The hospital inspects 50 doses of this drug at random from a very shipment and finds the mean of these doses to be 99.75cc. If the hospital sets a 0.10 significance level and asks us whether the dosages in this shipment are too small, how can we find the answer.

- $H_0 : \mu \geq 100 \rightarrow$ Null hypothesis
- $H_1 : \mu < 100 \rightarrow$ Alternative hypothesis
- $\alpha = 0.10 \rightarrow$ level of significance for testing this hypothesis.
- Choose the appropriate distribution and find the critical value.
  - Here population S.D. is known and n is larger than 30 we use the normal distribution.
  - From the table we can determine that the value of z for 40% (50% - 10% significance) of the area under the curve is 1.28
  - So the critical value for lower tailed test is -1.28 i.e. left tailed test
- Compute the standard error and standardize the sample statistics.

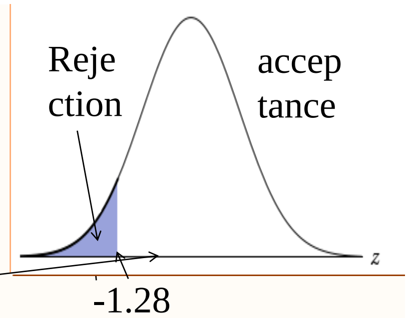
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{50}} = 0.2829cc$$

- Now we use the equation .

$$z = \frac{\bar{x} - \mu_{H_0}}{\sigma_{\bar{x}}}$$

$$= \frac{99.75 - 100}{0.2829}$$

$$= -0.88$$



- Sketch the distribution and mark the sample values and the critical value.
- Placing the standardized value on the z scale shows that this sample mean falls well within the acceptance region.
- Interpret the results : Hospital should accept the null hypothesis.

- A personnel specialist of a major corporation is recruiting a large number of employees for an overseas assignment. During the testing process, management asks how things are going and she replies. "Fine, I think the average score on the aptitude test will be around 90". When management reviews 20 of the test results compiled, it finds that the mean score is 84 and the S.D. of this score is 11. If management wants to test her hypothesis at the 0.1 level of significance, what is the procedure?

- State your hypothesis type of test, and significance level.
  - $H_0 : \mu = 90 \rightarrow$ Null hypothesis
  - $H_1 : \mu \neq 90 \rightarrow$ Alternative hypothesis
  - $\alpha = 0.10 \rightarrow$ level of significance for testing this hypothesis.
- Choose the appropriate distribution and find the critical value
  - Because the management is interested in knowing whether the true mean score, a two-tailed test is appropriate one to use.
  - The significance level is 0.1, so two area, each containing 0.05 of the area under the t distribution.

- Because the sample size is 20 i.e. n = 20 the appropriate number of degrees of freedom i.e. n-1 = 19, that is 20 -1.
- From the t table the critical value is 1.729
- Here population S.D. is not known so estimate it using the sample S.D. and equation

$$\hat{\sigma} = S = 11$$

- Compute the standard error and standardize the sample statistics.
- $$\hat{\sigma}_{\bar{x}} = \frac{\hat{\sigma}}{\sqrt{n}}$$
  $$= \frac{11}{\sqrt{20}}$$
  $$= 2.46$$

t distribution critical values

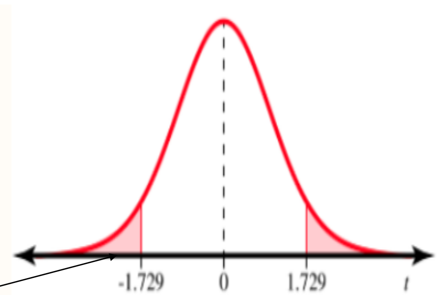
	Upper-tail probability *p*											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850

- Standardize statistics

$$t = \frac{\bar{x} - \mu_{H_0}}{\hat{\sigma}_{\bar{x}}}$$

$$= \frac{84 - 90}{2.46}$$

$$= -2.44$$



- Sketch the distribution and mark the sample value and the critical value.
- Drawing the results on a sketch of the sampling distribution, we see that the sample mean falls outside the acceptance region.
- Interpret the result : Management should reject the null hypothesis.


THANKS!
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