Probability

Computer Oriented Numerical and Statistical Methods

Outline

- Introduction
- Meaning
- Basic Definitions
- Types
- Basic Probability Rules

Introduction

- So far we have studied the methods of collection, description and analysis of data.
- This does not cover the entire operational field of statistics.
- Modern statistics has also to provide for :
  1. Estimation of population ‘parameters’ on the basis of sample ‘statistics’
  2. Drawing inferences about the sample ‘statistics’ on the basis of population ‘parameters’
  3. Testing of hypothesis with regards to (1) and (2) above and

Introduction

4. Decision-making under risk and uncertainty be estimating the degree of risk and the likely effect on business objectives in terms of pay off and expected values of a decision.

- The theory of probability helps in all these areas.
- Probability helps a person to make ‘educated guesses’ on matters, where either full facts are not known or there is uncertainty about the outcome.
- The decision-makers always face some degree of risk while selecting a particular decision (course of action or strategy) to solve a decision problem.

Introduction

- It is because each strategy can lead to a number of different possible outcomes (or results).
- Thus it is necessary for the decision-makers to enhance their capability of grasping the probabilistic situation so as to gain a deeper understanding of the decision problem and base their decision on rational considerations.

Meaning

- Probability means the number of occasion that a particular event is likely to occur in a large population of events.
- The particular event may be expressed positively where the event is likely to happen or negatively where the event is not likely to happen.

Meaning

- Broadly, there are three possible states of expectations :
  - Certainty
  - Impossibility
  - Uncertainty
- The probability theory describe certainty by 1, impossibility by 0 and the various grades of uncertainties by coefficients ranging between 0 and 1

Probability in Everyday Life

- Possible, it will rain tonight.
- Probably you will catch the train.
- There is a high chance of your getting the job in August.
- This year's demand for the product is likely to exceed that of the last year's.

Definitions

- **Probability** : It means the number of occasion that a particular event is likely to occur in a large population of events.
  - The particular event may be expressed positively where the event is likely to happen or negatively where the event is not likely to happen.

Definitions

- **Random Experiment or Trial**: An experiment is said to be a random experiment (or trial or act or operation or process), if it's out-come can't be predicted with certainty.
- Example :
  - If a coin is tossed, we can't say, whether head or tail will appear. So it is a random experiment.
  - Drawing a card from a pack of cards

Definitions

- **Sample Space (Possible outcomes)**: The set of all possible out-comes of an experiment is called the sample space. It is denoted by 'S' or 'U' and its number of elements are n(s).
- **Example:**
  - In throwing a dice, the number that appears at top is any one of 1,2,3,4,5,6. So here: S ={1,2,3,4,5,6} and n(s) = 6
  - In the case of a coin, S={Head,Tail} or {H,T} and n(s)=2.
- The elements of the sample space are called sample point or event point.

Definitions

- **Event**: Every subset of a sample space is an event. It is denoted by 'E'.
  - The empty set ∅ is called impossible event and the sample space U is called certain event.
  - Clearly E is a sub set of S
- **Example:**
  - In throwing a dice S={1,2,3,4,5,6}, the appearance of an event number will be the event E={2,4,6}.
- **Simple event** : An event, consisting of a single sample point is called a simple event.
- **Example**:
  - In throwing a dice, S={1,2,3,4,5,6}, so each of {1},{2},{3},{4},{5} and {6} are simple events.

Definitions

- **Compound event:** A subset of the sample space, which has more than one element is called a mixed event (compound event).
- Example:
  – In throwing a dice, the event of appearing of odd numbers is a compound event, because E={1,3,5} which has '3' elements.

Definitions

- **Equally likely events**: Events are said to be equally likely, if we have no reason to believe that one is more likely to occur than the other. OR The outcomes are said to be equally likely or equally probable if none of them is expected to occur in preference to other.
- Example:
  – When a dice is thrown, all the six faces {1,2,3,4,5,6} are equally likely to come up.

Definitions

- **Exhaustive events**: When every possible out come of an experiment is considered.
- **Example:**
  – A dice is thrown, cases 1,2,3,4,5,6 form an exhaustive set of events.
- **Collectively Exhaustive events**: The total number of possible outcomes of a random experiment is called the collective exhaustive events.
- **Example:**
  – A dice is thrown, cases 1,2,3,4,5,6 form an exhaustive set of events and number of cases is 6.
  – In toss of a single coin exhaustive number of cases is 2

Definitions

- **Complementary events**: The set of all elements of the sample space U except the elements of event A is called the complementary event A. It is denoted by A' or $\bar{A}$ it means that 'the event A does not occur.
- **Union events**: Let A and B be two events. The set of elements which are either in A or B is called the union event of event A and B. it is denoted by $A \cup B$. $A \cup B$ means the event 'either A occurs or B occurs'.
- **Intersection events**: Let A and B be two events. The set of elements which are in A and in B is called the intersection event of event A and B. it is denoted by $A \cap B$. $A \cap B$ means the event in which A and B occurs simultaneously.

Definitions

- **Differnce events**: Let A and B be two events. The set of elements all elements which are in A but not in B is called the difference event of event A and B. it is denoted by A - B. A - B means the event in which 'the event A occurs but the event B does not occurs'.
- **Mutually exclusively events /disjoint event** : If the intersection event of two events is the impossible event then these two events are said to be mutually exclusive. Thus it is $A \cap B = \varnothing$ then A and B are mutually exclusive events.
- **Example:**
  - When a coin is tossed, the event of occurrence of a head and the event of occurrence of a tail are mutually exclusive events.

Definitions

- **Dependent or Independent events**: Two or more events are considered to be independent if the occurrence of one event is no way affects the occurrence of the other.
- **Example :**
  - Tossing a coin a trial is not affected by the result of the previous trial
- If the occurrence of one event influences the occurrence of the other event, the events are said to be dependent events.
- **Example :**
  - If a card is drawn from a pack of shuffled cards, and not replaced before drawing the second card, then the second card drawn is dependent on the first one.

Definitions

- **Probability Set Function**: Let U be a finite sample space and let S(U) be its power set. Let P: S(U) $\rightarrow$ R be a set function satisfying the following postulates.
  1. P(A) $\geq$ 0 for every A $\in$ S(U)
  2. P(U) = 1
  3. If A and B are mutually exclusive events, then P(A $\cup$ B) = P(A) + P(B)

  Then function P is said to be probability set function on S(U) and the real number P(A) is said to be the probability of event A

Definitions

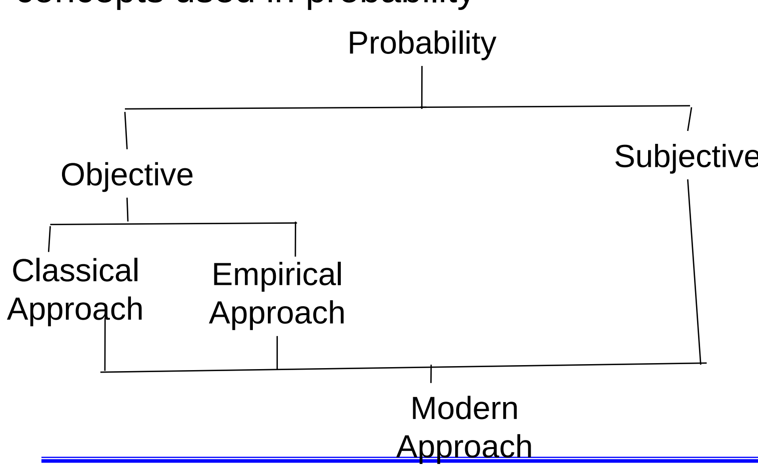
- **Elementary event**: If U = $\{x_1, x_2, \ldots, x_n\}$ is a sample space, then the single element subset $\{x_1\}, \{x_2\}, \ldots, \{x_n\}$ of U are called elementary events or primary events.
- If the probability of each primary event is same, then the primary event are called equi-probable
- If the above n primary events are equi-probable then the probability of each primary event is 1/n.

Definitions

- **Probability of an event / Classical definition of probability:**
  If 'S' be the sample space, then the probability of occurrence of an event 'E' is defined as:
- $P(E) = n(E)/N(S) = \dfrac{\text{(number of elements in 'E')}}{\text{(number of elements in sample space 'S')}}$
- **Example:**
  – Find the probability of getting a tail in tossing of a coin.
- Solution:
  – Sample space S = {H,T} and n(s) = 2
  – Event 'E' = {T} and n(E) = 1 therefore P(E) = n(E)/n(S) = ½
- Note: This definition is not true, if (a) The events are not equally likely. (b) The possible outcomes are infinite.

Types

- The following is the broad classification of the concepts used in probability



Classical Approach

- If a random experiment results in N exhaustive mutually exclusive and equally likely outcomes out of which m are favourable to the happening of an event A, then the probability of occurrence of A, usually denoted by P(A) is given by : $P(A) = m / N$
- Example : What is the chance of getting king in a draw from the pack of 52 cards?
- Solution : Total number of cases that can happen = 52
  No. of favourable case = 4

  $\therefore$ Probability of drawing a king = 4 / 52 = 1/ 13

Empirical Approach

- Empirical concept the probability of an event ordinary represents the proportion of times, under identical circumstances, the outcome can be expected to occur.
- The value refers to the event's long run frequency of occurrence.
- The main assumptions are :
  - The experiments or observations are random. As there is no bias in favour or any outcome all elements enjoy equal chance of selection.
  - There are a large number of observations.

Empirical Approach

- "If the experiment be repeated a large number of times under essentially identical conditions, the limiting value of the ratio of the number of times the event A happens to the total number of trails of the experiments as the number of trails increases indefinitely, is called the probability of the occurrence of A."

Empirical Approach (Example)

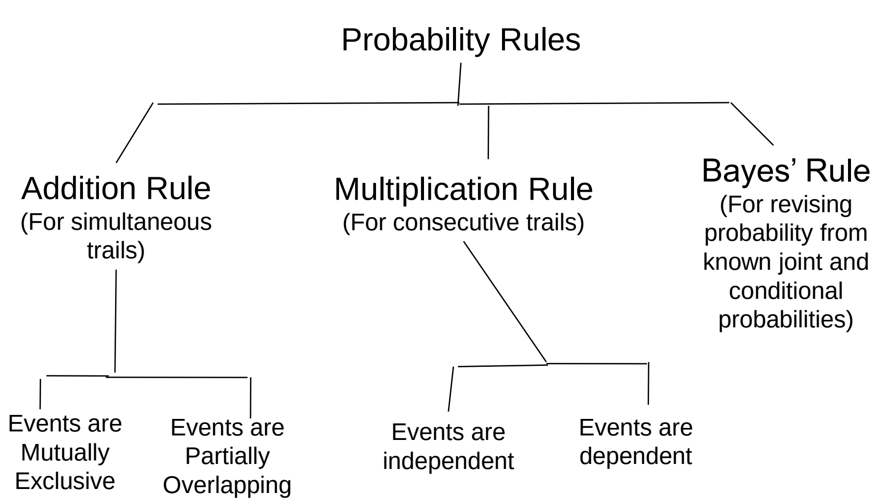
- A foreman in a factory examines the lots of 100 parts each after an interval of half hour during the day and records the number of defective parts. The day's record of 16 lots reveals the following number of defective parts.

No. of Defective parts	No. of lots
0	1
1	4
2	5
3	3
4	2
5	1

Subjective Probability

- It measures the confidence that an individual has in the truth of a particular proposition. It is bound to vary with person to person and is therefore called subjective probability.

Basic Probability Rules



Addition Rule (Mutually Exclusive Events)

- If A and B are mutually exclusive events, then
  $P(A \cup B) = P(A) + P(B)$
  In other words : P(A or B) = P(A) + P(B)
- Example
  The probability that a company executive will travel by train in 2/3 and the he will travel by plane is 1/5. the probability of his travelling by train or plane is :
- Solution :
  P(T or P) = P(T) + P(P)
  = 2/3 + 1/5 = 13/15
  the probability of not travelling by either train or plane =
  1- P(T or P) = 1 – 13 / 15 = 2 /15.

Addition Rule (Not Mutually Exclusive Events)

- If A and B are any two events, then
  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  In other words : P(A or B) = P(A) + P(B) - $P(A \cap B)$
- Generalization : it can be shown that for any 3 events A, B, C
  $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
- In general, for any events $A_1, A_2, \ldots, A_n$ we have
  $P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$ – (sum of probabilities of all possible intersections taken two at a time) + (sum of probabilities of all possible intersections taken three at a time) + ….. + $(-1)^{n-1}P(A_1 \cap A_2 \cap \ldots \cap A_n)$

Addition Rule (Not Mutually Exclusive Events)

- Deduction : If the two events A and B are mutually exclusive i.e. disjoint, then the addition rule of probability reduces to $P(A \cup B) = P(A) + P(B)$
- In general, for any events $A_1, A_2, \ldots, A_n$ we have $P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$

Multiplication Rule (Events Are Independent)

- The probability that both independent events A and B will occur is $P(A \cap B) = P(A) * P(B)$
  In other words : P(A and B) = P(A) * P(B)

Multiplication Rule (Events Are Dependent)

- If event A and B are so related that the occurrence of B is affected by the occurrence of A, then A and B are called dependent events. The probability of event B depending on the occurrence of event A is called conditional probability and is written as P(B / A) which may be read, "the probability of B given A."
- The probability that both the dependent events A and B will occur is $P(A \cap B) = P(A) * P(B/A)$
  In other words : P(A and B) = P(A) * P(B/A)

Conditional Probability

- If it is given that a particular event B has already occurred then the probability of occurrence of event A is the conditional probability of A. It is denoted by the symbol P(A/B). If P(B) > 0 then

$$P(A/B) = \frac{P(A \cap B)}{P(B)}; \ P(B/A) = \frac{P(A \cap B)}{P(A)}$$

- If P(B) = 0 then B does not occur at all and the question of defining P(A/B) does not arise.
- If the events are independent, the happening of B shall not affect the probability of A and therefore

$$P(A/B) = P(A)$$

Joint And Marginal Probability

- Joint probability is the probability of the occurrence of two or more events.
- Normally the probabilities can be expressed in a 2x2 table or joint probabilities table.
- Example : Two events A and B, there are four joint probabilities possible as explained below :

Joint Event	Sample points belonging to the joint event	Probability
$A \cap B$	$n(A \cap B)$	$n(A \cap B) / n(s)$
$A \cap B'$	$n(A \cap B')$	$n(A \cap B') / n(s)$
$A' \cap B$	$n(A' \cap B)$	$n(A' \cap B) / n(s)$
$A' \cap B'$	$n(A' \cap B')$	$n(A' \cap B') / n(s)$

Bayes' Rule

- Joint and marginal probabilities can be used to revise the probability of a particular event in the light of additional information.
- Example we have two box containing defective and non-defective items.
- One item is picked at random from any one of the boxes and is found defective, and now we might like to known the probability that it came from box one.
- To answer questions of this sort, we use Bayes' Rule which may be considered as an application of conditional probabilities.

Bayes' Rule

- The popularity of the theorem has been mainly because of its usefulness in revising a set of old probabilities called prior probabilities (derived subjectively or objectively) in the light of additional information made available and derive a set of new probabilities called the posterior probabilities.
- Although Bayes' Rule may be applied to more than two mutually exclusive and exhaustive events, we shall for the sake of simplicity confine generally to the application of Bayes' rule for two mutually exclusive and exhaustive events.

Bayes' Rule

- We know that the marginal probabilities are the sum of the two relevant joint probabilities as indicated below.
  - $P(A) = P(A \cap B) + P(A \cap B')$
  - $P(B) = P(A \cap B) + P(A' \cap B)$
- We can restate them in terms of conditional and marginal probabilities as follows
  - $P(A) = P(A/B) * P(B) + P(A / B') * P(B')$
  - $P(B) = P(B / A) * P(A) + P(B / A') * P(A')$
- Now recollect the original formulation of conditional probabilities viz $P(A/B) = P(A \cap B) / P(B) = P(B \cap A) / P(B)$ which can be written as

Bayes' Rule

$$P(A/B) = \frac{P(B/A) * P(A)}{P(B/A) * P(A) + P(B/\overline{A}) * P(\overline{A})}$$

- Similarly

$$P(B/A) = \frac{P(A/B) * P(B)}{P(A/B) * P(B) + P(A/\overline{B}) * P(\overline{B})}$$

- Proceeding on the same lines, other conditional probabilities viz P(A'/B) and P(B/A') can be revised.

Bayes' Rule

- Generalization : If an event B can only occur in conjunction with one of the n mutually exclusive and exhaustive events A_1, A_2, ….., A_n and if B actually happens, then the probability that it was preceded by the particular event A_i (i = 1, 2, …., n) is given by
  $P(A_i / B) = P(A_i \cap B) / P(B)$ i = 1, 2, …., n
  and $P(B) = P(A_1 \cap B) + P(A_2 \cap B) + \ldots\ldots P(A_n \cap B)$

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