TASK 8: Grover's algorithm for a 3-qubits database

Aim: To implement Grover's quantum search algorithm for a 3-qubit search space (8 items) using Cirq, and demonstrate that the marked item (target state) can be found with high probability after the optimal number of iterations

Algorithm - Grover's algorithm for a 3-qubits database:

1. Initialize 3 qubits to |0⟩.
2. Create uniform superposition by Hadamard gates H⊗3.
3. Repeat k = 2 times:

- Apply oracle marking the target bit string with phase flip on the marked state.
- Apply diffusion operator (inversion about the mean).

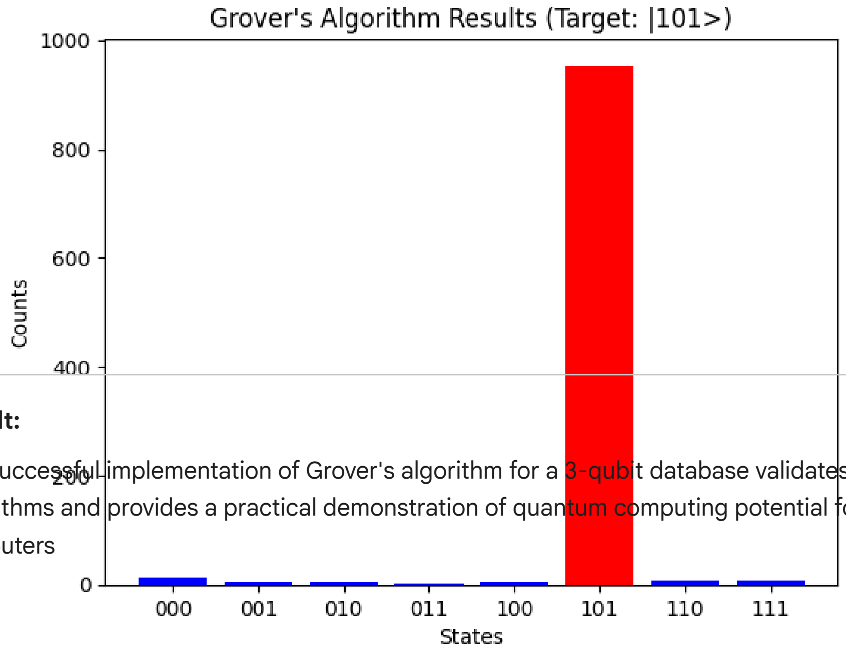
4. Measure the qubits to obtain results peaked at the target state with high probability

```
import cirq
import numpy as np
import matplotlib.pyplot as plt
def grover_3_qubit(target_binary):
    qubits = [cirq.GridQubit(0, i) for i in range(3)]
    circuit = cirq.Circuit()
    # Initialize superposition
    circuit.append(cirq.H.on_each(*qubits))
    # Removed: circuit.append(cirq.Barrier(*qubits)) # Barrier after initialization
    # Number of Grover iterations
    N = 2 ** 3
    iterations = int(np.floor(np.pi/4 * np.sqrt(N)))
    for iteration in range(iterations):
        # Oracle
        apply_oracle(circuit, qubits, target_binary)
        # Removed: circuit.append(cirq.Barrier(*qubits)) # Barrier after oracle
        # Diffusion
        apply_diffusion(circuit, qubits)
        # Removed: circuit.append(cirq.Barrier(*qubits)) # Barrier after diffusion
    # Measurement
    circuit.append(cirq.measure(*qubits, key='result'))
    return circuit, qubits
def apply_oracle(circuit, qubits, target_binary):
    # Apply X gates where target bit is 0
    for i, bit in enumerate(target_binary):
        if bit == '0':
            circuit.append(cirq.X(qubits[i]))
    # Multi-controlled Z using H and CCX
    circuit.append(cirq.H(qubits[-1]))
    circuit.append(cirq.CCX(qubits[0], qubits[1], qubits[2]))
    circuit.append(cirq.H(qubits[-1]))
    # Undo X gates
```

```
        for i, bit in enumerate(target_binary):
            if bit == '0':
                circuit.append(cirq.X(qubits[i]))
    def apply_diffusion(circuit, qubits):
        circuit.append(cirq.H.on_each(*qubits))
        circuit.append(cirq.X.on_each(*qubits))
        circuit.append(cirq.H(qubits[-1]))
        circuit.append(cirq.CCX(qubits[0], qubits[1], qubits[2]))
        circuit.append(cirq.H(qubits[-1]))
        circuit.append(cirq.X.on_each(*qubits))
        circuit.append(cirq.H.on_each(*qubits))
    def analyze_results(counts, target):
        total = sum(counts.values())
        success = counts.get(int(target, 2), 0)
        success_rate = success / total * 100
        print(f"Measurement results for target |{target}>:")
        for state in range(8):
            bitstr = format(state, '03b')
            count = counts.get(state, 0)
            pct = count / total * 100
            marker = "<-- Target" if bitstr == target else ""
            print(f"State |{bitstr}>: {count} times ({pct:.2f}%) {marker}")
        print(f"\nSuccess rate: {success_rate:.2f}% (optimal ~94% after 2 iterations)")
        states = [format(i, '03b') for i in range(8)]
        values = [counts.get(i, 0) for i in range(8)]
        colors = ['red' if s == target else 'blue' for s in states]
        plt.bar(states, values, color=colors)
        plt.title(f"Grover's Algorithm Results (Target: |{target}>)")
        plt.xlabel("States")
        plt.ylabel("Counts")
        plt.show()
    if __name__ == "__main__":
        target = "101"
        circuit, qubits = grover_3_qubit(target)
        print("Circuit diagram:")
        print(circuit)
        simulator = cirq.Simulator()
        result = simulator.run(circuit, repetitions=1000)
        counts = result.histogram(key='result')
        analyze_results(counts, target)
```

```
Circuit diagram:
(0, 0): ───H───────@───H───X─────────@───X───H──────────@───H───X─────────@───X───H───────M('result')───
                   │                 │                  │                 │               │
(0, 1): ───H───X───@───X───H───X─────@───X───H───X──────@───X───H───X─────@───X───H───────M─────────────
                   │                 │                  │                 │               │
(0, 2): ───H───H───X───H───H───X───H─X───H───X───H───H──X───H───H───X───H─X───H───X───H───M─────────────
Measurement results for target |101>:
State |000>: 14 times (1.40%)
State |001>: 4 times (0.40%)
State |010>: 4 times (0.40%)
State |011>: 3 times (0.30%)
State |100>: 6 times (0.60%)
State |101>: 954 times (95.40%) <-- Target
State |110>: 8 times (0.80%)
State |111>: 7 times (0.70%)

Success rate: 95.40% (optimal ~94% after 2 iterations)
```



Result:

The successful implementation of Grover's algorithm for a 3-qubit database validates the theoretical foundations of quantum search algorithms and provides a practical demonstration of quantum computing potential for solving problems more efficiently than classical computers