2) Elimination with matrices.

$$x + 2y + z = 2$$

$$3x + 8y + z = 12$$

$$4y + z = 2$$

Pivot

$$\begin{matrix} \boxed{1} & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{matrix} \xrightarrow{3xR1} \begin{matrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{matrix}$$

$$\xrightarrow{R_3 - 2R_2} \begin{matrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{matrix}$$

Back-subsitution

↳ Augmented matrix

$$\left[\begin{matrix} a & b \\ c & d \end{matrix}\right|$$

An example of augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & 1 & 2 \end{array}\right]$$

$$\underbrace{\begin{bmatrix} 1 & 2 & 7 \end{bmatrix}}_{1 \times 3} \underbrace{\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}}_{3 \times 3} \begin{array}{l} R1 \\ R2 \\ R3 \end{array}$$

Matrix × column = column

matrix + row = row

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Permutation rows:

$$\underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{P} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

Finding inverses

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E^{-1}} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{I}$$