① Geometry of Linear Equations

$$2x - y = 0$$
$$-x + 2y = 3$$

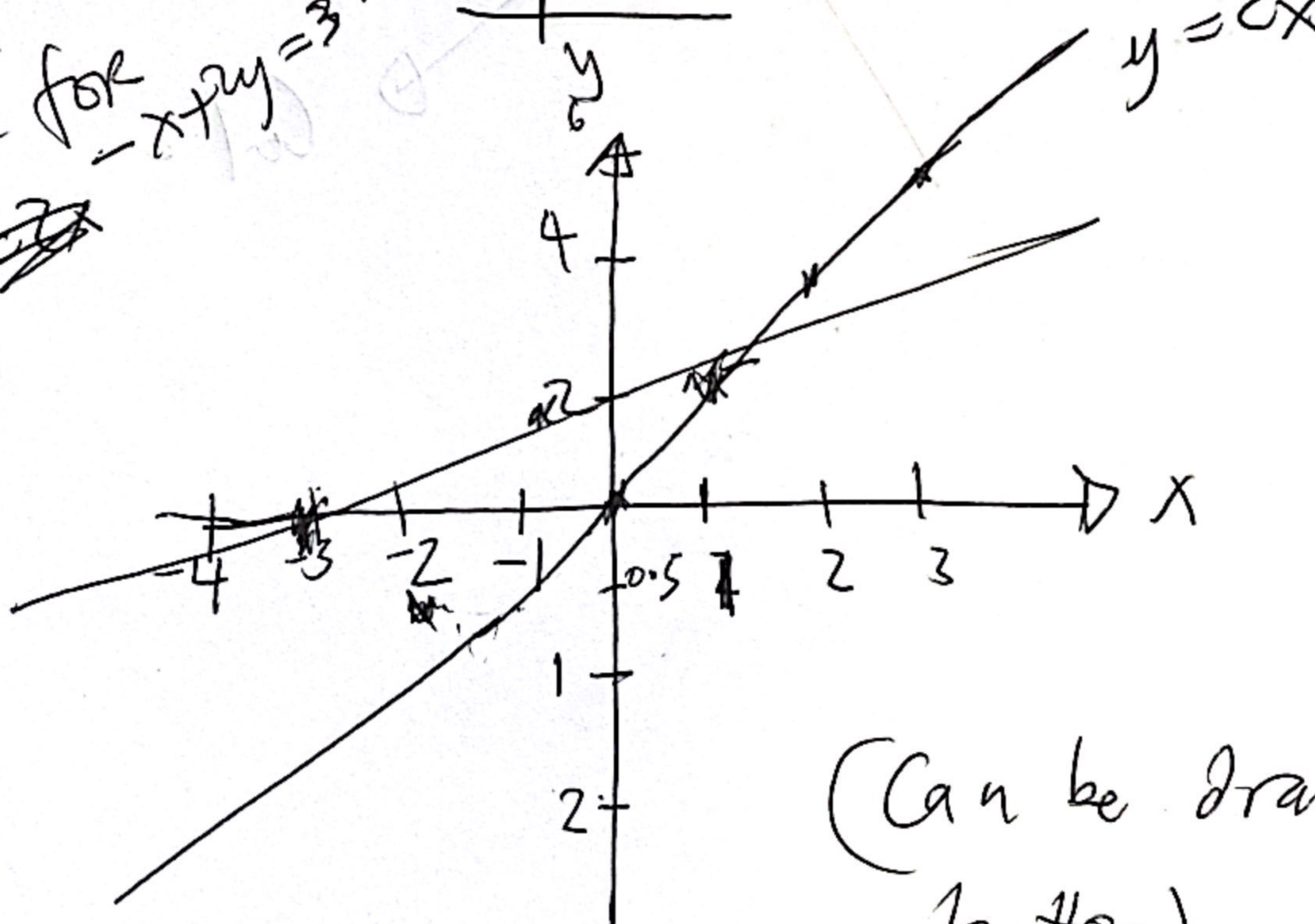
$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$A \quad\quad x = b$$

linear equations → $Ax = b$

Row picture

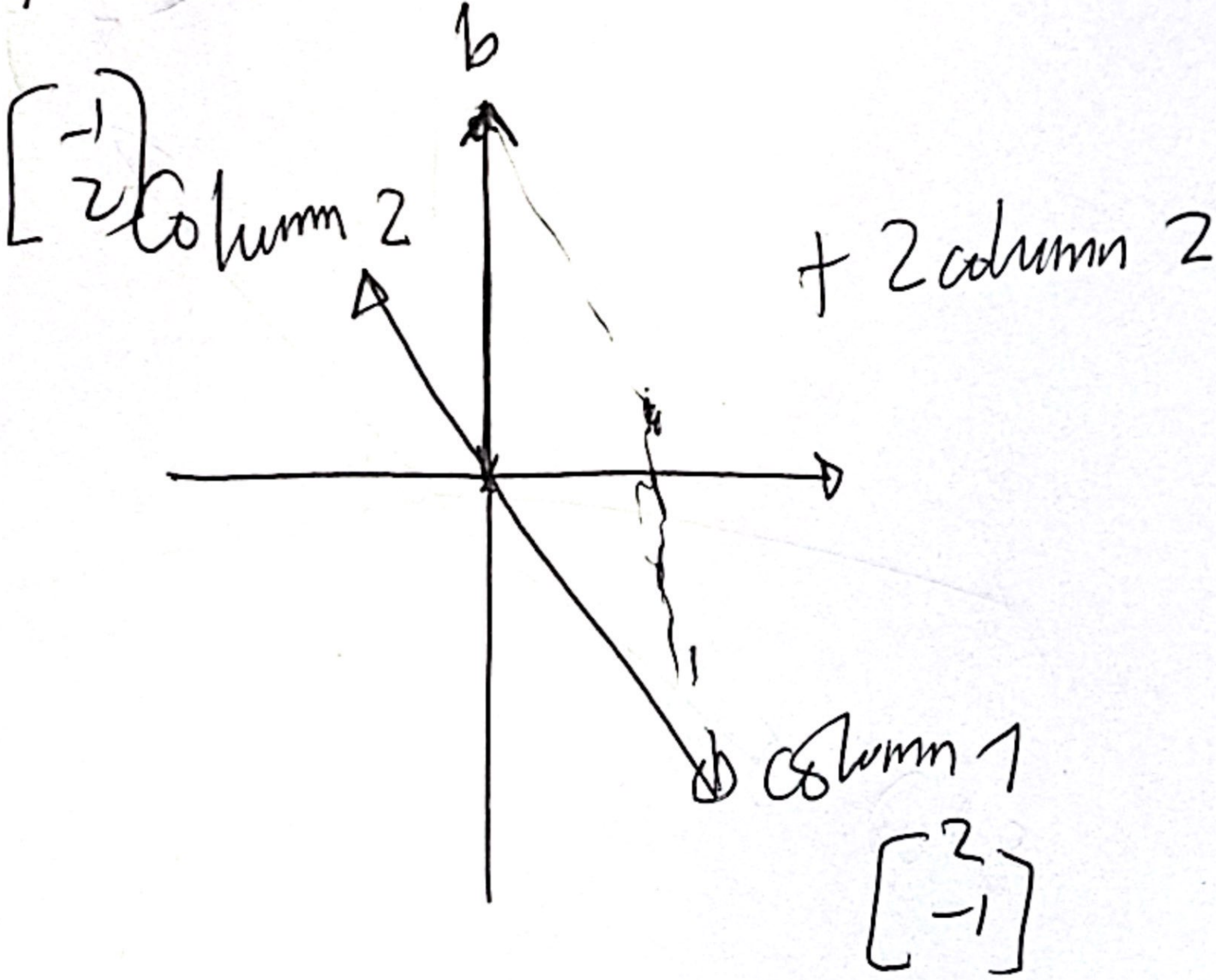
line for $-x+2y=3$



(Can be drawn better)

$$x\begin{bmatrix}2\\-1\end{bmatrix} + y\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}0\\3\end{bmatrix}$$

This is a <u>linear combination</u> of the columns.



We need the best combinatio of x,y to get

example $(x,y) = (1,2)$.

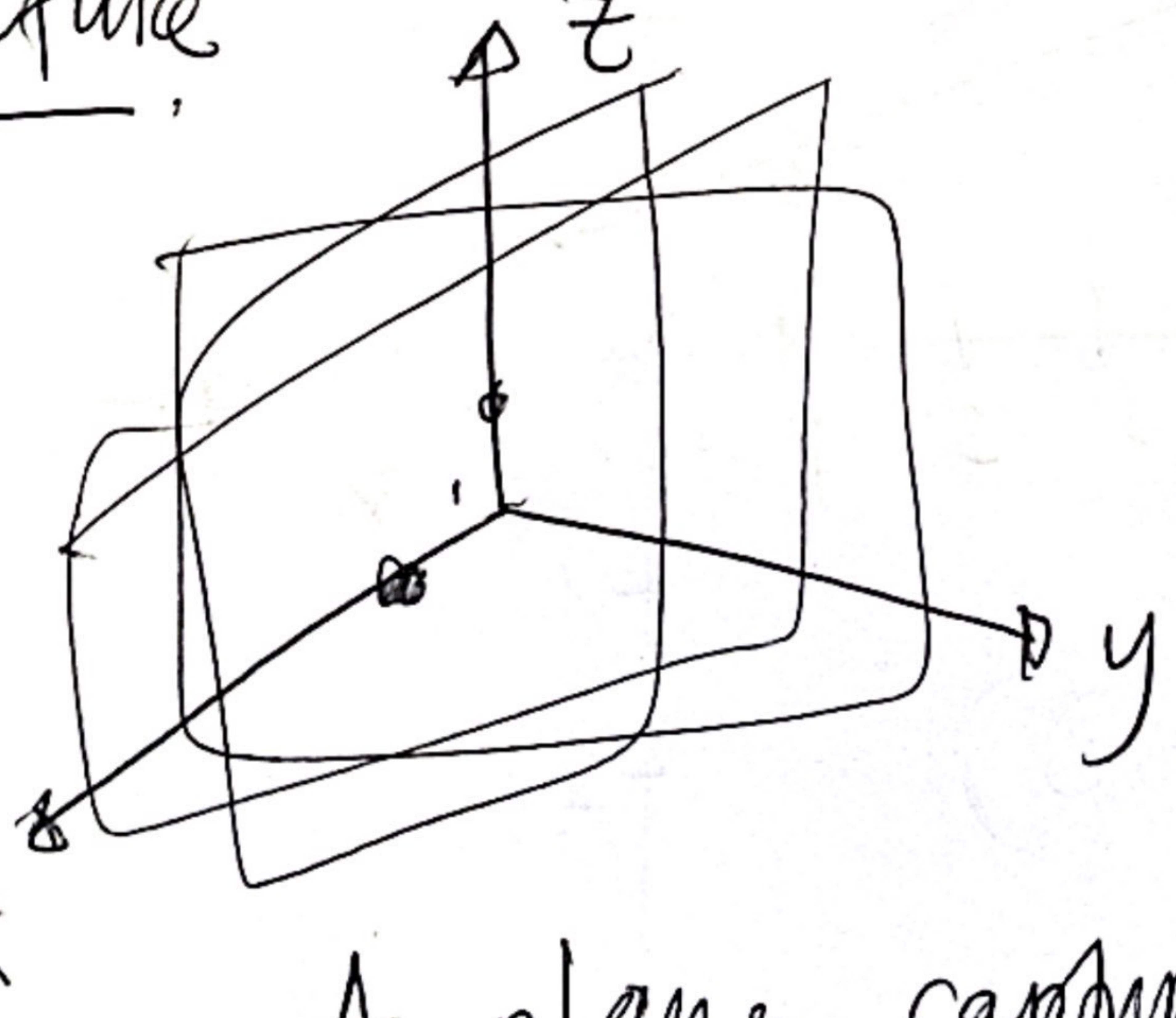
Let's go 3D:

$$2x - y = 0$$
$$-x + 2y - z = -1$$
$$-3y + 4z = 4$$

Matrix form

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

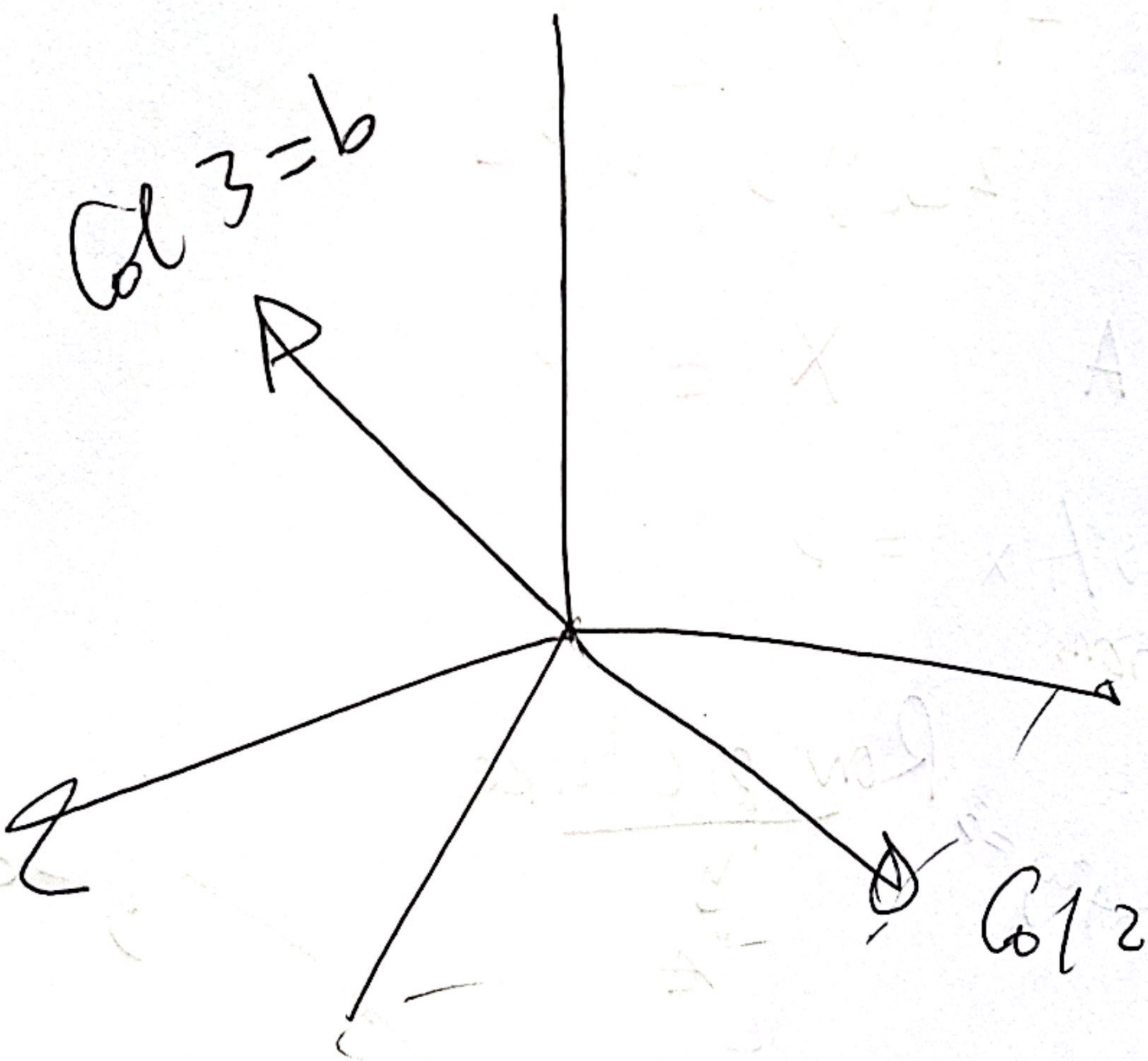
Row picture



A plane captures all the points that solves the problem.

Column Picture

$$x\begin{bmatrix}2\\-1\\0\end{bmatrix}+y\begin{bmatrix}-1\\2\\-3\end{bmatrix}+z\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}0\\-1\\4\end{bmatrix}$$



Can I solve $Ax=b$ for every b?

In better language.

Do the linear combo's of the columns fill 3-D space? Yes!

Non-singular matrix?

If all 3 columns lie in the same, their combinations would lie in the same plane.

hr: col 3 = col 1 + col 2

That'd be a singular case.