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The crash of October 1987 seen as a phase transition: amplitude and universality

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Abstract

We analyze the evolution of several financial indices before the crash of October 1987. The amplitude of the crash varies from one index to another. However, assuming that the crash is similar to a phase transition and particularly to a specific heat jump, we find that the crash amplitude can be well estimated by assuming a simple background which differs from market to market. We show that the divergence near the crash event is logarithmic and extends between 2 weeks and 4 years before the october 1987 crash on both S&P500 and Dow Jones indices. The behavior is like that found for the $d = 2$ Ising model specific heat. The latter result is in contrast to previous works which have considered a power law behavior of the index near the crash. Finally, we confirm the presence of log-periodic oscillations and discuss briefly their origin. ©

1. Introduction

Even though stock market crashes are considered as highly rare and unpredictable events, it should be noted that they do take place systematically during periods of generalized economic euphoria. Are we able to quantify that euphoria? Are we able to observe precursors of a crash?

In recent independent works, Sornette et al. [1] and Feigenbaum et al. [2] underlined the similarities between an economic crash and a phase transition. This idea has led these authors to search for the rupture point t_c, i.e. the prediction of the date of the crash itself, through the analysis of the precursor pattern using physical means and specifically "discrete scale invariance" [3].

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It is known that time series analysis is a very subtle matter and much caution must be taken before a definite conclusion is reached [4]. Beside mathematical aspects and criteria, physicists can nevertheless use some analogy for sorting out seemingly valid results.

In the present paper, we reinvestigate the crash of October 1987 from the classical phase transition point of view. We do not try here to forecast the crash event as previous work attempted [1]. In the present work, we try however to make a bridge between the crash and the jump of a "physical quantity" at a critical point t_c. We are also interested in the fundamental question of universality of the "crash" transition. In order to do so, we apply usual techniques to sort out critical exponents, i.e. taking into account the background first. This is known to be necessary in many cases as recently shown also in high-T_c superconductors [5].

2. Economic crash as a phase transition

At some phase transition point t_c, it is known [6] that the specific heat C presents some finite jump ΔC or behaves like

$$C = \begin{cases} \frac{A}{\alpha}(t_c - t)^{-\alpha} + C_{mf}(t) + C_{bg}(t) & \text{for } t < t_c\,, \\ \frac{A'}{\alpha'}(t - t_c)^{-\alpha'} + C'_{mf}(t) + C_{bg}(t) & \text{for } t > t_c\,, \end{cases} \tag{1}$$

where the first term represents the fluctuation contribution, the second term represents the usual mean-field contribution and the third term represents the background which is often due to "impurities" and is t_c-independent. The critical exponents α and α' can take positive or negative values. The mean-field contributions C_{mf} and C'_{mf} are analytic in t. C is generally asymmetric around t_c as it is the case e.g. in the XY model [6]. In the so-called mean-field approximation, the jump $|C_{mf}(t \to t_c^-) - C'_{mf}(t \to t_c^+)|$ is a measure of the number of relevant components of the system and is independent of the interaction scale [7]. One should note that the first and second terms are sometimes considered as a single contribution [5].

A schematic representation of a specific heat discontinuity is shown in Fig. 1. The background $C_{bg}(t)$ is exponential in the illustrative case of Fig. 1, but can have other usually simple analytic form. The contribution C_{bg} is denoted by the dotted line. The mean-field contribution C_{mf} is denoted by the dashed line while the total (C) is represented by the continuous line.

Other types of singularities exist at phase transition points but need not be discussed here. Indeed, they are arising for quantities, like the susceptibility or the compressibility derived in the presence of a small external field. In order to avoid the introduction of such a field in the analogy, we concentrate our attention to a quantity for which the field (i.e. temperature) is directly related to the internal disorder, i.e. a change in entropy

$$C = T\frac{\partial S}{\partial T}\,. \tag{2}$$

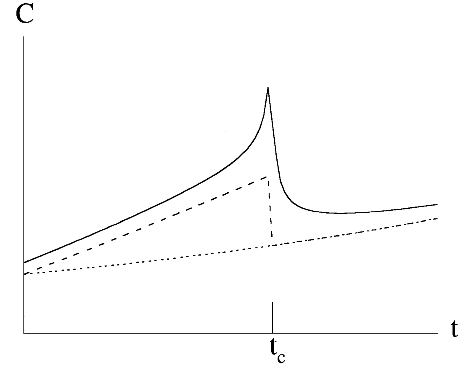

Fig. 1. Schematic illustration of the specific heat (continuous curve) near a phase transition at t_c. The jump of the specific heat can be decomposed into three additive contributions due to the impurities (dotted curve), mean-field (dashed curve) and fluctuations.

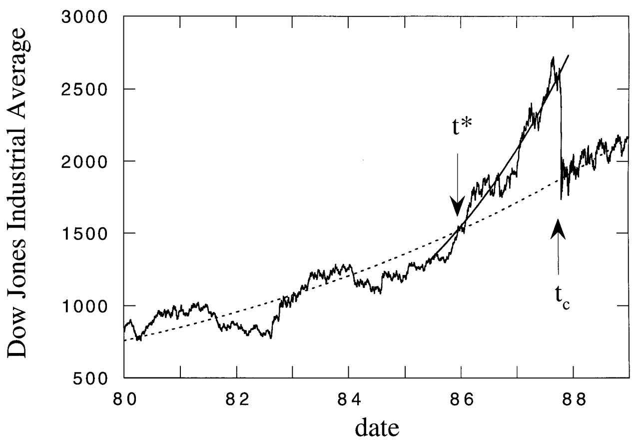

Fig. 2. The daily evolution of the Dow Jones Industrial Average from January 1980 to December 1988. Note the similarities of this evolution with Fig. 1. The dotted curve represents an exponential background with a rate $r \approx 0.1\,\mathrm{yr}^{-1}$ and the continuous curve that of an "anomalous" growth rate with $r \approx 0.3\,\mathrm{yr}^{-1}$ during the period from t^* to t_c.

Moreover, the following discussion will show that the analysis is powerful enough without introducing any external field.

Turning to "experimental data", Fig. 2 exhibits the daily evolution [8] of the Dow Jones Industrial Average from January 1980 till December 1988. At the so-called *black* monday, on October 19th, 1987 the Dow Jones index dropped by about 20%. By comparing Fig. 2 with Fig. 1, one can recognize some similarities between the specific heat and an economic index at t_c, i.e. some sort of divergence and obviously the jump.

What kind of useful information can be extracted from this analogy between both phenomena is the subject of the next sections.

3. The jump ΔC

If a crash is predictible as suggested by Sornette et al. [1], the next step is to predict the amplitude ΔC of the event. Indeed, this information should allow economic agents to hedge portofolios on derivative markets through call or put options. This problem reduces in fact to extract the true background contribution C_{bg} of an index C. Indeed, C_{bg} represents the "natural" evolution of stock markets out of any euphoric ("anomalous") evolution. This is a classical step in phase transitions before searching for critical exponents.

We assume first that the natural evolution of a financial index is given by

$$C_{bg} = a \exp(rt) \tag{3}$$

with time t in units of years and $r \ll 1$, i.e. an exponential growth with a slow rate r of e.g. a few percent ($\approx$ 7–11%) a year. Other cases like a stretched exponential or a high order polynomial would introduce extra parameters and would hide the basic arguments. Nevertheless, a proof that (2) is wholly correct will require further work. Such an exponential background is illustrated in Fig. 2 by the dotted line. At t^*, a crossover between the slow growth rate ($r \approx 0.1\,\mathrm{yr}^{-1}$) to a high growth rate ($r \approx 0.3\,\mathrm{yr}^{-1}$) is also indicated in Fig. 2 by a continuous line. During the period from t^* till t_c, the market shares were anomalously overestimated. We will study in Section 4 the evolution of the markets during this period.

We have fitted the exponential background C_{bg} on several index data from 1980 to t^*, and we have extrapolated this exponential evolution for $t > t_c$. Taking the hypothesis that such indices recover their "natural" value after the crash, the jump ΔC can be "predicted".

Fig. 3 collects *a posteriori* predictions/measures for ΔC of the major indices near the crash of October 1987. The amplitude is given in percent. The amplitude results from the difference between a one week average of the index before the jump and of a one week average after the crash. A linear background has also been used for comparison. For the majority of indices, the prediction resulting from substracting an exponential background from the data is seen to be close to that measured at the true jump. The extrapolation with a linear background results in a worse finding than the exponential one. However, the agreement is not so good in the case of the Asian indices for which the "theoretical" jump is overestimated. In other words, we should conjecture that the Asian indices have not dropped to their respective "natural" value at t_c. Thus, Asian markets should be considered as a "separate class" with a different mean field background as that of Eq. (2). Indeed, the Asian markets are known to have increased rapidly after 1987 and have reached another rupture point in 1989 through a "speculative bubble" leading to huge volatility of these indices [9]. These huge fluctuations are known to be still present nowadays and quite annoying from a world economic stability point of view.

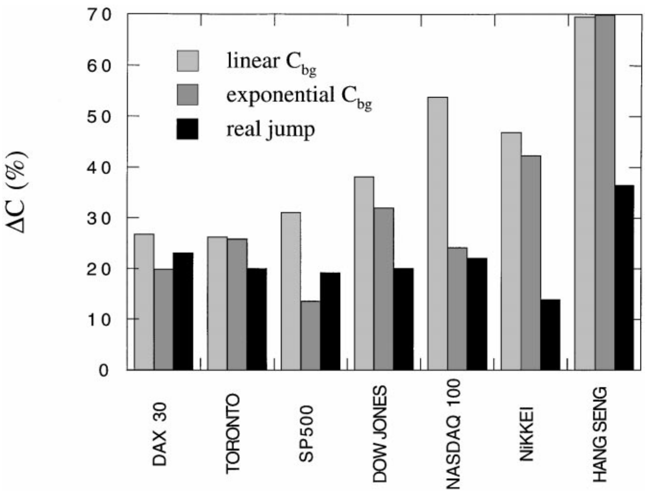

Fig. 3. Histogram of predicted jump amplitudes and real jumps for some major financial indices. Linear and exponential backgrounds distinguished by different grey levels have been used in order to extrapolate the evolution of indices.

4. The precursory divergence

We have studied hereabove the jump of C at t_c and seen that the jump ΔC is non-universal, i.e. varies from an index to another. The divergence of C close to t_c in terms of critical exponents is of physical interest. The existence of universal exponents will be envisaged also in this section.

In order to obtain a good estimate of the parameters a and r in Eq. (2), we have used post-crash data $(t > t_c)$ as well as $t < t^*$ data for the background fit. Having extracted the "best" exponential background contribution C_{bg} from the various indices, we have plotted $C - C_{bg}$ as a function of $(t - t_c)$. As a consequence, the $C - C_{bg}$ data drops to zero at t_c.

Fig. 4a and b presents such data in either (a) a semi-log or (b) a log–log plot in the case of the Standard & Poor 500 index while the same analysis is made in the case of the Dow Jones Industrial Average in Figs. 5a and b. The plots in Figs. 4a and 5a, i.e. the semi-log plots, appear at first better on the average even though structures (oscillations) are seen. The straight line for $t < t_c$ indicates a logarithmic divergence.

In the spirit of phase transition considerations, it can then be expected that taking into account fluctuations thus going beyond a mean field approximation C is infinite at t_c. Obviously, this cannot be the case in reality since stock markets fall before reaching $C = \infty$. This is due to inhomogeneities [11] or impurities [12]. These "finite-size" effects are observed for $t_c - t < 15$: $C - C_{bg}$ decreases as t approaches t_c.

The oscillations are denoted by downwards arrows in Fig. 4. Even with the presence of these oscillations, the logarithmic divergence seems quite plausible. The logarithmic

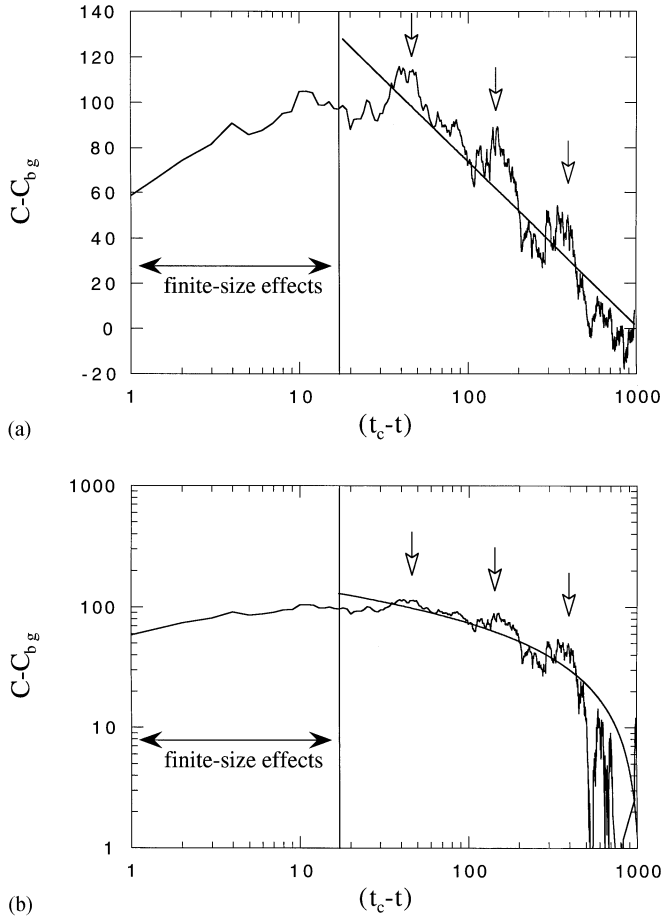

Fig. 4. The Standard & Poor 500, data represented as $C - C_{bg}$ as a function of $(t_c - t)$ in (a) a semi-log plot and in (b) a log–log plot between 01/01/1984 and 19/10/1987. "Finite size effects" are emphasized at low $t_c - t$. The continuous line represents the fit of $C - C_{bg}$ by a logarithmic divergence on both graphs. Downwards arrows denote the maxima of log-periodic oscillations of the index (see text).

divergence (denoted by the continuous line in both Figs. 4 and 5) holds in fact over a long period extending from 2–3 weeks to 4 years before t_c! For all indices studied in the present work, except Asian indices, we have found a better fit for a logarithmic divergence as if a stock market index behaves like the Ising model in $d = 2$ [13].

These findings are neither in agreement with the results of Sornette et al. [1] who have suggested $\alpha \neq 0$ nor with Feigenbaum and Freund [2] who have reported various values of α ranging from 0.53 to 0.06 for various indexes and events (upsurges and crashes). In [1,2], these authors do not have considered the background effect and variations of α as resulting from such a background as here. In a more recent work [10], Sornette and Johansen have removed an exponential drift of the index before

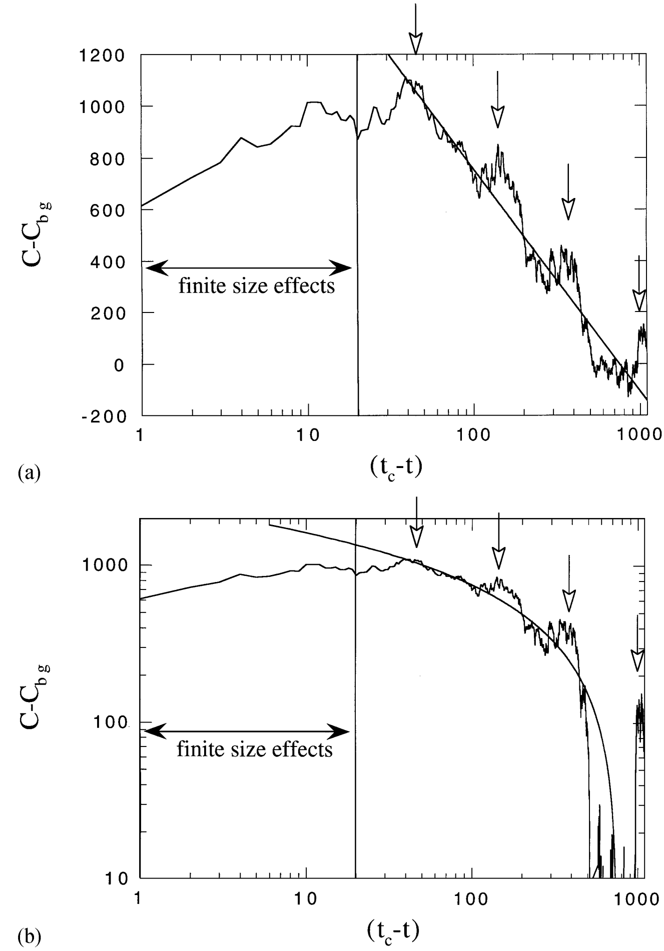

Fig. 5. The Dow Jones Industrial Average, data represented as $C - C_{bg}$ as a function of $(t_c - t)$ in (a) a semi-log plot and in (b) a log–log plot between 01/01/1984 and 19/10/1987. Finite size effects are emphasized at low $t_c - t$. The continuous line represents the fit of $C - C_{bg}$ by a logarithmic divergence. Downwards arrows denote the maxima of log-periodic oscillations of the index.

performing power law fits. They have obtained a value of α close to -0.63 for a different power law presenting no divergence but rather a convergence for $t \to t_c$ [10].

We believe that the logarithmic divergence seems more plausible than a power law behavior. This suggests also the existence of "universality" in the statistical physics sense with $\alpha \to 0$ for any stock market crash. One should recall that a log-divergence $(\alpha = 0)$ is encountered in low dimensional systems [14] such as the $d = 2$ Ising model, the $d = 3$ XY model. The XY models are often used to study vortices or complex flows like the cooperative flight of birds [15] a.s.o. Thus, the modelling of stock market crashes seems to be possible through the analogy with spin models.

5. The log-periodic oscillations

The oscillations denoted by the arrows in Figs. 4 and 5 can be further analyzed. It is found that they are log-periodic. Such oscillations have been associated to an underlying discrete scale invariance of the stock market described by a complex exponent $\alpha + i\omega$ [10]. It means that a hidden hierarchical structure (like a Cantor set) exists "behind" the stock markets. One should remind that the relevant parameter of the "crash transition" is the time t. Are financial markets characterized by a discrete set of time scale? The answer is "yes". Two arguments follow.

First, different types of investors can be found in the economic market ranging from banks to individuals. Table 1 lists different categories of investors and their relative weight on the stock market of London in 1993 [16]. Each class of investors is characterized by a specific horizon [17]: traders try to win money on daily (and intraday) fluctuations while individuals are known to be long-term investors and are looking e.g. for annual returns. Besides the "discreteness" of the hidden structure of stock markets, scale invariance is also needed. It should be emphasized that the weight of investor types as a function of their rank is roughly a power law as shown in Fig. 6. This is similar to the results of Lévy and Solomon about "the power law distribution of wealth" [18]. For the investor types, we have found an exponent close to -1.8 characterizing some scale invariance.

A second argument for the hypothesis of a possible discrete scale invariance in time is the fact that the markets are punctuated by periodic events as e.g. the opening–closing of markets, the expiry of options every 3rd Friday of each month, the results of companies published every quarter or semester, the regular meetings of central banks boards, etc. Thus, there exists an intrinsic hierarchy of discrete time scales (periods) behind the markets.

Table 1
The different categories of investors and their relative weight on the London stock market in 1993 (after [16])

Rank	Investor type	Weight
1	Pension Funds	34.2
2	Individuals	17.7
3	Insurance companies	17.3
4	Foreign	16.3
5	Unit Trusts	6.6
6	Others	2.3
7	Other personal sector	1.6
8	Industrial and commercial companies	1.5
9	Public Sector	1.3
10	Other Financial Institutions	0.6
11	Banks	0.6

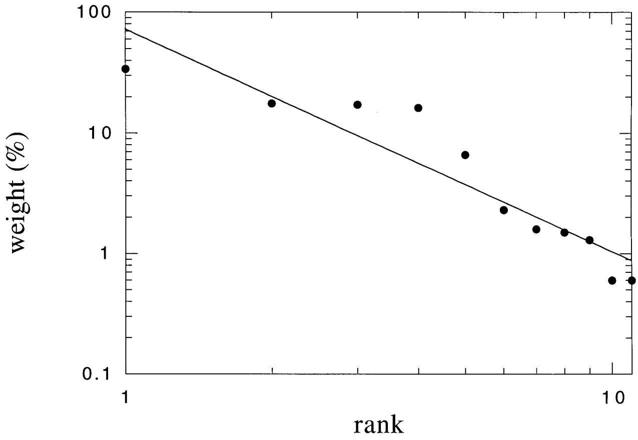

Fig. 6. The so-called weight of different categories of common investor types in the financial markets of London in 1993 as a function of their respective rank (after [16]). These categories of investors are listed in Table 1.

Taking into account the log-periodic corrections (due to the discreteness contribution iω), we propose the following general law

$$C - C_{bg} = C - ae^{rt} \approx A + B\ln(t - t_c)[1 + D\cos(\omega\ln(t - t_c) + \phi)]\,. \qquad (4)$$

Notice that this formula reduces the number of free parameters of the formula of Sornette et al. [1] in order to forecast t_c since $\alpha = 0$. We have used this law to forecast *a posteriori* the crash of 1987 and to predict the possibility of a crash for the fall of 1997 [19], as proved thereafter [20]. This should lead to some discussion, but outside the scope of the present paper.

6. Conclusion

We have investigated along physical feature lines the crash of October 1987 following the ideas used at a phase transition. We have analyzed the background contribution which has not been considered in previous works. We have shown that stock market indices have different underlying backgrounds. The mean-field jump has been estimated. We have shown next that the logarithmic divergence of indices before the crash event is a plausible law. This has been demonstrated by removing the background contribution. This results suggest that Ising-like spin models are relevant to model financial markets before “crashes”. Moreover, the logarithmic divergence has been verified on various indices meaning that the crash itself has “universal” features. We have briefly argued that the specific heat is the most simple thermodynamic quantity to be used in an analogy between phase transition physics and stock market index. At this stage there has been no need for the introduction of any external field.

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