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Traders' Long-Run Wealth in an Artificial Financial Market

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Abstract. In this paper, we study the long-run wealth distribution of agents with different trading strategies in the framework of the Genoa Artificial Stock Market. The Genoa market is an agent-based simulated market able to reproduce the main stylised facts observed in financial markets, i.e., fat-tailed distribution of returns and volatility clustering. Various populations of traders have been introduced in a 'thermal bath' made by random traders who make random buy and sell decisions constrained by the available limited resources and depending on past price volatility. We study both trend following and trend contrarian behaviour; fundamentalist traders (i.e., traders believing that stocks have a fundamental price depending on factors external to the market) are also investigated. Results show that the strategy alone does not allow forecasting which population will prevail. Trading strategies yield different results in different market conditions. Generally, in a closed market (a market with no money creation process), we find that trend followers lose relevance and money to other populations of traders and eventually disappear, whereas in an open market (a market with money inflows), trend followers can survive, but their strategy is less profitable than the strategy of other populations.

Key words: artificial financial markets, market simulations, wealth distribution, trading strategies, trading behaviour, asset prices, econophysics

1. Introduction

Understanding if there are winning and losing market strategies and determining their characteristics is an important question for investors and regulators alike. It seems obvious that different investors exhibit different investing behaviours that are, at least partially, responsible for the time evolution of market prices. But it is difficult to reconcile the regular functioning of financial markets with the coexistence of different populations of investors. If there were a consistently winning market strategy, then it is reasonable to assume that the losing population would disappear in the long run. Riedman first advanced in 1953 the hypothesis that, in the long run, irrational investors cannot survive, as they tend to lose wealth and disappear. However, an operational definition of rational investors presents conceptual difficulties since all investors are boundedly rational. No agent can realistically claim to have the kind of supernatural knowledge needed to formulate rational expectations. The fact that different populations of agents with different strategies prone to forecast errors can coexist in the long run is a fact that still requires explanation. In this paper, we will first describe the Genoa Artificial Stock Market (GASM) and introduce the general setting of our computational experiments. We then present experimental results on the price processes and the wealth distribution of agents, which provides a possible interpretation of results.

2. The Genoa Artificial Stock Market

The Genoa Artificial Stock Market (GASM) is a computational laboratory conceived to offer a simulated experimental facility with realistic trading features. In the last decade, an increasing number of agent-based computer-simulated artificial financial markets have been proposed, for a review see LeBaron (2000) and Levy et al. (2000). A special mention is devoted to the pioneering work done at the Santa Fe Institute (Palmer et al., 1994; Arthur et al., 1997; LeBaron, 1999). Generally speaking, the goal of building artificial financial markets is to explain the emergence of the characteristic statistical properties of asset prices on the basis of hypotheses on traders' behaviour, market microstructure, and economic environment. These problems are usually too complex to be treated analytically, so methods of microscopic simulations are employed.

We have built an agent-based computer simulator of a financial market, the Genoa Artificial Stock Market[1] (GASM). In the past, research has been focused mainly on modelling agent optimisation and learning capabilities. Little effort has been devoted (in our opinion) to studying how the market microstructure and the macroeconomic environment affect market prices. The GASM has been conceived to address these problems, and we use it in this paper to study the interplay of different trading strategies in a changing market environment. Results show that a trading strategy cannot be judged solely on the basis of the strategy itself. Its success depends also on the market conditions.

While an early release of GASM has been presented (Raberto et al., 2001), the GASM simulator has been conceived as an evolving system, able to be continuously modified and updated. It is implemented using object-oriented technology and extreme programming (Beck, 1999; Succi and Marchesi, 2001) as a development process. This makes it possible to develop complex systems and make substantial modifications quickly and without jeopardizing quality. We present here the current release of GASM. A number of features deserve special mention:

- The trading mechanism of the GASM is based on a realistic auctiontype order matching mechanism that allows defining a demandsupply schedule;
- Agents have only limited financial resources;
- The number of agents engaged in trading at each moment is a small fraction of the total number of agents.

The trading mechanism described in detail in the next section permits explicit constructing a demand-supply schedule, which is essential since price fluctuations are due to an imbalance between demand and supply. As the market must clear, somehow the 'intention' to buy or sell must be modified to allow orders to match. This is the essence of the demand-supply schedule. As demonstrated in many studies on market microstructure, e.g., see O'Hara (1995), the details of the order matching process have a bearing on both price setting and price-volume relationships.

The finiteness of agents' financial resources is another essential feature of the GASM. It is an important feature of real markets and poses significant constraints on possible trading strategies if different populations of agents should coexist indefinitely. Actually, the study of the interplay between different trading populations is one of the major scientific objectives of the GASM.

Another feature of real markets is that prices are set by transactions that involve only a small fraction of the market population. The notion that the entire population of investors is continuously engaged in trading is simply unrealistic; trading costs would skyrocket. Even professional fund managers tend to limit their trading activity to a few trades per day, and these simply to tune portfolios that remain substantially stable for several weeks. This implies that the consideration of the 'thermodynamic limit' of markets, i.e., an infinite number of traders, is simply unrealistic and possibly misleading. Finite size effects in real markets are not an artifact but a real feature. The interaction of a small set of agents sets the 'wealth' of the entire market.

Let us now consider the trading strategies implemented in the GASM. The notion that financial markets are purely speculative is somewhat misleading. A significant fraction of market trading is done for real needs and not for speculation. We claim that, in the long run, it is the interplay between the flow of cash in and out of financial markets and the creation/destruction of stocks that determine the price trends of the market. In this work, we assume only one security in the market plus cash. At every instant, agents hold a fraction of their wealth in stocks and the rest in cash.

The assumption of our first experimental setting is that there is no net flow of cash in or out of the market. At every trading period we assume that two percent of traders, randomly selected, is engaged in trading. This is a simple representation of background trading, which has two structural elements: a volatility feedback and the wealth distribution of agents. Although both trading and trading strategies are random (as will be discussed in the next section), we allow the market to be in different states of volatility. Volatility is uncertainty. The state of uncertainty of the market is modelled by the order limit prices issued by traders. We impose that, in periods of high volatility, traders are more nervous and therefore allow for wider price limits to get their trades done quickly. The presumption is that trades must be done for exogenous reasons, and therefore traders wish to execute trades at the best possible price. Fearing large market movements, they allow more freedom in setting the limit prices.

The other important structural element of the GASM is the agents' wealth distribution. It is well known empirically that wealth distribution tends to follow a Pareto inverse power law (Pareto, 1897; Levy and Solomon, 1997). It has been demonstrated theoretically that autocatalytic processes naturally lead to an inverse power law distribution of wealth (Montroll and Schlesinger, 1982; Bouchaud and Mezard, 2000; Huang and Solomon, 2001a, b). The wealth of agents in the GASM is indeed governed by an auto-catalytic process, which explains the emergence and conservation of inverse power law distributions.

3. Traders' Population

At every discrete trading moment h, a generic trader i holds an amount $c_i(h)$ of cash and an amount $a_i(h)$ of stock. Traders are segmented into four population-types, depending on their respective trading behaviour: random, momentum, contrarian, and fundamentalist. Random traders are characterized by very simple trading strategies: no intelligence and random trading constrained by limited resources and past volatility.

Though very simple, these ingredients are sufficient to build an artificial financial market, see Raberto et al. (2001), that is able to reproduce the main stylised facts of real markets: volatility clustering and fat tails in the distribution of price returns. In this paper, we introduce, in the framework of the GASM, three population-types that have already been described in the literature and that represent more realistic trading behaviours. The aim is twofold: first, we want to study the behaviour of these stylised populations in a realistic environment characterized by limited resources and a market clearing mechanism; second, we want to address the important issue about the existence of winning strategies.

3.1. RANDOM TRADERS

At each simulation step, each random trader issues an order with probability equal to 0.02. The order can be a buy or a sell, with probability 0.5. Suppose the ith trader issues a order to sell a_i^s shares of stock at time $h+1$. We assume a_i^s is a random fraction of the quantity of stock owned at time h by the ith trader. A limit price s_i accompanies the sell order, so it cannot be executed at prices below s_i. The limit price s_i is computed as follows: $s_i = p(h)/N_i(\mu, \sigma_i)$, where $N_i(\mu, \sigma_i)$ is a random draw from a Gaussian distribution with average $\mu = 1.01$ and standard deviation σ_i.

The value of σ_i is proportional to the historical volatility $\sigma(T_i)$ of the asset price through the equation $\sigma_i = k \cdot \sigma(T_i)$, where k is a constant and $\sigma(T_i)$ is the standard deviation of log-returns (Raberto et al., 2001), calculated in a time window T_i time steps long. Tuning the system through numerous simulations with different parameters, we find that the values for k that give the best correspondence with real world market data are rougly in the range between 3 and 4. In the following simulations we used 3.85. T_i is randomly drawn for each trader from a uniform distribution of integers from 10 to 100 at the beginning of the simulation. As highlighted in Section 2, linking limit orders to volatility takes into account a realistic aspect of trading psychology: when volatility is high, uncertainty on the 'true' price of a stock grows, and traders place orders with a broader distribution of limit prices.

Buy orders are generated fairly symmetrically relative to sell orders. If the ith trader issues a buy order at time $h+1$, the amount of cash employed is a fraction of available cash at time h. A limit price b_i is associated with each buy order, and we stipulate that buy orders cannot be executed at prices higher than the limit price. In the case of buy orders, limit prices are computed as follows: $b_i = p(h) \cdot N_i(\mu, \sigma_i)$, where $N_i(\mu, \sigma_i)$ is a random draw from a Gaussian distribution with average μ, and standard deviation σ_i. As for sell orders, $\mu = 1.01$ and $\sigma_i = k \cdot \sigma(T_i)$, where k varies between 3 and 4. T_i varies randomly from 10 to 100 time steps for each trader. The quantity of assets ordered, a_i^b, is therefore given by the integer part of the ratio between the amount of cash employed in the buy order and the limit price b_i.

It is worth noting that, the choice of $\mu = 1.01$ ensures that the mean value of all b_i is likely to be greater than $p(h)$ and that the mean value of all s_i is likely to be smaller than $p(h)$. In other words, we introduce a spread between the average value of buy/sell limit prices to represent the fact that a trader placing an order wants to increase the chance of the order's being executed. Hence, for a buy order the trader is likely to be willing to pay slightly more than $p(h)$; for a sell order, the trader is likely to offer the stock at a lower price than $p(h)$.

In conclusion, each random trader exhibits heterogeneous random behaviour subject to two constraints: the historical price volatility (which is included in σ_i) and the finiteness of the resources available to him.

Random traders represent the bulk of traders who perform trades for reasons linked to their needs and not to speculative reasons. For instance, they represent investors who need to sell stocks to pay personal expenses, or people who receive some compensation and decided to invest it in the stock market. For this reason, their trades are completely random.

3.2. MOMENTUM TRADERS

The momentum trader is a trend follower who makes decisions depending on the trend of past prices. The momentum trader speculates that, if prices are rising, they will keep rising, and if prices are falling, they will keep falling. They represent, in a simplified way, traders following technical analysis rules and traders following a herd behavior. In the literature, they are often referred to as noise or chartist traders (Black, 1986; De Long et al., 1990, 1991; Lux, 1997; Lux and Marchesi, 1999).

At each simulation step, a momentum trader places an order with probability 0.02. The order is a buy order if the past price trend is positive, a sell order if the trend is negative. To be more precise, at time step $h + 1$, the ith momentum trader computes the trend of past prices in the following way: $\mathcal{D}(h + 1, T_i) = (p(h) - p(h - T_i))/T_i$. T_i is the time window used in the calculation of trends, the number of time steps backward from the last simulated price. If $\mathcal{D}(h + 1, T_i) > 0$, the trader issues a buy order whose limit price is $b_i = p(h) + \mathcal{D}(h + 1, T_i) \cdot T_i$. Conversely, if $\mathcal{D}(h + 1, T_i) > 0$, the order is a sell order and the limit price is $s_i = p(h) + \mathcal{D}(h + 1, T_i) \cdot T_i$. Therefore, in the case of buy (sell) orders, we have $b_i > p(h)(s_i < p(h))$ and the difference $b_i - p(h)(p(h) - s_i)$ is greater when the past trend is sharper. This is to say that when prices are increasing (decreasing) in a fast way, the momentum trader forecasts an identical increase (decrease) in the future. A time window T_i is assigned to each trader at the beginning of the simulation through a random draw from a uniform distribution of integers in the range 10 to 50. If T_i were the same for all traders, momentum traders would behave in the same way. In the case of a buy order, the trader's order size is a random fraction of available cash and the quantity of stock demanded, a_i^b is given by the ratio between the cash employed in the order and the value of b_i. If the trader issues a sell order, the quantity of stocks offered for sale as is a random fraction of the amount of stocks owned.

3.3. CONTRARIAN TRADERS

The contrarian traders are structured simularly to the momentum traders except in their trading behaviour. A contrarian trader speculates that, if the stock price is rising, it will stop rising soon and decrease, so it is better to sell near the maximum, and *vice versa*. This trading behavior, while present, is probably less popular than trend-following strategies. In a closed market, the rationale of the contrarian strategy is that, when prices steadily increase, the total value of stocks exceeds the total value of cash in the market. This leads to an imbalance that will eventually bring prices down again. The opposite happens when the price steadily decreases. For both momentum and contrarian traders, we compute the trend on a time interval chosen randomly between 10 and 50 time steps, so they do not work on immediate price variations, but on steadier trends.

In detail, the contrarian behaviour is the following: If $\mathcal{D}(h + 1, T_i) > 0$, the contrarian trader issues a sell order whose limit price s_i is determined by: $s_i = p(h) - \mathcal{D}(h + 1, T_i) \cdot T_i$. If $\mathcal{D}(h + 1, T_i) < 0$, the order is a buy order and its limit price b_i is $b_i = p(h) - \mathcal{D}(h + 1, T_i) \cdot T_i$. Therefore, also in this case we have that $b_i > p(h)$ and $s_i < p(h)$. The distribution of the parameter T_i and the mechanics of order placement are the same as for momentum traders. At each simulation step, a contrarian places an order with probability 0.02.

3.4. FUNDAMENTALIST TRADERS

The fundamentalist trader believes that stocks have a fundamental price, due to factors external to the market, like the price-earning ratio, profitability, ROI, ROE, and economic characteristics of the firm's business. They believe that, in the long run, the price of the stock will revert to its fundamental price. Consequently, they sell stocks if the price is higher than fundamental price and buy stocks in the opposite case. In real markets, fundamentalists represent traders who do not follow short-term speculative behaviors, but try to make steady gains filtering out speculative movements of the market. In this model, fundamentalists at each time step 'decide' whether to trade with a probability of 0.02. If a fundamentalist decides to trade, he places a buy or a sell order if the present price is lower or higher than the fundamental price p_f, respectively. The fundamental price is the same for all fundamentalists. The order limit price is always p_f, while its magnitude (in stocks for sell orders and in cash for buy orders) is computed randomly as a fraction of the present amount of stocks or cash owned by the trader, the same as for momentum and contrarian traders.

4. Market Clearing

The price process is determined at the intersection of the demand and the supply curves. We compute the two curves at the time step $h + 1$ as follows: Suppose that at time $h + 1$ traders have issued U buy orders and V sell orders. For each buy order, let the pair (a_u^b, b_u), $u = 1, \ldots, U$, indicate, respectively, the quantity of stocks to buy and the associated limit price. For each sell order in the same time step, let the pair (a_v^s, s_v), $v = 1, \ldots, V$, denote, respectively, the quantity of stocks to sell and the associated limit price. Define the functions

$$f_{h+1}(p) = \sum_{u|b_u \geq p} a_u^b\,, \tag{1}$$

$$g_{h+1}(p) = \sum_{v|s_v \leq p} a_v^s\,, \tag{2}$$

$f_{h+1}(p)$ represents the total amount of stocks that would be bought at price p (demand curve). It is a decreasing step function of p, i.e., the bigger p, the fewer the buy orders that can be satisfied. If p is lower than the minimum value of b_u, $u = 1, \ldots, U$, then $f_{k+1}(p)$ is the sum of all stocks to buy. Conversely, $g_{k+l}(p)$ represents the total amount of stocks that would be sold at price p (supply curve) and is an increasing step function of p. Its proprierties are symmetric to those of $f_{k+1}(p)$. The clearing price computed by the system is the p^* at which the two function cross, i.e., $f_{h+1}(p^*) = g_{h+1}(p^*)$. We define the new market price at time step $h + 1$, $p(h + 1) = p^*$. Buy and sell orders with limit prices compatible with p^* are executed. Following transactions, traders' cash and portfolio are updated. Orders that do not match the clearing price are discarded.

5. Market Initialisation

At the beginning of the simulation ($h = 0$), the price $p(0)$ is set at \$100,00 and each trader i is endowed with an amount of cash $c_i(0)$ and an amount of stocks $a_i(0)$. The distribution of $c_i(0)$ and $a_i(0)$ follow a Pareto law with exponent $\alpha = 2$. A random variable X is said to follow a Pareto law if: $P(X \geq x) = 1/x^\alpha$, for $x \geq a$ where a is an appropriate positive constant (Pareto, 1897). To create this type of distribution we use the ranking property of the Pareto law (Takayasu, 1990). This means that if we order the cash of agents in decreasing order, the cash of the nth agent is $n^{-\frac{1}{\alpha}}$ the cash of the first agent. We therefore artificially create populations of agents whose cash, number of stocks, and wealth at time step 0 follows the ranking property of the Pareto law.

The population of random traders is made of 10,000 individuals and their cash and stock portfolio are initialised as follows: $c_i(0) = 10,000,000 \cdot i^{-\frac{1}{\alpha}}$ and $a_i(0) = 100,000 \cdot i^{-\frac{1}{\alpha}}$ for $i = 1, \ldots, 10,000$. As there are 10,000 agents in the market, the range of wealth spans four orders of magnitude. The populations of momentum, contrarian, and fundamentalist traders are made of 100 traders each. In this case, we have: $c_i(0) = 100,000 \cdot i^{-\frac{1}{\alpha}}$ and $a_i(0) = 1,000 \cdot i^{-\frac{1}{\alpha}}$ for $i = 1, \ldots, 100$.

For all four populations, the poorest trader has 10 stocks and \$1,000 of cash, while the richest random trader is far wealthier than the richest momentum, contrarian, or fundamentalist trader. Moreover, the number of random traders is one hundred times the number of traders belonging to the other three populations. The reason for these choices is that we want random traders to act as a 'thermal bath'. Thus, particular trading strategies can be studied with a minimum effect on the stability of the system and on the statistical properties of prices. Traders' portfolio initialisations follow (subject to rounding) the identity: $c_i(0) \cong p(0) \cdot a_i(0)$ for any i. According to this, the aggregate amount of cash at the beginning of the simulation $C(0) = \sum_i c_i(0)$ is approximately equal to the aggregate value of stocks, i.e., $p(0) \cdot A(0)$ where $A(0) = \sum_i a_i(0)$. The equality between the aggregate value of cash and the aggregate value of stocks, i.e., $C(h) \cong p(h).\ A(h)$ holds on average during the entire simulation. This results from the order matching process responsible for setting the stock price.

In a closed market, the total value of cash and the total number of stocks are constant during the simulation, i.e., $C(h) \equiv C(0)$ and $A(h) \equiv A(0)$. Let $\bar{p}$ be the price at which the aggregate value of stocks equals the total value of cash: $\bar{p} = C(0)/A(0)$. Then, the total value of stocks at time h depends only on the current price $p(h)$. Moreover, considering only random traders (who are the majority and are responsible for background trading), the trading mechanism implies that, at every time step, the average number of stocks available for sell orders and the average amount of cash available for buy orders are constant.

Suppose at time h the price is higher than $\bar{p}$, i.e., $p(h) > \bar{p}$. In this case, the aggregate value of stocks to sell is higher than the value of cash available to buy them. As a consequence, average sell orders for stocks exceed average buy orders.

The order matching mechanism therefore tends to reduce the stock price to match orders (on average). The reverse holds if the price is higher than $\bar{p}$. The price therefore oscillates around $\bar{p}$. Simulations confirm this mean-reverting behavior. We have initialized traders' wealth so that $p(0) = \$100,00$ is the equilibrium price for the closed market. In the case of external inflows or outflows of cash or stocks (open market), the equilibrium price varies and can be determined at every h as $\bar{p}(h) = C(h)/A(h)$. As noted above, we set the fundamental price p_f used by fundamentalists to $\bar{p}$. This is the most plausible and unbiased value that a trader adverse to speculative behavior can give to the fundamental price in our closed market.

6. Simulation Results and Interpretation

In this section we describe the results of the computational experiments performed on the GASM. First we explore market behaviour when only random traders are present and without inflow or outflow of cash. We then add different populations of traders. Finally we simulate a progressive increase in the amount of cash available to traders. For all these different market conditions, we explore the evolution of the distribution of wealth of the populations involved in the market.

6.1. CLOSED MARKET WITH RANDOM TRADERS ONLY

We let the market run with a population of random traders for 10,000 time steps. To give an idea of the timing involved, assume that a time step is of the order of a day. Then, 10,000 time steps corresponds roughly to 40 years of market activity. As with our previous work (Raberto et al., 2001), the price process exhibits the main features of real markets: fat tails for the returns distribution, zero autocorrela tion for the returns, slow decay of the autocorrelation function of the absolute values of logarithmic returns. Figure 1 shows the price path and the logarithmic returns $r(h) = \log p(h) - \log p(h-1)$ from a typical simulation. Following the methodology in Engle and Granger (1987), we find that the price path and the horizontal line at value \$100.00 are cointegrated at the 1% significance level. This fact corroborates our previous assertions that $\bar{p} = C(0)/A(0)$ is the equilibrium price in a closed market. It is also worth noting that the artificial price series is characterized by the well known real-markets phenomenon of persistence in the magnitudes of fluctuations.

Figure 2 shows the survival probability distribution of the standardized logarithmic returns $R(h)$, i.e., logarithmic returns $r(h)$ detrended by its mean and resealed by its standard deviation. For comparison, the solid line represents the survival probability distribution of the standard normal distribution $N(0, 1)$. It is possible to observe a clear deviation from Gaussian behaviour with approximate by power law scaling in the tail.

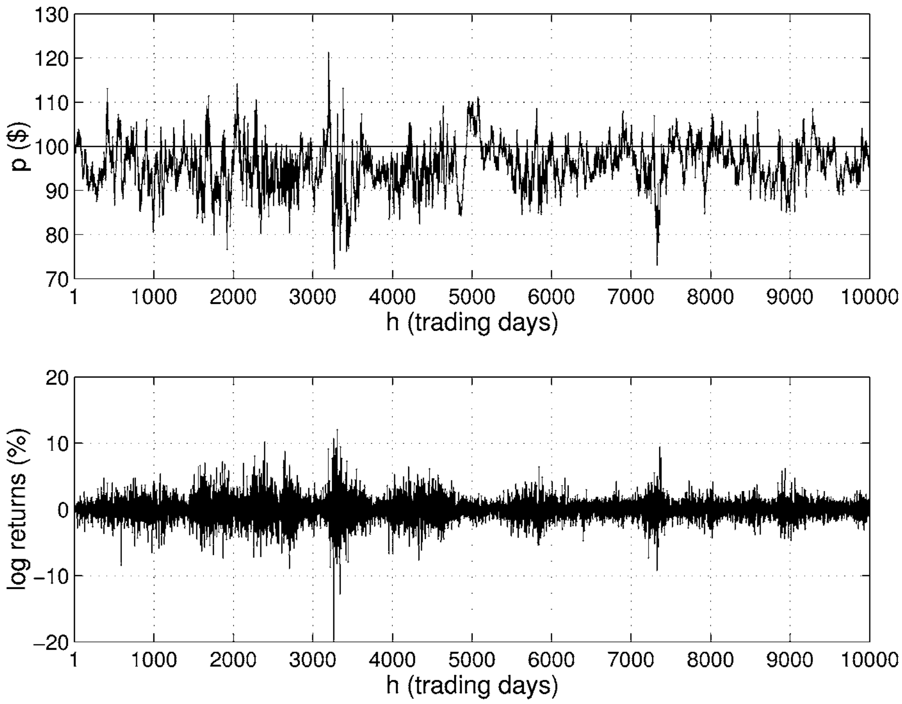

Figure 1. Simulated stock price path (above) and logarithmic returns (below) of a closed market with random traders only.

A log-log regression of points that satisfy the condition $|R| > 2$ gives the slope: -3.66 ± 0.02, in agreement with values found in various financial daily time series (Mantegna and Stanley, 1999; Bouchaud and Mezard, 2000). This result is similar to the value: -3.69 ± 0.02 found with an early release of GASM (Raberto et al., 2001). In that work, the fat-tailed shape of the log-returns distribution results both form an agent aggregation mechanism, similar to the herding model by Cont and Bouchaud (2000), and a functional dependence of limit prices on past price volatility. In this new release of GASM, we have not included the herding phenomenon. Fat tails are due to the link between market volatility and limit prices and to the distribution of wealth between agents following a Pareto law with exponent equal to 2. The dependence of limit prices on market volatility is a microscopic implementation of the GARCH model, which is notorious in yielding fat-tailed distributions (Roman et al., 2001). An initial distribution of wealth of a Pareto law type is a realistic assumption (Levy and Solomon, 1997) and contributes to the fat tails of the returns distribution. As there are 10,000 agents in the market, the range of wealth spans four orders of magnitude. Only 2% of traders act in the market at the same time. Thus when a rich trader 'decides' to issue an order, there is low probability that another rich trader will be chosen by the system to make an opposite order of the same size. This fact can cause large price variations.

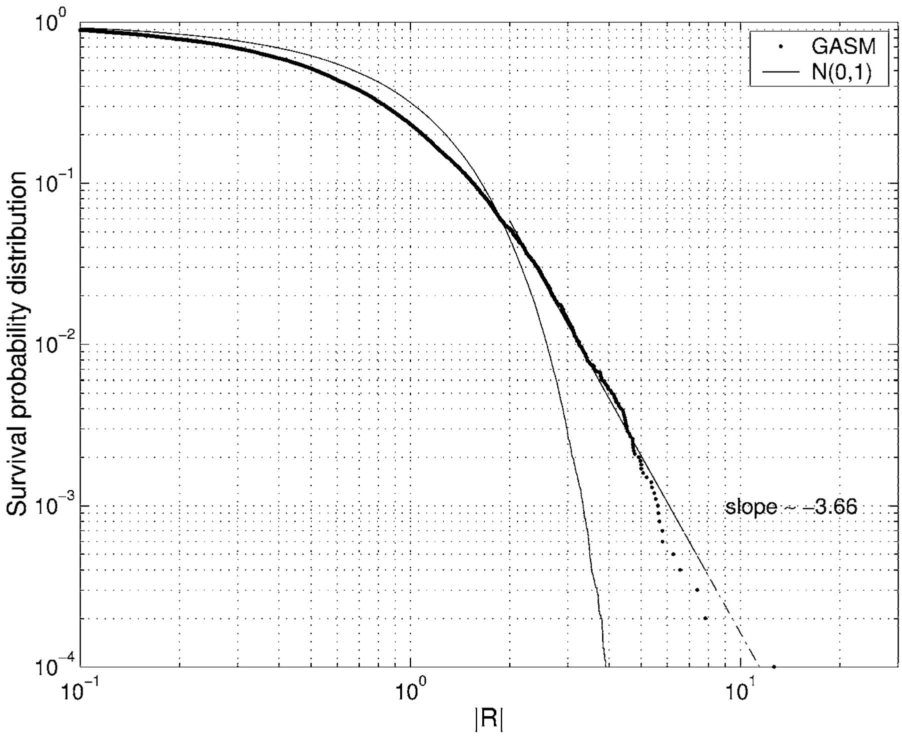

Figure 2. Survival probability distribution of standardized logarithmic returns $R(h)$. Considering logarithmic returns $r(h) = \log p(h) - \log p(h-1)$, $R(h)$ are computed as $R(h) = (r(h) - m_r)/\sigma_r$, where m_r and σ_r are the mean and the standard deviation of $r(h)$ over h, respectively. The dots represent an estimate of the cumulative distribution of $R(h)$ related to a simulation of GASM. The solid line represents the cumulative distribution of a random variable drawn from a normal distribution. The positive and the negative tails were merged by employing absolute returns. The dot-dashed line is the power law fit $P = K \cdot |R|^{-\tau}$ with $\tau = 3.66$ of the tail of the empirical survival probability distribution for $|R| > 2$.

In Figure 3, we present the autocorrelation function $C(\tau)$ of the absolute returns $|r|$ and raw returns r at different time lags τ. While the autocorrelation of raw returns decays immediately, the autocorrelation of the absolute value of returns shows the presence of long-range correlations. Both exponential and power law decays have been tested for the descend curve of absolute returns. The exponential fit gives an exponent equal to $(0.17 \pm 0.01)10^{-2}$ with 0.03 as prediction error.[2] The value found in the power law fit is (0.10 ± 0.01) with a prediction error of 0.04. While in the early release of GASM (Raberto et al., 2001), the exponential fit was better, here the power law fit is found to perform roughly closely to the exponential one, in agreement with real markets (Liu et al., 1999). The reason lies in the memory of random traders that vary in a range between 10 and 100, allowing the interplay between many time scales.

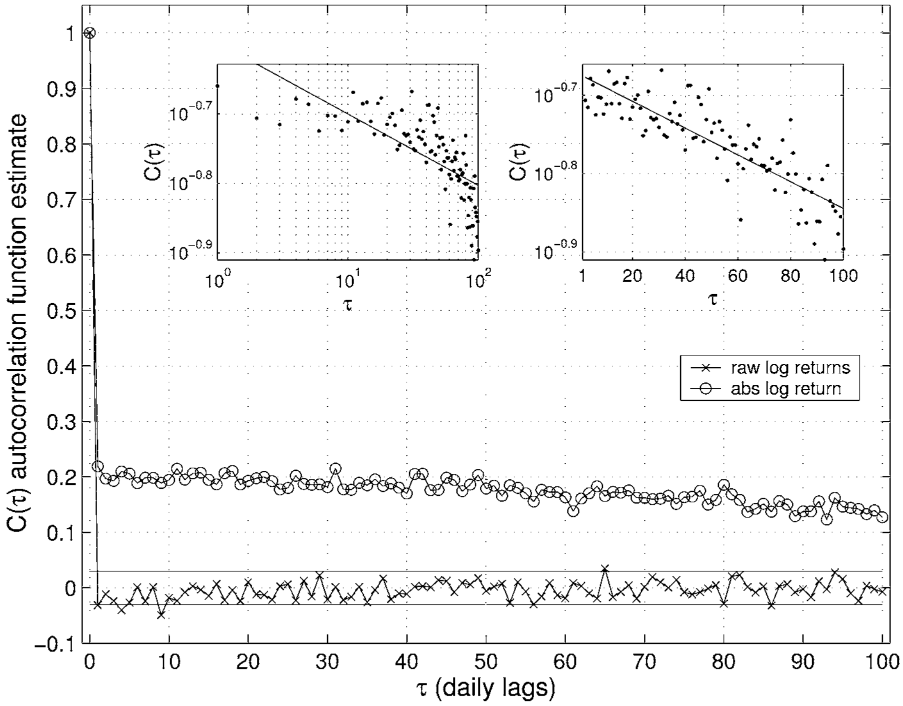

Figure 3. Estimate of the autocorrelation function of logarithmic returns. Circles represent the autocorrelation of absolute returns $|r(h)|$, crosses are related to the autocorrelation function of raw returns $r(h)$. Noise levels are computed as $\pm 3/\sqrt{(M)}$, where M is the length of the time series ($M = 10,000$). Absolute returns are fitted with an exponential decay (inset on the right) and a power law decay (inset on the left).

Figure 4 shows the survival probability distribution of wealth among random traders. The wealth distribution is plotted at $h = 0$ and after 10,000 steps. The wealth $w_i(h)$ of trader i at time step h is defined as $w_i(h) = c_i(h) + p(h) \cdot a_i(h)$. It is worth noting that, though the global amount of cash and the total number of stocks are constant in a closed market, the global wealth $W(h) = \sum_i w_i(h) = C(0) + p(h) \cdot A(0)$ is a time-varying quantity that is obviously a linear function of the stock price. In Figure 4, the solid line represents the survival probability distribution of wealth for $h = 0$. Note the characteristic straight line of a log-log plot of slope –2, representing the Pareto law with exponent 2. The dots give the values of the survival distribution of wealth after 10,000 steps. The central part of the Pareto distribution is conserved after 10,000 steps, which is an important computational finding that is in agreement with the theoretical analysis of auto-catalytic processes (Montroll and Schlesinger, 1982; Huang and Solomon, 2001b).

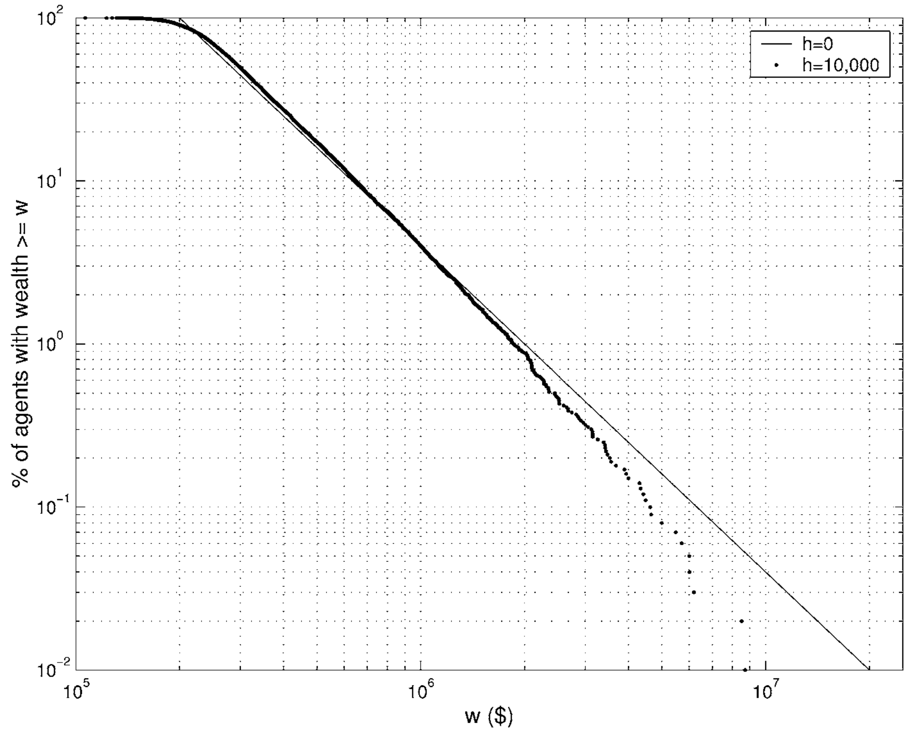

Figure 4. Survival probability distribution of wealth for random traders at the beginning of the simulation and after 10,000 steps in the case of closed market.

6.2. CLOSED MARKET WITH FOUR POPULATIONS OF TRADERS

Using a population of random traders as a 'thermal bath', we add fundamentalist, momentum, and contrarian traders to study the behavior of the wealth distribution of these populations under the assumption of a closed market without inflow or outflow of cash. We add 100 fundamentalist, 100 momentum, and 100 contrarian traders to the 'thermal bath' of 10,000 random traders. Given the large number of random traders, the approximation of the 'thermal bath' holds. Momentum, fundamentalist, and contrarian traders are not able to influence the price process, and the market produces prices having the same statistical properties discussed in the previous section. With respect to the average dynamics of wealth, we find that, on average, momentum traders lose wealth, while fundamentalists and contrarians gain wealth. This result does not depend on the presence of separate or contemporary single populations, as is expected with this approximation to a 'thermal bath'. The wealth of random traders remains approximately the same during the simulation. Figure 5 presents this result in the case of a typical simulation 10,000 steps long in which all four populations are present. The figure shows the average wealth $1/N \sum_i w_i(h)$ of all N traders belonging to a single population. The wealth is computed at the current price $p(h)$ and is plotted against the simulation time h.

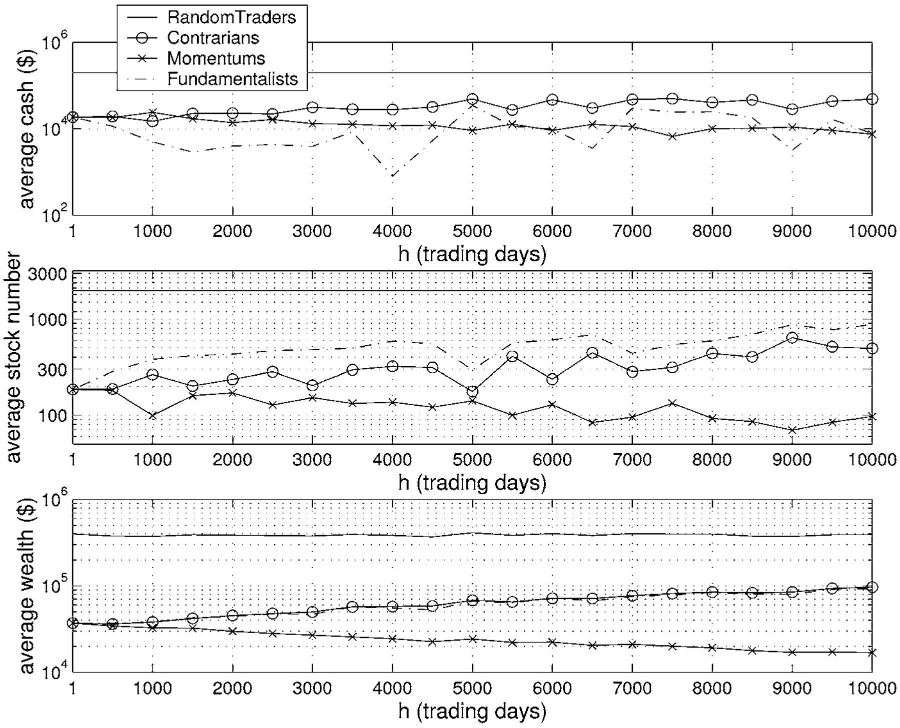

Figure 5. Dynamics of cash, number of stocks, and wealth of the four populations of traders for a typical simulation 10,000 steps long with a geometric cash inflow of 0.231% every 10 time steps.

A qualitative, heuristic explanation of these results is the following: The population of fundamentalists wins and gains wealth because their strategy is exactly in line with the mean reverting behavior of the market. We recall that the fundamental price p_f is set to the mean reverting price $\bar{p}$. Fundamentalists always sell stocks at prices higher than $\bar{p}$ and buy at prices lower than $\bar{p}$. Given the trading mechanism and given that they rarely issue orders randomly situated in the price path, the mean reverting behavior of stocks make them earn a profit on all trades. The success of the strategy to sell if $p(h) > \bar{p}$ and to buy if $p(h) < \bar{p}$ is empirically confirmed by Figure 5 and by the other simulations we run in a closed market with random traders and with any combination of the other trader populations.

The reason why momentum traders progressively lose wealth while contrarian traders gain it is subtler. Recall that both kinds of traders compute a price trend by subtracting from the current price $p(h)$ the price at time $h - T_i$. Due to the mean reverting behaviour of the market around the price $\bar{p}$, the probability that a price $p(h)$ (randomly drawn from the experimental sequence) be higher or lower than $\bar{p}$ is generally the same and is equal to 0.5. Conversely, if the trend at time h is positive, i.e., if $p(h) - p(h - T_i) > 0$, the probability that $p(h) > \bar{p}$ is higher than 0.5. This is due to the fact that there has been an upward price trend in the previous

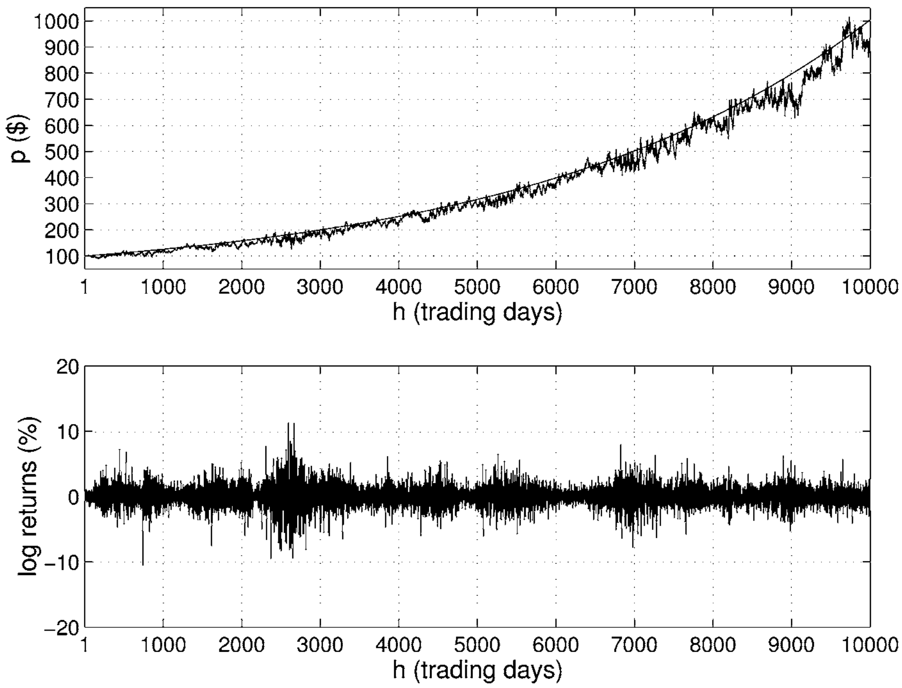

Figure 6. Simulated stock price path (above) and logarithmic returns (below) of a market with a geometric cash inflow of 0.231% every 10 time steps.

T_i time steps. Of course, if $p(h) - p(h - T_i) < 0$, the probability that $p(h) > \bar{p}$ is lower than 0.5. Consequently, if the price trend is ascending, it is more likely that the price is higher than $\bar{p}$, and the correct strategy should be to sell, as said above – but momentum traders will buy, thus making, on average, an error. Contrarians, on the other hand, will correctly sell a part of their stocks. If the trend is downward, the opposite is true.

It is worth noting that the distribution of wealth of single populations continues to follow approximately a Pareto law, which is shifted downward or upward depending on the gains or loses of wealth.

7. Open Market

We open the market by allowing cash to increase geometrically by a fixed percent every period. The cash inflow is distributed to agents proportionally to their total wealth. Figure 6 shows the price path and the logarithmic returns related to a geometric cash inflow of 0.231% every 10 steps. This particular value has been chosen to increase the global value of cash ten fold during the simulation: $(1 + 0.00231)^{1,000} \cong 10$. In this case, prices exhibits a clear exponential trend with the same rate of growth of cash. Again, following the methodology in Engle and Granger (1987), we find that the price path and the exponential trend (solid line) –

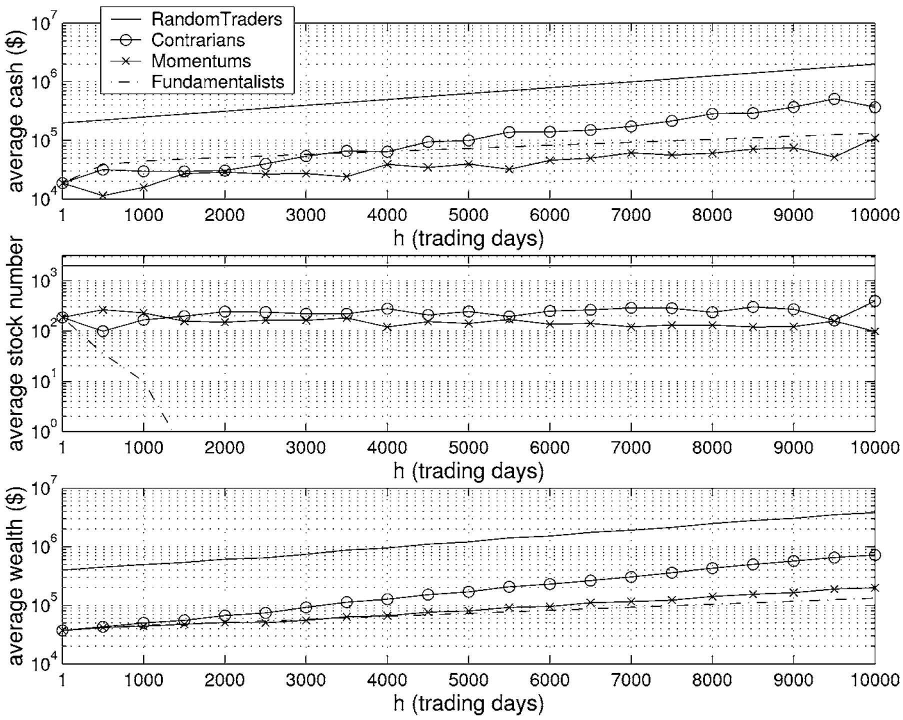

Figure 7. Dynamics of cash, number of stocks, and wealth of the four populations of traders for a typical simulation 10,000 steps long with a geometric cash inflow of 0.231% every 10 time steps.

which mimics the rate of growth of cash – are cointegrated at the 1% significance level. We believe that the cointegration relationship between prices and cash inflow exhibited in our artificial market is a feature of any competitive economy with finite resources. The exploration of the relationship between prices and global financial inflow could be an interesting subject of research on real markets.

In this case, see Figure 7, fundamentalists, who have a constant fundamental price $p_f = \$100,00$, soon sell all stocks they own and retain only cash. Thus, the increase in their wealth occurs only because of the injection of cash. All other populations increase their average wealth, but with different rates. Contrarians increase their wealth more rapidly than random traders, who, in turn, increase their wealth more than momentum traders. Measuring the slopes of lines representing the wealth dynamics in the log-log plot of Figure 7, the daily rates of growth are respectively: 0.010% for random traders, 0.013% for contrarian traders, 0.007% for momentum traders, and 0.005% for fundamentalists.

8. Conclusion

The computational experiments performed for this work show a number of important results. First, in an artificial market with finite resources, the average price level and the trends are set by the amount of cash present and eventually injected in the market. In a market with a fixed amounts of stocks, a cash injection creates an inflationary pressure on prices.

The other important finding of this work is that different populations of traders, characterized by simple but fixed trading strategies, cannot coexist in the long run. One population prevails and the others progressively lose weight and disappear. This result is in agreement with the observations of Friedman (1953), but appears in contrast with results in De Long et al. (1990, 1991).

Indeed, De Long et al. (1990, 1991) follow a different approach to the problem, presenting an analytical model in which irrational or noise traders are characterized by erroneous stochastic beliefs. Which population will prevail and which will lose cannot be decided on the basis of the strategies alone.

Trading strategies yield different results under different market conditions. In real life, different populations of traders with different trading strategies do coexist. These strategies are boundedly rational and thus one cannot really invoke rational expectations in any operational sense. Though market price processes in the absence of arbitrage can always be described as the rational activity of utility maximizing agents, the behaviour of these agents cannot be operationally defined. This work shows that the coexistence of different trading strategies is not a trivial fact but requires explanation. One could randomize strategies by having traders shift statistically from one strategy to another. It is, however, difficult to explain why a trader embracing a winning strategy would switch to a losing one. Perhaps markets change continuously and make trading strategies randomly more or less successful. More experimental work is necessary to gain an understanding of the conditions that allow the coexistence of different trading populations.

Notes

[1] The name is devoted to the beautiful city where most of this work was performed. Moreover, in the Middle Ages, Genoa was a major financial centre, where the I.o.u. and the derivatives were inventes (Briys and de Varenne, 2000).

[2] The prediction error of a linear fit is the standard deviation of the difference between the observed value and the theoretical value given by the result of the fit (Wild and Seber, 2000).

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